Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 14 Issue 4 Version 1.0 Year 2014
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Fixed Points for Cyclic Contractions in Dislocated Metric Spaces

By K. Prudhvi

Osmania University, India

Abstract- In this work, we study fixed point theorems for cyclic contractions in dislocated metric spaces. Our results, extension, improve and generalize some recent results in the literature.

Keywords: *dislocated metric spaces, fixed point, d-convergent, d-cyclic contraction.*

GJSFR-F Classification : MSC 2010: 30L05 , 31E05

FIXEDPOINTSFORCYCLICCONTRACTIONSINDISLOCATEDMETRICSPACES

:

Strictly as per the compliance and regulations of :

[1] Aage, C.T. and J.N. Salunke, The Results on fixed points in dislocated and dislocated quasi-metric Space, Applied Math. Sci., **2**(59)(2008), 2941 - 2948.

Fixed Points for Cyclic Contractions in Dislocated Metric Spaces

K. Prudhvi

Abstract- In this work, we study fixed point theorems for cyclic contractions in dislocated metric spaces. Our results, extension, improve and generalize some recent results in the literature.

Keywords: dislocated metric spaces, fixed point, d-convergent, d-cyclic contraction.

I. Introduction and Preliminaries

P.Hitzler et.al.[1] have introduced "the notion of dislocated metric space". Recently, Kirk et.al. [4] have introduced "the notion of cyclic contraction and prove fixed point theorems on this contraction". Later on many authors have proved fixed point theorems on cyclic contractions in dislocated metric spaces (see, [3], [5]). In this paper, we obtain, fixed point theorems for cyclic contractions in dislocated metric spaces which improve,extends and generalises results of [3].

The following definitions are needful to prove main results.

Definition 1.1 [1]. Let X be a non empty set and let $d: X \times X \rightarrow [0, \infty)$ be a function called a distance function .Consider the following conditions:
(i) $d(x, x) = 0$, for all $x \in X$.
(ii) $d(x, y) = 0$ then $x = y$, for all $x,y \in X$.
(iii) $d(x, y) = d(y,x)$, for all $x,y \in X$.
(iv) $d(x,y) \leq d(x,z) + d(z,y)$, for all $x,y,z \in X$.
If d satisfies (i) to (iv), then it is called metric. If it satisfies (ii) to (iv), then d is called dislocated metric (or simply d-metric).

Definition 1.2 [1]. A sequence $\{x_n\}$ in a d-metric space(X,d) converges with respect to d, if there exists a point $x \in X$ such that $d(x_n, x)$ converges to 0 as $n \rightarrow \infty$.

Proposition 1.1 [1]. Limits in d-metric spaces are unique.

Definition 1.3[1]. A sequence $\{x_n\}$ in a d-metric space(X,d) is called a Cauchy sequence , if for each $\varepsilon > 0$ there exists $n_0 \in N$ such that for all $m,n \geq n_0$, we have $d(x_m, x_n) < \varepsilon$.

Proposition 1.2[1]. Every convergent sequence in d-metric spaces is a Cauchy sequence.

Author: *Department of Mathematics, University College of Science, Saifabad, Osmania University, Hyderabad, Telangana State India.*
e-mail: prudhvikasani@rocketmail.com

Definition 1.4[1]. A function f: X → X in a d-metric space is called a contraction if there exists $0 \le \lambda < 1$ such that $d(fx, fy) \le \lambda\, d(x, y)$ for all $x,y \in X$.

Definition 1.5[4]. Let A and B be two non empty subsets of a metric space (X, d) and T: $A \cup B \to A \cup B$, T is called a cyclic map if $T(A) \subseteq B$ and $T(B) \subseteq A$.

Definition 1.6[4]. Let A and B be two non empty subsets of a metric space (X,d) and a cyclic map T: $A \cup B \to A \cup B$ is said to be a cyclic a cyclic contraction if there exists $k \in (0,1)$ such that $d(Tx, Ty) \le \lambda\, d(x, y)$ for all $x \in A$ and $x \in B$.

II. Main Results

In this section, we prove fixed point thoerems for cyclic contractions in dislocated metric spaces. Our results improve, extends and generalises the results of [3].

Theorem 2.1 . Let (X, d) be a complete dislocated metric space. Let A and B be two non empty closed subsets of X and $f : A \cup B \to A \cup B$ be such that

$d(fx,fy) \le \alpha[d(fx, x) + d(fy, y) + d(fy, x) + d(y, fx)]$, where $\alpha \in [0, 1/6)$.

Then f has a unique fixed point in $A \cap B$.

Proof: Let $\{f^n\} \subseteq X$, $\{f^{2n}\} \subseteq A$ and $\{f^{2n-1}\} \subseteq B$. Fix $x \in A$. By above definition there exists $\alpha \in [0, 1/6)$ such that

$$d(f^2x, fx) \le \alpha\, [d(f^2x, fx) + d(fx, x) + d(fx, fx) + d(x, f^2x)],$$

$$d(f^2x, fx) \le \alpha\, [d(f^2x, fx) + d(fx, x) + d(fx,x) + d(x, fx) + d(x,fx)+d(fx, f^2x)],$$

$$\le \alpha\, [2d(f^2x, fx) + 4d(fx, x)],$$

$$(1-2\alpha)\, d(f^2x, fx) \le 4\alpha d(fx, x)$$

$$d(f^2x, fx) \le 4\alpha / 1-2\alpha \;\; d(fx,x).$$

Put $k = 4\alpha / 1-2\alpha < 1$.

$$d(f^2x, fx) \le kd(fx, x).$$

By induction we have,

$$d(f^{n+1}x, f^nx) \le k^n d(fx, x).$$

More generally, for $m > n$ we have,

$$d(f^mx, f^nx) \le d(f^mx, f^{m-1}x) + d(f^{m-1}x, f^{m-2}x) + \dots + d(f^{n+1}x, f^nx)$$
$$\le (k^{m-1}x + k^{m-2}x + \dots + k^n x)d(fx,x)$$
$$= k^n (1+k+ k^2 + \dots + k^{m-n-1})d(fx,x).$$

Since, $k<1$ so as $m,n \to \infty$ we have $k^n (1+k+ k^2 + \dots + k^{m-n-1}) \to 0$.

Hence, $d(f^mx, f^nx) \to 0$.

Therefore, $\{f^nx\}$ is a Cauchy sequence. Since (X,d) is complete, so $\{f^nx\}$ converges to some point $z \in X$.

Since, $\{f^{2n}\} \subseteq A$ and and $\{f^{2n-1}\} \subseteq B$ so $z \in A \cap B$.

We claim that $fz = z$.

$$d(fz,z) = d(fz, f^{2n}x)$$

Global Journal of Science Frontier Research (F) Volume XIV Issue IV Version I Year 2014

Ref

[3] George Reny, R.Rajgopalan and S.Vinayagam, Cyclic contractions and fixed points in dislocated metric spaces, *Int. J. Math.Anal*, 7(9),(2013),403-411.

$$\leq \alpha\,[d(fz, z) + d(f^{2n}x\,,\ f^{2n-1}x) + d(f^{2n}x\,, z) + d(\ f^{2n-1}x, fz)]$$

Letting, as $n \to \infty$ we get that,

$$\begin{aligned} d(fz, z) &\leq \alpha\,[d(fz, z) + d(z\,, z) + d(z, z) + d(z, z)] \\ &\leq \alpha\,[d(fz, z) + 3d(z\,,z)] \\ &\leq \alpha\,[d(fz, z) + 3d(z\,, fz) + 3d(fz\,, z)] \\ &\leq 7\alpha d(fz, z), \quad \text{since } \alpha \in [0, 1/6). \end{aligned}$$

$(1-7\alpha)\, d(fz, z) \leq 0.$

Implies , $d(fz, z) = 0$.

Hence, $fz = z$.

Uniqueness, let u and v be two fixed points of f that is, $fu = u$ and $fv = v$. Then,

$$\begin{aligned} d(u,v) = d(fu,fv) &\leq \alpha\,[d(fu,u) + d(fv,v) + d(fv,u) + d(v,fu)] \\ &\leq \alpha\,[d(u,u) + d(v,v) + d(v,u) + d(v,u)] \\ &\leq \alpha\,[d(u,v) + d(u,v) + d(u,v) + d(u,v) + d(v,u) + d(v,u)] \\ &\leq 6\alpha\, d(u,v). \end{aligned}$$

Implies, $(1-6\alpha)\, d(u,v) \leq 0$. Since, $\alpha \in [0, 1/6)$.

We have, $d(u,v) = 0$. Implies, $u = v$.

Therefore, f has a unique fixed point in $A \cap B$.

Example 2.1. Let $X = R$, $A = [-1, 0]$, $B = [0, 1]$. Define $d(x, y) = |x - y| + 4|x| + 4|y|$ Then d is the dislocated metric.Define $f : A \cup B \to A \cup B$, by $fx = -x/6$, then f is a cyclic mapping. For any two points in A and B, the contractive condition is satisfied and 0 is the unique fixed point of the function f.

Theorem 2.2. Let (X, d) be a complete dislocated metric space. Let A and B be two non empty closed subsets of X and $f : A \cup B \to A \cup B$ be a cyclic mapping satisfying

(i). there exists a number $k \in [0,1/2)$.

(ii) . $d(fx,fy) \leq \max\{\, d(x,y), d(x,fx), d(y,fy), d(x, fy), d(y,fx)\}$

for all $x \in A$ and $y \in B$ then f has a unique fixed point in $A \cap B$.

Proof: Let $\{f^n\} \subseteq X$, $\{f^{2n}\} \subseteq A$and $\{f^{2n-1}\} \subseteq B$. Fix $x \in A$ If $f^n x = f^{n+1}x$ for some n, then $f^{n+1}x = f^{n+2}x$ then $\{f^n x\}$ converges to some $z \in X$. So suppose $f^n x \neq f^{n+1}x$. Now by using above contractive condition of the theorem, we have

$$\begin{aligned} d(f^2x, fx) &\leq k \max\{d(fx,x), d(fx, f^2x), d(x, fx), d(fx,fx), d(x, f^2x)\} \\ &= k \max\{d(fx, x), d(fx,fx), d(x, f^2x)\} \\ &\leq k \max\{d(fx,x), d(fx, x) + d(fx, x), d(x,fx) + d(fx, f^2x)\} \\ &\leq k \max\{d(fx,x)\,, 2\,d(fx, x)\} \\ &\leq k\, d(fx,x) \text{ or } 2k\, d(fx, x) \\ &\leq 2k\, d(fx,x) \\ &\leq h d(fx, x) \text{ where, } h = 2k. \end{aligned}$$

Now by induction we have,

$$d(f^2x, fx) \leq h^n x\, d(fx,x).$$

Following the same process as in the above theorem we show that $\{f^n x\}$ is a cauchy sequence. Since (X,d) is complete, so $\{f^n x\}$ converges to some point $z \in X$. Since $\{f^{2n}\} \subseteq A$ and $\{f^{2n-1}\} \subseteq B$ so $z \in A \cap B$. We claim that $fz = z$.

$$d(fz,f^{2n}x) \leq k \max\{d(z, f^{2-1}x), d(fz,z), d(f^{2n}x,\ f^{2n-1}x), d(z, f^{2n}x), d(f^{2n}x,z)\}.$$

Letting, as $n \to \infty$, we have

$$d(fz, z) \le k \max \{ d(z,z), d(fz, z), d(z, z), d(z, z)\}$$
$$\le k\, d(fz,z), \text{ which is a contradiction.}$$

Hence, $d(fz,z) = 0$.

$\Rightarrow$ $fz = z$.

Therefore, f has a fixed point.

Uniqueness, Let us assume that there exists fixed points u and v, that is $fu = u$ and $fv = v$.

$$d(fu, fv) \le k \max \{ d(u,v), d(u,fu), d(v,fv), d(u,fv), d(v,fu)\}$$
$$\le k \max \{ d(u,v), d(u, u), d(v,v), d(u,v), d(v,u)\}$$
$$= k \max \{ d(u,v), d(u, u), d(v,v), \}$$
$$\le k\ d(u,v) \text{ or } k\, d(u, u) \text{ or } k\, d(v,v)$$
$$(1-k)\, d(u,v) \le 0 \text{ or } (1-2k)\, d(u,v) \le 0.$$

This implies that, $u = v$.

Therefore, f has a unique fixed point in $A \cap B$.

References Références Referencias

[1] P.Hitzler and A.K.Seda, Dislocated topologies, J.Electr.Engin.51(12),3-7,(2000)

[2] P. Hitzler. Generalized Metrics and Topology in Logic Programming Semantics. Ph.d. thesis, *National University of Ireland,University College Cork*, 2001

[3] George Reny, R.Rajgopalan and S.Vinayagam, Cyclic contractions and fixed points in dislocated metric spaces, *Int. J. Math.Anal*, 7(9),(2013),403-411.

[4] Kirik W.A., P.S.Shrinivasan and P.Veeramani, Fixed points for mappings satisfying cyclic contractive condition, *Fixed Point Theory* , 4(2003),78-89.

[5] Zoto.K, Hoxha.E, Isfati.A, Some new results in dislocated and dislocated quasi-metric spaces,.Appl.Math.Sci .Vol .6, (2012),no.71,3519-3526.

Global Journals Inc. (US) Guidelines Handbook 2014

www.GlobalJournals.org