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Homeotopy Groups of 2–Dimensional Manifolds with One Boundary Component

By David J. Sprows
Villanova University, United States

Introduction- Let Y be a compact, connected 2—dimensional manifold with boundary. The homeotopy group of Y, denoted H(Y), is defined to be the group of isotopy classes in the space of all homeomorphisms of Yonto Y. This group (also known as the mapping class group) has been studied for various manifolds (see, for example, [2] and [3]). It is also possible to consider "subhomeotopy groups" where there are restrictions placed on the action of the homeomorphisms on the boundary of Y (see, for example, [7] and [8]). In this note we will consider the special case of a compact, connected manifold with exactly on boundary component. For the remainder of this paper we will assume Y represents a compact, connected manifold with exactly one boundary component and we willlet X denote the closed 2—manifold obtained by sewing a disk to the boundary of Y. Let Aut $\pi_1(X,x_0)$ denote the group of automorphisms of $\pi_1(X,x_0)$ where $x_0 \varepsilon$— Bd(Y). In this paper we establish the following result.
Theorem. If Y is not aMoebius band or a disk, then H(Y)=Aut π_1 (X, x0).

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Homeotopy Groups of 2—Dimensional Manifolds with One Boundary Component

David J. Sprows

I. Introduction

Let Y be a compact, connected 2—dimensional manifold with boundary. The homeotopy group of Y, denoted H(Y), is defined to be the group of isotopy classes in the space of all homeomorphisms of Y onto Y. This group (also known as the mapping class group) has been studied for various manifolds (see, for example, [2] and [3]). It is also possible to consider "subhomeotopy groups" where there are restrictions placed on the action of the homeomorphisms on the boundary of Y (see, for example, [7] and [8]). In this note we will consider the special case of a compact, connected manifold with exactly on boundary component. For the remainder of this paper we will assume Y represents a compact, connected manifold with exactly one boundary component and we willlet X denote the closed 2—manifold obtained by sewing a disk to the boundary of Y. Let Aut $\pi_1(X,x_0)$ denote the group of automorphisms of $\pi_1(X,x_0)$ where $x_0 \varepsilon$ X—Bd(Y). In this paper we establish the following result.

Theorem. If Y is not aMoebius band or a disk, then H(Y) =Aut π_1 (X, x_0)

II. Proof of the Theorem

Let $[(X, x_0),(X,x_0)]$ denote the set of homotopy classes (rel x_0) of maps from (X, x_0) to (X,x_0) and let [f] denote the homotopy class (rel x_0) of a mapping f from (X,x_0) to (X,x_0). Let End π_1 (X, x_0) denote the set of endomorphisms of $\pi_1(X,x_0)$ with the operation of composition.

Lemma 1. If X is a closed 2-manifo1d with $\pi_2(X, x_0) = 0$ and $\phi : [(X, x_0), (X, x_0)] \rightarrow$ End $\pi_1(X,x_0)$ is given by $\phi([f]) = f_*$, then ϕ is a bijection which preserves the operation of composition.

Proof of lemma 1.

1. Clearly ϕ is well defined and $\phi([f\circ g]) = \phi([f]) \circ \phi([g])$
2. Claim ϕ is a surjection.

Let F in END $\pi_1(X,x_0)$. We define f: $(X,x_0) \rightarrow (X, x_0)$ as follows:

Author: Villanova University. e-mail: david.sprows@villanova.edu

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7. D. Sprows, "Local sub homotopy groups of bounded surfaces" International Journal of Math & Math Sci. 24 (2000), 251-255.

Take x_0 to be vertex of the triangulation of X and let T be a maximal tree in X. If x ϵ T, we let f(x) = x_0. Now, suppose s is a 1—simplex not in T with h: [0,1] = s. Let Γ_i, be a path in T from x_0 to h(i), i = 0, 1. Define the loop α_s at x_0 by letting α_s (t) = Γ_0(t) if $-1\leq t\leq 0$, α_s (t) = h(t) if $0\leq t\leq 1$, and α_s (t) = Γ_0^{-1}(t) if $1\leq t\leq 2$. If F[α_s] = [β], we let f/s = $\beta\circ h^{-1}s$. This defines f on the 1-skeleton of X.

Finally, if Δ is a 2-simplex with edges s1, s2, and s3, then $[\alpha_{S1} * \alpha_{S2} * \alpha_{S3}] = 1$. This means f/δΔ is null homotopic, i.e., f extends to Δ. Hence, the mapping of f defined on the 1-skeleton as above, extends to a mapping defined on all of the 2-dimensional manifold X. Note that by construction, $f_*[\alpha_s]$ = F[α_s] and since {[α_s]: S is a 1-simplex of X″ generates $\pi_1(X,x_0)$ we have f_* = F.

III. Claim ϕ is an Injection

Suppose $f_* = g_*$. As in Part 2), let T be a maximal tree. f/T is homotopic (rel x_0) to a map which sends T to x_0 (just use the retraction of T to x_0). Hence, by the homotopy extension property, f is homotopic (rel x_0) to a map f' with f'(T) = x_0. Therefore, we can assume f(T) = g(T) = x_0. In particular, for each 1-simplex s, f/s and g/s are loops at x_0. Since $f_*= g_*$ this means f/s is homotopic to g/s (rel x_0). Thus, for each 2—simplex Δ we have a map H: δ(ΔxI) → X where H/Δx0 = f/Δ, H/Δxl = f/Δ and for each 1-simplex s in δΔ, H/SxI is a homotopy from f/s to g/s. Since $\pi 2(X,x_0) = 0$ and δ(ΔxI) = S^2 , this map H can be extended to all of ΔxI. Fitting together each of these H's we get a homotopy (rel x_0) from f to g, i.e., [f] = [g].

Lemma 2 : If X is a closed 2—manifold and h:(X,x_0) → (X,x_0) is a homeomorphism which is homotopic to the identity (rel x_0) then h is isotopic to the identity (rel x_0).

Proof. This is a special case of Theorem 6.3 of [1].

Lemma 3. If X is a closed 2—manifold with x_0 ϵ X and G is an automorphism of π1(X,x_0), then there exists a homeomorphism h: (X,x_0) + (X,x_0) with h_* = G.

Proof. This result is proved in [4].

Let H(X,x_0) denote the group of isotopy classes (rel x_0) in the space of all homeomorphisms of (X,x_0) onto (X,x_0).

By Lemma 2 the function from H(X,x_0) to [(X,x_0)] which sends the isotopy class (rel x_0) of a homeomorphism to its homotopy class (rel x_0) is an injection. By Lemma 1 the composition H(X,x_0) → [(X,x_0) , (X,x_0)] → End π1(X,x_0) is a monomorphism of the group H(X,x_0) onto a subgroup of Aut π1(X,x_0). Finally Lemma 3 shows that this monomorphism is an isomorphism onto Aut $\pi_1(X,x_0)$. Thus the proof of the theorem reduces to showing that H(Y) is isomorphic to H(X,x_0). This is done by the following lemma.

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Lemma 4. If Y is a compact, connected 2—dimensional manifold with one boundary component and X is the closed 2—manifold obtained by sewing a disk to the boundary of Y and $x_0 \in X\text{-}Bd(Y)$then $H(X,x_0) = H(Y)$.

Proof. This result is a special case of Theorem 6 of [6].

Remark 3. If Y is a Moebius band or disk, then H(Y) is not isomorphic to Aut $\pi1(X,x0)$. In the case Y is a disk, so that $X = S^2$, we have Aut $\pi1\ (X,x_0) = 1$ while $H(Y) = Z_2$(see Theorem 4.2 of [4]). In the case Y is a Moebius band, so that $X = P^2$, we have Aut $\pi1$, $(X,x_0) = 1$ while $H(Y) = Z_2$ (see Theorem 8.1 of[4]).

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