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Further Results on Modified Variational Iteration Method for the Analytical Solution of Nonlinear Advection Equations

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Abstract- In this paper, the result shows that the method is elegant and reliable with less computational efforts. This method is strongly recommended for the solution of strongly nonlinear partial differential equations and systems of differential equations further to our results in [12] on the solution of nonlinear advection equations, we present a further results on the nonlinear non-homogeneous advection equations using a modified variational iteration method.

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Ref

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Further Results on Modified Variational Iteration Method for the Analytical Solution of Nonlinear Advection Equations

M.O.Olayiwola [α], K.O.Kareem [σ] & A.W.Gbolagade [ρ]

Abstract- In this paper, the result shows that the method is elegant and reliable with less computational efforts. This method is strongly recommended for the solution of strongly nonlinear partial differential equations and systems of differential equations further to our results in [12] on the solution of nonlinear advection equations, we present a further results on the nonlinear non-homogeneous advection equations using a modified variational iteration method.

Keywords: lagrange multiplier, non-homogeneous equations, advection equations.

I. Introduction

A Modified Variational Iteration Method (MVIM) for the solution of nonlinear advection equations is presented. The method is an elegant combination of the Taylor's approximation and the Variational Iteration Method (VIM). The method is seen to be a very reliable alternative to some existing techniques for the nonlinear advection equations.

This paper outlines a reliable method among many others; the method gives rapidly convergent series with specific significant features for the problem.

The nonlinear non-homogeneous partial differential equation problem is of the form:

$$U_t + UU_x = 2x^2t + 2xt^2 + 2x^3t^4 \tag{1}$$

with the initial condition:

$$U(x,0) = 1$$

Many authors have worked on different numerical approaches for the solution of differential equations [1-23]. The nonlinear non-homogeneous advection equations plays a crucial role in applied mathematics and physics. A substantial amount of research work has been directed for the study of the solution of nonlinear non-homogeneous problems and on partial differential equations in particular.

In this paper, further to our results in [12] , a Modified Variational Iteration Method (MVIM) which accurately compute the solution of nonlinear non-homogeneous partial differential equations is presented. The main advantage of this method is that it can be applied directly to partial differential equation without any linearization.

II. Modified Variational Iteration Method

The variational Iteration Method was proposed by He [1-4]. In this paper a Modified Variational Iteration Method (MVIM) proposed by Olayiwola [5-8] is presented for the solution of nonlinear non-homogeneous partial differential equations.

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We consider the following general nonlinear partial differential equation of the form:

$$LU(x,t)+RU(x,t)+NU(x,t)=g(x,t) \tag{2}$$

Where: L is a linear time derivative operator,
R is a linear operator which has partial derivative with respect to x
N is a nonlinear operator and
g is an inhomogeneous term.

According to Variational Iteration Method (VIM), we can apply the correction functional as follows:

$$U_{n+1}(x,t)=U_n(x,t)+\int_0^t \lambda[LU_n+RU_n+NU_n-g]d\tau \tag{3}$$

Where $\lambda=-1$ is a Lagrange multiplier which can be identified optimally via Variational Iteration Method.

The subscript n denoted nth approximation, U_n is considered as a restricted variation i:e $\partial U_n=0$. The successive approximation U_{n+1},n≥0 of the solution U will be readily obtained upon using the determined Lagrange multiplier and any selective functionU_0 .Consequently, the solution is given by:

$$U=\lim_{n\to\infty}U_n \tag{4}$$

In a Modified Variational Iteration Method, $U_0(x,t)$ in equation (3) becomes:

$$U_0(x,t)=\sum_{i=0}^{3} g_i(x)t^i \tag{5}$$

Where $g_i(x)=k_i(x)$ can be found by substituting for $U_0(x,t)$ in (2) at $t=0$

III. Numerical Examples

a) Problem 1

$$U_t+UU_x=2x^2t+2xt^2+2x^3t^4, U(x,0)=1 \tag{6}$$

Let

$$U(x,0)=1$$
$$U^+=1+kt$$
$$U^+{}_t=k$$
$$U^+{}_x=0$$

Then

$$U^+(x,t)=1+0(t)=1 \tag{7}$$

$$U^{++}(x,t)=1+kt^2 \tag{8}$$

$$U^{++}{}_t=2kt \tag{9}$$

$$U^{++}{}_x=0$$

Substitute for (9) in (6)

$$U^{++}{}_t+U^{++}U^{++}{}_x=2x^2t+2xt^2+2x^3t^4 \tag{10}$$

$$2kt+(1+kt^2)(0)=2x^2t+2xt^2+2x^3t^4$$

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$$2kt = 2x^2t + 2xt^2 + 2x^3t^4$$

$$k = x^2 + xt + x^3t^3$$

When t=0,

$$k = x^2 \tag{11}$$

Notes

Then

$$U^{++}(x,t) = 1 + x^2t^2$$

Let

$$U^{+++}(x,t) = 1 + x^2t^2 + kt^3 \tag{12}$$

$$U^{+++}{}_t = 2x^2t + 3kt^2 \tag{13}$$

$$U^{+++}{}_x = 2xt^2 \tag{14}$$

Substitute for (13) and (14) in (6)

$$U^{+++}{}_t + U^{+++}U^{+++}{}_x = 2x^2t + 2xt^2 + 2x^3t^4 \tag{15}$$

$$(2x^2t + 3kt^2) + (1 + x^2t^2 + kt^3)(2xt^2) = 2x^2t + 2xt^2 + 2x^3t^4$$

$$2x^2t + 3kt^2 + 2xt^2 + 2x^3t^4 + 2kxt^5 = 2x^2t + 2xt^2 + 2x^3t^4$$

$$3kt^2 + 2kxt^5 = 0$$

$$k(3t^2 + 2xt^5) = 0$$

$$k = 0 \tag{16}$$

$$U^{+++}(x,t) = 1 + x^2t^2 + 0(t^3)$$

$$U^{+++}(x,t) = 1 + x^2t^2 \tag{17}$$

Let

$$U^{++++}(x,t) = 1 + x^2t^2 + kt^4 \tag{18}$$

$$U^{++++}{}_t = 2x^2t + 4kt^3 \tag{19}$$

$$U^{++++}{}_x = 2xt^2 \tag{20}$$

Substitute for (19) and (20) in (6)

$$U^{++++}{}_t + U^{++++}U^{++++}{}_x = 2x^2t + 2xt^2 + 2x^3t^4 \tag{21}$$

$$(2x^2t + 4kt^3) + (1 + x^2t^2 + kt^4)(2xt^2) = 2x^2t + 2xt^2 + 2x^3t^4$$

$$2x^2t + 4kt^3 + 2xt^2 + 2x^3t^4 + 2kxt^6 = 2x^2t + 2xt^2 + 2x^3t^4$$

$$4kt^3 + 2kxt^6 = 0$$

$$k(4t^3 + 2xt^6) = 0$$

$$k = 0 - \tag{22}$$

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$$U_0{}^{++++}(x,t)=1+x^2t^2=U_0\ (x,t)=1+x^2t^2 \tag{23}$$

By using Modified Variational Iteration Method(MVIM) Formula:

$$U_{n+1}(x,t)=U^{++++}(x,t)-\int_0^t[\frac{\partial U_n^{++++}(x,t)}{\partial\xi}+U_n(x,\xi)\frac{\partial U_n^{++++}(x,\xi)}{\partial x}-2x^2\xi-2x\xi^2-2x^3\xi^4]d\xi \tag{24}$$

$$U^{++++}{}_1(x,t)=1+x^2t^2$$

$$U^{++++}{}_1(x,t)=1+x^2t^2$$

$$\frac{\partial U^{++++}{}_1(x,\xi)}{\partial\xi}=2x^2\xi$$

$$\frac{\partial U^{++++}{}_1(x,\xi)}{\partial x}=2x\xi^2$$

Substitute for $U^{++++}{}_1(x,t)$, $U^{++++}{}_1(x,\xi)$, $\frac{\partial U^{++++}{}_1(x,t)}{\partial\xi}$ and $\frac{\partial U^{++++}{}_1(x,\xi)}{\partial x}$ in (24)

$$U_1(x,t)=1+x^2t^2-\int_0^t[2x^2\xi+(1+x^2\xi^2)(2x\xi^2)-2x^2\xi-2x\xi^2-2x^3\xi^4]d\xi$$

$$U_1(x,t)=1+x^2t^2-\int_0^t[2x^2\xi+2x\xi^2+2x^3\xi^4-2x^2\xi-2x\xi^2-2x^3\xi^4]d\xi$$

$$U_1(x,t)=1+x^2t^2-\int_0^t(0)d\xi$$

$$U_1(x,t)=1+x^2t^2 \tag{25}$$

This is the exact solution.

b) Problem 2

$$U_t+UU_x=1+tCosx+\frac{1}{2}Sin2x,\ U(x,0)=Sinx \tag{26}$$

Let

$$U(x,0)=Sinx$$

$$U^+(x,t)=Sinx+t \tag{27}$$

$$U^{++}(x,t)=Sinx+t+kt^2 \tag{28}$$

Ssubstitute for (27 and 28) in (26)

$$U^{++}{}_t+U^{++}U^{++}{}_x=1+tCosx+\frac{1}{2}Sin2x$$

$$1+2kt+(Sinx+t+kt^2)(Cosx)=1+tCosx+\frac{1}{2}Sin2x$$

$$1+2kt+tCosx+SinxCosx+kt^2Cosx=1+tCosx+\frac{1}{2}Sin2x$$

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$$2kt + kt^2 Cosx = \frac{1}{2} Sinx - SinxCosx$$

$$k = \lim_{t=0} \left[\frac{\frac{1}{2} Sinx - SinxCosx}{2t + t^2 Cosx} \right] \tag{29}$$

Let

$$U^{+++}(x,t) = t + Sinx = U^{+}(x,t)$$

For Modified Variational Iteration Method (MVIM)

$$U_{n+1}^{+++}(x,t) = U^{+++}(x,t) - \int_0^t [\frac{\partial U_n^{+++}(x,t)}{\partial \xi} + U_n(x,\xi)\frac{\partial U_n(x,\xi)}{\partial x} - 1 - \xi Cosx - \frac{1}{2} Sin2x] d\xi \tag{30}$$

$$U^{+++}{}_1(x,t) = t + Sinx$$

$$U_1(x,\xi) = \xi + Sinx$$

$$\frac{\partial U_n(x,t)}{\partial \xi} = 1$$

$$\frac{\partial U_n(x,\xi)}{\partial x} = Cosx$$

Substitute for $U^{+++}{}_1(x,t)$, $U^{+++}{}_1(x,\xi)$, $\frac{\partial U^{+++}{}_1(x,t)}{\partial \xi}$ and $\frac{\partial U^{+++}{}_1(x,\xi)}{\partial x}$ in (30)

$$U_1(x,t) = (t + Sinx) - \int_0^t [1 + (\xi + Sinx)(Cosx) - 1 - \xi Cosx - \frac{1}{2} Sin2x] d\xi \tag{31}$$

$$U_1(x,t) = t + Sinx - \int_0^t [1 + \xi Cosx + SinxCosx - 1 - \xi Cosx - \frac{1}{2} Sin2x] d\xi$$

$$U_1(x,t) = t + Sinx - \int_0^t [SinxCosx - \frac{1}{2} Sin2x] d\xi$$

$$U_n{}^{+++}(x,t) = t + Sinx - [\xi SinxCosx - \frac{\xi}{2} Sin2x]_0^t$$

$$U_1(x,t) = t + Sinx \tag{32}$$

This also gives the exact solution.

IV. Conclusion

In this paper, further to our recent results in [12], we presented a Modified Variational Iteration Method proposed in [5-8] to the solution of nonlinear non-homogeneous advection equations. The result shows that the method is elegant and reliable with less computational efforts. This method is strongly recommended for the solution of strongly nonlinear partial differential equations and systems of differential equations.

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