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Solution of Complex Differential Equation System by using Differential Transform Method

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SOLUTIONOFCOMPLEXDIFFERENTIALEQUATIONSYSTEMBYUSINGDIFFERENTIALTRANSFORMMETHOD

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[5] I.A. Salehbhai, A.K. Shukla The Numerical Solutions of Differential Transform Method And The Laplace Transform Method For A System of Differential Equations Nonlinear Analysis:Hybrid Systems (2010) 425-431

Solution of Complex Differential Equation System by using Differential Transform Method

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Abstract- In this study, using differential transform method second order complex differential equation system was solved. Firstly we seperated real and imaginer parts these equations system. Thus from two unknown equation system four equality was obtained. Later using two dimensional differential transform we obtained real and imaginer parts of solutions.

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I. Introduction

The concept of differential transform (one dimension) was first proposed and applied to solve linear and non linear initial value problems in electric circuit analysis by Zhou [1] :Solving partial differential equations by two dimensional differential transform method was proposed by Chao Kuang Chen and Shing Huei Ho by [2] : Partial differential equations was solved by using two dimensional DTM in [2]-[3]. System of differential equation was solved using two dimensional DTM in [4]. The numerical solutions of differential transform method and the Laplace transform method for a system of differential equations was compared in [5]. By using differential transform method was solved that integral equations, fractional differential equations, difference equations, integral and integro differential equations in [6], [7], [8],[9],[10] .

In this paper using [1] complex partial differential equations was solved. Let w = w (z; z) be a complex function. Here $z = x + iy$, $w(z, \bar{z}) = u(x, y) + iv(x, y)$. Derivative according to z and $\bar{z}$ of $w(z, \bar{z})$ is defined as follows:

$$\frac{\partial w}{\partial z} = \frac{1}{2}\left(\frac{\partial w}{\partial x} - i\frac{\partial w}{\partial y}\right) \tag{1}$$

$$\frac{\partial w}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + i\frac{\partial w}{\partial y}\right) \tag{2}$$

Here

$$\frac{\partial w}{\partial x} = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} \tag{3}$$

$$\frac{\partial w}{\partial y} = \frac{\partial u}{\partial y} + i\frac{\partial v}{\partial y} \tag{4}$$

Similarly second order derivatives according to z and $\bar{z}$ of $w(z, \bar{z})$ is defined as follows:

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$$\frac{\partial^2 w}{\partial z^2} = \frac{1}{4}\left(\frac{\partial^2 w}{\partial x^2} - 2i\frac{\partial^2 w}{\partial x \partial y} - \frac{\partial^2 w}{\partial y^2}\right) \tag{5}$$

$$\frac{\partial^2 w}{\partial \bar{z}^2} = \frac{1}{4}\left(\frac{\partial^2 w}{\partial x^2} + 2i\frac{\partial^2 w}{\partial x \partial y} - \frac{\partial^2 w}{\partial y^2}\right) \tag{6}$$

$$\frac{\partial^2 w}{\partial z \partial \bar{z}} = \frac{1}{4}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = \frac{1}{4}\Delta w \tag{7}$$

Notes

II. Two Dimensional Differential Transform

Definition 1: Two dimensional differential transform of function f (x,y) is defined as follows

$$F(k,h) = \frac{1}{k!.h!}\left[\frac{\partial^{k+h} f(x,y)}{\partial x^k \partial y^h}\right]_{x=0,y=0} \tag{8}$$

In Equation(8) , $f(x,y)$ is original function and $F(k,h)$ is transformed function, which is called T-function is brief.

Definition 2: Differential inverse transform of $F(k,h)F(k,\ h)$ is defined as follows

$$f(x,y) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} F(k,h)x^k y^h$$

$$f(x,y) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} \frac{1}{k!.h!}\left[\frac{\partial^{k+h} f(x,y)}{\partial x^k \partial y^h}\right]_{x=0,y=0} x^k y^h \tag{9}$$

Equation 9 implies that the concept of two dimensional differential transform is derived from two dimensional Taylor series expansion.

Theorem 3 [2] : If $w(x,y) = u(x,y) \pm v(x,y)$ then $W(k,h) = \lambda U(k,h)$

Theorem 4 [2] : If $w(x,y) = \lambda u(x,y)$ then $W(k,h) = \lambda U(k,h)$

Theorem 5 [2] **:** If $w(x,y) = \frac{\partial u(x,y)}{\partial x}$ then $W(k,h) = (k+1)U(k+1,h)$

Theorem 6 [2] **:** If $w(x,y) = \frac{\partial u(x,y)}{\partial y}$ then $W(k,h) = (h+1)U(k,h+1)$

Theorem 7 [2] **:** If $w(x,y) = \frac{\partial^{r+s} f(x,y)}{\partial x^r \partial y^s}$ then

$$W(k,h) = (k+1)(k+2)\ldots(k+r)(h+1)(h+2)\ldots(h+s)U(k+r,h+s)$$

Theorem 8 [2] : If $w(x,y) = u(x,y).v(x,y)$ then $W(k,h) = \sum_{r=0}^{k}\sum_{s=0}^{h} U(r,k-s)V(k-r,s)$

Theorem 9 [2] : If $w(x,y) = x^m y^n$ then $W(k,h) = \delta(k-m,h-n)$

Example 1 : Solve the following complex differential equation system

$$\frac{\partial w_1}{\partial z} + \frac{\partial w_2}{\partial \bar{z}} = 2z + 3 \tag{10}$$

$$\frac{\partial w_1}{\partial \bar{z}} + \frac{\partial w_2}{\partial z} = 7 \tag{11}$$

with initial conditions

$$w_1(x,0) = x^2 + 2x \tag{12}$$

$$w_2(x,0) = 8x \tag{13}$$

Since $w_1 = u_1 + iv_1$, $w_2 = u_2 + iv_2$ and from (1),(2), system (10)-(11) is equivalent that :

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} + \frac{\partial u_2}{\partial x} - \frac{\partial v_2}{\partial y} = 4x + 6 \tag{14}$$

$$\frac{\partial v_1}{\partial x} - \frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial y} + \frac{\partial v_2}{\partial x} = 4y \tag{15}$$

$$\frac{\partial u_1}{\partial x} - \frac{\partial v_1}{\partial y} + \frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} = 14 \tag{16}$$

$$\frac{\partial v_1}{\partial x} + \frac{\partial u_1}{\partial y} + \frac{\partial v_2}{\partial x} - \frac{\partial u_2}{\partial y} = 0 \tag{17}$$

Notes

From (12) and (13) initial conditions we have that:

$U_1(0,0) = 0,\ U_1(1,0) = 2,\quad U_1(2,0) = 1,\quad U_1(i,0) = 0\ (i > 2),\ \ V_1(i,0) = 0\qquad (i\epsilon N)$

$$U_2(0,0) = 0,\qquad U_2\ (1,0) = 8,\qquad U_2(i,0) = 0\ \ (i \geq 2)\quad V_2(i,0) = 0\quad (i\epsilon N) \tag{18}$$

From differential transform of (14)- (17) is get that:

$$\begin{aligned}(k+1)U_1(k+1,h) + (h+1)V_1(k,h+1) + (k+1)U_2(k+1,h) - (h+1)V_2(k,h+1)\\ = 4\delta(k-1,h) + 6\delta(k,h)\end{aligned} \tag{19}$$

$$\begin{aligned}(k+1)V_1(k+1,h) - (h+1)U_1(k,h+1) + (h+1)(k+1)U_2(k,h+1) + (k+1)V_2(k+1,h)\\ = 4\delta(k,h-1)\end{aligned} \tag{20}$$

$$\begin{aligned}(k+1)U_1(k+1,h) - (h+1)V_1(k,h+1) + (k+1)U_2(k+1,h) + (h+1)V_2(k,h+1)\\ = 14\delta(k,h)\end{aligned} \tag{21}$$

$$\begin{aligned}(k+1)V_1(k+1,h) + (h+1)U_1(k,h+1) + (k+1)V_2(k+1,h) - (h+1)U_2(k,h+1)\\ = 0\end{aligned} \tag{22}$$

In (19) equality if we write $k = 0, h = 0$ than by using (18) is obtained

$$V_1(0,1) - V_2(0,1) = -4,\ V_1(0,1) = a,\ V_2(0,1) = a + 4\ \ (a \in R) \tag{23}$$

By sum of (19) with (21) we get that:

$$U_1(k+1,h) + U_2(k+1,h) = \frac{4\delta(k,h-1) + 20\delta(k,h)}{2(k+1)} \tag{24}$$

By mines of (20) with (22) we get that:

$$U_2(k,h+1) - U_1(k,h+1) = \frac{4\delta(k,h-1)}{2(h+1)} \tag{25}$$

By mines of (19) with (21) we get that:

$$V_1(k,h+1) - V_2(k,h+1) = \frac{4\delta(k-1,h) - 8\delta(k,h)}{2(h+1)} \tag{26}$$

By sum of (20) with (22) we get that:

$$V_1(k+1,h) + V_2(k+1,h) = \frac{4\delta(k,h-1)}{2(k+1)} \tag{27}$$

If we write in place of $h, h+1$ in (24) than we get

$$U_1(k+1,h+1) + U_2(k+1,h+1) = \frac{4\delta(k,h) + 20\delta(k,h+1)}{2(k+1)} \tag{28}$$

If we write in place of $k, k+1$ in (25) than we get

$$U_2(k+1,h+1) - U_1(k+1,h+1) = \frac{4\delta(k+1,h-1)}{2(h+1)} \tag{29}$$

If we write in place of $k, k+1$ in (26) than we get

$$V_1(k+1,h+1) - V_2(k+1,h+1) = \frac{4\delta(k,h) - 8\delta(k+1,h)}{2(h+1)} \tag{30}$$

If we write in place of h , $h+1$ in (27) than we get

$$V_1(k+1,h+1)+V_2(k+1,h+1)=\frac{4\delta(k,h)}{2(k+1)} \tag{31}$$

By sum of (28) with (29) we get that:

$$U_2(k+1,h+1)=\frac{\delta(k,h)+5\delta(k,h+1)}{k+1}+\frac{\delta(k+1,h-1)}{h+1} \tag{32}$$

By mines of (29) from (28) we get that:

$$U_1(k+1,h+1)=\frac{\delta(k,h)+5\delta(k,h+1)}{k+1}-\frac{\delta(k+1,h-1)}{h+1} \tag{33}$$

By sum of (30) with (31) we get that:

$$V_1(k+1,h+1)=\frac{\delta(k,h)-2\delta(k+1,h)}{h+1}+\frac{\delta(k,h)}{k+1} \tag{34}$$

By mines of (30) from (31) we get that

$$V_2(k+1,h+1)=\frac{\delta(k,h)}{k+1}-\frac{\delta(k,h)-2\delta(k+1,h)}{h+1} \tag{35}$$

In (34) equality if we write $k=0,h=0$ than is obtained

$$V_1(1,1)=2 \tag{36}$$

In (20) equality if we write $k=0,h=1$ than by using (31) is obtained

$$-U_1(0,2)+U_2(0,2)=1, U_1(0,2)=b, U_2(0,2)=b\ , b\in R \tag{37}$$

If we write $k=0,h=0$ in (25) than we get

$$U_1(0,1)=U_2(0,1)=\mathrm{c}, \mathrm{c}\epsilon\mathrm{R} \tag{38}$$

By using (32),(33),(34) and (35) it is seen that all the other components of u_1, v_1, u_2 and v_2 is zero. Thus it is obtained

$$u_1(x,y)=\sum_{k=0}\sum_{h=0}U_1(k,h)x^k y^h$$

$$=x^2+by^2+2x+cy \tag{39}$$

$$v_1(x,y)=\sum_{k=0}\sum_{h=0}V_1(k,h)x^k y^h$$

$$=2xy+ay \tag{40}$$

$$u_2(x,y)=\sum_{k=0}\sum_{h=0}U_2(k,h)x^k y^h$$

$$=8x+(b+1)y^2+cy \tag{41}$$

$$v_2(x,y)=\sum_{k=0}\sum_{h=0}V_2(k,h)x^k y^h$$

$$=(a+4)y \tag{42}$$

From (39)-(42) we get

$$w_1(x,y)=u_1(x,y)+v_1(x,y)$$

$$=x^2+by^2+2x+cy+i(2xy+ay)$$

$$w_2(x,y)=u_2(x,y)+v_2(x,y)$$

Notes

Notes

Example 2 : Solve the following complex differential equation system

$$2\frac{\partial w_1}{\partial z}-\frac{\partial w_2}{\partial \bar{z}}=6z^2-4z \tag{43}$$

$$\frac{\partial w_1}{\partial \bar{z}}+3\frac{\partial w_2}{\partial z}=12\bar{z}-2 \tag{44}$$

with initial conditions

$$w_1(x,0)=x^3-2x \tag{45}$$

$$w_2(x,0)=4x^2 \tag{46}$$

If we write system in (43)-(44) system $w_1=u_1+iv_1$, $w_2=u_2+iv_2$ □□□□ we get following equations.

$$2\left(\frac{\partial u_1}{\partial x}+i\frac{\partial v_1}{\partial x}-i\frac{\partial u_1}{\partial y}+\frac{\partial v_1}{\partial y}\right)-\left(\frac{\partial u_2}{\partial x}+i\frac{\partial v_2}{\partial x}+i\frac{\partial u_2}{\partial y}-\frac{\partial v_2}{\partial y}\right)=12(x+iy)^2-8(x+iy)$$

$$\frac{\partial u_1}{\partial x}-\frac{\partial v_1}{\partial y}+i\frac{\partial u_1}{\partial x}+i\frac{\partial v_1}{\partial x}+3\frac{\partial u_2}{\partial x}+3i\frac{\partial v_2}{\partial x}-3i\frac{\partial u_2}{\partial y}+3\frac{\partial v_2}{\partial y} \tag{47}$$
$$=24(\mathrm{x}-\mathrm{iy})-4$$

If (47) equalities is seperated into real and imaginary parts then it is get following egualities.

$$2\frac{\partial u_1}{\partial x}+2\frac{\partial v_1}{\partial y}-\frac{\partial u_2}{\partial x}+\frac{\partial v_2}{\partial y}=12(x^2-y^2)-8x \tag{48}$$

$$2\frac{\partial v_1}{\partial x}-2\frac{\partial u_1}{\partial y}-\frac{\partial v_2}{\partial x}-\frac{\partial u_2}{\partial y}=24xy-8y \tag{49}$$

$$\frac{\partial u_1}{\partial x}-\frac{\partial v_1}{\partial y}+3\frac{\partial u_2}{\partial x}+3\frac{\partial v_2}{\partial y}=24x-4 \tag{50}$$

$$\frac{\partial u_1}{\partial y}+\frac{\partial v_1}{\partial x}+3\frac{\partial v_2}{\partial x}-3\frac{\partial u_2}{\partial y}=-24y \tag{51}$$

From (45)-(46) initial conditions we have that:

$U_1(0{,}0)=0,\ U_1(1{,}0)=-2,\quad U_1(2{,}0)=0,\quad U_1(3{,}0)=1,\ U_1(i,0)=0(i>3),\ \ V_1(i,0)=0\ \ (i\epsilon N)$

$$U_2(0{,}0)=0,\ U_2\ (1{,}0)=0,\ \ U_2(2{,}0)=4,\ \ U_2(i,0)=4(i\geq 3)\ ,\ \ V_2(i,0)=0\quad (i\epsilon N) \tag{52}$$

From differential transform of (48)- (51) is get following equalities:

$$2(k+1)U_1(k+1,h)+2(h+1)V_1(k,h+1)-(k+1)U_2(k+1,h)+(h+1)V_2(k,h+1)$$
$$=12\delta(k-2,h)-12\delta(k,h-2)$$
$$-8\delta(k-1,h) \tag{53}$$

$$2(k+1)V_1(k+1,h)-2(h+1)U_1(k,h+1)-(k+1)V_2(k+1,h)-(h+1)U_2(k,h+1)$$
$$=24\delta(k-1,h-1)-\delta(k,h-1) \tag{54}$$

$$(k+1)U_1(k+1,h)-(h+1)V_1(k,h+1)+3(k+1)U_2(k+1,h)+3(h+1)V_2(k,h+1)$$
$$=24\delta(k-1,h)-4\delta(k,h) \tag{55}$$

$$(h+1)U_1(k,h+1)+(k+1)V_1(k+1,h)+3(k+1)V_2(k+1,h)-3(h+1)U_2(k,h+1)$$
$$=-24\ \delta(k,h-1) \tag{56}$$

If we write $h=0$ in (54) and (56) from (52) we get that:

$$U_1(k,1)=U_2(k,1)=0 \tag{57}$$

If we write $k = 0, h = 0$ in (53) and (55) from (52) we get that:

$$V_1(0,1) = 2, V_2(0,1) = 0 \quad (58)$$

If we write $h = 0$ in (54) and (56) from (52) we get that:

$$U_1(k,1) = U_2(k,1) = 0 \quad (59)$$

If we write $k = 1, h = 0$ in (53) and (55) from (52) we get that:

$$V_1(1,1) = 0, V_2(1,1) = 0 \quad (60)$$

If we write $k = 0, h = 1$ in (54) and (56) from (52) we get that:

$$U_1(0,2) = 0, U_2(0,2) = 4 \quad (61)$$

If we write $h = 1$ in (53) and (55) from (52) and (57) we get that:

$$V_1(k,2) = V_2(k,2) = 0 \quad (62)$$

If we write $h = 2$ in (54) and (56) from (62) we get that:

$$U_1(k,3) = U_2(k,3) = 0 \quad (63)$$

If we write k=2,h=0 1 in (53) and (55) we get that

$$V_1(2,1) = 3, V_2(2,1) = 0 \quad (64)$$

If we write k=1,h=1 1 in (54) and (56) we get that

$$U_1(1,2) = -3, U_2(1,2) = 0 \quad (65)$$

By using equalities (57)-(65) we see other components of U_1, U_2, V_1 and V_2 are equal zero.

Thus it is obtained

$$u_1(x,y) = \sum_{k=0}\sum_{h=0} U_1(k,h)x^k y^h$$
$$= x^3 - 3xy^2 - 2x$$

$$v_1(x,y) = \sum_{k=0}\sum_{h=0} V_1(k,h)x^k y^h$$
$$= 3x^2y - y^3 + 2y$$

$$u_2(x,y) = \sum_{k=0}\sum_{h=0} U_2(k,h)x^k y^h$$
$$= 4x^2 + 4y^2$$

$$v_2(x,y) = \sum_{k=0}\sum_{h=0} V_2(k,h)x^k y^h$$
$$= 0$$

Therefore

$$w_1(x,y) = u_1(x,y) + v_1(x,y)$$
$$= x^3 - 3xy^2 - 2x + i(3x^2y - y^3 + 2y)$$
$$= z^3 - 2\bar{z}$$

$$w_2(x,y) = u_2(x,y) + v_2(x,y)$$
$$= 4x^2 + 4y^2$$
$$= 4z\bar{z}$$

Example:

Solve the following complex diferential equation system

$$\frac{\partial^2 w_1}{\partial z^2} + \frac{\partial^2 w_2}{\partial z \partial \bar{z}} = 3 \quad (66)$$

$$\frac{\partial^2 w_2}{\partial z^2} + \frac{\partial w_1}{\partial z} - \frac{\partial w_2}{\partial \bar{z}} = z \qquad (67)$$

with initial conditions

$$w_1(x,0) = x^2 \qquad (68)$$

$$\frac{\partial w_1}{\partial y}(x,0) = 2ix \qquad (69)$$

$$w_2(x,0) = x^2 \qquad (70)$$

$$\frac{\partial w_2}{\partial y}(x,0) = 0 \qquad (71)$$

Notes

Since $w_1 = u_1 + iv_1$, $w_2 = u_2 + iv_2$ and from (2),(5) and (7) system (66)-(67) is equivalent that :

$$\frac{1}{4}\left(\frac{\partial^2 w_1}{\partial x^2} - 2i\frac{\partial^2 w_1}{\partial x \partial y} - \frac{\partial^2 w_1}{\partial y^2}\right) + \frac{1}{4}\left(\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2}\right) = 3 \qquad (72)$$

$$\frac{1}{4}\left(\frac{\partial^2 w_2}{\partial x^2} - 2i\frac{\partial^2 w_2}{\partial x \partial y} - \frac{\partial^2 w_2}{\partial y^2}\right) + \frac{1}{2}\left(\frac{\partial w_1}{\partial x} - i\frac{\partial w_1}{\partial y}\right) - \frac{1}{2}\left(\frac{\partial w_2}{\partial x} + i\frac{\partial w_2}{\partial y}\right) = z \qquad (73)$$

$$\left(\frac{\partial^2 u_1}{\partial x^2} + i\frac{\partial^2 v_1}{\partial x^2}\right) - 2i\left(\frac{\partial^2 u_1}{\partial x \partial y} + i\frac{\partial^2 v_1}{\partial x \partial y}\right) - \left(\frac{\partial^2 u_1}{\partial y^2} + i\frac{\partial^2 v_1}{\partial y^2}\right) + \left(\frac{\partial^2 u_2}{\partial x^2} + i\frac{\partial^2 v_2}{\partial x^2}\right) + \left(\frac{\partial^2 u_2}{\partial y^2} + i\frac{\partial^2 v_2}{\partial y^2}\right) = 12 \qquad (74)$$

$$\left(\frac{\partial^2 u_2}{\partial x^2} + i\frac{\partial^2 v_2}{\partial x^2}\right) - 2i\left(\frac{\partial^2 u_2}{\partial x \partial y} + i\frac{\partial^2 v_2}{\partial x \partial y}\right) - \left(\frac{\partial^2 u_2}{\partial y^2} + i\frac{\partial^2 v_2}{\partial y^2}\right) + 2\left(\frac{\partial u_1}{\partial x} + i\frac{\partial v_1}{\partial x}\right) - 2i\left(\frac{\partial u_1}{\partial y} + i\frac{\partial v_1}{\partial y}\right)$$

$$-2\left(\frac{\partial u_2}{\partial x} + i\frac{\partial v_2}{\partial x}\right) - 2i\left(\frac{\partial u_2}{\partial y} + i\frac{\partial v_2}{\partial y}\right) = 4z \qquad (75)$$

$$\frac{\partial^2 u_1}{\partial x^2} + 2\frac{\partial^2 v_1}{\partial x \partial y} - \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2} = 12 \qquad (76)$$

$$\frac{\partial^2 v_1}{\partial x^2} - 2\frac{\partial^2 u_1}{\partial x \partial y} - \frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_2}{\partial x^2} + \frac{\partial^2 v_2}{\partial y^2} = 0 \qquad (77)$$

$$\frac{\partial^2 u_2}{\partial x^2} + 2\frac{\partial^2 v_2}{\partial x \partial y} - \frac{\partial^2 u_2}{\partial y^2} + 2\frac{\partial u_1}{\partial x} + 2\frac{\partial v_1}{\partial y} - 2\frac{\partial u_2}{\partial x} + 2\frac{\partial v_2}{\partial y} = 4x \qquad (78)$$

$$\frac{\partial^2 v_2}{\partial x^2} - 2\frac{\partial^2 u_2}{\partial x \partial y} - \frac{\partial^2 v_2}{\partial y^2} + 2\frac{\partial v_1}{\partial x} - 2\frac{\partial u_1}{\partial y} - 2\frac{\partial v_2}{\partial x} - 2\frac{\partial u_2}{\partial y} = 4y \qquad (79)$$

From (68)-(71) initial conditions we have that:

$$U_1(0,0) = 0,\ \ U_1(1,0) = 0,\ \ U_1(2,0) = 1,\ \ U_1(i,0) = 0(i > 2, i \in N), V_1(i,0) = 0(i\epsilon N)$$

$$U_1(i,1) = 0(i \in N),\quad V_1(0,1) = 0,\quad V_1(1,1) = 2,\quad V_1(i,1) = 0,(i > 1)$$

$$U_2(0,0) = 0,\quad U_2(1,0) = 0,\quad U_2(2,0) = 1\quad V_2(i,0) = 0 = U_2(i,1) = V_2(i,1) = 0(i\epsilon N) \qquad (80)$$

From diferential transform of (76)- (79) is get that:

$$(k+2)(k+1)U_1(k+2,h) + 2(k+1)(h+1)V_1(k+1,h+1) - (h+2)(h+1)U_1(k,h+2) + (k+2)(k+1)U_2(k+2,h) + (h+2)(h+1)U_2(k,h+2) = 12\delta(k,h) \qquad (81)$$

$$(k+2)(k+1)V_1(k+2,h) - 2(k+1)(h+1)U_1(k+1,h+1) - (h+2)(h+1)V_1(k,h+2) + (k+2)(k+1)V_2(k+2,h) + (h+2)(h+1)V_2(k,h+2) = 0 \qquad (82)$$

$$(k+2)(k+1)U_2(k+2,h) + 2(k+1)(h+1)V_2(k+1,h+1) - (h+2)(h+1)U_2(k,h+2) + 2(k+1)U_1(k+1,h) + 2(h+1)V_1(k,h+1) - 2(k+1)U_2(k+1,h) + 2(h+1)V_2(k,h+1) = 4\delta(k-1,h) \qquad (83)$$

$(k+2)(k+1)V_2(k+2,h) - 2(k+1)(h+1)V_2(k+1,h+1) - (h+2)(h+1)V_2(k,h+2) +$
$2(k+1)V_1(k+1,h) - 2(h+1)U_1(k,h+1) - 2(k+1)V_2(k+1,h) - 2(h+1)U_2(k,h+1) =$
$4\delta(k,h-1)$ (84)

When $h = 0$ is written in the equality (80),(81),(82), (83)

$U_1(0,2) = -1, U_2(0,2) = 1,\ U_2(i,2) = 0,\ \ U_1(i,2) = 0(i \geq 1),\ V_1(i,2) = \ V_2(i,2) = 0(i \geq 0)$ (85)

are obtained.

It is clear that all of the other components U_i and V_i are zero. Thus

$$u_1(x,y) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} U_1(k,h)x^k y^h$$
$$= x^2 - y^2$$
$$v_1(x,y) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} V_1(k,h)x^k y^h$$
$$= 2xy$$
$$u_2(x,y) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} U_2(k,h)x^k y^h$$
$$= x^2 + y^2$$
$$v_2(x,y) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} V_2(k,h)x^k y^h$$
$$= 0$$

are obtained.
Therefore

$$w_1(z) = u_1(x,y) + iv_1(x,y)$$
$$w_1(z) = x^2 - y^2 + i2xy = z^2$$

and

$$w_2(z) = u_2(x,y) + iv_2(x,y)$$
$$w_2(z) = x^2 + y^2 = (x+iy)(x-iy) = z.\bar{z}$$

References Références Referencias

[1] Zhou, J.K., Differential Transformation and Its Application for Electrical Circuits, Huazhang University Press, Wuhan, China, 1986.

[2] C.K Chen, S.H. Ho, Solving Partial Differential Equations By Two Dimensional Differential Transform, Appl. Math. Comput.106, 171-179, 1999.

[3] Ayaz, F., On The Two-Dimensional Differential Transform Method, Appl. Math. Comput. 143, 361-374, 2003

[4] Ayaz, F.,Solutions of The System of Differential Equations By Differential Transform Method, Appl.Math. Comput. 147(2004),547-567

[5] I.A. Salehbhai, A.K. Shukla The Numerical Solutions of Differential Transform Method And The Laplace Transform Method For A System of Differential Equations Nonlinear Analysis:Hybrid Systems (2010) 425-431

[6] J.Biazar, M.Eslami, M.R.Islam Differential Transform Method For Special Systems of Integral Equations , Journal of King Saud University - Science Volume 24, Issue 3, July 2012, Pages 211–214

Notes

[9] A. Arikhoglu, I. Ozkol "Solutions of Integral and Integro-differential equations system by usin differential transform method" Computers and Mathematics with Applications 56 (2008) 2411–2417

[10] A. Arikoglu, I. Ozkol, Solution of boundary value problems for integro -differential equations by using differential transform method, Appl. Math.Comput. 168 (2005) 1145–1158.

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