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An Wonderful Summation Formula of Half Argument Involving Contiguous Relation

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GJSFR-F Classification : *MSC 2010: 40A25*

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An Wonderful Summation Formula of Half Argument Involving Contiguous Relation

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2010 MSC NO : 33C05 , 33C20 , 33D15 , 33D50 , 33D60
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I. Introduction

Generalized Gaussian Hypergeometric function of one variable is defined by

$$ {}_AF_B\left[\begin{array}{ll} a_1, a_2, \cdots, a_A & ; \\ & \\ b_1, b_2, \cdots, b_B & ; \end{array} z\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!} \tag{1} $$

where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and A, B are non-negative integers and $\mid z \mid = 1$

Contiguous Relation is defined by

[Andrews p.363(9.16), E. D. p.51(10)]

$$ (a-b)\ {}_2F_1\left[\begin{array}{ll} a, & b\ ; \\ c & ; \end{array} z\right] = a\ {}_2F_1\left[\begin{array}{ll} a+1, & b\ ; \\ c & ; \end{array} z\right] - b\ {}_2F_1\left[\begin{array}{ll} a, & b+1\ ; \\ c & ; \end{array} z\right] \tag{2} $$

Gauss second summation theorem is defined by [Prudnikov., 491(7.3.7.8)]

$$ {}_2F_1\left[\begin{array}{ll} a, & b\ ; \\ \frac{a+b+1}{2} & ; \end{array} \frac{1}{2}\right] = \frac{\Gamma(\frac{a+b+1}{2})\ \Gamma(\frac{1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} \tag{3} $$

$$ = \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{a+b+1}{2})}{\Gamma(b)\ \Gamma(\frac{a+1}{2})} \tag{4} $$

In a monograph of Prudnikov et al., a summation theorem is given in the form [Prudnikov., p.491(7.3.7.8)]

$$ {}_2F_1\left[\begin{array}{ll} a, & b\ ; \\ \frac{a+b-1}{2} & ; \end{array} \frac{1}{2}\right] = \sqrt{\pi}\left[\frac{\Gamma(\frac{a+b+1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} + \frac{2\ \Gamma(\frac{a+b-1}{2})}{\Gamma(a)\ \Gamma(b)}\right] \tag{5} $$

Now using Legendre's duplication formula and Recurrence relation for Gamma function, the above theorem can be written in the form

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$$ {}_2F_1\left[\begin{array}{c} a,\ b \\ \frac{a+b-1}{2} \end{array};\ \frac{1}{2}\right] = \frac{2^{(b-1)}\ \Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} + \frac{2^{(a-b+1)}\ \Gamma(\frac{a}{2})\ \Gamma(\frac{a+1}{2})}{\{\Gamma(a)\}^2} + \frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\right] \tag{6} $$

Recurrence relation is defined by

$$ \Gamma(\zeta+1) = \zeta\ \Gamma(\zeta) \tag{7} $$

II. Main Summation Formula

$$ {}_2F_1\left[\begin{array}{c} a,\ b \\ \frac{a+b+52}{2} \end{array};\ \frac{1}{2}\right] = \frac{2^b\ \Gamma(\frac{a+b+52}{2})}{(a-b)\ \Gamma(b)\Big[\prod_{\tau=1}^{25}\{a-b-2\tau\}\Big]\Big[\prod_{\upsilon=1}^{25}\{a-b+2\upsilon\}\Big]} $$

$$ \Big[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a}{2})}\Big\{33554432(-52046984263666662269308108800000 + a^{25} $$

$$ +13640091550684680677132913868800000b - 1422743599044869675233377779712000b^2 $$

$$ +89713897629018090738557292380160 0b^3 - 34686930263033813436422789529600 0b^4 $$

$$ +10537653409428387338411621901926 4b^5 - 21454722011693905387175458897920b^6 $$

$$ +38742175046897617658373262540 80b^7 - 47803783310149822200209735680 0b^8 $$

$$ +56654588151086887435417681920b^9 - 45460707106105853664801587 20b^{10} $$

$$ +37318182993347219902169088 0b^{11} - 20194930448705937178009600b^{12} $$

$$ +11826415815423358523596 80b^{13} - 4379528767094444357632 0b^{14} + 1853638447567267092480b^{15} $$

$$ -468022761057968512 00b^{16} + 143011191927350400 0b^{17} - 24034443398763520b^{18} $$

$$ +5212824867340 80b^{19} - 5521231861600b^{20} + 81628506960b^{21} - 485121520b^{22} + 4481880b^{23} $$

$$ -11050b^{24}+51b^{25}+25a^{24}(-26+51b)+100a^{23}(2002-2652b+2499b^2)+460a^{22}(-84500+185198b $$

$$ -64974b^2+39151b^3)+2530a^{21}(2110264-3645616b+3657108b^2-581672b^3+251685b^4) $$

$$ +17710a^{20}(-31163600+72380152b-38575992b^2+24139388b^3-2181270b^4+721497b^5) $$

$$ +2300a^{19}(19390040384-37869421408b+39415220936b^2-10203690224b^3+4514501810b^4 $$

$$ -2626249 08b^5+69023213b^6)+8740a^{18}(-330455840000+796316947392b-518916043856b^2 $$

$$ +332176816104b^3-50876724500b^4+17020114230b^5-6902321 30b^6+1479068 85b^7) $$

$$ +2185a^{17}(69789853509376-146863394961408b+156544599068928b^2-51655821883008b^3 $$

$$ +23139727485168 4b^4-2344230332640b^5+617895802776b^6-184587792 48b^7+3283532847b^8) $$

$$ +37145a^{16}(-178814098777600+4420412961026 56b-326043316343808b^2+211636160789760b^3 $$

$$ -424411851138 24b^4+142595247852 96b^5-102687052036 8b^6+219152761464b^7-50218737 66b^8 $$

$$ +751134965b^9)+15640a^{15}(15383673087527680-34078827668715520b+369084626510584 32b^2 $$

$$ -141836941443133 44b^3+63920606761186 56b^4-864220759200960b^5+227240073284784b^6 $$

$$ -122341893360 96b^7+21543838456 14b^8-39059018180b^9+4957490769b^{10}) $$

$$ +109480a^{14}(-66546397551296000+167579244365076736b-134688825529456128b^2 $$

$$ +88134640433903872b^3-209884324230374 40b^4+70473859986988 80b^5-687149634511200b^6 $$

Notes

$$+145430620473840b^7 - 6076467256860b^8 + 8947703654506b^9 - 13152526530b^{10} + 1425623505b^{11})$$
$$+340a^{13}(5438758782734394050 56 - 125134062086781951795 2b + 136963463787553875404 8b^2$$
$$-585790021790811299840b^3 + 2643692051613697763 84b^4 - 430634565815737651 20b^5$$
$$+112501179228414036 48b^6 - 829489361267409024b^7 + 144068766719533656b^8$$
$$-48081651379580000b^9 + 597642037172180b^{10} - 7264977381480b^{11} + 675270333535b^{12})$$
$$+68a^{12}(-57891485665006958848000+1477930358928023932262 40b-126616282707392263152640b^2$$
$$+831664671947412790789 12b^3 - 223755416027547008783 36b^4 + 747986785942970616806 4b^5$$
$$-888687861460172067840b^6 + 185865876600461680896b^7 - 107349545276614848 48b^8$$
$$+155100894225191751 2b^9 - 42304119528430760b^{10} + 4467234591872052b^{11} - 456482745469 66b^{12}$$
$$+36464598010896b^{13}) + 136a^{11}(5161223524887023706112 00 - 122266503508490776381440 0b$$
$$+134712777867622688552448 0b^2 - 624384680472188982900736b^3 + 281174089484815331587328b^4$$
$$-523439350584128889630 72b^5 + 135407410491834602867 20b^6 - 122781186743318051020 8b^7$$
$$+209649585286086114624b^8 - 9747684004211325536b^9 + 118271683524494500 0b^{10}$$
$$-268600743748078 56b^{11} + 24193585509891 98b^{12} - 2106843440629 2b^{13} + 1447007857575b^{14})$$
$$+1496a^{10}(-69833545925787544576 0000 + 18009884755130787482882 80b$$
$$-16215539192339819404800000b^2 + 10655344757965731102791 68b^3 - 31445741625661089065 8304b^4$$
$$+10431835262334620734745 6b^5 - 14296859989592591756800b^6 + 29455999919714442314 24b^7$$
$$-21084904802364811699 2b^8 + 29796435316570523328b^9 - 11398541717565402 40b^{10}$$
$$+11678471781424808b^{11} - 2244635844964604b^{12} + 172785378722866b^{13} - 1297317389550b^{14}$$
$$+765084614 35b^{15}) + 5a^9(25811759982327997430874767 36 - 62580431402756893399053 76256b$$
$$+69192138054046350841801605 12b^2 - 34162339343013697568963 62496b^3$$
$$+15303827216377599779646627 84b^4 - 31550806256663017227400 8064b^5$$
$$+80574190195760192772212736b^6 - 84934097257340086913597 44b^7$$
$$+14213431085476704989107 20b^8 - 82452450586173649329 3312b^9 + 97337922400584350284 80b^{10}$$
$$-31188529925295834291 2b^{11} + 27127086010213084896b^{12} - 44635258599778246 4b^{13}$$
$$+29362226326633200b^{14} - 191991813933920b^{15} + 9691894453395b^{16})$$
$$+7a^8(-18750211068729592083906560000 + 48715214312117462420433 1417 60b$$
$$-45652368736003060515402547200b^2 + 29924735769246499429843 2061 44b^3$$
$$-95035553898432849637826641 92b^4 + 31195532801058196611951 9436 8b^5$$
$$-47722913716694790617831424 0b^6 + 96565086727541027651008 512b^7$$
$$-80895428813241227508111 36b^8 + 11145154306016774751319 04b^9 - 53509897670705946961920b^{10}$$
$$+53089779437641981731 84b^{11} - 14461858117196382547 2b^{12} + 10685144436609403488b^{13}$$
$$-152327779533540000b^{14} + 8546129120240400b^{15} - 490888160626 50b^{16} + 21101527266 75b^{17})$$
$$+4a^7(27161118659360298611748372 4800 - 67100953574301314561217658 8800b$$
$$+74253726250564691682415083 5200b^2 - 38596244973438649515343950 6432b^3$$

$$+1715181990034538166711724277776b^4-38366813415135487548897705984b^5$$
$$+964395233319582493938632704 0b^6-1142480222425883380596350976b^7$$
$$+18677435118594633896588236 8b^8-12755895614615627304742912b^9$$
$$+14599589837019929360819 20b^{10}-5902633895206991842099 2b^{11}+4937735270104755279616b^{12}$$
$$-1158089807435699523 84b^{13}+7263148214179472160b^{14}-905852285742768 00b^{15}$$
$$+4316543225775690b^{16}-219455883574 20b^{17}+7971688078 55b^{18})$$
$$+4a^6(-1801777082265492124949544960000+4705295225173373286331050885120b$$
$$-4557750243407517762479225569280b^2+2972359487446009853821201678336b^3$$
$$-1002601203494527952633354846208b^4+3246965552221263372059038187 52b^5$$
$$-5427368447185151966761644032 0b^6+1075086828199824083123288314 88b^7$$
$$-1018311284895403961234433024b^8+1362843212676021611744168 96b^9$$
$$-7745409165666979057771520b^{10}+7405168080044018997672 96b^{11}-2558081107893576703692 8b^{12}$$
$$+1805575454138254885632b^{13}-3686224405669574496 0b^{14}+1956546120608308320b^{15}$$
$$-215299124834921 40b^{16}+865664492098770b^{17}-3921208730530b^{18}+1190650424 25b^{19})$$
$$+2a^5(1875396822088160736778000 7411712-4705327364455760962871680303104 0b$$
$$+5198834277758696319847256555520 0b^2-2821200354155822000337911283712 0b^3$$
$$+1240191906486022646035852034048 0b^4-2968266311271297338692899962880b^5$$
$$+7320158566117644508819187302 40b^6-9534358840778930056534573056 0b^7$$
$$+1517115974614771015221660672 0b^8-1178042776563322567881052160b^9$$
$$+1301702129257838295056742 40b^{10}-6261749868483139430400000b^{11}$$
$$+5013338355508884941030 40b^{12}-1498308623885952313344 0b^{13}+8907510840747639321 60b^{14}$$
$$-1598237687560617600 0b^{15}+7139663829989670 00b^{16}-6984080838670320b^{17}+234669260950180b^{18}$$
$$-952520339400b^{19}+23813008485b^{20})+2a^4(-7431692780289410746516431175 6800$$
$$+1946876548594641199547368970 64960b-1939339484984464499941098651648 00b^2$$
$$+1254955534711063036747353402900 48b^3-4454701631684822519252423960166 4b^4$$
$$+1418803296150781433619175545241 6b^5-2553999485189716495501831045120b^6$$
$$+4934090928269108291146261463 04b^7-5167956506444961021557254963 2b^8$$
$$+6689358860110427498742499328b^9-4345484704254551052491878 40b^{10}$$
$$+3983023312602175636652748 8b^{11}-1645331321028515241698304b^{12}$$
$$+1102723246996943690460 16b^{13}-2882608118845349529600b^{14}+1437015461232844416 00b^{15}$$
$$-2285514984990274800b^{16}+851966183670042 00b^{17}-7458389138858 00b^{18}+206573008565 00b^{19}$$
$$-75504661050b^{20}+1521156175b^{21})+4a^3(1073107756402550250380470517760 00$$
$$-2727286486676356989209857228800 00b+3001394383508105935648095574425 60b^2$$
$$-1690217557903130324837306770391 04b^3+7326812661661042649971809268531 2b^4$$
$$-1858205431021375545273605737676 8b^5+4478731498377030241256147517440b^6$$

Notes

$$-6320180137369413006519679057 92b^7+9743478861044483692526901657 6b^8$$
$$-8412931376869168190174142464b^9+8925696740672573027800166 40b^{10}$$
$$-4933826918121795147939430 4b^{11}+3756115885754254624189952b^{12}$$
$$-1349302395401127295098888b^{13}+7544883035012338122240b^{14}-1742133105684636672 00b^{15}$$
$$+722709844085136096 0b^{16}-1026868297073908 80b^{17}+3154219002555320b^{18}-248620595300 00b^{19}$$
$$+556627384950b^{20}-1839537700b^{21}+28943775b^{22})+28a^2(-3010317038899539073491271 6800000$$
$$+7895938621438013627318371614720 0b-8058475700292895069849446776832 0b^2$$
$$+5158486694641724282461058865561 6b^3-1914670919130635432112583134412 8b^4$$
$$+5973414332894411456012491948032b^5-1146763745373159912248725995520b^6$$
$$+2150094051331171083674640711 68b^7-2454382889649849267865003622 4b^8$$
$$+3054398461825311081809575936b^9-2218904521786778382791270 40b^{10}$$
$$+1935956501806355097089433 6b^{11}-9241050125567948212295 68b^{12}+5830646475375966391347 2b^{13}$$
$$-1842268763764152652800b^{14}+8536366176106081024 0b^{15}-175698814220972928 0b^{16}$$
$$+599455227392784 00b^{17}-766522310449840b^{18}+190512146409 20b^{19}-135958559100b^{20}$$
$$+2391399010b^{21}-7167030b^{22}+83895b^{23})+35a(2837273074690669772132 6862336000$$
$$-7289417119652877147746001223 6800b+7967910059244261953705028354 0480b^2$$
$$-4637439284488715656570591405 6704b^3+1974152067591897650840512285 90 08b^4$$
$$-5272051559468802725848012554 240b^5+1235283313014379920045983662 0 80b^6$$
$$-1871032405931408529847463444 48b^7+2776165606090724523007541248 0b^8$$
$$-2628786139979958718631772160b^9+2656363293749731923090472 96b^{10}$$
$$-1652300848553745699618816 0b^{11}+1184461743479967024246784b^{12}$$
$$-4947914890954103037132 8b^{13}+2571557404807467722752b^{14}-7223727172308375961 6b^{15}$$
$$+2742555462044058880b^{16}-507668771369226 24b^{17}+13996848900820 48b^{18}-162246315580 16b^{19}$$
$$+3176062533 28b^{20}-20629660 48b^{21}+27285544b^{22}-74256b^{23}+595b^{24}))\Big\}$$
$$-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a+1}{2})}\Big\{33554432(-5204698426366666226930810880000 00+51a^{25}$$
$$+9930455761417344202464401817600 00b-8428887708918709405775560704000000b^2$$
$$+4292431025610201001521882071040 00b^3-1486338556057882149303286235136 00b^4$$
$$+3750793644176321473556001482342 4b^5-7207108329061968499798179840000b^6$$
$$+1086444746374411944469934899200b^7-1312514774811071445873459200000b^8$$
$$+1290587999116399871543738368 0b^9-1044709847049781666856960000b^{10}$$
$$+7019263993846352240312320 0b^{11}-3936621025220473201664000b^{12}$$
$$+1849177986129693977190 40b^{13}-7285499603915886080000b^{14}+2406006470889329152 00b^{15}$$
$$-6642049699093952000b^{16}+152490829917986560b^{17}-28881840416000000b^{18}+445970928832 00b^{19}$$
$$-5519073560000b^{20}+53389679 20b^{21}-38870000b^{22}+200200b^{23}-650b^{24}+b^{25}+425a^{24}(-26+49b)$$

$$+1020a^{23}(4394-2548b+2303b^2)+7820a^{22}(-62036+122122b-25662b^2+14805b^3)$$
$$+301070a^{21}(271128-239824b+222404b^2-24440b^3+10105b^4)+27370a^{20}(-201725680+406146104b$$
$$-139088040b^2+81348540b^3-5517330b^4+1740081b^5)+7820a^{19}(66660164544-726166374408b$$
$$+682140677768b^2-12717166000b^3+5283197150b^4-243611340b^5+60902835b^6)$$
$$+148580a^{18}(-161760959744+3297144376960b-144451640144b^2+84916381816b^3-10039560020b^4$$
$$+3158827042b^5-105564914b^6+2146099b^7)+6555a^{17}(218171154732800-2710664683130880b$$
$$+25606020392064b^2-626616809808640b^3+25994391568880b^4-2130917113248b^5+52824682965 6b^6$$
$$-1339166337 6b^7+225340489 5b^8)+1955a^{16}(-23939783174320640+490994583997657 60b$$
$$-2516402454315724 8b^2+1478690218077004 8b^3-233812274679312 0b^4+730400391814800b^5$$
$$-44050971833232b^6+8831801996472b^7-175765581810b^8+24787453845b^9)$$
$$+109480a^{15}(16931297475038976-230937569447198 72b+2183213855781606 4b^2$$
$$-6365119129282560b^3+2625165256179840b^4-291968887022400b^5+7148506103793 6b^6$$
$$-330965394864 0b^7+54642769311 0b^8-8768351020b^9+1045457237b^{10})$$
$$+4760a^{14}(-9200690687173202432+18908510329466674432b-10836875080965603840b^2$$
$$+6340237844548183296b^3-12111798818677939 20b^4+3742651613759512 32b^5-309766756778955 84b^6$$
$$+610348589426846 4b^7-22401144049050 0b^8+30842674712850b^9-40772832243 0b^{10}$$
$$+4134308164 5b^{11})+68a^{13}(17391787963857880181760-25467208997557883279360b$$
$$+24008544310371626317312b^2-79370729141242782064 64b^3+32433036676380696778 24b^4$$
$$-44067900702528009216 0b^5+10621032083166205209 6b^6-68122929849158795 52b^7$$
$$+10999413390627327 12b^8-32820043088072240b^9+3801278331903052b^{10}-421368688125 84b^{11}$$
$$+364645980108 9b^{12})+68a^{12}(-29698427130449907614720 0+60964942679115949777408 0b$$
$$-38051382869985669109452 8b^2+22094799327966203671705 6b^3-48392097677309271814656b^4$$
$$+14745112810320249826560b^5-15047535928785745315 84b^6+29045501588851501644 8b^7$$
$$-14887206885349217328b^8+19946386772215503 60b^9-493819885892212 88b^{10}$$
$$+48387171019783 96b^{11}-45648274546966b^{12}+3376351667675b^{13})$$
$$+136a^{11}(27439840436284720516300 80-42522448308368455504896 00b$$
$$+39857927978366134351841 28b^2-14511255641534691611586 56b^3+58573872244149641715 4816b^4$$
$$-92084556889457932800000 0b^5+21779906117776526463744b^6-17360687927079387770 88b^7$$
$$+27325621769374549420 8b^8-11466371296064644960b^9+12856631895956728 88b^{10}$$
$$-2686007437480785 6b^{11}+223361729593602 6b^{12}-181624434537 00b^{13}+11476269215 25b^{14})$$
$$+680a^{10}(-66853981038390961271767 04+13672458129594208427671552b$$
$$-91366656779455580467875 84b^2+52504098474544547222353 92b^3-12780837365454561919 09376b^4$$
$$+38285356742877596913433 6b^5-45561230386276347398656b^6+85879940217764290357 76b^7$$
$$-55083718190432592460 8b^8+71572001765135551680b^9-25076791778643885 28b^{10}$$
$$+23654336704898900 0b^{11}-4230411952843076b^{12}+29882101858609 0b^{13}-2117556771330b^{14}$$

Notes

$$+1140222876876^{15}) + 119a^9(476088976059553675927879680 - 773172394111752564303462400b$$
$$+7186819910177202545434296326^2 - 28278760930652666185459302 4b^3$$
$$+1124261993295870167855882246^4 - 19799038261568446519009280b^5$$
$$+4580985588826963400820736b^6 - 428769600491281590075392b^7 + 655597312118633808901126^8$$
$$-3464388680091329802240b^9 + 3745837582654580075526^{10} - 1114021029052722918 4b^{11}$$
$$+8862908241439528646^{12} - 137376146798800006^{13} + 8231887362140006^{14} - 513347096080006^{15}$$
$$+234461414075b^{16})+119a^8(-401712464791174976472350720 0+8165192959090366244139827200b$$
$$-5775018563881998277329420288b^2 + 3275118944888902081521647616b^3$$
$$-8685641187302455498415554566^4 + 2549774747251715991969177606^5$$
$$-3422895075278668777258598 4b^6+6278129451628448368601088b^7-4758554636073013382830086^8$$
$$+5972029867847355037440 0b^9 - 2650673746583004899328b^{10} + 2395995260412412738566^{11}$$
$$-6134259730092277056b^{12}+4116250477700961606^{13}-559034987631120 0b^{14}+283147591137840b^{15}$$
$$-15675420255306^{16} + 602900779056^{17}) + 68a^7(5697378683367296714466656256 0$$
$$-9630313854058720374214885376 0b + 8853328446657763285719108812 8b^2$$
$$-3717753021982007650893928857 6b^3 + 1451203214196796556219488665 6b^4$$
$$-2804223188464391193098403840b^5 + 6324040165881318136019312 64b^6$$
$$-6720471896622843415272652 8b^7 + 9940523633717458728780288b^8$$
$$-6245154210098535802470 40b^9 + 6480319982337177309132 8b^{10} - 2455623734866361020416^{11}$$
$$+1858658766004616808 96b^{12} - 4147446806337045120^{13} + 2341432989628824 00b^{14}$$
$$-28138635473020806^{15} + 1197121959497106^{16} - 59312400966 0b^{17} + 190103849256^{18})$$
$$+340a^6(-631021235638056040799278202 88 + 1271615175161861682400277299 20b$$
$$-9443936726602493394989508198 4b^2 + 5269095880443564989713114726 4b^3$$
$$-1502352638346892056177547673 6b^4 + 4305975627128026181658345472b^5$$
$$-6385139349629590549131345 92b^6 + 1134582627434802934045450 24b^7$$
$$-9825305765201868656612352b^8+1184914561702355776061952b^9-6290618395420740372992 0b^{10}$$
$$+5416296419673384114688b^{11} - 1777375722920344135 68b^{12} + 1125011792284140364 8b^{13}$$
$$-2212621823126064 00b^{14} + 104530433711000 64b^{15} - 112185604350204b^{16} + 3970889203134b^{17}$$
$$-17743025930b^{18} + 466921735b^{19}) + 34a^5(3099309826302466864238712324096$$
$$-5427111899453179276608248217600b + 4919282391795397669657346310144b^2$$
$$-2186124036495735935616006750208b^3 + 8345901742063420197759856148 48b^4$$
$$-1746039006630174905113470566 40b^5 + 3819959473201486320069456691 2b^6$$
$$-4513742754721822064576200704b^7 + 6422609694335511067166576 64b^8$$
$$-4639824449509267239323648 0b^9 + 4590007515427233123288064b^{10}$$
$$-2093757402336515558522 88b^{11} + 1495973571885941233612 8b^{12} - 4306345658157376512 00b^{13}$$
$$+2269258291581039360 0b^{14} - 39754154923244160 0b^{15} + 155785308279358 80b^{16}$$

$$-1506512728476006^{17}+43751705403006^{18}-17765802600b^{19}+3758150556^{20})$$
$$+34a^4(-10202038312657003951889055744000+2032215363697541699394645000019206$$
$$-15767878157546409440927155224576b^2+861977960195416782349624619827 2b^3$$
$$-26204127245204838348543670353926^4+729524650874130968256383549440b^5$$
$$-11795308276406211207451233484 86^6+2017861164746515490249087385 6b^7$$
$$-19566143449677351396023132166^8+22505628259378823205362688 0b^9$$
$$-13836126315290879188965376b^{10}+11246963579392613263493126^{11}-44751083205509401756672b^{12}$$
$$+26436920516136977638406^{13}-675827524021805568006^{14}+2940347911014581760b^{15}$$
$$-4636699473685272 0b^{16}+14870677810321 20b^{17}-130783109450006^{18}+3053927695006^{19}$$
$$-1136185050b^{20}+18728325b^{21})+476a^3(1884745748508783418877254041600$$
$$-34098818268299379827724936806406+3034403938024543695565328744448b^2$$
$$-14203508889942271637288292188 16b^3+5272922414752365700619131944966^4$$
$$-11853783000654714287134081024 0b^5+2497781081887403238505211494 4b^6$$
$$-32433819305410629844826849286^7+4400696436653896974976942086^8$$
$$-35884810234258085681684480b^9+3348822638217801203734528b^{10}$$
$$-17839562299205399511449 6b^{11}+11880923884963039868416b^{12}-41842144413629378560 0b^{13}$$
$$+20270967299797890560b^{14}-4660356647417241 60b^{15}+165151789759152006^{16}$$
$$-2371175857444806^{17}+6099212967960b^{18}-49303545200b^{19}+898127230b^{20}-3091660b^{21}+37835b^{22})$$
$$+2380a^2(-59779142817011330892158730240 0+1171751479300626757897798287360b$$
$$-94805596474034059645287609139 2b^2+5044360308416984765795118612486^3$$
$$-16296970462054323528916795392 0b^4+4368768300637559932644753408 0b^5$$
$$-76600844427017105251751690246^6+1247961785723776330796892160b^7$$
$$-13427167275295017798647808 0b^8+14536163456732426647437312b^9$$
$$-10192624635185029340160006^{10}+7697873021007010774425 6b^{11}-3617608077354064661504b^{12}$$
$$+19566209112507696486 4b^{13}-619568597435498188 8b^{14}+2425413259266969 6b^{15}$$
$$-5088604615794432b^{16}+1437184659519 36b^{17}-19055992534886^{18}+380903395606^{19}-287050764b^{20}$$
$$+3887598b^{21}-12558b^{22}+105b^{23})+5a(27280183101369361354265827737 6000$$
$$-5102591983757014003422200856576006+4421725628005287631298288104243 20b^2$$
$$-2181829189341085591367885783040006^3+7787506194378564798189475882598 4b^4$$
$$-1882130945782304385148672121 24166^5+3764236180138698629064840708096b^6$$
$$-5368076285944105164897412710406^7+6820130003696444738806639846 4b^8$$
$$-6258043140275689339905376256b^9+5388557518735131614862376966^{10}$$
$$-3325648895430949117575168 0b^{11}+2009985288142112547876864b^{12}$$
$$-8509116221901172722073 6b^{13}+3669315134617720211456b^{14}-1065985729477421465606^{15}$$
$$+3283924788746631424b^{16}-641793035981352 96b^{17}+13919620240412 16b^{18}-1741993384768 0b^{19}$$
$$+25637049838 4b^{20}-1844681696b^{21}+17038216b^{22}-53040b^{23}+255b^{24}))\Big\}\Big] \qquad (8)$$

Notes

III. Derivation of the Summation Formula

Substituting $c = \frac{a+b+52}{2}$ and $z = \frac{1}{2}$ in equation (2), we get

$$(a-b)\ {}_2F_1\left[\begin{matrix} a,\ b \\ \frac{a+b+52}{2} \end{matrix};\ \frac{1}{2}\right] = a\ {}_2F_1\left[\begin{matrix} a+1,\ b \\ \frac{a+b+52}{2} \end{matrix};\ \frac{1}{2}\right] - b\ {}_2F_1\left[\begin{matrix} a,\ b+1 \\ \frac{a+b+52}{2} \end{matrix};\ \frac{1}{2}\right]$$

Now involving the derived formula [Salahuddin et. al.], the summation formula is obtained.

Notes

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