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Physics and Space Science

Highlights

High Cosmological Constant

Semiclosed Friedman Universe

Solution of Orthogonal Plane

Conception of Elementary Particles

Discovering Thoughts, Inventing Future

VOLUME 14 ISSUE 7 VERSION 1.0

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Contents of the Issue

Global Journal of Science Frontier Research: A
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Holographic Origin of High Cosmological Constant Related to Large Mass Defect in Semiclosed Friedman Universe

By Noboru Hokkyo
Senjikan Institute, Japan

Abstract- An extremely high value of Einstein's cosmological constant $\Lambda \sim 10^{122}$ in quantum cosmology compared to astronomical observations, $0 \leq \Lambda \leq 1$, is related to the extremely large general relativistic mass defect of massive semiclosed universe.

Keywords: cosmology; general relativity, quantum theory; holography; black hole; semiclosd universe.

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Keywords: cosmology; general relativity, quantum theory; holography; black hole; semiclosd universe.

I. Introduction

Inflationary cosmology succeeded in explaining the origin of the large-scale structure of observed universe evolving from a singular Big Bang or large quantum fluctuations of pre-existing spacetime metric in causally related small region, answering why the present universe appeares flat, homogeneous and isotropic.Yet, apart from problems of the fine tuning of initial conditions and the unitarity of the expansion history,[1,2] there have been continued interpretational controversies regarding the extremely high quantum vacuum energy density to support inflation. We here show that about 120 order of magnitude supression of calculated quantum vacuum density to observed values can be understood as a general relativistic mass defect of an expanding and supermassive semiclosed universe.

II. Dimensional Cosmological Constant

Hubble parameter H was originally used to represent Hubble law, $v = Hd$, relating the relative velocity v of the extragalactic objects at a distance d receding away from the Earth. In the Friedman universe H is defined as $H(t)= (da(t)/dt)/a(t)$, where $a(t)$ is a scale factor normalized to a = 1 at the present epoch t = t_0 ~ 14Gyr, and obeys the equation[2]

$$H(t)^2 = 8\pi G/3[\rho_{m0}/a(t)^3 + \rho_{r0}/a(t)^4 + \rho_{\Lambda 0}/a(t)^{3(\omega+1)}]. \quad (1)$$

Here ρ_{m0} is the nonrelativistic matter (dark and luminous) densitiy, ρ_{r0} the relativistic radiation density, and $\rho_{\Lambda 0}$ the dark energy density at $t = t_0$ where $\omega = p_\Lambda/\rho_\Lambda c^2 < -1$ is the equation of state relating ρ_Λ and pressure ρ_Λ in inflationary cosmology. The most direct evidence of dark energy comes from obsevations of supernpovae with uniform energy density.

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In the Friedman-Lemaître cosmology the expansion history is determined by a set of dimensionless parameters at present epoch whose sum is normalized to uinity. As the observed energy density in the cosmic background radiation shows a minor contribution $\Omega_r \sim 1 \times 10^{-4}$ to H_0^2 the following numerical relations between dimensionless density parameters $\Omega_{\Lambda,m}$ and the critical densities $\rho_{c\Lambda,m}$ are considered significant:

$$\Omega_\Lambda = 8\pi\rho_\Lambda G/3H_0^2 = \rho_{\Lambda 0}/\rho_{c\Lambda 0} = 0.72, \quad (2)$$

$$\Omega_m = 8\pi\rho_m G/3H_0^2 = \rho_{m0}/\rho_{cm0} = 0.28 \quad (3)$$

Only from dimensional cosideration, we put $\Lambda = c^5/\hbar G = c^2/l_{pl}^2 \sim 10^{88}\mathrm{sec}^{-2}$, where $l_{pl} = c^2/\hbar G = 10^{-33}$ cm is the Planck length, $m_{pl} = \hbar/cl_{pl} \sim 10^{-5}$ g the planck mass. Using the Planck constant $\hbar = 1.026 \times 10^{-27}$ erg・sec and the cosmological unit h defined in H_0 = 100 hkm/sec Mpc, we get $\Lambda \sim 10^{122} h^{-2}$.

III. Holographic Cosmological Constant

Holographic principle in string theory states that the description of events in a volume of spacetime can be encoded on the boundary to the region like a gravitational horizon. The principle suggests that the entire universe can be seen as a two-dimensional information structure on the cosmological horizon with possible quantum fluctuations. From the observed cosmic background microwave temperature T = 3 K the entropy density s of the universe at $t = t_0$ is estimted by $s \sim gT^3$. Using g = 2 for photon we have $s_\gamma(t_0) \sim 1.5$ A volume estimate $V = (4\pi/3)R^3$ with R = 10^{28}cm gives a total radiation entropy $S_\gamma \sim 6.3 \times 10^{87}$. The entropy contribution frombaryons is smaller than S_γ. Inclusion of neutrino contribution increases S_γ to $S_{\gamma+\nu} \sim 10^{88}$. This is well below the holographic bound of the present universe dictated by the area in terms of the Planck units l_{pl} giving

$$S_{holog}(t_0) \sim (R/l_{pl})^2 \sim 10^{122}. \quad (4)$$

It is suggested that 34 orders of magnitde difference may come from supermassive black holes.[3] In the following we attribute the difference to the mass defect of the semiclosed Friedman universe.

IV. Semiclosed Universe and Mass Defect

The possibility of joining-on of the dust-filled semi-closed Friedman universe to an asymptotically flat space through Schwarzschid throat was first pointed out by Tolman[4] and used by Oppenheimer and Snyder[5] in their study of the dust motion in massive steller objects, followed by Milne,[6] Zel'dovich[7] and Novikov[8] independently in the form extendible to the electrically charged universe joined onto asymptotically flat outer space through double-valued Reissner-Nordström (RN) bottle-neck prevented from gravitatinal pinch-off by the gauge field lines of force extending to infinity or to an oppostely charged anti-universe.[9]

We here consider the embedding of the quantum uncertainty between position r and momentum $\hbar/r = Gm_{pl}^2/cr = r_{pl}/r$ in the radial line element ds of de Sitter universe:

$$ds^2 = c^2 g_{tt} dt^2 - g_{rr} dr^2,$$

$$g_{rr} = g_{tt}^{-1} = (1 - \Lambda r^2/c^2 + l_{pl}^2/r^2)^{-1} dr^2. \qquad (5)$$

We find that the light velocity $dr/dt = c(g_{tt}/g_{rr})^{1/2} = c(1 - \Lambda^2 r/c^2 + r^2/r_{pl}^2)$ exceeds c at $r \sim l_{pl}$. But,with the increase of r, $dr/dt = c$ is reached at $r \sim (cl_{pl})^{1/2}\Lambda^{-1/4} = 10^{-23}\Lambda^{-1/4} \sim 10^{-23}$cm if $\Lambda \sim 1$. In the inflationary model the radius of the causally related small region in electroweak and grand unification period extends from $r = l_{pl} \sim 10^{-33}$cm to $r \sim 10^{-25}$cm, followed by a brief interlude of re-heating, returning to the pre-inflationary period of the universe. After the inflationary period, further evolution of the universe is described by the standard Friedman model starting the radiation dominated phase of the Hubble's expansion history.

The semiclosed Friedman universe is a spherical but unisotropic universe joined onto an asymptotically flat space, and expands with increasing proper time $\tau = \int g_{tt} d\tau$ and radius $R = \int g_{rr} dr$ until the maximum radius R_{max} is reached with the proper volume $V_p = 2\pi\int r^2 g_{rr} dr$ and mass $M_p = \rho V_p$ filling the lower hemisphere of the closed universe with $\Omega_\Lambda = 0.5$. With further increase of r from $r = R_{max}$, R_p begins to decrease towards $R_p \sim l_P$, forming a Planck scale gravitational semiclosure with $\Omega_\Lambda = 1$. In quantum cosmlogy it is likely that the gavitational semiclosure develop black holes at $r = R_p = 0$ evaporating a dark energy[1] until the semiclosed universe reaches $\Omega_\Lambda = 0.5$, liberating the half of the total mass energy $M_p c^2$ with holographic information content $(R_{max}/l_{pl})^2$.

V. Conclusion

We have seen that the extremely high value of the cosmological constant $\Lambda \sim 10^{122}$ in quantum cosmology compared to astronomical observations, $0 \leq \Lambda \leq 1$, can be related to the extremely large general reltivistic mass defect of the semiclosed universe with $0.5 \leq \Omega_\Lambda \leq 1$.

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Elemental Profiling of Rice and Husk by Laser-Induced Breakdown Spectroscopy (Libs)

By Firoza Kabir & Jannatul Ferdous Ema
University of Dhaka, Bangladesh

Abstract- Laser induced breakdown spectroscopy (LIBS) was used to determine the elemental content of Rice and Husk which were collected from Bangladesh Rice Research Institute (BRRI) in Gazipur, Bangladesh (latitude: 24.0958^0 N, longitude: 90.4125^0 E). Samples were collected from one experimental plot. Several elements such as Ni, Cu, Zn, Fe, Ca, Al, Si, Na, K, Ti, Mn, Li, Mo, Co, Mg, C were identified in Rice and husk samples by Laser Induced Breakdown Spectroscopy.

Keywords: *LIBS, rice, husk.*

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Abstract: **Laser induced breakdown spectroscopy (LIBS) was used to determine the elemental content of Rice and Husk which were collected from Bangladesh Rice Research Institute (BRRI) in Gazipur, Bangladesh (latitude: 24.0958^0 N, longitude: 90.4125^0 E). Samples were collected from one experimental plot. Several elements such as Ni, Cu, Zn, Fe, Ca, Al, Si, Na, K, Ti, Mn, Li, Mo, Co, Mg, C were identified in Rice and husk samples by Laser Induced Breakdown Spectroscopy.**

Keywords: LIBS, rice, husk.

I. Introduction

LIBS have been established as a fast spectroscopic technique for multi-elemental detection of elements in many diverse situations (Cremers et al., 2006, Miziolek et al., 2006, Singh and Thakur, 2007). In this technique a high power laser pulse is used for the generation of weakly ionized plasma from the samples. The spectral emission lines from the excited atoms/ions of the plasma are then spectroscopically analyzed for both qualitative and quantitative detection of multiple elements in the sample. The technique is extremely versatile and powerful, and is capable of analyzing solid, liquid and gaseous samples. For example, LIBS has been used for the detection of trace elements of soils (Martin et al., 2003 and Acra et al., 1996), geological analysis (Harmon et al., 2006), aerosol analysis (Samuel et al. 2003 and Radziemski et al. 1983), analyses of industrial waste water (Rai and Rai, 2008 and Gondal et al.,2007) etc, to name a few.

In this paper, we used LIBS technique for the analysis of rice and husk collected from the agricultural land in Gazipur in Bangladesh (latitude: 24.0958^0 N, longitude: 90.4125^0 E) with a view to detect important elements and to determine their enormous uses in future.

II. Experimental Details

In the present LIBS experiment intense, transient plasma was produced by focusing the fundamental beam at 1064 nm from a Q-switched Nd: YAG laser (Spectra-Physics LAB-170-10) on the sample by a convex lens of 100 mm focal length. The laser pulse had a temporal width of 8 nsec and repetition rate of 10 Hz. The beam has a Gaussian profile in the far-field and has a beam divergence of less than 0.5 m rad. The experiments were performed in air. The laser pulse energy used was 40 mJ. The spot size at the sample position was about 200 μm. The light emitted from the plasma was focused by a fused quartz lens (f=50 mm) and collected by a 3 m long multimode silica optical fiber and was then transmitted through the fiber to the entrance slit of a 750 mm focal length computerized Czerny-Turner spectrograph (Acton Model SP-2758). The spectrograph was equipped with three ruled gratings: 2400, 600 and 300 grooves/mm blazed at 240, 500 and 300 nm, respectively, which were interchangeable under computer control. The schematic diagram of the experimental set up and more details are available elsewhere (Haider et al., 2011).

The spectrum was detected by an intensified and gated CCD camera (Unigen II coated Princeton PI-MAX camera with 1024X1024 pixels). The ICCD camera was electrically triggered by the Nd: YAG laser pulse after a software-controlled, adjustable time delay. In this way, the intense background initially created by the high-temperature plasma was largely eliminated, and the atomic/ionic emission lines of the elements were more clearly observed. In the present experiments, a delay time t_d of 1.5 μs and a gate width t_w of 50 μs were selected for the optimum signal. Usually, spectra from a number of laser shots (about 40–80) were acquired and averaged to increase the signal-to-noise ratio. Samples were manually moved between exposures to prevent crater formation and to avoid other deleterious effects. The spectrum, captured by the ICCD camera, was transferred to the personal computer by USB cable. All the functions of the ICCD camera and the Acton spectrograph were fully controlled by the WinSpec/32 software provided by the manufacturer (Haider et al., 2012).

III. Sample Processing

Rice and Husk samples from one experimental plots of Bangladesh Rice Research Institute (BRRI), Gazipur, were collected. Varieties of fertilizers were used in different plots. The fertilizers that were applied to the fields were NPK (Nitrogen, Phosphorous Potassium), NPKSZn (Nitrogen, Phosphorous, Potassium, Sulphur, zinc) and NPKSZnCu (Nitrogen, Phosphorous, Potassium, Sulphur, Zinc, and Copper).

The collected samples were powdered by hand mortar and pestle. Then the powder of each sample was

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passed separately through a 75 micron sieving machine which makes the sample most homogeneous to carry out LIBS experiment. Small pellets were made by using a hand press with sufficient pressure (80 bars).

IV. Results and Discussions

For every sample, spectra were acquired in the UV to IR region (190 to 900nm) using two gratings. A 600 grooves/mm grating blazed at 500nm was used to take spectra in the range of 360-880nm. Another grating of 2400g/mm blazed at 240nm was used for 190-360nm spectral range.

Some representative LIBS spectra for the rice and husk samples are shown in the figures (1- 20) where different emission lines are labeled with the charge state of the elements. Here, figure (9-11) shows the presence of Silicon in Husk samples.

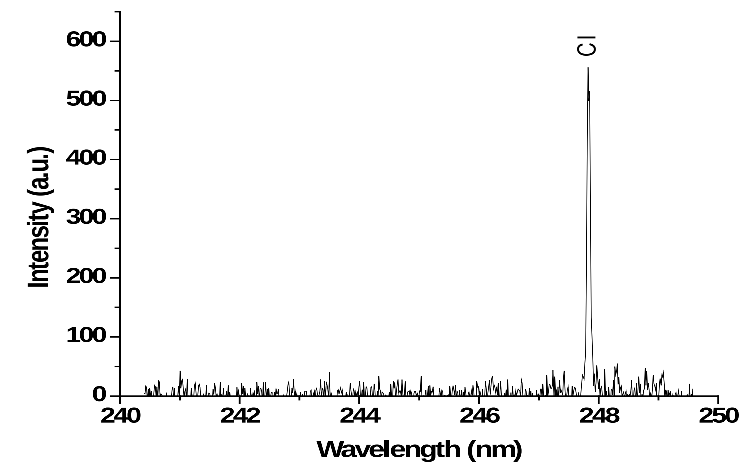

Figure 1 : LIBS spectrum of rice sample in the spectral range of 240 nm to 250 nm

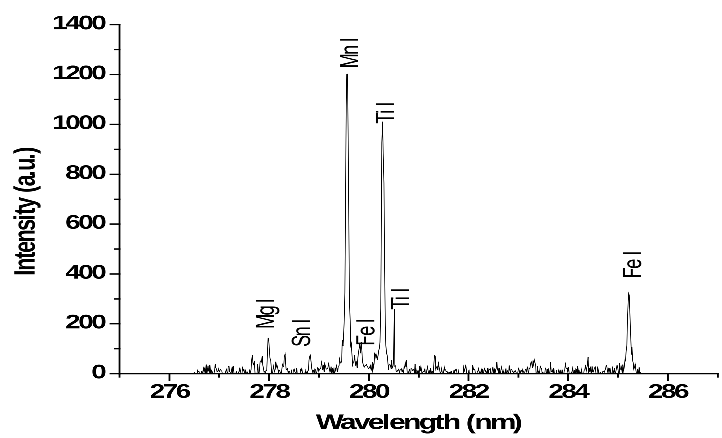

Figure 2 : LIBS spectrum of rice sample in the wavelength limit 276 nm-286 nm

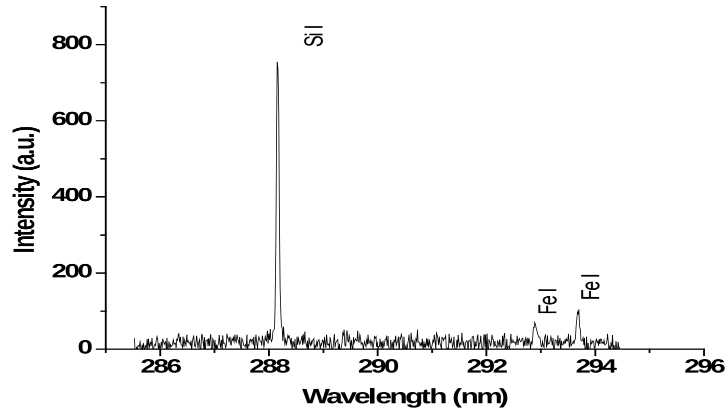

Figure 3 : LIBS spectrum of rice sample in the wavelength limit 286-296 nm

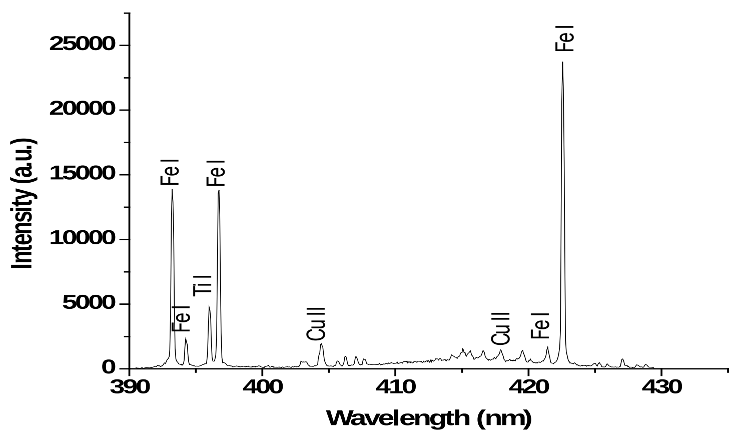

Figure 4 : LIBS spectrum of rice sample (NPK) in the wavelength limit 390-435 nm

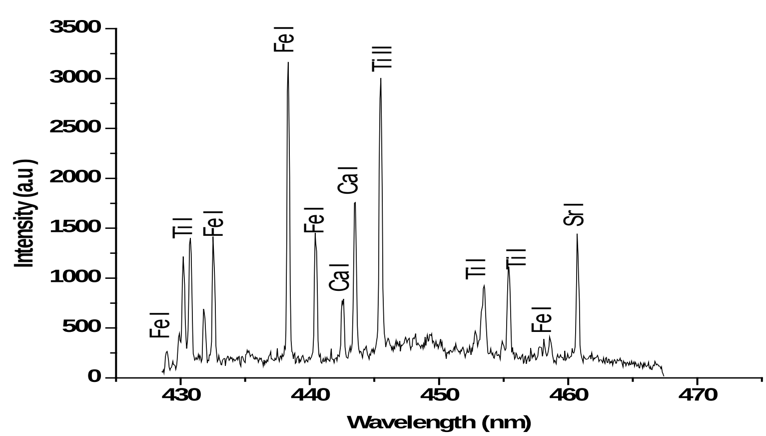

Figure 5 : LIBS spectrum of rice sample in the wavelength limit 430-470 nm

Global Journal of Science Frontier Research (A) Volume XIV Issue VII Version I Year 2014

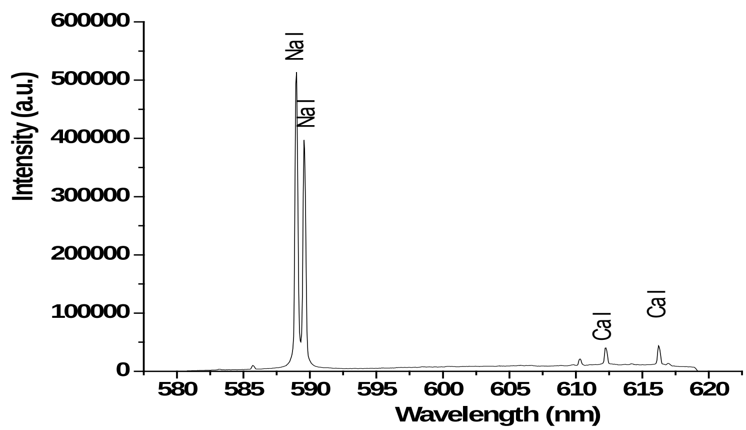

Figure 6 : LIBS spectrum of rice sample in the wavelength limit 580-620 nm

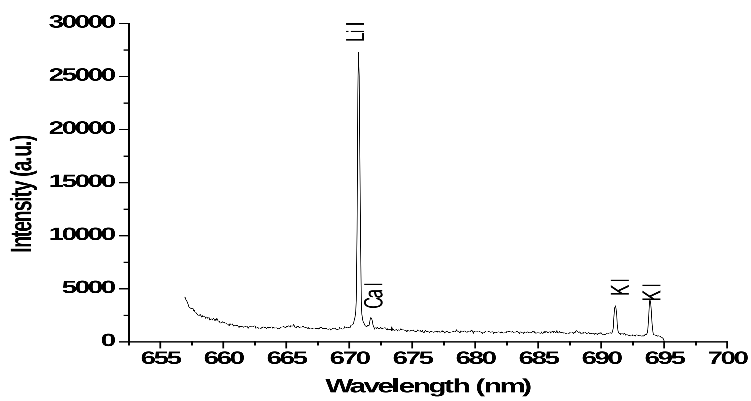

Figure 7 : LIBS spectrum of rice sample in the wavelength limit 655-700 nm

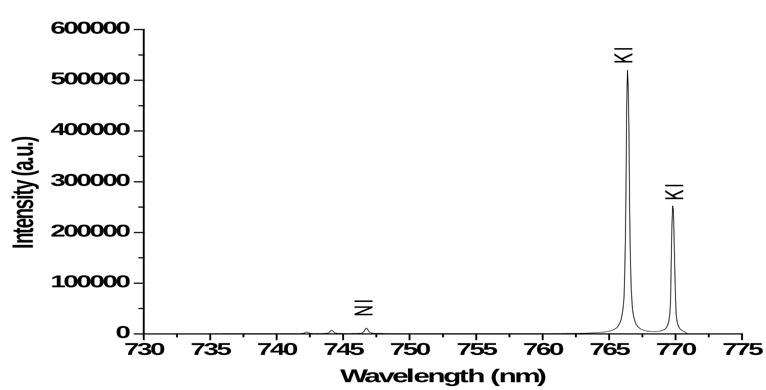

Figure 8 : LIBS spectrum of rice sample in the wavelength limit 730-775 nm

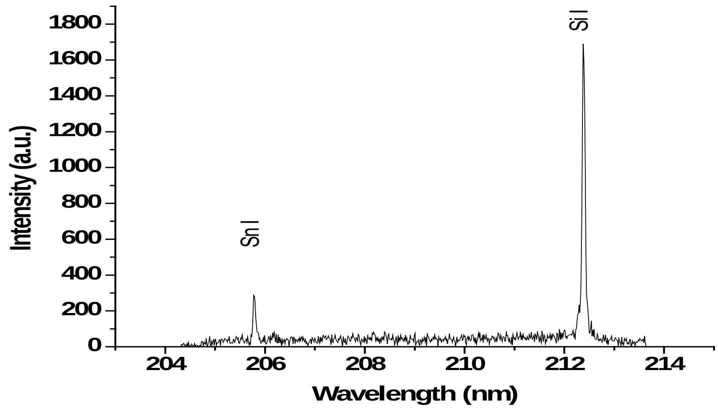

Figure 9 : LIBS spectrum of husk sample in the wavelength limit 203-215 nm

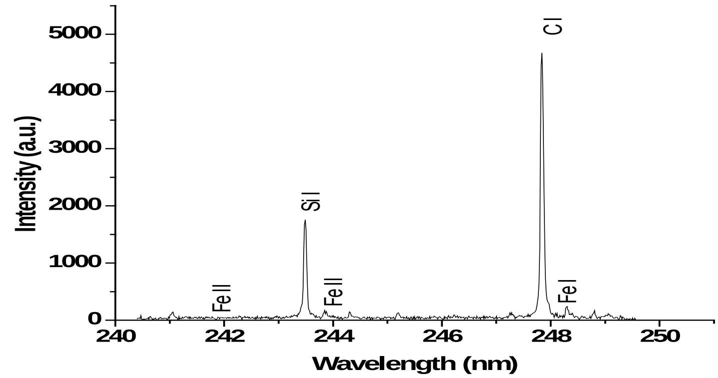

Figure 10 : LIBS spectrum of husk sample in the wavelength limit 240-250 nm

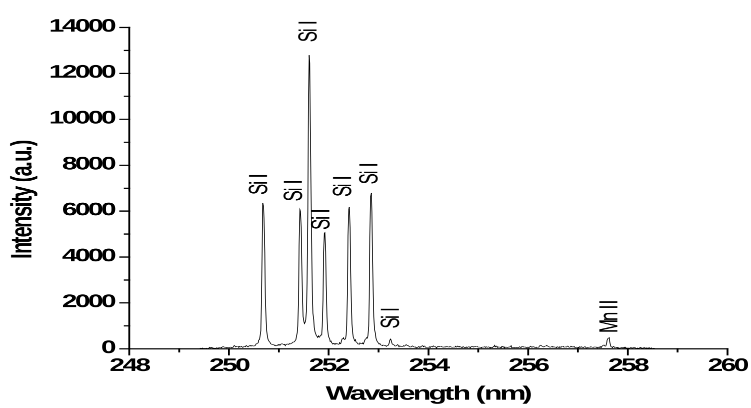

Figure 11 : LIBS spectrum of husk sample in the wavelength limit 248-260 nm

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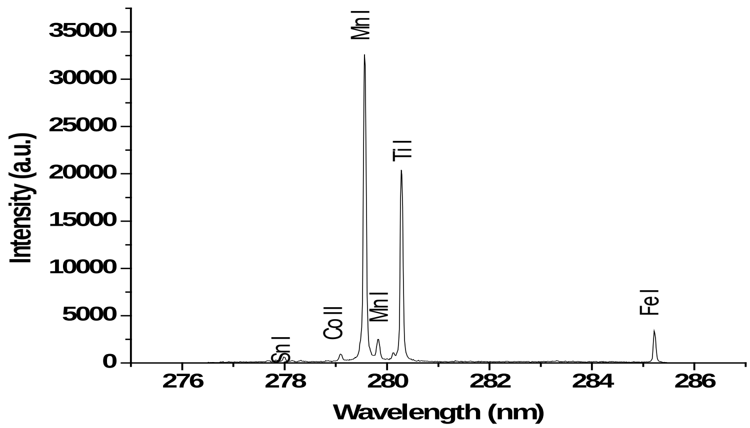

Figure 12 : LIBS spectrum of husk sample in the wavelength limit 276-286 nm

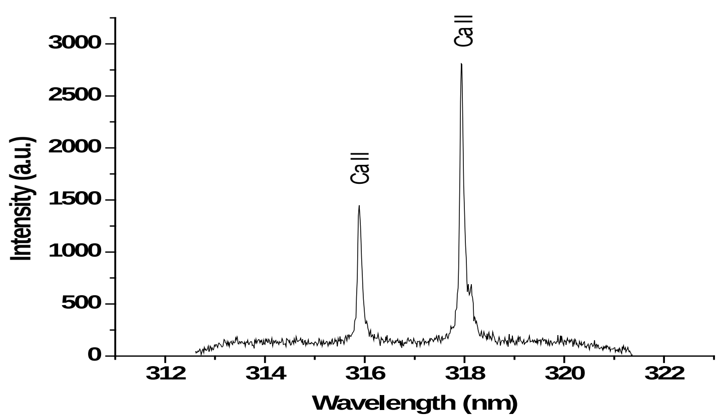

Figure 13 : LIBS spectrum of husk sample in the wavelength limit 311-322 nm

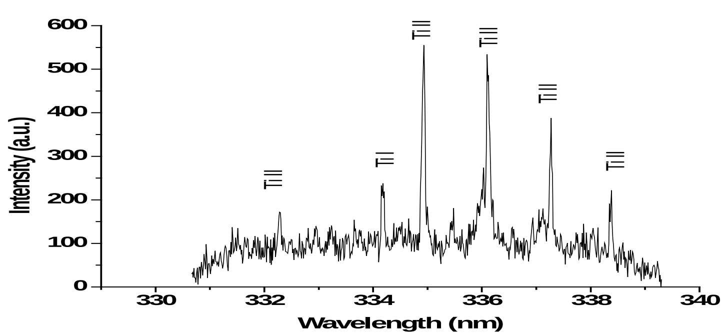

Figure 14 : LIBS spectrum of husk sample (NPK) in the wavelength limit 329-340 nm

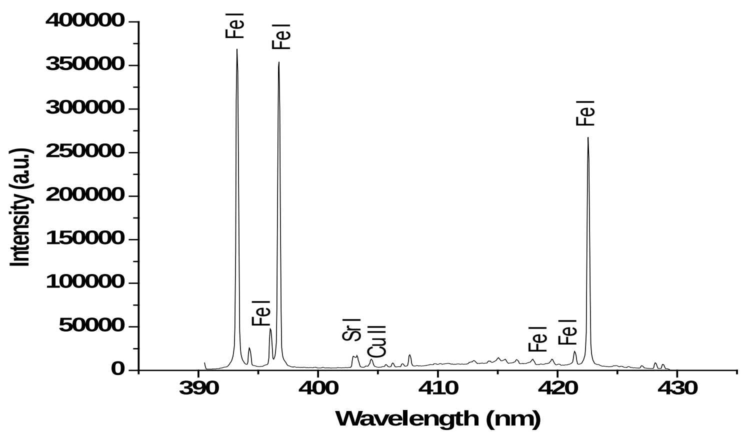

Figure 15 : LIBS spectrum of husk sample in the wavelength limit 385-435 nm

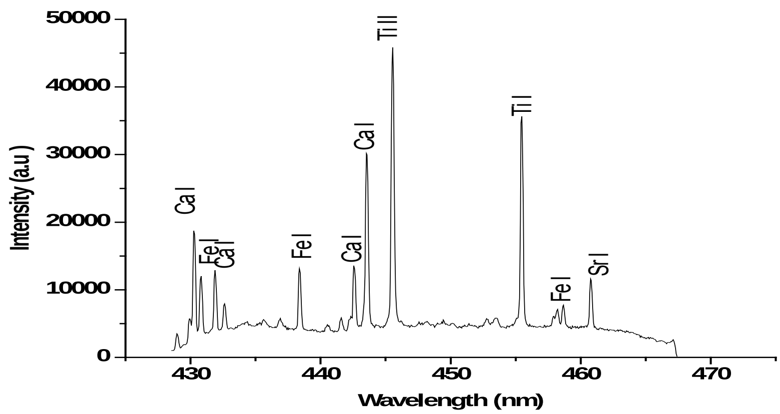

Figure 16 : LIBS spectrum of husk sample in the wavelength limit 425 - 475 nm

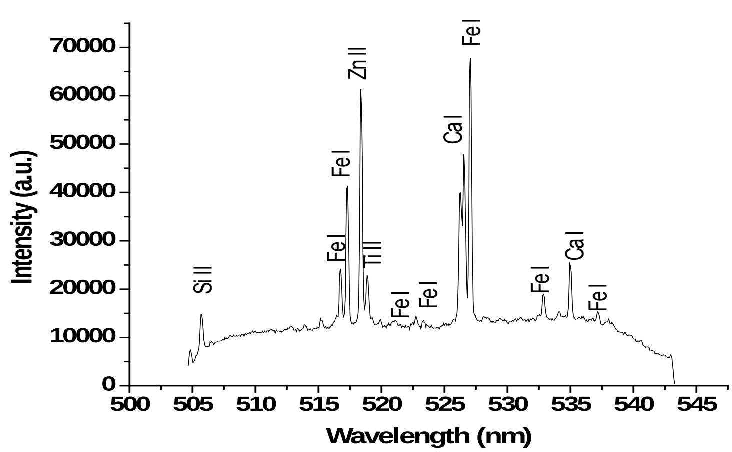

Figure 17 : LIBS spectrum of husk sample in the wavelength limit 500-545 nm

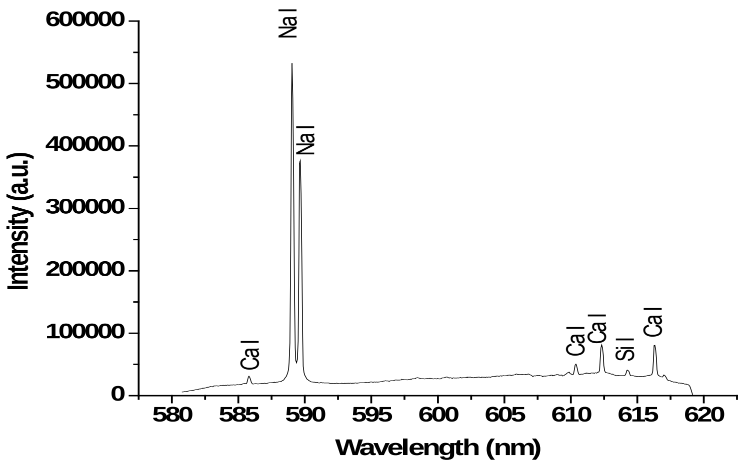

Figure 18 : LIBS spectrum of husk sample (NPK) in the wavelength limit 580-620 nm

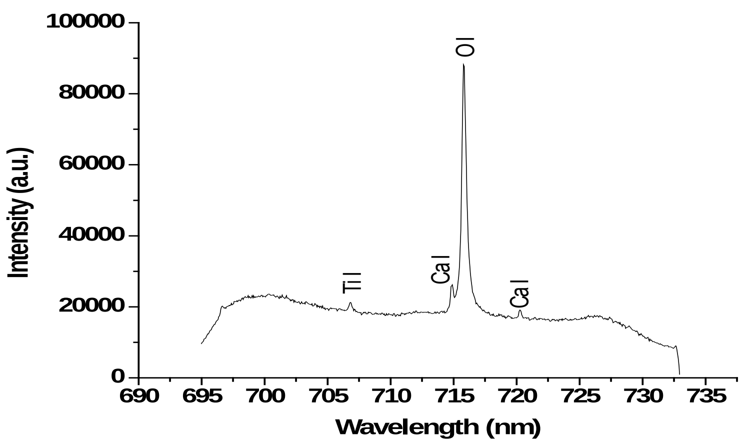

Figure 19 : LIBS spectrum of husk sample in the wavelength limit 690-735 nm

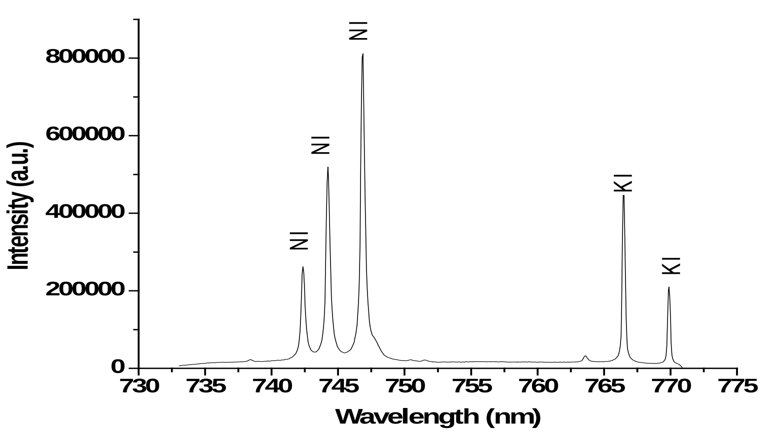

Figure 20 : LIBS spectrum of husk sample in the wavelength limit 730-775 nm

The standard database of atomic emission lines of the US National Institute of Standard and Technology (NIST,http://physics.nist.gov/PhysRefData/ASD/lines_form.html.) was used for the identification of the elements from the observed spectrum (Abedin et al., 2011). Table 1 summarizes the elements detected in the samples of rice and husk from the experimental plot NPK.

Table 1 : Elements found in the samples of rice and husk. A tick indicates the presence of the element in the sample whereas a cross indicates the absence of the element in the sample.

Elements	Rice	Husk
Fe	√	√
Cu	√	√
Na	√	√
Ca	√	√
Ti	√	√
Si	√	√
Co	√	√
Ni	√	√
Al	√	√
Zn	√	√
Li	√	√
K	√	√
Sn	×	×
Mn	√	√
Sr	√	√
Mo	√	√
P	√	√
C	√	√
Mg	√	√

From table 1, it is found that the elements Ni, Cu, Zn, Fe, Ca, Al, Si, Na, K, Ti, Mn, Li, Mo, Co are present in Rice and Husk samples in addition to Sr, P, C and Mg.

This experiment has detected the presence of micronutrients (such as Ni, Cu, Zn, Fe, Mn, Na) as well as some macro-nutrients (such as Ca, K, Si) in all samples. However no quantitative determination of these nutrients was attempted in the present study.

V. Conclusion

The present LIBS technique has been applied to get an elemental profile of the rice and husk samples of agricultural lands.

Rice is a staple food for more than one third of the world's population. Every year, rice growers produce about 422 million metric tons. Rice husks, a waste product produced during cultivation, make up about 20 percent of this. Because these husks are very abrasive, agricultural manufacturers use them in cheap items, such as fertilizer additives, stockbreeding rugs and bed soil, which take advantage of this quality.

But rice husks could have a more valuable use. They contain large amounts of silica, which engineers can convert to silicon for use in LIBs. Silica comprises between 15 and 20 percent of a rice husk's weight. This silica has evolved to be Nano porous, so that air and moisture can enter rice kernels but bacteria and insects cannot.

Pure silicon can be extracted from rice husk silica by adding acid and heat to remove metallic impurities and organic components and then using magnesium to reduce the silica to silicon. This treatment preserved the three-dimensional porous nanostructure. They then coated this silicon with carbon and used it in anodes in lithium coin cells.

Silicon extracted from rice husks could help meet the increasing demand for silicon in batteries used to run portable electronic devices and hybrid electric vehicles. This would allow a waste product from one of the world's most popular crops to contribute to the development of advanced technologies.

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Why Tidal Effects are Local and What it Means for Physics

By Robert Spoljaric

Abstract- The basis of relativistic dynamics is rendered consistent with the Planck-Einstein relations in a novel concept termed the Light. What the Light shows is that what is thought of as the rest mass of a particle of matter is in fact frequency. In turn the Light inevitably leads us to the Equivalence Identity and the conclusion that (a) the Newtonian concept of mass does not exist, and (b) tidal effects are local. Even though this was shown in [2] it was not emphasized and articulated to the extent that it perhaps should have been, and based upon some comments by Dunning-Davies [1] we shall give a detailed justification of the Light, and both (a) and (b). Both the Light and Equivalence Identity are the necessary foundations for a new paradigm of physics. The significance of this for Newtonian mechanics, Maxwell's equations, special relativity, general relativity, and quantum mechanics will be discussed.

Keywords: *The Light, Equivalence Identity, tidal effects, mass, laws of physics.*

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Keywords: The Light, Equivalence Identity, tidal effects, mass, laws of physics.

I. Introduction

The reason tidal effects are local was presented in [2] but not emphasized and articulated to the extent that it perhaps should have been. Thus, we shall take the opportunity to do so here by paying careful attention to the derivation of two concepts; the Light and Equivalence Identity.

We shall also take the opportunity in Section 4 to correct an error made in Section 4 of [2].

This paper is restricted to demonstrating tidal effects are local, and what will be argued is not beyond what we can know empirically. What this means for Newtonian mechanics, Maxwell's equations, special relativity, general relativity, and quantum mechanics will be discussed.

II. The Problem Posed by Tidal Effects

Einstein's principle of equivalence from 1907 has been simply but usefully phrased for our purpose by Thorne as follows:

> *"In any small, freely falling reference frame anywhere in our real, gravity-endowed Universe, the laws of physics must be the same as they are in an inertial reference frame in an idealized, gravity-free universe [3]."*

Implicit is this statement is that tidal effects are nonlocal, and thus if tidal effects *were* local then the absence of an idealized gravity-free universe implies the absence of those laws of physics that depend upon that idealization.

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In [2] it was shown that only the laws of classical physics in conjunction with the energy of a *photon* are needed to show that tidal effects are local. Furthermore, this could have been shown prior to the advent of quantum mechanics in 1925. With the exception of Newton's law of gravity all the following theories assume a gravity-free universe: Newtonian mechanics, Maxwell's equations, statistical mechanics, thermodynamics, and special relativity. Of course general relativity begins with Einstein's principle of equivalence, but if tidal effects are local then there is no vestige of a gravity-free universe, which implies the advent of general relativity is premature as it does not stand alone as *the* theory of physics.

Are tidal effects local? Sartori, based on a paper by Ohanian [4] writes that:

> *"If the radius of the earth were to shrink to zero, its density remaining constant, the shape of the tidal bulges would remain unchanged. In view of this result, it is hard to argue that tidal effects are nonlocal. In principle, an observer in a freely falling elevator could deduce that he is in a real gravitational field by detecting tidal bulges in a liquid drop [5]."*

III. The Reason Tidal Effects are Local

Why is a body's acceleration under gravity independent of its mass? As will now be shown the answer to this question is the reason *why* tidal effects are local!

Many practical applications of special relativity are found in the theory of relativistic mechanics. The basis of relativistic mechanics are the following four equations:

$$M = m_0/\sqrt{1-\ (v/c)^2} \tag{1}$$

which is a function of velocity v where M is the relativistic mass, m_0 the rest mass and c the speed of light. Multiplying both sides of Eq. (1) by the velocity vector $\boldsymbol{v}$ gives us the expression for relativistic momentum

$$\boldsymbol{p} = M\boldsymbol{v} = m_0\boldsymbol{v}/\sqrt{1-\ (v/c)^2} \tag{2}$$

and multiplying both sides of Eq. (1) by c^2 gives the total energy of a particle

$$E = Mc^2 = m_0c^2/\sqrt{1-\ (v/c)^2} \tag{3}$$

Ignoring M in Eqs. (2) and (3) and combining the two equations gives us

$$E = \sqrt{(m_0c^2)^2 + (pc)^2}$$

where we have ignored the negative root. Many would claim that of these four equations only Eqs. (2) and (3) are necessary, and as the basis of relativistic *dynamics* they are routinely confirmed in elementary-particle physics. This suggests that the basis of relativistic mechanics is arbitrary.

We now introduce the energy of a *photon* $E = hf$ where h is Planck's constant, and f is frequency, and equating this with the expression above gives us

$$hf = \sqrt{(m_0c^2)^2 + (pc)^2}$$

Now, if $m_0 = 0$ then using $f = c/\lambda$ we derive the de Broglie hypothesis

$$\lambda = h/p \tag{4}$$

where λ is wavelength, and if $p = 0$ we similarly derive the Compton wavelength

$$\lambda_c = h/m_0c \tag{5}$$

All the expressions presented thus far were known of prior to the advent of quantum mechanics in 1925.

For contrast and contrary to what is derived in [2], let us ignore Eq. (1) and substitute Eq. (5) into Eq. (3) to derive

$$E = hc^2/\lambda\sqrt{c^2 - v^2} = hfc/\sqrt{c^2 - v^2}$$

According to Dunning-Davies on page 111 of [1] this expression only holds when $\boldsymbol{p}$ is zero, which for Eq. (2) implies either $m_0 = 0$ or $v = 0$. But since $m_0 = 0$ would be problematic for Eq. (5) we must assume that $v = 0$ and that leaves $E = hf$, " which simply brings one back to the starting point." However, this analysis is beside the point, for if we substitute Eq. (5) into Eq. (2) as well, and then use the wave vector $\boldsymbol{k}$ where $k = 2\pi/\lambda$, the Dirac constant $\hbar = h/2\pi$, and the angular frequency $\omega = 2\pi f = kc$ to render these new expressions consistent with Eq. (4) we derive the following relations:

$$\begin{aligned} \boldsymbol{p} &= \hbar\boldsymbol{k} \\ &= \hbar\omega c/\sqrt{c^2 - v^2} \\ \boldsymbol{p} &= \hbar\omega\boldsymbol{v}/c\sqrt{c^2 - v^2} \qquad v > 0 \end{aligned}$$

Now we see that $\boldsymbol{p} = \boldsymbol{0}$ in the bottom expression if and only if $v = 0$, and rather than going "back to the starting point" we are left with the Planck-Einstein relations. As mentioned these relations are contrary to what is derived in [2], but since they are presented here we shall show that not using Eq. (1) is misguided. For this reason we shall refer to these relations as the pseudo-Light.

To show why tidal effects are local it is necessary to generalize the Compton wavelength, so as per Section 2 of [2] we proceed by rewriting Eq. (5) in terms of m_0 and substituting for m_0 in Eq. (1) to obtain the temporary expression

$$\lambda = h/M\sqrt{c^2 - v^2} \tag{6}$$

But M is also a function of velocity v, and so we have

$$\lambda = \frac{h}{\dfrac{m_0\sqrt{c^2 - v^2}}{\sqrt{1 - (v/c)^2}}} \tag{6a}$$

If we cancel off terms, then we undo Eq. (6) and are unable to show why tidal effects are local. But if we let $v = 0$ in Eq. (6a) we have the Compton wavelength. In this case what remains of M used in Eq. (6) is m_0, which leaves the case of $v > 0$ where the M used in Eq. (6) corresponds to

$$m_0/\sqrt{1 - (v/c)^2} \tag{\#}$$

in Eq. (6a). Therefore, if Eq. (6a) is the Compton wavelength when $v = 0$, then holding m_0 fixed and *discarding* (#) when $v > 0$ gives us the *generalized* Compton wavelength

$$\lambda_{GC} = h/m_0\sqrt{c^2 - v^2} \tag{7}$$

Suspending judgement on (#) for a moment, and rewriting Eq. (7) in terms of m_0 and substituting into Eq. (2), we find a *qualitatively* different expression where frequency has replaced the concept of rest mass

$$\boldsymbol{p} = hf\boldsymbol{v}/(c^2 - v^2) \tag{8}$$

To use Eq. (8) we have only to translate rest mass to frequency as follows:

$$\begin{aligned} \boldsymbol{p} &= m_0\boldsymbol{v}/\sqrt{1 - (v/c)^2} \\ \lambda_{GC} &= h/m_0\sqrt{c^2 - v^2} \\ \boldsymbol{p} &= hc\boldsymbol{v}/\lambda_{GC}(c^2 - v^2) = hf\boldsymbol{v}/(c^2 - v^2) \end{aligned}$$

Excluding Eq. (3), then, leaves (#) as the only expression using rest mass, and (#) in itself is meaningless, and so it is incumbent upon us to account for its absence. Therefore, starting with Eq. (4)

$$\lambda = h/p$$

and substituting Newton's definition of momentum $p = mv$ for p, we get the de Broglie equation

$$\lambda = h/mv \tag{9}$$

Substituting (#) for m puts (#) in context giving the relativistic expression

$$\lambda = \frac{h\sqrt{1 - (v/c)^2}}{m_0v} \tag{10}$$

Finally, rewriting Eq. (10) in terms of m_0 and substituting into the magnitude of Eq. (2) we find

$$p = \frac{hv\sqrt{1-(v/c)^2}}{\lambda v\sqrt{1-(v/c)^2}} = \frac{h}{\lambda}$$

Thus, the nonexistence of rest mass means we have *no* choice but to use Eq. (8). Furthermore, the nonexistence of Eq. (9) leaves no basis for wave mechanics, and consistently the nonexistence of $p = mv$ leads to the Equivalence Identity below. However, to include the *photon* we must render Eqs. (3) and (4) consistent with Eq. (8), which again requires the wave vector $\boldsymbol{k}$ where $k = 2\pi/\lambda$, the Dirac constant $\hbar = h/2\pi$, and the angular frequency $\omega = 2\pi f = kc$, to give us:

$$\boldsymbol{p} = \hbar \boldsymbol{k}$$
$$E = \hbar\omega c^2/(c^2 - v^2)$$
$$\boldsymbol{p} = \hbar\omega \boldsymbol{v}/(c^2 - v^2) \qquad v > 0$$

The basis of relativistic dynamics ($v > 0$) is now consistent with the Planck-Einstein relations ($v = 0$), and so incorporating the *photon* implies it is really *these* relations that are being routinely confirmed in elementary-particle physics! In hindsight the reason for the apparent arbitrariness of the basis of relativistic mechanics is that it was incomplete. Now, if $v = 0$ we have the energy and momentum of a *photon*, however, the term "photon" refers to the smallest unit of *radiant* energy, but the relations just derived imply the term should be extended to include the energy of *matter* as well. To avoid confusion we can drop the term "photon" and simply define the Light as the electromagnetic energy and momentum of a particle of *radiation,* or a particle of *matter.*

We are now ready to derive the Equivalence Identity and show why tidal effects are local. But first let us consider Newton's second law of motion ($F = ma$) as it contains all three of Newton's laws of motion, i.e., the third law ($F = -F$); and first law ($F = 0$), or law of inertia. Inertial frames are those in which the law of inertia holds. Thus if it could be shown that $F = ma$ does not exist, then it follows that neither do inertial frames, and tidal effects *are* local.

Consider, then, that as the derivation of the Light accounts for the absence of $p = mv$ from physics, this in turn implies the absence of Newton's second law, for mathematically we have

$$F = \frac{d(mv)}{dt} = m_I a$$

where F is force, m_I is inertial mass and a is acceleration. We also have

$$W = m_G g$$

where W is the weight of a terrestrial body, m_G is its gravitational mass, and g is the local acceleration of free fall. If we ignore air resistance then by Galileo's *empirical* law of falling bodies we put $F = W$ and obtain

$$a = \frac{m_G}{m_I} g$$

To be absolutely certain that the local acceleration of free fall in a gravitational field is the same for all bodies we would need to cancel out the factor of m. And finally $m_G = m_I$ *a priori* as the Light entails locally there *is* only

$$a = g \qquad \text{(Equivalence Identity)}$$

Thus, the reason a body's acceleration under gravity is independent of its mass is because in reality the Newtonian concept of mass does *not* exist! And the absence of $F = ma$ implies the observer in the freely falling elevator *must* detect tidal bulges in a liquid drop. That is, consistent with Ohanian's [4] result stated in Section 2 it is now impossible to argue that tidal effects are nonlocal!

In contrast, the pseudo-Light leaves $p = mv$ intact, but the use of mass m in $p = mv$ is inconsistent with those relations. Furthermore, *choosing* to derive $a = g$ starting with $p = mv$ is not an option, given that with the derivation of the Light the absence of $p = mv$ mathematically *necessitates* the absence of $F = ma$ and is finally realized with the derivation of $a = g$. Thus, using Eq. (1) to inevitably derive $a = g$ renders not using Eq. (1) misguided.

It stands to reason, then, that rather than two theories of relativity (special and general) we now have the necessary foundations for just one theory of Relativity. Furthermore, unlike Maxwell's equations, the Light holds in the subatomic realm where the nonuniformity of the gravitational field is negligible. In contrast, Maxwell's equations, which leads to a *wave* theory of light (radiation), assumes a gravity-free universe, and thus it follows that all that remains of Maxwell's unrealistic theory is the constant c as encoded in the Light. Similarly, the gravitational constant, G, is used in general relativity, and if we assume the same holds true in the theory of Relativity, then all that remains of Newtonian mechanics is G. Finally, the absence of Eq. (9) leaves no basis for wave mechanics, but it should be clear that the advent of *non*relativistic quantum mechanics in 1925 in general could have been avoided, and the nonexistence of the Newtonian concept of mass renders that advent misguided.

IV. A Correction to Section 4 of [2]

In Section 4 of [2] I claimed that as the velocity of Eq. (7) is defined in the range $0 \leq v < c$, its wavelength mutually excludes the wavelength of Eq. (9). I erroneously argued that it was the generalized Compton wavelength for an electron that was being confirmed in the Davisson-Germer experiment [6]. Whilst this section has no real bearing upon the Light and Equivalence Identity it is mentioned for we cannot numerically argue with the experimental outcome, which at 54eV is 0.165 nm. To determine the generalized Compton wavelength for an electron at 54eV we use the following values:

$$h = 6.626\,076 \times 10^{-34}\ \mathrm{J\ s}$$
$$c = 2.997\,924\,58 \times 10^{8}\ \mathrm{m\ s^{-1}}$$
$$m_0 = 9.109\,3897 \times 10^{-31}\ \mathrm{kg} = 0.511\,0034\ \mathrm{MeV/c^2}$$

First rearranging the expression for relativistic kinetic energy $K = (\gamma - 1)m_0c^2$ in terms of γ we have $\gamma = {}^{K}/_{m_0c^2} + 1$, which gives us the dimensionless value

$$\gamma = {}^{54\mathrm{eV}}/_{511003.4\mathrm{eV}} + 1 = 1.000\,105\,674$$

Secondly expressing relativistic gamma in terms of v we have $v = \left(\sqrt{1 - {}^{1}/_{\gamma^2}}\right).c$, which yields $v = 4\,357\,989.829\ \mathrm{m\ s^{-1}}$ Finally using this data for Eq. (7) we find λ_{GC}=0.00242 nm, which is far smaller than 0.165 nm.

As shown in Section 3 above, however, both Eqs. (7) and (9) have been used for the final time in the derivation of the Light, and if mass is nonexistent and tidal effects are local, then the true explanation for the Davisson-Germer experiment can only begin with the *matter* aspect of the Light.

V. Conclusion

We have used the laws of classical physics for the final time to demonstrate that mass is nonexistent and tidal effects are local. The above constitutes the body of [2] and it is inevitable that this has implications for the remaining two classical theories of statistical mechanics and thermodynamics, given their dependence on Newton's three 'laws' of motion. This much can be demonstrated empirically.

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A Class of Solution of Orthogonal Plane MHD Flow Through Porous Media in A Rotating Frame

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Abstract- Steady, two dimensional, incompressible electrically conducting, MHD fluid through porous media in a rotating reference frame under the presence of magnetic field which is mutually perpendicular to the velocity field has been considered. Complex variable technique has been employed and exact solutions for three different flow problems have been determined. Lastly, flows with isometric geometry have been studied.

Keywords: steady flow, stream function, complex variable technique, rotating reference frame, exact solution, MHD, porous media.

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Strictly as per the compliance and regulations of :

A Class of Solution of Orthogonal Plane MHD Flow through Porous Media in A Rotating Frame

Sayantan Sil [α] & Manoj Kumar [σ]

Abstract- **Steady, two dimensional, incompressible electrically conducting, MHD fluid through porous media in a rotating reference frame under the presence of magnetic field which is mutually perpendicular to the velocity field has been considered. Complex variable technique has been employed and exact solutions for three different flow problems have been determined. Lastly, flows with isometric geometry have been studied.**

Keywords: steady flow, stream function, complex variable technique, rotating reference frame, exact solution, MHD, porous media.

I. Introduction

The broad subjects of oceanography, meteorology, atmospheric science and limnology all contain some important and essential features of rotating fluids. The rotating flow of an electrically conducting fluid in the presence of a magnetic field is encountered in cosmical and geophysical fluid dynamics. It can provide an explanation for the observed maintenance and secular variation of geomagnetic field[1]. It is also important in the solar physics involved in the sunspot development, the solar cycle and the structure of rotating magnetic stars[2]. The theory of rotating fluid is highly important because of its occurrence in various natural phenomena and for its applications in various technological solutions which are directly governed by the action of coriolis force due to earth's rotation. The effect of coriolis force is found to be significant compared to the inertial and viscous forces in the equations of motion. The coriolis and the magnetic forces are of comparable magnitude. The coriolis force exerts a strong influence on the hydromagnetic flow in the earth's liquid core which plays an important role in the mean geomagnetic field[1].

Many researchers have studied MHD flows in rotating systems. C. S. Bagewadi and Siddabasappa[3, 4] studied variably inclined steady plane rotating MHD flows and plane rotating viscous MHD flows. K. K. Singh and D. P. Singh[5] studied steady plane MHD flows through porous media with constant speed along each streamline. S. N. Singh, H. P. Singh and Rambabu[6] applied hodograph transformation to study steady plane rotating hydromagnetic flows. S. N. Singh and D. D. Tripathi[7] used hodograph transformation to find exact solution for plane rotating MHD flows with orthogonal magnetic and velocity field. H. P. Singh and D. D. Tripathi[8] used legendre transformation and found out exact solutions for variably inclined plane rotating MHD fluid flows with orthogonal magnetic and velocity field. C. Thakur and Manoj Kumar[9] studied plane rotating viscous MHD flows through porous media using Martin's method. G. S. Seth et.al.[10] studied unsteady MHD convective flow within a parallel plate rotating channel with thermal source/sink in porous medium. Dileep Singh Chauhan and Priyanka Rastogi[11] studied viscous incompressible electrically conducting fluid in a parallel plate horizontal rotating channel in the presence of inclined magnetic field. Krishan Dev Singh[12] studied an unsteady MHD Poiseuille flow in a rotating system. M. A. Imran et.al.[13] found out exact solutions for the MHD rotating flow of a second grade fluid in a porous medium using integral transform technique. A. M. Rashid[14] studied unsteady MHD flow of a rotating fluid from stretching surface in porous medium and effects of radiation and variable viscosity on it.

The complex variable technique is well known for the analysis of fluid flow problems. Wan-Lee Yin[15] and M. P. Stallybrass[16] have used this technique to find the solution of fluid flow problems. P. V Nguyen and O. P. Chandna[17] have used this technique to study plane orthogonal MHD second grade fluid having infinite electrical conductivity. C. Thakur and Manoj Kumar[18] employed this technique to study orthogonal MHD flows through porous media. O. P. Chandna and H. Toews[19] and O. P. Chandna and M. R. Garg[20] have studied MHD flows of Newtonian fluid, when the streamlines and their orthogonal trajectories form an isometric net.

In this paper we consider two dimensional motion of a steady orthogonal MHD flows of an incompressible electrically conducting fluid through porous media having infinite electrical conductivity in a rotating reference frame. We determine the system of equations in the velocity field that the flow must satisfy and introduce the stream function required to satisfy a system of two equations for our flows. We employ the complex variables $z = x + iy$ and $\bar{z} = x - iy$ as our new

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independent variables. Further, we determine exact solutions for different flow problems. Finally we study flows with isometric geometry.

II. Basic Equations

The basic equations governing the motion of a steady, homogenous, infinite electrically conducting, viscous MHD fluid in a rotating frame through porous media are

$$\nabla \bullet \mathbf{V} = 0, \tag{1}$$

$$\rho\left[(\mathbf{V}\bullet\nabla)\mathbf{V} + 2\boldsymbol{\Omega}\times\mathbf{V} + \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})\right] = -\nabla P + \eta\nabla^2\mathbf{V} + \mu\mathbf{J}\times\mathbf{H} - \frac{\eta}{K}\mathbf{V} \tag{2}$$

$$\nabla\times(\mathbf{V}\times\mathbf{H}) = 0, \tag{3}$$

and

$$\nabla \bullet \mathbf{H} = 0 \ , \tag{4}$$

where, $\mathbf{V} =$ velocity vector, $\mathbf{H} =$ magnetic field vector, $\mathbf{J} =$ current density vector, $P =$ fluid pressure, $\rho =$ fluid density, $\eta =$ coefficient of viscosity, $\boldsymbol{\Omega} =$ angular velocity vector, $\mu =$ magnetic permeability and $K =$ permeability of the medium.

We consider the flow to be two-dimensional so that $\mathbf{V}$ and H lie in a plane defined by the rectangular co-ordinates (x, y) and all the flow variables are functions of x and y. Also we introduce the functions:

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}, \quad \text{(Vorticity function)} \tag{5}$$

$$j = \frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y}, \quad \text{(Current density function)} \tag{6}$$

$$h = \frac{1}{2}\rho V^2 + P' + \frac{1}{2}\rho\left|\boldsymbol{\Omega}\times\mathbf{r}\right|^2, \quad \text{(Bernoulli function)} \tag{7}$$

where $V^2 = u^2 + v^2$, P' is the reduced pressure given by $P' = P - \frac{1}{2}\rho\left|\boldsymbol{\Omega}\times\mathbf{r}\right|^2$ and the last term being the centrifugal contribution of pressure, u, v are the components of velocity vector also H_1, H_2 are the components of magnetic field vector.

Now the above system of equations (1)-(4) is replaced by the following system:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{8}$$

$$\eta\frac{\partial \omega}{\partial y} - \rho\omega v - 2\rho\Omega v + \mu j H_2 + \frac{\eta}{K}u = -\frac{\partial h}{\partial x}, \tag{9}$$

$$\eta\frac{\partial \omega}{\partial x} - \rho\omega u - 2\rho\Omega u + \mu j H_1 - \frac{\eta}{K}v = \frac{\partial h}{\partial y}, \tag{10}$$

$$uH_2 - vH_1 = A, \tag{11}$$

$$\frac{\partial H_1}{\partial x} + \frac{\partial H_2}{\partial y} = 0, \tag{12}$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \omega, \tag{13}$$

where A is an arbitrary constant of integration obtained from the diffusion equation (3)

III. Steady Plane Orthogonal Flow

A plane flow is said to be an orthogonal or crossed flow if the velocity and magnetic vectors are mutually perpendicular everywhere in the flow region. Therefore, we have

$$uH_1 + vH_2 = 0. \tag{14}$$

Solving (11) and (14) simultaneously, we obtain

$$\mathbf{H} = \frac{A}{|V|^2}(\hat{e}\times\mathbf{V}), \tag{15}$$

where $\hat{e} = (0,0,1)$.

Equations (9) and (10) can be written in vector form as

$$\nabla h - \rho\left[\mathbf{V}\times(\boldsymbol{\omega} + 2\boldsymbol{\Omega})\right] = \eta\nabla^2\mathbf{V} + \mu(\nabla\times\mathbf{H})\times\mathbf{H} - \frac{\eta}{K}\mathbf{V} \tag{16}$$

Employing (15) in (16) and making use of (8) we get

$$\nabla h - \rho\left[\mathbf{V}\times(\boldsymbol{\omega} + 2\boldsymbol{\Omega})\right] = \eta\nabla^2\mathbf{V} + \frac{\mu A^2}{|\mathbf{V}|^2}\left[\nabla\left(\frac{1}{|\mathbf{V}|^2}\right)\bullet\mathbf{V}\right]\mathbf{V} - \frac{\eta}{K}\mathbf{V} \tag{17}$$

Taking curl of equation (17), we obtain the integrability condition for $h(x, y)$ to be

$$\nabla\times\left[\mathbf{V}\times(\boldsymbol{\omega}+2\boldsymbol{\Omega})\right]+\frac{\eta}{\rho}\nabla^2\boldsymbol{\omega}+\frac{\mu}{\rho}\nabla\times\left[\frac{A^2}{|\mathbf{V}|^2}\left\{\nabla\left(\frac{1}{|\mathbf{V}|^2}\right)\bullet\mathbf{V}\right\}\mathbf{V}\right]-\frac{\eta}{\rho K}\boldsymbol{\omega}=\mathbf{0}\ , \tag{18}$$

where $\boldsymbol{\omega}=\nabla\times\mathbf{V}$.

Using (15) in (4), we have

$$\nabla\times\left(\frac{\mathbf{V}}{|\mathbf{V}|^2}\right)=\mathbf{0}. \tag{19}$$

Therefore, for the steady, plane, orthogonal flow of an incompressible fluid of infinite electrical conductivity through porous media in a rotating reference frame, the velocity field satisfies equation (8), (18) and (19).

Equation (8) implies the existence of a stream function $\psi(x,y)$, so that

$$u=\psi_y\ ,\ v=-\psi_x\ . \tag{20}$$

Using (20) in (13), we get

$$\boldsymbol{\omega}=-\nabla^2\psi\hat{e}\ . \tag{21}$$

Employing (20) and (21) in (18) and (19), we have

$$\frac{\partial(\psi,\nabla^2\psi)}{\partial(x,y)}+\frac{\eta}{\rho}\nabla^4\psi+\frac{\mu A^2}{2\rho}\left[\psi_x\left\{\frac{\partial\left(\psi,\left(1\big/|\nabla\psi|^4\right)\right)}{\partial(x,y)}\right\}_x+\psi_y\left\{\frac{\partial\left(\psi,\left(1\big/|\nabla\psi|^4\right)\right)}{\partial(x,y)}\right\}_y+\frac{\partial\left(\psi,\left(1\big/|\nabla\psi|^4\right)\right)}{\partial(x,y)}\nabla^2\psi\right]+\frac{\eta}{\rho K}\nabla^2\psi=0\ , \tag{22}$$

$$\nabla^2\frac{\psi}{|\nabla\psi|^2}+\psi_x\left(\frac{1}{|\nabla\psi|^2}\right)_x+\psi_y\left(\frac{1}{|\nabla\psi|^2}\right)_y=0\ , \tag{23}$$

where $|\nabla\psi|^2=\psi_x^2+\psi_y^2$.

We introduce the variables

$$z=x+iy\ ,\quad \bar{z}=x-iy\ ,$$

where $i=\sqrt{-1}$ and the following relations can be derived

$$m_x=m_z+m_{\bar{z}}\quad,\ m_y=i\left(m_z-m_{\bar{z}}\right),$$

$$4m_zm_{\bar{z}}=m_x^2+m_y^2\quad,\quad 4m_{z\bar{z}}=m_{xx}+m_{yy}\ ,$$

$$\frac{\partial(m,n)}{\partial(x,y)}=-4\mathrm{Im}\left\{m_{\bar{z}}m_z\right\},$$

$$m_xn_x+m_yn_y=4\mathrm{Re}\left\{m_zn_{\bar{z}}\right\}=4\mathrm{Re}\left\{m_{\bar{z}}n_z\right\}, \tag{24}$$

where m and n are scalar functions.

If $f(z)=\alpha(x,y)+i\beta(x,y)$ is a complex analytic function, then

$$f'(z)=\alpha_x+i\beta_x=\alpha_x-i\alpha_y$$
$$=2\alpha_z=2i\beta_z\ .$$

and $\overline{f'(z)}=2\alpha_{\bar{z}}=-2i\beta_{\bar{z}}$, (25)

where prime denotes differentiation with respect to z.

Applying (24) in (22) and (23), we obtain the following system of equations:

$$\psi_{zz\bar{z}\bar{z}}-\frac{\rho}{\eta}\mathrm{Im}\left\{\psi_{\bar{z}}\psi_{zz\bar{z}}\right\}-\frac{\mu A^2}{32\eta}\left\{\mathrm{Re}\left\{\left[\mathrm{Im}\left\{\left(\frac{1}{\psi_z^2\psi_{\bar{z}}^2}\right)_z\psi_{\bar{z}}\right\}\right]\right.\right.$$
$$\left.\left.+\psi_{z\bar{z}}\,\mathrm{Im}\left\{\left(\frac{1}{\psi_z^2\psi_{\bar{z}}^2}\right)_z\psi_{\bar{z}}\right\}\right\}\right\}-\frac{1}{4K}\psi_{z\bar{z}}=0\ , \tag{26}$$

and
$$\frac{\psi_{z\bar{z}}}{\psi_z\psi_{\bar{z}}}+\mathrm{Re}\left\{\left(\frac{1}{\psi_z\psi_{\bar{z}}}\right)_z\psi_{\bar{z}}\right\}=0\ . \tag{27}$$

Summing up we have the following theorem:

Theorem 1. If $\psi(z\bar{z})$ be the stream function of a steady plane orthogonal flow of a viscous incompressible fluid having infinite electrical conductivity through porous media in a rotating reference frame, then $\psi(z\bar{z})$ must satisfy equations (26) and (27).

IV. Straight, Parallel, Radial and Vortex Flows

As application of Theorem 1, we determine the solutions for the straight parallel, radial and vortex problems in this section.

a) Straight parallel flows

We let $\psi=\psi(\alpha)\,,\psi'(\alpha)\neq 0$ (28)

to be the stream function, where

$$\alpha(z,\bar{z})=B_1(z+\bar{z})+iB_2(\bar{z}-z) \tag{29}$$

and B_1, B_2 are arbitrary real constants.

From (28) and (29) we get

$$\psi_z=\bar{B}\psi'\,,\ \psi_{\bar{z}}=B\psi' \text{ and } \psi_{z\bar{z}}=B\bar{B}\psi'' \tag{30}$$

where $B=B_1+iB_2$ and prime denotes differentiation with respect to α.

Employing (30) in (27), we have

$$\frac{\psi''}{\psi'^2}-2\,\mathrm{Re}\left\{\frac{\psi''}{\psi'^2}\right\}=0.$$

Since $\psi(\alpha)$ is real, it follows that

$$\frac{\psi''(\alpha)}{\psi'^2(\alpha)}=0\ . \tag{31}$$

Integrating (31) with respect to α, we get

$$\psi(\alpha)=B_3\,\alpha+B_4\,, \tag{32}$$

where B_3 and B_4 are arbitrary real constants. Using (32) in (26) we find that (26) is identically satisfied. Therefore, the stream function of the flow in physical co-ordinate is given by

$$\psi(x,y)=B_5\,x+B_6 y+B_4\,, \tag{33}$$

where $B_5=2B_1B_3$, $B_6=2B_2B_3$.

Using (33) in (20), we get

$$\mathbf{V}=(u,v)=(B_6,-B_5)\,, \tag{34}$$

$$\omega=0\,, \tag{35}$$

From (15), $\mathbf{H}=\dfrac{A}{|\mathbf{V}|^2}(-v,u)=\dfrac{A}{B_5^2+B_6^2}(B_5,B_6)$. (36)

Using (34)-(36) in (9) and (10) and integrating, we obtain

$$h=-\left[2\rho\Omega\left(B_5\,x+B_6 y\right)+\frac{\eta}{K}\left(B_6\,x-B_5\,y\right)\right]+B_7\,, \tag{37}$$

and from (7) we get

$$P=-\left[2\rho\Omega\left(B_5\,x+B_6 y\right)+\frac{\eta}{K}\left(B_6\,x-B_5\,y\right)+\frac{1}{2}\rho\left(B_6^2+B_5^2\right)\right]+B_7\,, \tag{38}$$

where B_7 is an arbitrary constant of integration.

b) Radial flows

Letting $\psi=\psi(\alpha)\,,\psi'(\alpha)\neq 0$ (39)

to be the stream function where

$$\alpha(z,\bar{z})=D_1\tan^{-1}\left[\frac{i(\bar{z}-z)}{z+\bar{z}}\right]+D_2\,,$$

and D_1, D_2 are arbitrary real constants.

Proceeding as in the previous case we obtain

$$\psi(\alpha)=D_3\alpha+D_4\,, \tag{40}$$

$$\psi(x,y)=D_5\tan^{-1}\left(\frac{y}{x}\right)+D_6\,, \tag{41}$$

$$\mathbf{V}=(u,v)=D_5\left(\frac{x}{x^2+y^2},\frac{y}{x^2+y^2}\right), \tag{42}$$

$$\omega=0\,, \tag{43}$$

$$\mathbf{H}=\frac{A}{D_5}(-y,x)\,, \tag{44}$$

$$h(x,y)=2\rho\Omega D_5\left[\tan^{-1}\frac{x}{y}-\tan^{-1}\frac{y}{x}\right]-\frac{2\mu A^2}{D_5^2}\frac{(x^2+y^2)}{2}-\frac{\eta}{K}D_5\ln(x^2+y^2)+D_7\,,$$

$$P(x,y)=2\rho\Omega D_5\left[\tan^{-1}\frac{x}{y}-\tan^{-1}\frac{y}{x}\right]-\frac{2\mu A^2}{D_5^2}\frac{(x^2+y^2)}{2}-\frac{\eta}{K}D_5\ln(x^2+y^2)-\frac{1}{2}\rho D_5^2\left(\frac{1}{x^2+y^2}\right)+D_7, \quad (45)$$

where $D_5=D_1D_3$, $D_6=D_2D_3+D_4$ and D_7 are arbitrary real constants.

c) *Vortex flows*

Letting $\psi=\psi(\alpha)$, $\psi'(\alpha)\neq 0$ (46)

to be the stream function where

$$\alpha(z,\bar{z})=C_1z\bar{z}+C_2\ . \quad (47)$$

Equations (46) and (47) yield

$$\psi_z=C_1\bar{z}\psi',\quad \psi_{\bar{z}}=C_1z\psi',$$

$$\psi_{z\bar{z}}=C_1^2\bar{z}z\psi''+C_1\psi'.$$

Proceeding as before, we find that the stream function of the flow is given by

$$\psi(\alpha)=C_3\alpha+C_4, \quad (48)$$

where C_3 and C_4 are arbitrary real constants.

Using (48) in (26), we find that the equation (26) is not identically satisfied hence (48) is not the solution of the system of equations (26) and (27). Therefore, in the case of the vortex flows, the solution of the system of equation is not possible.

V. Isometric Flows

In this section, we study flows in which streamlines and their orthogonal trajectories (i.e. magnetic lines) coincide with the curves in an isometric net. G. Hamel[21] investigated this problem for non-MHD flows of an incompressible viscous fluid. M. H. Martin[22] also investigated the non-MHD flow problems as an illustration of this new approach in studying Navier-Stokes equations for steady plane flows. In these works, it is proved that parallel straight lines, concentric circles, concurrent straight lines and spirals are possible streamlines. Also P. V. Nguyen and O. P. Chandna[17] investigated this problem for MHD orthogonal flow of an incompressible second grade fluid and proved that possible streamlines are straight parallel lines, concentric circles and concurrent straight lines. In this study, we prove that for MHD orthogonal flow of an incompressible fluid of infinite electrical conductivity through porous media in a rotating frame the possible streamlines are straight parallel lines and concurrent straight lines.

We assume

$$\psi=\psi(\xi),\ \psi=\psi'(\xi)\neq 0, \quad (49)$$

where the curves $\xi(x,y)=\text{constant}$, generate an isometric net.

Thus,

$$f(z)=\xi(x,y)+i\lambda(x,y), \quad (50)$$

is an analytic complex function and

$$\xi_{xx}+\xi_{yy}=4\xi_{z\bar{z}}=0.$$

From (49), we get

$$\begin{aligned}
&\psi_z=\psi'\xi_z,\quad \psi_{\bar{z}}=\psi'\xi_{\bar{z}},\\
&\psi_{z\bar{z}}=\psi''\xi_z\xi_{\bar{z}},\\
&\psi_{z\bar{z}z}=\psi'''\xi_z^2\xi_{\bar{z}}+\psi''\xi_{zz}\xi_{\bar{z}},\\
&\psi_{z\bar{z}z\bar{z}}=\psi^{iv}\xi_z^2\xi_{\bar{z}}^2+\psi'''\left(\xi_z^2\xi_{\bar{z}\bar{z}}+\xi_{\bar{z}}^2\xi_{zz}\right)+\psi''\xi_{zz}\xi_{\bar{z}\bar{z}},\\
&\psi_{z\bar{z}z\bar{z}z}=\psi^{v}\xi_z^3\xi_{\bar{z}}^2+\psi^{iv}\left(3\xi_z\xi_{\bar{z}}^2\xi_{zz}+\xi_z^3\xi_{\bar{z}\bar{z}}\right)+\psi'''\left(3\xi_z\xi_{zz}\xi_{\bar{z}\bar{z}}+\xi_{\bar{z}}^2\xi_{zzz}\right)+\psi''\xi_{zzz}\xi_{\bar{z}\bar{z}},
\end{aligned} \quad (51)$$

where prime denotes differentiation with respect to ξ.

Using (51) in (26) and (27) and making use of the fact that $\psi(\xi),\xi_z,\xi_{\bar{z}},\xi_{zz}\xi_{\bar{z}\bar{z}}$ and $\left(\xi_z\xi_{\bar{z}}^3\xi_{zz}+\xi_{\bar{z}}\xi_z^3\xi_{\bar{z}\bar{z}}\right)$ are real, we get

$$\psi^{iv}(\xi)+\left(\frac{\xi_{zz}}{\xi_z^2}+\frac{\xi_{\bar{z}\bar{z}}}{\xi_{\bar{z}}^2}\right)\psi'''(\xi)+\frac{\xi_{zz}\xi_{\bar{z}\bar{z}}}{\xi_z^2\xi_{\bar{z}}^2}\psi''(\xi)-\frac{\rho}{\eta}\mathrm{Im}\left\{\frac{\xi_{zz}}{\xi_z^2}\right\}\psi'(\xi)\psi''(\xi)$$

$$-\frac{\mu A^2}{16\eta\psi'^2(\xi)\xi_z^2\xi_{\bar{z}}^2}\left[\mathrm{Re}\left\{\left(\frac{1}{\xi_z}\right)_z\right\}\mathrm{Im}\left\{\left(\frac{1}{\xi_z}\right)_z\right\}+\mathrm{Re}\left\{\left[\left(\frac{1}{\xi_z}\right)_z\right]_z\right\}\right]$$

$$-\frac{2\psi''(\xi)}{\psi(\xi)}\mathrm{Im}\left\{\left(\frac{1}{\xi_z}\right)_z\right\}\Bigg]-\frac{1}{4K\xi_z^2\xi_{\bar z}^2}\psi''(\xi)\xi_z\xi_{\bar z}=0. \tag{52}$$

$$\mathrm{Re}\left\{\left(\frac{1}{\xi_z}\right)_z\right\}=\frac{\psi''(\xi)}{\psi'(\xi)}. \tag{53}$$

Letting

$$a=\mathrm{Re}\left\{\frac{\xi_{zz}}{\xi_z^2}\right\}=\frac{1}{2}\left[\frac{\xi_{zz}}{\xi_z^2}+\frac{\xi_{\bar z\bar z}}{\xi_{\bar z}^2}\right], \tag{54}$$

$$b=\mathrm{Im}\left\{\frac{\xi_{zz}}{\xi_z^2}\right\}=\frac{1}{2i}\left[\frac{\xi_{zz}}{\xi_z^2}+\frac{\xi_{\bar z\bar z}}{\xi_{\bar z}^2}\right], \tag{55}$$

we have

$$a^2+b^2=\frac{\xi_{zz}\xi_{\bar z\bar z}}{\xi_z^2\xi_{\bar z}^2}. \tag{56}$$

Applying (25) in (50) and making use of (55) and (56), we obtain

$$\begin{aligned}&\psi^{iv}(\xi)+2a\psi'''(\xi)+\left(a^2+b^2\right)\psi''(\xi)-\frac{\rho b}{\eta}\psi'(\xi)\psi''(\xi)\\&-\frac{\mu A^2}{16\eta\psi'^2(\xi)\xi_z^2\xi_{\bar z}^2}\left[ab-\mathrm{Re}\left\{\frac{1}{\xi_z}b_z\right\}+\frac{2\psi''(\xi)}{\psi(\xi)}b\right]-\frac{1}{4K\xi_z\xi_{\bar z}}\psi''(\xi)=0,\end{aligned} \tag{60}$$

$$-a=\frac{\psi''(\xi)}{\psi(\xi)}. \tag{61}$$

Equation (61) yields

$$a=a(\xi). \tag{62}$$

From (62) and (59), we get

$$b=b(\lambda). \tag{63}$$

Using (62) and (63) in (58), we get

$$a'(\xi)=b'(\lambda), \tag{64}$$

where prime denotes differentiation with respect to argument.

Equation (64) gives

$$a(\xi)=E\xi+E_1, \tag{65}$$

$$b(\lambda)=E\lambda+E_2, \tag{66}$$

$$\frac{2f''(z)}{f'^2(z)}=\frac{\xi_{zz}}{\xi_z^2}=a+ib. \tag{57}$$

Since f(z) is an analytic function of $z=x+iy$, it follows from (57), that $a+ib$ is an analytic function of z, and since z is an analytic function of $\xi+i\lambda$, $a+ib$ is also an analytic function of $\xi+i\lambda$.

Hence we have the following relations

$$a_\xi=b_\lambda, \tag{58}$$

$$a_\lambda=-b_\xi. \tag{59}$$

Conversely, $f(z)=\xi+i\lambda$ is an analytic function of

$$\frac{f''(z)}{f'^2(z)}=a+ib.$$

Using (54) to (56) in (52) and (53), we get, respectively

where E, E_1 and E_2 are arbitrary real constants.

Substituting (65) in (61) and integrating with respect to ξ, we obtain

$$\psi'(\xi)=\exp\left[-\left(\frac{E}{2}\xi^2+E_1\xi+E_3\right)\right]. \tag{67}$$

Employing (65) and (66) in (57), we have

$$2\frac{f''(z)}{f'(z)}=Ef(z)f'(z)+(E_1+iE_2)f'(z). \tag{68}$$

Integrating with respect to z, we find

$$f'(z)=F\exp\left[\frac{E}{4}f^2(z)+\frac{E_1+iE_2}{2}f(z)\right],$$

where F is an arbitrary complex constant.

Since $f'(z)=2\xi_z$, it follows that

$$\xi_z=\frac{F}{2}\exp\left[\frac{E}{4}\left(\xi^2-\lambda^2+2i\xi\lambda\right)+\frac{E_1+iE_2}{2}(\xi+i\lambda)\right]. \tag{69}$$

From (69) we have

$$\xi_{\bar{z}} = \frac{\bar{F}}{2}\exp\left[\frac{E}{4}\left(\xi^2-\lambda^2+2i\xi\lambda\right)+\frac{E_1-iE_2}{2}(\xi-i\lambda)\right] ,$$

$$\xi_z\xi_{\bar{z}} = \frac{F\bar{F}}{4}\exp\left[\frac{E}{4}\left(\xi^2-\lambda^2\right)+E_1\xi-E_2\lambda\right],$$

$$\xi_{zz} = (a+ib)\xi_z^2,\ \xi_{\bar{z}\bar{z}} = (a-ib)\xi_{\bar{z}}^2 ,$$

$$\xi_{zzz} = 2\left[(a+ib)^2+E\right]\xi_z^3 \quad . \qquad (70)$$

Differentiating (66) with respect to z and using (25), we find

$$b_z = E\lambda_z = -iE\xi_z \ .$$

Therefore

$$\mathrm{Re}\left\{\frac{1}{\xi_z}b_z\right\} = \mathrm{Re}\{-iE\} = 0 \ . \qquad (71)$$

Using (67), (70) and (71) in (60), we obtain

$$\psi^{iv}(\xi)+2a\,\psi'''(\xi)+\left(a^2+b^2\right)\psi''(\xi)-\frac{\rho b}{\eta}\psi'(\xi)\psi''(\xi)+\frac{\mu A^2}{\eta(F\bar{F})^2\psi'^2(\xi)}W(\xi,\lambda)ab$$
$$-\frac{1}{KF\bar{F}}\exp\left[\frac{E}{2}\left(\lambda^2-\xi^2\right)+E_2\lambda-E_1\xi\right]\psi''(\xi)=0 \quad . \qquad (72)$$

Where $W(\xi,\lambda)=\exp\left[E\left(\lambda^2-\xi^2\right)+2E_2\lambda-2E_1\xi\right]$.

Differentiating (72) three times with respect to λ , we get

$$\frac{\mu A^2}{\eta(F\bar{F})^2\psi'^2(\xi)}\left\{2a\left(4b^4+12b^2b_\lambda+3b_\lambda^2\right)\right\}W(\xi,\lambda)$$
$$-\frac{\psi'(\xi)a}{KF\bar{F}}\left(3bb_\lambda+b^3\right)\exp\left[\frac{E}{2}\left(\lambda^2-\xi^2\right)+E_2\lambda-E_1\xi\right]=0 \ . \qquad (73)$$

Multiplying (73) by $\dfrac{\psi'^2(\xi)}{W(\xi,\lambda)}$ and using (67) and (70), we get

$$\frac{\mu A^2}{\eta(F\bar{F})^2}\chi_2(\xi,\lambda)-\frac{\chi_1(\xi,\lambda)}{KF\bar{F}}\frac{\psi'(\xi)\psi'^2(\xi)}{W(\xi,\lambda)}\exp\left[\frac{E}{2}\left(\lambda^2-\xi^2\right)+E_2\lambda-E_1\xi\right]=0 \ , \qquad (74)$$

where

$$\chi_1(\xi,\lambda)=a\left(3bb_\lambda+b^3\right), \qquad (75)$$

and

$$\chi_2(\xi,\lambda)=2a\left(4b^4+12b^2b_\lambda+3b_\lambda^2\right) \qquad (76)$$

$\chi_1(\xi,\lambda)\neq 0$ then (74) yields

$$\frac{\psi'(\xi)\psi'^2(\xi)}{W(\xi,\lambda)}\exp\left[\frac{E}{2}\left(\lambda^2-\xi^2\right)+E_2\lambda-E_1\xi\right]=\frac{\mu A^2\left(KF\bar{F}\right)}{\eta(F\bar{F})^2}\frac{\chi_2(\xi,\lambda)}{\chi_1(\xi,\lambda)} . \qquad (77)$$

Which is impossible since the left hand side of equation (77) is an exponential function while the right hand side of (77) is an algebraic function in ξ and λ .

Therefore,

$$\chi_1(\xi,\lambda)=0 \ , \qquad (78)$$

and

$$\chi_2(\xi,\lambda)=0 \ . \qquad (79)$$

Using (79), we find that either

$$a=0 \ , \qquad (80)$$

or,

$$\left(4b^4+12b^2b_\lambda+3b_\lambda^2\right)=0. \qquad (81)$$

Employing (65) and (66) in (80) and (81) respectively, we have

$$E\xi + E_1 = 0 \,, \tag{82}$$

$$4E^4\lambda^4 + 2E^3E_2\lambda^3 + 12\left(E^2E_2^2 + E^3\right)\lambda^2 + \left(16EE_2^3 + 24E^2E_2\right)\lambda + \left(4E_2^4 + 12EE_2^2 + 3E^2\right) = 0. \tag{83}$$

Since (λ, ξ) is a linearly independent set, it follows that equation (82) holds true if and only if

$$E = 0 \quad , \quad E_1 = 0. \tag{84}$$

Similarly equation (83) yields

$$E = 0 \quad , \quad E_2 = 0. \tag{85}$$

Combining (84) and (85), we find that equation (79) holds if and only if

(i) $E = 0,\ E_1 = E_2 = 0$,

(ii) $E = 0,\ E_1 \neq 0,\ E_2 = 0$,

(iii) $E = 0,\ E_1 = 0,\ E_2 \neq 0.$

Using the values of E, E_1 and E_2 given in (75), we find that (78) identically satisfied for first two [(i) and (ii)] conditions. Hence, from (65), (66) and (67), we obtain three cases

(i) $a = 0,\ b = 0,\ \psi'(\xi) = \exp(-E_3)$,

(ii) $a = E_1,\ b = 0,\ \psi'(\xi) = \exp(-E_1\xi - E_3)$,

(iii) $a = 0,\ b = E_2,\ \psi'(\xi) = \exp(-E_3)$.

Substituting the values of a, b and $\psi'(\xi)$ given above in equation (72) we find that (72) is identically satisfied only for case (i) and (iii) and the geometric flow pattern corresponding to these cases is determined as follows:

i. *Straight parallel flows:*

In this case, we have

$$a = 0,\ b = 0,\ \psi'(\xi) = \exp(-E_3)\,. \tag{86}$$

Equation (57) takes the form

$$2\frac{f''(z)}{|f'(z)|^2} = 0 \,. \tag{87}$$

Integrating (87) with respect to z, we obtain

$$f'(z) = 2\xi_z = \xi_z - i\xi_y = \frac{2\bar{G}}{G\bar{G}} \,, \tag{88}$$

where $G = G_1 + iG_2$ is an arbitrary complex constant.

Separating real and imaginary parts and integrating the resulting equations, we have

$$\xi(x, y) = \frac{2G_1}{G_1^2 + G_2^2}x + \frac{2G_2}{G_1^2 + G_2^2}y + G_3 \,, \tag{89}$$

where G_3 is an arbitrary real constant.

Equation (86) yields

$$\psi(\xi) = G_4\,\xi + G_5 \,. \tag{90}$$

where $G_4 = \exp(-E_3)$ and G_5 is an arbitrary real constant.

From (89) and (90), we obtain the stream function in physical plane as

$$\psi(x, y) = G_6x + G_7y + G_8 \,, \tag{91}$$

where

$$G_6 = \frac{2G_1G_4}{G_1^2 + G_2^2},\ G_7 = \frac{2G_2G_4}{G_1^2 + G_2^2} \text{ and } G_8 = G_3G_4 + G_5 .$$

Using (92) in (20), we get

$$\mathbf{V} = (u, v) = (G_7, -G_6) \,, \tag{92}$$

$$\omega = 0 \,,$$

$$\mathbf{H} = \frac{A}{|\mathbf{V}|^2}(-v, u) = \frac{A}{G_6^2 + G_7^2}(G_6, G_7) \,.$$

$$h = -\left[2\rho\Omega\left(G_6x + G_7y\right) + \frac{\eta}{K}\left(G_7\,x - G_6y\right)\right] + G_8 \,,$$

$$P = -\left[2\rho\Omega\left(G_6x + G_7y\right) + \frac{\eta}{K}\left(G_7\,x - G_6y\right) + \frac{1}{2}\rho\left(G_7^2 + G_6^2\right)\right] + G_8 \,, \tag{93}$$

where G_8 is an arbitrary constant of integration.

ii. *Radial flows*

Proceeding as in case (i) we obtain the stream function in this case as

$$\psi(x, y) = F_3 \tan^{-1}\frac{(x + F_1)}{(y + F_2)} + F_4 \,, \tag{94}$$

Global Journal of Science Frontier Research (A) Volume XIV Issue VII Version I Year 2014

where F_1, F_2, F_3 and F_4 are arbitrary real constants. Hence, we get

$$\mathbf{V} = \left\{ \frac{-F_3(x+F_1)}{(x+F_1)^2+(y+F_2)^2}, \frac{-F_3(y+F_2)}{(x+F_1)^2+(y+F_2)^2} \right\}$$

$$\omega = 0,$$

$$\mathbf{H} = \frac{A}{F_3}\{(y+F_2), -(x+F_1)\},$$

$$h = 2\rho\Omega F_3\left[\tan^{-1}\frac{(y+F_2)}{(x+F_1)} - \tan^{-1}\frac{(x+F_1)}{(y+F_2)}\right] - \frac{2\mu A^2}{F_3^2}\left[\frac{x^2+y^2}{2} + F_1 x + F_2 y\right]$$
$$+\frac{\eta}{K}F_3 \ln\left[(x+F_1)^2+(y+F_2)^2\right] + F_5 .$$

$$P = 2\rho\Omega F_3\left[\tan^{-1}\frac{(y+F_2)}{(x+F_1)} - \tan^{-1}\frac{(x+F_1)}{(y+F_2)}\right] - \frac{2\mu A^2}{F_3^2}\left[\frac{x^2+y^2}{2} + F_1 x + F_2 y\right]$$
$$+\frac{\eta}{K}F_3 \ln\left[(x+F_1)^2+(y+F_2)^2\right] - \frac{1}{2}\rho F_3\left\{\frac{1}{(x+F_1)^2+(y+F_2)^2}\right\} + F_5 , \tag{95}$$

where F_5 is an arbitrary real constant of integration. Summing up we have the following theorem:

Theorem 2. If the stream lines of the steady plane orthogonal MHD flow of an incompressible fluid having infinite electrical conductivity through porous media in a rotating reference frame coincide with a curve in isometric net then the streamlines are restricted to straight parallel lines and concurrent straight lines.

VI. Conclusion

In this paper we consider steady, two dimensional, incompressible electrically conducting, MHD fluid in a rotating reference frame through porous media under the presence of magnetic field which is mutually perpendicular to the velocity field. We employ complex variable technique to find out exact solutions for three different flow problems. We also study flows with isometric geometry. In the absence of rotating reference frame i.e. if $\mathbf{\Omega} = 0$ we recover the results of C. Thakur and Manoj Kumar[18]. Also when the porous media is absent i.e. the term $\frac{\eta}{K} \to 0$ and in a non-rotating system our results will tally with P. V. Nguyen and O. P. Chandna[17].

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Structural Stability of Long Lived Superheavy Nucleus $^{298}114$

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Abstract- Synthesis of superheavy nuclei has been achieved recently through hot fusion reactions. A systematic theoretical calculation of determining the stability of the superheavy nucleus Z=114 is studied in the context of angular momentum and temperature. The level density can provide clues to the applicability of the statistical model which is only correct for a high density of excited states. The collapse of pairing correlation at moderately higher spin and at low temperature causes a shape change and stability at higher spin is also expected. The drop in separation energy is closely associated with the structural changes in the rotating nuclei; relative increase in neutron emission probability around certain values of temperature may be construed as evidence for the shape transition. Such effects are not obtained for $^{298}114$. Hence this statistical study reveals a higher stability for $^{298}114$ against temperature and angular momentum.

Keywords: *superheavy nucleus; level density; separation energy; nucleon emission; shape transition.*

GJSFR-A Classification : *FOR Code: 020199*

STRUCTURALSTABILITYOFLONGLIVEDSUPERHEAVYNUCLEUS298114

Strictly as per the compliance and regulations of :

Structural Stability of Long Lived Superheavy Nucleus $^{298}114$

S. Santhosh Kumar [α] & A. Victor Babu [σ]

Abstract- Synthesis of superheavy nuclei has been achieved recently through hot fusion reactions. A systematic theoretical calculation of determining the stability of the superheavy nucleus Z=114 is studied in the context of angular momentum and temperature. The level density can provide clues to the applicability of the statistical model which is only correct for a high density of excited states. The collapse of pairing correlation at moderately higher spin and at low temperature causes a shape change and stability at higher spin is also expected. The drop in separation energy is closely associated with the structural changes in the rotating nuclei; relative increase in neutron emission probability around certain values of temperature may be construed as evidence for the shape transition. Such effects are not obtained for $^{298}114$. Hence this statistical study reveals a higher stability for $^{298}114$ against temperature and angular momentum.

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I. Introduction

We are living in an exciting period of time when new superheavy elements (SHE) are being discovered, one after another. This journey across the sea of instability has been possible because of a tremendous progress in theory, experiments and accelerator technologies. Upcoming radioactive ion beam (RIB) facilities now promise to lead us to the ultimate magic island where the neutron-rich SHE resides. Search for SHE in nature is the current interest of nuclear physicists, and are expected from terrestrial matter, meteorites and cosmic rays (Aleksandrov, 2013).

Microscopic nuclear theories suggest a significant enhancement in nuclear stability when approaching the closed spherical shells with $Z = 114$, or, $Z = 120$, 122 and $N = 184$. Still there is a dilemma whether 114 or/and 126, 120 is the proton shell closure and 172 or/and 184 is the neutron shell closure.

Several theoretical investigations have been carried out using the microscopic– macroscopic method and the self-consistent mean field in both the relativistic and non-relativistic formalisms. The primary aim in early studies has been to predict the combination of neutron number (N) and proton number (Z) where Spherical shell closure may occur. An ''island of stability'' had been predicted around the hypothetical doubly magic $^{298}114$ ($N = 184$) about 30 years ago. More recently, nuclei in this vicinity are expected to be spherical or almost so with longer half-lives. Most theories do predict $N = 184$ as being magic, however, there is no consensus on the location of the proton magic number due to differences in the treatment of the large Coulomb term and the spin–orbit interaction.

Microscopic-Macroscopic models, which assume a prior knowledge about the densities and single-particle potentials, include the Finite Range Droplet Model with folded Yukawa single-particle potentials (FRDM + FY) (Moller et al., 1997) and the Yukawa plus Exponential model with Woods–Saxon single-particle potentials (YPE + WS) (Muntian et al., 2003a, 2003b), both of which confirm the prediction of $^{298}114_{184}$ as being the next spherical doubly magic nucleus. Non-relativistic microscopic models such as the Skyrme–Hartree–Fock–Bogoliubov method (Cwiok et al., 1999), where the spin–orbit term has to be manually introduced, predict $Z = 120$ may be as probable as $Z = 114$, indicating that magic shells in this region are isotope dependent (Greiner, 1995; Rutz et al., 1997). Such techniques tend to overestimate the splitting of levels due to the spin–orbit coupling which may effect predictions for shell closures. With the large density of single-particle states which in turn characterizes this mass region, the SHE serve as a sensitive probe for distinguishing between the various theories that attempt to predict shell structure, especially when these models describe stable nuclei with comparable accuracy. Also, it has been known for some time that deformation effects are important to the understanding of stability in this region(Cwiok et al., 1999). Bohr and Mottelson (1969) have observed that deformation may enhance stability.

II. Importance of the Nucleus

The issue of the existence of super heavy nuclei is of utmost importance for understanding the properties of nuclear matter. It is highly interesting to verify the prediction of a significantly increasing stability of nuclei in the vicinity of the magic numbers Z=114 and N = 184, which could lead to the existence of “stability islands” of the relatively stable superheavy nuclei . The lifetimes of some isotopes of this superheavy nucleus is several seconds and even minutes, which exceeds tens

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of thousands of times the lifetimes of nuclei with smaller charges.

The knowledge of nuclear level densities is a crucial input in various fields/applications such as the description of excited nucleus properties and the nuclear reaction cross-section calculations for many branches of nuclear physics, nuclear astrophysics, nuclear medicine, and applied areas (medical physics, etc.) (Bethe, 1936; Gilbert and Cameron, 1965; Von Egidy and Bucurescu, 2005; Ignatyuk and Yu Shubin, 1969; Ahmadov et al., 2002; Okuducu et al., 2003; 2006a; 2006b; 2009; 2010; 2011; Bucurescu and Von Egidy, 2005; Kataria et al., 1978). The neutron capture cross-sections, required for both design and nuclear model calculations in nuclear science and technologies, are approximately proportional to the corresponding level densities around the neutron resonance region. In nuclear medicine, the cross-section data obtained from nuclear level density approaches are needed to optimize production of radioactive isotopes for therapeutic purposes, for example, biomedical applications such as production of medical radioisotopes and cancer therapy and accelerator-driven incineration/transmutation of the long-lived radioactive nuclear wastes.

Nuclear reactions calculations based on standard nuclear reaction models play an important role in determining the accuracy of various parameters of theoretical models and experimental measurements. Especially, the calculations of nuclear level density parameters for the isotopes can be helpful in the investigation of reaction cross-sections. The analytical expressions used for the nuclear level density calculations (Bethe, 1936; Gilbert and Cameron, 1965; Ignatyuk and Yu Shubin, 1969) are based on the Fermi gas model. The most widely used description of the nuclear level density is the Bethe formula, based on the thermodynamic relation between entropy and the average energy of a system considered in the framework of non-interacting particles of the Fermi gas. The traditional Bethe theory of the nuclear level density calculation, which uses the assumption that the individual neutrons and protons occupy a set of low energy levels in the ground state and fill up the higher individual states at any excitation energy, has been successfully used so far, with different contributions made to this model in the form of shell, pairing, deformation effects (Von Egidy and Bucurescu, 2005; Huang et al.,2000; Newton, 1956; Ericson, 1958), finite size effects (Bohr and Mottelson, 1969), and thermal and quantal effects, as well as improvements in the determination of the spin cutoff factors (Santhosh Kumar, 2009). However, such contributions do not take into account the collective effects, which may play a basic role in describing the nuclear level density of some deformed nuclides.

The level density is a fundamental property of a many-body system as all thermo-dynamical quantities can be derived from it. In nuclear physics, level densities are important because, according to Fermi's golden rule, they are critical for estimating nuclear reaction rates. The calculation of statistical nuclear reaction rates requires knowledge of the angular momentum distribution of the nuclear level density. An empirical formula for the angular momentum distribution of the level density at fixed excitation energy assumes uncorrelated and randomly coupled single-particle spins.

The compound nucleus formed either through cold (Pb or Bi targets) or hot(actinide targets) fusion reactions are in excited state and hence their decay will be greatly influenced by thermal and collective excitations. Hence a statistical model approach will be more suitable and the code is developed pertaining to the evaluation of sp level density, separation energy and emission probability, and the sp energies are obtained by cranked Nilsson model.

III. Theoretical Formalism

The statistical quantities like excitation energy, level density parameter and nuclear level density which play the important roles in the nuclear structure and nuclear reactions can be calculated theoretically by means of the Statistical or Partition function method. In this work we have followed the statistical model approach to probe the dynamical properties of the nucleus in the microscopic level.

The nuclei formed in collision may be in excited states and hence their decay or emission for stability will greatly influenced by thermal and collective excitation. Hence a thermo dynamical approach, which incorporates thermal and rotational excitations, is the appropriate methodology.

The statistical theory of hot rotating nucleus can be easily obtained from the grand canonical partition function

$$Q(\alpha_{Z,}\alpha_N,\beta,\gamma) = \sum \exp\left(-\beta E_i + \alpha_Z Z_i \alpha_N N_i + \gamma M_i\right.$$

The Lagrangian multiplier γ plays the same role as the rotational frequency as in the cranking term ω.JZ. The pair breaking term γ-mj is temperature dependent and will generate the required angular momentum. The temperature effect creates particle hole excitation.
The total excitation energy is obtained using

$$E^* = U(M,T) = U_{eff}(T) + E_{rot}(M)$$

The level density parameter a(M,T) as a function of angular momentum and temperature is extracted using the equation

$$a(M,T) = \frac{S^2(M,T)}{4U(M,T)}$$

where S is the entropy and U is the total excitation energy. The neutron or proton separation energy is obtained from (Rajasekaran, 1988),

$$S_n = -T\frac{(\partial \ell_n Q)}{(\partial \propto_N)}\frac{(\partial \propto_N)}{(\partial N)}$$

where N is the number of neutrons or protons. The dependence of the nuclear level density ρ, on angular momentum M, can be written as

$$\rho(U,M) = \left\{\frac{(2M+1)}{2\sigma^2}\right\} exp\left\{\frac{[-M(M+1)]}{2\sigma^2}\right\}\rho(U)$$

where ρ(U) is the level density and is given by

$$\rho(U) = exp\frac{[2(a(U-E_i)^{1/2}]}{12(2\sigma^2)^{1/2}a^{1/4}(U-E_i)^{5/4}}$$

IV. Results and Discussion

In this work we have studied the structural stability of hot rotating SHN $^{298}114$ using statistical theory. The angular momentum impact at a particular temperature is mainly considered here especially on excitation energy, single neutron separation energy and nuclear level density. The shape deformation due to spin and temperature effect is also determined.

The system shows a spherical shape at low spin and temperature T< 0.8 MeV. From spin J=18η it shows an oblate deformation (γ=-180°; δ=0.1) and which increases with temperature. From the excitation energy Vs spin plot (Fig.1) it is evident that there is a drop in excitation energy at certain angular momentum and temperature. At about T = 0.8MeV, the excitation energy becomes a smooth Gaussian (Fig.2), and the shape of the nucleus become spherical (δ=0.0).

The neutron separation energy is an important parameter in determining the stability of the nucleus against particle decay/emission. The nucleus $^{298}114$ shows a decreasing effect of neutron separation energy to angular momentum up to T≤0.8MeV (Fig.3), and which decreases gradually with temperature. At higher temperatures, ie., T≥1.0MeV, the neutron separation energy becomes almost constant, which indicates the plasma state of the nucleus.

Since the SHN formed through fusion reaction will reach the stable state mainly via alpha decay and terminate with spontaneous fission, the proton separation energy also gets equal importance in determining the stability against angular momentum. In fig.3, we have shown its effect at T=1.0MeV, in comparison with the effect of neutron decay possibility and we found both are in similar track while the excitation energy shows an exponential growth with spin in the energy range from 42MeV to 49MeV, and hence this nucleus will not undergo any single particle decay instead possibility of alpha decay is highly probable.

The level density parameter against spin is plotted in Fig.4, for low temperatures. The shift in level density parameter at J=16ℏ, 20ℏ and 22ℏ for temperatures T=0.3MeV, 0.4MeV and 0.5MeV respectively, may a signature for collapse of pairing correlation.

V. Conclusion

For the system $^{298}114$, the observed changes in excitation energy, separation energy (neutron & proton) and level density parameter, the increase in deformation from spherical to oblate may a signature of increased stability as stated by Bohr & Mottelson (1969), at low temperatures. The shift in single particle binding energy reveals its phase change at higher temperatures. Due to the collapse of pairing correlation at spin J>14ℏ at low temperatures causes a shape change and provides stability at higher spin.

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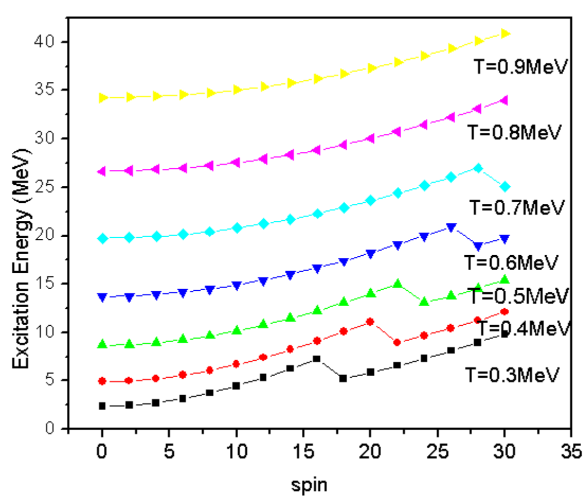

Figure 1 : Excitation energy Vs. spin($\hbar$); The drop in excitation energy is due to the shape transition.

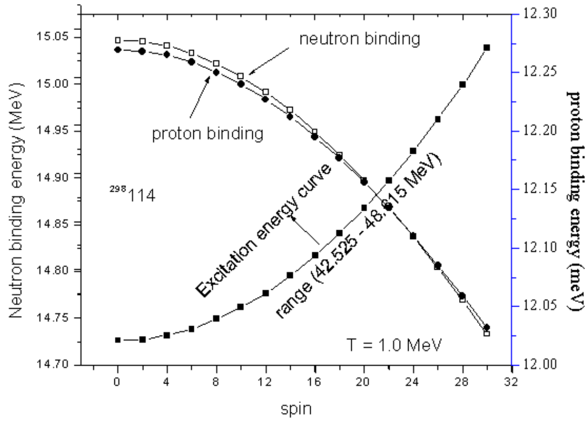

Figure 2 : Particle separation energy Vs. spin($\hbar$) (Excitation energy plot is not to axis scale)

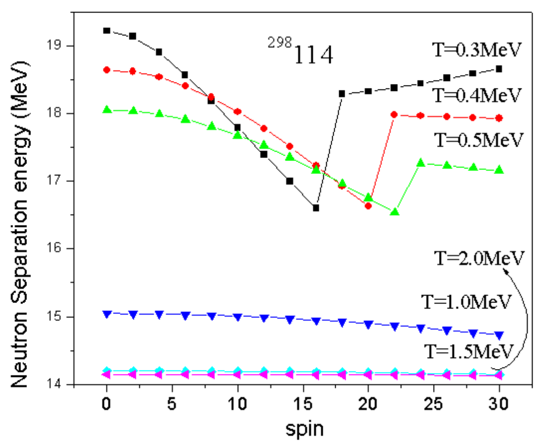

Figure 3 : Influence of spin (in units of $\hbar$) on Neutron separation energy at different Temperatures

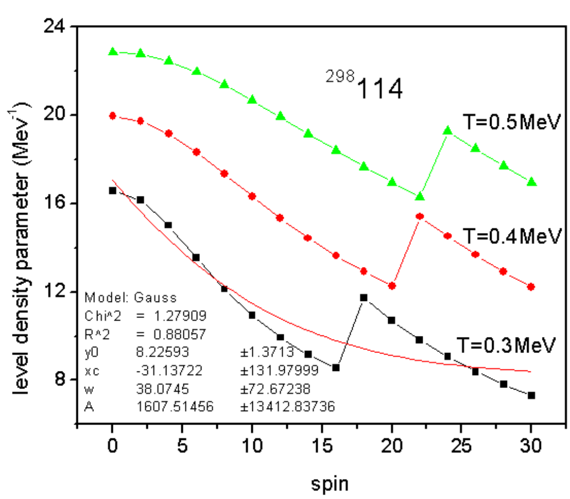

Figure 4 : Pairing phase transition at different spin with respect to temperature

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Numerical Simulation of Wave Equation

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Abstract- Wave equation is a very important equation in applied mathematics. This equation is used to simulate large destructive waves in fjord, lake, or the ocean generated by slides, earthquakes, subsea volcanoes, meteorites. It has analytical solution but it is time consuming. Therefore one needs to use numerical methods for solving this equation. For this we investigate finite difference method and present explicit upwind difference scheme for one dimensional wave equation, central difference scheme for second order wave equation. We implement the numerical scheme by computer programming for initial boundary value problem and verify the qualitative behavior of the numerical solution of the wave equation.

Keywords: linear advection equation, equation of continuity, wave equation, central difference scheme, explicit upwind difference scheme.

GJSFR-A Classification : *FOR Code: 029999*

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Md. Saiduzzaman [α], Md. Yeadul Islam Shaikh [σ] & SM Sala Uddin Samrat [ρ]

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Keywords: linear advection equation, equation of continuity, wave equation, central difference scheme, explicit upwind difference scheme.

I. Introduction

Wave equation is used to understanding of tsunamis, assists warning systems, assists building of harbor protection (break maters), recognize critical coastal areas, hind cost historical tsunami (assist geologists). It has analytical solution but it is time consuming. How over with rapid developments of numerical methods and computer technology the system can be solved numerically. Numerical simulation[1] is very much challenging. Many scientific groups are involved in dealing with this problem. The aim of this thesis will be provide an easy way to solving wave equation. In this thesis we use finite difference scheme known as central difference scheme [3], [4], explicit upwind difference scheme [8],[6].

In the first section we introduced symbols and notations. In the second section we introduced the first and second order wave equation, method of characteristics, D'Alembert solution and analytical solution of the wave equation. In the third section we discussed numerical methods. In the last we discussed about our numerical experiments and results.

II. Mathematical Models and Methods

a) Symbols and Notation

Let $\Omega \subset R^d, d \in \{1,2\}$ be a region occupied by a fluid flow, and let $[t_0, T]$ be a time interval with $0 \le t_0 \le T$. An arbitrary point in Ω is denoted by $X = (x_1, x_2, ..., x_d)^T$. For the description of a general unsteady compressible fluid flow, we introduce the quantities:

The density $\rho = \rho(x,t)$, the velocity vector $V = V(x,t) = (v_1(x,t), ...v_d(x,t))^T$, the pressure $P = P(x,t)$ the energy density $E = E(x,t)$. We denote the density of the external force by $f = f(x,t) = (f_1(x,t), f_2(x,t), ... f_d(x,t))^T$, the mass flux by $q = q(x,t)$ for the description of the viscous flow, let λ and μ denote the coefficient of kinetic viscosity respectively.

b) The equation of continuity [5] is given by

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho V) = 0$$

c) First order wave equation

The first -order wave (advection) equation [6] is $(c > 0)$

$$\frac{\partial u}{\partial t} + c\frac{\partial y}{\partial x} = 0, u(x,0) = u_0(x)$$

d) Wave equation in 1d

One dimensional wave equation [2] is

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \tag{1}$$

Initial condition:

$$u(0,x) = u_0(x) \quad \text{and} \quad u_t(0,x) = v_0(x)$$

Formally, we can write Laplace equation as:

$$u_{tt}(t,x) - c^2 u_{xx}(t,x) = \left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial x}\right)\left(\frac{\partial}{\partial t} - c\frac{\partial}{\partial x}\right)u(t,x) \tag{2}$$

e) D'Alembert's solution

We get the characteristics system from (2) as

$$\frac{d}{dx}(x) = \pm c \tag{3}$$

Integrating both side of (3) then we get

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$$x = ct + \xi \text{ (when c is positive)} \tag{4}$$

Solving (2.24) and (2.25) then we get

$$x = \frac{1}{2}(\eta + \xi) \quad \text{and} \quad t = \frac{1}{2c}(\eta - \xi)$$

$$x = -ct + \eta \text{ (when c is negative)} \tag{5}$$

Introducing new variable $\eta = -x + ct$ and $\xi = x - ct$

We consider,

$$w(\eta,\xi) = u(t,x) = u[\frac{1}{2c}(\eta - \xi), \frac{1}{2}(\eta + \xi)] \tag{6}$$

$$\frac{\partial w}{\partial \eta} = \frac{\partial u}{\partial t}.\frac{\partial t}{\partial \eta} + \frac{\partial u}{\partial x}.\frac{\partial x}{\partial \eta} = \frac{1}{2c}\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial u}{\partial x}$$

$$\frac{\partial^2 w}{\partial \xi \partial \eta} = -\frac{1}{4c^2}\frac{\partial^2 u}{\partial t^2} + \frac{1}{4c}\frac{\partial^2 u}{\partial x \partial t} - \frac{1}{4c}\frac{\partial^2 u}{\partial x \partial t} + \frac{1}{4}\frac{\partial^2 u}{\partial x^2}$$

$$= -\frac{1}{4c^2}[\frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2}] = 0 \text{ (using 1)} \tag{7}$$

Integrating (6) with respect to ξ we get,

$$w_\xi = f(\eta)$$

Again Integrating (6) with respect to η we get,

$$w = f(\eta) + g(\xi) \tag{8}$$

Now we get from (6) and (8)

$$w = f(\eta) + g(\xi)$$

$$= f(x+ct) + g(x-ct) \tag{9}$$

This is the general form of solution of (1).

We use initial condition to determine f and g. For t=0 we get,

$$u(0,x) = u_0(x) = f(x) + g(x) \tag{10}$$

$$u_t(0,x) = v_0(x) = [\frac{\partial f}{\partial \eta}.\frac{\partial \eta}{\partial t} + \frac{\partial g}{\partial \xi}.\frac{\partial \xi}{\partial t}]_{t=0}$$

$$cf'(x) - cg'(x) \tag{11}$$

Integrating (11) we get,

$$f(x) - g(x) = \frac{1}{c}\int_0^x v_0(s)ds + k \tag{12}$$

Solving (10)and(12)we get,

$$f(x) = \frac{1}{2}u_0(x) + \frac{1}{2c}\int_0^x v_0(s)ds + k \tag{13}$$

$$f(x) = \frac{1}{2}u_0(x) + \frac{1}{2c}\int_0^x v_0(s)ds + k \tag{14}$$

From (9) we get,

$$u(t,x) = \frac{1}{2}\{u_0(x+ct) + u_0(x-ct) + \frac{1}{2c}\int_{x\ \ ct}^{x+ct} v_0(s)ds$$

This is called D'Alemberts solution for 1-D wave equation [7].

f) Method of characteristic

Consider the IVP

$$u_x(t,x) + cu_x(t,x) = 0, -\infty < x < \infty, t > 0$$

$$u(0,x) = f(x), -\infty < x < \infty,$$

Where c=constant. If we measure the rate of change of u from moving position given by the chain rule,

$$\frac{d}{dt}u(t,x(t)) = u_t(x(t),t) + u_x(t,x(t))x'(t)$$

The first term on the right-hand side above is the change in u at a fixed point x while the second one is the change in u resulting from the movement of the observation position.

Assuming that $x'(t) = c, c$ from the PDE we see that

$$\frac{d}{dt}u(t,x(t)) = u_t(t,x(t)) + cu_x(t,x(t)) = 0$$

That is $u =$ constant as perceived from the moving observation point. The position of this point is obtained by integrating its velocity $x'(t) = c\ \ c$

$$x = x_0 + ct, x_0 = x(0)$$

This formula defines a family of lines in the (x,t)-plane, which are called characteristics (Fig.1.1). As mentioned above, the characteristics have the property that $u(t,x)$ takes a constant value along each one of them (but in the integral, different constant values on different characteristics)

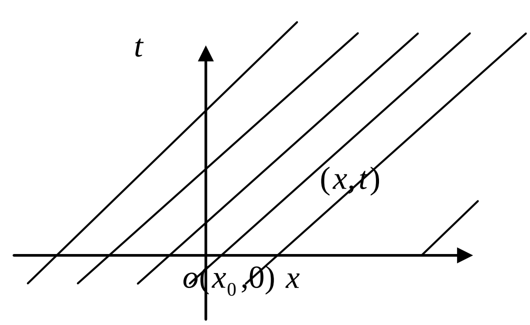

Figure 1.1 : Characteristic plane

Global Journal of Science Frontier Research (A) Volume XIV Issue VII Version I Year 2014

Hence, to find the value of the solution u at (x,t) we consider the characteristics through (x,t) of equation $x = ct + x_0$ which intersects the x axis at $(x_0,0)$ .Since u is a constant on this line, its value at (x,t) is the same as at $(x_0,0)$ .But the latter is known from the IC, so

$$u(t,x_0) = f(x_0)$$

The parameter is now replaced from the equation of the characteristics line: $x_0 = x - ct$ .Thus the solution of the given IVP is

$$u(t,x) = f(x-ct)$$

This formula shows that at a fixed time t the shape of the solution is the same as at $t = 0$,0, but is shifted by ct along the x axis. In other words, the shape of the initial data function travels in the positive (negative) x direction with velocity c if $c > 0 (c < 0)$ which means that the solution is a wave.

III. Numerical Methods

a) Explicit Upwind Difference Scheme

We now describe the explicit upwind difference scheme for example we take linear advection equation.

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0 \qquad (15)$$

With initial condition $u(x,0) = u_0(x)$

and left-hand boundary condition $u(a,t) = u_a(t)$

b) Central difference scheme for Wave equation

In this chapter we investigate the finite difference scheme for the Wave equation [7].For a given velocity u the general form of wave equation is

$$u_{tt}(t,x) = c^2 u_{xx}(t,x), 0 \le x \le a, 0 \le t \le b \qquad (16)$$

With boundary condition $u(0,t) = 0$ and $u(a,t) = 0$ for $0 \le t \le b$ and initial condition is $u(x,0) = f(x)$ and $u_t(0,x) = g(x)$ for $0 < x < a$.

For equidistance grid, with temporal step size Δt and spatial step size Δx the discretization of $u_{tt}(t,x)$ and $u_{xx}(t,x)$ we use the central difference formula then we get

$$u_{tt}(t,x) = \frac{u(t+\Delta t,x) - 2u(t,x) + u(t-\Delta t,x)}{\Delta t^2} + o(\Delta t^2) \qquad (17)$$

And

$$u_{xx}(t,x) = \frac{u(t,x+\Delta x) - 2u(t,x) + u(t,x-\Delta x)}{\Delta x^2} + o(\Delta x^2) \ldots.(18)$$

We consider uniform grid that is $x_{i+1} = x_i + \Delta x$ and $t^{n+1} = \Delta t$ .Drop the terms $o(\Delta t^2)$ and $o(\Delta x^2)$ and use the approximation u_i^n for $u(x_i,t^n)$ in (17) and (18). Then for equation (15) we get

$$\frac{u_i^{n+1} - 2u_i^n + u_i^{n-1}}{\Delta t^2} = c^2 \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2}$$

$$u_i^{n+1} = (2 - 2\gamma^2)u_i^n + \gamma^2(u_{i+1}^n + u_{i-1}^n) - u_i^{n-1}$$

Where $\gamma = c\dfrac{\Delta t}{\Delta x}$

This is the central difference scheme for wave equation.

Stencil:

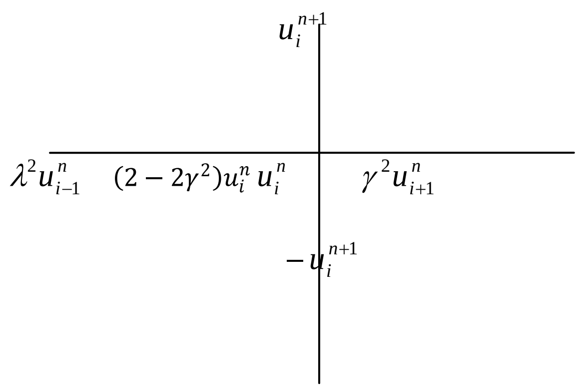

Figure 1.2 : Stencil of wave equation

IV. Numerical Experiments and Results

We develop a computer program (code) in Matlab scientific programming language and the implement the central difference scheme for wave equation.

a) Inserting data

We implemented the central difference scheme for the numerical simulation for the wave equation. We implemented the scheme for artificial initial and boundary data and verify the qualitative behavior of the numerical solution of wave equation. The main parts of the implementation of our numerical schemes are given in the following algorithm.

Input: nt and nx the number of grid points of time and space respectively.

If T the right end point of [0,T]

x_b the right end point of [0,b]

u_0 the initial velocity apply as initial condition.

u_b the velocity at the boundary point, apply as a boundary value.

Output: $u(t, x)$ the solution matrix.

Initialization: $dt = \dfrac{t-0}{nt}$ the temporal grid size.

$dx = \dfrac{b-0}{nx}$ the spatial grid size.

$c = 0.5$ a constant

$gm = c * \dfrac{dt}{dx}.$

$gma = 2 - 2 * gm * gm.$

Step 1: calculation for the scheme

for $j = 3 : nt\, nt$

for $i = 2 : nx - 1$

$u(j,i) = gma * u(j-1.i) + gm * gm * (u(j-1,i+1)$
$u(j-1,i-1)) - u(j-2,i)$

Step 2: Output $u(t, x)$.

Step 3: Graph presentation.

Step 3: Stop.

b) Results

To test the accuracy of the implementation of the numerical scheme for wave equation, we discuss our experiment and results under generating the cases.

Case 1: In this case we considered the first order wave equation (linear advection) $\dfrac{\partial u}{\partial t} + c\dfrac{\partial u}{\partial x} = 0$.

We considered the analytical solution of one dimensional first order wave equation and performed the experiment in matlab. We choose the parameters as $\Delta t = 0.8, \Delta x = 6.28, c = 0.5, x = (0,2\pi), t = (0,101)$ with initial condition $u(x,0) = \sin x$ and boundary condition $u(x,0) = t^2$ and $u(x,t) = 0$ and run the propagation for 101 ts. In this case we see the figure in plot form and mesh form as follows.

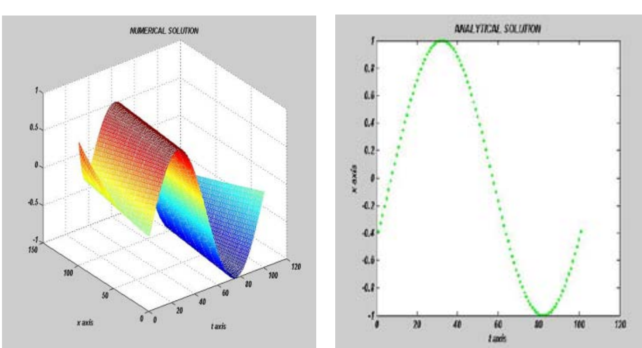

Figure 1 : Analytical solution of first order wave equation in plot form and mesh form

Case 2: In this case we consider the first order wave equation(linear advection) $\dfrac{\partial u}{\partial t} + c\dfrac{\partial u}{\partial x} = 0$ and perform the numerical experiment for three time step. We choose $\Delta t = 0.8, \Delta x = 6.28, c = 0.5, x = (0,2\pi), t = (0,101)$ with initial condition $u(x,0) = \sin x$ and boundary condition $u(x,0) = t^2$ and $u(x,t) = 0$ and run the propagation for 101 ts. We present the solution for the three different time step as shown in the figure. As expected we observer that the initial value in moving forward with respect to time.

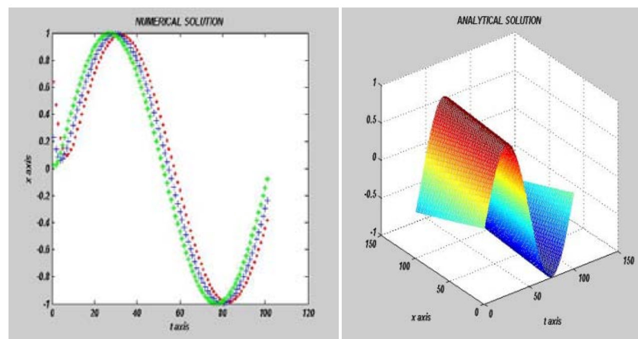

Figure 2 : Numerical solution of first order wave equation in plot form and mesh form using explicit upwind difference scheme

Case 3: In this case we consider the analytical solution of the wave equation $\dfrac{\partial^2 u}{\partial t^2} = c\dfrac{\partial^2 u}{\partial x^2}$ and perform the experiment in matlab. We choose the parameters as $\Delta t = 0.1, \Delta x = 6.28, c = 0.5, x = (0,2\pi), t = (0,101)$ with initial condition $u(x,0) = \sin x$ and boundary condition $u(x,0) = t^2$ and $u(x,t) = 0$ and run the propagation for 101 ts. In this case we see the figure in plot form and mesh form as follows.

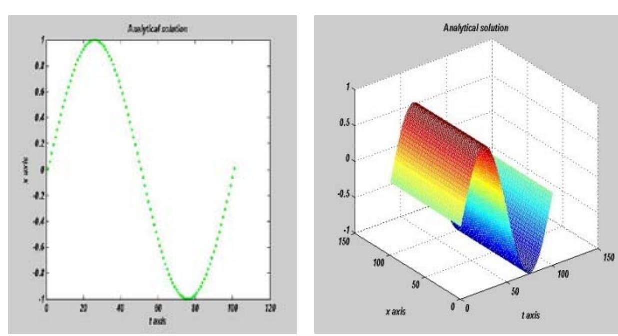

Figure 3 : Analytical solution of 2D wave equation in plot form and mesh form

Case 4: In this case we perform the numerical experiment for the equation $\frac{\partial^2 u}{\partial t^2} = c\frac{\partial^2 u}{\partial x^2}$ and perform the experiment in matlab. We choose the parameters as $\Delta t = 0.1, \Delta x = 6.28, c = 0.5, x = (0,2\pi), t = (0,101)$ with initial condition $u(x,0) = \sin x$ and boundary condition $u(x,0) = t^2$ and $u(x,t) = 0$. In this case the initial configuration is not moving forward but only smears out which is typical behavior of the wave equation.

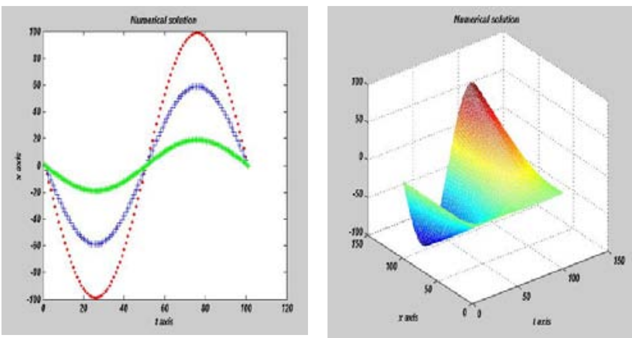

Figure 4 : Numerical solution of 2D wave equation in plot form and mesh form using central difference scheme

V. CONCLUSION

In this paper we considered wave equation which is a fundamental partial differential equation in fluid mechanics. First we showed equation of continuity, first order wave equation, second order wave equation, method of characteristics, D'Alembert solution. Finally we showed the numerical result of first order wave equation based on explicit upwind difference scheme and second order wave equation based on central difference scheme agrees with basic qualitative behavior of this equation.

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Dynamic Parameters of the Space Environment (Space Ether's Dynamics)

By Stanislav Ivanovich. Konstantinov

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Abstract- This paper presents a new dynamic approach to studies of the space environment. The author has explored a mechanism to produce vortex flows of muons and π -mesons in the space, their stability at superlight velocities. It submits numerical values of parameters for the ether's dynamic space environment and presents more details for the principle of mass equivalence.

Keywords: *neo-ether, ether's dynamics, dipole, medium density, pressure, temperature, dynamic viscosity, kinematic viscosity, muon, π-meson, reynolds number, equivalence of mass equivalence, roche limit.*

GJSFR-A Classification : *FOR Code: 020199p*

DYNAMICPARAMETERSOFTHESPACEENVIRONMENTSPACEETHERSDYNAMICS

Strictly as per the compliance and regulations of :

Dynamic Parameters of the Space Environment (Space Ether's Dynamics)

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I. Introduction

Today, science has irrefutable arguments to say that a concept of the space environment goes far beyond a concept of the physical vacuum. Within a standard model, recent astronomical observations made by Chinese scientists [1] have not found their explanation. From the perspective of the general relativity theory, it is impossible to explain anisotropy of three-degree background thermal radiation [2] discovered by American scientists from the NASA. A nature of superluminal radiation by N. Kozyrev also has remained a mystery, as well as of torsion radiation by G. Shipov [3]. We need a new approach to assessing a role of the space environment in dynamics of celestial bodies and its nature.

The neo-ether theory by A. Rykov [6], where electron-positron dipoles form a gas-alike medium, allows applying laws of hydro and aerodynamics for a preliminary numerical calculation of its dynamic parameters. There are medium density, pressure, temperature, dynamic viscosity, kinematic viscosity, and other gas-dynamic parameters. Comparing estimates for planets' motion in neo-ether, got with their help, with results of astronomical observations, one can better understand a nature of the universe laws. As anobject to be observed, let's choose two planets within the Solar System, our Earth rotating along its stationary orbit around the Sun, and the Mercury, the orbit of which is subject to strong perturbations due to a close location of the planet to the gravity centre. Along with their orbital motion around the Sun, planets, together with the entire Solar System, are moving toward the centre of the Galaxy. At the same time, a spiral motion of planets in the space forms a toroid. As a planet is moving in the space environment, a toroidal spiral vortex is blowing from its mid – its central channel – a spiral flow of the ether. A progressive motion of the flow is converted into a toroidal motion of the ether around the planet. This motion follows the Biot-Savart law and extends to an area of the space quite remote from the planet [4]. As a central channel of a celestial body is not always oriented towards the Earth, the radiation, found by N. Kozyrev, is difficult to be reproduced. [3]. A rotational motion of the planet follows ether-dynamics laws. In case of a toroidal motion, one volume of the ether involves another by exposing it to a direct pressure, while in case of a ring motion neighbour layers are involved due to the ether's viscosity. This is a cause for the toroidal motion that will cover the entire surrounding environment, while the ring motion may have two conditions. One covers the surrounding area and another one is localized within a certain boundary layer. A location of planets close to a spherical form facilitates numerical assessments of an impact of the space environment on their motion. At that, it is necessary to take into account polarization of the ether next to massive celestial bodies that leads to appearance of extra ethereal atmosphere around planets. Such extra ethereal atmosphere rotates together with the planet. [6] This allows us to standardize calculations of dynamic parameters for the neo-ether and proceed to considerations of a structural element of the space environment, i.e. π -meson (muon), made with a cluster of 137 dipoles of virtual pairs: electron + positron as a trial sphere. [6]

a) Ether density - ρ [kg · m $^{-3}$]

Rotation of a structural element of the ether with r radius produces a rotation field around π -meson. Mechanical energy of a velocity field with constant ether's density is [5]:

$$\mathrm{Wv} = 2\pi\rho v^2\, r^{3} \qquad (1)$$

where ρ is ether's density,
r is a π-meson radius,
v is a motion velocity of ether's structural elements.

For π-meson with q charge, electric energy We is [5]:

$$\mathrm{We} = \frac{q^2}{8\pi\varepsilon_0\,\varepsilon\, r}, \qquad (2)$$

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where εo is the electric constant,
ε is relative permittivity.

Having compared formulae for mechanical energy of the velocity field Wv (1) with electric energy of the field for π-meson charge We (2), we have:

$$2\pi\rho v^2 r^3 = \frac{q^2}{8\pi\varepsilon o\, \varepsilon\, r} \quad (3)$$

Hence,

$$v = \frac{q}{4\pi r\sqrt{\varepsilon o \varepsilon \rho}} \quad (4)$$

From the equation (4) we have found that

$$\rho\,(v \cdot S)^2 = \varepsilon o \varepsilon \left(\frac{q}{\varepsilon o \varepsilon}\right)^2 \quad (5)$$

S is a sphere surface for π-meson, around v velocity.

Thus, values **εoε** and q in formula (5) receive their simple interpretation:

εoε corresponds to **ρ**,
q corresponds to **ρ**·v·S,

Where dielectric permittivity is the ether's density, while a charge is a surface circulation of ether's rings [5].

$$\rho = 8.85 \cdot 10^{-12}\ \mathrm{kg/m^3}$$

b) Ether's pressure - $P\ [n \cdot m^{-2}]$.

To determine the ether's pressure (P) let us apply a ratio of a photoelectric effect for the ether [6]. A photon with energy W=1 MeV $=1.6\cdot10^{-13}$ Joule turns into a pair electron-positron, i.e. the photon destroys a dipole structure in the ether. At the same time, limit deformation of dr dipole, with which an interaction between a virtual electron and positron in the dipole reduces to zero, is $dr=1.02\cdot10^{-17}$. Hence, a force arising from the electron separated from the positron,

$$F = \frac{dW}{dr} \quad F = 1.6 \cdot 10^4 N \quad (6)$$

As for π- meson of the space ether, it has a looser structure than a nuclear one, To a first approximation, we can take its area size equal to the area of a dipole cross section $Sd=\pi r^2$, where r=1, $4\cdot10^{-15}$ m is a structural element of the cosmic ether, equal to a distance between centres of a pair of charges in the dipole [6]. Hence, the pressure in the ether will be

$$P = \frac{F}{S} \quad P = 0{,}2 \cdot 10^{34}\ N \cdot m^{-2} \quad (7)$$

c) Speed of ether's structural elements - V [$m\cdot s^{-1}$].

A motion speed of π-meson in the ether can be determined from formula (4)

$v = e/4\pi r^2 \rho$, $\quad v=3\cdot10^{21}\ m\cdot s^{-1}$,

where e is π-meson charge, $e = 1.6\cdot10^{-19}$ kl
r is radius of the ether's π-meson, $r=1.4\cdot10^{-15}$ m
ρ is the ether's density, $\rho=8.85\cdot10^{-12}$ kg/m³

d) Ether's dynamic viscosity - η [$n\cdot m^{-1}\cdot s^{-1}$]

Dynamic viscosity (internal friction coefficient) η can be found from the Newton's equation [5]

$$dF = \eta \cdot \left(\frac{dv}{dx}\right) \cdot dS \qquad \eta = P \cdot \frac{dr}{dv} \quad (8)$$

where: dr is dipole limit deformation,
v is a relative velocity of ether's structural units,
P is ether's pressure

The ether's dynamic viscosity $\eta=6\cdot10^{-6}\ n\cdot m^{-1}\cdot s^{-1}$

e) Kinematic viscosity [$m^2\cdot s^{-1}$]

Kinematic viscosity χ equals to a ratio of dynamic viscosity (η) to ether's density (ρ)

$$\chi = \frac{\eta}{p} \qquad \chi = 7 \cdot 10^5 \quad (9)$$

f) Temperature of the space ether – T

The temperature of the space ether was found in 1965 by A. Penzias and R. Wilson while making measurements for background radiation of the space environment in a microwave range 10GHz-33 GHz and equals to T=2.7 degrees Kelvin [2]. A value of the ether's temperature obtained from astronomical observations, T = 2.7 K, is significantly higher than the temperature obtained with calculations based on gas representations $T=7\cdot10^{-51}$ K. This suggests that the space environment cannot be in full compared with the gas. There are electro-dynamic processes described with solutions of equations within the Unitary Quantum Theory by Professor L. Sapogin [9].

f) In addition, the state of superconductivity and superfluidity can significantly change the dynamic characteristics of the space environment [11].

Now it is possible to assess sustainability of the space environment, in particular its structural elements. It should be noted that according to certain data, besides π -mesons muons may act as structural elements of the space ether, while π-mesons in nuclear and nucleon ether. Under experimental conditions, with V<C these particles are unstable; an average lifetime of a muon is 10^{-6} s, while of a π-meson is 10^{-8} s. However, from experiments we have established that with increasing velocity of particles their lifetime also increases.

There is a dependence between a nature of structural elements making a gaseous or liquid medium and the Reynolds number. In this regard, it would be interesting to determine the Reynolds number that characterizes a muon (π-meson) behaviour in the space ether. [5]. A particle diameter is a value of about d $(1.3\text{-}1.4)\cdot10^{-15}$ m, which can be taken as an indicative size. A motion speed of structural elements around a muon (π-meson) is about $3\cdot10^{21} m\cdot s^{-1}$. Taking that into account, kinematic viscosity of the ether $\chi=7\cdot10^5\ m^2\cdot s^{-1}$, we find Re

$$Re = \frac{v \cdot d}{\chi} \quad Re = 6 \quad (10)$$

The theory of liquids and gases recognises a statement that vortices start appearing when the

Reynolds number is over 1000. Hence, a muon (π-meson) in the near Earth's ether can be regarded as a stable vortex entity in the space environment with an established structure, which does not depend on either initial or edge conditions available in time of its appearance.

Having defined ether's dynamic parameters of the space environment and its stability, one can analyse some specifics of Kepler-Newton's celestial mechanics. In particular, a difference in a value of the Kepler constant for the Mercury (K=3.33) and terrestrial planets (K = 3.35).

The ancestor of celestial mechanics J. Kepler in the early 17th century, based on multi-year astronomical observations by Tycho Breguet, established empirically three laws of planetary motion relative to the Sun.

Unless evolutionary processes in the Solar System disturb stability of planets' rotation, these laws would not change.

Kepler's third law states, "The ratio of period squares of any two planets is a ratio of cubes of their large semi-axes of elliptical orbits, along which they rotate around a central body." [7]. This implies that the ratio of the cube of the orbit radius to the square of the orbit time of the planet is constant

$$K = \frac{R^3}{T^2} \tag{11}$$

where K is Kepler's constant.

Kepler calculated K values for all planets known to him in the Solar System:

$$K = (3.33 - 3.35)10^{24}\ \text{km}^3 \cdot \text{year}^{-2}$$

Half a century after Kepler, Newton introduced forces into the spatial model of the universe [7]. The space of the universe produces gravity and inertia forces acting following quadratic laws of interaction between bodies (laws by Coulomb and Cavendish).

Having articulated his laws of dynamics and universal gravitation, Newton got Kepler's third law as consequence of the universal gravitation law and the second law of dynamics as follows:

$$K = GM\frac{m\ gr.}{m\ in.} = \frac{R^3}{T^2}. \tag{12}$$

where

m gr. is the planet gravitational mass, interacting with the Sun, the M mass, produces a centripetal force of gravity;

m in. is the inertial mass of the planet. It is rotating around a circle of R radius and producing a centrifugal force of repulsion,

R is a distance from the centre of the planet to the centre of the Sun,

T is a period of the planet rotation around the Sun,

G is the gravitational constant.

In time of planets' motion along their stationary orbits, resistance of the space environment almost only depends on friction forces. Classical physics to describe such a motion applies the Stokes' law. Stokes found that the resistance force in this case is proportional to dynamic viscosity coefficient η, velocity of a body in relation to V environment, and a distinctive size of a body L: F ~ u v L.

The proportionality factor depends on a shape of a body. For a sphere, if as a radius L we take a radius of the sphere r, then the proportionality factor is equal to 6π [4]. Hence, the force of resistance to the motion of the sphere in the medium (ether), in accordance with the Stokes' formula, is:

$$F = 6\pi\,\eta\, r\, v \tag{13}$$

Having known orbital velocities of the Mercury and the Earth in the Solar System and planets' radii, let us calculate the friction force based on the Stokes' law (13).

Mercury' speed Vm = $7.5 \cdot 10^4$ m · s^{-1}.

Mercury' radius Rm = $2.5 \cdot 10^6$m [7]

Earth's velocity Ve = $3 \cdot 10^4$ m · s^{-1}.

Earth's radius Re = $6.3 \cdot 10^6$m [7]

$$Fm = 6 \cdot (3.14) \cdot (7.5 \cdot 10^4) \cdot (2.5 \cdot 10^6) \cdot (10 \cdot 6^{-6}) = 21 \cdot 10^6\ \text{N}.$$

$$Fe = 6 \cdot (3.14) \cdot (3 \cdot 10^4) \cdot (6.3 \cdot 10^6) \cdot (10 \cdot 6^{-6}) = 30 \cdot 10^6\ \text{N}$$

Let us define the energy, spent by the Earth to overcome the ether's friction per second of orbital motion Efr=Fe·Ve, Efr = $9 \cdot 10^{11}$ J. One can compare this energy with a tidal deceleration of the Earth by the Moon per second Etd = $25 \cdot 10^{11}$ J. As can be seen from a comparison of Efr and Etd, the energy of the Earth's deceleration with the ether is 35% of the energy of the Moon's deceleration of the Earth, while a contribution of the Moon tide does not exceed 1% in the total Earth's energy [10].

Results of calculations show that in case of an equilibrium nature of planets motion along their orbits, the resistance force for the Mercury and the Earth are of the same order, but in case of strong disturbances imposed by the Sun on a trajectory of Mercury's motion, the picture changes. In case of a non-equilibrium state of the system, a speed of a body increases, its vector is constantly changing, there are vortices appearing behind the body. At the same time, an energy of vortices is actively influencing the system "from the outside" (from a side of the environment). Pressure in a vortex area formed behind the body, will be reduced, so the resultant of pressure forces will be non-zero, determining in its turn any resistance. As a result, frontal resistance consists of frictional resistance and pressure resistance. The ratio between the frictional resistance and the pressure resistance depends on the Reynolds number (Re). The more Re is, the more a role of pressure resistance is. Hence, a transition of the system from a stable state to an unstable one, its non-equilibrium state, would be accompanied with a growth of ether's vortices. The growth would counteract a change to the state of the system, i.e. generating an additional field of inertia, which is stronger, when the greater disturbance influences the environment [4].

Let us pay attention to a difference in a value of the Kepler constant K for terrestrial planets, such as the Venus, the Earth, the Mars, rotating along stable, seldom-perturbed orbits, for which value K = 3.35, and the Mercury, the orbit of which is subject to strong perturbations due its close location to the Sun. For the Mercury, value K is 3.33, that is 1% less than that for planets with stable orbits. Perhaps, this results from vortex ether-dynamic forces in the space environment responding to its perturbation by the Mercury. At the same time, the inertial field increases and because a value of K depends on the ratio of masses, gravitational to inertial (12), we can conclude about a growth of the inertial mass of the Mercury.

The equivalence principle stated by Einstein based on numerous experiments by R. von Eötvös, who had found that under the Earth's conditions the inertial mass and gravitational mass were always equal, requires clarification. [8]

An example of the Mercury shows that for none-equilibrium systems, a difference of the inertial mass from the gravitational mass can reach 1%. As far as in the Solar System the Mercury is the closest planet to the centre of gravity, we can assume that violating the principle of equivalence by more than 1% leads to irreversible consequences and the system becomes unsustainable. In astronomy, this minimum radius of a circular orbit, where a satellite is not destroyed, was called the Roche limit [7].

Einstein's paper, in which he had tried to explain the motion of Mercury's perihelion from a standpoint of the general relativity theory and seemingly achieved remarkable results, has turned to be wrong.

Member of Academy Hua Di found that calculating the integral, Einstein had made a mistake with fatal consequences, as results of repeated calculations turned to be very distinctive from results of multi-year astronomical observations of the Mercury's motion [1].

Quite a curious case when, having made an error, A. Einstein got a correct result, saying that "in one turn of a planet, its orbit rotates by a part of a complete turn-around, equal to fractions $d=3\cdot v^2/c^2$, where v is orbital velocity of the planet." Within the accuracy of one arc second, this result meets astronomical observations. [8] There is a link between the ratio obtained by A. Einstein, Kepler constant (K) and the ether. Inserting a value of an average speed of a planet orbital rotation v $= 2\pi R/T$ into the expression for the Kepler constant (11), we would obtain:

$$v^2 = 4\pi^2 \frac{K}{R} \tag{14}$$

Then, the Einstein relation would have the following form:

$$d = \frac{12\,\pi^2 K}{RC^2} \tag{15}$$

The amount of displacement of the orbit of Mercury will be much more than offset the Earth's orbit. This is because the orbit of mercury is in a polarized ethereal sphere Sun.

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The Way to Skeleton Conception of Elementary Particles

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Abstract- The tachyon model of neutrino is constructed, basing on the statement that quantum description is a statistical description of stochastically moving particles. Besides, the tachyon model contains two conceptual points: (1) universal formalism of particle dynamics, describing uniformly all particles: deterministic, stochastic and quantum, (2) discrete space-time geometry and skeleton conception of particle dynamics. The universal formalism is a result of a logical reloading, when the statistical ensemble becomes to be the basic object of particle dynamics instead of a single particle. Such a reloading admits one to describe uniformly the quantum, stochastic and deterministic particles in terms of a statistical ensemble without a reference to principles of quantum mechanics. Besides, one uses a relativistic state of a particle, when the state is described by the particle skeleton (several space-time points) instead of the point in the phase space, what is nonrelativistic concept of the particle state. Representing the Dirac equation in terms of the statistical ensemble, one concludes that in the deterministic approximation the world line of the Dirac particle may be a spacelike helix with timelike axis. The rotational component of the relativistic Dirac particle is described nonrelativistically. It shows that the world line may be spacelike, and the Dirac particle may be a tachyon. Neutrino is a Dirac particle, and it is a tachyon.

Keywords: *structural approach; united formalism of dynamics; multivariant geometry, dynamic disquantization; tachyon, tachyon gas, tachyon dynamics, dark matter.*

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Strictly as per the compliance and regulations of :

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Abstract- The tachyon model of neutrino is constructed, basing on the statement that quantum description is a statistical description of stochastically moving particles. Besides, the tachyon model contains two conceptual points: (1) universal formalism of particle dynamics, describing uniformly all particles: deterministic, stochastic and quantum, (2) discrete space-time geometry and skeleton conception of particle dynamics. The universal formalism is a result of a logical reloading, when the statistical ensemble becomes to be the basic object of particle dynamics instead of a single particle. Such a reloading admits one to describe uniformly the quantum, stochastic and deterministic particles in terms of a statistical ensemble without a reference to principles of quantum mechanics. Besides, one uses a relativistic state of a particle, when the state is described by the particle skeleton (several space-time points) instead of the point in the phase space, what is nonrelativistic concept of the particle state. Representing the Dirac equation in terms of the statistical ensemble, one concludes that in the deterministic approximation the world line of the Dirac particle may be a spacelike helix with timelike axis. The rotational component of the relativistic Dirac particle is described nonrelativistically. It shows that the world line may be spacelike, and the Dirac particle may be a tachyon. Neutrino is a Dirac particle, and it is a tachyon. Free quantum particles appear to move stochastically, and this bring up the question, what is the reason of stochastic motion of free quantum particles. It appears, that the discrete space-time geometry is a multivariant geometry. It is a reason of stochastic particle motion. If the elementary length λ_0 of the discrete space-time geometry is connected with the quantum constant $\hbar$ by the relation $\lambda_0^2 = \hbar/bc$, where b is some universal constant, then statistical description of the free particle motion coincides with the quantum description in terms of the Schrödinger equation.

Keywords: structural approach; united formalism of dynamics; multivariant geometry, dynamic disquantization; tachyon, tachyon gas, tachyon dynamics, dark matter.

I. Introduction

The particles moving with the velocity, which is greater than the speed of the light, are called tachyons [1]-[4]. We shall use this name for particle, whose world line is spacelike. Both definitions mean the same, if the world line is smooth, and one can define a derivative along the world line. This derivative is known as a velocity. We shall show that the world line of tachyons is not smooth. This property differ tachyons from tardions which are particles moving with velocity less, than the speed of the light, and the world line of a tardion is smooth.

Neutrino is a tachyon, whose world line is a spacelike helix with timelike axis. However, most physicists believe that tachyons do not exist, in particular, tachyons with helical world line do not exist. Our model of neutrino is based on the skeleton conception of elementary particles [5]. The skeleton conception is a new conception based on such unusual fundamentals as (1) refuse from quantum principles, which are replaced by a discrete space-time geometry, (2) description of the particle state by its skeleton (several space-time points) instead of a point in the phase space of coordinates and momenta.

It is useful to describe characteristic features of these fundamentals, using method of the book by Lee Smolin [6]. It is an excellent book, where all problems of the elementary particle theory are presented without any formula. Lee Smolin distinguishes principle theories and constructive theories. The principle theory is to be valid for all physical phenomena, whereas the constructive theory is valid only for some class of physical phenomena. The constructive theory is created on the basis of some experimental data, and it is valid for the class of the physical phenomena

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close to phenomena verified by experiment. For instance, the special relativity and the general relativity are principle theories. The elementary particle theory is a constructive theory.

Lee Smolin formulated five unsolved important problems of contemporary theoretical physics:

Problem 1: Unification of general relativity and quantum theory (quantum gravitation)

Problem 2: Rationale of quantum mechanics.

Problem 3: Unification of particles and fields.

Problem 4: Explanation how to choose free constants in the standard model of elementary particle physics.

Problem 5: Explanation of the phenomenon of dark matter and dark energy.

Besides, Lee Smolin describes new and old fundamental conceptions as unifications. For instance, he formulates the special relativity theory as an unification of space and time. The inertia law is formulated as an unification of the rest and motion. The general relativity is formulated as unification of space-time and gravitation.

The tachyon model of neutrino is constructed on the basis of a principle theory (skeleton conception). This principle theory is formulated on the basis of two unifications:

1. Unification of the deterministic particle motion with the stochastic particle motion

2. Unification of the continuous space-time geometry with the discrete space-time geometry.

The two unifications concern space-time geometry and the particle dynamics. These disciplines relate to all physical phenomena. The two unifications are more fundamental, than problems formulated by Lee Smolin. They solve four of five Smolin's problems (the fourth problem is not solved, because it a specific problem of the standard model of elementary particles). The first problem (the quantum gravitation) is solved in the sense, that the gravitation field does not need to be quantized, as well as other geometrical fields.

Both unifications are produced on the basis of a logical reloading, which means a change of basic statements of a theory.

In the unification of the deterministic particle motion with the stochastic particle motion it means as follows. A deterministic particle is a dynamical system $\mathcal{S}_{\mathrm{d}}$, and one can write dynamic equations for the deterministic particle $\mathcal{S}_{\mathrm{d}}$. Stochastic particle $\mathcal{S}_{\mathrm{st}}$ is not a dynamic system. It is a stochastic system $\mathcal{S}_{\mathrm{st}}$, and there are no dynamic equations for a single stochastic particle $\mathcal{S}_{\mathrm{st}}$. One can describe only mean motion of a stochastic particle $\mathcal{S}_{\mathrm{st}}$. To describe the mean motion, one considers a statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$ of stochastic particles $\mathcal{S}_{\mathrm{st}}$. The statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$ is a dynamic system of the type of continuous medium, and one can write dynamic equations for $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$. Statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\mathrm{d}}\right]$ of deterministic dynamic systems $\mathcal{S}_{\mathrm{d}}$ can be constructed also. Ensemble $\mathcal{E}\left[\mathcal{S}_{\mathrm{d}}\right]$ is also a dynamic system of the type of continuous medium. Any statistical ensemble ($\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$ and $\mathcal{E}\left[\mathcal{S}_{\mathrm{d}}\right]$) may be considered as some fluid (continuous medium). In the Lagrange representation dynamic equations for $\mathcal{E}\left[\mathcal{S}_{\mathrm{d}}\right]$ coincide with dynamic equations for $\mathcal{S}_{\mathrm{d}}$. Only the number of dynamic equations is different. If the number of degrees of freedom for

$\mathcal{S}_{\rm d}$ is equal to n, the number of the freedom degrees for $\mathcal{E}[\mathcal{S}_{\rm d}]$ is equal to nN, where N is the number of dynamic system $\mathcal{S}_{\rm d}$ in $\mathcal{E}[\mathcal{S}_{\rm d}], N \to \infty$.

If the dynamical equations for $\mathcal{S}_{\rm d}$ are known, the dynamical equations for $\mathcal{E}[\mathcal{S}_{\rm d}]$ are also known. Vice versa, if dynamic equations for $\mathcal{E}[\mathcal{S}_{\rm d}]$ are known, one can write dynamic equations for a single particle $\mathcal{S}_{\rm d}$. In other words, it is not essential, what is the basic object ($\mathcal{E}[\mathcal{S}_{\rm d}]$ or $\mathcal{S}_{\rm d}$) at description of the deterministic particle $\mathcal{S}_{\rm d}$. However, at description of stochastic particle $\mathcal{S}_{\rm st}$ it is essential, because there are dynamic equations for $\mathcal{E}[\mathcal{S}_{\rm st}]$, whereas there are no dynamic equations for $\mathcal{S}_{\rm st}$. If the statistical ensemble is a basic object of the particle dynamics, then dynamic equations exist for all sorts of basic objects $\mathcal{E}[\mathcal{S}_{\rm d}]$ and $\mathcal{E}[\mathcal{S}_{\rm st}]$. The difference between $\mathcal{S}_{\rm d}$ and $\mathcal{S}_{\rm st}$ consists in the circumstance, that dynamic equations for $\mathcal{S}_{\rm d}$ can be obtained from dynamic equations for $\mathcal{E}[\mathcal{S}_{\rm d}]$, but dynamic equations for $\mathcal{S}_{\rm st}$ cannot be obtained from dynamic equations for $\mathcal{E}[\mathcal{S}_{\rm st}]$. How it may be possible, will be shown later in a simple example.

The logical reloading leads to creation of united formalism for description of deterministic, stochastic and quantum particles in terms of the statistical ensemble [7]. It appears, that quantum particles are stochastic particles, described in terms of the statistical ensemble. The wave function ψ appears at such a description, because it is simply a way of the ideal fluid description [8]. The wave function ψ is used, because describing statistical ensemble of quantum particles, the internal energy of the "quantum fluid" appears to be such one, that dynamic equations in terms of ψ are linear (Schrödinger equation) for non-rotational flow of the "quantum fluid", describing this ensemble.

Thus, one does not need quantum principles, if the basic object of particle dynamics is a statistical ensemble. Quantum particles are simply stochastic particles. It appears that the quantum principles are not fundamental principles of nature, and there is no necessity to quantize the gravitational field, especially if one takes into account that the dynamic equations of the gravitational field do not contain the quantum constant.

Explanation of quantum theory as a statistical description of stochastic particles brings up the question: "Why free elementary particles move stochastically?" The answer to this question is as follows. The real space-time geometry is discrete. There is a minimal space-time distance between the events (points) of space-time. This distance is called elementary length λ_0. Condition of the space-time discreteness Must be written in the form

$$|\rho(P,Q)| \notin (0,\lambda_0), \quad \forall P,Q \in \Omega \tag{1.1}$$

where Ω is the set of the space-time points and $\rho(P,Q)$ is the space-time interval between points P and Q. Note that $\rho(P,Q) = 0$ is possible, and it is compatible with (1.1), for instance, if $P = Q$.

Usually one considers the restriction (1.1) as a restriction on the set Ω, and the distance function ρ is defined as follows

$$\rho(P,Q) = \sqrt{2\sigma(P,Q)} \tag{1.2}$$

where σ is the world function $\sigma_{\rm M}$ of the space-time of Minkowski. In the inertial coordinate system the world function $\sigma_{\rm M}$ has the form

$$\sigma_{\mathrm{M}}\left(x,x'\right)=\frac{1}{2}g_{ik}\left(x^{i}-x'^{i}\right)\left(x^{k}-x'^{k}\right),\quad g_{ik}=\mathrm{diag}\left(c^{2},-1,-1,-1\right) \tag{1.3}$$

Consideration of (1.1) as restriction on Ω leads to a geometry on a lattice. Geometry of Minkowski on a lattice is not uniform and isotropic. It is not invariant with respect to Lorentz transformations. Nevertheless it is used by theorists for approximate calculations.

It is more correct to consider (1.1) as a restriction on the form of the distance function ρ and the world function $\sigma=\frac{1}{2}\rho^{2}$. A use of the world function is more convenient, than a use of distance, because it is always real (σ is positive for timelike distances, and it is negative for spacelike ones). World function σ_{d} for a discrete geometry $\mathcal{G}_{\mathrm{d}}$ can be taken in the form

$$\sigma_{\mathrm{d}}\left(P,Q\right)=\sigma_{\mathrm{M}}\left(P,Q\right)+\frac{\lambda_{0}^{2}}{2}\mathrm{sgn}\left(\sigma_{\mathrm{M}}\left(P,Q\right)\right)\quad\forall P,Q\in\Omega \tag{1.4}$$

where Ω is the same point set (continuum) which is used in the space-time geometry of Minkowski. The relation (1.4) is compatible with restriction (1.1).

Multivariance is the most unexpected and important property of the discrete geometry $\mathcal{G}_{\mathrm{d}}$. Geometrical vector (g-vector) $\mathbf{AB}$ is defined as the ordered set $\{A,B\}$ of two points A and B. Multivariance of a geometry means that a g-vector $\mathbf{AB}$ at the point A has many equivalent g-vectors $\mathbf{CD}$, $\mathbf{CD}'$, $\mathbf{CD}''$,...at the point C, but these g-vectors are not equivalent between themselves. Contemporary theorists do not accept the property of multivariance in a geometry and try to remove it, if it appears by accident in geometry. For instance, when it appears in the Riemannian geometry, one removes this property, connecting any of numerous g-vectors $\mathbf{CD}$, $\mathbf{CD}'$, $\mathbf{CD}''$,...at the point C with the path of its parallel transport from the point A and asserting absence of absolute parallelism in the Riemannian geometry.

Multivariance for spacelike g-vectors takes place in the space-time geometry of Minkowski. Ignoring this multivariance, one cannot describe motion of tachyons. Conventional viewpoint, that tachyons do not exist is connected with disregard of the spacelike g-vectors multivariance.

Such a relation to multivariance is connected with the fact that beginning from Euclid one studied only proper Euclidean geometry, assuming that the space-time geometry cannot have any additional properties which are absent in the Euclidean geometry. The multivariance is denied in the Riemannian geometry, because one considers absence of absolute parallelism as a less defect of the geometry, than multivariance of the g-vector equivalence. Besides, the multivariance is incompatible with contemporary methods of differential geometry. The operations of the linear vector space $\mathcal{L}_{n}$ (summation of vectors $u\in\mathcal{L}_{n}$ and multiplication of a vector $u\in\mathcal{L}_{n}$ by a real number) are not adequate in the multivariant geometry. The fact is that any linvector $u\in\mathcal{L}_{n}$ exist in one copy. We use the name linear vector (linvector) for vectors $u\in\mathcal{L}_{n}$, in order to distinguish it from the geometric vector (g-vector) $\mathbf{AB}$, which is defined as the ordered set of two points $\mathbf{AB}=\{A,B\}\in\Omega\times\Omega$. There are many equivalent g-vectors in any space-time geometry. In the Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ the set $\Omega_{\mathbf{AB}}$ of g-vectors $\mathbf{CD}\in\Omega\times\Omega$ form the equivalence class $[\mathbf{AB}]$ of the g-vector $\mathbf{AB}$. All the g-vectors of $[\mathbf{AB}]$ are equivalent between themselves, and all $[\mathbf{AB}]$ may be set in correspondence with all linvectors of $\mathcal{L}_{n}$. As a result the linear operations of $\mathcal{L}_{n}$ can be used for g-vectors of Euclidean geometry $\mathcal{G}_{\mathrm{E}}$. In the multivariant geometry the set of g-vectors $\Omega_{\mathbf{AB}}$ does not form the equivalence class, because $\Omega_{\mathbf{AB}}$ contains the g-vectors, which are not equivalent between themselves (they are equivalent only to g-vector $\mathbf{AB}$). As a result operations of the linear space $\mathcal{L}_n$ are not adequate in the multivariant geometry. They can be introduced, but these operations appear to be ambiguous.

Multivariance of the space-time geometry generates a random wobbling of particle world lines. In the geometry of Minkowski $\mathcal{G}_{\mathrm{M}}$ the equivalence of timelike g-vectors is not multivariant, whereas the equality (equivalence) of spacelike g-vectors is multivariant. Wobbling of spacelike world line has infinite amplitude, and a single tachyon cannot be detected due to infinite random wobbling of its world line. As a result it is used to think that tachyons do not exist.

In the discrete geometry $\mathcal{G}_{\mathrm{d}}$ both timelike and spacelike world lines wobble. However, the wobbling amplitude of timelike world lines is restricted by the elementary length λ_0, and it vanishes in the geometry of Minkowski $\mathcal{G}_{\mathrm{M}}$, where $\lambda_0 = 0$. The wobbling amplitude of spacelike world lines of tachyons is infinite in $\mathcal{G}_{\mathrm{M}}$ and in $\mathcal{G}_{\mathrm{d}}$. In the real space-time geometry $\mathcal{G}_{\mathrm{d}}$ the restricted wobbling of timelike world lines is a reason of stochastic motion of particles. If λ_0^2 is proportional to the quantum constant $\hbar$, the statistical description of wobbling world lines (dynamic equations for the statistical ensemble) leads to the Schrödinger equation [10].

In the discrete geometry $\mathcal{G}_{\mathrm{d}}$ all geometric quantities are functions of the world function σ_{d}. In particular, the dimension n of a geometry is determined by its world function σ. In $\mathcal{G}_{\mathrm{d}}$ the dimension has no definite value. It is rather unusual, because in the Riemannian geometry the dimension of the geometry is a definite natural number.

If the quantum particles $\mathcal{S}_{\mathrm{q}}$ are stochastic particles, described in terms of statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$, dynamic equations for any quantum particle can be reduced to dynamic equations for a statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ of stochastic particles $\mathcal{S}_{\mathrm{st}}$. In particular, the Dirac equation is to be reduced to the dynamic equations for some statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{Dst}}]$ of stochastic particles $\mathcal{S}_{\mathrm{Dst}}$. One can introduce a "deterministic model" $\mathcal{S}_{\mathrm{d}}$ of a quantum particle $\mathcal{S}_{\mathrm{st}}$ by means of dynamic disquantization (D-disquantization) of the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ of the stochastic particle $\mathcal{S}_{\mathrm{st}}$ [9]. Dynamic disquantization is a dynamic operation which does not use quantum principles. As a result of the dynamic disquantization all derivatives $\partial_k \equiv \partial/\partial x^k$ in the dynamic equations are replaced by derivatives which are in parallel with 4-vector j^k of the particle current

$$\partial_k \rightarrow \frac{j_k j^l}{j_s j^s}\partial_l \tag{1.5}$$

As a result of the dynamic disquantization the dynamic equations for the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ of stochastic particles $\mathcal{S}_{\mathrm{st}}$ turn to dynamic equations for the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{d}}]$ of deterministic particles $\mathcal{S}_{\mathrm{d}}$. In the Lagrange representation the dynamic equations for $\mathcal{S}_{\mathrm{d}}$ are ordinary differential equations, because they contain derivatives only along the direction of the vector j^k. The dynamic system $\mathcal{S}_{\mathrm{d}}$ has finite number of the freedom degrees. The dynamic system $\mathcal{S}_{\mathrm{d}}$ can be interpreted as a deterministic model of the stochastic particle $\mathcal{S}_{\mathrm{st}}$. Dynamic equations for $\mathcal{S}_{\mathrm{d}}$ may contain the quantum constant $\hbar$, because in the dynamic disquantization one uses only procedure (1.5), but one does not use the limit $\hbar \rightarrow 0$. In particular, if the stochastic particle $\mathcal{S}_{\mathrm{st}}$ is the Dirac particle $\mathcal{S}_{\mathrm{D}}$ described by the Dirac equation, the deterministic model of the Dirac particle is a dynamic system having ten degrees

Global Journal of Science Frontier Research (A) Volume XIV Issue VII Version I Year 2014

of freedom [11]. It may be interpreted as a rotator (two rigidly connected pointlike particles). If one follows only one particle of the rotator, one concludes that the world line of the deterministic Dirac particle appears to be a helix (spacelike or timelike) with timelike axis. Neutrino is believed to be a Dirac particle. As a result neutrino appears to be a tachyon moving along the spacelike helix with timelike axis. (Timelike world line of neutrino is improbable, because in this case the regular velocity of neutrino appears to be essentially less, than the speed of the light). Note that conventionally the deterministic model $\mathcal{S}_{\rm Dd}$ of the Dirac particle $\mathcal{S}_{\rm Dst}$ is considered as a pointlike tardion equipped with spin (angular momentum). The term "tardion" means a particle having timelike world line, whereas the term "tachyon" means a particle having spacelike world line.

As we shall see, the Dirac particle is described by three-point skeleton $\mathcal{P}^2 = \{P_0, P_1, P_2\}$, or by three connected g-vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_0\mathbf{P}_2$, $\mathbf{P}_1\mathbf{P}_2$. One of these g-vectors is spacelike and two of them are timelike. The timelike vector wobbling amplitude is restricted by the elementary length λ_0, whereas the spacelike vector wobbling amplitude is not restricted (infinite). All points of the skeleton $\mathcal{P}^2 = \{P_0, P_1, P_2\}$ are connected rigidly. The world chain wobbling of such a particle is a mixture of the unrestricted tachyon wobbling and of the tardion wobbling, restricted by value of λ_0 (or by $\hbar$). As a result of dynamical disquantization of $\mathcal{S}_{\rm Dst}$ one obtains the dynamic system $\mathcal{S}_{\rm Dd}$, which is described in the space-time geometry of Minkowski by a helical world line (spacelike or timelike) with the timelike axis. The dynamic equations for $\mathcal{S}_{\rm Dd}$ contain the quantum constant $\hbar$ (in the expression for spin). It means that the dynamic system $\mathcal{S}_{\rm Dd}$ is not a classical approximation of the stochastic Dirac particle $\mathcal{S}_{\rm Dst}$.

The deterministic model $\mathcal{S}_{\rm Dd}$ of the stochastic particle $\mathcal{S}_{\rm Dst}$ is a dynamic system $\mathcal{S}_{\rm Dd}$, which can be considered in the space-time geometry of Minkowski. The dynamic system $\mathcal{S}_{\rm Dd}$ has finite number of the freedom degrees. The deterministic model $\mathcal{S}_{\rm Dd}$ describes the arrangement of the particle, described by the Dirac equation. One should take into account that usually one does not consider the deterministic model $\mathcal{S}_{\rm Dd}$, which contains the quantum constant $\hbar$ and explains the particle spin by a rotation of a particle due to its helical world line. Instead, one considers the dynamic system $\mathcal{S}_{\rm Dcl}$, which is described by a straight world line (not helix), and spin is introduced axiomatically. From the dynamic system $\mathcal{S}_{\rm cl}$ one cannot obtain any information on arrangement of $\mathcal{S}_{\rm Dst}$.

Attempts of obtaining information on arrangement of $\mathcal{S}_{\rm Dst}$ from the contemporary elementary particle theory remind attempts of investigating the atom arrangement on the basis of the periodical system of chemical elements and of chemical reactions between chemical elements. One obtains a lot of information on the atom properties of different chemical elements and no information on the planetary model of atoms.

Replacement of quantum theory by a statistical description together with the dynamic disquantiazation admit one to construct a new approach to description of elementary particles. We qualify this approach as structural approach. The conventional approach to the elementary particles description (standard model) is qualified as empirical approach. Empirical approach to theory of elementary particles is based on the quantum theory. The empirical approach considers elementary particles as pointlike objects equipped by quantum numbers. It cannot determine the connection of quantum numbers with the elementary particles arrangement. The structural approach admits one to determine arrangement (structure) of the elementary particle.

The difference between the structural approach and the empirical one can be seen in the theory of chemical elements. The empirical approach is realized by chemical methods, when chemical elements are systematized by the periodic system of chemical elements, and one investigates reactions between different chemical substances. Empirical approach cannot determine the atom structure (nucleus, electronic envelope). On the contrary, the ctructural approach uses methods of quantum mechanics and of atomic physics. It admits one to discover the atom arrangement.

Thus, we have presented briefly the way to the tachyon model of neutrino, which is based on two logical reloadings (in dynamics and in the space-time geometry). It is the most short way, but in reality we went to the deterministic model of neutrino by another way. We search defects and mistakes in the existing theory of microcosm physics and eliminate them step by step. Such an investigation strategy is the best one in the case, when a theory continues to be in crisis. As far as I know, nobody uses such a strategy. Furthermore I was criticized for such a strategy, because nobody believes that there may be mistakes in the existing theory of microcosm physics. All researchers dreamed about new happy ideas, which could help us to go out of crisis. Further we shall present the way to the deterministic model of neutrino. It was a long way, which took thirty years. Happily, it was the way not only to the deterministic model of neutrino. It was the way to the skeleton conception of elementary particles [12].

II. United Formalism for Particle Dynamics

After explanation of heat phenomena by means of the kinetic gas theory it was reasonable to think, that quantum effects may be explained as some stochastic motion of microparticles. Some researchers [13, 14] tried to obtain quantum mechanics as a statistical description of stochastically moving microparticles. They failed to explain the quantum mechanics as a statistical description of stochastically moving particles. Moyal [13] tried to reduce quantum dynamic equations to the form, which is characteristic for dynamic equations of stochastic processes. Fenyes [14] tried to obtain statistical description, using similarity between the Schrödinger equation and the Fokker equation for diffusion processes. Both authors used the concept of the wave function without understanding, what it means. Explanation of quantum phenomena is hardly possible without understanding, what is the wave function. However, then nobody knew, what is the wave function.

The fact, that the Schrödinger equation may be reduced to irrotational flow of some quantum fluid was shown by Madelung [15]. However, representation of the hydrodynamic equations for ideal fluid in terms of the wave function needs a complete integration of hydrodynamic equations.

For transition from the Schrödinger equation to the system of four hydrodynamic equations, the complex Schrödinger equation for the wave function $\psi = \sqrt{\rho}\exp(i\varphi/\hbar)$ is represented in the form of two real equations for amplitude $\sqrt{\rho}$ and for the phase φ. To obtain hydrodynamic equations, it is sufficient to take gradient from the equation for the phase φ. As a result one obtains four dynamic equations, which turn into hydrodynamic equations after introducing proper designations. In other words, for transition from dynamic equations in terms of the wave function to the hydrodynamic form of these equations, one needs to differentiate dynamic equations. On the contrary, to pass from hydrodynamic form of dynamic equations to their representation in terms of the wave function, one needs to integrate dynamic equations. In the case of the irrotational flow this integration is carried out rather simply, whereas in the case of vortical flow the way of integration became to be known only in the end of twentieth century [8].

Bohm [16] used the hydrodynamic representation of the Schrödinger equation for interpretation of quantum mechanics. He started from the wave function and quantum principles and interpreted them in hydrodynamic terms. However, he could not found quantum mechanics on the basis of hydrodynamics, because for such a foundation he would start from hydrodynamic concepts and equations, in order to obtain the wave function in hydrodynamic terms. He could not make this, because in this case he would be forced to integrate hydrodynamic equations in general case, but not only for irrotational flows. Integration of the hydrodynamic equations was not known almost during the whole twentieth century.

Information on other attempts of a statistical foundation of quantum mechanics can be found in the book by Holland [17]. All authors tried to found the nonrelativistic quantum phenomena on the basis of nonrelativistic statistical description. This circumstance was the main reason of failures. The nonrelativistic quantum mechanics describes a mean motion of particles, and the mean motion is nonrelativistic. However, the nonrelativistic character of the mean motion does not mean, that the exact particle motion is also nonrelativistic. Stochastic component of the particle motion may be relativistic, and this component disappear at the averaging. To obtain a correct description one should use a relativistic statistical description.

Nonrelativistic statistical description is produced usually in terms of the probability density. It uses nonrelativistic concept of particle state as a point in the phase space of coordinates and momenta. At proper normalization the nonnegative density ρ of particles in the phase space is used as a probability density.

In the relativistic physics the state of a particle is determined by its world line (not as a point in the phase space). As a result the state density of a statistical ensemble of relativistic particles at some space-time point x is determined by the vector $j^k(x)$ of the 4-current [18]. This vector cannot be described in terms of the probability density. As a result the statistical description of relativistic stochastic particle differs from that of the nonrelativistic particles. The relativistic statistical description of stochastically moving particles is a consideration of many stochastic particles (statistical ensemble), and it is the primary definition of the statistical description. Consideration of the statistical ensemble of stochastic particles is a consideration of some continuous medium, consisting of infinite number of independent stochastic particles [18, 19, 20]. Thus, a statistical ensemble of stochastic particles is a dynamic system, which is described by some dynamic equations, whereas a single stochastic particle is not a dynamic system, and there are no dynamic equations, describing a single stochastic particle.

Consideration of the statistical ensemble admits one to obtain a dynamic system, whose evolution can be investigated. Of course, the relativistic statistical description in terms of statistical ensemble and that in terms of a fluid are connected. However, one prefers to use nonrelativistic statistical description in terms of the probability density. The Brownian particles are described by means of the nonrelativistic statistical description. Such an approach is true, because the stochastic component of the Brownian particle motion is nonrelativistic, and the state of the Brownian particle may be described as a point in the usual space.

However, application of nonrelativistic statistical description to quantum particle is incorrect, because *the nonrelativistic quantum mechanics is in reality a relativistic conception*. This statement looks rather unexpected. But note, that if one knows nothing about the stochastic component of a particle motion, one should consider the general (relativistic) case. If one considers the nonrelativistic quantum mechanics as a relativistic conception, but the quantum mechanics appears to be a nonrelativistic conception, such a consideration of quantum mechanics as a relativistic conception will be true, because a nonrelativistic conception is a special case of a relativistic conception. However, if one considers the nonrelativistic quantum mechanics as a nonrelativistic conception, but it appears to be a relativistic conception, the nonrelativistic consideration will be incorrect, in general. The difference lies in the concept of the particle state.

Thus, if one tries to obtain a statistical foundation of quantum mechanics as a statistical description of stochastically moving particles, one should use adequate relativistic concepts. Formalism of nonrelativistic quantum mechanics is nonrelativistic. To produce a statistical foundation of quantum mechanics, one should carry out a logical reloading, i.e. a transition from inadequate (nonrelativistic) concepts to adequate (relativistic) concepts. It means that the probability density $\rho(x)$ should be replaced by the "probability vector" $j^k(x)$ (world lines density). Introduction of 4-vector $j^k(x)$ means a consideration of some "quantum fluid". The wave function ψ is a way of the fluid description [8], and it appears as a result of description of the "quantum fluid", which describes the state of the statistical ensemble. As a result the main concept of the quantum mechanics (the wave function) appears to be a secondary derivative concept. The wave function may be introduced and interpreted in terms of concepts of the statistical ensemble. This fact admits one to found the quantum mechanics as a statistical description of stochastically moving particles.

Relativistic character of the nonrelativistic quantum mechanics makes to be useless the construction of relativistic quantum theory as a result of uniting of quantum and relativistic principles. Such an uniting is inconsistent, because nonrelativistic quantum mechanics is already a nonrelativistic approximation of a relativistic conception. Such an uniting reminds an unification of axiomatic conception of thermodynamics with the model conception of the kinetic gas theory. Relativistic quantum theory should be obtained as a refuse from the nonrelativistic approximation of the relativistic statistical foundation of the quantum mechanics. It means that the conventional conception of the relativistic quantum theory is doomed to fitting instead of logical development of the existing relativistic statistical description.

The main difference between the quantum mechanics and statistical description of stochastic particles lies in a use of the von Neumann formula for calculations of mean values

$$\langle f\rangle = \int \psi^* f \psi d\mathbf{x} \qquad (2.1)$$

According to statistical approach this formula is valid, if f is an arbitrary function of coordinate $\mathbf{x}$, or it is an additive quantity (energy, momentum, angular momentum). According to the von Neumann interpretation the formula (2.1) is valid for arbitrary function of coordinates and momentum $f(\mathbf{x}, \mathbf{p})$, $\mathbf{p} = -i\hbar\boldsymbol{\nabla}$. The statement of the von Neumann theorem that one cannot introduce hidden variables in quantum mechanics is based on application of formula (2.1) to arbitrary functions $f(\mathbf{x}, \mathbf{p})$ [21]. The statistical description of stochastic particles my be considered as an introduction of hidden variables, but in this case formula (2.1) is not valid for arbitrary functions $f(\mathbf{x}, \mathbf{p})$, and there is no conflict with the theorem on hidden variables.

It is worth to note, that the logical reloading of statistical description to a relativistic conception does not need any new hypothesis. The probability density is not used simply, because it is an attribute of nonrelativistic description. As far as the quantum mechanics is a dynamics of a statistical ensemble of stochastic particles, it follows that the wave function ψ describes a state of the dynamic system $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ [8]. This dynamic system $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ is statistical ensemble of stochastic particles $\mathcal{S}_{\mathrm{st}}$. If the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ is normalized to one particle, it can be interpreted as a statistically averaged particle $\langle\mathcal{S}_{\mathrm{st}}\rangle$. The statistically averaged particle $\langle\mathcal{S}_{\mathrm{st}}\rangle$ has energy, momentum and other total characteristics of a single particle $\mathcal{S}_{\mathrm{d}}$, but its motion is a motion of a statistical ensemble. For instance, $\langle\mathcal{S}_{\mathrm{st}}\rangle$ may move through two open slits at once, whereas a single deterministic particle $\mathcal{S}_{\mathrm{d}}$ may move only through one of two open slits.

The Copenhagen interpretation of quantum mechanics, where the wave function describes a single particle is incompatible with the formalism of quantum mechanics [21, 22]. As far as the quantum mechanics is a statistical theory (dynamics of a statistical ensemble), there are two different kinds of quantum measurements: (1) a massive measurement (M-measurement) which is produced over all elements (particles) $\mathcal{S}_{\mathrm{st}}$ of the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$, and (2) a single measurement (S-measurement) which is produced over a single particle $\mathcal{S}_{\mathrm{st}}$ of the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$. These measurements have different properties, and one may not mix up them.

S-measurement of a quantity R gives a random quantity R', which cannot be obtained, generally speaking, at a repeated S-measurement. In the S-measurement one deals with a single stochastic system $\mathcal{S}_{\mathrm{st}}$. The state of the stochastic system $\mathcal{S}_{\mathrm{st}}$ may be changed after S-measurement, which is a dynamical effect on $\mathcal{S}_{\mathrm{st}}$. However, the state of $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ cannot be changed by this effect, because $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ contains infinite number of stochastic particles $\mathcal{S}_{\mathrm{st}}$. A repeated S-measurement of the same quantity R can be produced on other stochastic particle $\mathcal{S}_{\mathrm{st}}$. It gives, generally speaking, another value R'' of the measured quantity R.

M-measurement is a set of N S-measurements ($N \to \infty$). M-measurement of a quantity R gives a distribution $F(R)$, which can be obtained at repeated M-measurement. M-measurement of the quantity R at the state ψ of the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ can change the state of the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$, because in this case one deals with N ($N \to \infty$) stochastic systems $\mathcal{S}_{\mathrm{st}}$. Any stochastic system changes after S-measurement produced over it. N changed system $\mathcal{S}'_{\mathrm{st}}$ form a statistical ensemble $\mathcal{E}[\mathcal{S}'_{\mathrm{st}}]$ at $N \to \infty$. As a result the state ψ of the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ changes.

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Is it possible to obtain a definite value R' at a M-measurement of the quantity R (instead of the distribution $F(R)$)? It is possible, provided the measurement is accompanied by a discriminating operation, which removes from the statistical ensemble $\mathcal{E}[\mathcal{S}'_{\mathrm{st}}]$ all stochastic systems $\mathcal{S}'_{\mathrm{st}}$, where the measured value of the quantity R is not equal to R'. The M-measurement accompanied by some discriminating operation will be referred to as a selective M-measurement (SM-measurement). The SM-measurement of the quantity R may give a definite value R' and change the state ψ of the statistical ensemble. In other words, the SM-measurement may have properties of the S-measurement and of M-measurement.

At the Copenhagen interpretation of quantum mechanics, where ψ is a state of a quantum particle $\mathcal{S}_{\mathrm{q}}$, there is only one kind of measurement. In some situation it is interpreted as M-measurement, in other situation it is interpreted as S-measurement. It is supposed that such a measurement of the quantity R can give a definite random value R' of the quantity R and simultaneously change the state $\psi \rightarrow \psi_{R'}$. In other words, in the Copenhagen interpretation a measurement is supposed to have properties of SM-measurement. A use of one term for different kinds of measurement (S-,M-,SM-) leads to numerous paradoxes.

We consider only one of paradoxes: "action of a measurement at a distance". Let us consider a system $\mathcal{S} = \mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ at the state ψ. Let $\mathcal{S}$ at the state ψ can decay into two systems (particles): $\mathcal{S}_1$ and $\mathcal{S}_2$. Let the two particles states $\mathcal{S}_1$ and $\mathcal{S}_2$ appear to be correlated in the sense, that if a S-measurement of the dichotomic quantity s (spin) in $\mathcal{S}_1$ gives the result $s' = 1/2$, the S-measurement of the same quantity s (spin) in $\mathcal{S}_2$ gives the result $s'' = -1/2$. Let these particles $\mathcal{S}_1$ and $\mathcal{S}_2$ move, and at some moment they appear to be divided by the distance L. According to Copenhagen viewpoint, when one measures the quantity s in $\mathcal{S}_1$ and obtains the result $s' = 1/2$, the state of $\mathcal{S}_1$ changes (SM-measurement) $\psi_1 \rightarrow \psi'_1$. At the same time the state of $\mathcal{S}_2$ is to be changed $\psi_2 \rightarrow \psi'_2$, because in $\mathcal{S}_2$ the quantity s takes the value $s'' = -1/2$. As a result a measurement of the quantity s in $\mathcal{S}_1$ changes instantly the state of the particle $\mathcal{S}_2$, although the distance L between the particles $\mathcal{S}_1$ and $\mathcal{S}_2$ may be large ("action of a measurement at a distance"). Such a situation is incompatible with the special relativity principles, and it is considered as a paradox.

The paradox is resolved by a reference, that in the given case there is a SM-measurement, which is accompanied by a discriminating operation, and information on this operation is to be transmitted from point A_1, , to the point A_2, where $\mathcal{S}_2$ is located. Indeed, if one speaks on influence of measurement in $\mathcal{S}_1$ on the wave function of $\mathcal{S}_2$, one should consider ensembles $\mathcal{E}[\mathcal{S}_1]$ and $\mathcal{E}[\mathcal{S}_2]$, because the wave function relates to the statistical ensemble $\mathcal{E}[\mathcal{S}_1]$, but not to the single particle $\mathcal{S}_1$ In the SM-measurement one considers N $(N \rightarrow \infty)$ stochastic systems $S'_1, S'_2, ...S'_N$ of the statistical ensemble $\mathcal{E}[\mathcal{S}_1]$ and N $(N \rightarrow \infty)$ stochastic systems $S''_1, S''_2, ...S''_N$ of the statistical ensemble $\mathcal{E}[\mathcal{S}_2]$. The stochastic system S'_k of $\mathcal{E}[\mathcal{S}_1]$ correlates with stochastic system S''_k of $\mathcal{E}[\mathcal{S}_2]$. It means that if the quantity s has the value $s' = 1/2$ in S'_k of $\mathcal{S}_1$, then the quantity s has the value $s'' = -1/2$ in S''_k of $\mathcal{S}_2$. One measures the quantity s in all N stochastic systems S'_k, $k = 1, 2, ...N$ and obtains that the value $s' = 1/2$ appears in stochastic systems $S'_{(k_1)}, S'_{(k_2)}, ...S'_{(k_m)}$ of $\mathcal{E}[\mathcal{S}_1]$. Then due to correlation the quantity s has the value $s' = -1/2$ in stochastic systems

$S''_{(k_1)}, S''_{(k_2)}, ...S''_{(k_m)}$ of $\mathcal{E}[\mathcal{S}_2]$, one can form the statistical ensemble $\mathcal{E}[\mathcal{S}'_1] = \mathcal{E}\left[S'_{(k_l)}\right]$. Its state is described by the wave function ψ'_2, where $s = 1/2$. One can form the statistical ensemble $\mathcal{E}[\mathcal{S}'_2] = \mathcal{E}\left[S''_{(k_l)}\right]$ .of the stochastic systems $S''_{(k_1)}, S''_{(k_2)}, ...S''_{(k_m)}$. Its state is described by the wave function ψ''_2, where $s = -1/2$. However, the numbers $(k_1), (k_2), ...(k_m)$ are not known at the point A_2, where the system $\mathcal{S}_2$ is found. In order to construct $\mathcal{E}[\mathcal{S}'_2] = \mathcal{E}\left[S''_{(k_l)}\right]$ with $s = -1/2$, one needs to transmit these numbers from A_1 to A_2. This transmission cannot be realized with the speed, which is greater, than the speed of the light.

The united method of description of dynamic systems and stochastic ones is presented in [22]. Here we present only a short scheme of this method application in the example of a free quantum particle.

Statistical ensemble $\mathcal{E}[\mathcal{S}_{\rm cl}]$ of free nonrelativistic classical particles $\mathcal{S}_{\rm cl}$ is described by the action

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\rm cl}]}[\mathbf{x}] = \int\int_{V_\xi} \frac{m}{2}\dot{\mathbf{x}}^2 \rho_0(\boldsymbol{\xi})\, dt d\boldsymbol{\xi}, \qquad \dot{\mathbf{x}} \equiv \frac{d\mathbf{x}}{dt} \tag{2.2}$$

where $\mathbf{x} = \mathbf{x}(t, \boldsymbol{\xi})$, $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$ are parameters, labelling the particles of the statistical ensemble, and ρ_0 is a weight factor.

If the particles of the ensemble are stochastic, the stochasticity is taken into account by additional dynamical variables in the action. The action for the statistical ensemble $\mathcal{E}[\mathcal{S}_{\rm st}]$ of stochastic particles $\mathcal{S}_{\rm st}$ Must be written in the form

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\rm st}]}[\mathbf{x}, \mathbf{u}] = \int\int_{V_\xi} \left\{\frac{m}{2}\dot{\mathbf{x}}^2 + \frac{m}{2}\mathbf{u}^2 - \frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}\right\} \rho_0(\boldsymbol{\xi})\, dt d\boldsymbol{\xi}, \qquad \dot{\mathbf{x}} \equiv \frac{d\mathbf{x}}{dt} \tag{2.3}$$

The variable $\mathbf{x} = \mathbf{x}(t, \boldsymbol{\xi})$ describes the regular component of the particle motion. The variable $\mathbf{u} = \mathbf{u}(t, \mathbf{x})$ describes the mean value of the stochastic velocity component, $\hbar$ is the quantum constant. The second term in (2.3) describes the kinetic energy of the stochastic velocity component. The third term describes interaction between the stochastic component $\mathbf{u}(t, \mathbf{x})$ and the regular component $\dot{\mathbf{x}}(t, \boldsymbol{\xi})$. The operator

$$\boldsymbol{\nabla} = \left\{\frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3}\right\} \tag{2.4}$$

is defined in the space of coordinates $\mathbf{x}$.

Description of a stochastic physical system distinguishes from that of a deterministic physical system only by additional terms and by additional dynamic variables in the Lagrangian function. The additional dynamic variables describe stochasticity of the particle motion.

Dynamic equations for the dynamic system $\mathcal{E}[\mathcal{S}_{\rm st}]$ are obtained as a result of variation of the action (2.3) with respect to dynamic variables $\mathbf{x}$ and $\mathbf{u}$.

To obtain the action functional for $\mathcal{S}_{\rm st}$ from the action (2.3) for $\mathcal{E}[\mathcal{S}_{\rm st}]$, we should omit integration over $\boldsymbol{\xi}$ in (2.3). We obtain

$$\mathcal{A}_{\mathcal{S}_{\rm st}}[\mathbf{x}, \mathbf{u}] = \int \left\{\frac{m}{2}\dot{\mathbf{x}}^2 + \frac{m}{2}\mathbf{u}^2 - \frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}\right\} dt, \qquad \dot{\mathbf{x}} \equiv \frac{d\mathbf{x}}{dt} \tag{2.5}$$

where $\mathbf{x} = \mathbf{x}(t)$ and $\mathbf{u} = \mathbf{u}(t, \mathbf{x})$ are dependent dynamic variables. The action functional (2.5) is not well defined for $\hbar \neq 0$, because the operator $\boldsymbol{\nabla}$ is defined in some 3-dimensional vicinity of point $\mathbf{x}$, but not at the point $\mathbf{x}$ itself. As far as the action functional (2.5) is not well defined, one cannot obtain dynamic equations for $\mathcal{S}_{\rm st}$. By definition it means that the particle $\mathcal{S}_{\rm st}$ is stochastic. Setting $\hbar = 0$ in (2.3), we transform the action (2.3) into the action (2.2), because in this case $\mathbf{u} = 0$ in virtue of dynamic equations.

The quantum constant $\hbar$ has been introduced in the action (2.3), in order the description by means of the action (2.3) be equivalent to the quantum description by means of the Schrödinger equation. If we substitute the term $-\hbar\boldsymbol{\nabla}\mathbf{u}/2$ by some function $f(\mathbf{u}, \boldsymbol{\nabla}\mathbf{u})$, we obtain statistical description of other stochastic system with other form of stochasticity, which does not coincide with the quantum stochasticity. In other words, the form of the last term in (2.3) describes the type of the stochasticity.

To obtain dynamic equations for the statistical ensemble $\mathcal{E}[\mathcal{S}_{\rm st}]$ of stochastic systems $\mathcal{S}_{\rm st}$, one needs to vary the action (2.3). Variation of (2.3) with respect to $\mathbf{u}$ gives

$$\begin{aligned}
\delta\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\rm st}]}[\mathbf{x}, \mathbf{u}] &= \int\int\limits_{V_\xi} \left\{ m\mathbf{u}\delta\mathbf{u} - \frac{\hbar}{2}\boldsymbol{\nabla}\delta\mathbf{u} \right\} \rho_0(\boldsymbol{\xi})\, dt d\boldsymbol{\xi} \\
&= \int\int\limits_{V_\mathbf{x}} \left\{ m\mathbf{u}\delta\mathbf{u} - \frac{\hbar}{2}\boldsymbol{\nabla}\delta\mathbf{u} \right\} \rho_0(\boldsymbol{\xi}) \frac{\partial(\xi_1, \xi_2, \xi_3)}{\partial(x^1, x^2, x^3)} dt d\mathbf{x} \\
&= \int\int\limits_{V_\mathbf{x}} \delta\mathbf{u} \left\{ m\mathbf{u}\rho + \frac{\hbar}{2}\boldsymbol{\nabla}\rho \right\} dt d\mathbf{x} - \int\oint \frac{\hbar}{2}\rho\delta\mathbf{u} dt d\mathbf{S}
\end{aligned}$$

where

$$\rho = \rho_0(\boldsymbol{\xi}) \frac{\partial(\xi_1, \xi_2, \xi_3)}{\partial(x^1, x^2, x^3)} = \rho_0(\boldsymbol{\xi}) \left(\frac{\partial(x^1, x^2, x^3)}{\partial(\xi_1, \xi_2, \xi_3)} \right)^{-1} \tag{2.6}$$

We obtain the following dynamic equation

$$\delta\mathbf{u}: \qquad m\rho\mathbf{u} + \frac{\hbar}{2}\boldsymbol{\nabla}\rho = 0 \tag{2.7}$$

where $\rho = \rho(t, \mathbf{x})$ is defined by the relation (2.6). Resolving (2.7) with respect to $\mathbf{u}$, we obtain the equation

$$\mathbf{u} = \mathbf{u}(t, \mathbf{x}) = -\frac{\hbar}{2m}\boldsymbol{\nabla}\ln\rho, \tag{2.8}$$

which reminds the expression for the mean velocity of the Brownian particle with the diffusion coefficient $D = \hbar/2m$.

Variation of the action (2.3) with respect to $\mathbf{x}$ is produced at fixed form of $\mathbf{u}$, but $\mathbf{u} = \mathbf{u}(t, \mathbf{x})$, and argument $\mathbf{x}$ of the function $\mathbf{u}$ should be varied. Variation of (2.3) with respect to $\mathbf{x}$ gives

$$\delta\mathcal{A}_{\mathcal{S}_{\rm st}}[\mathbf{x}, \mathbf{u}] = \int \left\{ m\dot{\mathbf{x}}\delta\dot{\mathbf{x}} + \delta\left(\frac{m}{2}\mathbf{u}^2 - \frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u} \right) \right\} \rho_0(\boldsymbol{\xi})\, dt d\boldsymbol{\xi}, \tag{2.9}$$

One obtains dynamic equation

$$\delta \mathbf{x}: \qquad -m\frac{d^2\mathbf{x}}{dt^2}+\boldsymbol{\nabla}\left(\frac{m}{2}\mathbf{u}^2-\frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}\right)=0 \tag{2.10}$$

Substituting (2.8) in (2.10) and considering ρ as a function of $t,\mathbf{x}$, one obtains

$$m\frac{d^2\mathbf{x}}{dt^2}=-\boldsymbol{\nabla}U_{\mathrm{B}} \tag{2.11}$$

where d/dt means the substantial derivative with respect to time t

$$\frac{dF}{dt}\equiv\frac{\partial\left(F,\xi_1,\xi_2,\xi_3\right)}{\partial\left(t,\xi_1,\xi_2,\xi_3\right)}$$

$\boldsymbol{\nabla}$ is gradient in the space of coordinates x, and U_{B} is so-called Bohm potential

$$\begin{aligned} U_{\mathrm{B}}\left(t,\mathbf{x}\right) &= -\frac{m}{2}\mathbf{u}^2+\frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}=U\left(\rho,\boldsymbol{\nabla}\rho,\boldsymbol{\nabla}^2\rho\right) \\ &= \frac{\hbar^2}{8m}\frac{\left(\boldsymbol{\nabla}\rho\right)^2}{\rho^2}-\frac{\hbar^2}{4m}\frac{\boldsymbol{\nabla}^2\rho}{\rho}=-\frac{\hbar^2}{2m}\frac{1}{\sqrt{\rho}}\boldsymbol{\nabla}^2\sqrt{\rho} \end{aligned} \tag{2.12}$$

where for calculation of U_{B} one uses the relation (2.8)

One obtains

$$m\frac{d^2\mathbf{x}}{dt^2}=\frac{\hbar^2}{2m}\boldsymbol{\nabla}\left(\frac{1}{\sqrt{\rho}}\boldsymbol{\nabla}^2\sqrt{\rho}\right) \tag{2.13}$$

However, the relation (2.6) determines the variable ρ as a function of variables $x^{\alpha,\beta}\equiv\partial x^{\alpha}/\partial\xi_{\beta}$, and one needs to take into account this circumstance in the dynamic equation (2.13).

In the Euler representation (in terms of independent variables $t,\mathbf{x}$) the equation (2.11) can be written in the form

$$\frac{d\mathbf{v}}{dt}=\frac{\partial\mathbf{v}}{\partial t}+\left(\mathbf{v}\boldsymbol{\nabla}\right)\mathbf{v}=-\frac{1}{m}\boldsymbol{\nabla}U_{\mathrm{B}}, \qquad \mathbf{v}=\mathbf{v}\left(t,\mathbf{x}\right) \tag{2.14}$$

Using the relation (2.6), let us represent the quantity $\rho\mathbf{v}$ in the form

$$\rho\mathbf{v}\left(t,\mathbf{x}\right)=\rho_0\left(\boldsymbol{\xi}\right)\frac{\partial\left(t,\xi_1,\xi_2,\xi_3\right)}{\partial\left(t,x^1,x^2,x^3\right)}\frac{\partial\left(\mathbf{x},\xi_1,\xi_2,\xi_3\right)}{\partial\left(t,\xi_1,\xi_2,\xi_3\right)}=\rho\frac{\partial\left(\mathbf{x},\xi_1,\xi_2,\xi_3\right)}{\partial\left(t,x^1,x^2,x^3\right)} \tag{2.15}$$

Then using identity

$$\frac{\partial}{\partial t}\left(\rho_0\left(\boldsymbol{\xi}\right)\frac{\partial\left(\xi_1,\xi_2,\xi_3\right)}{\partial\left(x^1,x^2,x^3\right)}\right)+\frac{\partial}{\partial x^{\alpha}}\left(\rho\frac{\partial\left(x^{\alpha},\xi_1,\xi_2,\xi_3\right)}{\partial\left(t,x^1,x^2,x^3\right)}\right)\equiv 0 \tag{2.16}$$

one obtains the continuity equation for variables $\rho=\rho\left(t,\mathbf{x}\right)$ and $\mathbf{v}=\mathbf{v}\left(t,\mathbf{x}\right)$

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x^{\alpha}}\left(\rho v^{\alpha}\right)=0 \tag{2.17}$$

Equations (2.14), (2.17) together with (2.12) form dynamic equations for the statistical ensemble of stochastic particles in Euler representation, when independent dynamic variables are $t,\mathbf{x}$.

Any reference to the stochastic velocity distribution or to some other probability distribution is absent. Influence of this distribution on the mean motion of the particles is described by the form of Bohm potential U_{B} (2.12). The situation reminds the case of the gas dynamics, where the action of the Maxwell velocity distribution on the gas motion is described by the internal gas energy. Of course, such a description is not comprehensive, however, it is sufficient for a description of the mean motion of the stochastic particle. As a result we obtain a *purely dynamic description* of the mean motion of a stochastic particle.

The fluid described by dynamic equations (2.14), (2.17) can be described in terms of two-component wave function [8] or [7]. One obtains the following dynamic equation for wave function ψ

$$i\hbar\partial_0\psi + \frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\psi + \frac{\hbar^2}{8m}\boldsymbol{\nabla}^2 s_\alpha \cdot (s_\alpha - 2\sigma_\alpha)\,\psi - \frac{\hbar^2}{4m}\frac{\boldsymbol{\nabla}\rho}{\rho}\boldsymbol{\nabla} s_\alpha\sigma_\alpha\psi = 0 \qquad (2.18)$$

where

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \qquad \rho = \psi^*\psi, \qquad s_\alpha = \frac{\psi^*\sigma_\alpha\psi}{\rho}, \qquad \alpha = 1,2,3 \qquad (2.19)$$

σ_α are 2×2 Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \qquad (2.20)$$

In the case of non-rotational flow the wave function becomes to be one component, because at $\psi = \psi_1 = a\psi_2$, a =const and s_α =const, $\alpha = 1,2,3$. In this case the equation (2.18) turns to the linear equation (Schrödinger equation)

$$i\hbar\partial_0\psi + \frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\psi = 0 \qquad (2.21)$$

One should note a specificity of description in terms of the wave function. Any ideal fluid may be described in terms of the wave function [8]

III. The Case of Relativistic Particles

The form of stochasticity of nonrelativistic stochastic particle in (2.5) is defined by two last terms. In the relativistic case the action for the statistical ensemble (2.5) is replaced by the action [23]

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{st}}]}[x,\kappa] = -\int\int_{V_\xi} mcK\sqrt{g_{ik}\dot{x}^i\dot{x}^k}\rho_0(\boldsymbol{\xi})\,d\tau d\boldsymbol{\xi}, \qquad \dot{\mathbf{x}} \equiv \frac{d\mathbf{x}}{d\tau} \qquad (3.1)$$

$$K = \sqrt{1+\lambda^2\left(g_{kl}\kappa^k\kappa^l + \partial_k\kappa^k\right)}, \qquad \lambda = \frac{\hbar}{mc} \qquad (3.2)$$

where $x = \left\{x^k\right\} = \left\{x^k(\tau,\boldsymbol{\xi})\right\}, k = 0,1,2,3$. The quantity g_{kl} =diag$\{c^2,-1,-1,-1\}$ is the metric tensor. The independent variables $\boldsymbol{\xi} = \{\xi_1,\xi_2,\xi_3\}$ label the particles of the statistical ensemble. The dependent variables $\kappa^k = \kappa^k(x)$, $k = 0,1,2,3$ form some force field, connected with the mean stochastic component u^l of the parti-

Global Journal of Science Frontier Research (A) Volume XIV Issue VII Version I Year 2014

cle 4-velocity by the relation $\kappa^l = \frac{m}{\hbar}u^l$, and λ is the Compton wave length of the particle.

In the nonrelativistic approximation one may neglect the temporal component $\kappa^0 = \frac{m}{\hbar}u^0$ with respect to the spatial one $\boldsymbol{\kappa} = \frac{m}{\hbar}\mathbf{u}$. Setting $\tau = t = x^0$ in (3.1), (3.2) we obtain in the nonrelativistic approximation instead (3.1)

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{st}}]}[\mathbf{x},\mathbf{u}] = \int\int_{V_{\xi}} \left\{-mc^2 + \frac{m}{2}\dot{\mathbf{x}}^2 + \frac{m}{2}\mathbf{u}^2 - \frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}\right\}\rho_0(\boldsymbol{\xi})\,dtd\boldsymbol{\xi}, \qquad \dot{\mathbf{x}} \equiv \frac{d\mathbf{x}}{dt} \quad (3.3)$$

The action (3.3) coincides with the action (2.3) except for the first term, which does not contribute to dynamic equations.

Let us add to the action (3.1) the term describing interaction with the electromagnetic field and write it in the form

$$\mathcal{A}[x,\kappa] = \int \left\{-mcK\sqrt{g_{ik}\dot{x}^i\dot{x}^k} - \frac{e}{c}A_k\dot{x}^k\right\}d^4\xi, \quad d^4\xi = d\xi_0 d\boldsymbol{\xi} \quad (3.4)$$

$$K = \sqrt{1+\lambda^2\left(\kappa_l\kappa^l + \partial_l\kappa^l\right)}, \qquad \lambda = \frac{\hbar}{mc}, \quad \tau = \boldsymbol{\xi}_0 \quad (3.5)$$

Here $x = \{x^i(\xi_0,\boldsymbol{\xi})\}$, $i = 0,1,2,3$ are dependent variables. $\xi = \{\xi_0,\boldsymbol{\xi}\} = \{\xi_k\}$, $k = 0,1,2,3$ are independent variables, and $\dot{x}^i \equiv dx^i/d\xi_0$. The quantities $\kappa^l = \{\kappa^l(x)\}$, $l = 0,1,2,3$ are dependent variables, describing stochastic component of the particle motion, $A_k = \{A_k(x)\}$, $k = 0,1,2,3$ is the potential of electromagnetic field. We shall refer to the dynamic system, described by the action (3.4), (3.5) as $\mathcal{S}_{\mathrm{KG}}$, because irrotational flow of $\mathcal{S}_{\mathrm{KG}}$ is described by the Klein-Gordon equation [24]. We present here this transformation to the Klein-Gordon form. Here and farther a summation is produced over repeated Latin indices $(0-3)$ and over Greek indices $(1-3)$.

Let us consider variables $\xi = \xi(x)$ in (3.4) as dependent variables and variables x as independent variables. Let the Jacobian

$$J = \frac{\partial(\xi_0,\xi_1,\xi_2,\xi_3)}{\partial(x^0,x^1,x^2,x^3)} = \det\left|\left|\xi_{i,k}\right|\right|, \qquad \xi_{i,k} \equiv \partial_k\xi_i, \qquad i,k = 0,1,2,3 \quad (3.6)$$

be considered to be a multilinear function of $\xi_{i,k}$ $J = J\left(\xi_{i,k}\right)$. Then

$$d^4\xi = Jd^4x, \qquad \dot{x}^i \equiv \frac{dx^i}{d\xi_0} \equiv \frac{\partial(x^i,\xi_1,\xi_2,\xi_3)}{\partial(\xi_0,\xi_1,\xi_2,\xi_3)} = J^{-1}\frac{\partial J}{\partial\xi_{0,i}} \quad (3.7)$$

After transformation to dependent variables ξ the action (3.4) takes the form

$$\mathcal{A}[\xi,\kappa] = \int\left\{-mcK\sqrt{g_{ik}\frac{\partial J}{\partial\xi_{0,i}}\frac{\partial J}{\partial\xi_{0,k}}} - \frac{e}{c}A_k\frac{\partial J}{\partial\xi_{0,k}}\right\}d^4x, \quad (3.8)$$

Let us introduce new variables

$$j^k = \frac{\partial J}{\partial\xi_{0,k}}, \qquad k = 0,1,2,3 \quad (3.9)$$

by means of Lagrange multipliers p_k

$$\mathcal{A}\left[\xi,\kappa,j,p\right]=\int\left\{-mcK\sqrt{g_{ik}j^ij^k}-\frac{e}{c}A_kj^k+p_k\left(\frac{\partial J}{\partial\xi_{0,k}}-j^k\right)\right\}d^4x, \qquad (3.10)$$

The variable ξ_0 is fictitious. Variation with respect to ξ_i gives

$$\frac{\delta\mathcal{A}}{\delta\xi_i}=-\partial_l\left(p_k\frac{\partial^2J}{\partial\xi_{0,k}\partial\xi_{i,l}}\right)=0, \qquad i=0,1,2,3 \qquad (3.11)$$

Using identities

$$\frac{\partial^2J}{\partial\xi_{0,k}\partial\xi_{i,l}}\equiv J^{-1}\left(\frac{\partial J}{\partial\xi_{0,k}}\frac{\partial J}{\partial\xi_{i,l}}-\frac{\partial J}{\partial\xi_{0,l}}\frac{\partial J}{\partial\xi_{i,k}}\right) \qquad (3.12)$$

$$\frac{\partial J}{\partial\xi_{i,l}}\xi_{k,l}\equiv J\delta_k^i, \qquad \partial_l\frac{\partial^2J}{\partial\xi_{0,k}\partial\xi_{i,l}}\equiv 0 \qquad (3.13)$$

one can test by direct substitution that the general solution of linear equations (3.11) has the form

$$p_k=b_0\left(\partial_k\varphi+g^\alpha\left(\boldsymbol{\xi}\right)\partial_k\xi_\alpha\right), \qquad k=0,1,2,3 \qquad (3.14)$$

where $b_0\neq 0$ is a constant, $g^\alpha\left(\boldsymbol{\xi}\right), \quad \alpha=1,2,3$ are arbitrary functions of $\boldsymbol{\xi}=\{\xi_1,\xi_2,\xi_3\}$, and φ is the dynamic variable ξ_0, which ceases to be fictitious. Let us substitute (3.14) in (3.10). The term of the form $\partial_k\varphi\partial J/\partial\xi_{0,k}$ is reduced to Jacobian and does not contribute to dynamic equations. The terms of the form $\xi_{\alpha,k}\partial J/\partial\xi_{0,k}$ vanish due to identities (3.13). We obtain

$$\mathcal{A}\left[\varphi,\boldsymbol{\xi},\kappa,j\right]=\int\left\{-mcK\sqrt{g_{ik}j^ij^k}-j^k\pi_k\right\}d^4x, \qquad (3.15)$$

where quantities π_k are determined by the relations

$$\pi_k=b_0\left(\partial_k\varphi+g^\alpha\left(\boldsymbol{\xi}\right)\partial_k\xi_\alpha\right)+\frac{e}{c}A_k, \qquad k=0,1,2,3 \qquad (3.16)$$

Integration of (3.11) in the form (3.14) is that integration, which admits one to introduce a wave function. Note that coefficients in the system of equations (3.11) for p_k are constructed of minors of the Jacobian (3.6). It is the circumstance that admits one to produce a general integration.

Variation of (3.15) with respect to κ^l gives

$$\frac{\delta\mathcal{A}}{\delta\kappa^l}=-\frac{\lambda^2mc\sqrt{g_{ik}j^ij^k}}{K}\kappa_l+\partial_l\frac{\lambda^2mc\sqrt{g_{ik}j^ij^k}}{2K}=0 \qquad (3.17)$$

It can be written in the form

$$\kappa^l=g^{lk}\partial_k\kappa, \qquad \kappa=\frac{1}{2}\ln\frac{\lambda^2mc\sqrt{g_{ik}j^ij^k}}{2K\rho_0} \qquad (3.18)$$

where ρ_0 is a constant of integration. It means that the stochastic component of velocity $u^l=\frac{m}{\hbar}\kappa^l$ can be presented in the form

$$u_l = \frac{\hbar}{m}\kappa_l = \frac{\hbar}{m}\partial_l\kappa = \frac{\hbar}{2m}\partial_l \ln\frac{\lambda^2 mc\sqrt{j_s j^s}}{2K\rho_0} = \frac{\hbar}{2m}\partial_l \ln\frac{\lambda^2 mc\sqrt{j_s j^s}}{2\rho_0\sqrt{1+\lambda^2 e^{-\kappa}\partial_s\partial^s e^{\kappa}}} \tag{3.19}$$

Substituting (3.5) in (3.18), we obtain dynamic equation for κ

$$\hbar^2\left(\partial_l\kappa\cdot\partial^l\kappa+\partial_l\partial^l\kappa\right)=\frac{e^{-4\kappa}j_s j^s}{\rho_0^2}-m^2c^2 \tag{3.20}$$

It can be transformed to the form

$$\begin{aligned} j_s j^s &= m^2c^2\rho_0^2 e^{4\kappa}\left(1+\lambda^2 e^{-\kappa}\partial_l\partial^l e^{\kappa}\right) \\ &= m^2c^2\rho_0^2 e^{4\kappa}\left(1-\lambda^2\partial_l\kappa\partial^l\kappa+\frac{\lambda^2}{2}e^{-2\kappa}\partial_l\partial^l e^{2\kappa}\right) \end{aligned} \tag{3.21}$$

Variation of (3.15) with respect to j^k gives

$$\pi_k = -\frac{mcKj_k}{\sqrt{g_{ls}j^l j^s}} \tag{3.22}$$

or

$$\pi_k g^{kl}\pi_l = m^2c^2K^2 \tag{3.23}$$

It follows from (3.20), (3.22) and (3.21) that

$$j_k = -\frac{\sqrt{g_{ls}j^l j^s}}{mcK}\pi_k = -\rho_0 e^{2\kappa}\pi_k, \tag{3.24}$$

Now we eliminate the variables j^k from the action (3.15), using relation (3.24) and (3.21). We obtain

$$\mathcal{A}[\varphi,\boldsymbol{\xi},\kappa]=\int m^2c^2\rho_0 e^{2\kappa}\left\{-K\sqrt{\left(1-\lambda^2\partial_l\kappa\partial^l\kappa+\frac{\lambda^2}{2}e^{-2\kappa}\partial_l\partial^l e^{2\kappa}\right)}+\pi^k\pi_k\right\}d^4x$$

or

$$\mathcal{A}[\varphi,\boldsymbol{\xi},\kappa]=\int m^2c^2\rho_0 e^{2\kappa}\left\{-\left(1-\lambda^2\partial_l\kappa\partial^l\kappa+\frac{\lambda^2}{2}e^{-2\kappa}\partial_l\partial^l e^{2\kappa}\right)+\pi^k\pi_k\right\}d^4x \tag{3.25}$$

where π_k is determined by the relation (3.16). The bracket in the action (3.25) can be transformed as follows.

$$\begin{aligned} &-m^2c^2e^{2\kappa}\left(1-\lambda^2\partial_l\kappa\partial^l\kappa+\frac{\lambda^2}{2}e^{-2\kappa}\partial_l\partial^l e^{2\kappa}\right) \\ =\; &-m^2c^2e^{2\kappa}+\hbar^2 e^{2\kappa}\partial_l\kappa\partial^l\kappa-\frac{\hbar^2}{2}\partial_l\partial^l e^{2\kappa} \end{aligned}$$

Let us take into account that the last term has the form of divergence. It does not contribute to dynamic equations and can be omitted. Omitting this term, we obtain instead of (3.25)

$$\mathcal{A}\left[\varphi,\boldsymbol{\xi},\kappa\right]=\int\rho_{0}e^{2\kappa}\left\{-m^{2}c^{2}+\hbar^{2}\partial_{l}\kappa\partial^{l}\kappa+\pi^{k}\pi_{k}\right\}d^{4}x, \tag{3.26}$$

Instead of dynamic variables $\varphi,\boldsymbol{\xi},\kappa$ we introduce n-component complex function

$$\psi=\{\psi_{\alpha}\}=\left\{\sqrt{\rho}e^{i\varphi}w_{\alpha}\left(\boldsymbol{\xi}\right)\right\}=\left\{\sqrt{\rho_{0}}e^{\kappa+i\varphi}w_{\alpha}\left(\boldsymbol{\xi}\right)\right\},\qquad\alpha=1,2,...n \tag{3.27}$$

Here w_{α} are functions of only $\boldsymbol{\xi}=\{\xi_1,\xi_2,\xi_3\}$, having the following properties

$$\sum_{\alpha=1}^{\alpha=n}w_{\alpha}^{*}w_{\alpha}=1,\qquad-\frac{i}{2}\sum_{\alpha=1}^{\alpha=n}\left(w_{\alpha}^{*}\frac{\partial w_{\alpha}}{\partial\xi_{\beta}}-\frac{\partial w_{\alpha}^{*}}{\partial\xi_{\beta}}w_{\alpha}\right)=g^{\beta}\left(\boldsymbol{\xi}\right) \tag{3.28}$$

where $(^{*})$ denotes complex conjugation. The number n of components of the wave function ψ is chosen in such a way, that equations (3.28) have a solution. Then we obtain

$$\psi^{*}\psi\equiv\sum_{\alpha=1}^{\alpha=n}\psi_{\alpha}^{*}\psi_{\alpha}=\rho=\rho_{0}e^{2\kappa},\qquad\partial_{l}\kappa=\frac{\partial_{l}\left(\psi^{*}\psi\right)}{2\psi^{*}\psi} \tag{3.29}$$

$$\pi_{k}=-\frac{ib_{0}\left(\psi^{*}\partial_{k}\psi-\partial_{k}\psi^{*}\cdot\psi\right)}{2\psi^{*}\psi}+\frac{e}{c}A_{k},\qquad k=0,1,2,3 \tag{3.30}$$

Substituting relations (3.29), (3.30) in (3.26), we obtain the action, written in terms of the wave function ψ

$$\begin{aligned}\mathcal{A}\left[\psi,\psi^{*}\right]=&\int\left\{\left[\frac{ib_{0}\left(\psi^{*}\partial_{k}\psi-\partial_{k}\psi^{*}\cdot\psi\right)}{2\psi^{*}\psi}-\frac{e}{c}A_{k}\right]\left[\frac{ib_{0}\left(\psi^{*}\partial^{k}\psi-\partial^{k}\psi^{*}\cdot\psi\right)}{2\psi^{*}\psi}-\frac{e}{c}A^{k}\right]\right.\\&\left.+\hbar^{2}\frac{\partial_{l}\left(\psi^{*}\psi\right)\partial^{l}\left(\psi^{*}\psi\right)}{4\left(\psi^{*}\psi\right)^{2}}-m^{2}c^{2}\right\}\psi^{*}\psi d^{4}x\end{aligned} \tag{3.31}$$

Let us use the identity

$$\begin{aligned}&\frac{\left(\psi^{*}\partial_{l}\psi-\partial_{l}\psi^{*}\cdot\psi\right)\left(\psi^{*}\partial^{l}\psi-\partial^{l}\psi^{*}\cdot\psi\right)}{4\psi^{*}\psi}+\partial_{l}\psi^{*}\partial^{l}\psi\\\equiv&\frac{\partial_{l}\left(\psi^{*}\psi\right)\partial^{l}\left(\psi^{*}\psi\right)}{4\psi^{*}\psi}+\frac{g^{ls}}{2}\psi^{*}\psi\sum_{\alpha,\beta=1}^{\alpha,\beta=n}Q_{\alpha\beta,l}^{*}Q_{\alpha\beta,s}\end{aligned} \tag{3.32}$$

where

$$Q_{\alpha\beta,l}=\frac{1}{\psi^{*}\psi}\left|\begin{array}{cc}\psi_{\alpha}&\psi_{\beta}\\\partial_{l}\psi_{\alpha}&\partial_{l}\psi_{\beta}\end{array}\right|,\qquad Q_{\alpha\beta,l}^{*}=\frac{1}{\psi^{*}\psi}\left|\begin{array}{cc}\psi_{\alpha}^{*}&\psi_{\beta}^{*}\\\partial_{l}\psi_{\alpha}^{*}&\partial_{l}\psi_{\beta}^{*}\end{array}\right| \tag{3.33}$$

Then we obtain

$$\begin{aligned}\mathcal{A}\left[\psi,\psi^{*}\right]=&\int\left\{\begin{array}{c}\left(ib_{0}\partial_{k}+\frac{e}{c}A_{k}\right)\psi^{*}\left(-ib_{0}\partial^{k}+\frac{e}{c}A^{k}\right)\psi\\+\frac{b_{0}^{2}}{2}\sum\limits_{\alpha,\beta=1}^{\alpha,\beta=n}g^{ls}Q_{\alpha\beta,l}Q_{\alpha\beta,s}^{*}\psi^{*}\psi\end{array}\right.\\&\left.-m^{2}c^{2}\psi^{*}\psi+\left(\hbar^{2}-b_{0}^{2}\right)\frac{\partial_{l}\left(\psi^{*}\psi\right)\partial^{l}\left(\psi^{*}\psi\right)}{4\psi^{*}\psi}\right\}d^{4}x\end{aligned} \tag{3.34}$$

Let us consider the case of irrotational flow, when $g^{\alpha}(\boldsymbol{\xi}) = 0$ and the function ψ has only one component. It follows from (3.33), that $Q_{\alpha\beta,l} = 0$, and only the last term in (3.34) is not bilinear with respect to ψ, ψ^*. The constant b_0 is an arbitrary integration constant. One may set $b_0 = \hbar$. Then we obtain instead of (3.34)

$$\mathcal{A}[\psi, \psi^*] = \int \left\{ \left(i\hbar\partial_k + \frac{e}{c}A_k \right) \psi^* \left(-i\hbar\partial^k + \frac{e}{c}A^k \right) \psi - m^2c^2\psi^*\psi \right\} d^4x \qquad (3.35)$$

Variation of the action (3.35) with respect to ψ^* generates the Klein-Gordon equation

$$\left(-i\hbar\partial_k + \frac{e}{c}A_k \right) \left(-i\hbar\partial^k + \frac{e}{c}A^k \right) \psi - m^2c^2\psi = 0 \qquad (3.36)$$

Thus, description in terms of the Klein-Gordon equation is a special case of the stochastic system description by means of the action (3.4), (3.5).

In the case of rotational flow the wave function is two-component, and the dynamic equation has the form (see for details in [24]):

$$\left(-i\hbar\partial_k + \frac{e}{c}A_k \right) \left(-i\hbar\partial^k + \frac{e}{c}A^k \right) \psi - \left(m^2c^2 + \frac{\hbar^2}{4} \left(\partial_l s_\alpha \right) \left(\partial^l s_\alpha \right) \right) \psi$$
$$= -\hbar^2 \frac{\partial_l \left(\rho \partial^l s_\alpha \right)}{2\rho} \left(\sigma_\alpha - s_\alpha \right) \psi \qquad (3.37)$$

where 3-vector $\mathbf{s} = \{s_1, s_2, s_3,\}$ is defined by the relation

$$\rho = \psi^*\psi, \qquad s_\alpha = \frac{\psi^*\sigma_\alpha\psi}{\rho}, \qquad \alpha = 1, 2, 3 \qquad (3.38)$$

$$\psi = \binom{\psi_1}{\psi_2}, \qquad \psi^* = \left(\psi_1^*, \psi_2^* \right), \qquad (3.39)$$

and Pauli matrices $\boldsymbol{\sigma} = \{\sigma_1, \sigma_2, \sigma_3\}$ have the form

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad (3.40)$$

The gradient of the unit 3-vector $\mathbf{s} = \{s_1, s_2, s_3\}$ describes rotational component of the fluid flow. Equation (3.37) turns to the conventional Klein-Gordon equation (3.36), if $\mathbf{s}$ =const. Curl of the vector field π_k, determined by the relation

$$\partial_k\pi_l - \partial_l\pi_k = -4b_0 \left[\partial_k\mathbf{n} \times \partial_l\mathbf{n} \right] \mathbf{z} + \frac{e}{c} \left(\partial_k A_l - \partial_l A_k \right), \qquad k, l = 0, 1, 2, 3 \qquad (3.41)$$

Here the quantities $\mathbf{n}$ and $\mathbf{z}$ are obtained from the wave function, presented in the form

$$\psi = \sqrt{\rho}e^{i\varphi} \left(\mathbf{n}\boldsymbol{\sigma} \right) \chi, \qquad \psi^* = \sqrt{\rho}e^{-i\varphi}\chi^* \left(\boldsymbol{\sigma}\mathbf{n} \right), \qquad \mathbf{n}^2 = 1, \qquad \chi^*\chi = 1 \qquad (3.42)$$

by means of relations

$$\mathbf{s} = 2\mathbf{n} \left(\mathbf{nz} \right) - \mathbf{z}, \qquad \mathbf{n} = \frac{\mathbf{s} + \mathbf{z}}{\sqrt{2 \left(1 + (\mathbf{sz}) \right)}} \qquad (3.43)$$

Global Journal of Science Frontier Research (A) Volume XIV Issue VII Version I Year 2014

$$\mathbf{z} = \chi^* \boldsymbol{\sigma} \chi, \qquad \mathbf{z}^2 = \chi^* \chi = 1 \tag{3.44}$$

The fundamental difference between the nonrelativistic description (2.8) and the relativistic description (3.19) is as follows. The nonrelativistic equation (2.8) does not contain temporal derivatives, and the field $\mathbf{u}$ is determined uniquely by its source (the particle density ρ). The relativistic equation (3.19) contains temporal derivatives, and the κ-field $u^k = \hbar\kappa^k/m$ can exist without its source. The relativistic κ-field $u^k = \hbar\kappa^k/m$ can escape from its source. Besides, the κ-field changes the effective particle mass, as one can see from the relations (3.1), (3.2). If $\boldsymbol{\kappa}^2$ is large enough, or $\partial_k\kappa^k < 0$ and $\left|\partial_k\kappa^k\right|$ is large enough, the effective particle mass may be imaginary. In this case the mean world line may turn-round in the time direction, and this turn-round may appear to be connected with the pair production, or with the pair annihilation.

In the nonrelativistic case the mean stochastic velocity $\mathbf{u}$ may be eliminated and replaced by its source (the particle density ρ). In the relativistic case the κ-field has in addition its own degrees of freedom, which cannot be eliminated, replacing the κ-field by its source. The κ-field can travel from one space-time region to another one.

The uniform formalism of the particle dynamics (with the statistical ensemble as a basic object of dynamics) admits one to describe such a physical phenomena, which cannot be described in the framework of the conventional dynamic formalism, when the basic object is a single particle. In particular, one can describe the pair production effect, which cannot been described in the framework of the conventional relativistic mechanics, as well as in the framework of the nonrelativistic quantum mechanics.

IV. Deterministic Models of Elementary Particles

Stochastic (and quantum) particles $\mathcal{S}_{\mathrm{st}}$ are described by the statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$. Dynamic equations for $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$ form a system of partial differential equations (PDE). Is it possible to simplify description of stochastic particle, reducing the system of PDE to a system of ODE, describing a statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\mathrm{d}}\right]$ of deterministic particles $\mathcal{S}_{\mathrm{d}}$? It is possible. One needs only to project all derivatives in the system of PDE onto direction of the particle current j^k by means of (1.5). After such a projection the system of PDE turns to the system of ODE. This system of ODE form a dynamic equations for a statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\mathrm{d}}\right]$ of deterministic particles $\mathcal{S}_{\mathrm{d}}$. The deterministic particle $\mathcal{S}_{\mathrm{d}}$ is called a *deterministic model of stochastic particle* $\mathcal{S}_{\mathrm{st}}$. Such a procedure is called dynamic disquantization [9]. The dynamic disquantization (D-disquantization) transforms the dynamic system $\mathcal{E}\left[\mathcal{S}_{\mathrm{st}}\right]$ to a simpler dynamic system $\mathcal{E}\left[\mathcal{S}_{\mathrm{d}}\right]$, where wobbling of the stochastic particle world line is removed. One can obtain dynamic equations for a single $\mathcal{S}_{\mathrm{d}}$ from dynamic equations for $\mathcal{E}\left[\mathcal{S}_{\mathrm{d}}\right]$. Introduction of deterministic model is founded on the fact, that in the coordinate system, where the state of the statistical ensemble is uniform, the stochastic component (2.8) does not contribute in the dynamic equations of the statistical ensemble. The dynamic disquantization is a purely dynamic procedure which removes stochastic fluctuations and generates a deterministic model. In general, the dynamic disquantization removes fluctuations of any kind, but not only quantum fluctuations. For the nonrelativistic equations (Schrödinger equation) the

D-disquantization is equivalent to a transition from nonrelativistic quantum particle to a nonrelativistic classical particle. However, for the relativistic quantum particle (Klein-Gordon equation) the D-disquantization leads to a transition to a relativistic classical particle equipped by a κ-field, which is responsible for the pair production.

To obtain deterministic model of a relativistic quantum particle, let us vary the action (3.4), (3.5) taken in the form

$$\mathcal{A}\left[x,\kappa\right]=\int\left\{-mcK\sqrt{g_{ik}\dot{x}^{i}\dot{x}^{k}}-\frac{e}{c}A_{k}\dot{x}^{k}\right\}d^{4}\xi,\qquad d^{4}\xi=d\xi_{0}d\boldsymbol{\xi},\quad \tau=\boldsymbol{\xi}_{0}\tag{4.1}$$

$$K=\sqrt{1+\lambda^{2}\left(\kappa_{l}\kappa^{l}+\partial_{l}\kappa^{l}\right)},\qquad\lambda=\frac{\hbar}{mc}\tag{4.2}$$

Here $x=\left\{x^{i}\left(\xi_{0},\boldsymbol{\xi}\right)\right\}$, $i=0,1,2,3$ are dependent variables. $\xi=\left\{\xi_{0},\boldsymbol{\xi}\right\}=\left\{\xi_{k}\right\}$, $k=0,1,2,3$ are independent variables, and $\dot{x}^{i}\equiv dx^{i}/d\xi_{0}$. The quantities $\kappa^{l}=\left\{\kappa^{l}\left(x\right)\right\}$, $l=0,1,2,3$ are dependent variables, describing stochastic component of the particle velocity, $A_{k}=\left\{A_{k}\left(x\right)\right\}$, $k=0,1,2,3$ is the potential of electromagnetic field. Variation of (4.1) gives

$$\begin{aligned}\frac{\delta\mathcal{A}\left[x,\kappa\right]}{\delta x^{k}} &= \frac{d}{d\tau}\frac{mcKg_{ik}\dot{x}^{i}}{\sqrt{\dot{x}_{s}\dot{x}^{s}}}-\frac{e}{c}\left(\frac{\partial A_{i}}{\partial x^{k}}-\frac{\partial A_{k}}{\partial x^{i}}\right)\dot{x}^{i}\\ &\quad -\frac{\lambda^{2}mc\sqrt{\dot{x}_{s}\dot{x}^{s}}}{K}\left(\kappa_{l,k}\kappa^{l}+\frac{1}{2}\partial_{k}\partial_{l}\kappa^{l}\right)=0\end{aligned}\tag{4.3}$$

$$\frac{\delta\mathcal{A}\left[x,\kappa\right]}{\delta\kappa^{k}}=\frac{mc\sqrt{\dot{x}_{s}\dot{x}^{s}}J}{K}\kappa_{k}-\frac{\partial}{\partial x^{k}}\frac{mc\sqrt{\dot{x}_{s}\dot{x}^{s}}J}{2K}=0\tag{4.4}$$

Here J is the Jacobian (3.6), which appears, because κ_{l} is a function of x, and one needs to go to integration over x in (4.1), in order to obtain (4.4). One obtains from (4.4)

$$\kappa_{k}=\partial_{k}\kappa=\frac{1}{2}\partial_{k}\ln\frac{mc\sqrt{\dot{x}_{s}\dot{x}^{s}}J}{K}=\frac{1}{2}\partial_{k}\ln\frac{mc\sqrt{j_{s}j^{s}}}{K},\quad j^{k}=\dot{x}^{k}J\tag{4.5}$$

Using (4.2), one can write (4.5) in the form of dynamic equation for the variable κ which can be written in the form

$$e^{3\kappa}\left(e^{\kappa}+\lambda^{2}\partial_{s}\partial^{s}e^{\kappa}\right)=C^{2}m^{2}c^{2}j_{s}j^{s}\tag{4.6}$$

where C is the integration constant. The quantity C does not depend on x, but it may depend on coordinates of other particles.

Introducing new variable

$$w=e^{\kappa}\tag{4.7}$$

the equation (4.6) can be written in the form

$$\hbar^{2}\partial_{s}\partial^{s}w+m^{2}c^{2}w=\frac{C^{2}m^{2}c^{2}j_{s}j^{s}}{w^{3}}\tag{4.8}$$

$$K=\sqrt{1+\frac{\lambda^{2}}{w}\partial_{s}\partial^{s}w}\tag{4.9}$$

Global Journal of Science Frontier Research (A) Volume XIV Issue VII Version I Year 2014

Equation (4.3) Must be written in the form

$$\frac{d}{d\tau}\frac{mcKg_{ik}\dot{x}^i}{\sqrt{\dot{x}_s\dot{x}^s}}-\frac{e}{c}\left(\frac{\partial A_i}{\partial x^k}-\frac{\partial A_k}{\partial x^i}\right)\dot{x}^i-mc\sqrt{\dot{x}_s\dot{x}^s}\partial_k K=0 \qquad (4.10)$$

The relativistic stochastic particle $\mathcal{S}_{\text{st}}$ is described by equations (4.8) - (4.10). Its stochasticity is conditioned by the field w, which depends on the state of the whole statistical ensemble $\mathcal{E}\left[\mathcal{S}_{\text{st}}\right]$ and maybe on other particles via the constant C. Operation of disquantization cannot be applied to the field w, because this field is an external field (at least, partly). Thus, the exact equations (4.8) - (4.10) are dynamic equations for deterministic model simultaneously.

V. Dirac Equation in Terms of Hydrodynamic Variables

The Dirac particle is a dynamic system $\mathcal{S}_{\text{D}}$, whose dynamic equation is the Dirac equation

$$i\gamma^k\partial_k\psi+mc\psi=0 \qquad (5.1)$$

The Dirac dynamic system $\mathcal{S}_{\text{D}}$ was investigated by many researchers. There is no possibility to list all them, and we mention only some of them. First, this is transformation of the Dirac equation on the base of quantum mechanics [26, 27]. The complicated structure of Dirac particle was discovered by Schrödinger [28], who interpreted it as some complicated quantum motion (zitterbewegung). Investigation of this quantum motion and different models of Dirac particle can be found in [29, 30, 31, 32, 33] and references therein. Our investigation differs in absence of any suppositions on the Dirac particle model and in absence of referring to the quantum principles. We use only dynamic methods and *investigate the Dirac particle* $\mathcal{S}_{\text{D}}$ *simply as a dynamic system.* To obtain the deterministic model of the Dirac particle, one needs to write the Dirac equation in terms of hydrodynamic variables.

The action for a free Dirac particle is written in the form

$$\mathcal{S}_{\text{D}}: \qquad \mathcal{A}_{\text{D}}[\bar{\psi},\psi]=\int(-m\bar{\psi}\psi+\frac{i}{2}\hbar\bar{\psi}\gamma^l\partial_l\psi-\frac{i}{2}\hbar\partial_l\bar{\psi}\gamma^l\psi)d^4x \qquad (5.2)$$

Here ψ is four-component complex wave function, ψ^* is the Hermitian conjugate wave function, and $\bar{\psi}=\psi^*\gamma^0$ is conjugate one. γ^i, $i=0,1,2,3$ are 4×4 complex constant matrices, satisfying the relation

$$\gamma^l\gamma^k+\gamma^k\gamma^l=2g^{kl}I, \qquad k,l=0,1,2,3. \qquad (5.3)$$

where I is the unit 4×4 matrix, and g^{kl} =diag$(c^{-2},-1,-1,-1)$ is the metric tensor. Considering dynamic system $\mathcal{S}_{\text{D}}$, we choose for simplicity such units, where the speed of the light $c=1$.

In our calculations we used the mathematical technique [35, 36], where γ-matrices are represented as hypercomplex numbers. Using designations

$$\gamma_5=\gamma^{0123}\equiv\gamma^0\gamma^1\gamma^2\gamma^3, \qquad (5.4)$$

$$\boldsymbol{\sigma}=\{\sigma_1,\sigma_2,\sigma_3,\}=\{-i\gamma^2\gamma^3,-i\gamma^3\gamma^1,-i\gamma^1\gamma^2\} \qquad (5.5)$$

we make the change of variables

$$\psi=Ae^{i\varphi+\frac{1}{2}\gamma_5\kappa}\exp\left(-\frac{i}{2}\gamma_5\boldsymbol{\sigma\eta}\right)\exp\left(\frac{i\pi}{2}\boldsymbol{\sigma}\mathbf{n}\right)\Pi \tag{5.6}$$

$$\psi^*=A\Pi\exp\left(-\frac{i\pi}{2}\boldsymbol{\sigma}\mathbf{n}\right)\exp\left(-\frac{i}{2}\gamma_5\boldsymbol{\sigma\eta}\right)e^{-i\varphi-\frac{1}{2}\gamma_5\kappa} \tag{5.7}$$

where (*) means the Hermitian conjugation, and

$$\Pi=\frac{1}{4}(1+\gamma^0)(1+\mathbf{z}\boldsymbol{\sigma}),\qquad \mathbf{z}=\{z^\alpha\}=\text{const},\qquad \alpha=1,2,3;\qquad \mathbf{z}^2=1 \tag{5.8}$$

is a zero divisor. The quantities A, κ, φ, $\boldsymbol{\eta}=\{\eta^\alpha\}$, $\mathbf{n}=\{n^\alpha\}$, $\alpha=1,2,3$, $\mathbf{n}^2=1$ are eight real parameters, determining the wave function ψ. These parameters may be considered as new dependent variables, describing the state of dynamic system $\mathcal{S}_\mathrm{D}$. The quantity φ is a scalar, and κ is a pseudoscalar. Six remaining variables A, $\boldsymbol{\eta}=\{\eta^\alpha\}$, $\mathbf{n}=\{n^\alpha\}$, $\alpha=1,2,3$, $\mathbf{n}^2=1$ can be expressed through the flux 4-vector

$$j^l=\bar\psi\gamma^l\psi,\qquad l=0,1,2,3 \tag{5.9}$$

and spin 4-pseudovector

$$S^l=i\bar\psi\gamma_5\gamma^l\psi,\qquad l=0,1,2,3 \tag{5.10}$$

Because of two identities

$$S^lS_l\equiv -j^lj_l,\qquad j^lS_l\equiv 0, \tag{5.11}$$

there are only six independent components among eight components of quantities j^l, and S^l.

After change of variables (5.6), (5.7) the γ-matrices disappear from the action and from dynamic equations. One obtains the action (5.2) in terms of hydrodynamic variables j,φ,ξ,κ (see details of calculations in [11, 34])

$$\mathcal{S}_\mathrm{D}:\qquad \mathcal{A}_D[j,\varphi,\kappa,\boldsymbol{\xi}]=\int\mathcal{L}d^4x,\qquad \mathcal{L}=\mathcal{L}_\mathrm{cl}+\mathcal{L}_\mathrm{q1}+\mathcal{L}_\mathrm{q2} \tag{5.12}$$

$$\mathcal{L}_\mathrm{cl}=-m\rho-\hbar j^i\partial_i\varphi-\frac{\hbar j^l}{2\left(1+\boldsymbol{\xi}\mathbf{z}\right)}\varepsilon_{\alpha\beta\gamma}\xi^\alpha\partial_l\xi^\beta z^\gamma,\qquad \rho\equiv\sqrt{j^lj_l} \tag{5.13}$$

$$\mathcal{L}_\mathrm{q1}=2m\rho\sin^2\left(\frac{\kappa}{2}\right)-\frac{\hbar}{2}S^l\partial_l\kappa, \tag{5.14}$$

$$\mathcal{L}_\mathrm{q2}=\frac{\hbar(\rho+j_0)}{2}\varepsilon_{\alpha\beta\gamma}\partial^\alpha\frac{j^\beta}{(j^0+\rho)}\xi^\gamma-\frac{\hbar}{2(\rho+j_0)}\varepsilon_{\alpha\beta\gamma}\left(\partial^0j^\beta\right)j^\alpha\xi^\gamma \tag{5.15}$$

Lagrangian is a function of 4-vector j^l, scalar φ, pseudoscalar κ, and unit 3-pseudovector $\boldsymbol{\xi}$, which is connected with the spin 4-pseudovector S^l (5.10) by means of the relations

$$\xi^\alpha=\rho^{-1}\left[S^\alpha-\frac{j^\alpha S^0}{(j^0+\rho)}\right],\qquad \alpha=1,2,3;\qquad \rho\equiv\sqrt{j^lj_l} \tag{5.16}$$

$$S^0 = \mathbf{j}\boldsymbol{\xi}, \qquad S^\alpha = \rho\xi^\alpha + \frac{(\mathbf{j}\boldsymbol{\xi})j^\alpha}{\rho + j^0}, \qquad \alpha = 1, 2, 3 \tag{5.17}$$

After change of variables the description of $\mathcal{S}_{\text{D}}$ ceases to be relativistically covariant, because the constant matrix 4-vector γ^k is transformed to dynamical variables (see discussion in [37, 39]).

VI. Dynamic Disquantization of Dirac Equation

Let us produce dynamical disquantization [9, 41] of the action (5.12)–(5.15), making the change (1.5). The action (5.12)–(5.15) takes the form

$$\begin{aligned}\mathcal{A}_{\text{Dqu}}[j, \varphi, \kappa, \boldsymbol{\xi}] \quad &= \quad \int \left\{ -m\rho\cos\kappa - \hbar j^i \left(\partial_i \varphi + \frac{\varepsilon_{\alpha\beta\gamma}\xi^\alpha \partial_i \xi^\beta z^\gamma}{2\left(1 + \boldsymbol{\xi}\mathbf{z}\right)} \right) \right. \\ &\quad \left. + \frac{\hbar j^k}{2(\rho + j_0)\rho} \varepsilon_{\alpha\beta\gamma} \left(\partial_k j^\beta\right) j^\alpha \xi^\gamma \right\} d^4x \end{aligned} \tag{6.1}$$

Note that the second term $-\frac{\hbar}{2}S^l\partial_l\kappa$ in the relation (5.14) is neglected, because 4-pseudovector S^k is orthogonal to 4-vector j^k, and the derivative $S^l\partial_{||l}\kappa = S^l\rho^{-2}j_l j^k\partial_k \kappa/j_s j^s$ vanishes. The action (6.1) is also non-relativistically invariant, because the dynamic disquantization (1.5) is a relativistic procedure.

Although the action (6.1) contains a non-classical variable κ, in fact this variable is a constant. Indeed, a variation with respect to κ leads to the dynamic equation

$$\frac{\delta\mathcal{A}_{Dqu}}{\delta\kappa} = m\rho\sin\kappa = 0, \qquad \rho \equiv \sqrt{j_s j^s} \tag{6.2}$$

which has solutions

$$\kappa = n\pi, \qquad n = \text{integer} \tag{6.3}$$

Thus, the effective mass $m_{\text{eff}} = m\cos\kappa$ has two values

$$m_{\text{eff}} = m\cos\kappa = \kappa_0 m = \pm m \tag{6.4}$$

where κ_0 is a dichotomic quantity $\kappa_0 = \pm 1$ introduced instead of $\cos\kappa$. The quantity κ_0 is a parameter of the dynamic system $\mathcal{S}_{\text{Dqu}}$. It is not to be varying. The action (6.1), turns into the action

$$\begin{aligned}\mathcal{A}_{\text{Dqu}}[j, \varphi, \boldsymbol{\xi}] \quad &= \quad \int \left\{ -\kappa_0 m\rho - \hbar j^i \left(\partial_i \varphi + \frac{\varepsilon_{\alpha\beta\gamma}\xi^\alpha \partial_i \xi^\beta z^\gamma}{2\left(1 + \boldsymbol{\xi}\mathbf{z}\right)} \right) \right. \\ &\quad \left. + \frac{\hbar j^k}{2(\rho + j_0)\rho} \varepsilon_{\alpha\beta\gamma} \left(\partial_k j^\beta\right) j^\alpha \xi^\gamma \right\} d^4x \end{aligned} \tag{6.5}$$

Let us introduce Lagrangian coordinates $\tau = \{\tau_0, \boldsymbol{\tau}\} = \{\tau_i(x)\}$, $i = 0, 1, 2, 3$ as functions of coordinates x in such a way that only coordinate τ_0 changes along the direction j^l. The action (6.5) is transformed to the form

$$\mathcal{A}_{\text{Dqu}}[x, \boldsymbol{\xi}] = \int \mathcal{A}_{\text{Dd}}[x, \boldsymbol{\xi}]d\boldsymbol{\tau}, \qquad \mathbf{d}\boldsymbol{\tau} = d\tau_1 d\tau_2 d\tau_3 \tag{6.6}$$

Global Journal of Science Frontier Research (A) Volume XIV Issue VII Version I Year 2014

where

$$\mathcal{S}_{\mathrm{Dd}}: \quad \mathcal{A}_{\mathrm{Dd}}[x,\boldsymbol{\xi}] = \int \left\{ \begin{array}{c} -\kappa_0 m\sqrt{\dot{x}^i\dot{x}_i} + \hbar\frac{(\dot{\boldsymbol{\xi}}\times\boldsymbol{\xi})\mathbf{z}}{2(1+\boldsymbol{\xi}\mathbf{z})} \\ +\hbar\frac{(\dot{\mathbf{x}}\times\ddot{\mathbf{x}})\boldsymbol{\xi}}{2\sqrt{\dot{x}^s\dot{x}_s}(\sqrt{\dot{x}^s\dot{x}_s}+\dot{x}^0)} \end{array} \right\} d\tau_0 \tag{6.7}$$

After dynamic disquantization the Dirac particle is a statistical ensemble of dynamic systems $\mathcal{S}_{\mathrm{Dd}}$, as it follows from (6.6) and (6.7). Any dynamic system $\mathcal{S}_{\mathrm{Dd}}$ has 10 degrees of freedom. Six degrees of freedom describe a progressive motion of a particle and 4 degrees of freedom describe the rotational motion of the particle. It is a deterministic model of the Dirac particle, which contains the quantum constant. The quantum constant appears in classical dynamic equations, because these equations are to contain magnetic moment. But the magnetic moment, (classical quantity!) depends on the quantum constant. The variables $\boldsymbol{\xi}$ describe rotation, which is a deterministic analog of so-called "zitterbewegung". The Dirac particle is not a pointlike particle [34]. Description of internal degrees of freedom in terms of $\boldsymbol{\xi}$ appears to be nonrelativistic [39, 37], although the translational degrees of freedom in terms of x are described relativistically.

It is easy to see that the action (6.7) is invariant with respect to transformation $\tau_0 \to \tilde{\tau}_0 = F(\tau_0)$, where F is an arbitrary monotone function. This transformation admits one to choose the variable $t = x^0$ as a parameter τ_0, or to choose the parameter τ_0 in such a way that $\dot{x}^l\dot{x}_l = \dot{x}_0^2 - \dot{\mathbf{x}}^2 = 1$. In the last case the parameter τ_0 is the proper time along the world line of deterministic Dirac particle. Besides, invariance with respect to transformation $\tau_0 \to \tilde{\tau}_0 = F(\tau_0)$ leads to a connection between the components of the canonical momentum

$$p_k = \frac{\partial L}{\partial \dot{x}^k} - \frac{d}{d\tau_0}\frac{\partial L}{\partial \ddot{x}^k}, \qquad k = 0,1,2,3$$

where L is the Lagrange function for the action (6.7).

We shall not consider here problems connected with relativistic non-invariance of terms, describing internal degrees of freedom, referring to [9], where these problems are discussed. We obtain dynamic equations generated by the action (6.7), solve them and try to interpret the obtained solution.

Variation of the action (6.7) with respect to $\mathbf{x}$ gives the dynamic equation

$$\frac{d}{d\tau_0}\left(-\kappa_0 m\frac{\dot{\mathbf{x}}}{\sqrt{\dot{x}^s\dot{x}_s}} + \frac{\hbar Q}{2}(\boldsymbol{\xi}\times\ddot{\mathbf{x}}) - \frac{\hbar}{2}\frac{\partial Q}{\partial \dot{\mathbf{x}}}(\dot{\mathbf{x}}\times\ddot{\mathbf{x}})\boldsymbol{\xi} + \frac{\hbar}{2}\frac{d}{d\tau_0}(Q(\boldsymbol{\xi}\times\dot{\mathbf{x}}))\right) = 0 \tag{6.8}$$

where

$$Q = Q(\dot{x}) = \left(\sqrt{\dot{x}^s\dot{x}_s}(\sqrt{\dot{x}^s\dot{x}_s}+\dot{x}^0)\right)^{-1}, \qquad \dot{x}^s\dot{x}_s = \dot{x}_0^2 - \dot{\mathbf{x}}^2 \tag{6.9}$$

Varying the action (6.7) with respect to x^0, we obtain

$$\frac{d}{d\tau_0}\left(\kappa_0 m\frac{\dot{x}^0}{\sqrt{\dot{x}^s\dot{x}_s}} - \frac{\hbar}{2}\frac{\partial Q}{\partial \dot{x}^0}(\dot{\mathbf{x}}\times\ddot{\mathbf{x}})\boldsymbol{\xi}\right) = 0 \tag{6.10}$$

Varying the action (6.7) with respect to $\boldsymbol{\xi}$, one should take into account the side constraint $\boldsymbol{\xi}^2 = 1$. Setting

$$\xi^\alpha = \frac{\zeta^\alpha}{\sqrt{\boldsymbol{\zeta}^2}}, \qquad \alpha = 1,2,3 \tag{6.11}$$

Global Journal of Science Frontier Research (A) Volume XIV Issue VII Version I Year 2014

where $\boldsymbol{\zeta}$ is an arbitrary 3-pseudovector, one obtains

$$\frac{\delta\mathcal{A}_{\rm dcl}}{\delta\zeta^{\mu}}=\frac{\delta\mathcal{A}_{\rm dcl}}{\delta\xi^{\alpha}}\frac{\delta\xi^{\alpha}}{\delta\zeta^{\mu}}=\frac{\delta\mathcal{A}_{\rm dcl}}{\delta\xi^{\alpha}}\frac{\delta^{\alpha\mu}-\xi^{\alpha}\xi^{\mu}}{\sqrt{\boldsymbol{\zeta}^{2}}}=0 \tag{6.12}$$

It means that there are only two independent equations among three dynamic equations (6.12). They are orthogonal to 3-pseudovector $\boldsymbol{\xi}$ and can be obtained from equation $\delta\mathcal{A}_{\rm dcl}/\delta\xi^{\alpha}=0$ by means of vector product with $\boldsymbol{\xi}$.

$$-\hbar\frac{\left(\dot{\boldsymbol{\xi}}\times\mathbf{z}\right)\times\boldsymbol{\xi}}{2(1+\mathbf{z}\boldsymbol{\xi})}+\hbar\left(-\frac{d}{d\tau_{0}}\frac{(\boldsymbol{\xi}\times\mathbf{z})}{2(1+\mathbf{z}\boldsymbol{\xi})}-\frac{(\dot{\boldsymbol{\xi}}\times\boldsymbol{\xi})\mathbf{z}}{2(1+\mathbf{z}\boldsymbol{\xi})^{2}}\mathbf{z}\right)\times\boldsymbol{\xi}+\hbar\frac{(\dot{\mathbf{x}}\times\ddot{\mathbf{x}})\times\boldsymbol{\xi}}{2}Q=0 \tag{6.13}$$

After transformations this equation reduces to the equation (see Appendix)

$$\dot{\boldsymbol{\xi}}=-\boldsymbol{\xi}\times(\dot{\mathbf{x}}\times\ddot{\mathbf{x}})Q, \tag{6.14}$$

which does not contain the vector $\mathbf{z}$. It means that $\mathbf{z}$ determines a fictitious direction in the space-time.

Using invariance of the action (6.7) with respect to transformation of the parameter τ_0, we choose τ_0 in such a way, that

$$\sqrt{\dot{x}^{s}\dot{x}_{s}}=\sqrt{\dot{x}_{0}^{2}-\dot{\mathbf{x}}^{2}}=1,\qquad \dot{x}_{0}=\sqrt{1+\dot{\mathbf{x}}^{2}} \tag{6.15}$$

Then, using condition (6.15), we obtain from (6.9) for quantities Q, $\partial Q/\partial\dot{x}_0$, $\partial Q/\partial\dot{\mathbf{x}}$

$$Q=\frac{1}{1+\dot{x}_{0}},\qquad \frac{\partial Q}{\partial\dot{x}_{0}}=-1,\qquad \frac{\partial Q}{\partial\dot{\mathbf{x}}}=\frac{\dot{\mathbf{x}}\left(2+\dot{x}_{0}\right)}{\left(1+\dot{x}_{0}\right)^{2}} \tag{6.16}$$

Integration of equation (6.10) leads to

$$\kappa_{0}m\dot{x}_{0}+\frac{\hbar}{2}\left(\dot{\mathbf{x}}\times\ddot{\mathbf{x}}\right)\boldsymbol{\xi}=-p_{0} \tag{6.17}$$

where p_0 is an integration constant. This constant p_0 describes the time component of the dynamic system $\mathcal{S}_{\rm Dd}$ canonical 4-momentum.

Integration of equation (6.8) gives

$$-\kappa_{0}m\frac{\dot{\mathbf{x}}}{\sqrt{\dot{x}^{s}\dot{x}_{s}}}+\frac{\hbar Q}{2}(\boldsymbol{\xi}\times\ddot{\mathbf{x}})-\frac{\hbar}{2}\frac{\partial Q}{\partial\dot{\mathbf{x}}}(\dot{\mathbf{x}}\times\ddot{\mathbf{x}})\boldsymbol{\xi}+\frac{\hbar}{2}\frac{d}{d\tau_{0}}\left(Q(\boldsymbol{\xi}\times\dot{\mathbf{x}})\right)=-\mathbf{p}=\text{const} \tag{6.18}$$

where $\mathbf{p}$ is the 3-momentum of the dynamic system $\mathcal{S}_{\rm Dd}$ as a whole.

Using the gauge (6.9) and relations (6.16), we rewrite the equation (6.18) in the form

$$-m\dot{\mathbf{x}}+\frac{\hbar}{2}\frac{(\boldsymbol{\xi}\times\ddot{\mathbf{x}})}{1+\dot{x}_{0}}-\frac{\hbar}{2}\frac{\dot{\mathbf{x}}\left(2+\dot{x}_{0}\right)}{\left(1+\dot{x}_{0}\right)^{2}}(\dot{\mathbf{x}}\times\ddot{\mathbf{x}})\boldsymbol{\xi}+\frac{\hbar}{2}\frac{d}{d\tau_{0}}\left(\frac{(\boldsymbol{\xi}\times\dot{\mathbf{x}})}{1+\dot{x}_{0}}\right)=-\mathbf{p} \tag{6.19}$$

If we set $\hbar=0$ in (6.19), we obtain conventional connection $\mathbf{p}=m\dot{\mathbf{x}}$ between the velocity $\dot{\mathbf{x}}=d\mathbf{x}/d\tau_0$ and the momentum of a free particle. But the quantum constant $\hbar$ is a coefficient before the highest time derivative, and setting $\hbar = 0$, we suppress some degrees of freedom.

If these additional degrees of freedom are not excited (or suppressed), the classical Dirac particle has six degrees of freedom. We shall see that characteristic energy associated with additional degrees of freedom is of the order of the particle rest energy m. At low energetic processes (calculation of atomic spectra, quantum electrodynamics) one may neglect these degrees of freedom, remaining only numerical characteristics (spin, magnetic momentum) of these degrees of freedom. However, in the case of high energies (ultrarelativistic collisions, structure of elementary particles), one cannot neglect these degrees of freedom. Of course, using the Dirac equation, we take into account these additional degrees of freedom automatically. But it is important also to take into account these additional degrees of freedom in our interpretation of the high energetic processes.

Transformation and solution of equation (6.18) is rather bulky. Many efforts were used to prove that the 3-vectors $\boldsymbol{\xi}$, $\dot{\mathbf{x}}$, and $\ddot{\mathbf{x}}$ are mutually orthogonal and their modules are constant [9] in the coordinate system, where $\mathbf{p} = 0$. We shall not spend time for this proof. Instead, we choose the coordinate system in such a way that $\mathbf{p} = 0$

$$\boldsymbol{\xi} = \{0, 0, \varepsilon_0\}, \qquad \varepsilon_0 = \pm 1 \tag{6.20}$$

and impose constraints

$$\dot{\mathbf{x}}^2 = \text{const}, \qquad (\dot{\mathbf{x}}\boldsymbol{\xi}) = 0, \qquad (\ddot{\mathbf{x}}\boldsymbol{\xi}) = 0, \qquad (\dot{\mathbf{x}} \times \ddot{\mathbf{x}})\,\boldsymbol{\xi} = \text{const} \tag{6.21}$$

We use constraints (6.21) in solution of the system of dynamic equations (6.14), (6.17), (6.19) and show that the constraints (6.21) are compatible with dynamic equations (6.14), (6.17), (6.19).

Taking into account (6.21) and (6.15), we introduce new variables

$$\mathbf{y} = \frac{\dot{\mathbf{x}}}{\sqrt{1+\dot{x}_0}} = \frac{\dot{\mathbf{x}}}{\sqrt{1+\sqrt{1+\dot{\mathbf{x}}^2}}}, \qquad \dot{\mathbf{x}} = \mathbf{y}\sqrt{(\mathbf{y}^2+2)} \tag{6.22}$$

$$\dot{x}_0 = \sqrt{1+\mathbf{y}^2\left(\mathbf{y}^2+2\right)} = \mathbf{y}^2 + 1 \tag{6.23}$$

Introducing designation

$$\mathbf{y}^2 = \gamma - 1 = \text{const}, \tag{6.24}$$

we obtain

$$\dot{x}_0 = \sqrt{1+\mathbf{y}^2\left(\mathbf{y}^2+2\right)} = \mathbf{y}^2 + 1 = \gamma = \text{const} \tag{6.25}$$

Then at $\mathbf{p} = 0$ the equation (6.19) takes the form

$$-\kappa_0 m\mathbf{y}\left(\gamma+1\right) + \frac{\hbar}{2}(\boldsymbol{\xi} \times \dot{\mathbf{y}}) - \frac{\hbar}{2}\left(\gamma+2\right)\left((\mathbf{y} \times \dot{\mathbf{y}})\boldsymbol{\xi}\right)\mathbf{y} + \frac{\hbar}{2}\frac{d}{d\tau_0}\left((\boldsymbol{\xi} \times \mathbf{y})\right) = 0 \tag{6.26}$$

The equation (6.14) takes the form

$$\dot{\boldsymbol{\xi}} = -(\mathbf{y} \times \dot{\mathbf{y}}) \times \boldsymbol{\xi} = 0 \tag{6.27}$$

because of constraints (6.21). In terms of variables $\mathbf{y}$ conditions (6.21) have the form

Global Journal of Science Frontier Research (A) Volume XIV Issue VII Version I Year 2014

$$\mathbf{y}^2 = \gamma - 1, \qquad (\boldsymbol{\xi}\mathbf{y}) = 0, \qquad (\boldsymbol{\xi}\dot{\mathbf{y}}) = 0, \qquad (\mathbf{y}\dot{\mathbf{y}}) = 0 \tag{6.28}$$

where γ is a constant of integration. In accordance with (6.25) and (6.28) we obtain

$$(\mathbf{y} \times \dot{\mathbf{y}})\boldsymbol{\xi} = \varepsilon_0\omega(\gamma - 1) \tag{6.29}$$

where ω is an indefinite constant (some angular velocity).

Substituting (6.29) in (6.26), we obtain after simplification

$$(\boldsymbol{\xi} \times \dot{\mathbf{y}}) - \left(\frac{1}{2}(\gamma+2)(\gamma-1)\varepsilon_0\omega + \frac{\kappa_0 m}{\hbar}(\gamma+1)\right)\mathbf{y} = 0 \tag{6.30}$$

As far as $\mathbf{y}^2 = \gamma - 1$, the equation (6.29) describes rotation of the vector $\mathbf{y}$ with the angular frequency ω. Equation (6.30) describes rotation of the vector $\mathbf{y}$ around the vector $\boldsymbol{\xi}$ with the angular frequency $\frac{1}{2}(\gamma+2)(\gamma-1)\varepsilon_0\omega + \frac{\kappa_0 m}{\hbar}(\gamma+1)$. Equations (6.29) and (6.30) are compatible, if these frequencies coincide. According to (6.28) vectors $\mathbf{y}$ and $\dot{\mathbf{y}}$ are orthogonal to $\boldsymbol{\xi}$. Then in accordance with (6.20) the vectors $\mathbf{y}$ and $\dot{\mathbf{y}}$ can be represented in the form

$$\mathbf{y} = \left\{\sqrt{\gamma-1}\cos\Phi, \sqrt{\gamma-1}\sin\Phi, 0\right\} \tag{6.31}$$

$$\dot{\mathbf{y}} = \left\{-\sqrt{\gamma-1}\omega\sin\Phi, \sqrt{\gamma-1}\omega\cos\Phi, 0\right\}, \qquad \omega = \frac{d\Phi}{d\tau_0} \tag{6.32}$$

By means of (6.31), and (6.32) the equations (6.30) take the form

$$-\varepsilon_0\omega y_1 - \left(\frac{1}{2}(\gamma+2)(\gamma-1)\varepsilon_0\omega + \frac{\kappa_0 m}{\hbar}(\gamma+1)\right)y_1 = 0 \tag{6.33}$$

$$-\varepsilon_0\omega y_2 - \left(\frac{1}{2}(\gamma+2)(\gamma-1)\varepsilon_0\omega + \frac{\kappa_0 m}{\hbar}(\gamma+1)\right)y_2 = 0 \tag{6.34}$$

Equations (6.33), (6.34) are satisfied, provided

$$\varepsilon_0\omega + \left(\frac{1}{2}(\gamma+2)(\gamma-1)\varepsilon_0\omega + \frac{\kappa_0 m}{\hbar}(\gamma+1)\right) = 0 \tag{6.35}$$

Solution of (6.35) has the form

$$\omega = -\frac{2\varepsilon_0\kappa_0 m}{\hbar\gamma} \tag{6.36}$$

According to (6.22) and (6.23) the dynamic equation (6.17) takes the form

$$-p_0 = \kappa_0 m\gamma + \frac{\hbar}{2}(\mathbf{y} \times \dot{\mathbf{y}})\boldsymbol{\xi}(\gamma+1) \tag{6.37}$$

Using relations (6.29) and (6.36) we obtain from (6.37)

$$-p_0 = \kappa_0 m\left(\gamma - \frac{\gamma^2-1}{\gamma}\right) = \frac{\kappa_0 m}{\gamma}, \qquad \kappa_0 = \pm 1 \tag{6.38}$$

Then we obtain for the total mass M_{Dd} of the dynamic system $\mathcal{S}_{\mathrm{Dd}}$.

$$M_{\mathrm{Dd}}=\sqrt{p_0^2-\mathbf{p}^2}=|p_0|=\frac{m}{\gamma} \tag{6.39}$$

Note, that writing the relation (6.39), we do not act quite consequently. Writing the relation (6.39), we suppose that the dynamic equations (6.17) and (6.18) are relativistically invariant, and solution of equations (6.17), (6.18) for arbitrary $\mathbf{p}$ can be obtained from the solution for $\mathbf{p}=0$ by means of a corresponding Lorentz transformation. Unfortunately, dynamic equations (6.17), (6.18) are not relativistically invariant, and for arbitrary $\mathbf{p}$ the solution is not a helix, in general, although it is a helix for $\mathbf{p}=0$. World line is a helix approximately in the nonrelativistic case, when $|\mathbf{p}|\ll m$.

Let us transit from independent variable τ_0 to the independent variable $x^0=t$. We have

$$\Omega t=-\varepsilon_0\kappa_0\omega\tau_0,\qquad -\varepsilon_0\kappa_0\omega=\Omega\dot{x}_0=\Omega\gamma=\frac{2m}{\hbar\gamma},\qquad \Omega=\frac{2m}{\hbar\gamma^2} \tag{6.40}$$

Returning from variables $\mathbf{y}$ to variables $\dot{\mathbf{x}}$, we obtain instead of (6.31) and (6.32)

$$\frac{d\mathbf{x}}{dt}=\left\{\frac{\sqrt{\gamma^2-1}}{\gamma}\cos\left(\Omega t\right),-\frac{\sqrt{\gamma^2-1}}{\gamma}\sin\left(\Omega t\right),0\right\},\qquad \Omega=\frac{2m}{\hbar\gamma^2} \tag{6.41}$$

$$\mathbf{x}=\left\{\frac{\hbar\gamma\sqrt{\gamma^2-1}}{2m}\sin\left(\frac{2m}{\hbar\gamma^2}t\right),\frac{\hbar\gamma\sqrt{\gamma^2-1}}{2m}\cos\left(\frac{2m}{\hbar\gamma^2}t\right),0\right\} \tag{6.42}$$

where $\gamma\geq 1$ is an arbitrary constant.

Thus, in the coordinate system, where the canonical momentum four-vector has the form

$$P_k=\{p_0,\mathbf{p}\}=\left\{-\frac{\kappa_0 m}{\gamma},0,0,0\right\} \tag{6.43}$$

the world line of the deterministic Dirac particle is a helix, which is described by the relation

$$\{t,\mathbf{x}\} = \{t,a_{\mathrm{Dd}}\sin\left(\Omega t\right),a_{\mathrm{Dd}}\cos\left(\omega_{\mathrm{Dd}}t\right),0\} \tag{6.44}$$

$$a_{\mathrm{Dd}} = \frac{\hbar\gamma\sqrt{\gamma^2-1}}{2m},\qquad \omega_{\mathrm{Dd}}=\frac{2m}{\hbar\gamma^2} \tag{6.45}$$

It follows from (6.41) that the classical Dirac particle velocity $\mathbf{v}=d\mathbf{x}/dt$ is expressed as follows

$$\mathbf{v}^2=1-\frac{1}{\gamma^2},\qquad \gamma=\frac{1}{\sqrt{1-\mathbf{v}^2}} \tag{6.46}$$

In other words, the quantity γ is the Lorentz factor of the classical Dirac particle.

We see that the characteristic frequency, connected with the internal degrees of freedom is $2m/\gamma^2$, and the characteristic energy is of the order $|-m\gamma+m\gamma^{-1}|$.

Parameters γ and ω_{Dd} as functions of the radius a_{Dd} and the Dirac mass m have the form

$$\gamma=\sqrt{\frac{1}{2}\left(1+\sqrt{1+\zeta^2}\right)},\qquad \omega_{\mathrm{Dd}}=\frac{4m}{\hbar\left(1+\sqrt{1+\zeta^2}\right)}\qquad \zeta=\frac{4ma_{\mathrm{Dd}}}{\hbar} \tag{6.47}$$

Global Journal of Science Frontier Research (A) Volume XIV Issue VII Version I Year 2014

VII. Discrete Space-Time Geometry

Foundation of quantum mechanics as a statistical description of stochastically moving particles brings up the question: Why do free microparticles move stochastically? It appears that the space-time is discrete, and particles of small mass feel this discreteness. As a result the particles of small mass move stochastically. World function σ_{d} of the discrete space-time geometry $\mathcal{G}_{\mathrm{d}}$ is restricted by the relation (1.1).

In the nonrelativistic physics the particle state is described as a point in the phase space of coordinates and momenta. The particle world line is supposed to be smooth and the particle 4-momentum p_k is described by the relation

$$p_k = g_{kl}\frac{dx^l}{d\tau} = g_{kl}\lim_{d\tau\to 0}\frac{x^l(\tau+d\tau)-x^l(\tau)}{d\tau} \tag{7.1}$$

where $x^l = x^l(\tau)$, $l = 0,1,2,3$ is an equation of the world line. In the relativistic case the particle state is described by the world line. In $\mathcal{G}_{\mathrm{d}}$ a world line cannot be smooth, because the limit (7.1) does not exist in $\mathcal{G}_{\mathrm{d}}$. In $\mathcal{G}_{\mathrm{d}}$ a smooth world line is replaced by sequential set of points $...P_0, P_1, P_2, ...$, or by a broken line, whose links are straight segments of the same length

$$\mathcal{C} = \bigcup_s \mathbf{P}_s\mathbf{P}_{s+1}, \quad |\mathbf{P}_s\mathbf{P}_{s+1}| = \mu, \quad s = ...0,1,.. \tag{7.2}$$

where the length $|\mu| \geq \lambda_0$, and λ_0 is a parameter (elementary length) of the world function σ_{d}, defined by (1.4). The term "world chain" will be used for such a broken world line. The quantity μ is the geometric mass of the particle. It is connected with the usual mass m by the relation

$$m = b\mu \tag{7.3}$$

where b is some universal constant.

For a free particle the adjacent g-vectors $\mathbf{P}_s\mathbf{P}_{s+1}$ and $\mathbf{P}_{s+1}\mathbf{P}_{s+2}$ are in parallel

$$(\mathbf{P}_s\mathbf{P}_{s+1}.\mathbf{P}_{s+1}\mathbf{P}_{s+2}) = |\mathbf{P}_s\mathbf{P}_{s+1}|\cdot|\mathbf{P}_{s+1}\mathbf{P}_{s+2}|, \quad s = ...0,1,.. \tag{7.4}$$

where the scalar product $(\mathbf{P}_s\mathbf{P}_{s+1}.\mathbf{P}_{s+1}\mathbf{P}_{s+2})$ is defined by relation

$$(\mathbf{AB}.\mathbf{CD}) = \sigma(A,D) + \sigma(B,C) - \sigma(A,C) - \sigma(B,D) \tag{7.5}$$

In the geometry of Minkowski, when $\lambda_0 \to 0$, the timelike world chain turns to a smooth world line. The wobbling of a spacelike world chain (7.2) does not disappear, because the multivariance of the spacelike g-vectors equivalence remains at $\lambda_0 \to 0$.

The discrete geometry $\mathcal{G}_{\mathrm{d}}$ is a multivariant geometry, and it is the most essential property of $\mathcal{G}_{\mathrm{d}}$. Furthermore, the discrete geometry is a nonaxiomatizable geometry, which cannot be constructed on the basis of a finite number of axioms. As any generalized geometry, the discrete geometry $\mathcal{G}_{\mathrm{d}}$ is a generalization of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$.

The proper Euclidean geometry as well as the geometry of Minkowski are continuous geometries. They are described by methods of differential geometry. However, there may exist discrete geometries, where the distance between any two points of the space-time is larger, than some elementary length λ_0. If characteristic scale of the problem is much larger, than the elementary length λ_0, one may set $\lambda_0 = 0$ and consider the space-time geometry as a continuous geometry. However, in microcosm, where characteristic scale is of the order of λ_0, one should consider a discrete space-time geometry, because the real space-time geometry may be discrete, and such a possibility is to be investigated.

At the conventional construction of the Euclidean geometry one uses such concepts as manifold, dimension, coordinate system, linear vector space, which might be used only in continuous (differential) geometries. A discrete geometry is considered as a generalization of the proper Euclidean geometry, because it is the only geometry, whose consistency has been proved. Constructing a discrete geometry as a generalization of the proper Euclidean geometry, one may not use above-mentioned concepts. The only concept, which may be used in the continuous geometry and in the discrete one, is the distance ρ. But the distance ρ is to be introduced as a fundamental quantity. In the Riemannian geometry the distance ρ is introduced as an integral along the geodesic from the infinitesimal distance

$$ds = \sqrt{g_{ik} dx^i dx^k}$$

Such a method of introduction of the distance ρ is inadequate in the discrete geometry, because it uses infinitesimal distance, which does not exist in the discrete geometry. Besides, in the case, when there are several geodesics, connecting two points, one obtains many-valued expressions for the distance or for the world function. Many-valued world function is inadmissible in a geometry.

To construct a discrete geometry, one needs to use the metric approach to geometry. One represents the proper Euclidean geometry in terms of the distance ρ (or in terms of the world function $\sigma = \frac{1}{2}\rho^2$) and uses this representation for generalization of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ on the case of a discrete geometry $\mathcal{G}_{\mathrm{d}}$. Such a replacement of basic concepts of the Euclidean geometry means a logical reloading of the Euclidean geometry conception. Representation of a geometry in terms of a world function will be referred to as σ-immanent representation. The σ-immanent representation of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ is always possible.

The distance function ρ_{d} of a discrete geometry $\mathcal{G}_{\mathrm{d}}$ satisfies the condition (1.1) It means that in the geometry $\mathcal{G}_{\mathrm{d}}$ there are no distances, which are shorter, than the elementary length λ_0. The distance $\rho_{\mathrm{d}}(P, Q) = 0$ is admissible. This condition takes place, if $P = Q$.

Note, that the condition (1.1) is a restriction on the values of the distance function, but not on values of its argument (points of Ω), although one considers usually a discrete geometry as a geometry on a lattice. It is true, that the geometries on a lattice are discrete geometries (they satisfy the relation (1.1)), but they form a very special case of the discrete geometries. Such a geometry is essentially a conventional differential geometry, given on a countable set of points, where the distances are the same as in the differential geometry, given on a continual set of points. Besides, such a discrete geometry cannot be uniform and isotropic. A general case of a discrete geometry takes place, when restrictions are imposed on the admissible values of the world function (distance function).

Global Journal of Science Frontier Research (A) Volume XIV Issue VII Version I Year 2014

The simplest case of a discrete space-time geometry $\mathcal{G}_{\rm d}$ is obtained, if $\mathcal{G}_{\rm d} = \{\sigma_{\rm d}, \Omega_{\rm M}\}$ is given on the manifold $\Omega_{\rm M}$, where the geometry of Minkowski $\mathcal{G}_{\rm M} = \{\sigma_{\rm M}, \Omega_{\rm M}\}$ is given. The world function $\sigma_{\rm d}$ is chosen in the form (1.4), It is easy to verify, that $\rho_{\rm d} = \sqrt{2\sigma_{\rm d}}$, defined by (1.4) satisfies the constraint (1.1). Such a discrete geometry is uniform and isotropic as well as the geometry of Minkowski.

VIII. Metric Approach to Geometry

There is another circumstance, which prevents from constructing a discrete geometry. The proper Euclidean geometry is an axiomatizable geometry. It means, that all statements of the proper Euclidean geometry can be deduced from a system of several axioms (basic statements of the geometry). Usually one considers the axiomatizability of a geometry as an inherent property of any geometry. One believes that there are no nonaxiomatizable geometries. The reason of such a belief is rather simple. During two thousand years we knew the only geometry - the proper Euclidean geometry, which is axiomatizable. All differential geometries, constructed as a generalization of the proper Euclidean geometry, are also axiomatizable. One knows no other method of a geometry construction other, than the Euclidean method of the geometry deduction from some system of axioms. All differential geometries are constructed by means of this method. Mathematicians believe that any geometry is a logical construction. Such a discipline as the symplectic geometry is used in dynamics, but not for description of the geometric objects properties. Nevertheless it is called a geometry, because its structure reminds the structure of the Euclidean geometry.

In reality any geometry investigates a shape and a mutual disposition of geometrical objects in the space, or in the space-time. This property is an original property of a geometry. However, one used the only Euclidean method of the geometry construction during two thousand years, and as a result the axiomatizability of a geometry is considered now as an inherent property of any geometry, whereas a description of geometrical objects is considered as a secondary property of discipline, called geometry.

In general, there is a metrical approach to geometry, when a geometry is considered as a science, investigating a shape and a mutual disposition of geometrical objects. Such a geometry is known as a metric geometry (metric space), if it uses the triangle axiom. If the triangle axiom is not used, the geometry is called the distant geometry [42, 43]. It is supposed, that the distant geometry $\mathcal{G}_{\rm ds} = \{\sigma, \Omega\}$ is described completely by the world function $\sigma = \frac{1}{2}\rho^2$

$$\sigma: \quad \Omega \times \Omega \rightarrow \mathbb{R}, \quad \sigma(P,Q) = \sigma(Q,P), \quad \forall P,Q \in \Omega \tag{8.1}$$

where Ω is the point set, where the geometry is given. The world function σ is used instead of the distance function ρ, because in the geometry of Minkowski the distance ρ may be either positive, or pure imaginary, whereas $\sigma = \frac{1}{2}\rho^2$ is always real.

At the metric approach to geometry, a geometry can be constructed on any point set (but not necessarily on a manifold) without a use of coordinates. In the metric space the distance function ρ satisfies additional constraints

$$\rho(P,Q) \geq 0, \quad \forall P,Q \in \Omega, \quad \rho(P,Q) = \sqrt{2\sigma(P,Q)} \tag{8.2}$$

Global Journal of Science Frontier Research (A) Volume XIV Issue VII Version I Year 2014

$$\rho(P,Q)+\rho(P,R)\geq\rho(Q,R), \quad \forall P,Q,R\in\Omega \tag{8.3}$$

The condition (8.3) is known as the triangle axiom. This axiom admits one to introduce a straight line in the metric space as a shortest line between two points. In the distant geometry, where the constraint (8.3) is absent, one failed to introduce the straight line in terms of the distance function ρ. Blumental [43] introduced a curve as a continuous mapping $(0,1)\rightarrow\Omega$. The continuous mapping is an operation, which cannot be expressed only in terms of the distance function. As a result a purely metric approach to geometry, when geometry is described completely in terms of the distance function ρ, failed. The reason of this failure lies in the fact, that Blumental believed that the straight line has no thickness, whereas in reality in the distant geometry $\mathcal{G}_{\mathrm{d}}$ the straight line is a hollow tube. In reality the distant geometry is nonaxiomatizable geometry, which cannot be constructed by the Euclidean method.

What is on the bottom of the Euclidean method of the geometry construction? Let us get outside of this method. One cannot perceive the distance directly. One can perceive physical bodies. Geometrical object is an abstraction of space-time properties of a physical body. A physical body, evolving in the space-time, may pass from one space-time region with the space-time geometry $\{\sigma_1,\Omega_1\}$ to another space-time region with the space-time geometry $\{\sigma_2,\Omega_2\}$. We must have a possibility to recognize and to identify the same geometrical object in different space-time geometries. In order, that it should be possible, any geometrical object is to be described in terms of the distance function ρ and only in terms of ρ. Any geometrical object is described by its skeleton and its envelope. We consider a simple examples of geometrical objects. (The general definition of a geometrical object will be given later).

The simplest geometrical object is a sphere $\mathcal{SP}_{P_0P_1}$, determined by two points P_0,P_1 (skeleton). The point P_0 is a center of the sphere, P_1 is some point on the surface of the sphere. The points $\{P_0,P_1\}$ form the sphere skeleton. The surface of the sphere (its envelope) is a set of points

$$\mathcal{SP}_{P_0P_1}=\{R|\rho(P_0,R)=\rho(P_0,P_1)\}, \quad \rho=\sqrt{2\sigma} \tag{8.4}$$

The sphere is a hollow geometrical object in the sense, that there are internal points of the sphere, which do not belong to the sphere surface (envelope).

Another simple geometrical object is an ellipsoid $\mathcal{EL}_{F_1F_2P}$, determined by three points F_1,F_2,P. The points F_1,F_2 are focuses of the ellipsoid, and the point P is some point on the surface of the ellipsoid

$$\mathcal{EL}_{F_1F_2P}=\{R|\rho(F_1,R)+\rho(F_2,R)=\rho(F_1,P)+\rho(F_2,P)\}, \quad \rho=\sqrt{2\sigma} \tag{8.5}$$

If $F_1\neq P\wedge F_2\neq P$, the ellipsoid $\mathcal{EL}_{F_1F_2P}$ is a hollow geometrical object.

If $F_1=P\vee F_2=P$, the ellipsoid degenerates into a straight line segment $\mathcal{T}_{[P_0P_1]}$

$$\mathcal{T}_{[P_0P_1]}\equiv\mathcal{EL}_{P_0P_1P_1}=\mathcal{EL}_{P_0P_1P_0}=\{R|\rho(P_0,R)+\rho(P_1,R)=\rho(P_0,P_1)\} \tag{8.6}$$

The degenerate ellipsoid $\mathcal{EL}_{P_0P_1P_1}$ is a straight line segment $\mathcal{T}_{[P_0P_1]}$ by definition. This name is used, because in the proper Euclidean geometry a degenerate ellipsoid is a straight line segment. In other geometries the geometric object (8.6) may be a hollow geometrical object. It means, that it is not one-dimensional point set, as in the proper Euclidean geometry, but nevertheless we shall refer to it as a straight line segment.

The segment $\mathcal{T}_{[P_0P_1]}$ is determined by two points. All points of $\mathcal{T}_{[P_0P_1]}$ are points of the envelope, which consists of boundary points only. In the proper Euclidean geometry it is not a hollow geometrical object, because it does not contain internal points.

Is the straight line segment $\mathcal{T}_{[P_0P_1]}$ a hollow geometrical object in other distant geometries? It depends on the constraints (8.2),(8.3). If they are satisfied, the segment $\mathcal{T}_{[P_0P_1]}$ is entire (not hollow). If the distance function ρ does not satisfy the triangle axiom (8.3) the segment $\mathcal{T}_{[P_0P_1]}$ may be hollow. In other words, the segment $\mathcal{T}_{[P_0P_1]}$ may be a hollow tube.

Why is the segment entire, if the triangle axiom (8.3) is fulfilled? Let us consider a closed surface $\mathcal{S}$ defined by the relation

$$\mathcal{S}: \quad S_{P_0P_1}(R)=0, \quad S_{P_0P_1}(R)=\rho(P_0,R)+\rho(P_1,R)-\rho(P_0,P_1) \tag{8.7}$$

Internal points R' (points inside the closed surface $\mathcal{S}$) satisfy the relation $S_{P_0P_1}(R')<0$. External points R'' satisfy the relation $S_{P_0P_1}(R'')>0$. If the triangle axiom is fulfilled, it may be written in the form

$$\rho(P_0,R)+\rho(P_1,R)\geq\rho(P_0,P_1), \quad \forall P_1,P_2,R\in\Omega \tag{8.8a}$$

It follows from (8.7) and (8.8a), that $S_{P_0P_1}(R')\geq 0, \quad \forall R'\in\Omega$. It means that the surface $\mathcal{S}$, which coincides with the segment $\mathcal{T}_{[P_0P_1]}$, cannot contain internal points.

Why it is important, whether or not the segment $\mathcal{T}_{[P_0P_1]}$ is hollow? Geometry is reduced to construction of geometrical objects and to investigation of their properties. In the proper Euclidean geometry all geometrical objects are constructed of blocks (point, straight segment). Blocks are to be simple entire (not hollow) geometrical objects. The segment $\mathcal{T}_{[P_0P_1]}$ is determined by two points, and it is entire in the proper Euclidean geometry. It may be used as a constructive block for construction of geometrical objects. For instance, in the proper Euclidean geometry a cube can be filled by straight segments placed in parallel with the cube edge in such a way, that any point of a cube belongs to one and only one segment. Such a situation is impossible, if the blocks are hollow geometrical objects. If the blocks are hollow tubes, one cannot fill the cube by these tubes in such a way, that any point of a cube belongs to one and only one tube. It means, that a cube cannot be constructed of hollow blocks. The same relates to any geometrical object.

The Euclidean method of the geometric object construction is based on the possibility of construction of any geometrical object from blocks. There is a finite number of rules, describing the blocks properties, and there is a finite number of rules for description of the blocks combinations at a construction of a geometrical object. Euclid formulated these rules in the form of axioms of a logical construction. Thus, the axiomatics of the proper Euclidean geometry describes the procedure of a construction of geometrical objects from blocks. If the segment $\mathcal{T}_{[P_0P_1]}$ is entire, the distant geometry is an axiomatizable geometry, because it can be realized as a geometry, where any geometric object can be constructed of blocks, i.e. by means of the Euclidean method.

If blocks are hollow, they cannot be used for construction of geometrical objects. In this case the distant geometry is nonaxiomatizable, because in this case one cannot use the Euclidean method for construction of geometric objects. Formally the segment $\mathcal{T}_{[P_0P_1]}$ is hollow, if the equivalence relation is intransitive (and the geometry is multivariant). If the equivalence relation is transitive, the segment $\mathcal{T}_{[P_0P_1]}$ may be entire.

The constructive block $\mathcal{T}_{[P_0P_1]}$ is a directed object, whose direction is described by the g-vector $\mathbf{P}_0\mathbf{P}_1 = \overrightarrow{P_0P_1} = \{P_0, P_1\}$, which is an ordered set of two points. The point P_0 is the origin of the g-vector, the point P_1 is the end of the g-vector. Any g-vector $\mathbf{P}_0\mathbf{P}_1$ is described by its module

$$|\mathbf{P}_0\mathbf{P}_1| = \rho(P_0, P_1) = \sqrt{2\sigma(P_0, P_1)} \tag{8.9}$$

Geometric vectors are directed quantities, and interrelation of two g-vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ is described by an angle φ between them. In the proper Euclidean geometry there is a lot of g-vectors $\mathbf{Q}_0\mathbf{Q}_1$, which form the angle $\varphi \neq 0$ with the g-vector $\mathbf{P}_0\mathbf{P}_1$. However, in the proper Euclidean geometry there is only one g-vector $\mathbf{Q}_0\mathbf{Q}_1$ at the point Q_0 with fixed length $|\mathbf{Q}_0\mathbf{Q}_1|$, which forms with the g-vector $\mathbf{P}_0\mathbf{P}_1$ the angle $\varphi = 0$. By definition such a g-vector $\mathbf{Q}_0\mathbf{Q}_1$ is called the g-vector, which is parallel ($\mathbf{Q}_0\mathbf{Q}_1 \upuparrows \mathbf{P}_0\mathbf{P}_1$) to the g-vector $\mathbf{P}_0\mathbf{P}_1$.

Instead of the angle φ the mutual direction of two g-vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ may be described by the scalar product $(\mathbf{Q}_0\mathbf{Q}_1.\mathbf{P}_0\mathbf{P}_1)$ of these g-vectors, defined by the relation

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = |\mathbf{P}_0\mathbf{P}_1| \cdot |\mathbf{Q}_0\mathbf{Q}_1| \cos\varphi \tag{8.10}$$

In the proper Euclidean geometry the definition of the scalar product may be expressed in terms of the world function

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = \sigma(P_0, Q_1) + \sigma(P_1, Q_0) - \sigma(P_1, Q_1) - \sigma(P_0, Q_0) \tag{8.11}$$

As far as the definition of the scalar product is produced in terms of the world function, this definition may be used for any distant geometry.

Then condition of the g-vectors parallelism is obtained from (8.10) at $\varphi = 0$. It Must be written in the form

$$(\mathbf{Q}_0\mathbf{Q}_1 \upuparrows \mathbf{P}_0\mathbf{P}_1): \quad (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = |\mathbf{P}_0\mathbf{P}_1| \cdot |\mathbf{Q}_0\mathbf{Q}_1| \tag{8.12}$$

In the proper Euclidean geometry all g-vectors $\mathbf{P}_0\mathbf{P}_1$,$\mathbf{P}_0\mathbf{P}_1'$, $\mathbf{P}_0\mathbf{P}_1''$, which are parallel to g-vector $\mathbf{Q}_0\mathbf{Q}_1$, are parallel between themselves. Such a situation is rather special. It is connected with a degenerate character of the proper Euclidean geometry. In the distant geometry g-vectors $\mathbf{P}_0\mathbf{P}_1$,$\mathbf{P}_0\mathbf{P}_1'$, $\mathbf{P}_0\mathbf{P}_1''$, which are parallel to g-vector $\mathbf{Q}_0\mathbf{Q}_1$, are not parallel between themselves, in general. This circumstance generates hollowness of straight segments $\mathcal{T}_{[P_0P_1]}$. It depends on properties of the world function σ, which describes a distant geometry completely.

In the proper Euclidean geometry two g-vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{Q}_0\mathbf{Q}_1$ are equivalent by definition, if they are parallel ($\mathbf{P}_0\mathbf{P}_1 \upuparrows \mathbf{Q}_0\mathbf{Q}_1$) and their lengths are equal $|\mathbf{P}_0\mathbf{P}_1| = |\mathbf{Q}_0\mathbf{Q}_1|$

$$(\mathbf{P}_0\mathbf{P}_1\mathrm{eqv}\mathbf{Q}_0\mathbf{Q}_1): \quad (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = |\mathbf{P}_0\mathbf{P}_1| \cdot |\mathbf{Q}_0\mathbf{Q}_1| \wedge |\mathbf{P}_0\mathbf{P}_1| = |\mathbf{Q}_0\mathbf{Q}_1| \tag{8.13}$$

This definition of two g-vectors equivalency (equality) together with the definitions (8.9), (8.11) formulates the equivalence of two g-vectors in terms of the world function and only in these terms. It does not refer to a dimension, to a coordinate system and other means of description. This definition of two g-vectors equivalence should be used in any distant geometry.

There are such distant geometries, where the straight segments $\mathcal{T}_{[P_0 P_1]}$ are hollow tubes. Then the definition (8.13), (8.12) appears to be intransitive, and the distant geometry appears to be nonaxiomatizable. Some mathematicians object, that the definition (8.13), (8.11) cannot be used as an equivalence relation, because the equivalence relation is transitive by definition. They insist, that one should use another term for the definition (8.13), (8.11), (for instance, general equivalency). The reason of such an objection lies in the fact, that the mathematicians dealt only with axiomatizable geometries, which are logical constructions. Indeed, if one uses a logical construction, one can deduce conclusions, only if the equivalence relation is transitive, and from $a \backsim b$ and $b \backsim c$ it follows, that $a \backsim c$. If the the equivalence relation has not this property, one cannot deduce corollaries of axioms and theorems. Thus, if one insists on the transitivity of the equivalence relation, one insists on impossibility of nonaxiomatizable geometries, in particular, on impossibility of discrete space-time geometries, where the straight segments $\mathcal{T}_{[P_0 P_1]}$ are hollow tubes. We believe that imperfection of the description methods cannot be a reason of the discrete geometry discard. Nonaxiomatizability of the discrete geometry $\mathcal{G}_{\rm d}$ does not mean that $\mathcal{G}_{\rm d}$ does not exist .

The transitivity of the equivalence relation has been obtained from our experience of work with axiomatizable geometries (Euclidean geometry and its modifications). We have no authority to generalize this property to all space-time geometries. Whether or not the real space-time geometry is discrete, is a question of experimental data, but not a question of mathematical scholasticism. Another problem lies in the fact, that we could construct only axiomatizable geometries, and we could not construct discrete geometries. As a result we constructed only geometries on a lattice, which are not rigorous discrete geometries. How to construct discrete (nonaxiomatizable) geometries, we consider a few later.

IX. Description of Geometric Objects

If the distant geometry includes indefinite metrics (as in the geometry of Minkowski), the condition (8.2) is to be omitted, and description of the geometry is produced in terms of the world function. The geometry described completely by the world function (8.1) will be referred to as a physical geometry.

A geometrical object is a geometrical image of a physical body. Any geometrical object is some subset of points in the space-time. However, geometrical object is not an arbitrary set of points. Geometrical object is to be defined in the physical geometry in such a way, that similar geometrical objects (which are images of similar physical bodies) could be recognized in different space-time geometries.

Definition 1: A geometrical object $g_{\mathcal{P}_n,\sigma}$ of the geometry $\mathcal{G} = \{\sigma, \Omega\}$ is a subset $g_{\mathcal{P}_n,\sigma} \subset \Omega$ of the point set Ω. This geometrical object $g_{\mathcal{P}_n,\sigma}$ is a set of roots $R \in \Omega$ of the function $F_{\mathcal{P}_n,\sigma}$

$$g_{\mathcal{P}_n,\sigma} = \{R | F_{\mathcal{P}_n,\sigma}(R) = 0\}, \quad F_{\mathcal{P}_n,\sigma} : \quad \Omega \rightarrow \mathbb{R} \tag{9.1}$$

where $F_{\mathcal{P}_n,\sigma}$ depends on the point R via world functions of arguments $\{\mathcal{P}_n, R\} = \{P_0, P_1, ...P_n, R\}$

$$F_{\mathcal{P}_n,\sigma} : \quad F_{\mathcal{P}_n,\sigma}(R) = G_{\mathcal{P}_n,\sigma}(u_1, u_2, ...u_s), \qquad s = \frac{1}{2}(n+1)(n+2) \tag{9.2}$$

$$u_l = \sigma(w_i, w_k), \quad i,k = 0,1,...n+1, \quad l = 1,2,...\frac{1}{2}(n+1)(n+2) \tag{9.3}$$

$$w_k = P_k \in \Omega, \qquad k = 0,1,...n, \qquad w_{n+1} = R \in \Omega \tag{9.4}$$

Here $\mathcal{P}_n = \{P_0, P_1, ..., P_n\} \subset \Omega$ are $n+1$ points which are parameters, determining the geometrical object $g_{\mathcal{P}_n,\sigma}$

$$g_{\mathcal{P}_n,\sigma} = \{R | F_{\mathcal{P}_n,\sigma}(R) = 0\}, \qquad R \in \Omega, \qquad \mathcal{P}_n \in \Omega^{n+1} \tag{9.5}$$

$F_{\mathcal{P}_n,\sigma}(R) = G_{\mathcal{P}_n,\sigma}(u_1, u_2, ...u_s)$ is a function of $\frac{1}{2}(n+1)(n+2)$ arguments u_k and of $n+1$ parameters $\mathcal{P}_n$. The set $\mathcal{P}_n = \{P_0, P_1, ...P_n\} \in \Omega^{n+1}$ of the geometric object parameters will be referred to as the skeleton of the geometrical object. The subset $g_{\mathcal{P}_n,\sigma} \subset \Omega$ will be referred to as the envelope of the skeleton. The skeleton is an analog of a frame of reference, attached rigidly to a physical body. Tracing the skeleton motion, one can trace the motion of the physical body. When a particle is considered as a geometrical object, its motion in the space-time is described by the skeleton $\mathcal{P}_n$ motion. At such an approach (the rigid body approximation) the shape of the envelope is of no importance.

Remark: An arbitrary subset Ω' of the point set Ω is not a geometrical object, in general. It is supposed, that physical bodies may have only a shape of a geometrical object, because only in this case one can identify identical physical bodies (geometrical objects) in different space-time geometries.

Existence of the same geometrical objects in different space-time regions, having different geometries, brings up the question on equivalence of geometrical objects in different space-time geometries. Such a question did not brought up before, because one does not consider such a situation, when a physical body moves from one space-time region to another space-time region, having another space-time geometry. In general, mathematical technique of the conventional space-time geometry (differential geometry) is not applicable for simultaneous consideration of several different geometries of different space-time regions.

We can perceive the space-time geometry only via motion of physical bodies in the space-time, or via construction of geometrical objects corresponding to these physical bodies. As it follows from the *definition 1* of the geometrical object, the function $G_{\mathcal{P}_n,\sigma}$ as a function of its arguments u_k, $k = 1,2,...n(n+1)/2$ (of world functions of different points) is the same in all physical geometries. It means, that a geometrical object $\mathcal{O}_1$ in the geometry $\mathcal{G}_1 = \{\sigma_1, \Omega_1\}$ is obtained from the same geometrical object $\mathcal{O}_2$ in the geometry $\mathcal{G}_2 = \{\sigma_2, \Omega_2\}$ by means of the replacement $\sigma_2 \rightarrow \sigma_1$ in the definition of this geometrical object.

Definition 2: Geometrical object $g_{P'_n,\sigma'}$ ($\mathcal{P}'_n = \{P'_0, P'_1, ..P'_n\}$) in the geometry $\mathcal{G}' = \{\sigma', \Omega'\}$ and the geometrical object $g_{P_n,\sigma}$ ($\mathcal{P}_n = \{P_0, P_1, ..P_n\}$) in the geometry $\mathcal{G} = \{\sigma, \Omega\}$ are similar geometrical objects, if

$$\sigma'(P'_i, P'_k) = \sigma(P_i, P_k), \qquad i,k = 0,1,..n \tag{9.6}$$

and the functions $G'_{\mathcal{P}'_n,\sigma'}$ for $g_{P'_n,\sigma'}$ and $G_{\mathcal{P}_n,\sigma}$ for $g_{P_n,\sigma}$ in the formula (9.2) are the same functions of arguments $u_1, u_2, ...u_s$

$$G'_{\mathcal{P}'_n,\sigma'}(u_1, u_2, ...u_s) = G_{\mathcal{P}_n,\sigma}(u_1, u_2, ...u_s) \tag{9.7}$$

In this case

$$u_l \equiv \sigma(P_i, P_k) = u'_l \equiv \sigma'(P'_i, P'_k), \quad i,k = 0,1,...n, \quad l = 1,2,..n(n+1)/2 \tag{9.8}$$

The functions $F'_{\mathcal{P}'_n,\sigma'}$ for $g_{P'_n,\sigma'}$ and $F_{\mathcal{P}_n,\sigma}$ for $g_{P_n,\sigma}$ in the formula (9.2) have the same roots, if the relation (9.7) is fulfilled. As a result one-to-one connection between the geometrical objects $g_{P'_n,\sigma'}$ and $g_{P_n,\sigma}$ arises.

As far as the physical geometry is determined by its geometrical objects construction, a physical geometry $\mathcal{G} = \{\sigma, \Omega\}$ can be obtained from some known standard geometry $\mathcal{G}_{\text{st}} = \{\sigma_{\text{st}}, \Omega\}$ by means of a deformation of the standard geometry $\mathcal{G}_{\text{st}}$. Deformation of the standard geometry $\mathcal{G}_{\text{st}}$ is realized by the replacement $\sigma_{\text{st}} \to \sigma$ in all definitions of the geometrical objects in the standard geometry. The proper Euclidean geometry $\mathcal{G}_{\text{E}}$ is an axiomatizable geometry. It has been constructed by means of the Euclidean method as a logical construction. Simultaneously the proper Euclidean geometry is a physical geometry. It may be used as a standard geometry $\mathcal{G}_{\text{st}}$. Construction of a physical geometry as a deformation of the proper Euclidean geometry will be referred to as the deformation principle [44, 45]. The most physical geometries are nonaxiomatizable geometries. They can be constructed only by means of the deformation principle.

X. General Geometric Relations

Describing a physical geometry in terms of the world function, one should distinguish between general geometric relations and specific geometric relations. The general geometric relations are definitions of the proper Euclidean geometry, which are written in terms and only in terms of the world function. The general geometric relations are valid for any physical geometry.

The first general geometric definition is the definition of the scalar product of two g-vectors (8.11). Definition of the two g-vector equivalence (8.13) is also a general geometric relation.

Linear dependence of n g-vectors $\mathbf{P}_0\mathbf{P}_1, \mathbf{P}_0\mathbf{P}_2, ...\mathbf{P}_0\mathbf{P}_n$ is defined by the relation,

$$F_n(\mathcal{P}_n) = 0, \quad F_n(\mathcal{P}_n) \equiv \det ||(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_k)||, \qquad i,k = 1,2,...n \tag{10.1}$$

where $\mathcal{P}_n = \{P_0, P_1, ...P_n\}$ and $F_n(\mathcal{P}_n)$ is the Gram's determinant. Vanishing of the Gram's determinant $F_n(\mathcal{P}_n)$ is the necessary and sufficient condition of the linear dependence of n g-vectors. Condition of linear dependence is considered usually as properties of the linear vector space $\mathcal{L}_n$, because it is defined via operations in $\mathcal{L}_n$. It seems rather meaningless to use it, if the linear vector space cannot be introduced. Nevertheless, the relation (10.1) written as a general geometric relation describes some general geometric properties of g-vectors, which are transformed to the property of linear dependence in the proper Euclidean geometry . In particular, the metric dimension of the proper Euclidean geometry is defined in terms of the world function by means of the relations of the type (10.1) as a maximal number of linear independent vectors, which is possible in the Euclidean space. This circumstance seems to be rather unexpected, because in conventional presentation of the Euclidean geometry the geometry dimension is postulated in the beginning of the presentation.

As we have seen, a definition of geometrical objects in the form of general geometric relations (i.e. in terms of the world function) is necessary to recognize the same physical body (and corresponding geometrical object) in different space-time geometries.

The general geometric relations are parametrized by the form of the world function σ. Changing the form of the world function σ, one obtains the general geometric relations at a new value of the parameter σ (new form of the world function).

XI. Specific Properties of the N-Dimensional Euclidean Space

Along of general geometric properties, describing mainly definitions of the linear vector space, there are special geometric relations, describing properties of the world function. For instance, there are relations, which are necessary and sufficient conditions of the fact, that the world function $\sigma = \sigma_{\mathrm{E}}$ is the world function of n-dimensional Euclidean space. They have the form [46]:

I. Definition of the dimension:

$$\exists \mathcal{P}^n \equiv \{P_0, P_1, ...P_n\} \subset \Omega, \quad F_n(\mathcal{P}^n) \neq 0, \quad F_k\left(\Omega^{k+1}\right) = 0, \qquad k > n \tag{11.1}$$

where $F_n(\mathcal{P}^n)$ is the n-th order Gram's determinant (10.1) Geometric vectors $\mathbf{P}_0\mathbf{P}_i$, $i = 1, 2, ...n$ are basic g-vectors of the rectilinear coordinate system K_n with the origin at the point P_0. The metric tensors $g_{ik}(\mathcal{P}^n)$, $g^{ik}(\mathcal{P}^n)$, $i, k = 1, 2, ...n$ in K_n are defined by the relations

$$\sum_{k=1}^{k=n} g^{ik}(\mathcal{P}^n) g_{lk}(\mathcal{P}^n) = \delta_l^i, \quad g_{il}(\mathcal{P}^n) = (\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}_l), \quad i, l = 1, 2, ...n \tag{11.2}$$

$$F_n(\mathcal{P}^n) = \det ||g_{ik}(\mathcal{P}^n)|| \neq 0, \qquad i, k = 1, 2, ...n \tag{11.3}$$

II. Linear structure of the Euclidean space:

$$\sigma(P, Q) = \frac{1}{2}\sum_{i,k=1}^{i,k=n} g^{ik}(\mathcal{P}^n)(x_i(P) - x_i(Q))(x_k(P) - x_k(Q)), \quad \forall P, Q \in \Omega \tag{11.4}$$

where coordinates $x_i(P)$, $x_i(Q)$, $i = 1, 2, ...n$ of the points P and Q are covariant coordinates of the g-vectors $\mathbf{P}_0\mathbf{P}$, $\mathbf{P}_0\mathbf{Q}$ respectively in the coordinate system K. The covariant coordinates are defined by the relation

$$x_i(P) = (\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}), \qquad i = 1, 2, ...n \tag{11.5}$$

III: The metric tensor matrix $g_{lk}(\mathcal{P}^n)$ has only positive eigenvalues g_k

$$g_k > 0, \qquad k = 1, 2, ..., n \tag{11.6}$$

IV. The continuity condition: the system of equations

$$(\mathbf{P}_0\mathbf{P}_i.\mathbf{P}_0\mathbf{P}) = y_i \in \mathbb{R}, \qquad i = 1, 2, ...n \tag{11.7}$$

considered to be equations for determination of the point P as a function of coordinates $y = \{y_i\}$, $i = 1, 2, ...n$ has always one and only one solution.

Conditions I – IV contain a reference to the dimension n of the Euclidean space, which is defined by the relations (11.1). All relations I – IV are written in terms of the world function. They are constraints on the form of the world function of the proper Euclidean geometry. Constraints (11.1), determining the dimension via the form of the world function, look rather unexpected. They contain a lot of constraints imposed on the world function of the proper Euclidean geometry, and they are necessary. At the conventional approach to geometry one uses a very simple supposition: "Let the dimension of the Euclidean space be n." Conventionally one uses this very short postulate instead of numerous constraints (11.1), used in the σ-representation (description in terms of the world function) of a geometry.

In the vector representation of the proper Euclidean geometry, which is based on a use of the linear vector space, the dimension is considered as a primordial property of the linear vector space and as a primordial property of the Euclidean geometry. Situation, when the geometry dimension is different at different points of the space Ω, or when the dimension is indefinite, is not considered. In the vector representation of the Euclidean geometry one does not distinguish between the general geometric relations and the specific relations of the geometry.

Instead of constraints (11.1) – (11.7) one may use an explicit form of the world function

$$\sigma_{\mathrm{E}}(x, x') = \frac{1}{2}\sum_{k=1}^{k=n}\left(x^k - x'^k\right)^2 \tag{11.8}$$

where $x^k, x'^k \in \mathbb{R}$, $k = 1, 2, ...n$ are Cartesian coordinates of points P and P' respectively. The relation (11.8) satisfies all constraints (11.1) – (11.7). It uses concepts of dimension and of coordinates as primordial concepts of geometry. Using the world function only in such an explicit form, one cannot imagine a generalized geometry without such concepts as a dimension and a coordinate system, although these concepts are only means of a geometry description.

In general, after the logical reloading to σ-representation the proper Euclidean geometry looks rather unexpected. Some concepts look very simple in the vector representation. The same concepts look complicated in the σ-representation and vice versa. As a result the proper Euclidean geometry in the σ-representation is perceived hardly.

In the vector representation one has several fundamental concepts and quantities: dimension, coordinate system, linear dependence, whereas in the σ-representation there is only one fundamental quantity: world function. The dimension, the coordinate system and the linear dependence are derivative quantities and concepts. Agreement between these quantities is achieved in any physical geometry, because they are defined as some attributes of the world function.

XII. Skeleton Conception of Particle Dynamics

An elementary particle is a physical body. In the discrete space-time geometry a position of a physical body is described by its skeleton $\mathcal{P}_n = \{P_0, P_1, ..P_n\}$. Of

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course, such a description of a physical body position may be used in any space-time geometry. The skeleton is an analog of the frame of reference attached rigidly to the particle (physical body). Tracing the skeleton motion, one traces the physical body motion. Direction of the skeleton displacement is described by the leading g-vector $\mathbf{P}_0\mathbf{P}_1$.

The skeleton motion is described by a world chain $\mathcal{C}$ of connected skeletons

$$\mathcal{C} = \bigcup_{s=-\infty}^{s=+\infty} \mathcal{P}_n^{(s)} \tag{12.1}$$

Skeletons $\mathcal{P}_n^{(s)}$ of the world chain are connected in the sense, that the point P_1 of a skeleton is a point P_0 of the adjacent skeleton. It means

$$P_1^{(s)} = P_0^{(s+1)}, \qquad s = ...0, 1, ... \tag{12.2}$$

The g-vector $\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)} = \mathbf{P}_0^{(s)}\mathbf{P}_0^{(s+1)}$ is the leading g-vector, which determines the direction of the world chain. The case (7.2), when the skeleton $\mathcal{P}_1 = \{P_s, P_{s+1}\}$ of a pointlike particle is described by two points is a special case of (12.1).

If the particle motion is free, the adjacent skeletons are equivalent

$$\mathcal{P}_n^{(s)} \mathrm{eqv} \mathcal{P}_n^{(s+1)}: \qquad \mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)} \mathrm{eqv} \mathbf{P}_i^{(s+1)}\mathbf{P}_k^{(s+1)}, \qquad i,k = 0,1,...n, \qquad s = ..0,1,.. \tag{12.3}$$

If the particle is described by the skeleton $\mathcal{P}_n^{(s)}$, the world chain (12.1) has $n(n+1)/2$ invariant quantities

$$\mu_{ik} = \left|\mathbf{P}_i^{(s)}\mathbf{P}_k^{(s)}\right|^2 = 2\sigma\left(P_i^{(s)}, P_k^s\right), \qquad i,k = 0,1,...n, \qquad s = ...0,1,... \tag{12.4}$$

which are constant along the whole world chain.

Equations (12.3) form a system of $n(n+1)$ difference equations for evolution of nD coordinates of n skeleton points $\{P_1, ..P_n\}$, where D is the dimension of the space-time. The number of dynamical variables, which are liable for determination of the world chain, distinguishes, in general, from the number of dynamic equations. It is the main difference between the skeleton conception of particle dynamics and the conventional conception of particle dynamics, where the number of dynamic variables coincides with the number of dynamic equations.

In the case of pointlike particle, when $n = 1$, $D = 4$, the number of equations $n_{\mathrm{e}} = 2$, whereas the number of variables $n_{\mathrm{v}} = 4$. The number of equations is less, than the number of dynamic variables. In the discrete space-time geometry (1.4) the position of the adjacent skeleton is not determined uniquely. As a result the world chain wobbles. In the nonrelativistic approximation a statistical description of the stochastic world chains leads to the Schrödinger equations [10], if the elementary length λ_0 has the form $\lambda_0^2 = \hbar/bc$, where $\hbar$ is the quantum constant, c is the speed of the light and b is a universal constant, connecting the particle mass m with the length (geometric mass) μ of the world chain link

$$m = b\mu$$

Dynamic equations (12.3) are difference equations. At the large scale, when one may go to the limit $\lambda_0 = 0$, the dynamic equations (12.3) turn to the differential dynamic equations. In the case of pointlike particle ($n = 1$) and of the Kaluza-Klein five-dimensional space-time geometry these equation describe the motion of a charged particle in the given electromagnetic field. One can see in this example, that the space-time geometry "assimilates" the electromagnetic field. It means that one may consider only a free particle motion, keeping in mind, that the space-time geometry can "assimilate" all force fields.

Dynamic equations (12.3) realize the skeleton conception of particle dynamics in the microcosm. The skeleton conception of dynamics distinguishes from the conventional conception of particle dynamics in the relation, that the number of dynamic equations may differ from the number of dynamic variables. In the case of the conventional particle dynamics a unique solution of dynamic equations is to be determined, because in this case the number of dynamic equations (first order) coincides always with the number of dynamic variables, which are to be determined. As a result the motion of a particle appears to be deterministic. In the case of quantum particles, whose motion is stochastic (indeterministic), the dynamic equations are written for a statistical ensemble of indeterministic particles (or for the statistically averaged particle).

In the conventional conception of dynamics one can obtain dynamic equation for the statistically averaged particle (i.e. statistical ensemble normalized to one particle), but there are no dynamic equations for a single stochastic particle. In the skeleton conception of the particle dynamics there are dynamic equations for a single particle. These equations are many-valued (multivariant), but they do exist. In the conventional conception of the particle dynamics one can derive dynamic equations for the statistically averaged particle, which are a kind of equations for a fluid (continuous medium). But one cannot obtain dynamic equations for a single indeterministic particle [7].

The skeleton conception of the particle dynamics realizes a more detailed description of elementary particle. One may hope to obtain some information on the elementary particle structure.

We have now only two examples of the skeleton conception application. Considering compactification in the 5-dimensional discrete space-time geometry of Kaluza-Klein, and imposing condition of uniqueness of the world function, one obtains that the value of the electric charge of a stable elementary particle is restricted by the elementary charge [47]. This result has been known from experiments, but it could not be explained theoretically, because in the continuous space-time geometry nobody considers the world function as a fundamental quantity, and one does not demand its uniqueness.

Another example concerns structure of Dirac particles (fermions). Consideration in framework of skeleton conception [48] shows that a world chain of a fermion is a (spacelike or timelike) helix with timelike axis. The averaged world chain of a free fermion is a timelike straight line. The helical motion of a skeleton generates an angular moment (spin) and magnetic moment. Such a result looks rather reasonable. In the conventional conception of the particle dynamics the spin and magnetic moment of a fermion are postulated without a reference to its structure. Thus, deterministic model of the Dirac particle gives a more detailed information on arrangement of the Dirac particle. In the classical model the spin and the magnetic moment are axiomatic quantities containing quantum constant. The classical model gives no information on arrangement of spin and magnetic moment.

To obtain the helical world chain in the skeleton conception, one consider space-time geometry $\mathcal{G}_{\rm g}$ described by the quasi-discrete world function

$$\sigma_{\rm g} = \sigma_{\rm M} + d\left(\sigma_{\rm M}\right), \qquad d\left(\sigma_{\rm M}\right) = \frac{\lambda_0^2}{2} f\left(\frac{\sigma_{\rm M}}{\sigma_0}\right), \qquad f\left(x\right) = \begin{cases} 1 & \text{if} \quad x \geq 1 \\ x^3 & \text{if} \quad -1 < x < 1 \\ -1 & \text{if} \quad x \leq -1 \end{cases} \tag{12.5}$$

where $d\left(\sigma_{\rm M}\right)$ is a distortion describing deflection of the world function $\sigma_{\rm g}$ from the world function $\sigma_{\rm M}$ of the geometry of Minkowski $\mathcal{G}_{\rm M}$. The quantity λ_0 is the elementary length and σ_0 is some constant. The geometry $\mathcal{G}_{\rm g}$ does not pretend to be a real space-time geometry. The geometry $\mathcal{G}_{\rm g}$ is a granulated geometry, i.e. $\mathcal{G}_{\rm g}$ is the space-time geometry, which is discrete only partly. It is considered as a possible space-time geometry, where the particle world chain may be a helix. The particle skeleton consists of three points $\mathcal{P}_2 = \{P_0, P_1, P_2\}$. The leading vector $\mathbf{P}_0\mathbf{P}_1$ may be timelike or spacelike. The vector $\mathbf{P}_0\mathbf{P}_2$ is timelike. It is directed along the helix axis. The vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_0\mathbf{P}_2$ satisfy the restrictions

$$\left||\mathbf{P}_0\mathbf{P}_1|^2\right| < \sigma_0, \quad |\mathbf{P}_0\mathbf{P}_2|^2 < \sigma_0 \tag{12.6}$$

Under these restrictions the world chain wobbles, but the wobbling amplitude can be restricted, even if the vector $\mathbf{P}_0\mathbf{P}_1$ is spacelike. Details of this investigation may be found in [48].

XIII. Tachyons

Tachyons are particles with spacelike world chain. They were not detected experimentally and they are not envisaged by the Standard model of elementary particles. Impossibility of the tachyon existence is conditioned by a usage of linear vector space operations in the case, when they are not adequate. Operations of the linear vector space are applied to spacelike g-vectors of the space-time geometry of Minkowski. It is a mistake, because the equivalence relation is intransitive and multivariant for spacelike g-vectors in $\mathcal{G}_{\rm M}$. For instance, all g-vectors $\{r, r\cos\phi, r\sin\phi, z\}$, where r and ϕ are arbitrary number, are equivalent to the g-vector $\{0,0,0,z\}$, but they are not equivalent between themselves. It is a reason, why the tachyon world chain wobbles with infinite amplitude. Such a tachyon cannot been detected because of this wobbling. Impossibility of a single tachyon detection does not mean that tachyons do not exist. A single tachyon cannot be detected, but the tachyon gas may be detected by its gravitational field. Properties of the the tachyon gas are such, that the tachyon gas is the best candidate for the dark matter [49, 50].

According to (7.2) the world chain for two-point skeleton $\mathcal{P}_1 = \{P_0, P_1\}$ have the form

$$\mathcal{C} = \bigcup_s \mathbf{P}_s\mathbf{P}_{s+1}, \quad |\mathbf{P}_s\mathbf{P}_{s+1}| = \mu = \text{const}, \quad s = ...0, 1, 2, ... \tag{13.1}$$

The adjacent g-vectors $\mathbf{P}_s\mathbf{P}_{s+1}$ and $\mathbf{P}_{s+1}\mathbf{P}_{s+2}$ are equivalent ($\mathbf{P}_s\mathbf{P}_{s+1}\text{eqv}\mathbf{P}_{s+1}\mathbf{P}_{s+2}$) for a free particle. The equivalence conditions (7.4), (7.5) can be written in the form

$$\begin{aligned}\sigma\left(P_s,P_{s+2}\right) &= 4\sigma\left(P_s,P_{s+1}\right), \quad \sigma\left(P_s,P_{s+1}\right)=\sigma\left(P_{s+1},P_{s+2}\right) \\ s &= 0,\pm1,\pm2,...\end{aligned} \qquad (13.2)$$

If there exist the limit $\mu\rightarrow 0$, the world chain (13.1) turns into a smooth world line. Keeping in mind that world function $\sigma\left(P_s,P_{s+1}\right)=\frac{1}{2}\rho^2\left(P_s,P_{s+1}\right)$, where ρ is the distance between the points P_s and P_{s+1}, one can see, that in the proper Euclidian geometry $\mathcal{G}_{\mathrm{E}}$ the relation (13.2) describes the rule of the straight line construction by means of only compasses.

In the case of tachyon $\sigma\left(P_s,P_{s+1}\right)<0$ and μ is imaginary $\mu^2=-\left|\mu^2\right|$. We consider three adjacent points P_0, P_1, P_2 of the world chain

$$P_0=\left\{x_0,\mathbf{x}\right\}, \quad P_1=\left\{x_0+p_0,\mathbf{x}+\mathbf{p}\right\}, \quad P_2=\left\{x_0+2p_0+\alpha_0,\mathbf{x}+2\mathbf{p}+\boldsymbol{\alpha}\right\} \qquad (13.3)$$

The 4-vector $\alpha=\left\{\alpha_0,\boldsymbol{\alpha}\right\}$ is a discrete analog of the acceleration vector. We write equations (13.2) for the points (13.3). The quantities $x=\left\{x_0,\mathbf{x}\right\}$ and $\left\{x_0+p_0,\mathbf{x}+\mathbf{p}\right\}$ are supposed to be given, and the four components of the 4-vector $\alpha=\left\{\alpha_0,\boldsymbol{\alpha}\right\}$ are to be determined from two equations (13.2) (acceleration is determined from the dynamic equations).

One considers the space-time geometry with the world function

$$\sigma\left(x,x'\right)=\frac{1}{2}\left(\left(c^2-2V\left(\mathbf{y}\right)\right)\left(x_0-x_0'\right)^2-\left(\mathbf{x}-\mathbf{x}'\right)^2\right), \quad \mathbf{y}=\frac{\mathbf{x}+\mathbf{x}'}{2} \qquad (13.4)$$

where $\left\{x^0,\mathbf{x}\right\}=\left\{x^0,x^1,x^2,x^1\right\}$ are coordinates in some inertial coordinate system, $V=V\left(x\right)$ is the gravitational potential ($V\ll c^2$).

One obtains the following nonunique solution [49, 50]

$$\alpha_{\|}=\frac{r\sqrt{c^2-2V}}{\sqrt{\left(v^2-c^2+2V\right)}}, \quad v=\frac{p}{p_0} \qquad (13.5)$$

$$\alpha_{\perp1}=r\cos\phi, \quad \alpha_{\perp2}=r\sin\phi \quad v=\frac{p}{p_0}=\frac{p\sqrt{\left(c^2-2V\right)}}{\sqrt{\mathbf{p}^2-\left|\mu\right|^2}} \qquad (13.6)$$

$$\alpha_0=\frac{\boldsymbol{\alpha}\mathbf{p}}{p_0\left(c^2-2V\right)}=\frac{p}{p_0}\left(\frac{r}{\sqrt{\left(v^2-c^2+2V\right)\left(c^2-2V\right)}}\right) \qquad (13.7)$$

where r,ϕ are arbitrary real numbers $r\geq 0$. The length $\left|\boldsymbol{\alpha}\right|$ of multivariant 3-vector $\boldsymbol{\alpha}$ is of the order r and components of $\boldsymbol{\alpha}$ are defined by the relations

$$\boldsymbol{\alpha}_{\|}=\mathbf{p}\frac{\left(\boldsymbol{\alpha}\mathbf{p}\right)}{\mathbf{p}^2}, \quad \boldsymbol{\alpha}_{\perp}=\boldsymbol{\alpha}-\boldsymbol{\alpha}_{\|}, \quad \alpha_{\|}^2=\frac{\left(\boldsymbol{\alpha}\mathbf{p}\right)^2}{\mathbf{p}^2}, \quad \alpha_{\|}=\frac{\boldsymbol{\alpha}\mathbf{p}}{p}, \quad p=\left|\mathbf{p}\right| \qquad (13.8)$$

Here $\boldsymbol{\alpha}_{\|}$ is the component of 3-vector $\boldsymbol{\alpha}$ which is in parallel with the 3-vector $\mathbf{p}$, whereas $\boldsymbol{\alpha}_{\perp}$ is the component of 3-vector $\boldsymbol{\alpha}$, which is perpendicular to the 3-vector $\mathbf{p}$. As far as the quantity r may be infinite, the wobbling of the tachyon world chain may have infinite amplitude.

Averaging over r and ϕ, one obtains macroscopic parameters of the tachyon gas (the mean components of the tachyon gas velocity) [50].

$$\langle u_{\parallel}\rangle = \frac{\sqrt{c^2-2V}}{p}\sqrt{\mathbf{p}^2-|\mu^2|} = c\sqrt{1-2\frac{V}{c^2}}\sqrt{1-\frac{|\mu^2|}{\mathbf{p}^2}} \tag{13.9}$$

$$\langle \mathbf{u}_{\perp}\rangle = 0 \tag{13.10}$$

$$\langle u_{\parallel}^2\rangle = \langle u_{\parallel}\rangle^2 = \langle \mathbf{u}\rangle^2 = c^2\left(1-2\frac{V}{c^2}\right)\left(1-\frac{|\mu^2|}{\mathbf{p}^2}\right) < c^2 \tag{13.11}$$

$$\begin{aligned}\langle \mathbf{u}_{\perp}^2\rangle &= \frac{p_0^2\left(v^2-c^2+2V\right)\left(c^2-2V\right)}{p^2} = \left(c^2-2V\right)\left(1-\frac{p_0^2\left(c^2-2V\right)}{p^2}\right)\\ &= \frac{|\mu|^2}{p^2}\left(c^2-2V\right) = c^2-2V-\langle u_{\parallel}^2\rangle\end{aligned} \tag{13.12}$$

$$\langle \mathbf{u}^2\rangle = \langle u_{\parallel}^2\rangle + \langle \mathbf{u}_{\perp}^2\rangle = c^2-2V \tag{13.13}$$

One can see from (13.10), (13.13) that results for $\langle \mathbf{u}_{\perp}\rangle$ and $\langle \mathbf{u}^2\rangle$ do not depend on the geometric mass μ of tachyon.

The energy-momentum tensor has the form [50]

$$T^{00} = \rho, \quad T^{\alpha 0} = T^{0\alpha} = \rho\langle u^{\alpha}\rangle \tag{13.14}$$

$$T^{\alpha\beta} = \rho\langle u^{\alpha}\rangle\langle u^{\beta}\rangle + P^{\alpha\beta}, \quad \alpha,\beta = 1,2,3 \tag{13.15}$$

$$P^{\alpha\beta} = \frac{1}{2}\rho\left(\delta^{\alpha\beta}-\frac{\langle u^{\alpha}\rangle\langle u^{\beta}\rangle}{\langle \mathbf{u}\rangle^2}\right)\left(c^2-2V-\langle \mathbf{u}\rangle^2\right) \tag{13.16}$$

In other words, macroscopic parameters of the tachyon gas are the same, as for usual gas of very high pressure. One may work with the tachyon gas as with usual gas, whose molecules cannot be detected. One can detect only the gravitational field of the tachyon gas.

XIV. Tachyon World Chain with Two-Point Skeleton

We investigate now, whether a world chain with a spacelike leading vector may form a helix with timelike axis. If it is possible, then we try to investigate, under which world function such a situation is possible. We consider the world function σ_{g} of the form

$$\sigma_{\mathrm{g}} = \sigma_{\mathrm{M}} + \frac{\lambda_0^2}{2}f\left(\frac{\sigma_{\mathrm{M}}}{\sigma_0}\right), \qquad f(x) = \begin{cases} \operatorname{sgn}(x) & \text{if} \quad |x|>1 \\ Cx+\varepsilon g(x) & \text{if} \quad |x|\le 1\end{cases}, \tag{14.1}$$

$$\sigma_0 = \text{const} > 0, \quad g(x) = -g(-x), \quad 0 \le \varepsilon \ll 1 \tag{14.2}$$

where C is a constant, which is determined from the relation

$$C + \varepsilon g(1) = 1$$

Such a choice of the space-time geometry does not pretend to a real space-time. This is only a model, which is easy for investigation. The function $f\left(\frac{\sigma_{\mathrm{M}}}{\sigma_0}\right)$ should be determined from the condition that the world chain with spacelike leading g-vectors

$\mathbf{P}_0^{(s)}\mathbf{P}_1^{(s)}$ forms a helix with timelike axis. The shape of the chain is determined by leading g-vectors.

To estimate the form of $\sigma_{\rm g}$ as a function of $\sigma_{\rm M}$ at $\sigma_{\rm M} < \sigma_0$, it is useful to consider the world chain, consisting only of spacelike leading g-vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_1\mathbf{P}_2$, $\mathbf{P}_2\mathbf{P}_3$,...Other g-vectors of the skeleton will be considered later, when one needs to reduce the chain wobbling. The chain describes the free particle motion, and its links satisfy the equations (12.3). We suppose that the chain is a helix with timelike axis in the space-time. Let the points $...P_0, P_1, ...$ have the coordinates

$$P_k = \{kl_0, R\cos(k\varphi), R\sin(k\varphi), 0\}, \qquad k = ...0, 1, 2, ... \tag{14.3}$$

All points (14.3) lie on a helix with timelike axis. The quantities R, l_0, φ are parameters of the chain.

We investigate, if it is possible such a space-time geometry (14.1), that the world chain, consisting of connected g-vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_1\mathbf{P}_2$, $\mathbf{P}_2\mathbf{P}_3$,... form a helix with the radius R. The parameters l_0, $l_1 = 2R\sin\frac{\varphi}{2}$ are small in the sense, that

$$|l_0|, |l_1| < \sqrt{2\sigma_0}, \quad l_1 = 2R\sin\frac{\varphi}{2} \tag{14.4}$$

To obtain connection between parameters l_0, l_1, φ, it is sufficient to solve equations, connecting adjacent leading g-vectors $\mathbf{P}_0\mathbf{P}_1$, $\mathbf{P}_1\mathbf{P}_2$. The dynamic equations have the form

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{P}_2)_{\rm g} = |\mathbf{P}_0\mathbf{P}_1|_{\rm g}^2 \tag{14.5}$$

$$|\mathbf{P}_0\mathbf{P}_1|_{\rm g}^2 = |\mathbf{P}_1\mathbf{P}_2|_{\rm g}^2 \tag{14.6}$$

Here index "g" means that the quantities are calculated in the space-time geometry $\mathcal{G}_{\rm g}$, whose world function $\sigma_{\rm g}$ is chosen in the form (14.1) where g is some function $g(x) = -g(-x)$, $x \in (-1, 1)$ and $\varepsilon \ll 1$.

We are to verify that two adjacent g-vectors $\mathbf{P}_0\mathbf{P}_1$ and $\mathbf{P}_1\mathbf{P}_2$ satisfy the relations (14.5), (14.6), if

$$P_0 = \{0, 0, 0, 0\}, \quad P_1 = \{l_0, l_1, 0, 0\}, \quad P_2 = \{2l_0, l_1\cos\varphi, l_1\sin\varphi, 0\} \tag{14.7}$$

and $l_0^2 < l_1^2$. If parameter $l_1 = 2R\sin\frac{\varphi}{2}$, the points (14.7) correspond to three points of the helix (14.3). It is sufficient to verify, that the points (14.7) satisfy equations (14.5), (14.6), because in this case all other pairs of adjacent points (14.3) will satisfy equations of the form (14.5), (14.6).

It is important to keep in mind that the g-vectors

$$\mathbf{P}_0\mathbf{P}_1 = \{l_0, l_1, 0, 0\}, \quad \mathbf{P}_1\mathbf{P}_2 = \{l_0, l_1(\cos\varphi - 1), l_1\sin\varphi, 0\} \tag{14.8}$$

are not unique solution of the equations (14.5), (14.6). There is a lot of other solutions, which lead to unpredictable wobbling of the world chain (14.3). Amplitude of this wobbling is infinite. The world chain of a pointlike particle, described by two-point skeleton $\mathcal{P}_2 = \{P_0, P_1\}$ with spacelike vector $\mathbf{P}_0\mathbf{P}_1$, is unobservable, because it is impossible to trace such a world chain. One cannot trace the world chain, because the spatial distance between points P_s and P_{s+1} may be infinite in any coordinate system. It means that the statement of the relativity theory on impossibility of the tachyons existence is strongly overstated. Tachyons may exist, but they are unobservable.

Considering equations (14.5), (14.6), we write them in the Minkowski space-time, setting

$$\sigma_{\mathrm{g}}\left(P_0,P_1\right)=\sigma_{\mathrm{M}}\left(P_0,P_1\right)+d\left(P_0,P_1\right),\quad d\left(P_0,P_1\right)\equiv\frac{\lambda_0^2}{2}f\left(\frac{\sigma_{\mathrm{M}}\left(P_0,P_1\right)}{\sigma_0}\right)\tag{14.9}$$

Then equations (14.5), (14.6) take the form

$$\left(\mathbf{P}_0\mathbf{P}_1.\mathbf{P}_1\mathbf{P}_2\right)_{\mathrm{M}}+w\left(P_0,P_1,P_1,P_2\right)=\left|\mathbf{P}_0\mathbf{P}_1\right|_{\mathrm{M}}^2+2d\left(P_0,P_1\right)\tag{14.10}$$

$$\left|\mathbf{P}_0\mathbf{P}_1\right|_{\mathrm{M}}^2=\left|\mathbf{P}_1\mathbf{P}_2\right|_{\mathrm{M}}^2\tag{14.11}$$

where

$$w\left(P_0,P_1,P_3,P_4\right)=d\left(P_0,P_4\right)+d\left(P_1,P_3\right)-d\left(P_0,P_3\right)-d\left(P_1,P_4\right)\tag{14.12}$$

Dynamic equations (14.10), (14.11) may be treated as a description of the particle motion in the space-time geometry of Minkowski under influence of force fields w and d. In other words, we pass from description in $\mathcal{G}_{\mathrm{g}}$ to description in the Minkowski space-time geometry $\mathcal{G}_{\mathrm{M}}$, introducing additional force fields, generated by the geometry $\mathcal{G}_{\mathrm{g}}$. Such a passage admits one to use conventional mathematical technique of the Minkowski geometry.

Further we shall use the scalar product only in the space-time of Minkowski. Index "M" will be omitted for brevity. We present points (14.7) in the form

$$P_0=\{0,0,0,0\},\qquad P_1=l,\qquad P_2=l+q+\alpha\tag{14.13}$$

$$\mathbf{P}_0\mathbf{P}_1=l,\quad \mathbf{P}_1\mathbf{P}_2=q+\alpha,\quad \mathbf{P}_0\mathbf{P}_2=l+q+\alpha\tag{14.14}$$

Here

$$l=\{l_0,l_1,0,0\},\quad q=\{l_0,l_1\cos\varphi,l_1\sin\varphi,0\}\tag{14.15}$$

$$\alpha=\{\alpha_0,\alpha_1,\alpha_2,\alpha_3\}=\{\alpha_0,\boldsymbol{\alpha}\}\tag{14.16}$$

Vector α describes wobbling of the point P_2 near the "helical" position of the point $P_2=l+q$.

To determine the form of the world function , we set $\alpha=0$ in (14.13), (14.14). For $\left|\mathbf{P}_0\mathbf{P}_1\right|^2$, $\left|\mathbf{P}_1\mathbf{P}_2\right|^2$, $\left|\mathbf{P}_0\mathbf{P}_2\right|^2$ and w in (14.10) one obtains dynamic equations

$$\left|\mathbf{P}_0\mathbf{P}_1\right|_{\mathrm{M}}^2=\left|\mathbf{P}_1\mathbf{P}_2\right|_{\mathrm{M}}^2=2\sigma_{\mathrm{M}}\left(P_0,P_1\right)=l_0^2-l_1^2\equiv l^2,\quad l_0^2<l_1^2\tag{14.17}$$

$$\left|\mathbf{P}_0\mathbf{P}_2\right|_{\mathrm{M}}^2=4l^2+4l_1^2\sin^2\frac{\varphi}{2},\quad l^2<0,\quad l_0^2,l_1^2<\sigma_0\tag{14.18}$$

$$w\left(P_0,P_1,P_1,P_2\right)=\frac{\lambda_0^2}{2}\left(f\left(\frac{2l_1^2\sin^2\frac{\varphi}{2}+2\left(l_0^2-l_1^2\right)}{\sigma_0}\right)-2f\left(\frac{l_0^2-l_1^2}{2\sigma_0}\right)\right)\tag{14.19}$$

Setting

$$l^2=l_0^2-l_1^2=-2\nu\sigma_0,\qquad \nu>0\tag{14.20}$$

$$a=\frac{2l_1^2}{\sigma_0}\sin^2\frac{\varphi}{2},\qquad \varkappa=\frac{\sigma_0}{\lambda_0^2} \tag{14.21}$$

dynamic equation (14.5) may be written in the form

$$a\varkappa+f\left(a-4\nu\right)=-4f\left(\nu\right) \tag{14.22}$$

Here the function f is an antisymmetric function, defined by the relation (14.1). Dynamic equation (14.6) transforms to the identity.

After a use of (14.1) equation (14.22) turns into

$$a\left(\varkappa+1\right)-\varepsilon g\left(1\right)-\varepsilon g\left(4\nu-a\right)+4\varepsilon g\left(\nu\right)=0 \tag{14.23}$$

$$a=\frac{\varepsilon\left(g\left(4\nu\right)-4g\left(\nu\right)\right)}{\varkappa+1-\varepsilon g\left(1\right)-\varepsilon g'\left(4\nu\right)}=\frac{\varepsilon\left(g\left(4\nu\right)-4g\left(\nu\right)\right)}{\varkappa+1}+O\left(\varepsilon^2\right) \tag{14.24}$$

It follows from (14.24), that a may be a small quantity, if $\varepsilon\ll 1$. According to (14.21) a must be positive. It is possible, if

$$g\left(4\nu\right)>4g\left(\nu\right),\quad \nu>0,\quad 0<\varepsilon\ll 1 \tag{14.25}$$

According to (14.4) and (14.21) one obtains

$$R=\frac{l_1}{2\sin\frac{\varphi}{2}}=\frac{l_1^2}{\sqrt{2a\sigma_0}}=\frac{l_1}{\sqrt{\varepsilon}}\frac{\frac{l_1}{\sqrt{2\sigma_0}}\sqrt{1+\frac{\sigma_0}{\lambda_0^2}}}{\sqrt{\left(g\left(4\nu\right)-4g\left(\nu\right)\right)}} \tag{14.26}$$

It means that the radius R of helix may be macroscopic, if ε is small enough.

The result obtained

$$\mathbf{P}_1\mathbf{P}_2=q,\quad \mathbf{P}_0\mathbf{P}_2=l+q \tag{14.27}$$

corresponds to position of the point P_2 on the helix (14.3). However, there are another solutions of equations (14.5), (14.6), where the point P_2 is described by relations (14.13) and g-vectors (14.14)

$$\mathbf{P}_1\mathbf{P}_2=q+\alpha,\quad \mathbf{P}_0\mathbf{P}_2=l+q+\alpha \tag{14.28}$$

Here vector α describes wobbling of the point P_2. It satisfies the dynamic equations

$$l^2=\left(q+\alpha\right)^2 \tag{14.29}$$

$$\left(l.q+\alpha\right)+w\left(P_0,P_1,P_1,P_2\right)=l^2+2d\left(\frac{l^2}{2}\right) \tag{14.30}$$

which are reduced to the form

$$\alpha^2+2\left(q.\alpha\right)=0 \tag{14.31}$$

$$2l_1^2\sin^2\frac{\varphi}{2}+\left(l.\alpha\right)+\frac{\lambda_0^2}{2}f\left(\frac{2l^2+2l_1^2\sin^2\frac{\varphi}{2}+\left(l.\alpha\right)}{\sigma_0}\right)-2\lambda_0^2f\left(\frac{l^2}{2\sigma_0}\right)=0 \tag{14.32}$$

Supposing that $(l.\alpha) = l_0\alpha_0 - \mathbf{l}\boldsymbol{\alpha}$ is a small quantity, one expands (14.32) over $(l.\alpha)$. As far as the zeroth term of expansion coincides with (14.22), the first term of expansion of (14.32) has the form

$$(l.\alpha) + \varepsilon\frac{\lambda_0^2}{2}g'\left(\frac{2l^2 + 2l_1^2\sin^2\frac{\varphi}{2}}{\sigma_0}\right)\frac{(l.\alpha)}{\sigma_0} = 0 \qquad (14.33)$$

or

$$(l.\alpha) = l_0\alpha_0 - l_1\alpha_1 = 0, \quad \alpha_0 = \frac{l_1\alpha_1}{l_0} \qquad (14.34)$$

Substituting α_0 from (14.34) in (14.31), one obtains

$$2\left(l_1 - l_1\cos\varphi\right)\alpha_1 - 2l_1\sin\varphi\alpha_2 + \left(\frac{l_1\alpha_1}{l_0}\right)^2 - \alpha_1^2 - \alpha_2^2 - \alpha_3^2 = 0 \qquad (14.35)$$

Taking into account that φ is small and setting for simplicity $\varphi = 0$, one obtains for spatial components of vector α

$$\left(\left(\frac{l_1}{l_0}\right)^2 - 1\right)\alpha_1^2 - \alpha_2^2 - \alpha_3^2 = 0 \qquad (14.36)$$

As far as $l_1^2 > l_0^2$, the first term in (14.36) is positive, components of 3-vector $\boldsymbol{\alpha}$ may be infinitely large. It means that the wobbling amplitude is infinite. Thus, the helical world chain (14.3) with the two-point spacelike skeleton $\mathcal{P}_1^{(s)} = \left\{P_0^{(s)}, P_1^{(s)}\right\}$ is unstable with respect to the wobbling.

XV. Helical World Chain with Three-Point Skeleton

Reduction of the wobbling of the world chain, consisting of spacelike vectors, can be achieved, if we consider the world chain with more complicated links, whose skeleton consists of three points $\{P_k, P_{k+1}, Q_{k+1}\}$, $k = ...1.2, ...$ Let $\mathbf{P}_k\mathbf{P}_{k+1}$ be a spacelike g-vector, whereas the vector $\mathbf{P}_k\mathbf{Q}_{k+1}$ be a timelike g-vector in $\mathcal{G}_{\mathrm{M}}$. To investigate the effect of stabilization, it is sufficient to consider the points P_0, P_1, P_2, Q_1, Q_2, having coordinates

$$\begin{aligned} P_0 &= \{0\}, \qquad P_1 = \{l\}, \qquad P_2 = \{l+q+\alpha\}, \\ Q_1 &= \{s\}, \qquad Q_2 = \{s+q+\beta\}, \end{aligned} \qquad (15.1)$$

Corresponding vectors have the form

$$\begin{aligned}
\mathbf{P}_0\mathbf{P}_1 &= l, \qquad \mathbf{P}_1\mathbf{P}_2 = q+\alpha, \qquad \mathbf{P}_0\mathbf{P}_2 = l+q+\alpha, && (15.2)\\
\mathbf{P}_0\mathbf{Q}_1 &= s, \quad \mathbf{P}_1\mathbf{Q}_2 = s+q-l+\beta, \quad \mathbf{P}_0\mathbf{Q}_2 = s+q+\beta, && (15.3)\\
\mathbf{P}_1\mathbf{Q}_1 &= s-l, \qquad \mathbf{P}_2\mathbf{Q}_2 = s-l+\gamma, \qquad \mathbf{Q}_1\mathbf{Q}_2 = q+\beta, && (15.4)\\
\mathbf{Q}_1\mathbf{P}_2 &= l+q-s+\alpha, \quad \gamma = \beta-\alpha && (15.5)
\end{aligned}$$

Here l, q, s are 4-vectors of the Minkowski space-time

$$l = \{l_0, l_1, 0, 0\} \qquad q = \{l_0, l_1\cos\varphi, l_1\sin\varphi, 0\}, \qquad s = \{s_0, s_1, s_2, 0\} \qquad (15.6)$$

Geometric vectors α, β, $\gamma = \beta - \alpha$ are g-vectors describing wobbling, connected with points P_2 and Q_2. On needs to write six dynamic equations corresponding to equalities $\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{P}_1\mathbf{P}_2$, $\mathbf{P}_0\mathbf{Q}_1\text{eqv}\mathbf{P}_1\mathbf{Q}_2$, and $\mathbf{P}_1\mathbf{Q}_1\text{eqv}\mathbf{P}_2\mathbf{Q}_2$. Two equations, corresponding to $\mathbf{P}_0\mathbf{P}_1\text{eqv}\mathbf{P}_1\mathbf{P}_2$, have been written and investigated (equations (14.5), (14.6))

In the case $\mathbf{P}_0\mathbf{Q}_1\text{eqv}\mathbf{P}_1\mathbf{Q}_2$ one obtains

$$s^2 = (s+q-l+\beta)^2 \tag{15.7}$$

$$s^2 + (\beta.s) + w(P_0, Q_1, P_1, Q_2) = s^2 + 2d\left(\frac{s^2}{2}\right) \tag{15.8}$$

where according to (14.12) and (15.2) - (15.5)

$$w(P_0, Q_1, P_1, Q_2) = d(P_0, Q_2) + d(Q_1, P_1) - d(P_0, P_1) - d(Q_1, Q_2)$$

$$= \frac{\lambda_0^2}{2}\left(f\left(\frac{(s+q+\beta)^2}{2\sigma_0}\right) + f\left(\frac{(s-l)^2}{2\sigma_0}\right) - f\left(\frac{l^2}{2\sigma_0}\right) - f\left(\frac{(q+\beta)^2}{2\sigma_0}\right)\right) \tag{15.9}$$

We define s in such a way, that

$$2(s.q-l) = -(q-l)^2 = 4l_1^2\sin^2\frac{\varphi}{2} \tag{15.10}$$

Then

$$s = \{s_0, l_1(1-\cos\varphi), l_1\sin\varphi, 0\} \tag{15.11}$$

Equations (15.7), (15.8) are transformed to the form

$$2(\beta.s+q-l) + \beta^2 = 0 \tag{15.12}$$

$$\begin{aligned}(s.\beta) + 2l_1^2\sin^2\frac{\varphi}{2} + \frac{\lambda_0^2}{2}&\left(\begin{array}{c} f\left(\frac{(s+q+\beta)^2}{2\sigma_0}\right) + f\left(\frac{(s-l)^2}{2\sigma_0}\right) \\ -f\left(\frac{l^2}{2\sigma_0}\right) - f\left(\frac{(q+\beta)^2}{2\sigma_0}\right) + 2f\left(\frac{s^2}{2\sigma_0}\right)\end{array}\right) \\ = \quad -2d\left(\frac{s^2}{2}\right)&\end{aligned} \tag{15.13}$$

The necessary condition of the fact, that equation (15.13) has the solution $\beta = 0$, has the form

$$2l_1^2\sin^2\frac{\varphi}{2} + \frac{\lambda_0^2}{2}\left(\begin{array}{c} f\left(\frac{(s+q)^2}{2\sigma_0}\right) + f\left(\frac{(s-l)^2}{2\sigma_0}\right) \\ -2f\left(\frac{l^2}{2\sigma_0}\right) - 4f\left(\frac{s^2}{2\sigma_0}\right)\end{array}\right) = 0 \tag{15.14}$$

Substituting f from (14.1) in (15.14), one obtains

$$\begin{aligned}&\frac{4l_1^2}{\lambda_0^2}\sin^2\frac{\varphi}{2} + \varepsilon g\left(\frac{(s_0+l_0)^2 - l_1^2(1+4\sin^2\varphi)}{2\sigma_0}\right) + \varepsilon g\left(\frac{(s_0-l_0)^2 - l_1^2}{2\sigma_0}\right) \\ &-2\varepsilon g\left(\frac{l_0^2-l_1^2}{2\sigma_0}\right) - 4\varepsilon g\left(\frac{s_0^2}{2\sigma_0}\right) = -\frac{(1-\varepsilon g(1))}{\sigma_0}s_0^2 + \mathcal{O}(\varepsilon^2)\end{aligned} \tag{15.15}$$

This equation together with (14.22) determine parameters of the helical world chain: l_0, l_1, s_0, R, where R is defined by equation (14.21), (14.26) $R = l_1\left(2\sin\frac{\varphi}{2}\right)^{-1}$). These parameters depend on the form of function g.

In the case $\mathbf{P}_1\mathbf{Q}_1\mathrm{eqv}\mathbf{P}_2\mathbf{Q}_2$ we obtain

$$(s-l)^2 = (s-l+\gamma)^2 \tag{15.16}$$

$$(s-l.s-l+\gamma) + w(P_1, Q_1, P_2, Q_2) = (s-l)^2 + 2d\left(\frac{(s-l)^2}{2}\right) \tag{15.17}$$

where according to (14.12) and (15.2) - (15.5)

$$\begin{aligned} & w(P_1, Q_1, P_2, Q_2) \\ = \; & d(\sigma_{\mathrm{M}}(P_1, Q_2)) + d(\sigma_{\mathrm{M}}(Q_1, P_2)) - d(\sigma_{\mathrm{M}}(P_1, P_2)) - d(\sigma_{\mathrm{M}}(Q_1, Q_2)) \\ = \; & f\left(\frac{(s+q-l+\beta)^2}{2\sigma_0}\right) + f\left(\frac{(l+q-s+\alpha)^2}{2\sigma_0}\right) - f\left(\frac{l^2}{2\sigma_0}\right) - f\left(\frac{(q+\beta)^2}{2\sigma_0}\right) \end{aligned}$$

Equations (15.16) and (15.17) take the form

$$\gamma^2 + 2((s-l).\gamma) = 0, \quad \gamma = \beta - \alpha \tag{15.18}$$

$$\begin{aligned} & ((s-l).\gamma) + \frac{\lambda_0^2}{2} f\left(\frac{(s+q-l+\beta)^2}{2\sigma_0}\right) + \frac{\lambda_0^2}{2} f\left(\frac{(l+q-s+\alpha)^2}{2\sigma_0}\right) \\ & -\frac{\lambda_0^2}{2} f\left(\frac{l^2}{2\sigma_0}\right) - \frac{\lambda_0^2}{2} f\left(\frac{(q+\beta)^2}{2\sigma_0}\right) - \lambda_0^2 f\left(\frac{(s-l)^2}{2\sigma_0}\right) = 0 \end{aligned} \tag{15.19}$$

In the case $\alpha = \beta = \gamma = 0$ equation (15.19) turns to the equation

$$\varepsilon\left(g\left(\frac{(s+q-l)^2}{2\sigma_0}\right) + g\left(\frac{(l+q-s)^2}{2\sigma_0}\right) - 2g\left(\frac{l^2}{2\sigma_0}\right) - 2g\left(\frac{(s-l)^2}{2\sigma_0}\right)\right) = 0$$

or

$$\begin{aligned} & \varepsilon g\left(\frac{s_0^2 - 4l_1^2\sin^2\varphi}{2\sigma_0}\right) + \varepsilon g\left(\frac{(2l_0 - s_0)^2 - 4l_1^2\cos^2\varphi}{2\sigma_0}\right) \\ = \; & 2\varepsilon g\left(\frac{l_0^2 - l_1^2}{2\sigma_0}\right) + 2g\left(\frac{(s-l)^2}{2\sigma_0}\right) \end{aligned} \tag{15.20a}$$

One supposes that the function g has such a form, that system of three equations (14.22), (15.15), (15.20a), considered as system of equations for variables l_0, l_1, s_0, R ($l_0^2, l_1^2, s_0^2 < \sigma_0$) has a solution. Thus, one supposes that parameters l_0, l_1, s_0, R are not arbitrary. They satisfy equations (14.22), (15.15), (15.20a). There may be other solutions with $\alpha, \beta \neq 0$

Let us return to equations (15.18), (15.19)

$$\gamma^2 + 2((s-l).\gamma) = 0, \quad \gamma = \beta - \alpha \tag{15.21}$$

Global Journal of Science Frontier Research (A) Volume XIV Issue VII Version I Year 2014

Expanding them over γ, and taking into account (15.20a), one obtains

$$\begin{aligned}
&((s-l).\gamma)+\varepsilon\frac{\lambda_0^2}{2}g'\left(\frac{s^2}{2\sigma_0}\right)\left(2\left(s+q-l.\beta\right)+\beta^2\right)\\
&+\varepsilon\frac{\lambda_0^2}{2}g'\left(\frac{(l+q-s)^2}{2\sigma_0}\right)\left(\alpha^2+2\left(l+q-s+\alpha\right)\right)\\
&-\varepsilon\frac{\lambda_0^2}{2}f\left(\frac{q^2}{2\sigma_0}\right)\left(\beta^2+2\left(q.\beta\right)\right)=0
\end{aligned}\tag{15.22}$$

Taking into account (15.12) and (14.31) one obtains from (15.22)

$$\begin{aligned}
&((s-l).\gamma)+\varepsilon\lambda_0^2 g'\left(\frac{(s_0-2l_0)^2-4l_1^2\cos^2\varphi}{2\sigma_0}\right)((l-s.\alpha))\\
&-\varepsilon\lambda_0^2 g'\left(\frac{(l_0)^2-l_1^2}{2\sigma_0}\right)(l-s.\beta)=0
\end{aligned}\tag{15.23}$$

Or

$$((s-l).\gamma)=\mathcal{O}(\varepsilon)\tag{15.24}$$

Supposing that β is small and expanding (15.13) over β, one obtains

$$(s.\beta)+2l_1^2\sin^2\frac{\varphi}{2}+\varepsilon\frac{\lambda_0^2}{2}\left(\begin{array}{c} g'\left(\frac{(s+q)^2}{2\sigma_0}\right)\left(2\left(\beta.l\right)\right)\\ -g'\left(\frac{q^2}{2\sigma_0}\right)\left(\beta^2+2\left(\beta.l-s\right)\right)\end{array}\right)=0$$

As far as $\sin^2\frac{\varphi}{2}=\mathcal{O}(\varepsilon)$, one obtains

$$(s.\beta)=\mathcal{O}(\varepsilon)\tag{15.25}$$

It follows from (15.25) and (15.11)

$$\beta_0=\frac{\boldsymbol{\beta}\mathbf{s}}{s_0}+\mathcal{O}(\varepsilon)=\frac{l_1\left(\beta_1\left(1-\cos\varphi\right)+\beta_2\sin\varphi\right)}{s_0}+\mathcal{O}(\varepsilon)=\mathcal{O}\left(\sqrt{\varepsilon}\right)\tag{15.26}$$

Substituting (15.26) in (15.12) and taking into account that

$$s+q-l=\{s_0,0,2l_1\sin\varphi,0\}\tag{15.27}$$

one obtains

$$\left(\frac{\boldsymbol{\beta}\mathbf{s}}{s_0}\right)^2-\boldsymbol{\beta}^2-4l_1\sin\varphi\beta_2=\mathcal{O}(\varepsilon)\,,\quad \boldsymbol{\beta}^2=-4l_1\sin\varphi\beta_2+\mathcal{O}(\varepsilon)=\mathcal{O}\left(\sqrt{\varepsilon}\right)\tag{15.28}$$

$$\beta_1^2+\left(1-\frac{4l_1^2\sin^2\varphi}{s_0^2}\right)\beta_2^2+4\beta_2 l_1\sin\varphi+\beta_3^2=\mathcal{O}\left(\sqrt{\varepsilon}\right)\tag{15.29}$$

Then

$$\beta_1,\beta_3=\mathcal{O}\left(\sqrt{\varepsilon}\right),\quad \beta_2=\mathcal{O}(1)\,,\quad \text{if } s_0^2>4l_1^2\sin^2\varphi\tag{15.30}$$

Let us consider equations for γ (15.24), (15.21). It follows from (15.24) and (15.11)

$$\gamma_0 = \frac{\boldsymbol{\gamma}(\mathbf{s}-\mathbf{l})}{s_0 - l_0} = \frac{-l_1 \cos\varphi\gamma_1 + l_1 \sin\varphi\gamma_2}{s_0 - l_0} = \frac{-l_1\gamma_1}{s_0 - l_0} + \mathcal{O}\left(\sqrt{\varepsilon}\right) \tag{15.31}$$

Substituting (15.31) in (15.21), one obtains

$$\left(\frac{l_1}{s_0 - l_0}\right)^2 \gamma_1^2 - \gamma_1^2 - \gamma_2^2 - \gamma_3^2 = \mathcal{O}\left(\sqrt{\varepsilon}\right) \tag{15.32}$$

It follows from (15.32) that

$$\gamma_1, \gamma_2, \gamma_3 = \mathcal{O}\left(\varepsilon^{1/4}\right), \quad \text{if} \ \ l_1^2 < (s_0 - l_0)^2 \tag{15.33}$$

Restriction (14.36) on $\boldsymbol{\alpha}$ is valid, but $\boldsymbol{\alpha} = \boldsymbol{\gamma} + \boldsymbol{\beta}$, and $\boldsymbol{\gamma}$ and $\boldsymbol{\beta}$ are restricted by the conditions (15.30) and (15.33), then $\boldsymbol{\alpha}$ is restricted by the condition

$$\alpha_1, \alpha_3 = \mathcal{O}\left(\sqrt{\varepsilon}\right), \quad \alpha_2 = \mathcal{O}(1), \ \text{if} \ \ l_1^2 < (s_0 - l_0)^2, \quad \varepsilon \ll 1 \tag{15.34}$$

Thus, in the case of three-point skeleton wobbling of the helical world chain is restricted, provided s_0 component of the timelike vector $\mathbf{P}_s\mathbf{Q}_{s+1}$ is large enough. According to (14.26) the radius of the helix is of the order $\varepsilon^{-1/2}$, whereas the amplitude of wobbling is of the order 1. It means, that in the case, when $\varepsilon \ll 1$ and $l_1^2 < (s_0 - l_0)^2$, the wobbling of the world chain violates slightly the shape of helix.

This property of the tachyon with three point skeleton reminds discrete states of atomic electrons. The discreteness of the atomic electron states is conditioned by the electromagnetic emanation of the atom. It radiates until the charge density of the electron envelope changes in time. As soon as the electric charge density ceases to change, the atom ceases to radiate and the electron state becomes stable. In the case of the tachyon helix there is wobbling of the tachyon world chain. At some values of parameters l, q, s the world chain wobbling reduces, and a quasi-stable tachyon world chain arises.

XVI. Conclusion

In our way to tachyon model of neutrino we followed to physical principles (not to arbitrary hypotheses), correcting mistakes and defects. At first we substituted nonrelativistic concept of the particle state by the relativistic one. As a result we succeeded to construct a united formalism for description of deterministic and stochastic relativistic particles. It appeared that nonrelativistic quantum mechanics is a relativistic conception in the sense that stochastic component of the quantum particle motion is relativistic. One should use relativistic description of the nonrelativistic quantum particle motion. This stochastic component vanishes after averaging. The mean regular component remains. It is nonrelativistic. As a result the relativistic quantum theory looks as a nonrelativistic conception, although in reality it can be understood only from the viewpoint of relativistic statistical description.

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The united formalism of dynamics admits one to interpret the quantum mechanics as a dynamics of relativistic stochastic particles. Then the idea of uniting of special relativity with principles of quantum theory appears to be unnecessary. The question on reasons of the free elementary particles stochasticity appears instead. Usually the stochastic behavior of quantum particle is explained by the quantum principles, i.e. axiomatically. Now, when the quantum principles are not used, one should find reasons of the elementary particle stochasticity. A reason of the free elementary particle stochasticity appears a discreteness of the space-time geometry. Exactly the reason of stochasticity is a multivariance of the discrete space-time geometry. Elementary length λ_0 of the space-time geometry is connected with the quantum constant $\hbar$. As a result the quantum constant appears to be a parameter of the discrete space-time geometry. This fact explains the overall character of the quantum constant (it is explained by properties of the space-time).

Using description of quantum mechanics, any quantum particle is labelled by a classical particle which is a simplified (classical) model of the quantum particle. For instance, from classical viewpoint a free electron has spin s and magnetic moment μ, which depend on $\hbar$. World line of the free electron is a straight line. Classical model for appearance of s and μ is absent. The quantities s and μ are simply quantum numbers ascribed to electron. The quantities s and μ are obtained from the concept of the quantum electron as a result of the limit $\hbar \to 0$, but nevertheless s and μ depend on $\hbar$.

Using statistical description of a free electron as a stochastic particle, one can label the stochastic particle by a deterministic ("classical") particle. However, in this case the world line of the deterministic particle is a helix. Spin s and magnetic moment μ are explained by the helical character of the world line. In this case the deterministic model admits one to construct a more detailed arrangement of the electron. In this example we see, that the statistical approach admits one to determine more detailed arrangement of the elementary particle. This becomes more clear, if we compare situation of the elementary particles arrangement with the situation at the investigation of the atoms arrangement.

In the investigation of the atoms properties there are two different approaches: (1) structural approach, (2) empirical approach. At the structural approach one investigates the atoms arrangement, its components (nucleus and electron envelope), dynamics of these components and their interaction. At the structural approach one uses quantum mechanics and atomic physics. At the empirical approach one investigates properties of different chemical elements, classification of chemical elements over their properties, and reactions between chemical elements. The empirical approach is used in chemistry.

If one knows arrangement of atoms, one can calculate in principle properties of chemical elements. But these calculations are very complicated, and one does not use them in practice. One prefer to use periodical system of chemical elements in order to classify and to investigate chemical reactions. The periodical system was obtained empirically. It is more simple technically, although in principle the chemical reaction can be calculated, if arrangement of atoms is known. However, one cannot to investigate arrangement of the atoms, basing on empirical data, obtained at empirical approach (periodical system of chemical elements). In this sense the structural approach is more fundamental, than the empirical approach.

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In the contemporary investigations of elementary particles one uses only empirical approach. One cannot hope to investigate the arrangement of elementary particles, using only empirical approach, which ascribes quantum numbers to elementary particles instead of investigation of their structure. Founded on quantum theory, the empirical approach cannot explain, from where these quantum numbers appear. Formalism of quantum theory does not admit one to obtain such an explanation. Disruption between the structural approach and the empirical approach is more in the elementary particle theory, than in the atomic theory.

Note that the structural approach uses a new formalism of the particle dynamics and a new formalism of the space-time geometry. A new operation such as the dynamical disquantization is used in the structural approach.

XVII. Appendix. Transformation of Equation for Variable $\boldsymbol{\xi}$

Multiplying equation (6.13) by $2(1+\mathbf{z}\boldsymbol{\xi})/\hbar$ and keeping in mind that $\boldsymbol{\xi}^2=1$ and $\mathbf{z}^2=1$, we obtain

$$\begin{aligned} & -\left(\dot{\boldsymbol{\xi}}\times\mathbf{z}\right)\times\boldsymbol{\xi}+\left(-(\dot{\boldsymbol{\xi}}\times\mathbf{z})+\frac{(\boldsymbol{\xi}\times\mathbf{z})\mathbf{z}\dot{\boldsymbol{\xi}}}{(1+\mathbf{z}\boldsymbol{\xi})}-\frac{(\dot{\boldsymbol{\xi}}\times\boldsymbol{\xi})\mathbf{z}}{(1+\mathbf{z}\boldsymbol{\xi})}\mathbf{z}\right)\times\boldsymbol{\xi} \\ = & -(\dot{\mathbf{x}}\times\ddot{\mathbf{x}})\times\boldsymbol{\xi}Q(1+\mathbf{z}\boldsymbol{\xi}) \end{aligned} \tag{17.1}$$

$$\begin{aligned} & -\left(\dot{\boldsymbol{\xi}}\times\mathbf{z}\right)\times\boldsymbol{\xi}-(\dot{\boldsymbol{\xi}}\times\mathbf{z})\times\boldsymbol{\xi}+\left(\frac{(\boldsymbol{\xi}\times\mathbf{z})\left(\mathbf{z}\dot{\boldsymbol{\xi}}\right)}{(1+\mathbf{z}\boldsymbol{\xi})}-\mathbf{z}\frac{\left((\boldsymbol{\xi}\times\mathbf{z})\,\dot{\boldsymbol{\xi}}\right)}{(1+\mathbf{z}\boldsymbol{\xi})}\right)\times\boldsymbol{\xi} \\ = & -(\dot{\mathbf{x}}\times\ddot{\mathbf{x}})\times\boldsymbol{\xi}Q(1+\mathbf{z}\boldsymbol{\xi}) \end{aligned} \tag{17.2}$$

The term in brackets is written as a double vector product

$$-2\left(\dot{\boldsymbol{\xi}}\times\mathbf{z}\right)\times\boldsymbol{\xi}+\frac{\dot{\boldsymbol{\xi}}\times((\boldsymbol{\xi}\times\mathbf{z})\times\mathbf{z})}{(1+\mathbf{z}\boldsymbol{\xi})}\times\boldsymbol{\xi}+(\dot{\mathbf{x}}\times\ddot{\mathbf{x}})\times\boldsymbol{\xi}Q(1+\mathbf{z}\boldsymbol{\xi})=0 \tag{17.3}$$

$$-2\left(\dot{\boldsymbol{\xi}}\times\mathbf{z}\right)\times\boldsymbol{\xi}-\frac{\dot{\boldsymbol{\xi}}\times(\boldsymbol{\xi}-\mathbf{z}(\boldsymbol{\xi}\mathbf{z}))}{(1+\mathbf{z}\boldsymbol{\xi})^2}\times\boldsymbol{\xi}+(\dot{\mathbf{x}}\times\ddot{\mathbf{x}})\times\boldsymbol{\xi}Q(1+\mathbf{z}\boldsymbol{\xi})=0 \tag{17.4}$$

Transforming the double vector products in the first and second terms, one obtains

$$\dot{\boldsymbol{\xi}}\left(\boldsymbol{\xi}\left(2\mathbf{z}-\frac{(\mathbf{z}\boldsymbol{\xi})\,\mathbf{z}-\boldsymbol{\xi}}{(1+\mathbf{z}\;\boldsymbol{\xi})}\right)\right)+(\dot{\mathbf{x}}\times\ddot{\mathbf{x}})\times\boldsymbol{\xi}Q(1+\mathbf{z}\boldsymbol{\xi})=0 \tag{17.5}$$

$$\dot{\boldsymbol{\xi}}\left((2\mathbf{z}\boldsymbol{\xi}-(\mathbf{z}\boldsymbol{\xi}-1))\right)+(\dot{\mathbf{x}}\times\ddot{\mathbf{x}})\times\boldsymbol{\xi}Q(1+\mathbf{z}\boldsymbol{\xi})=0 \tag{17.6}$$

$$\dot{\boldsymbol{\xi}}=-\left(\boldsymbol{\xi}\times(\dot{\mathbf{x}}\times\ddot{\mathbf{x}})\right)Q \tag{17.7}$$

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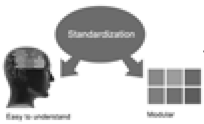

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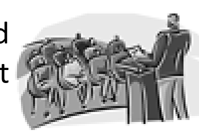

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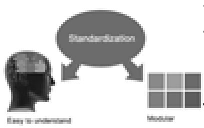

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"

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1. General,

2. Ethical Guidelines,

3. Submission of Manuscripts,

4. Manuscript’s Category,

5. Structure and Format of Manuscript,

6. After Acceptance.

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The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.

- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

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- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

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Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

INDEX

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