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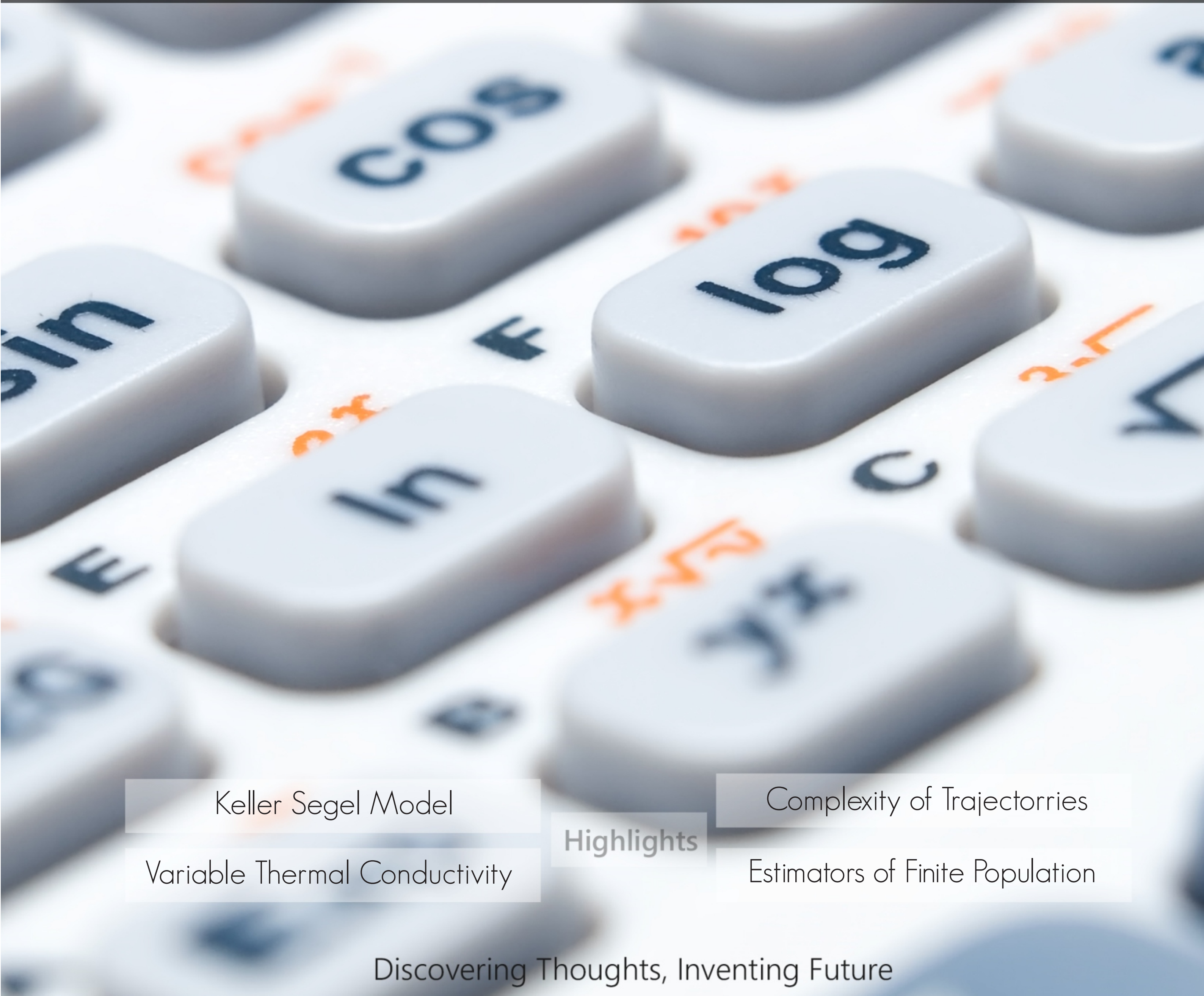

Highlights

Keller Segel Model

Variable Thermal Conductivity

Complexity of Trajectorries

Estimators of Finite Population

Discovering Thoughts, Inventing Future

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On Some Properties of the Rising Sun Function

By Vajha Srinivasa Kumar

Abstract- This paper studies a few interesting properties of the rising sun function of a bounded real function defined on a closed and bounded interval on the real line. An operator on the space of all bounded real functions defined on a closed and bounded interval is introduced and its properties are investigated.

Keywords: rising sun function, semi-continuity, darboux continuity, lower (upper) semicontinuity, lower (upper) semi-quasicontinuity, symmetric continuity, cliquishness, quasicontinuity, differentiability.

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ONSOMEPROPERTIESOFTHERISINGSUNFUNCTION

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3. Riesz, F. And B.Sz. Nagy, *Functional Analysis*, Frederick Ungar Publishing Co., New York, 1995.

On Some Properties of the Rising Sun Function

Vajha Srinivasa Kumar

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Keywords : rising sun function, semi-continuity, darboux continuity, lower (upper) semicontinuity, lower (upper) semi-quasicontinuity, symmetric continuity, cliquishness, quasicontinuity, differentiability.

I. Introduction

The rising sun function was used as a tool in the proof of the famous Lebesgue's theorem on the differentiability of a real valued monotone function without using the theory of integration [3]. In this paper, some properties of the rising sun function are presented and an operator on the space of all bounded real functions defined on a closed and bounded interval $[a,b]$ is introduced and its properties are investigated.

In what follows X, $\mathbb{R}$ and $\mathbb{N}$ stand for a topological space, the real line and the set of all positive integers respectively. Also $\mathcal{B}$ stands for the Banach space of all bounded real functions defined on a closed and bounded interval $[a,b]$ where $a,b \in \mathbb{R}$ and $a < b$ under the supremum norm.

II. Preliminaries

1.1 Definition [6]: The rising sun function of a function $f \in \mathcal{B}$ is defined by

$$f_{\odot}(x) = \sup\{f(y) / x \le y \le b\}.$$

1.2 Definition: For $f \in \mathcal{B}$ we define the following.

(i) $_{\odot}f(x) = \sup\{f(y) / a \le y \le x\}$

(ii) $f^{\odot}(x) = \inf\{f(y) / x \le y \le b\}$

(iii) $^{\odot}f(x) = \inf\{f(y) / a \le y \le x\}$

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1.3 Definition [5]: A function $f: X \to \mathbb{R}$ is said to be semi-continuous at a point $p \in X$ if for every $\varepsilon > 0$ and every neighborhood U of p in X there exists a non-empty open set $W \subset U$ such that $|f(x) - f(p)| < \varepsilon \ \forall\, x \in W$. We say that a function f is semi-continuous on X if it is semi-continuous at every point of X.

1.4 Definition [6]: A function $f: X \to \mathbb{R}$ is said to be lower semicontinuous (lsc) at a point $x \in X$ if for every $\varepsilon > 0$ there exists a neighborhood U of x such that

$$f(y) - f(x) > -\varepsilon \ \ \forall \ \ y \in U.$$

We say that a function f is *lsc* on X if it is *lsc* at every point of X.

1.5 Definition [6]: A function $f: X \to \mathbb{R}$ is said to be upper semicontinuous (usc) at a point $x \in X$ if for every $\varepsilon > 0$ there exists a neighborhood G of x such that

$$f(y) - f(x) < \varepsilon \ \ \forall \ \ y \in G.$$

We say that a function f is *usc* on X if it is *usc* at every point of X.

1.6 Definition [4]: A function $f: X \to \mathbb{R}$ is said to be lower semi-quasicontinuous $(lsqc)$ at a point $x \in X$ if for every $\varepsilon > 0$ and every neighborhood U of x there exists a non-empty open set $W \subset U$ such that $f(y) - f(x) > -\varepsilon \ \ \forall \ \ y \in W$.

We say that a function f is *lsqc* on X if it is *lsqc* at every point of X.

1.7 Definition [4]: A function $f: X \to \mathbb{R}$ is said to be upper semi-quasicontinuous $(usqc)$ at a point $x \in X$ if for every $\varepsilon > 0$ and every neighborhood U of x there exists a non-empty open set $W \subset U$ such that $f(y) - f(x) < \varepsilon \ \ \forall \ \ y \in W$.

We say that a function f is *usqc* on X if it is *usqc* at every point of X.

1.8 Definition [5]: A function $f: X \to \mathbb{R}$ is said to be cliquish at a point $x \in X$ if for every $\varepsilon > 0$ and every neighborhood U of x there exists a non-empty open set $W \subset U$ such that $|f(y) - f(z)| < \varepsilon \ \ \forall \ \ y, z \in W$

We say that a function f is cliquish on X if it is cliquish at every point of X.

1.9 Definition: Let $f: [a,b] \to \mathbb{R}$. We define $f(a-) = f(a)$ and $f(b+) = f(b)$.

We say that $f(p+)$ exists at $p \in [a,b)$ and we write $f(p+) = L$, where $L \in \mathbb{R}$ if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that $|f(x) - L| < \varepsilon \ \ \forall \ \ x \in (p, p+\delta) \subset [a,b]$. Similarly for $p \in (a,b]$ we write

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$f(p-) = l \in \mathbb{R}$ if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that

$$|f(x) - l| < \varepsilon \ \ \forall x \in (p-\delta, p) \subset [a,b].$$

1.10 Definition: A function $f:[a,b] \to \mathbb{R}$ is said to be quasicontinuous at a point $p \in [a,b]$ if $f(p+)$ and $f(p-)$ exist.

1.11 Definition: A function $f:[a,b] \to \mathbb{R}$ is said to be symmetrically continuous at a point $x \in [a,b]$ if $\lim_{h \to 0}[f(x+h) - f(x-h)] = 0$.

1.12 Definition: A function $f:[a,b] \to \mathbb{R}$ is Darboux continuous if for all $p, q \in [a,b]$ and for each c between $f(p)$ and $f(q)$ there is an x between p and q such that $f(x) = c$.

1.13 Definition[2]: An operator P on a linear space L is said to be sublinear if (i) $P(x+y) \le P(x) + p(y)$ $\forall$ $x, y \in$ L and (ii) $P(\lambda x) = \lambda P(x)$ for any positive real number λ and every $x \in$ L**.**

III. Relations among $f_{\odot}, {}_{\odot}f, f^{\odot}$ and ${}^{\odot}f$

In this section the relations between the rising sun function and its analogues that are introduced are presented in the following propositions.

2.1 Proposition: For $f \in \mathcal{B}$, (a) $f^{\odot} = -(-f)_{\odot}$ and (b) ${}^{\odot}f = -{}_{\odot}(-f)$.

2.2 Proposition: For $f \in \mathcal{B}$, (a) $(f_{\odot})_{\odot} = f_{\odot}$ $\quad (b)$ ${}_{\odot}({}_{\odot}f) = {}_{\odot}f$

$$(c)\ \left(f^{\odot}\right)^{\odot} = f^{\odot} \qquad (d)\ {}^{\odot}\left({}^{\odot}f\right) = {}^{\odot}f$$

2.3 Proposition: For $f \in \mathcal{B}$ and $x \in [a,b]$,

(i) ${}_{\odot}\left(f_{\odot}\right)(x) = f_{\odot}(a)$ $\quad (ii)$ $\left(f_{\odot}\right)^{\odot}(x) = f_{\odot}(b)$ $\quad (iii)$ ${}^{\odot}\left(f_{\odot}\right)(x) = f_{\odot}(x)$

(iv) $\left({}_{\odot}f\right)_{\odot}(x) = {}_{\odot}f(b)$ $\quad (v)$ $\left({}_{\odot}f\right)^{\odot}(x) = {}_{\odot}f(x)$ $\quad (vi)$ ${}^{\odot}\left({}_{\odot}f\right)(x) = {}_{\odot}f(a)$

(vii) $\left(f^{\odot}\right)_{\odot}(x) = f^{\odot}(b)$ $\quad (viii)$ ${}_{\odot}\left(f^{\odot}\right)(x) = f^{\odot}(x)$ $\quad (ix)$ ${}^{\odot}\left(f^{\odot}\right)(x) = f^{\odot}(a)$

(x) $\left({}^{\odot}f\right)_{\odot}(x) = {}^{\odot}f(x)$ $\quad (xi)$ ${}_{\odot}\left({}^{\odot}f\right)(x) = {}^{\odot}f(a)$ $\quad (xii)$ $\left({}^{\odot}f\right)^{\odot}(x) = {}^{\odot}f(b)$

2.4 Remark: In view of the previous propositions it is enough to investigate the properties of the rising sun function and the properties of ${}_{\odot}f$, $f^{\odot}$ and ${}^{\odot}f$ follow analogously.

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IV. Characterisations of $f_{\odot}, {}_{\odot}f, f^{\odot}$ and ${}^{\odot}f$

3.1 proposition: For $f \in \mathscr{B}$, $f_{\odot}$ is the smallest decreasing function dominating f

More precisely,

(a) $f_{\odot}(x) \geq f(x)\ \ \forall\ x \in [a,b]$

(b) $f_{\odot}$ is decreasing on $[a,b]$

(c) If g satisfies (a) and (b) above, then $f_{\odot}(x) \leq g(x)\ \ \forall\ x \in [a,b]$.

3.2 Proposition: For $f \in \mathscr{B}$, ${}_{\odot}f$ is the smallest increasing function dominating f. More precisely,

(a) ${}_{\odot}f(x) \geq f(x)\ \ \forall\ x \in [a,b]$

(b) ${}_{\odot}f$ is increasing on $[a,b]$

(c) If g satisfies (a) and (b) above, then ${}_{\odot}f(x) \leq g(x)\ \ \forall\ x \in [a,b]$.

3.3 Proposition: For $f \in \mathscr{B}$,

(a) $f^{\odot}$ is the largest increasing function such that $f^{\odot}(x) \leq f(x)\ \ \forall\ x \in [a,b]$

(b) ${}^{\odot}f$ is the largest decreasing function such that ${}^{\odot}f(x) \leq f(x)\ \ \forall\ x \in [a,b]$

V. The Rising Sun Operator

4.1 Definition: Define $T:\mathscr{B} \to \mathscr{B}$ by $Tf = f_{\odot}$. We call this operator T, the rising sun operator on $\mathscr{B}$.

4.2 Proposition: The rising sun operator T is sublinear on $\mathscr{B}$. More precisely,

$(a)\,T(f+g) \leq Tf + Tg\ \ \forall\ \ f,g \in \mathscr{B}$

(b) $T(\lambda f) = \lambda Tf$ for every real number $\lambda > 0$ and every $f \in \mathscr{B}$.

Proof: Let $f,g \in \mathscr{B}$ and $x \in [a,b]$.

(a) For $y \in [x,b]$, $(f+g)(y) = f(y) + g(y) \leq f_{\odot}(x) + g_{\odot}(x) = (f_{\odot} + g_{\odot})(x)$

$$\Rightarrow (f+g)(y) \leq (f_{\odot} + g_{\odot})(x)\ \ \forall\ \ y \in [x,b]$$
$$\Rightarrow (f+g)_{\odot}(x) \leq (f_{\odot} + g_{\odot})(x)\ \ \forall\ \ x \in [a,b]$$

Hence $T(f+g) \leq Tf + Tg\ \ \forall\ \ f,g \in \mathscr{B}$.

(b) Suppose that λ is a positive real number and $f \in \mathscr{B}$

Then
$$(\lambda f)_{\odot}(x) = \sup\{(\lambda f)(y) / x \leq y \leq b\}$$
$$= \lambda \sup\{f(y) / x \leq y \leq b\}$$
$$= \lambda\ f_{\odot}(x)$$
$$T\ \lambda f\ = \lambda Tf.$$

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4.3 Remark: From the following example, it is clear that $T(f+g) \neq Tf + Tg$.

4.4 Example: Define $f:[0,1] \to \mathbb{R}$ and $g:[0,1] \to \mathbb{R}$ by $f(x) = x$ and

$$g(x) = 1 - x \quad \forall\, x \in [0,1] \quad \Rightarrow (f+g)(x) = 1. \forall\, x \in [0,1]$$

Then $(f+g)_{\odot}(x) = 1$ and $(f_{\odot} + g_{\odot})(x) = 2 - x \quad \forall\, x \in [0,1]$.

Hence $T(f+g) \neq Tf + Tg$.

4.5 Proposition: For $f \in \mathcal{B}$ and $k \in \mathbb{R}$, $T(f+k) = Tf + k$.

4.6 Proposition: For $f, g \in \mathcal{B}$, (a) $f \leq g \Rightarrow Tf \leq Tg$ (b) $T(f \vee g) = Tf \vee tg$

where $(f \vee g)(x) = \max\{f(x), g(x)\}$.

4.7 Proposition: If $\{f_\alpha / \alpha \in \Delta\}$ is a collection of functions in $\mathcal{B}$ and if

$\bigvee_{\alpha\in\Delta} f_\alpha = \sup\{f_\alpha / \alpha \in \Delta\}$ exists in $\mathcal{B}$ then $T\left(\bigvee_{\alpha\in\Delta} f_\alpha\right) = \bigvee_{\alpha\in\Delta} T(f_\alpha)$.

4.8 Remark: From the following example it can be observed that

$T(f \wedge g) \neq Tf \wedge Tg$, where $(f \wedge g)(x) = \min\{f(x), g(x)\}$.

4.9 Example: Define $f:[0,1] \to \mathbb{R}$ and $g:[0,1] \to \mathbb{R}$ by

$$f(x) = \begin{cases} 1-2x & \text{if } 0 \leq x \leq \frac{1}{2} \\ 2x-1 & \text{if } \frac{1}{2} \leq x \leq 1 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq \frac{1}{2} \\ -2x+2 & \text{if } \frac{1}{2} \leq x \leq 1 \end{cases}$$

Then $f_{\odot}(x) = 1 \;\forall\, x \in [0,1]$ and $g_{\odot}(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq \frac{1}{2} \\ -2x+2 & \text{if } \frac{1}{2} \leq x \leq 1 \end{cases}$

$$\Rightarrow (f_{\odot} \wedge g_{\odot})(x) = g_{\odot}(x) \quad \forall\, x \in [0,1]$$

Also $(f \wedge g)(x) = \begin{cases} 2x & \text{if } 0 \leq x \leq \frac{1}{4} \\ -2x+1 & \text{if } \frac{1}{4} \leq x \leq \frac{1}{4} \\ 2x-1 & \text{if } \frac{1}{2} \leq x \leq \frac{3}{4} \\ -2x+2 & \text{if } \frac{3}{4} \leq x \leq 1 \end{cases}$

Notes

$$\Rightarrow (f \wedge g)_{\odot}(x) = \begin{cases} \frac{1}{2} & if \quad 0 \le x \le \frac{3}{4} \\ -2x+2 & if \quad \frac{3}{4} \le x \le 1 \end{cases}$$

Hence $(f \wedge g)_{\odot}(x) \neq (f_{\odot} \wedge g_{\odot})(x)$.

4.10 Proposition: T is continuous on $\mathcal{B}$.

Proof: Let $f_n \in \mathcal{B}$ for $n \in \mathbb{N}$ and $f_n \to f$ uniformly on $[a,b]$. Then $f \in \mathcal{B}$.

Let $\varepsilon > 0$ be given. Then there exists a positive integer N such that

$n \ge N \Rightarrow |f_n(x) - f(x)| < \varepsilon \; \forall \; x \in [a,b]$

$\Rightarrow -\varepsilon < f_n(x) - f(x) < \varepsilon \quad \forall \; x \in [a,b]$

$\Rightarrow f(x) - \varepsilon < f_n(x) < f(x) + \varepsilon \quad \forall \; x \in [a,b]$.

Let $x \in [a,b]$ and choose $y \in [x,b]$. Then $y \in [a,b]$.

$\Rightarrow f(y) - \varepsilon < f_n(y) < f(y) + \varepsilon \quad \forall \; n \ge N$

$n \ge N \Rightarrow f_n(y) < f(y) + \varepsilon \le f_{\odot}(x) + \varepsilon$

$\Rightarrow f_n(y) < f_{\odot}(x) + \varepsilon \; \forall \; y \in [x,b]$

$\Rightarrow (f_n)_{\odot}(x) \le f_{\odot}(x) + \varepsilon$.

Similarly $f_{\odot}(x) - \varepsilon < (f_n)_{\odot}(x)$.

Hence $|(f_n)_{\odot}(x) - f_{\odot}(x)| \le \varepsilon \quad \forall \; n \ge N$ and for every $x \in [a,b]$.

Hence $(f_n)_{\odot} \to f_{\odot}$ uniformly on $[a,b]$.

Hence $f_n \to f$ in $\mathcal{B}$ $\Rightarrow Tf_n \to Tf$ in $\mathcal{B}$.

$\Rightarrow$ T is continuous on $\mathcal{B}$.

4.11Proposition: (a) $T^n f = f_{\odot} \quad \forall \; f \in \mathcal{B}$ and for every $n \in \mathbb{N}$

(b) For $f \in \mathcal{B}$, the cycle of T is the set $\{f, f_{\odot}\}$.

4.12 Proposition: If $f \in \mathcal{B}$ and f is monotonically decreasing then f is a fixed point of T.

4.13 Remark: The set of all fixed points of T is the set of all monotonically decreasing functions on $[a,b]$. Let F denote the set of all fixed points of T

Then $F = \{f \in \mathcal{B} / Tf = f\} = \{f \in \mathcal{B} / f \text{ is decreasing}\}$.

Notes

4.14 Proposition: The set F of all fixed points of the rising sun operator T is closed in $\mathscr{B}$.

4.15 Proposition: Fix $f \in F$. Let $F^* = \{ f \in \mathscr{B} / Tf = f \}$.

Then (a) F^* is closed in $\mathscr{B}$.

(b) $(F^*, \leq)$ is a $\vee -$semilattice under the relation $\leq$ defined on F^* by

$$f \leq g \;\Leftrightarrow\; f(x) \leq g(x) \;\; \forall\, x \in [a,b]$$

4.16 Proposition: The operator T is bounded. More precisely,

(i) $\|Tf\| \leq \|f\| \;\; \forall \; f \in \mathscr{B}$ and (ii) $\|T\| = 1$

VI. Invariant Properties

5.1 Proposition: Let $f \in \mathscr{B}$. If f is *usc* at a point $x \in [a,b]$ then so is $Tf = f_{\odot}$.

Proof: Let $\varepsilon > 0$ be given and $x \in [a,b]$. Since f is *usc* at $x \in [a,b]$, there exists a $\delta > 0$ such that $f(t) - f(x) < \varepsilon \;\; \forall\, t \in (x-\delta, x+\delta) \cap [a,b] = U$.

$\Rightarrow f_{\odot}(x) + \varepsilon \geq f(x) + \varepsilon > f(t) \;\; \forall\, t \in U$

$\Rightarrow f_{\odot}(x) + \varepsilon > f(t) \;\; \forall\, t \in U$

Suppose that $x < y$.

$$\Rightarrow f_{\odot}(x) + \varepsilon > f_{\odot}(x) \geq f_{\odot}(y) \geq f(y)$$

$$\Rightarrow f_{\odot}(x) + \varepsilon > f(y).$$

If $x < t$ and $y \in [t,b] \Rightarrow x < y$

$$\Rightarrow f_{\odot}(x) + \varepsilon > f(y)$$
$$\Rightarrow f_{\odot}(x) + \varepsilon \geq f_{\odot}(t)$$

Suppose that $t \leq x$. Then $y \in [t,b] \Rightarrow t \leq y \leq x$ or $x < y < b$.

If $t \leq y \leq x$ then $y \in U \;\Rightarrow f_{\odot}(x) + \varepsilon > f(y) \;\Rightarrow f_{\odot}(x) + \varepsilon > f_{\odot}(t)$.

If $x < y \leq b$ then $f_{\odot}(x) + \varepsilon > f_{\odot}(t)$. Hence $f_{\odot}(x) + \varepsilon > f_{\odot}(t) \;\; \forall\, t \in U$.

$$\Rightarrow f_{\odot} \text{ is } usc \text{ at } x.$$

5.2 Proposition: Let $f \in \mathscr{B}$. If f is *lsc* at a point $x \in [a,b]$ then so is $Tf = f_{\odot}$.

Proof: Let $\varepsilon > 0$ be given and $x \in [a,b]$. Since f is *lsc* at $x \in [a,b]$, there exists a $\delta_1 > 0$ such that $f(t) - f(x) > \frac{-\varepsilon}{2}$ $\forall t \in (x-\delta_1, x+\delta_1) \cap [a,b] = U$

$$\Rightarrow f_{\odot}(t) + \frac{\varepsilon}{2} \geq f(t) + \frac{\varepsilon}{2} > f(x) \quad \forall t \in U$$

$$\Rightarrow f_{\odot}(t) + \frac{\varepsilon}{2} > f(x) \quad \forall t \in U$$

Since $f_{\odot}(x) - \frac{\varepsilon}{2}$ is not an upper bound of the set $\{f(y) / x \leq y \leq b\}$, there exists a point $y \in [x,b]$ such that $f(y) > f_{\odot}(x) - \frac{\varepsilon}{2}$.

If $y = x$ then $f_{\odot}(t) + \frac{\varepsilon}{2} > f(x) > f_{\odot}(x) - \frac{\varepsilon}{2}$ $\forall t \in U$

$$\Rightarrow f_{\odot}(t) + \varepsilon > f_{\odot}(x) \quad \forall t \in U.$$

$y \neq x$. If $x < z < y$ then $f_{\odot}(z) \geq f_{\odot}(y) \geq f(y) > f_{\odot}(x) - \frac{\varepsilon}{2}$

$$\Rightarrow f_{\odot}(z) > f_{\odot}(x) - \frac{\varepsilon}{2} > f_{\odot}(x) - \varepsilon.$$

If $z \leq x$ then $f_{\odot}(z) \geq f_{\odot}(x) > f_{\odot}(x) - \varepsilon$.

Choose $\delta > 0$ such that $a \leq x - \delta < x < x + \delta \leq y$.

Then $f_{\odot}(z) > f_{\odot}(x) - \varepsilon$ $\forall z \in (x-\delta, x+\delta) \cap [a,b]$. Hence $f_{\odot}$ is *lsc* at x.

5.3 Corollary: Let $f \in \mathscr{B}$. If f is continuous at a point $x \in [a,b]$, then $Tf = f_{\odot}$ is continuous at x.

5.4 Proposition: For any $f \in \mathscr{B}$, $Tf = f_{\odot}$ is *lsqc* at every $x \in (a,b]$.

Proof: Let $f \in \mathscr{B}$ and $x \in (a,b]$. Let $\varepsilon > 0$ be given and let U be a neighborhood of x in $[a,b]$. Then there exists a $\delta > 0$ such that $(x-\delta, x+\delta) \cap [a,b] \subset U$. Choose $W = (x-\delta, x) \cap [a,b]$.

W is a non-empty open subset of U.

$$y \in W \Rightarrow x - \delta < y < x$$

$$\Rightarrow f_{\odot}(y) \geq f_{\odot}(x)$$

$$\Rightarrow f_{\odot}(y) + \varepsilon > f_{\odot}(y) \geq f_{\odot}(x)$$

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Hence $f_{\odot}(y)+\varepsilon > f_{\odot}(x)\ \forall\ y \in W$.

Thus for every $\varepsilon > 0$ and every neighborhood U of x there exists a non-empty open set $W \subset U$ such that $f_{\odot}(y)+\varepsilon > f_{\odot}(x)\ \forall\ y \in W$.

5.5 Proposition: Let $f \in \mathscr{B}$. If $f:[a,b] \to \mathbb{R}$ is *lsqc* at the point a then $Tf = f_{\odot}$ is *lsqc* at a.

Proof: Let $f \in \mathscr{B}$ and $\varepsilon > 0$ be given. Let U be a neighborhood of a in $[a,b]$. Since f is *lsqc* at a, there exists a non-empty open set $W \subset U$ such that $f(t) - f(x) > -\varepsilon\ \forall\, t \in W$.

$$\Rightarrow f_{\odot}(t) \geq f(t) > f(a) - \varepsilon\ \ \forall\, t \in W$$

$$\Rightarrow f_{\odot}(t) > f(a) - \varepsilon\ \ \forall\, t \in W .$$

Since $f_{\odot}(a) - \varepsilon$ is not an upper bound of $\{f(y) / a \leq y \leq b\}$, there exists a point $y \in [a,b]$ such that $f_{\odot}(a) - \varepsilon < f(y)$.

If $y = a$ then $f_{\odot}(t) + \varepsilon > f(a) > f_{\odot}(a) - \varepsilon\ \ \forall\, t \in W$.

Suppose that $y \neq a$.

Since $a \in U$ and U is open in $[a,b]$, there exists a $\delta_1 > 0$ such that $[a, a+\delta_1) \subset U$. Choose $\delta_2 > 0$ such that $a < a + \delta_2 < y$.

Put $\delta = \min\{\delta_1, \delta_2\}$ and $W_1 = (a, a+\delta)$. Clearly, W_1 is a non-empty open set such that $W_1 \subset U$.

Then $z \in W_1 \Rightarrow z < y$

$$\Rightarrow f_{\odot}(z) \geq f_{\odot}(y) \geq f(y) > f_{\odot}(a) - \varepsilon$$

Hence $f_{\odot}(z) + \varepsilon > f_{\odot}(a)\ \forall\, z \in W_1$

$$\Rightarrow f_{\odot} \text{ is } lsqc \text{ at } a .$$

5.6 Proposition: Let $f \in \mathscr{B}$. Then

(a) $f_{\odot}$ is *usqc* at every $x \in [a,b)$.

(b) If $f:[a,b] \to \mathbb{R}$ is *usqc* at b then so is $f_{\odot}$.

5.7 Proposition [5]: Let $f:[a,b] \to \mathbb{R}$ and $p \in [a,b]$. If $f(p+)$ exists then f is cliquish at p.

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5.8 Corollary: For any $f \in \mathscr{B}$, $Tf = f_{\odot}$ is cliquish on $[a,b]$.

5.9 Proposition: For any $f \in \mathscr{B}$, $Tf = f_{\odot}$ is quasicontinuous on $[a,b]$.

VII. Variant Properties

6.1 Symmetric continuity: It is not necessary that the rising sun function of a symmetrically continuous function is symmetrically continuous. For example, define $f:[-1,1] \to \mathbb{R}$ by

$$f(x) = \begin{cases} x^2 \ if \ \ x \neq 0 \\ 0 \ \ if \ \ x = 0 \end{cases}$$

Then
$$f_{\odot}(x) = \begin{cases} x^2 \ \ if \ \ -1 \le x \le 0 \\ 1 \ \ \ if \ \ \ 0 < x \le 1 \end{cases}$$

Clearly f is symmetrically continuous on $[-1,1]$, but $f_{\odot}$ is not.

6.2 Semi-continuity: The semi-continuity of f need not imply the semi-continuity of $f_{\odot}$ as is evident from the following example.

Define $f:[-1,1] \to \mathbb{R}$ by

$$f(x) = \begin{cases} 0 & if \ \ x = 0 \\ (x+1)2^{\frac{-2}{x}} & if \ \ 0 < x \le 1 \\ x+1 & if \ \ -1 \le x < 0 \end{cases}$$

Then
$$f_{\odot}(x) = \begin{cases} 1 & if \ \ -1 \le x \le 0 \\ \frac{1}{2} & if \ \ 0 < x \le 1 \end{cases}$$

Clearly f is semi-continuous on $[-1,1]$. But $f_{\odot}$ is not semi-continuous at $x = 0$.

6.3 Darboux continuity: It is not necessary that the rising sun function of a Darboux continuous function is Darboux continuous. The function $f:[-1,1] \to \mathbb{R}$ defined in the above example is Dourbox continuous on $[-1,1]$ but its rising sun function is not Dourbox continuous.

6.4 Differentiability: The rising sun function of a differentiable function is not necessarily differentiable as can be observed from the following example.

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Define $f:[0,1]\to\mathbb{R}$ by

$$f(x)=\begin{cases}+\sqrt{\frac{2x}{5}-x^2} & if \;\; 0\le x\le \frac{2}{5}\\ -\sqrt{\frac{6x}{5}-\frac{8}{25}-x^2} & if \;\; \frac{2}{5}\le x\le \frac{4}{5}\\ +\sqrt{\frac{9x}{5}-\frac{4}{5}-x^2} & if \;\; \frac{4}{5}\le x\le 1\end{cases}$$

Then $$f_{\odot}(x)=\begin{cases}\frac{1}{5} & if \;\; 0\le x\le \frac{1}{5}\\ f(x) & if \;\; \frac{1}{5}\le x\le \frac{9}{10}\\ \frac{1}{10} & if \;\; \frac{2}{5}\le x\le \frac{9}{10}\\ f(x) & if \;\; \frac{9}{10}\le x\le 1\end{cases}$$

Clearly f is differentiable at $a=0.3732$, but $f_{\odot}$ is not differentiable at this point.

6.5 Pointwise Convergence: If $\{f_n\}$ converges pointwise to f on $[a,b]$, it is not necessary that $\{Tf_n\}$ converges to Tf as can be seen from the following example. Define $f_n:[0,1]\to\mathbb{R}$ by

$$f_n(x)=\begin{cases}1 & if \;\; x\in\left\{\frac{1}{n}/\,n\in\mathbb{N}\right\}\\ 0 & if \;\; x\notin\left\{\frac{1}{n}/\,n\in\mathbb{N}\right\}\end{cases}$$

Then $\{f_n\}$ converges pointwise to 0.

But $(f_n)_{\odot}(x)=\begin{cases}1 & if \;\; x=0\\ 0 & if \;\; 0<x\le 1\end{cases}$ does not converge to 0.

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Some Statistical Properties of Exponentiated Weighted Weibull Distribution

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Abstract- This article basically focused on some statistical properties of exponentiated-weighted weibull model which of course numerous authors have written one thing or the other on exponential weibull distribution and not on exponential weighted weibull. This model is established with a view to obtaining a model that is better than both weighted weibull and weibull distribution in terms of the estimate of their characteristics and their parameters using the logit of Beta by Jones (2004). The weighted weibull distribution is proposed by Mahdy (2013) with an additional parameter called "sensitive skewness parameter". Some basic properties of the proposed model including moments and moment generating function (first and second moments about the origin even with standard deviation are derive), survival rate function, hazard rate function, asymptotic behaviours, and the estimation of parameters have been studied. The result from the new model is better representativeness in data and its flexibility and shape.

Keywords: *exponentiated-weighted weibull, hazard rate, moments, weighted-weibull, survival rate.*

GJSFR-F Classification : *MSC 2010: 97K80, 35B40*

SOMESTATISTICALPROPERTIESOFEXPONENTIATEDWEIGHTEDWEIBULLDISTRIBUTION

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Some Statistical Properties of Exponentiated Weighted Weibull Distribution

Badmus, N. Idowu [α] & Bamiduro, T. Adebayo[σ]

Abstract- This article basically focused on some statistical properties of exponentiated-weighted weibull model which of course numerous authors have written one thing or the other on exponential weibull distribution and not on exponential-weighted weibull. This model is established with a view to obtaining a model that is better than both weighted weibull and weibull distribution in terms of the estimate of their characteristics and their parameters using the logit of Beta by Jones (2004). The weighted weibull distribution is proposed by Mahdy (2013) with an additional parameter called "sensitive skewness parameter". Some basic properties of the proposed model including moments and moment generating function (first and second moments about the origin even with standard deviation are derive), survival rate function, hazard rate function, asymptotic behaviours, and the estimation of parameters have been studied. The result from the new model is better representativeness in data and its flexibility and shape.

Keywords: exponentiated-weighted weibull, hazard rate, moments, weighted-weibull, survival rate.

I. Introduction

In recent time, numerous researchers had used weibull distribution as an alternative to some distribution e.g Gamma and Log-normal distribution in reliability engineering and life testing. The weibull distribution is a well known common distribution and has been a powerful probability distribution in reliability analysis, while weighted distributions are used to adjust the probabilities of the events as observed and recorded. Mahdy applied Azzalini's method to the weibull distribution that produced a new class of weighted weibull distribution as $WW(\lambda, \beta, \alpha)$ distribution with an additional parameter called "Sensitive Skewness Parameter" and the sensitive skewness parameter governs essentially the shape of the probability density function of the $WW(\lambda, \beta, \alpha)$ distribution.

The probability density and the cumulative density function (pdf and cdf) of the new class of weighted weibull distribution by Mahdy (2013) is given by

$$f_{y|\{\lambda,\beta,\alpha\}|}(y) = \frac{\lambda\beta(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}}, \text{ for } y > 0 \quad (1)$$

and

$$F_{y|\{\lambda,\beta,\alpha\}|}(y) = \frac{[(1+\alpha^{\beta})(1-e^{-\lambda y^{\beta}})+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1]}{\alpha^{\beta}} \quad (2)$$

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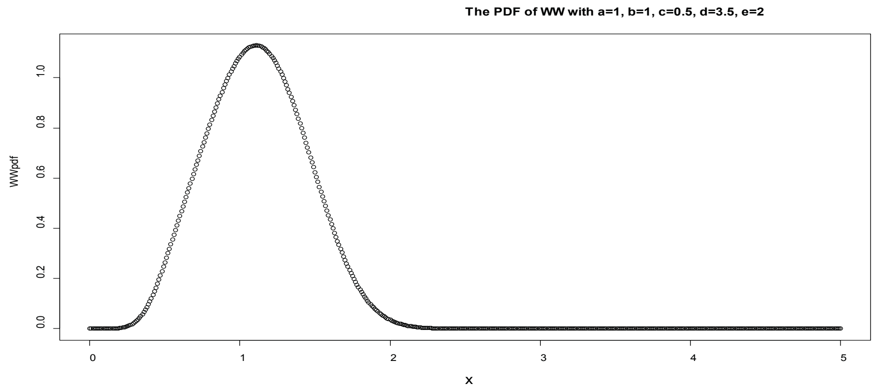

Figure 1 : The Probability Density Function of Weghted Weibull Distribution with a=1, b=1, λ =0.5, $\beta = 3.5$, $\alpha = 2$

Authors on exponenetiated-weighted weibull are very few. The aim of this article is to introduce and investigate this distribution on its statistical properties. The paper is divided as follows: In section 2, we present the proposed distribution exponentiated-weighted weibull distribution. Moments, and moment generating function is studied in section 3, section 4, discussed on the estimation of parameters mathematically and in section 5 we preset the real application to data set and section 6 concluded the research.

II. The Proposed Exponentiated-Weighted Weibull Distribution

Recently, many authors have studied the properties of exponentiated distributions. For instance, Gupta et al (2001) for exponential pareto, Nadarajah and Gupta (2007) for exponential gamma distribution, Mudholkar et al (1995) studied on exponentiated weibull distribution, Salem and Abo-Kasem (2011) based their research on estimation for the parameters of the exponentiated weibull distribution, Gupta and Kundu (2001) they put up a paper on exponentiated exponential etc. Azzalini (1985) first proposed a method of obtaining weighted and the method has been used extensively for several symmetric and non-symmetric distributions. Mahdy (2013) applied the method to study a new class of weighted weibull distribution with an additional parameter called "sensitive skewness parameter". More so, various extensions of weibull and exponential distribution have been proposed in literature. An extension of exponential distribution has been provided by Nadarajah and Kotz (2005) using the logit of Beta distribution and the logit of Beta distribution (the link function of the Beta generalized distribution) is introduced by Jones (2004). Since then extensive work has been done using the logit of beta distribution in literature. For instance, Gupta and Kundu (1999) proposed a generalized exponential distribution which provides an alternative to exponential and weibull distributions. Famoye et al (2005) also introduced the Beta-weibull distribution alongside its major properties and Cordeiro et al (2011) among others.

Now, letting y be a random variable form of the distribution with parameters and defined (1) and (2) using the logit of beta by Jones (2004), we then have

$$f_{EWW}{}^{(y)} = \frac{1}{B(a,b)}\,[F(y)]^{a-1}\;[1-F(y)]^{b-1}\;\; f(y) \tag{3}$$

by setting b=1, we get

$$f_{EWW}{}^{(y)} = a[F(y)]^{a-1}\;\; f(y) \tag{4}$$

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Putting expressions (1) and (2) in (4) to obtain the probability density function of Exponenetiated-weighted weibull distribution

$$f_{EWW}{}^{(y)} = a\left[\frac{[(1+\alpha^\beta)(1-e^{-\lambda y^\beta})+e^{-\lambda y^\beta(1+\alpha^\beta)}-1]}{\alpha^\beta}\right]^{a-1}\frac{\lambda\beta(1+\alpha^\beta)y^{\beta-1}e^{-\lambda y^\beta}(1-e^{-\lambda(\alpha y)^\beta})}{\alpha^\beta} \quad (5)$$

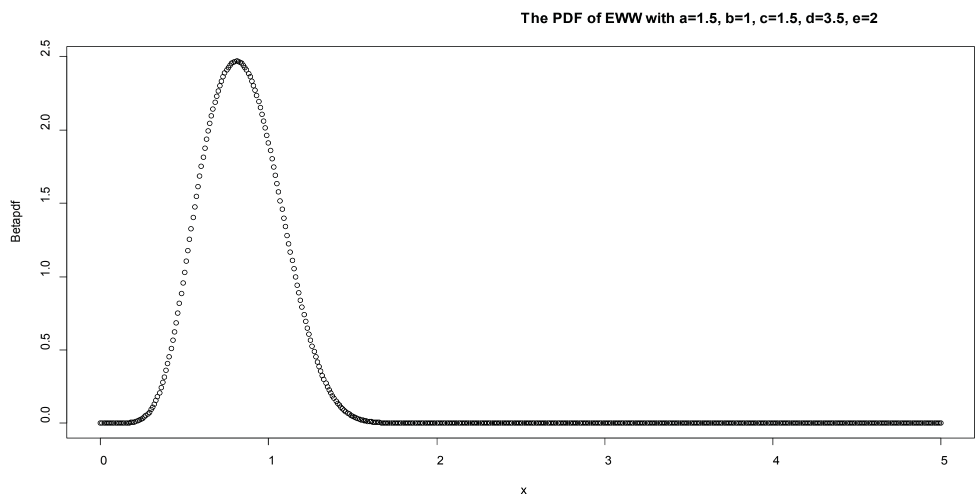

Figure 2 : The PDF of Exponentiated Weighted Weibull Distribution with values of the parameters $(a = 1.5, b = 1, c = \lambda = 1.5, d = \alpha = 3.5, e = \beta = 2.)$ and is rightly skewed

Where $a > 0, \lambda > 0, \beta > 0, \alpha > 0$ *and* $y > 0$ *such that* $Y \sim EWW(a, \lambda, \beta, \alpha)$. Equation (5) is the pdf of Exponenetiated-weighted weibull distribution.

$$\text{set } u(x) = \frac{[(1+\alpha^\beta)(1-e^{-\lambda y^\beta})+e^{-\lambda y^\beta(1+\alpha^\beta)}-1]}{\alpha^\beta}$$

$$\frac{du}{dy} = \frac{\lambda\beta(1+\alpha^\beta)y^{\beta-1}e^{-\lambda y^\beta}(1-e^{-\lambda(\alpha y)^\beta})}{\alpha^\beta}[\frac{1}{\alpha^\beta}\{(1+\alpha^\beta)\left(1-e^{-\lambda y^\beta}\right)+e^{-\lambda y^\beta(1+\alpha^\beta)}-1] \quad (6)$$

substituting dy into (5), we obtain

$$f_{EWW}{}^{(y)} = a\left[\frac{1}{\alpha^\beta}\{(1+\alpha^\beta)\left(1-e^{-\lambda y^\beta}\right)+e^{-\lambda y^\beta(1+\alpha^\beta)}-1\right]^{a-1} du \quad (7)$$

$$u = \frac{1}{\alpha^\beta}\{(1+\alpha^\beta)\left(1-e^{-\lambda y^\beta}\right)+e^{-\lambda y^\beta(1+\alpha^\beta)}-1$$

Now, (5) can becomes

$$f_{EWW}{}^{(y)} = a[U]^{a-1}\frac{du}{dy} \quad (8)$$

a) Cumulative Density Function (cdf)

The probability density function (pdf) of $EWW(a, \lambda, \beta, \alpha)$ given in (7), then expression (7) can be written as

$$F_{EWW}{}^{(y)} = P(Y \le y) = \int_0^y f(u)du$$

$$= \int_0^y a\left[\frac{1}{\alpha^\beta}\{(1+\alpha^\beta)\left(1-e^{-\lambda y^\beta}\right)+e^{-\lambda y^\beta(1+\alpha^\beta)}-1\right]^{a-1}\frac{\lambda\beta(1+\alpha^\beta)y^{\beta-1}e^{-\lambda y^\beta}(1-e^{-\lambda(\alpha y)^\beta})}{\alpha^\beta} du \quad (9)$$

$$F_{EWW}{}^{(y)} = P(Y \le y) = \int_0^y a[U]^{a-1}\, du$$

$= a\int_0^y [U]^{a-1}\, du$ and the cdf is obtained as

$$F_{EWW}{}^{(y)} = -(y)^a \tag{10}$$

b) The Survival Rate Function

The survival rate function of the Exponentiated-weighted weibull distribution is given by $S_{EWW}(y) = 1 - F_{EWW}(y) = 1 - \int_0^y f(u)du$

$$= 1 - a\int_0^y [U]^{a-1}\, du = 1 - [\,-\,(y)^a]$$

$$S_{EWW}(y) = 1 + (y)^a \tag{11}$$

c) The Hazard Rate Function

The hazard rate function of a random variable y with the pdf and cdf is defined by

$$h_{EWW}(y) = \frac{f_{EWW}(y)}{1 - F_{EWW}(y)}$$

Hence, the $EWW(a, \lambda, \beta, \alpha)$ with $f_{EWW}(y)$ and $F_{EWW}(y)$ respectively defined in (4) and (10), the hazard rate function can be expressed as:

$$= \frac{a(u)^{a-1}u'}{S_{EWW}(y)} \tag{12}$$

where U is expression in (5)

To show that $\lim_{y\to\infty} h_{EWW}(y) = 0$ *and* $\lim_{y\to 0} h_{EWW}(y) = 0$, we have the following

$$\lim_{y\to\infty} h_{EWW}(y) = \lim_{y\to\infty} \frac{a(u)^{a-1}u'}{1-(1+y)^a}$$

where, $u' = \frac{1}{\alpha^\beta}\{(1+\alpha^\beta)\left(1-e^{-\lambda y^\beta}\right) + e^{-\lambda y^\beta(1+\alpha^\beta)} - 1$

$$= \lim_{y\to\infty} \frac{a\left[\frac{1}{\alpha^\beta}\{(1+\alpha^\beta)\left(1-e^{-\lambda y^\beta}\right)+e^{-\lambda y^\beta(1+\alpha^\beta)}-1\right]^{a-1} \frac{\lambda\beta(1+\alpha^\beta)y^{\beta-1}e^{-\lambda y^\beta}(1-e^{-\lambda(\alpha y)^\beta})}{\alpha^\beta}}{1-(1+x)^a}$$

For simplification on the rigorous mathematics, we take the limit of the following:

When $y \to \infty = 0$ *and* $y \to 0 = 0$

$$= \lim_{y\to\infty} \frac{\lambda\beta(1+\alpha^\beta)y^{\beta-1}e^{-\lambda y^\beta}(1-e^{-\lambda(\alpha y)^\beta})}{\alpha^\beta} \quad = \lim_{y\to\infty} \frac{\lambda\beta(1+\alpha^\beta)\infty^{\beta-1}e^{-\lambda\infty^\beta}(1-e^{-\lambda(\alpha\infty)^\beta})}{\alpha^\beta}$$

$$= 0$$

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$$= \overset{lim}{y \longrightarrow 0} \frac{\lambda\beta\,(1+\alpha^{\beta})0^{\beta-1}e^{-\lambda 0^{\beta}}\,(1-e^{-\lambda(\alpha 0)^{\beta}})}{\alpha^{\beta}} = 0$$

As $y \to \infty = 0$ *and* $y \to 0 = 0$, expression (12) above tends to ∞ *and* 0 and equal to zero.

d) Asymptotic Behaviours

Following the steps in hazard function above taken $\overset{lim}{y \longrightarrow \infty}\, f_{EWW}(y)$ *and* $\overset{lim}{y \longrightarrow 0}\, f_{EWW}(y)$ of the $EWW(a,\lambda,\beta,\alpha)$ distribution is investigated as follows. Now from expression (5),
we have

$$\overset{lim}{y \longrightarrow \infty} f_{EWW}(y) = a\left[\frac{[(1+\alpha^{\beta})\left(1-e^{-\lambda y^{\beta}}\right)+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}}{\alpha^{\beta}}\right]^{a-1} \frac{\lambda\beta(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}}$$

taking the limit

$$= \overset{lim}{y \longrightarrow \infty} \frac{\lambda\beta(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}} \quad = \overset{lim}{y \longrightarrow \infty} \frac{\lambda\beta(1+\alpha^{\beta})\infty^{\beta-1}e^{-\lambda\infty^{\beta}}(1-e^{-\lambda(\alpha\infty)^{\beta}})}{\alpha^{\beta}}$$

$$= 0$$

$$= \overset{lim}{y \longrightarrow 0} \frac{\lambda\beta\,(1+\alpha^{\beta})0^{\beta-1}e^{-\lambda 0^{\beta}}\,(1-e^{-\lambda(\alpha 0)^{\beta}})}{\alpha^{\beta}} = 0$$

From the above results as $y \to \infty = 0$ *and* $y \to 0 = 0$, this shows that the distribution has at least a mode.

III. Moments and Moment Generating Function

Hosking (1990) described in their paper that when a random variable following a generalized beta generated distribution i.e $y \sim GBG(f,a,c)$ *then* $\mu_r^{'} = E[F^{-1}K^{\frac{1}{c}}]^r$ where $K \sim B(a,1), c$ is a constant and $F^{-1}(y)$ is the inverse of CDF of the weighted weibull distribution, since $EWW(a,\lambda,\beta,\alpha)$ distribution is a special form when a=c=1.We then derive the moment generating function (mgf) of the proposed distribution $m(t) = E(e^{ty})$ and the general rth moment of a beta generated distribution is defined by

$$\mu_r^{'} = \frac{1}{B(a,1)}\int_0^1 [F^{-1}(y)]^r\,[y]^{a-1}\,du \tag{13}$$

Also, using the taylor series expansion around the point $E(y_f) = \mu_f$ to obtain

$$\mu_r^{'} = \sum_{u=0}^{r}\binom{r}{k}[F^{-1}(\mu)]^{r-k}\,[F^{-1(1)}(\mu_f)]^k \sum_{k=0}^{n}(-1)^i\binom{r}{i} \tag{14}$$

Cordeiro et al (2011) gave an alternative series expansion for $\mu_r^{'}$ in terms of $r(r,U) = E(U^r\,F(U)^Y)$ where k follows the parent distribution then for u = 0, 1, …

$$\mu_r^{'} = \frac{1}{B(a,1)}\sum_{i=0}^{\infty}(-1)^i\binom{b-1}{i}\,r(r,i-1)$$

They further described another mgf of y for generated beta distribution as

$$M(t) = \frac{1}{B(a,1)} \sum_{i=0}^{\infty} (-1)^i \binom{a-1}{i} \rho(t, i-1) \quad (15)$$

Where,

$$\rho(t,r) = \int_{-\infty}^{\infty} e^{ty} [F(y)]^m f(y)\, dy$$

Therefore,

$$M_y^{(t)} = \frac{1}{B(a,1)} \sum_{i=0}^{\infty} (-1)^i \binom{a-1}{i} \int_{-\infty}^{\infty} e^{ty} [F(y)]^{(i+1)-1} f(y)\, dy \quad (16)$$

Substituting both probability density and cumulative density function of the weighted weibull distribution into (16), we obtain

$$M_{EWW(y)}^{(t)} = a \sum_{i=0}^{\infty} (-1)^i \binom{a-1}{i} \int e^{ty} \left[\frac{1}{\alpha^\beta}\{(1+\alpha^\beta)\left(1-e^{-\lambda y^\beta}\right) + e^{-\lambda y^\beta (1+\alpha^\beta)} - 1\right]^{(i+1)-1} \frac{\lambda\beta(1+\alpha^\beta) y^{\beta-1} e^{-\lambda y^\beta}(1-e^{-\lambda(\alpha y)^\beta})}{\alpha^\beta}\, dy \quad (17)$$

Equation (17) becomes the mgf of Exponentiated-weighted weibull distribution. Then, setting a=1 and i = 0, the same expression (17) is reduced to becomes the parent distribution. To obtain the rth moment of $EWW(a, \lambda, \beta, \alpha)$, the weighted weibull distribution by Mahdy (2013) and is given by

$$M_{(y)}^{(t)} = \int_0^{\infty} e^{ty} \frac{\lambda\beta(1+\alpha^\beta) y^{\beta-1} e^{-\lambda y^\beta}\left(1-e^{-\lambda(\alpha y)^\beta}\right)}{\alpha^\beta}\, dy$$

$$= \sum_{j=0}^{\infty} \frac{t^j}{j!\alpha^\beta} \{\lambda^{-\frac{j}{\beta}} (1+\alpha^\beta) \Gamma(\tfrac{j+\beta}{\beta})(1-(1+\alpha^\beta))^{-\left(\frac{j+\beta}{\beta}\right)}\} \quad (18)$$

Equation (17) can be re-written as

$$M_{EWW(y)}^{(t)} = a \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (-1)^i \binom{a-1}{i} \frac{t^j}{j!\alpha^\beta} \{\lambda^{-\frac{j}{\beta}} (1+\alpha^\beta) \Gamma(\tfrac{j+\beta}{\beta})(1-(1+\alpha^\beta))^{-\left(\frac{j+\beta}{\beta}\right)}\} \quad (19)$$

The rth moment of the $EWW(a, \lambda, \beta, \alpha)$ distribution can also be written from equation (19) as

$$\mu_r' = E(Y^r) = a \sum_{i=0}^{\infty} (-1)^i \binom{a-1}{i} \frac{t^r}{r!\alpha^\beta} \{\lambda^{-\frac{r}{\beta}} (1+\alpha^\beta) \Gamma(\tfrac{r+\beta}{\beta})(1-(1+\alpha^\beta))^{-\left(\frac{r+\beta}{\beta}\right)}\} \quad (20)$$

Again, putting a = 1 in expression (20) leads to the rth moment of the weighted weibull model by Mahdy (2013) and is given by

$$\mu_r' = E(Y^r) = \frac{1}{\alpha^\beta} \{\lambda^{-\frac{r}{\beta}} (1+\alpha^\beta) \Gamma(\tfrac{r+\beta}{\beta})(1-(1+\alpha^\beta))^{-\left(\frac{r+\beta}{\beta}\right)}\} \quad (21)$$

$$\mu_{EWW(r)}' = a \sum_{i=0}^{\infty} (-1)^i \binom{a-1}{i} \frac{r}{\alpha^\beta} \{\lambda^{-\frac{1}{\beta}} (1+\alpha^\beta) \Gamma(\tfrac{r+\beta}{\beta})(1-(1+\alpha^\beta))^{-\left(\frac{r+\beta}{\beta}\right)}\} \quad (22)$$

Notes

From (22), it is easy to obtain the first and the second mean about the origin e.g when r = 1and the second moment when r = 2, etc.
The first moment of $EWW(a,\lambda,\beta,\alpha)$ is obtain

$$\mu'_{EWW(1)} = a\sum_{i=0}^{\infty}(-1)^i\binom{a-1}{i}\frac{1}{\alpha^\beta}\{\lambda^{-\frac{1}{\beta}}(1+\alpha^\beta)\Gamma(\frac{1+\beta}{\beta})(1-(1+\alpha^\beta))^{-\left(\frac{1+\beta}{\beta}\right)}\} \tag{23}$$

Notes

The second moment can also be obtained as follows:

$$\mu'_2 = V(a,1,\lambda,\beta,\alpha)^{(y)} = V_1 - V_2 \tag{24}$$

where,

$$V_1 = E(a,\lambda,\beta,\alpha)^{(Y^2)} = a\sum_{i=0}^{\infty}(-1)^i\binom{a-1}{i}\frac{1}{\alpha^\beta}\{\lambda^{-\frac{2}{\beta}}(1+\alpha^\beta)\Gamma(\frac{2+\beta}{\beta})(1-(1+\alpha^\beta))^{-\left(\frac{2+\beta}{\beta}\right)}\}$$

$$V_2 = E(a,\lambda,\beta,\alpha)^{(Y)} = (a\sum_{i=0}^{\infty}(-1)^i\binom{a-1}{i}\frac{1}{\alpha^\beta}\{\lambda^{-\frac{2}{\beta}}(1+\alpha^\beta)\Gamma\left(\frac{1+\beta}{\beta}\right)\left(1-(1+\alpha^\beta)\right)^{-\left(\frac{1+\beta}{\beta}\right)}\})^2$$

Likewise, the standard deviation is given by

$$SD_{EWW}.(a,\lambda,\beta,\alpha)^{(Y)} = \sqrt{V_1 - V_2}$$

IV. Estimation of Parameter

We show the maximum likelihood estimate (MLEs) of the parameter of $EWW(a,\lambda,\beta,\alpha)$ distribution mathematically following Cordeiro et al (2011) and Shittu and Adepoju (2013) studied on the log-likelihood function for $\omega = (a,c,\varphi)$, where $\varphi = (\lambda,\beta,\alpha)$ and setting ω to be a vector of parameter and is given by

$$L(\omega) = nlogc - nlog[B(a,1)] + \sum_{i=1}^{n}\log[f(y;\varphi)] + (a-1)\sum_{i=1}^{n}\log[F(y;\varphi)] \tag{25}$$

Note that, b = c = 1 (24) becomes $\theta = (a,\varphi)$

$$L(\omega) = -nlog(a,1) + \sum_{i=1}^{n}\log[f(y;\varphi)] + (a-1)\sum_{i=1}^{n}\log[F(y;\varphi)] \tag{26}$$

where, $f(y;\varphi) = \frac{\lambda\beta(1+\alpha^\beta)y^{\beta-1}e^{-\lambda y^\beta}(1-e^{-\lambda(\alpha y)^\beta})}{\alpha^\beta}$ and

$$F(y;\varphi) = \frac{1}{\alpha^\beta}\{(1+\alpha^\beta)\left(1-e^{-\lambda y^\beta}\right) + e^{-\lambda y^\beta(1+\alpha^\beta)} - 1$$

$$L_{EWW}^{(\omega)} = -nlog(a,1) + \sum_{i=1}^{n} \log\left[\frac{\lambda\beta\,(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}\,(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}}\right] + (a-1)\sum_{i=1}^{n}\log\left[\frac{1}{\alpha^{\beta}}\{\left(1+\alpha^{\beta}\right)\left(1-e^{-\lambda y^{\beta}}\right) + e^{-\lambda y^{\beta}(1+\alpha^{\beta})} - 1\right] \quad (27)$$

For determining the MLE of a,λ,β,α, we take the partial derivative of the (27) with respect to (a,λ,β,α) as follows:

$$\frac{L_{EWW}^{(\omega)}}{\partial a} = -nlog(a,1) + (a-1)\sum_{y=1}^{n}\log\left[\frac{1}{\alpha^{\beta}}\{\left(1+\alpha^{\beta}\right)\left(1-e^{-\lambda y^{\beta}}\right) + e^{-\lambda y^{\beta}(1+\alpha^{\beta})} - 1\right] \quad (28)$$

$$\frac{L_{EWW}^{(\omega)}}{\partial \lambda} = \sum_{y=1}^{n} log\left[\frac{\frac{\partial}{\partial\lambda}\frac{\lambda\beta\,(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}\,(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}}}{\frac{\lambda\beta\,(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}\,(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}}}\right] +$$

$$(a-1)\sum_{y=1}^{n}\log\left[\frac{\frac{\partial}{\partial\lambda}\left(\frac{1}{\alpha^{\beta}}\{(1+\alpha^{\beta})(1-e^{-\lambda y^{\beta}})+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1\right)}{\frac{1}{\alpha^{\beta}}\{(1+\alpha^{\beta})(1-e^{-\lambda y^{\beta}})+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1}\right] \quad (29)$$

$$\frac{L_{EWW}^{(\omega)}}{\partial \beta} = \sum_{y=1}^{n} log\left[\frac{\frac{\partial}{\partial\beta}\frac{\lambda\beta\,(1+\alpha^{\beta})y\,e^{-\lambda y^{\beta}}\,(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}}}{\frac{\lambda\beta\,(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}\,(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}}}\right] +$$

$$(a-1)\sum_{y=1}^{n}\log\left[\frac{\frac{\partial}{\partial\beta}\left(1-\frac{1}{\alpha^{\beta}}\{(1+\alpha^{\beta})(1-e^{-\lambda y})+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1\right)}{\frac{1}{\alpha^{\beta}}\{(1+\alpha^{\beta})(1-e^{-\lambda y^{\beta}})+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1}\right] \quad (30)$$

$$\frac{L_{EWW}^{(\omega)}}{\partial \alpha} = \sum_{y=1}^{n} log\left[\frac{\frac{\partial}{\partial\alpha}\frac{\lambda\beta\,(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}\,(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}}}{\frac{\lambda\beta\,(1+\alpha^{\beta})y^{\beta-1}e^{-\lambda y^{\beta}}\,(1-e^{-\lambda(\alpha y)^{\beta}})}{\alpha^{\beta}}}\right] +$$

$$(a-1)\sum_{y=1}^{n}\log\left[\frac{\frac{\partial}{\partial\alpha}\left(1-\frac{1}{\alpha^{\beta}}\{(1+\alpha^{\beta})(1-e^{-\lambda y^{\beta}})+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1\right)}{\frac{1}{\alpha^{\beta}}\{(1+\alpha^{\beta})(1-e^{-\lambda y^{\beta}})+e^{-\lambda y^{\beta}(1+\alpha^{\beta})}-1}\right] \quad (31)$$

The equations derive above can also be solved using iteration method (Newton Raphson) to obtain the $\hat{a},\hat{\lambda},\hat{\beta},\hat{\alpha}$ the MLE of (a,λ,β,α) respectively.

Taking second derivatives of the said equations 28, 29, 30 and 31 with respect to the parameters above, it is possible to derive the interval estimate and hypothesis tests on the model parameter. This may be shown in further research.

V. Real Data Set

The data used in this section was studied by Lemonte in his BJPS Accepted Manuscript on the remission times (in months) of a random sample of 128 bladder cancer patients reported in Lee and Wang (2003) to compare and contrast the Exponentiated Weighted Weibull and Weighted Weibull distribution.

Notes

Notes

R (code) software is used to determine the maximum likelihood estimates and the log-likelihood for the Exponentiated Weighted Weibull distribution are: $\hat{\alpha} = 8.95941,\ \hat{\lambda} = 8.17089,\ \hat{\beta} = 8.36111,\ \hat{\alpha} = 7.91621\ and\ log_{EWW} = 448.5781$ while the maximum likelihood estimates and the log-likelihood for the Weighted Weibull distribution are: $\hat{\lambda} = 7.90434,\ \ \hat{\beta} = 8.44282,\ \hat{\alpha} = 8.70033$ and $log_{WW} = 448.4913$, where $lg_{LWW}\ and\ lg_{WW}$ denote log-likelihood of both Exponentiated Weighted Weibull distribution and Weighted Weibull distribution.

$$\begin{pmatrix} 0.802169648 & -0.02880174 & 0.002221849 & -0.02674447 \\ -0.028801742 & 0.65433442 & -0.025345533 & -0.02069611 \\ 0.002221849 & -0.02534553 & 0.705642669 & -0.02353514 \\ -0.026744474 & -0.02069611 & -0.023535137 & 0.60907454 \end{pmatrix}$$

VI. Conclusion

We investigated on the statistical properties of the proposed distribution e.g moments, moment generating function, estimation of parameters using R (Code) software for data analysis presented in this article. We also upgraded with an additional parameter to the existing three parameters in the weighted weibull distribution and the results from the estimated parameters show that the Exponentiated Weighted Weibull distribution has a better representation of data than weighted weibull distribution.

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Notes

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A Summation Formula Involving Certain Special Functions

By Salahuddin & Intazar Husain
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Abstract- The main aim of the present paper is to compute a summation formula involving Recurrence relation and Contiguous relation.

Keywords: *gauss second summation theorem, recurrence relation, prudnikov.*

GJSFR-F Classification : *MSC 2010: 33C05 , 33C20 , 33C45 ,33D50 ,33D60*

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A Summation Formula Involving Certain Special Functions

Salahuddin α & Intazar Husain σ

Abstract- The main aim of the present paper is to compute a summation formula involving Recurrence relation and Contiguous relation.

Keywords : *gauss second summation theorem, recurrence relation, prudnikov.*

I. Introduction

Generalized Gaussian Hypergeometric function of one variable is defined by

$$ {}_AF_B\left[\begin{array}{ccc} a_1, a_2, \cdots, a_A & ; & \\ & & z \\ b_1, b_2, \cdots, b_B & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!} \tag{1} $$

where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and $A,\ B$ are non-negative integers and $\mid z \mid = 1$

Contiguous Relation is defined by

[Andrews p.363(9.16), E. D. p.51(10)]

$$ (a-b)\ {}_2F_1\left[\begin{array}{ccc} a,\ b\ ; & & \\ c & ; & z \end{array}\right] = a\ {}_2F_1\left[\begin{array}{ccc} a+1,\ b & ; & \\ c & ; & z \end{array}\right] - b\ {}_2F_1\left[\begin{array}{ccc} a,\ b+1 & ; & \\ c\ ; & & z \end{array}\right] \tag{2} $$

Gauss second summation theorem is defined by [Prudnikov., 491(7.3.7.5)]

$$ {}_2F_1\left[\begin{array}{ccc} a,\ b & ; & \\ \frac{a+b+1}{2} & ; & \frac{1}{2} \end{array}\right] = \frac{\Gamma(\frac{a+b+1}{2})\ \Gamma(\frac{1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} \tag{3} $$

$$ = \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{a+b+1}{2})}{\Gamma(b)\ \Gamma(\frac{a+1}{2})} \tag{4} $$

In a monograph of Prudnikov et al., a summation theorem is given in the form [Prudnikov., p.491(7.3.7.8)]

$$ {}_2F_1\left[\begin{array}{ccc} a,\ b & ; & \\ \frac{a+b-1}{2} & ; & \frac{1}{2} \end{array}\right] = \sqrt{\pi}\left[\frac{\Gamma(\frac{a+b+1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} + \frac{2\ \Gamma(\frac{a+b-1}{2})}{\Gamma(a)\ \Gamma(b)}\right] \tag{5} $$

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Now using Legendre's duplication formula and Recurrence relation for Gamma function, the above theorem can be written in the form

$$ {}_2F_1\left[\begin{matrix} a, \ b & ; & \frac{1}{2} \\ \frac{a+b-1}{2} & ; & \end{matrix}\right] = \frac{2^{(b-1)}\,\Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} + \frac{2^{(a-b+1)}\,\Gamma(\frac{a}{2})\,\Gamma(\frac{a+1}{2})}{\{\Gamma(a)\}^2} + \frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\right] \tag{6} $$

Notes

Recurrence relation is defined by

$$ \Gamma(\zeta+1) = \zeta\,\Gamma(\zeta) \tag{7} $$

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II. Main Summation Formula

$$ {}_2F_1\left[\begin{matrix} a, \ b & ; & \frac{1}{2} \\ \frac{a+b+45}{2} & ; & \end{matrix}\right] = $$

$$ = \frac{2^b\,\Gamma(\frac{a+b+45}{2})}{(a-b)\,\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+1}{2})}\left\{\frac{1}{\left[\prod_{\zeta=1}^{21}\{a-b-(2\zeta-1)\}\right]\left[\prod_{\eta=1}^{22}\{a-b+(2\eta-1)\}\right]}\times\right.\right. $$

$$ \times\Big(2097152a(-131130704576879886034406225 + 32835960506324395192397625a $$

$$ -33100902479081461753011150a^2 + 18852771841456100048901630a^3 $$

$$ -6998152417792503243516261a^4 + 1830981648088050432124941a^5 $$

$$ -354501246907809948405480a^6 + 52462262753503539155240a^7 - 6068045552614043865426a^8 $$

$$ +557078686778195952226a^9 - 41014337504927410260a^{10} + 2436777594647459700a^{11} $$

$$ -117140064945846306a^{12} + 4552422878866226a^{13} - 142380641710440a^{14} + 3551773672360a^{15} $$

$$ -69638530941a^{16} + 1048818981a^{17} - 11702670a^{18} + 91070a^{19} - 441a^{20} + a^{21} $$

$$ +537459628826206088931678751b - 450537436682222240043441100ab $$

$$ +113826458136107026075141710a^2b - 32581896347358559999317420a^3b $$

$$ +2221353788317106815143953 1a^4b - 343788182112078258282945 6a^5b $$

$$ +106951340379166584535712 8a^6b - 102017962030642073210480a^7b $$

$$ +17605238308074264053574a^8b - 1094032303287108057912a^9b + 114934366191849037236a^{10}b $$

$$ -474270953130853476 0a^{11}b + 317038686494337262a^{12}b - 8612153544899408a^{13}b $$

$$ +370669544999304a^{14}b - 6357587526960a^{15}b + 172733256855a^{16}b - 1689533124a^{17}b + 27096622a^{18}b $$

$$ -114380a^{19}b + 903a^{20}b + 341874092984654036300445 0b^2 + 152628207170808204647275770ab^2 $$

$$ -296620575380129998953385 74a^2b^2 + 739246794890589180460159 38a^3b^2 $$

$$ -909526173243039232365717 6a^4b^2 + 647504538210537533588948 0a^5b^2 $$

$$ -523071699408050859929016a^6b^2 + 17124294235130363291053 6a^7b^2 $$

Notes

$$
\begin{aligned}
&-9336842670271654008484a^{8}b^{2} + 1689474770765990596076a^{9}b^{2} - 62588562419040481252a^{10}b^{2} \\
&+6843044908448608444a^{11}b^{2} - 169609751776172344a^{12}b^{2} + 11684556963374568a^{13}b^{2} \\
&-185095028637272a^{14}b^{2} + 8105951624872a^{15}b^{2} - 73945230446a^{16}b^{2} + 2006403306a^{17}b^{2} \\
&-7972286a^{18}b^{2} + 123410a^{19}b^{2} + 49302844553810665888575930b^{3} + 51962354398327511515172788ab^{3} \\
&+9173580472480782112 1609526a^{2}b^{3} - 6747725269089701729734368a^{3}b^{3} \\
&+1643842734922932898254589 6a^{4}b^{3} - 1038876114726270005445776a^{5}b^{3} \\
&+735240415476876038312632a^{6}b^{3} - 342418206975970887228 48a^{7}b^{3} \\
&+1122618694511849293985 2a^{8}b^{3} - 3710784022919059612 24a^{9}b^{3} + 67240314521518855860a^{10}b^{3} \\
&-1526743180734652192a^{11}b^{3} + 166445736776215624a^{12}b^{3} - 2459287660143888a^{13}b^{3} \\
&+167607787407128a^{14}b^{3} - 1445744454752a^{15}b^{3} + 61843300298a^{16}b^{3} - 235148940a^{17}b^{3} \\
&+6096454a^{18}b^{3} + 6523406582853698567485227b^{4} + 4368975768739841479054755 3ab^{4} \\
&+185819173756527654667236 0a^{2}b^{4} + 1956654920038543110397221 6a^{3}b^{4} \\
&-673581690360746582755756a^{4}b^{4} + 1596002262124887064872892a^{5}b^{4} \\
&-568646128827746268317 20a^{6}b^{4} + 390964730462947230209 36a^{7}b^{4} - 10996496652426170007 98a^{8}b^{4} \\
&+352022359346404662630a^{9}b^{4} - 715717976953844 8936a^{10}b^{4} + 1266877992724507592a^{11}b^{4} \\
&-17271058034689004a^{12}b^{4} + 1832663509135228a^{13}b^{4} - 14882731838680a^{14}b^{4} + 979484169144a^{15}b^{4} \\
&-3560764597a^{16}b^{4} + 145008513a^{17}b^{4} + 641298227578144236953444 7b^{5} \\
&+3020391903873160874624880ab^{5} + 1107254450671742504094332 0a^{2}b^{5} \\
&+251409502832929940580848a^{3}b^{5} + 1850900598832213039588308a^{4}b^{5} \\
&-333621835286836961431 84a^{5}b^{5} + 766351440781327984848 72a^{6}b^{5} - 161410710324278038395 2a^{7}b^{5} \\
&+106460057466283361383 4a^{8}b^{5} - 18285405023328014000a^{9}b^{5} + 5631584078513951272a^{10}b^{5} \\
&-68669517587706672a^{11}b^{5} + 11692563911936180a^{12}b^{5} - 87796234586480a^{13}b^{5} + 8925930851320a^{14}b^{5} \\
&-30677356528a^{15}b^{5} + 1917334783a^{16}b^{5} + 604573745934401191005912b^{6} \\
&+276153536122183796513666 4ab^{6} + 446409836411046907751368a^{2}b^{6} \\
&+1155941471891438860601896a^{3}b^{6} + 1562421487601667973935 2a^{4}b^{6} \\
&+873136801485651501118 32a^{5}b^{6} - 874649607643712372472a^{6}b^{6} + 194311571651293977680 8a^{7}b^{6} \\
&-2452088236644689042 4a^{8}b^{6} + 1539602176685380007 2a^{9}b^{6} - 157355447527654376a^{10}b^{6} \\
&+46179770718580408a^{11}b^{6} - 309271965286648a^{12}b^{6} + 50125620668424a^{13}b^{6} - 159286274280a^{14}b^{6} \\
&+15338678264a^{15}b^{6} + 231724339078175706150136b^{7} + 153811376527605888117264ab^{7} \\
&+374921775760059309890200a^{2}b^{7} + 288313472542322163984 32a^{3}b^{7} \\
&+581185453177003354134 00a^{4}b^{7} + 490468210861184892336a^{5}b^{7} + 218585142125509659500 0a^{6}b^{7} \\
&-124382363133364336 00a^{7}b^{7} + 2667252758559901168 8a^{8}b^{7} - 196990298922392976a^{9}b^{7}
\end{aligned}
$$

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$$+117137843987075912a^{10}b^{7}-652631982303392a^{11}b^{7}+181277011099912a^{12}b^{7}-512075874352a^{13}b^{7}$$
$$+78378960360a^{14}b^{7}+1512893281338087531211 0b^{8}+5688148102699391131057 8ab^{8}$$
$$+13039358045721374943420a^{2}b^{8}+22139104917821671572676a^{3}b^{8}+9214719430968641392 66a^{4}b^{8}$$
$$+15208787678367586162 86a^{5}b^{8}+809504348744234343 2a^{6}b^{8}+297209013764764369 84a^{7}b^{8}$$
$$-944270808969834 06a^{8}b^{8}+19511344911362561 4a^{9}b^{8}-774287072717444a^{10}b^{8}$$
$$+43441105228 2116a^{11}b^{8}-1013931747010a^{12}b^{8}+265182149218a^{13}b^{8}+30240089608763204202 46b^{9}$$
$$+23324084734017336985 04ab^{9}+45694770086352136541 48a^{2}b^{9}+49267825418369635448 8a^{3}b^{9}$$
$$+64594553721844467664 2a^{4}b^{9}+15329149883995675728a^{5}b^{9}+2135754030064486031 2a^{6}b^{9}$$
$$+69758491996612848a^{7}b^{9}+215842089833789562a^{8}b^{9}-353214178214040a^{9}b^{9}$$
$$+702101231167908a^{10}b^{9}-1152680301864a^{11}b^{9}+608359048206a^{12}b^{9}+1371048711540959426 04b^{10}$$
$$+4542220139072028439 00ab^{10}+120615054891875887772a^{2}b^{10}+161949836634482706524a^{3}b^{10}$$
$$+9180121926490903960a^{4}b^{10}+980721663952288658 4a^{5}b^{10}+132405723186144408a^{6}b^{10}$$
$$+158910354362756568a^{7}b^{10}+291326223911580a^{8}b^{10}+772341543530700a^{9}b^{10}$$
$$-503154100020a^{10}b^{10}+960566918220a^{11}b^{10}+161116280063842269532b^{11}$$
$$+132665240099341704 56ab^{11}+224686304942925367 88a^{2}b^{11}+274465411043053692 8a^{3}b^{11}$$
$$+281282920956346389 6a^{4}b^{11}+85999358210064208a^{5}b^{11}+7733116772117245 6a^{6}b^{11}$$
$$+552383916933472a^{7}b^{11}+579276741224844a^{8}b^{11}+457412818200a^{9}b^{11}+1052049481860a^{10}b^{11}$$
$$+504464808447965118b^{12}+1520074513892383306ab^{12}+422390481555014024a^{2}b^{12}$$
$$+48343181524244495 2a^{3}b^{12}+29546096072985492a^{4}b^{12}+2445838570812182 0a^{5}b^{12}$$
$$+381004267695688a^{6}b^{12}+294616546783832a^{7}b^{12}+864510226398a^{8}b^{12}+800472431850a^{9}b^{12}$$
$$+36842919196739110b^{13}+30556991340393392ab^{13}+46327319945209016a^{2}b^{13}$$
$$+5648934122340208a^{3}b^{13}+4881179381635732a^{4}b^{13}+144878605202576a^{5}b^{13}$$
$$+100261443987096a^{6}b^{13}+623957998160a^{7}b^{13}+421171648758a^{8}b^{13}+777692306266456b^{14}$$
$$+2164283489573848ab^{14}+578915870956584a^{2}b^{14}+585771144020552a^{3}b^{14}+32252520795880a^{4}b^{14}$$
$$+22313057280808a^{5}b^{14}+256037937176a^{6}b^{14}+151532656696a^{7}b^{14}+35857948972408b^{15}$$
$$+28280942023504ab^{1}5+39037774410808a^{2}b^{15}+4143128700320a^{3}b^{15}+3132630060840a^{4}b^{15}$$
$$+63714509712a^{5}b^{15}+36576848168a^{6}b^{15}+484185248451b^{16}+1250881854197ab^{16}$$
$$+288251667106a^{2}b^{16}+262400960302a^{3}b^{16}+9586673915a^{4}b^{16}+5752004349a^{5}b^{16}+13811946279b^{17}$$
$$+9489432124ab^{17}+11986499486a^{2}b^{17}+837826964a^{3}b^{17}+563921995a^{4}b^{17}+106564578b^{18}$$
$$+254376218ab^{18}+39191490a^{2}b^{18}+32224114a^{3}b^{18}+1743994b^{19}+839188ab^{19}+962598a^{2}b^{19}$$
$$+5719b^{20}+12341ab^{20}+43b^{21})\Big)+\frac{1}{\left[\prod\limits_{\mu=1}^{22}\{a-b-(2\mu-1)\}\right]\left[\prod\limits_{\xi=1}^{21}\{a-b+(2\xi-1)\}\right]}\times$$

Notes

$$\Big(2097152b(-1311307045768798860344062 5+ 5374596288262060889316787 5a$$

$$+3418740929846540363004450a^2 + 4930284455381066588857593 0a^3$$

$$+6523406582853698567485227a^4+6412982275781442369534447a^5+6045737459344011910059 12a^6$$

$$+2317243390781757061501 36a^7 + 1512893281338087531211 0a^8 + 3024008960876320420246a^9$$

$$+137104871154095942604a^{10} + 16116280063842269532a^{11} + 504464808447965118a^{12}$$

$$+36842919196739110a^{13} + 777692306266456a^{14} + 35857948972408a^{15} + 484185248451a^{16}$$

$$+13811946279a^{17}+106564578a^{18}+1743994a^{19}+5719a^{20}+43a^{21}+328359605063243951923976 25b$$

$$-4505374366822222400434410 0ab + 1526282071708082046472757 70a^2b$$

$$+5196235439832751151572788a^3b + 4368975768739841479054755 3a^4b$$

$$+3020391903873160874624880a^5b + 2761535361221837965136664a^6b$$

$$+1538113765276058881172 64a^7b+568814810269939113105 78a^8b+233240847340173369850 4a^9b$$

$$+4542220139072028439 00a^{10}b + 1326652400993417045 6a^{11}b + 1520074513892383306a^{12}b$$

$$+30556991340393392a^{13}b + 2164283489573848a^{14}b + 28280942023504a^{15}b + 1250881854197a^{16}b$$

$$+9489432124a^{17}b + 254376218a^{18}b + 839188a^{19}b + 12341a^{20}b - 3310090247908146175301115 0b^2$$

$$+11382645813610702607514171 0ab^2 - 2966205753801299989533857 4a^2b^2$$

$$+9173580472480782112160952 6a^3b^2 + 1858191737565276546672360a^4b^2$$

$$+1107254450671742504094332 0a^5b^2 + 446409836411046907751368a^6b^2$$

$$+3749217757600593098902 00a^7b^2+1303935804572137494342 0a^8b^2+4569477008635213654148a^9b^2$$

$$+1206150548918758877 72a^{10}b^2 + 224686304942925367 88a^{11}b^2 + 4223904815550140 24a^{12}b^2$$

$$+463273199452090 16a^{13}b^2 + 578915870956584a^{14}b^2 + 39037774410808a^{15}b^2 + 288251667106a^{16}b^2$$

$$+11986499486a^{17}b^2 + 39191490a^{18}b^2 + 962598a^{19}b^2 + 1885277184145610004890163 0b^3$$

$$-3258189634735855999931742 0ab^3 + 7392467948905891804601593 8a^2b^3$$

$$-6747725269089701729734368a^3b^3 + 1956654920038543110397221 6a^4b^3$$

$$+2514095028329299405808 48a^5b^3 + 1155941471891438860601896a^6b^3$$

$$+2883134725423221639843 2a^7b^3+2213910491782167157267 6a^8b^3+4926782541836963544 88a^9b^3$$

$$+1619498366344827065 24a^{10}b^3 + 274465411043053692 8a^{11}b^3 + 4834318152424449 52a^{12}b^3$$

$$+5648934122340208a^{13}b^3 + 585771144020552a^{14}b^3 + 4143128700320a^{15}b^3 + 262400960302a^{16}b^3$$

$$+837826964a^{17}b^3 + 32224114a^{18}b^3 - 6998152417792503243516261b^4$$

$$+2221353788317106815143953 1ab^4 - 9095261732430392323657176a^2b^4$$

$$+1643842734922932898254589 6a^3b^4 - 6735816903607465827557 56a^4b^4$$

$$+1850900598832213039588308a^5b^4 + 156242148760166797393 52a^6b^4$$

$$+5811854531770033541340 0a^7b^4 + 921471943096864139266a^8b^4 + 64594553721844467664 2a^9b^4$$

$$+9180121926490903960a^{10}b^{4} + 2812829209563463896a^{11}b^{4} + 295460960729854 92a^{12}b^{4}$$

$$+4881179381635732a^{13}b^{4} + 32252520795880a^{14}b^{4} + 3132630060840a^{15}b^{4} + 9586673915a^{16}b^{4}$$

$$+563921995a^{17}b^{4} + 18309816480880504321249 41b^{5} - 3437881821120782582829456ab^{5}$$

$$+6475045382105375335889480a^{2}b^{5} - 10388761147262700054457 76a^{3}b^{5}$$

$$+1596002262124887064872892a^{4}b^{5} - 333621835286836961431 84a^{5}b^{5}$$

$$+873136801485651501118 32a^{6}b^{5} + 490468210861184892336a^{7}b^{5} + 152087876783675861628 6a^{8}b^{5}$$

$$+1532914988399567572 8a^{9}b^{5} + 980721663952288658 4a^{10}b^{5} + 85999358210064208a^{11}b^{5}$$

$$+2445838570812182 0a^{12}b^{5} + 144878605202576a^{13}b^{5} + 22313057280808a^{14}b^{5} + 63714509712a^{15}b^{5}$$

$$+5752004349a^{16}b^{5} - 354501246907809948405480b^{6} + 1069513403791665845357128ab^{6}$$

$$-5230716994080508599290 16a^{2}b^{6} + 735240415476876038312632a^{3}b^{6}$$

$$-568646128827746268317 20a^{4}b^{6} + 766351440781327984848 72a^{5}b^{6} - 874649607643712372472a^{6}b^{6}$$

$$+2185851421255096595000a^{7}b^{6} + 80950434874423434 32a^{8}b^{6} + 213575403006448603 12a^{9}b^{6}$$

$$+1324057231861444 08a^{10}b^{6} + 77331167721172456a^{11}b^{6} + 381004267695688a^{12}b^{6}$$

$$+100261443987096a^{13}b^{6} + 256037937176a^{14}b^{6} + 36576848168a^{15}b^{6} + 5246226275350353915524 0b^{7}$$

$$-102017962030642073210480ab^{7} + 171242942351303632910536a^{2}b^{7}$$

$$-342418206975970887228 48a^{3}b^{7} + 390964730462947230209 36a^{4}b^{7} - 161410710324278038395 2a^{5}b^{7}$$

$$+1943115716512939776808a^{6}b^{7} - 1243823631333643360 0a^{7}b^{7} + 297209013764764369 84a^{8}b^{7}$$

$$+697584919966128 48a^{9}b^{7} + 158910354362756568a^{10}b^{7} + 552383916933472a^{11}b^{7}$$

$$+294616546783832a^{12}b^{7} + 623957998160a^{13}b^{7} + 151532656696a^{14}b^{7} - 6068045552614043865426b^{8}$$

$$+176052383080742640535 74ab^{8} - 933684267027165400848 4a^{2}b^{8} + 112261869451184929398 52a^{3}b^{8}$$

$$-1099649665242617000798a^{4}b^{8} + 106460057466283361383 4a^{5}b^{8} - 2452088236644689042 4a^{6}b^{8}$$

$$+2667252758559901168 8a^{7}b^{8} - 944270808969834 06a^{8}b^{8} + 215842089833789562a^{9}b^{8}$$

$$+291326223911580a^{10}b^{8} + 579276741224844a^{11}b^{8} + 864510226398a^{12}b^{8} + 421171648758a^{13}b^{8}$$

$$+557078686778195952226b^{9} - 1094032303287108057912ab^{9} + 1689474770765990596076a^{2}b^{9}$$

$$-3710784022919059612 24a^{3}b^{9} + 352022359346404662630a^{4}b^{9} - 182854050233280140 00a^{5}b^{9}$$

$$+1539602176685380007 2a^{6}b^{9} - 196990298922392976a^{7}b^{9} + 195113449113625614a^{8}b^{9}$$

$$-353214178214040a^{9}b^{9} + 772341543530700a^{10}b^{9} + 457412818200a^{11}b^{9} + 800472431850a^{12}b^{9}$$

$$-410143375049274102 60b^{10} + 1149343661918490372 36ab^{10} - 625885624190404812 52a^{2}b^{10}$$

$$+6724031452151885586 0a^{3}b^{10} - 715717976953844893 6a^{4}b^{10} + 5631584078513951272a^{5}b^{10}$$

$$-157355447527654376a^{6}b^{10} + 117137843987075912a^{7}b^{10} - 774287072717444a^{8}b^{10}$$

$$+702101231167908a^{9}b^{10} - 503154100020a^{1}0b^{10} + 1052049481860a^{11}b^{10} + 2436777594647459700b^{11}$$

$$-4742709531308534760ab^{11} + 6843044908448608444a^{2}b^{11} - 1526743180734652192a^{3}b^{11}$$

Notes

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$$+1266877992724507592a^4b^{11} - 686695175877066672a^5b^{11} + 46179770718580408a^6b^{11}$$
$$-652631982303392a^7b^{11} + 434411052282116a^8b^{11} - 1152680301864a^9b^{11} + 960566918220a^{10}b^{11}$$
$$-1171400649458463306b^{12} + 317038686494337262ab^{12} - 169609751776172344a^2b^{12}$$
$$+166445736776215624a^3b^{12} - 17271058034689004a^4b^{12} + 11692563911936180a^5b^{12}$$
$$-309271965286648a^6b^{12} + 181277011099912a^7b^{12} - 1013931747010a^8b^{12} + 608359048206a^9b^{12}$$
$$+45524228788662226b^{13} - 8612153544899408ab^{13} + 11684556963374568a^2b^{13}$$
$$-2459287660143888a^3b^{13}+1832663509135228a^4b^{13}-87796234586480a^5b^{13}+50125620668424a^6b^{13}$$
$$-512075874352a^7b^{13} + 265182149218a^8b^{13} - 1423806417104400b^{14} + 370669544999304ab^{14}$$
$$-185095028637272a^2b^{14} + 167607787407128a^3b^{14} - 14882731838680a^4b^{14} + 8925930851320a^5b^{14}$$
$$-159286274280a^6b^{14} + 78378960360a^7b^{14} + 35517736723600b^{15} - 6357587526960ab^{15}$$
$$+8105951624872a^2b^{15} - 1445744454752a^3b^{15} + 979484169144a^4b^{15} - 30677356528a^5b^{15}$$
$$+15338678264a^6b^{15}-696385309410b^{16}+172733256855ab^{16}-73945230446a^2b^{16}+61843300298a^3b^{16}$$
$$-3560764597a^4b^{16} + 1917334783a^5b^{16} + 10488189810b^{17} - 1689533124ab^{17} + 2006403306a^2b^{17}$$
$$-235148940a^3b^{17}+145008513a^4b^{17}-117026700b^{18}+27096622ab^{18}-7972286a^2b^{18}+6096454a^3b^{18}$$
$$+910700b^{19} - 114380ab^{19} + 123410a^2b^{19} - 4410b^{20} + 903ab^{20} + b^{21})\Big)\Bigg\}-$$

$$-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\Bigg\{\frac{4194304}{\left[\prod_{\zeta=1}^{21}\{a-b-(2\zeta-1)\}\right]\left[\prod_{\eta=1}^{22}\{a-b+(2\eta-1)\}\right]}\Big(131130704576879886034406250$$
$$+537459628826206088931678750a - 341874092984654036300445000a^2$$
$$+493028445538106658885759300a^3 - 652340658285369856748522700a^4$$
$$+641298227578144236953444700a^5-604573745934401191005912000a^6+231724339078175706150136000a^7$$
$$-151289328133808753121100a^8 + 302400896087632042024600a^9 - 1371048711540959426040a^{10}$$
$$+1611628006384226953200a^{11} - 504464808447965118a^{12} + 3684291919673911000a^{13}$$
$$-777692306266456a^{14}+35857948972408a^{15}-484185248451a^{16}+13811946279a^{17}-106564578a^{18}$$
$$+1743994a^{19}-5719a^{20}+43a^{21}+328359605063243951923976250b+450537436682222240043441000ab$$
$$+152628207170808204647275770a^2b - 519623543983275115157278800a^3b$$
$$+436897576873984147905475530a^4b - 302039190387316087462488000a^5b$$
$$+276153536122183796513666400a^6b - 153811376527605888117264a^7b$$
$$+568814810269939113105780a^8b - 233240847340173369850400a^9b + 454222013907202843900a^{10}b$$
$$+2164283489573848a^{14}b - 28280942023504a^{15}b + 1250881854197a^{16}b - 9489432124a^{17}b$$
$$+254376218a^{18}b - 839188a^{19}b + 12341a^{20}b + 331009024790814617530111500b^2$$

$$+11382645813610702607514171 0ab^2+2966205753801299989533857 4a^2b^2$$

$$+9173580472480782112160952 6a^3b^2-1858191737565276546672360a^4b^2$$

$$+1107254450671742504094332 0a^5b^2-446409836411046907751368a^6b^2$$

$$+374921775760059309890200a^7b^2-130393580457213749434 20a^8b^2$$

$$+456947700863521365414 8a^9b^2-1206150548918758877 72a^{10}b^2+224686304942925367 88a^{11}b^2$$

$$-4223904815550140 24a^{12}b^2+46327319945209016a^{13}b^2-578915870956584a^{14}b^2$$

$$+39037774410808a^{15}b^2-288251667106a^{16}b^2+11986499486a^{17}b^2-39191490a^{18}b^2+962598a^{19}b^2$$

$$+188527718414561000489016 30b^3+325818963473585599993174 20ab^3$$

$$+7392467948905891804601593 8a^2b^3+674772526908970172973436 8a^3b^3$$

$$+1956654920038543110397221 6a^4b^3-251409502832929940580848a^5b^3$$

$$+1155941471891438860601896a^6b^3-28831347254232216398432a^7b^3$$

$$+2213910491782167157267 6a^8b^3-492678254183696354488a^9b^3+16194983663448270652 4a^{10}b^3$$

$$-2744654110430536928a^{11}b^3+48343181524244495 2a^{12}b^3-5648934122340208a^{13}b^3$$

$$+585771144020552a^{14}b^3-4143128700320a^{15}b^3+262400960302a^{16}b^3-837826964a^{17}b^3$$

$$+32224114a^{18}b^3+699815241779250324351626 1b^4+2221353788317106815143953 1ab^4$$

$$+909526173243039232365717 6a^2b^4+164384273492293289825458 96a^3b^4$$

$$+673581690360746582755756a^4b^4+185090059883221303958830 8a^5b^4$$

$$-1562421487601667973935 2a^6b^4+5811854531770033541340 0a^7b^4-9214719430968641392 66a^8b^4$$

$$+645945537218444676642a^9b^4-9180121926490903960a^{10}b^4+2812829209563463896a^{11}b^4$$

$$-295460960729854 92a^{12}b^4+4881179381635732a^{13}b^4-32252520795880a^{14}b^4$$

$$+3132630060840a^{15}b^4-9586673915a^{16}b^4+563921995a^{17}b^4+183098164808805043212494 1b^5$$

$$+343788182112078258282945 6ab^5+64750453821053753358894 80a^2b^5$$

$$+1038876114726270005445776a^3b^5+159600226212488706487289 2a^4b^5$$

$$+333621835286836961431 84a^5b^5+873136801485651501118 32a^6b^5-490468210861184892336a^7b^5$$

$$+152087876783675861628 6a^8b^5-1532914988399567572 8a^9b^5+98072166395228865 84a^{10}b^5$$

$$-859993582100642 08a^{11}b^5+244583857081218 20a^{12}b^5-1448786052025 76a^{13}b^5$$

$$+22313057280808a^{14}b^5-63714509712a^{15}b^5+5752004349a^{16}b^5+354501246907809948405480b^6$$

$$+1069513403791665845357128ab^6+5230716994080508599290 16a^2b^6$$

$$+7352404154768760383126 32a^3b^6+568646128827746268317 20a^4b^6$$

$$+76635144078132798484872a^5b^6+874649607643712372472a^6b^6+218585142125509659500 0a^7b^6$$

$$-809504348744234343 2a^8b^6+213575403006448603 12a^9b^6-132405723186144408a^{10}b^6$$

$$+7733116772117245 6a^{11}b^6-381004267695688a^{12}b^6+100261443987096a^{13}b^6-256037937176a^{14}b^6$$

Notes

Notes

$$+36576848168a^15b^6 + 5246226275350353915524 0b^7 + 10201796203064207321048 0ab^7$$

$$+1712429423513036329105 36a^2b^7 + 3424182069759708872284 8a^3b^7$$

$$+3909647304629472302093 6a^4b^7 + 1614107103242780383952a^5b^7 + 1943115716512939776808a^6b^7$$

$$+1243823631333643360 0a^7b^7 + 297209013764764369 84a^8b^7 - 69758491996612848a^9b^7$$

$$+15891035436275656 8a^{10}b^7 - 552383916933472a^{11}b^7 + 294616546783832a^{12}b^7$$

$$-623957998160a^{13}b^7 + 151532656696a^{14}b^7 + 6068045552614043865426b^8$$

$$+1760523830807426405357 4ab^8 + 9336842670271654008484a^2b^8 + 1122618694511849293985 2a^3b^8$$

$$+1099649665242617000798a^4b^8 + 1064600574662833613834a^5b^8 + 245208823664468904 24a^6b^8$$

$$+2667252758559901168 8a^7b^8 + 944270808969834 06a^8b^8 + 21584208983378956 2a^9b^8$$

$$-291326223911580a^{10}b^8 + 579276741224844a^{11}b^8 - 864510226398a^{12}b^8 + 421171648758a^{13}b^8$$

$$+5570786867781959522 26b^9 + 10940323032871080579 12ab^9 + 16894747707659905960 76a^2b^9$$

$$+3710784022919059612 24a^3b^9 + 3520223593464046626 30a^4b^9 + 1828540502332801400 0a^5b^9$$

$$+15396021766853800072a^6b^9 + 196990298922392976a^7b^9 + 19511344911362561 4a^8b^9$$

$$+353214178214040a^9b^9 + 772341543530700a^{10}b^9 - 457412818200a^{11}b^9 + 800472431850a^{12}b^9$$

$$+410143375049274102 60b^{10} + 1149343661918490372 36ab^{10} + 6258856241904048125 2a^2b^{10}$$

$$+6724031452151885586 0a^3b^{10} + 7157179769538448936a^4b^{10} + 5631584078513951272a^5b^{10}$$

$$+157355447527654376a^6b^{10} + 117137843987075912a^7b^{10} + 774287072717444a^8b^{10}$$

$$+702101231167908a^9b^{10} + 503154100020a^10b^{10} + 1052049481860a^{11}b^{10} + 2436777594647459700b^{11}$$

$$+4742709531308534760ab^{11} + 6843044908448608444a^2b^{11} + 1526743180734652192a^3b^{11}$$

$$+1266877992724507592a^4b^{11} + 686695175877066 72a^5b^{11} + 461797707185804 08a^6b^{11}$$

$$+652631982303392a^7b^{11} + 434411052282116a^8b^11 + 1152680301864a^9b^{11} + 960566918220a^{10}b^{11}$$

$$+117140064945846306b^{12} + 317038686494337262ab^{12} + 169609751776172344a^2b^{12}$$

$$+166445736776215624a^3b^{12} + 17271058034689004a^4b^{12} + 11692563911936180a^5b^{12}$$

$$+309271965286648a^6b^{12} + 181277011099912a^7b^{12} + 1013931747010a^8b^{12} + 608359048206a^9b^{12}$$

$$+4552422878866226b^{13} + 8612153544899408ab^{13} + 11684556963374568a^2b^{13}$$

$$+2459287660143888a^3b^{13} + 1832663509135228a^4b^{13} + 87796234586480a^5b^{13}$$

$$+50125620668424a^6b^{13} + 512075874352a^7b^{13} + 265182149218a^8b^{13} + 142380641710440b^{14}$$

$$+370669544999304ab^{14} + 185095028637272a^2b^{14} + 167607787407128a^3b^{14} + 14882731838680a^4b^{14}$$

$$+8925930851320a^5b^{14} + 159286274280a^6b^{14} + 78378960360a^7b^{14} + 3551773672360b^{15}$$

$$+6357587526960ab^{15} + 8105951624872a^2b^{15} + 1445744454752a^3b^{15} + 979484169144a^4b^{15}$$

$$+30677356528a^5b^{15} + 15338678264a^6b^{15} + 69638530941b^{16} + 172733256855ab^{16}$$

$$+73945230446a^2b^{16} + 61843300298a^3b^{16} + 3560764597a^4b^{16} + 1917334783a^5b^{16} + 1048818981b^{17}$$

$$+1689533124ab^{17}+2006403306a^{2}b^{17}+235148940a^{3}b^{17}+145008513a^{4}b^{17}+11702670b^{18}$$

$$+27096622ab^{18}+7972286a^{2}b^{18}+6096454a^{3}b^{18}+91070b^{19}+114380ab^{19}+123410a^{2}b^{19}+441b^{20}$$

$$+903ab^{20}+b^{21}\Big)+\frac{4194304}{\left[\prod\limits_{\mu=1}^{22}\{a-b-(2\mu-1)\}\right]\left[\prod\limits_{\xi=1}^{21}\{a-b+(2\xi-1)\}\right]}\times$$

$$\times\Big(13113070457687988603440625+32835960506324395192397625a$$

$$+33100902479081461753011150a^{2}+18852771841456100048901630a^{3}$$

$$+6998152417792503243516261a^{4}+1830981648088050432124941a^{5}$$

$$+354501246907809948405480a^{6}+52462262753503539155240a^{7}+6068045552614043865426a^{8}$$

$$+557078686778195952226a^{9}+41014337504927410260a^{10}+2436777594647459700a^{11}$$

$$+117140064945846306a^{12}+4552422878866226a^{13}+142380641710440a^{14}+3551773672360a^{15}$$

$$+69638530941a^{16}+1048818981a^{17}+11702670a^{18}+91070a^{19}+441a^{20}+a^{21}$$

$$+53745962882620608893167875b+45053743668222224004344100ab$$

$$+113826458136107026075141710a^{2}b+32581896347358559999317420a^{3}b$$

$$+22213537883171068151439531a^{4}b+3437881821120782582829456a^{5}b$$

$$+1069513403791665845357128a^{6}b+102017962030642073210480a^{7}b$$

$$+17605238308074264053574a^{8}b+1094032303287108057912a^{9}b+114934366191849037236a^{10}b$$

$$+4742709531308534760a^{11}b+317038686494337262a^{12}b+8612153544899408a^{13}b$$

$$+370669544999304a^{14}b+6357587526960a^{15}b+172733256855a^{16}b+1689533124a^{17}b$$

$$+27096622a^{18}b+114380a^{19}b+903a^{20}b-3418740929846540363004450b^{2}$$

$$+152628207170808204647275770ab^{2}+29662057538012999895338574a^{2}b^{2}$$

$$+73924679489058918046015938a^{3}b^{2}+9095261732430392323657176a^{4}b^{2}$$

$$+6475045382105375335889480a^{5}b^{2}+523071699408050859929016a^{6}b^{2}$$

$$+171242942351303632910536a^{7}b^{2}+9336842670271654008484a^{8}b^{2}$$

$$+1689474770765990596076a^{9}b^{2}+62588562419040481252a^{10}b^{2}+6843044908448608444a^{11}b^{2}$$

$$+169609751776172344a^{12}b^{2}+11684556963374568a^{13}b^{2}+185095028637272a^{14}b^{2}$$

$$+8105951624872a^{15}b^{2}+73945230446a^{16}b^{2}+2006403306a^{17}b^{2}+7972286a^{18}b^{2}+123410a^{19}b^{2}$$

$$+49302844553810665888575930b^{3}-5196235439832751151572788ab^{3}$$

$$+91735804724807821121609526a^{2}b^{3}+6747725269089701729734368a^{3}b^{3}$$

$$+16438427349229328982545896a^{4}b^{3}+1038876114726270005445776a^{5}b^{3}$$

$$+735240415476876038312632a^{6}b^{3}+34241820697597088722848a^{7}b^{3}$$

$$+11226186945118492939852a^{8}b^{3}+371078402291905961224a^{9}b^{3}+67240314521518855860a^{10}b^{3}$$

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$$+15267431807346521 92a^{11}b^3 + 1664457367762156 24a^{12}b^3 + 2459287660143888a^{13}b^3$$

$$+167607787407128a^{14}b^3 + 1445744454752a^{15}b^3 + 61843300298a^{16}b^3 + 235148940a^{17}b^3$$

$$+6096454a^{18}b^3 - 652340658285369856748 5227b^4 + 4368975768739841479054 7553ab^4$$

$$-185819173756527654667 2360a^2b^4 + 195665492003854311039 72216a^3b^4$$

$$+673581690360746582755756a^4b^4 + 159600226212488706487 2892a^5b^4$$

$$+56864612882774626831720a^6b^4 + 39096473046294723020936a^7b^4$$

$$+1099649665242617000798a^8b^4 + 35202235934640466 2630a^9b^4 + 7157179769538448936a^{10}b^4$$

$$+1266877992724507592a^{11}b^4 + 172710580346890 04a^{12}b^4 + 1832663509135228a^{13}b^4$$

$$+148827318386 80a^{14}b^4 + 979484169144a^{15}b^4 + 3560764597a^{16}b^4 + 145008513a^{17}b^4$$

$$+641298227578144236953 44447b^5 - 302039190387316087462 4880ab^5$$

$$+110725445067174250409 43320a^2b^5 - 251409502832929940580 848a^3b^5$$

$$+185090059883221303958 8308a^4b^5 + 333621835286836961431 84a^5b^5$$

$$+7663514407813279848 4872a^6b^5 + 161410710324278038 3952a^7b^5 + 106460057466283361 3834a^8b^5$$

$$+182854050233280140 00a^9b^5 + 5631584078513951272a^{10}b^5 + 686695175877066 72a^{11}b^5$$

$$+116925639119361 80a^{12}b^5 + 87796234586480a^{13}b^5 + 8925930851320a^{14}b^5 + 30677356528a^{15}b^5$$

$$+1917334783a^{16}b^5 - 604573745934401191005912b^6 + 276153536122183796513 6664ab^6$$

$$-446409836411046907751368a^2b^6 + 115594147189143886060 1896a^3b^6$$

$$-156242148760166797393 52a^4b^6 + 873136801485651501118 32a^5b^6 + 874649607643712372472a^6b^6$$

$$+194311571651293977680 8a^7b^6 + 24520882366446890424a^8b^6 + 1539602176685380007 2a^9b^6$$

$$+157355447527654376a^{10}b^6 + 461797707185804 08a^{11}b^6 + 309271965286648a^{12}b^6$$

$$+50125620668424a^{13}b^6 + 159286274280a^{14}b^6 + 15338678264a^{15}b^6 + 231724339078175706150136b^7$$

$$-153811376527605888117264ab^7 + 374921775760059309890200a^2b^7$$

$$-288313472542322163984 32a^3b^7 + 581185453177003354134 00a^4b^7 - 490468210861184892336a^5b^7$$

$$+218585142125509659500 0a^6b^7 + 124382363133364336 00a^7b^7 + 266725275855990116 88a^8b^7$$

$$+196990298922392976a^9b^7 + 11713784398707591 2a^{10}b^7 + 652631982303392a^{11}b^7$$

$$+181277011099912a^{12}b^7 + 512075874352a^{13}b^7 + 78378960360a^{14}b^7 - 151289328133808753121 10b^8$$

$$+568814810269939113105 78ab^8 - 1303935804572137494342 0a^2b^8 + 221391049178216715726 76a^3b^8$$

$$-921471943096864139266a^4b^8 + 1520878767836758616286a^5b^8 - 80950434874423434 32a^6b^8$$

$$+2972090137647643698 4a^7b^8 + 9442708089698340 6a^8b^8 + 195113449113625614a^9b^8$$

$$+774287072717444a^{10}b^8 + 434411052282116a^{11}b^8 + 1013931747010a^{12}b^8 + 265182149218a^{13}b^8$$

$$+302400896087632042024 6b^9 - 233240847340173369850 4ab^9 + 456947700863521365414 8a^2b^9$$

$$-492678254183696354488a^3b^9 + 645945537218444676642a^4b^9 - 153291498839567572 8a^5b^9$$

$$
\begin{aligned}
&+2135754030064486031 2a^6b^9 - 697584919966128 48a^7b^9 + 215842089833789562a^8b^9 \\
&+353214178214040a^9b^9 + 702101231167908a^{10}b^9 + 1152680301864a^{11}b^9 + 608359048206a^{12}b^9 \\
&-13710487115409594260 4b^{10} + 45422201390720284390 0ab^{10} - 1206150548918758877 72a^2b^{10} \\
&+1619498366344827065 24a^3b^{10} - 918012192649090396 0a^4b^{10} + 98072166395228865 84a^5b^{10} \\
&-13240572318614440 8a^6b^{10} + 1589103543627565 68a^7b^{10} - 29132622391158 0a^8b^{10} \\
&+772341543530700a^9b^{10}+503154100020a^10b^{10}+960566918220a^{11}b^{10}+1611628006384226953 2b^{11} \\
&-1326652400993417045 6ab^{11} + 2246863049429253678 8a^2b^{11} - 274465411043053692 8a^3b^{11} \\
&+281282920956346389 6a^4b^{11} - 85999358210064208a^5b^{11} + 7733116772117245 6a^6b^{11} \\
&-552383916933472a^7b^{11} + 579276741224844a^8b^{11} - 457412818200a^9b^{11} + 1052049481860a^10b^{11} \\
&-504464808447965118b^{12} + 1520074513892383306ab^{12} - 42239048155501402 4a^2b^{12} \\
&+48343181524244495 2a^3b^{12} - 29546096072985492a^4b^{12} + 24458385708121820a^5b^{12} \\
&-381004267695688a^6b^{12} + 294616546783832a^7b^{12} - 864510226398a^8b^{12} + 800472431850a^9b^{12} \\
&+36842919196739110b^{13} - 30556991340393392ab^{13} + 46327319945209016a^2b^{13} \\
&-5648934122340208a^3b^{13} + 4881179381635732a^4b^{13} - 144878605202576a^5b^{13} \\
&+100261443987096a^6b^{13} - 623957998160a^7b^{13} + 421171648758a^8b^{13} - 777692306266456b^{14} \\
&+2164283489573848ab^{14} - 578915870956584a^2b^{14} + 585771144020552a^3b^{14} \\
&-32252520795880a^4b^{14} + 22313057280808a^5b^{14} - 256037937176a^6b^{14} + 151532656696a^7b^{14} \\
&+35857948972408b^{15} - 28280942023504ab^{15} + 39037774410808a^2b^{15} - 4143128700320a^3b^{15} \\
&+3132630060840a^4b^{15} - 63714509712a^5b^{15} + 36576848168a^6b^{15} - 484185248451b^{16} \\
&+1250881854197ab^{16} - 288251667106a^2b^{16} + 262400960302a^3b^{16} - 9586673915a^4b^{16} \\
&+5752004349a^5b^{16} + 13811946279b^{17} - 9489432124ab^{17} + 11986499486a^2b^{17} - 837826964a^3b^{17} \\
&+563921995a^4b^{17} - 106564578b^{18} + 254376218ab^{18} - 39191490a^2b^{18} + 32224114a^3b^{18} \\
&+1743994b^{19} - 839188ab^{19} + 962598a^2b^{19} - 5719b^{20} + 12341ab^{20} + 43b^{21}\Big)\Big\}\Big] \qquad (8)
\end{aligned}
$$

III. Derivation of the Summation Formula

Substituting $c = \frac{a+b+45}{2}$ and $z = \frac{1}{2}$ in equation (2), we get

$$
(a-b)\ {}_2F_1\left[\begin{matrix} a,\ b & ; \\ \frac{a+b+45}{2} & ; \end{matrix}\ \frac{1}{2}\right] = a\ {}_2F_1\left[\begin{matrix} a+1,\ b & ; \\ \frac{a+b+45}{2} & ; \end{matrix}\ \frac{1}{2}\right] - b\ {}_2F_1\left[\begin{matrix} a,\ b+1 & ; \\ \frac{a+b+45}{2} & ; \end{matrix}\ \frac{1}{2}\right]
$$

Now involving the formula [Salahuddin et. al. p.12-41(8)], the summation formula is obtained.

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Global Existence and Uniqueness of the Weak Solution in Keller Segel Model

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Abstract- This paper deals with the global existence, uniqueness and boundedness of the weak solution for the chemotaxis system **(P)** defined as

$$\begin{cases} u_t - \Delta u + \operatorname{div}(u\nabla c) = 0 & (t,x) \in \mathbb{R}^+ \times \Omega \\ -\Delta c + \tau c = 0 & x \in \Omega \\ u = 0, \ c = g & \Gamma \\ u(0,x) = u_0 & x \in \Omega \end{cases} \qquad \text{(P)}$$

The system **(P)** is under homogeneous Dirichlet boundary conditions in a convex smooth bounded domain $\Omega \in \mathbb{R}^n$ with smooth boundary Γ ($\in H^{\frac{3}{2}}(\Gamma)$and $u_0 \in H^{\frac{1}{2}}(\Omega)$ Based on Galerkin's method, Lax -Milgran's and maximum principle, a prove of the existence and uniqueness of a global solution for the system **(P)** is determined. Moreover we show that the unique solution is positive.

Keywords and Phrases: chemotaxis, global existence, boundedness, positive solution.

GJSFR-F Classification : MSC 2010: 35K58, 45G05, 65C35, 82C22, 82C31,82C80, 92C17

GLOBALEXISTENCEANDUNIQUENESSOFTHEWEAKSOLUTIONINKELLERSEGELMODEL

:

Strictly as per the compliance and regulations of

Ref

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Global Existence and Uniqueness of the Weak Solution in Keller Segel Model

C. Messikh [α], A. Guesmia [σ] & S. Saadi [ρ]

Abstract- This paper deals with the global existence, uniqueness and boundedness of the weak solution for the chemotaxis system **(P)** defined as

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The system **(P)** is under homogeneous Dirichlet boundary conditions in a convex smooth bounded domain $\Omega \in \mathbb{R}^n$ with smooth boundary Γ ($\in H^{\frac{3}{2}}(\Gamma)$and $u_0 \in H^{\frac{1}{2}}(\Omega)$ Based on Galerkin's method, Lax-Milgran's Theorem and maximum principle, a prove of the existence and uniqueness of a global solution for the system **(P)** is determined. Moreover we show that the unique solution is positive.

Keywords and Phrases: *chemotaxis, global existence, boundedness, positive solution.*

I. Introduction

Chemotaxis is an important means for cellular communication. It is the influence of chemical substances in the environment on the movement of mobile species. This can lead to strictly oriented movement or to partially oriented and partially tumbling movement. The movement towards a higher concentration of the chemical substance is called positive chemotaxis whereas the movement towards regions of lower chemical concentration is called negative chemotactical movement.

The classical chemotaxis model — the so-called Keller–Segel model — system defined in (0.1) was first introduced by Paltak [11] (1953), E. Keller and L. Segel [9] (1970)

$$\begin{array}{ll} u_t - \nabla(a\nabla u) + \nabla(\chi u \nabla c) = 0 & (t,x) \in \mathbb{R}^+ \times \mathbb{R}^d \\ \alpha c_t - \Delta c + \tau c + \beta u = 0 & x \in \mathbb{R}^d \end{array} \qquad (0.1)$$

where $u(t,x)$ denotes the density of bacteria in the position $x \in \mathbb{R}^d$ and at time t, c the concentration of chemical signal substance, $\alpha \geq 0$ the relaxation time, the parameter χ the sensitivity of cells to the chemoattractant and $a, \tau,\ \beta$ are given smooth functions. As it can be seen, when $\alpha \neq 0$ the model is

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called Parabolic-Parabolic while it is an Elliptic-Parabolic model when $\alpha = 0$. This modelling is very simple, it exhibits a profound mathematical structure and mostly only dimension 2 is understood, especially chemotactic collapse. The proposed model has been extensively studied in the last few years (see ([7]-[8],[12],[13]) for a recent survey articles).

The Parabolic-Parabolic model has been investigated by many authers (see for examples Refs [8] and [13]), I. Fatkullin [10] had developed numerical method (a composite particle-grid) with adaptive time stepping which allows us to resolve and propagates singular solutions when with Neumann boundary condition.

The Elliptic-Parabolic model has been investigated by many authers (see for examples Refs [3] and [4]). This model have been carried out where the main concern is whether the solution of model is bounded or blow-up. It has been proved that the solution strongly depends on the spatial dimension. It does not occur in one-dimensional problems, and it occurs conditionally in higher dimensional situations. More precisley, see [2] in case higher dimensions ($n \geq 3$), if the norm of initial condition u_0 is small in space $L^{\frac{n}{2}}(\mathbb{R}^n)$, then there are global weak solutions and if $\left(\int x^2 u_0\right)^{d-2} \leq C \|u_0\|^n_{L^1(\mathbb{R}^n)}$ with C is small ,then there is blow up in a finite time T^*. But in two dimension, (see [5]), if $\|u_0\|_{L^1(\mathbb{R}^2)} < \frac{8\pi}{\chi}$, there are smooth solutions, and if $\|u_0\|_{L^1(\mathbb{R}^2)} > \frac{8\pi}{\chi}$, there is creation of a singular measure (blow-up) in finite time.

In this paper we demonstrate the global existence and uniqueness of weak positive solution for the elliptic-parabolic model's problem defined as

$$(P)\begin{cases} (P_1)\begin{cases} u_t - \Delta u + \operatorname{div}(u \nabla c) = 0 & (t,x) \in \mathbb{R}^+ \times \Omega \\ u = 0 & \Gamma \\ u(0,x) = u_0 & x \in \Omega \end{cases} \\ (P_2)\begin{cases} -\Delta c + \tau c = 0 & x \in \Omega \\ c = g & \Gamma \end{cases} \end{cases}$$

Where $u(t,x)$ is a function denotes the density of bacteria in the position $x \in \Omega \subset \mathbb{R}^2$ or $\mathbb{R}^3$, Ω is a bounded convex domain with smooth boundary Γ, c denotes the concentration of chemical signal that stimulates the bacteria. The parameter τ is a time constant and it is expressed on the one hand the movement of bacteria (representing a random distribution side and a deterministic drift in the direction of high concentrations) and secondly the diffusion degradation of c.

To simplify the solution of the system (P), a decomposition of (P) into two subsystems (P_1) and (P_2) are adopted. Lax-Milgram's Theorem is very important theorem which we help us to demonstrate the existence and uniqueness of a weak solution for the system (P_2). However this theorem can not be applied directly because it is a nonhomogenous system. For this raison an adoptation of Trace Theorem is used to simplify the system (P_2), and together with Galerking method we can demonstrate the existence and uniquness of a weak solution for the system (P_1). Therefore we have the existence and uniquness for the problem (P). Moreover we show that the solution is positive. The following initial-boundary conditions on u_0 and g assumptions are used to prove the proposed solution of (P)

H_1 $\quad g \in L^{\frac{1}{2}}(\Gamma)$
H_2 $\quad g \in L^{\frac{3}{2}}(\Gamma)$
H_3 $\quad u_0 \in L^2(\Omega)$
H_4 $\quad u_0 \geq 0$ and $g \geq 0$.

Ref

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Notes

If the hypothesis H_1 is satisfies and using the theorem of trace, one can find a lifting of this trace which we denote $R(g) \in H_0^1(\Omega)$. Thus by definition it verifies $\gamma_0(R(g)) = g$. Now looking for c having the form $c = \widetilde{c} + R(g)$ reduces the problem (P_2) to $\widetilde{c}$.

$$\left(\widetilde{P}_2\right)\begin{cases} -\Delta\widetilde{c} + \tau\widetilde{c} - \Delta R(g) + \tau R(g) = 0 & \text{in } x \in \Omega \\ \widetilde{c} = 0 & \text{on } \Gamma \end{cases}$$

Definition 1 *We say* $(u, \widetilde{c}) \in L^2\left(0, T; H_0^1(\Omega)\right) \times H_0^1(\Omega)$ *with* $u_t \in L^2\left(0, T; H^{-1}(\Omega)\right)$ *is a weak solution of the problem (P) if and only if*

$$\langle u_t, v\rangle + B(u, v, t) = 0 \quad (0.2)$$

$$a(\widetilde{c}, q, t) = l(q) \quad (0.3)$$

where

$$\begin{cases} B(u, v, t) = \int_\Omega (\nabla u \nabla v + \nabla c \nabla u v + \tau c u v)\, dx \\ a(\widetilde{c}, q) = \int_\Omega (\nabla\widetilde{c}\nabla q + \tau\widetilde{c} q)\, dx \\ l(q) = -\int_\Omega (\nabla R(g)\nabla q + \tau R(g) q)\, dx \end{cases}$$

for all $(v, q) \in \left(H_0^1(\Omega)\right)^2$, $0 \le t \le T$*, and*

$$u(0, x) = u_0 \in L^2(\Omega) \quad (0.4)$$

Remark 2 *Note that* $u \in C\left([0, T]; L^2(\Omega)\right)$ *as* $u \in L^2\left(0, T; H_0^1(\Omega)\right)$ *and* $u_t \in L^2\left(0, T; H^{-1}(\Omega)\right)$*. Then equality (0.4) makes sense.*

II. Existence of Weak Solution of the Problem (P)

In this section, use the Theorem of Lax- Milgran to study the existence and uniqueness of weak solution of problem (P_2), which its variational formulat is given by equation (0.3) and use the method of Galerking to study the existence and uniqueness of weak solution of problem (P_1), which its variational formulat is given by equation (0.2). So we have the existence and uniqueness of weak solution of problem (P).

a) *Existence of weak solution of the problem (P2)*

Theorem 3 *If the hypothesis H_1 holds. Then the problem (P_2) has only one solution* $c \in H^1(\Omega)$ *for any* $q \in H^1(\Omega)$*.*

By applying the Theorem of Lax-Milgran, the solution $\widetilde{c}$ of the problem (0.3) exists and it is unique. So (P_2) has unique solution.

Remark 4 *Elliptic regularity Theorem remains valid provided that the boundary condition g is in the space* $L^{\frac{3}{2}}(\Gamma)$ *which is the image by the operator trace space* $H^2(\Omega)$*.*

Remark 5 [6] *If* $c \in H^2(\Omega)$ *and (c is a solution of problem* (P_2) *) this implies that* $c \in W^{1,q}(\Omega)$ $(H^2(\Omega) \hookrightarrow W^{1,q}(\Omega)$ *for* $1 \le q \le 2^*)$.

Using the Maximum Principle one can show that the solution of the problem (P_2) is positive as follows. Multiplying the first equation of (P_2) by $q \in H_0^1(\Omega)$, we obtain other variational formulat for problem (P_2)

$$\left(\widetilde{P}_3\right) \int_\Omega (\nabla c \nabla q + \tau c q)\, dx = 0.$$

Proposition 6 [1] *If* $g \in L^{\frac{3}{2}}(\Gamma)$ *and* $c \in H^1(\Omega) \cap C(\overline{\Omega})$ *then the problem* $\left(\widetilde{P}_3\right)$ *have a positive solution* c.

Proof. As Γ is smooth enough and $g \in L^{\frac{3}{2}}(\Gamma)$ then $c \in H^2(\Omega)$. And as $\Omega \subset \mathbb{R}^2$ or $\mathbb{R}^3$, by embedding of Sobolev spaces ($H^2(\overline{\Omega}) \hookrightarrow C(\overline{\Omega})$) this implies that $c \in C(\overline{\Omega})$. If $c = g \geq 0$ on Γ, then $c^- = \min(c, 0) \in H_0^1(\Omega)$. So, we have

$$\begin{aligned} \int_\Omega cc^- dx &= \int_\Omega (c^-)^2 dx \\ \int_\Omega \nabla c \nabla c^- dx &= \int_\Omega (\nabla c^-)^2 dx, \end{aligned}$$

Since the support of functions $c^-$and $c^+ = \max(c, 0)$ is set $A(x) = \{x/u(x) = 0\}$. This implies that $\nabla u = 0$ on $A(x)$. As $c = c^+ + c^-$, thus we have

$$0 = \int (\nabla c^-)^2 + \tau (c^-)^2 dx \geq \min(1, \tau) \left\|c^-\right\|^2_{H_0^1(\Omega)}$$

Finally, we find $c^- = 0$.

b) *Existence of a weak solution of the problem (P1)*

Before proving the existence and uniqueness of weak solution of problem (P_1), we need the following lemma

Lemma 7 *i) For all* $v \in H_0^1(\Omega)$ *then* $B(.,.,t)$ *is continuous in* $H_0^1(\Omega) \times H_0^1(\Omega)$, *there exists a constant positive* C *such that*

$$|B(u, v, t)| \leq C \|u\|_{H^1(\Omega)} \|v\|_{H^1(\Omega)} \tag{1.1}$$

ii) For any $u \in H_0^1(\Omega)$ *and* H_2 *is hold. Then there exists a constant positive* β *such that*

$$\beta \|u\|^2_{H_0^1(\Omega)} \leq B(u, u, t)$$

Proof. i) We use the Cauchy-Shwarz inequality and $c \in H^2(\Omega) \hookrightarrow L^q(\Omega)$ for any $q \in \left[1, \frac{2n}{n-2}\right[$ with $n = 2$ or $n = 3$, we obtain i) as follows

$$\begin{aligned} B(u, v, t) &\leq \|\nabla u\|_{L^2(\Omega)} \|\nabla v\|_{L^2(\Omega)} + \|\nabla c\|_{L^4(\Omega)} \|u\|_{L^2(\Omega)} \|v\|_{L^4(\Omega)} \\ &\quad + \tau \|c\|_{L^4(\Omega)} \|u\|_{L^2(\Omega)} \|v\|_{L^4(\Omega)} \\ &\leq C \|u\|_{H^1(\Omega)} \|v\|_{H^1(\Omega)} \end{aligned}$$

ii) Making use of $-\Delta c + \tau c = 0$ the expression of $B(u, u, t)$ becomes

$$\begin{aligned} B(u, u, t) &= \int (\nabla u)^2 + \frac{\nabla c}{2} \nabla u^2 + \tau c u^2 dx \\ &= \int (\nabla u)^2 + \left(\tau c - \frac{\Delta c}{2}\right) u^2 dx \\ &= \int (\nabla u)^2 + \frac{1}{2} \tau c u^2 dx \geq \|\nabla u\|^2_{L^2(\Omega)}. \end{aligned}$$

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Finally, by Poincarre inequality yields

$$B(u,u,t) \geq \beta \|u\|^2_{H_0^1(\Omega)}.$$

To demonstrate the existence of weak solution of (P_1) via the method of Galerking, we assume $w_k = w_k(x)$ are smooth functions verifying

$$\{w_k\}_{k=1}^{\infty} \quad \text{is an orthogonal basis of } H_0^1(\Omega) \tag{1.2}$$

and

$$\{w_k\}_{k=1}^{\infty} \quad \text{is an orthonormal basis of } L^2(\Omega). \tag{1.3}$$

Consider a positive integer m. We will look for a function $u_m : [0\ T] \to H_0^1(\Omega)$ of the form

$$u_m(t) := \sum_{k=1}^{m} d_m^k(t)\, w_k \tag{1.4}$$

which satisfies

$$d_m^k(0) = (u_0, w_k) \tag{1.5}$$

and

$$\langle u_m', w_k \rangle + B(u_m, w_k, t) = 0, \quad 0 \leq t \leq T \text{ and } \quad k = 1, ..., m \tag{1.6}$$

where $u' = u_t$ and here $(.,.)$ denotes the scalar product in $L^2(\Omega)$.

Theorem 8 *(construction of the approximate solution) For each integer m, there exists a unique function u_m of the form* (1.4) *satisfying* (1.5) *and* (1.6).

Proof. Assuming u_m has the structure (1.4). Substituting (1.4) into (1.5) and using (1.3) we obtained

$$d_m'^k(t) + \sum_{l=1}^{m} d_m^l B(w_l, w_k, t) = 0 \qquad 0 \leq t \leq T \text{ and } k = 1, ..., m \tag{1.7}$$

According to standard existence theory for ordinary differential equations, there exists a unique absolutely continuous functions $d_m(t) = \left(d_m^1, d_m^2, ..., d_m^m\right)$ satisfying (1.5) and (1.7). So u_m of the form (1.4) satisfies (1.5) and (1.6) for all $t \in [0\ T]$.

c) Energy estimates

We propose now to send m to infinity and show a subsequence of our solutions u_m of the approximation problems (1.5) and (1.6) converges to a weak solution of (P_1). For this we will need some uniform estimates.

Theorem 9 (*Energy estimates*) [6]. *There exists a constant C, depending only on Ω, T and c, such that*

$$\max_{0 \leq t \leq T} \|u_m\|_{L^2(\Omega)} + \|u_m\|_{L^2\left(0,T;\ H_0^1(\Omega)\right)} + \|u_m'\|_{L^2(0,T;\ H^{-1}(\Omega))} \leq C \|u_0\|_{L^2(\Omega)} \tag{1.8}$$

for $m = 1, 2, ...$

Proof. 1. Multiplying equation (1.6) by $d_m^k(t)$, summing for $k = 1, ..., m$, and then recalling (1.4) we find

$$(\acute{u}_m, u_m) + B(u_m, u_m, t) = 0 \tag{1.9}$$

for all $0 \leq t \leq T$. From Lemma 7, there exists constant $\beta > 0$ such that

$$\beta \|u_m\|^2_{H^1_0(\Omega)} \leq B(u_m, u_m, t) \tag{1.10}$$

for all $0 \leq t \leq T$. Consequently (1.10) yields the inequality

$$\frac{d}{dt}\left(\|u_m\|^2_{L^2(\Omega)}\right) + \beta \|u_m\|^2_{H^1_0(\Omega)} \leq 0 \text{ for all } 0 \leq t \leq T. \tag{1.11}$$

This implies that

$$\|u_m\|^2_{L^2(\Omega)} \leq \|u_m(0)\|^2_{L^2(\Omega)} \leq \|u_0\|^2_{L^2(\Omega)} \text{ for all } 0 \leq t \leq T. \tag{1.12}$$

So we have

$$\max_{0 \leq t \leq T} \|u_m\|_{L^2(\Omega)} \leq \|u_0\|_{L^2(\Omega)}. \tag{1.13}$$

2. Integrate inequality (1.11) from 0 to T and we employ the inequality (1.13) to find

$$\|u_m\|^2_{L^2(0,T;\ H^1_0(\Omega))} = \int_0^T \|u_m\|^2_{H^1_0(\Omega)} dt \leq C \|u_0\|^2_{L^2(\Omega)}.$$

3. Fix any $v \in H^1_0(\Omega)$, with $\|v\|^2_{H^1_0(\Omega)} \leq 1$, and write $v = v^1 + v^2$, where $v^1 \in span(w_k)^{k=m}_{k=1}$, and $(v^2, w_k) = 0$ $(k = 1, ..., m)$. We use (1.6), we deduce for all $0 \leq t \leq T$ that

$$(u'_m, v^1) + B(u_m, v^1, t) = 0.$$

Then (1.4) implies

$$\langle u'_m, v \rangle = (u'_m, v) = (u'_m, v^1) = -B(u_m, v^1, t),$$

consequently

$$|\langle u'_m, v \rangle| \leq C \|u_m\|_{H^1_0(\Omega)}.$$

Since $\|v^1\|^2_{H^1_0(\Omega)} \leq \|v\|^2_{H^1_0(\Omega)} \leq 1$. Thus

$$\|u'_m\|_{H^{-1}(\Omega)} \leq C \|u_m\|_{H^1_0(\Omega)},$$

and therefore

$$\|u'_m\|^2_{L^2(0,T;\ H^{-1}(\Omega))} = \int_0^T \|u'_m\|^2_{H^{-1}(\Omega)} dt \leq C \int_0^T \|u_m\|^2_{H^1_0(\Omega)} dt \leq C \|u_0\|^2_{L^2(\Omega)}.$$

d) *Existence and uniqueness*

Next we pass to limits as $m \to \infty$, to build a weak solution of our initial boundary-value problem (P_1).

Theorem 10 *(Existence of weak solution). Under hypothesis H_2 and H_3, there exists a weak solution of* (P_1).

Proof. *1. According to the energy estimates (1.8), we see that the sequence $\{u_m\}^\infty_{m=1}$ is bounded in $L^2(0,T;H^1_0(\Omega))$ and $\{u'_m\}^\infty_{m=1}$ is bounded in $L^2(0,T;H^{-1}(\Omega))$. Consequently there exists a subsequence which is also noted by $\{u_m\}^\infty_{m=1}$ and a function $u \in L^2(0,T;H^1_0(\Omega))$, with $u' \in L^2(0,T;H^{-1}(\Omega))$, such that*

$$\begin{aligned} u_m &\rightharpoonup u \quad \text{weakly in } L^2(0,T;H^1_0(\Omega)) \\ u'_m &\rightharpoonup u' \quad \text{weakly in } L^2(0,T;H^{-1}(\Omega)). \end{aligned} \tag{1.14}$$

2. Next fix an integer N and choose a function $v \in C^1(0,T;H^1_0(\Omega))$ having the form

$$v(t) = \sum_{k=1}^N d^k(t) w_k \tag{1.15}$$

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where $\{d^k\}_{k=1}^{N}$ *are given smooth functions. We choose* $m \geq N$*, multiply equation* (1.6) *by* $d^k(t)$*, sum for* $k = 1, ..., N$*, and then integrate with respect to* t *to find*

$$\int_0^T \langle u_m', v\rangle + B(u_m, v, t)\, dt = 0. \tag{1.16}$$

We recall (1.14) *to find upon passing to weak limits that*

$$\int_0^T \langle u', v\rangle + B(u, v, t)\, dt = 0 \qquad \forall v \in L^2\left(0, T; H_0^1(\Omega)\right). \tag{1.17}$$

As functions of the form (1.15) *are dense in* $L^2\left(0, T; H_0^1(\Omega)\right)$*. Hence in particular*

$$\langle u', v\rangle + B(u, v, t)\, dt = 0 \qquad \forall v \in H_0^1(\Omega) \text{ and } \forall t \in [0\ T], \tag{1.18}$$

and from Remark 2 we have $u \in C\left(0, T; L^2(\Omega)\right)$*.*
3. In order to prove $u(0) = u_0$*, we first note from* (1.17) *that*

$$\int_0^T -\langle u, v'\rangle + B(u, v, t)\, dt = (u(0), v(0)) \tag{1.19}$$

for each $v \in C^1\left(0, T; H_0^1(\Omega)\right)$ *with* $v(T) = 0$*. Similary, from* (1.16) *we deduce*

$$\int_0^T -\langle u_m, v'\rangle + B(u_m, v, t)\, dt = (u_m(0), v(0)). \tag{1.20}$$

We use again (1.14)*, we obtain*

$$\int_0^T -\langle u, v'\rangle + B(u, v, t)\, dt = (u_0, v(0)), \tag{1.21}$$

since $u_m(0) \to u_0$ *in* $L^2(\Omega)$*. Comparing* (1.19) *and* (1.21)*, we conclude* $u(0) = u_0$*.*

Theorem 11 *(Uniqueness of weak solutions) A weak solution of* (P_1) *is unique.*
Proof. *We suppose there exists two weak solution* u_1 *and* u_2*. We put*

$$U = u_2 - u_1$$

then U *is also a solution of* (P_1)*with* $U_0 = (u_2 - u_1)(0) \equiv 0$*. Setting* $v = U$ *in identity* (1.18) *we have*

$$\frac{d}{dt}\left(\frac{1}{2}\|U\|_{L^2(U)}^2\right) + B(U, U, t) = 0.$$

From Lemma 7 we have $B(U, U, t) \geq \beta \|U\|_{H_0^1(U)}^2 \geq 0$*, so* $\frac{d}{dt}\left(\frac{1}{2}\|U\|_{L^2(U)}^2\right) \leq 0$*, then integrate with respect to* t *to find*

$$\|U\|_{L^2(\Omega)}^2 \leq \|U_0\|_{L^2(\Omega)}^2 = 0,$$

thus $U \equiv 0$*.*

e) Global solution of problem (P)

Our main results in this paper are stated as follows.

Theorem 12 *i) if $c \geq c_0 > 0$. Then la solution (u, c) of problem (P) is global*
ii) if $c \geq c_0 > 0$. Then la solution (u, c) of problem (P) is global. Furthermore there exists $\tau_0 > 0$ such that $\|u\|_{L^2} \leq e^{-\tau_0 t} \|u_0\|_{L^2}$.

Proof. We put

$$E(t) = \frac{1}{2} \int_\Omega u^2 dx \tag{1.22}$$

We derivate the equation (1.22) and we use firsts equations of (P_1) and (P_2) to find
i) We have

$$\frac{dE}{dt} = -B(u, u, t) \leq 0,$$

therefore

$$E(t) \leq E(0).$$

ii) We have

$$\frac{dE}{dt} = -B(u, u, b, t) = -\int \left((\nabla u)^2 + \frac{1}{2} \tau c u^2 \right) dx \leq \frac{-1}{2} \tau c_0 \|u\|^2_{L^2(\Omega)} = -\tau_0 E(t).$$

This implies that

$$E(t) \leq E(0) e^{-\tau_0 t}.$$

Proposition 13 [1] *Let $u_0 \in L^2(\Omega)$ and $u \in C\left([0, T];\ L^2(\Omega)\right) \cap L^2\left(0, T;\ H_0^1(\Omega)\right)$ is the unique weak solution of (P_1). If $u_0 \geq 0$ in Ω, then $u \geq 0$ in $]0, T[\times \Omega$.*
Proof. *If $u_0 \geq 0$ on Γ. Therefore $u^- = \min(u, 0) \in L^2\left(]0, T[;\ H_0^1(\Omega)\right)$. A reasoning similar to the Proposition 6, we obtain for all $0 \leq t \leq T$*

$$\frac{1}{2} \frac{d}{dt} \int_\Omega \left(u^-\right)^2 dx + \int_\Omega B\left(u^-, u^-, t\right) dx = 0.$$

Using the Lemma 7 and integrating with respect to τ from 0 to t, we get

$$\frac{1}{2} \int_\Omega \left(u^-\right)^2 dx + \beta \int_0^t \|u(s)\|^2_{H_0^1(U)} ds \leq \frac{1}{2} \int_\Omega \left(u^-(0)\right)^2 dx = 0.$$

Since $u^-(0) = (u_0)^- = 0$. So $u^- = 0$.

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Effect of Variable Thermal Conductivity & Heat Source/Sink Near a Stagnation Point on a Linearly Stretching Sheet using HPM

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Abstract- Aim of the paper is to investigate effects of variable thermal conductivity on flow of a viscous incompressible fluid in variable free stream near a stagnation point on a non-conducting stretching sheet. The equations of continuity, momentum and energy are transformed into ordinary differential equations and solved numerically using Similarity transformation and Homotopy Perturbation Method. The velocity and temperature distributions are discussed numerically and presented through graphs. Skin-friction coefficient and the Nusselt number at the sheet are derived, discussed numerically and their numerical values for various values of physical parameter are presented through Tables.

Keywords: homotopy perturbation method, similarity transformation method, steady, boundary layer, variable thermal conductivity, stretching sheet, skin-friction coefficient and nusselt number.

GJSFR-F Classification : *MSC 2010: 00A69*

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Effect of Variable Thermal Conductivity & Heat Source/Sink Near a Stagnation Point on a Linearly Stretching Sheet using HPM

Vivek Kumar Sharma [α] & Aisha Rafi [σ]

Abstract- Aim of the paper is to investigate effects of variable thermal conductivity on flow of a viscous incompressible fluid in variable free stream near a stagnation point on a non-conducting stretching sheet. The equations of continuity, momentum and energy are transformed into ordinary differential equations and solved numerically using Similarity transformation and Homotopy Perturbation Method. The velocity and temperature distributions are discussed numerically and presented through graphs. Skin-friction coefficient and the Nusselt number at the sheet are derived, discussed numerically and their numerical values for various values of physical parameter are presented through Tables.

Keywords: homotopy perturbation method, similarity transformation method, steady, boundary layer, variable thermal conductivity, stretching sheet, skin-friction coefficient and nusselt number.

I. Introduction

Study of heat transfer in boundary layer find applications in extrusion of plastic sheets, polymer, spinning of fibers, cooling of elastic sheets etc. The quality of final product depends on the rate of heat transfer and therefore cooling procedure has to be controlled effectively. Liquid metals have small Prandtl number of order 0.01~ 0.1(e.g. Pr = 0.01 is for Bismuth, Pr = 0.023 for Mercury etc.) and are generally used as coolants because of very large thermal conductivity.

Aim of the present paper is to investigate effects of variable thermal conductivity, heat source/sink and variable free stream on flow of a viscous incompressible electrically conducting fluid and heat transfer on a non-conducting stretching sheet. Linear stretching of the sheet is considered because of its simplicity in modelling of the flow and heat transfer over stretching surface and further it permits the similarity solution, which are useful in understanding the interaction of flow field with temperature field. The heat source and sink is included in the work to understand the effect of internal heat generation and absorption [Chaim (1998)].

The Homotopy Perturbation Method is a combination of the classical perturbation technique and homotopy technique, which has eliminated the limitations of the traditional perturbation methods. This technique can have full advantage of the traditional perturbation techniques. J. H. He, Approximate analytical solution for seepage flow with fractional derivatives in porous media. J. H. He, A coupling method of homotopy technique and perturbation technique for nonlinear problems. To illustrate the basic idea of the Homotopy Perturbation Method for solving nonlinear differential equations, we consider the following nonlinear differential equation:

$$A(u) - f(r) = 0 \qquad (1)$$

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Subject to boundary condition

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0 \tag{2}$$

Where A is a general differential operator, B is a boundary operator, f(r) is a known analytic function, and Γ is the boundary of the domain Ω.

The operator A can, generally speaking, be divided into two parts: a linear part L and a nonlinear part N. Equation can be rewritten as follows:

$$L(u) + N(u) - f(r) = 0 \tag{3}$$

By the homotopy technique, we construct a homotopy V(r,p) : Ω*(0,1)→R which satisfy

$$H(V,p) = (1-p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0 \tag{4}$$

$$H(V,p) = L(v) - L(u_0) + p\,L(u_0) + p[N(v) - f(r)] = 0 \tag{5}$$

Where $p \in [0, 1]$ is an embedding parameter and u_0 is an initial approximation of which satisfies the boundary conditions.

$$H(V,0) = L(v) - L(u_0)$$

$$H(V,1) = A(v) - f(r)] \tag{6}$$

Thus, the changing process of p from zero to unity is just that of $v(r, p)$ from $u_0(r)$ to $u(r)$. In Topology, this is called deformation and $L(v) - L(u_0)$, $A(v) - f(r)$ are called homotopic. According to the HPM, we can first use the embedding parameter p as a "small parameter," and assume that the solution of can be written as a power series in p:

$$V = V_0 + pV_1 + p^2 V_2 + \ldots\ldots\ldots\ldots\ldots \tag{7}$$

Setting p=1 results in the approximate solution of

$$u = \lim_{p \to 1} V = V_0 + V_1 + \ldots\ldots\ldots\ldots\ldots\ldots \tag{8}$$

The series is convergent for most cases; however, the convergent rate depends upon the Nonlinear operator $A(V)$. The second derivative of $N(V)$ with respect to V must be small because the parameter may be relatively large; that is, $p \to 1$.

In this paper is to investigate effects of variable thermal conductivity on flow of a viscous incompressible fluid in variable free stream near a stagnation point on a non-conducting stretching sheet.

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II. Formulation of the Problem

Consider steady two-dimensional flow of a viscous incompressible electrically conducting fluid of variable thermal conductivity in the vicinity of a stagnation point on a non-conducting stretching sheet It is assumed that external field is zero, the electric field owing to polarization of charges and Hall Effect are neglected. Stretching sheet is placed in the plane $y = 0$ and x-axis is taken along the sheet The fluid occupies the upper half plane i.e. $y > 0$. The governing equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \,, \tag{9}$$

$$\mathrm{u}\frac{\partial \mathrm{u}}{\partial \mathrm{x}} + \mathrm{v}\frac{\partial \mathrm{u}}{\partial \mathrm{y}} = -\frac{1}{\rho}\frac{\partial \mathrm{p}}{\partial \mathrm{x}} + \upsilon\frac{\partial^2 \mathrm{u}}{\partial \mathrm{y}^2}, \tag{10}$$

$$\rho C_p \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(k^*\frac{\partial T}{\partial y}\right) + Q\,(T - T_\infty) \,, \tag{11}$$

where ε -perturbation parameter, η-similarity parameter $\{ = (c/\nu)^{1/2}\, y\}$, η_∞-value of η at which boundary conditions is achived, κ-uniform thermal conductivity, κ^*-variable thermal conductivity, ν-kinematic viscosity, ρ-density of fluid, ψ-stream function, σ-electrical conductivity, θ-dimensionless temperature$\{ = (T - T_\infty) / (T_w - T_\infty)\}$, τ_w-shear stress, S-heat source/sink parameter $\{= Q/\rho\, C_p\, c \}$, T-fluid temperature.

The second derivatives of u and T with respect to x have been eliminated on the basis of magnitude analysis considering that Reynolds number is high. Hence the Navier-Stokes equation modifies into Prandtl's boundary layer equation.
The boundary conditions are.

$$\left.\begin{aligned} &y = 0: \quad u = u_w(x) = c \quad v = 0, \quad T = T_w \\ &y \to \infty: \quad u = U(x) = bx\,, \quad T = T_\infty \end{aligned}\right\} \tag{12}$$

Introducing the stream function $\psi\,(x, y)$ as defined by

$$u = \frac{\partial \psi}{\partial y}\,, \qquad v = -\frac{\partial \psi}{\partial x}, \tag{13}$$

the similarity variable $\eta = (c / \nu)^{1/2}\, y$ and

$$\Psi\,(x, y) = (c\,\nu)^{1/2}\; x\, f(\eta), \tag{14}$$

into the equations (3) and (5), we get

$$f''' + ff'' - (f')^2 + \lambda^2 = 0\,, \tag{15}$$

And

$$(1+\varepsilon)\,\theta'' + \varepsilon\,(\theta')^2 + \Pr\theta' f + \Pr S\theta = 0.$$

The governing boundary layer and thermal boundary layer equations (15) and (16) with the boundary conditions (17) are solved using Homotopy Perturbation Method.

Equations (15) and (16) are non-linear coupled differential equation. To solve these equations, we introduce the following Homotopy.

$$D(f,p) = (1-p)\left[\left(\frac{d^3 f}{d\eta^3}\right) - \left(\frac{d^3 f_I}{d\eta^3}\right)\right] + p\left[\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} - \left(\frac{df}{dn}\right)^2 + \lambda^2\right] = 0 \quad (16)$$

$$D(\theta,p) = (1-p)\left[\left(\frac{\partial^2\theta}{\partial\eta^2} + P_r S\theta\right) - \left(\frac{\partial^2\theta_I}{\partial\eta^2} + P_r S\theta_I\right)\right]$$

$$+P\left[(1+\varepsilon\theta)\frac{\partial^2\theta}{\partial n^2} + \varepsilon\left(\frac{\partial\theta}{dn}\right)^2 + P_r\frac{\partial\theta}{\partial n}f + P_r S\theta\right] = 0 \quad (17)$$

With the following assumption

$$f = f_0 + pf_1 + p^2 f_2 + \cdots \quad (18)$$

$$\theta = \theta_0 + p\theta_1 + p^2\theta_2 \quad (19)$$

Using equation (18),(19) into equation (10) and (11) and on comparing the like powers of p, we get the zeoth order equation,

$$\left[\left(\frac{d^3 f_0}{d\eta^3}\right) - \left(\frac{d^3 f_I}{d\eta^3}\right)\right] + \left[\frac{d^3 f_0}{d\eta^3} + f_0\frac{d^2 f_0}{d\eta^2} - \left(\frac{df_0}{dn}\right)^2 + \lambda^2\right] = 0, \quad (21)$$

$$\left[\left(\frac{\partial^2\theta_0}{\partial\eta^2} + P_r S\theta_0\right) - \left(\frac{\partial^2\theta_I}{\partial\eta^2} + P_r S\theta_I\right)\right] + \left[(1+\varepsilon\theta_0)\frac{\partial^2\theta_0}{\partial n^2} + \varepsilon\left(\frac{\partial\theta_0}{dn}\right)^2 + P_r\frac{\partial\theta_0}{\partial n}f + P_r S\theta_0\right]$$

$$= 0, \quad (21)$$

with the corresponding boundary conditions are of zeroth order equations are:

$$\eta = 0:\ f_0 = 0,\ f_0' = 1, \theta_0 = 1\,; \quad (22)$$

$$\eta = \infty:\quad f_0' = \lambda, \quad \theta_0 = 0 \quad (23)$$

$$\frac{d^3 f_1}{d\eta^3} + (e^{-n} - \lambda e^{-\eta}) + (\lambda\eta - e^{-n} + \lambda e^{-n} - \lambda + 1)(\lambda e^{-\eta} - e^{-\eta}) - (\lambda + e^{-n} - \lambda e^{-n})(\lambda + e^{-n} - \lambda e^{-n}) + \lambda^2 = 0 \quad (24)$$

$$\frac{\partial^2\theta_I}{\partial\eta^2} + P_r S\theta_I + e^{-\eta}(1+\varepsilon e^{-\eta}) + \epsilon\, e^{-2\eta} - P_r e^{-\eta}(\lambda\eta - e^{-\eta} + \lambda e^{-\eta} - \lambda + 1) - P_r S e^{-\eta}$$

$$= 0 \quad (25)$$

With the corresponding boundary conditions are of first order equations are:

$$\left.\begin{array}{l} \boldsymbol{\eta = 0:\ f_0 = 0,\ f_0' = 0, \theta_0 = 0} \\ \boldsymbol{\eta = \infty:\quad f_0' = 0, \quad \theta_0 = 0\,;} \end{array}\right\} \quad (26)$$

Solving equations with corresponding boundary conditions, the following functions can be obtained successively, by summing up the results, and $p \to 1$ we write the f($\boldsymbol{\eta}$) , $\boldsymbol{\theta(\eta)}$, profile as:

Notes

$$f(\eta) = \lambda\eta - e^{-\eta} + \lambda e^{-\eta} - \lambda + 1 + (\eta e^{-\eta} + 4e^{-\eta} + 3\eta - 4) \quad (27)$$

$$\theta(\eta) = e^{-\eta} + \frac{A_2 e^{-\eta}}{(A_1)^2+1} + \frac{A_3 e^{-2\eta}}{(A_1)^2+4} + \frac{A_4 e^{-\eta}}{(A_1)^2}\left(\eta - \frac{\eta-2}{(A_1)^2}\right) - \left(\frac{2A_4}{(A_1)^4} + \frac{A_3}{(A_1)^2+4} + \frac{A_2}{(A_1)^2+1}\right) e^{-\eta} \quad (28)$$

Notes

where $A_1 = \sqrt{P_r S}$: $A_2 = P_r S + P_r - \lambda P_r - 1$: $A_3 = \lambda P_r - P_r - 2$: $A_4 = \lambda P_r$ (29)

Skin-Friction: Skin-friction coefficient at the sheet is given by

$$C_f = x f''(0) \quad (30)$$

Nusselt Number: The rate of heat transfer in terms of the Nusselt number at the sheet is given by

$$Nu = -\theta'(0) \quad (31)$$

III. Conclusion

It is observed from Table 1 as L increases, the numerical values of $f''(0)$ also increase. It is noted from Table 2 that the numerical values of $-\theta'(0)$ increase when λ increases and $-\theta'(0)$ decreases when ε increases. The skin-friction coefficient and Nusselt number are presented by equations (30) and (31) and they are directly proportional to $f''(0)$ and $-\theta'(0)$ respectively. The effects of ε, Pr and S on Nusselt number have been presented through Table 3 respectively.

Table 1

λ	f''(0)	λ	f''(0)
0.0	-1	0.1	-1.0800
0.01	-1.0098	1.0	0.0004
0.05	-1.0450	2.0	2.0175

Table 2

λ	$-\theta'(0)$	ε	$-\theta'(0)$
0.1	.81235	0.0	0.223558
0.5	.13629	0.05	0.215792
2.0	.24133	0.1	0.204672

Table 3

ε	S	P_r	Nu
0	0	0.5	2.010357
0.1	0	0.5	3.542315
0	-0.1	1	2.565423
0.1	-0.1	1	2.845633

Notes

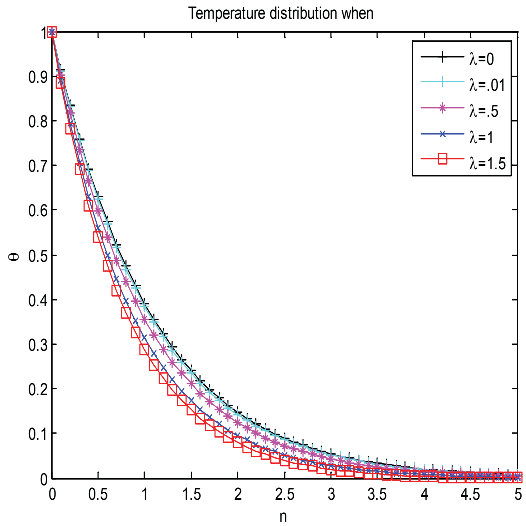

Figure 1

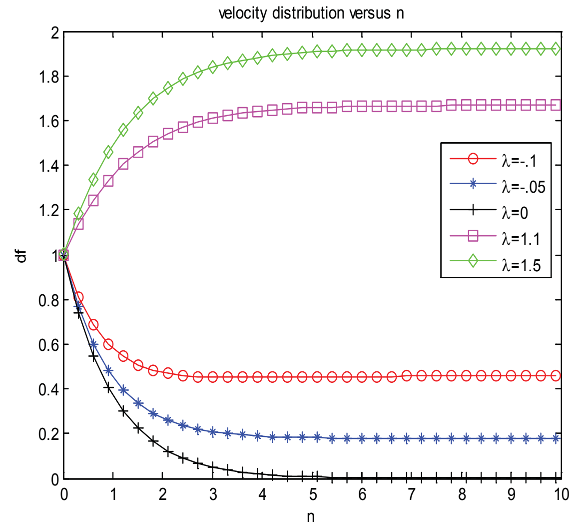

Figure 2

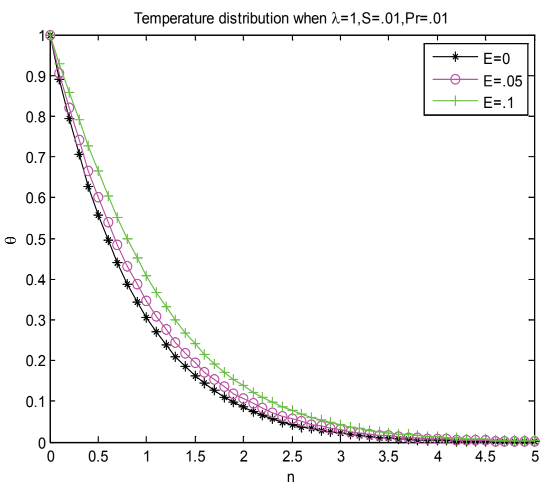

Figure 3

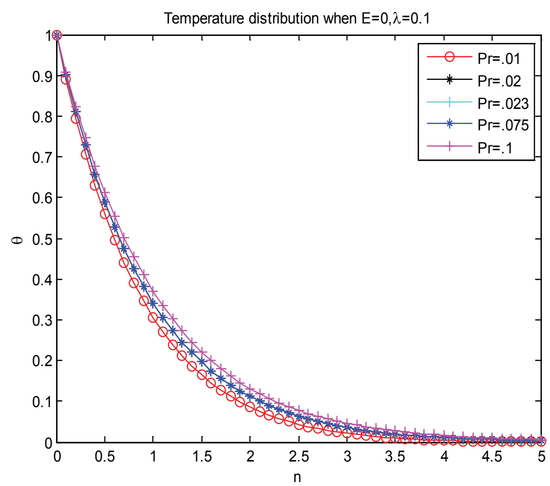

Figure 4

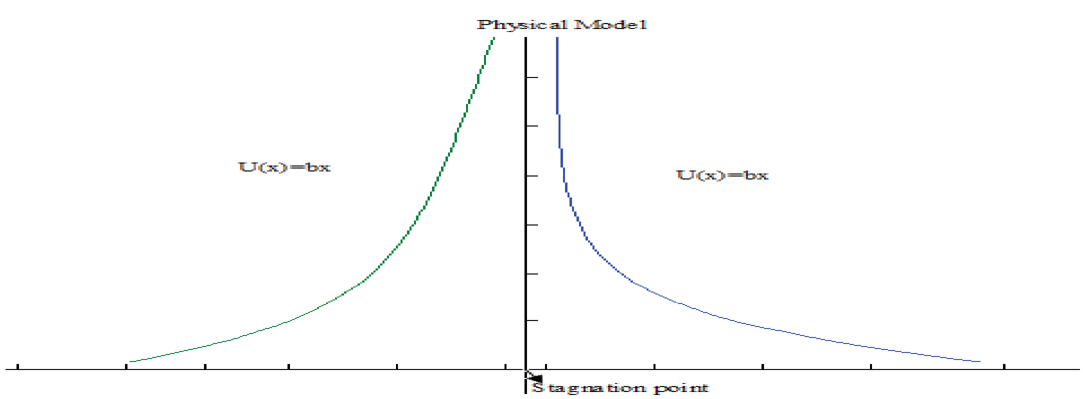

Figure 5

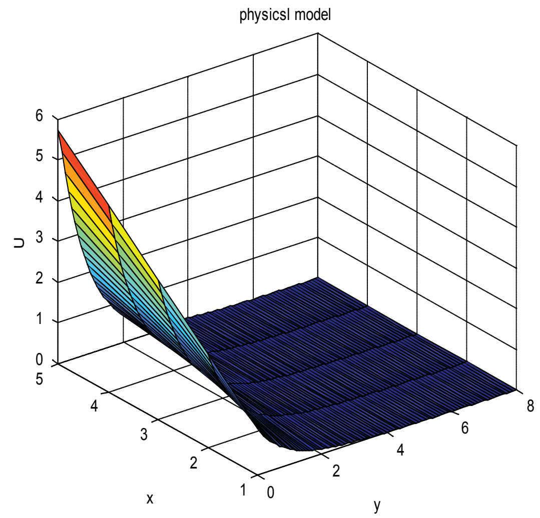

Figure 6

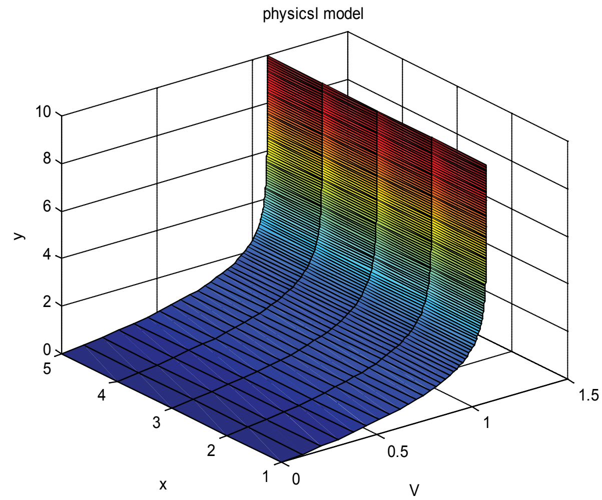

Figure 7

From figure 1, we observe that as λ increases, value of f' also increases. From figure 2 it is observed that when λ increase simultaneously θ also increases. In figure3, λ, S and Pr are constant but when ε increases θ will also increased.It is observed in figure 4, s, ε and λ are constant, when Pr increases, θ will also increase. Figure 5 is a physical model which becomes clearer from figure, 6 and 7.

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Homeotopy Groups of 2–Dimensional Manifolds with One Boundary Component

By David J. Sprows
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Introduction- Let Y be a compact, connected 2—dimensional manifold with boundary. The homeotopy group of Y, denoted H(Y), is defined to be the group of isotopy classes in the space of all homeomorphisms of Yonto Y. This group (also known as the mapping class group) has been studied for various manifolds (see, for example, [2] and [3]). It is also possible to consider "subhomeotopy groups" where there are restrictions placed on the action of the homeomorphisms on the boundary of Y (see, for example, [7] and [8]). In this note we will consider the special case of a compact, connected manifold with exactly on boundary component. For the remainder of this paper we will assume Y represents a compact, connected manifold with exactly one boundary component and we willlet X denote the closed 2—manifold obtained by sewing a disk to the boundary of Y. Let Aut $\pi_1(X,x_0)$ denote the group of automorphisms of $\pi_1(X,x_0)$ where $x_0 \varepsilon$— Bd(Y). In this paper we establish the following result.

Theorem. If Y is not aMoebius band or a disk, then H(Y)=Aut π_1 (X, x0).

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Homeotopy Groups of 2—Dimensional Manifolds with One Boundary Component

David J. Sprows

I. Introduction

Let Y be a compact, connected 2—dimensional manifold with boundary. The homeotopy group of Y, denoted H(Y), is defined to be the group of isotopy classes in the space of all homeomorphisms of Y onto Y. This group (also known as the mapping class group) has been studied for various manifolds (see, for example, [2] and [3]). It is also possible to consider "subhomeotopy groups" where there are restrictions placed on the action of the homeomorphisms on the boundary of Y (see, for example, [7] and [8]). In this note we will consider the special case of a compact, connected manifold with exactly on boundary component. For the remainder of this paper we will assume Y represents a compact, connected manifold with exactly one boundary component and we willlet X denote the closed 2—manifold obtained by sewing a disk to the boundary of Y. Let Aut $\pi_1(X,x_0)$ denote the group of automorphisms of $\pi_1(X,x_0)$ where $x_0 \varepsilon$ X—Bd(Y). In this paper we establish the following result.

Theorem. If Y is not aMoebius band or a disk, then H(Y) =Aut π_1 (X, x_0)

II. Proof of the Theorem

Let $[(X, x_0),(X,x_0)]$ denote the set of homotopy classes (rel x_0) of maps from (X, x_0) to (X,x_0) and let [f] denote the homotopy class (rel x_0) of a mapping f from (X,x_0) to (X,x_0). Let End π_1 (X, x_0) denote the set of endomorphisms of $\pi_1(X,x_0)$ with the operation of composition.

Lemma 1. If X is a closed 2-manifo1d with $\pi_2(X, x_0) = 0$ and $\phi : [(X, x_0), (X, x_0)] \to$ End $\pi_1(X,x_0)$ is given by $\phi([f]) = f_*$, then ϕ is a bijection which preserves the operation of composition.

Proof of lemma 1.

1. Clearly ϕ is well defined and $\phi([f\circ g]) = \phi([f]) \circ \phi([g])$
2. Claim ϕ is a surjection.

Let F in END $\pi_1(X,x_0)$. We define f: $(X,x_0) \to (X, x_0)$ as follows:

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Take x_0 to be vertex of the triangulation of X and let T be a maximal tree in X. If x ϵ T, we let $f(x) = x_0$. Now, suppose s is a 1—simplex not in T with h: [0,1] = s. Let Γ_i, be a path in T from x_0 to h(i), i = 0, 1. Define the loop α_s at x_0 by letting $\alpha_s(t) = \Gamma_0(t)$ if $-1 \le t \le 0$, $\alpha_s(t) = h(t)$ if $0 \le t \le 1$, and $\alpha_s(t) = \Gamma_0^{-1}(t)$ if $1 \le t \le 2$. If $F[\alpha_s] = [\beta]$, we let $f/s = \beta \circ h^{-1}s$. This defines f on the 1-skeleton of X.

Finally, if Δ is a 2-simplex with edges s1, s2, and s3, then $[\alpha_{S1} * \alpha_{S2} * \alpha_{S3}] = 1$. This means f/δΔ is null homotopic, i.e., f extends to Δ. Hence, the mapping of f defined on the 1-skeleton as above, extends to a mapping defined on all of the 2-dimensional manifold X. Note that by construction, $f_*[\alpha_s] = F[\alpha_s]$ and since $\{[\alpha_s]$: S is a 1-simplex of X″ generates $\pi_1(X,x_0)$ we have $f_* = F$.

III. Claim ϕ is an Injection

Suppose $f_* = g_*$. As in Part 2), let T be a maximal tree. f/T is homotopic (rel x_0) to a map which sends T to x_0 (just use the retraction of T to x_0). Hence, by the homotopy extension property, f is homotopic (rel x_0) to a map f' with $f'(T) = x_0$. Therefore, we can assume $f(T) = g(T) = x_0$. In particular, for each 1-simplex s, f/s and g/s are loops at x_0. Since $f_* = g_*$ this means f/s is homotopic to g/s (rel x_0). Thus, for each 2—simplex Δ we have a map H: δ(ΔxI) → X where H/Δx0 = f/Δ, H/Δxl = f/Δ and for each 1-simplex s in δΔ, H/SxI is a homotopy from f/s to g/s. Since $\pi 2(X,x_0) = 0$ and $\delta(\Delta xI) = S^2$, this map H can be extended to all of ΔxI. Fitting together each of these H's we get a homotopy (rel x_0) from f to g, i.e., [f] = [g].

Lemma 2 : If X is a closed 2—manifold and $h:(X,x_0) \to (X,x_0)$ is a homeomorphism which is homotopic to the identity (rel x_0) then h is isotopic to the identity (rel x_0).

Proof. This is a special case of Theorem 6.3 of [1].

Lemma 3. If X is a closed 2—manifold with x_0 ϵ X and G is an automorphism of $\pi 1(X,x_0)$, then there exists a homeomorphism h: $(X,x_0) + (X,x_0)$ with $h_* = G$.

Proof. This result is proved in [4].

Let $H(X,x_0)$ denote the group of isotopy classes (rel x_0) in the space of all homeomorphisms of (X,x_0) onto (X,x_0).

By Lemma 2 the function from $H(X,x_0)$ to $[(X,x_0)]$ which sends the isotopy class (rel x_0) of a homeomorphism to its homotopy class (rel x_0) is an injection. By Lemma 1 the composition $H(X,x_0) \to [(X,x_0)\ ,\ (X,x_0)] \to$ End $\pi 1(X,x_0)$ is a monomorphism of the group $H(X,x_0)$ onto a subgroup of Aut $\pi 1(X,x_0)$. Finally Lemma 3 shows that this monomorphism is an isomorphism onto Aut $\pi_1(X,x_0)$. Thus the proof of the theorem reduces to showing that H(Y) is isomorphic to $H(X,x_0)$. This is done by the following lemma.

Notes

Notes

Lemma 4. If Y is a compact, connected 2—dimensional manifold with one boundary component and X is the closed 2—manifold obtained by sewing a disk to the boundary of Y and $x_0 \in$ X-Bd(Y)then H(X,x_0) = H(Y).

Proof. This result is a special case of Theorem 6 of [6].

Remark 3. If Y is a Moebius band or disk, then H(Y) is not isomorphic to Aut π1(X,x0). In the case Y is a disk, so that X = S^2, we have Aut π1 (X,x_0) = 1 while H(Y) = Z_2(see Theorem 4.2 of [4]). In the case Y is a Moebius band, so that X = P^2, we have Aut π1, (X,x_0) = 1 while H(Y) = Z_2 (see Theorem 8.1 of[4]).

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Improved Class of Ratio -Cum- Product Estimators of Finite Population Mean in two Phase Sampling

By Waikhom Warseen Chanu & B. K. Singh

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Abstract- In the present study, we have proposed a class of ratio-cum-product estimators for estimating finite population mean of the study variable in two phase sampling. The bias and mean square error of the proposed estimator have been obtained. The asymptotically optimum estimator (AOE) in this class has also been identified along with its approximate bias and mean square error. Comparison of the proposed class of estimators with other estimators is also worked out theoretically to demonstrate the superiority of the proposed estimator over the other estimators.

Keywords: ratio-cum-product, mean, two phase sampling, asymptotically optimum estimator, bias, mean square error.

GJSFR-F Classification : 62D05

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Improved Class of Ratio -Cum- Product Estimators of Finite Population Mean in two Phase Sampling

Waikhom Warseen Chanu [α] & B. K. Singh [σ]

Abstract- In the present study, we have proposed a class of ratio-cum-product estimators for estimating finite population mean $\bar{Y}$ of the study variable y in two phase sampling. The bias and mean square error of the proposed estimator have been obtained. The asymtotically optimum estimator (AOE) in this class has also been identified along with its approximate bias and mean square error. Comparison of the proposed class of estimators with other estimators is also worked out theoretically to demonstrate the superiority of the proposed estimator over the other estimators.

Keywords: ratio-cum-product, mean, two phase sampling, asymptotically optimum estimator, bias, mean square error.

I. Introduction

The literature on survey sampling describes a great variety of techniques for using auxiliary information in order to obtain improved estimators for estimating some most common population parameter such as population total, population mean, population proportion, population ratio etc. More often we are interested in the estimation of the mean of a certain characteristic of a finite population on the basis of a sample taken from the population following a specified sampling procedure.

Use of auxiliary information has shown its significance in improving the efficiency of estimators of unknown population parameters. Cochran (1940) used auxiliary information in the form of population mean of auxiliary variate at estimation stage for the estimation of population parameters when study and auxiliary variate are positively correlated. In case of negative correlation between study variate and auxiliary variate, Robson (1957) defined product estimator for the estimation of population mean which was revisited by Murthy (1967). Ratio estimator performs better than simple mean estimator in case of positive correlation between study variate and auxiliary variate.

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For further discussion on ratio cum product estimator, the reader is referred to Singh (1967), Shah and Shah (1978), Singh and Tailor (2005), Tailor and Sharma (2009), Tailor and Sharma (2009), Sharma and Tailor (2010), Choudhury and Singh (2011) etc.

Notes

When the population mean $\bar{X}$ of the auxiliary variable x is unknown before start of the survey, it is estimated from a preliminary large sample on which only the auxiliary characteristic x is observed. The value of X in the estimator is then replaced by its estimate. After then a smaller second-phase sample of the variate of interest (study variate) y is then taken. This technique is known as double sampling or two-phase sampling. Neyman (1938) was the first to give the concept of double sampling in connection with collecting information on the strata sizes in a stratified sampling

Consider a finite population $U = (u_1, u_2, u_3, ..., u_N)$ of size N units, y and x are the study and auxiliary variate respectively. When the population mean $\bar{X}$ of x is not known, a first phase sample of size n_1 is drawn from the population on which only the x characteristic is measured in order to furnish a good estimate of $\bar{X}$. After then a second-phase sample of size n $(n < n_1)$ is drawn on which both the variates y and x are measured.

The usual ratio and product estimators in double sampling are:

$$\bar{Y_R}^d = \bar{y}\frac{\bar{x_1}}{\bar{x}}$$

and

$$\bar{Y_P}^d = \bar{y}\frac{\bar{z}}{\bar{z_1}}$$

where $\bar{x}, \bar{y}$ and $\bar{z}$ are the sample mean of x, y and z respectively based on the sample of size n out of the population N units and $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$, $\bar{x_1} = \frac{1}{n_1}\sum_{i=1}^{n_1} x_i$ and $\bar{z_1} = \frac{1}{n_1}\sum_{i=1}^{n_1} z_i$ denote the sample mean of x and z based on the first- phase sample of the size n_1.

Singh (1967) improved the ratio and the product methods of estimation by studying the ratio cum product estimator for estimating $\bar{Y}$ as

$$\bar{Y}_{RP} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\frac{\bar{z}}{\bar{Z}}\right)$$

Motivated by Singh (1967), Choudhury and Singh (2011) proposed a modified class of ratio cum product type of estimator for estimating population mean $\bar{Y}$ as

Notes

$$\bar{Y}_{RP}^{(\alpha)} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\frac{\bar{z}}{\bar{Z}}\right)^{\alpha}$$

Motivated by ?) and as an extension to the work of Choudhury and Singh (2011), we have developed an improved class of ratio-cum-product estimators in double sampling to estimate the population mean $\bar{Y}$ theoretically and studied the properties of the proposed estimator.

II. The Proposed Estimator

The proposed improved class of ratio-cum-product estimators of population mean $\bar{Y}$ in two-phase sampling is given as

$$\bar{Y}_{RP}^{w(d)} = \bar{y}\left(\frac{\bar{x}_1}{\bar{x}}.\frac{\bar{z}}{\bar{z}_1}\right)^{\alpha}. \quad (1)$$

where α is a suitably chosen constant.

To obtain the bias and MSE of $\bar{Y}_{RP}^{w(d)}$ to the first degree of approximation, we write

$$e_0 = \left(\bar{y}-\bar{Y}\right)/\bar{Y}, \quad e_1 = \left(\bar{x}-\bar{Y}\right)/\bar{X}, \quad e_2 = \left(\bar{x}_1-\bar{X}\right)/\bar{X}, \quad e_3 = \left(\bar{z}-\bar{Z}\right)/\bar{Z},$$

$$e_4 = \left(\bar{z}_1-\bar{Z}\right)/\bar{Z}.$$

Expressing $Y_{RP}^{\bar{w}(d)}$ in terms of e's and neglecting higher power of e's, we have

$$Y_{RP}^{\bar{w}(d)} = \bar{Y}\left(1+e_0\right)\left\{\left(1+e_2\right)\left(1+e_1\right)^{-1}\left(1+e_3\right)\left(1+e_4\right)^{-1}\right\}^{\alpha}.$$

Assuming the sample size to be large enough so that $|e_1| < 1$, $|e_4| < 1$ and expanding $(1+e_1)^{-1}$, $(1+e_4)^{-1}$ in powers of e_1, e_4, multiplying out and neglecting higher powers of $e's$, we have

$$\bar{Y}_{RP}^{w(d)} = \bar{Y}\left(1+e_0\right)\left[1-\left\{e_1-e_2-e_3+e_4-e_1^2-e_4^2+e_1e_2-e_1e_4 +e_2e_4+e_1e_3-e_2e_3+e_3e_4\right\}\right]^{\alpha}$$

$$\bar{Y}_{RP}^{w(d)} - \bar{Y} = \bar{Y}\left[e_0-\alpha\left(e_1-e_2-e_3+e_4-\frac{e_1^2}{2}+\frac{e_2^2}{2}+\frac{e_3^2}{2}-\frac{e_4^2}{2}+e_0e_1 -e_0e_2+e_0e_4-e_0e_3\right)+\alpha^2\left(\frac{e_1^2}{2}+\frac{e_2^2}{2}+\frac{e_3^2}{2}+\frac{e_4^2}{2}-e_1e_2 -e_1e_3+e_1e_4+e_2e_3-e_2e_4-e_3e_4\right)\right] \quad (2)$$

The following two cases will be considered separately.

Case I: When the second phase sample of size n is subsample of the first phase of size n_1.

Case II: when the second phase sample of size n is drawn independently of the first phase sample of size n_1.

CASE I

III. Bias, MSE and Optimum Value of $\bar{Y}_{RP}^{wd}$ in Case I

In this case, we have

$$E\left(e_0\right)=E\left(e_1\right)=E\left(e_2\right)=E\left(e_3\right)=E\left(e_4\right)=0;$$

$$E\left(e_0^2\right)=\left(\frac{1-f}{n}\right)C_Y^2;\quad E\left(e_1^2\right)=\left(\frac{1-f}{n}\right)C_X^2;\quad E\left(e_3^2\right)=\left(\frac{1-f}{n}\right)C_Z^2;$$

$$E\left(e_2^2\right)=E\left(e_1e_2\right)=\left(\frac{1-f^*}{n}\right)C_X^2;\quad E\left(e_4^2\right)=E\left(e_3e_4\right)=\left(\frac{1-f^*}{n}\right)C_Z^2;$$

$$E\left(e_0e_1\right)=\left(\frac{1-f}{n}\right)\rho_{YX}C_YC_X;\quad E\left(e_0e_2\right)=\left(\frac{1-f^*}{n}\right)\rho_{YX}C_YC_X;$$

$$E\left(e_0e_3\right)=\left(\frac{1-f}{n}\right)\rho_{YZ}C_YC_Z;\quad E\left(e_0e_4\right)=\left(\frac{1-f^*}{n}\right)\rho_{YZ}C_YC_Z;$$

$$E\left(e_1e_3\right)=\left(\frac{1-f}{n}\right)\rho_{XZ}C_XC_Z;$$

$$E\left(e_1e_4\right)=E\left(e_2e_3\right)=E\left(e_2e_4\right)=\left(\frac{1-f^*}{n}\right)\rho_{XZ}C_XC_Z \quad (3)$$

where $f=\frac{n}{N}$ is the sampling fraction, $f^*=\frac{n_1}{N}$, $C_X^2=\frac{S_X^2}{\bar{X}^2}$, $C_Y^2=\frac{S_Y^2}{\bar{Y}^2}$, $C_Z^2=\frac{S_Z^2}{\bar{Z}^2}$.

Notes

Taking expectations on both the sides and using the results of (3) in (2), we get the bias of $\bar{Y}_{RP}^{w(d)}$ as

$$B\left(\bar{Y}_{RP}^{w(d)}\right)_I = \bar{Y}\left(\frac{1-f_1}{2n}\right)\left[\alpha^2\left\{C_X^2 + C_Z^2\left(1-2K_{XZ}\right)\right\} + \alpha\left\{C_X^2\left(1-2K_{YX}\right) - C_Z^2\left(1-2K_{YZ}\right)\right\}\right] \quad (4)$$

$$= \bar{Y}\left(\frac{1-f_1}{2n}\right)\left[\alpha^2 K_3 + \alpha\left(K_1 - K_2\right)\right] \quad (5)$$

where $f_1 = \frac{n}{n_1}$, $K_{YX} = \rho_{YX}\frac{C_Y}{C_X}$, $K_1 = C_X^2\left(1-2K_{YX}\right)$, $K_2 = C_Z^2\left(1-2K_{YZ}\right)$, $K_3 = C_X^2 + C_Z^2\left(1-2K_{XZ}\right)$.

Now from equation (2), we have

$$\bar{Y}_{RP}^{w(d)} - \bar{Y} = \bar{Y}\left[e_0 - \alpha\left(e_1 - e_2 - e_3 + e_4\right)\right].$$

Squaring both the sides and taking expectations in the above equation and using the results of (3), we get the mean square error of $\bar{Y}_{RP}^{w(d)}$ to the first degree of approximation as

$$M\left(\bar{Y}_{RP}^{w(d)}\right)_I = \bar{Y}^2\left(\frac{1-f}{n}\right)C_Y^2 + \bar{Y}^2\left(\frac{1-f_1}{n}\right)\left[\alpha^2\left\{C_X^2 + C_Z^2\left(1-2K_{XZ}\right)\right\} - 2\alpha\left(C_{YX} - C_{YZ}\right)\right]$$

$$= \bar{Y}^2\left(\frac{1-f}{n}\right)C_Y^2 + \bar{Y}^2\left(\frac{1-f_1}{n}\right)\left[\alpha^2 K_3 - 2\alpha S_1\right] \quad (6)$$

where $S_1 = C_{YX} - C_{YZ}$, $C_{YZ} = \rho_{YZ}C_Y C_Z$, $C_{YX} = \rho_{YX}C_Y C_X$

Differentiating $M\left(\bar{Y}_{RP}^{w(d)}\right)$ w.r.t α and equating to zero, we get

$$\alpha = \frac{S_1}{K_3}. \quad (7)$$

Now putting the optimum value of α from (7) in the proposed estimator (1), we get the asymptotically optimum estimator(AOE) as

$$\left(\bar{Y}_{RP}^{w(d)}\right)_{I(opt)} = \bar{y}\left(\frac{\bar{x}_1}{\bar{x}}\frac{\bar{z}}{\bar{z}_1}\right)^{I\alpha(opt)}.$$

Therefore, after putting the value of α in (4) and (6), we obtain the optimum bias and MSE of $\bar{Y}_{RP}^{w(d)}$ respectively as

$$B\left(\bar{Y}_{RP}^{w(d)}\right)_{I\alpha(opt)} = \bar{Y}\left(\frac{1-f_1}{2n}\right)\frac{S_1}{K_3}\left[K_1' - K_2'\right]$$

where $K_1' = C_X^2\left(1-K_{YX}\right), \quad K_2' = C_Z^2\left(1-K_{YZ}\right).$

$$M\left(\bar{Y}_{RP}^{w(d)}\right)_{I\alpha(opt)} = \bar{Y}^2\left(\frac{1-f}{n}\right)C_Y^2 - \bar{Y}^2\left(\frac{1-f_1}{n}\right)\frac{S_1^2}{K_3}. \tag{8}$$

Notes

Remark 1 *For $\alpha = 1$, the estimator reduces to ratio cum product estimator in double sampling. The bias and MSE of $\bar{Y}_{RP}^{(d)}$ are obtained by putting $\alpha = 1$ in relation (4) and (6) as follows*

$$B\left(\bar{Y}_{RP}^{(d)}\right)_I = \bar{Y}\left(\frac{1-f}{n}\right)\left[K_1' - C_{XZ} + C_{YZ}\right] \tag{9}$$

where $K_1' = C_X^2\left(1-K_{YX}\right)$

$$\text{and} \quad MSE\left(\bar{Y}_{RP}^{(d)}\right)_I = \left(\frac{1-f}{n}\right)S_Y^2 + \bar{Y}^2\left(\frac{1-f_1}{n}\right)\left(K_1 + K_4\right) \tag{10}$$

where $K_4 = C_Z^2\left(1 - 2K_{XZ} + 2K_{YZ}\right).$

Remark 2 *For $\alpha = 1$ and when the auxiliary variate z is not used, i.e if z is non-zero constant, the proposed estimator reduces to the usual ratio estimator in two phase sampling. The bias and mean square error of $\bar{Y}_R^d$ can be obtained by putting $\alpha = 1$ and omitting the terms of z in equation (4) and (6), respectively as*

$$B\left(\bar{Y}_R^d\right)_I = \bar{Y}\left(\frac{1-f_1}{n}\right)K_1'$$

and

$$MSE\left(\bar{Y}_R^d\right)_I = \bar{Y}\left(\frac{1-f}{n}\right)C_Y^2 + \bar{Y}\left(\frac{1-f_1}{n}\right)K_1. \tag{11}$$

Notes

Remark 3 *For $\alpha = 1$ and when the auxiliary variate x is not used, i.e if x is non-zero, the proposed estimator reduces to the usual product estimator in two phase sampling. The bias and mean square error of $\bar{Y}_P^d$ can be obtained by putting $\alpha = 1$ and omitting the terms of x in equation (4) and (6), respectively as*

$$B\left(\bar{Y}_P^d\right)_I = \bar{Y}\left(\frac{1-f_1}{n}\right)C_{YZ}$$

$$\text{and} \quad M\left(\bar{Y}_P^d\right)_I = \bar{Y}^2\left(\frac{1-f}{n}\right)C_Y^2 + \bar{Y}^2\left(\frac{1-f_1}{n}\right)K_5 \tag{12}$$

where $K_5 = C_X^2\left(1 + 2K_{YX}\right)$.

IV. Efficiency Comparisons

Compairison of the optimum proposed estimator $\left(\bar{Y}_{RP}^{w(d)}\right)_{I\alpha(opt)}$

a) with sample mean per unit estimator $\bar{y}$

The MSE of sample mean $\bar{y}$ under SRSWOR sampling scheme is given by

$$V\left(\bar{y}\right) = \left(\frac{1-f}{n}\right)S_Y^2. \tag{13}$$

From equation (13) and (8), we observed

$$V\left(\bar{y}\right) - M\left(\bar{Y}_{RP}^{w(d)}\right)_{I\alpha I(opt)} = \frac{S_1^2}{K_3} > 0. \tag{14}$$

if $K_3 > 0$ i.e $K_{XZ} < 1/2$.

b) with ratio estimator in double sampling

From equation (11) and (8), we observed

$$M\left(\bar{Y}_R^d\right) - M\left[\bar{Y}_{RP}^{w(d)}\right]_{I\alpha(opt)} = \bar{Y}^2\left(\frac{1-f_1}{n}\right)\left(K_1 + \frac{S_1^2}{K_3}\right) > 0. \tag{15}$$

if $K_1 > 0$, $K_3 > 0$ i.e. $K_{YX} < 1/2$, $K_{XZ} < 1/2$.

c) with product estimator in double sampling

From (12) and (8), we observed

$$M\left(\bar{Y}_P^d\right) - MSE\left[\bar{Y}_{RP}^{w(d)}\right]_{I\alpha(opt)} = \bar{Y}^2\left(\frac{1-f_1}{n}\right)\left[K_5 + \frac{S_1^2}{K_3}\right] > 0 \tag{16}$$

if $K_3 > 0,\ K_5 > 0$ i.e. $K_{XZ} < 1/2$.

Notes

d) with ratio cum product estimators in double sampling

From (10) and (8), we observed

$$M\left(\bar{Y}_{RP}^d\right) - M\left[\bar{Y}_{RP}^{w(d)}\right]_{I\alpha(opt)} = \bar{Y}^2\left[K_1 + K_5 + \frac{S_1^2}{K_3}\right] > 0 \tag{17}$$

if $K_1 > 0,\ K_3 > 0,\ K_5 > 0$ i.e. $K_{YX} < 1/2,\ K_{XZ} < 1/2,\ K_{XZ} - K_{YZ} < 1/2$.

Now we state the theorem

Theorem 4 *To the first degree of approximation, the proposed class of estimators $\bar{Y}_{Rd}^{w(d)}$ under the optimality (7) is consider to be more efficient than $\bar{Y}_R^d$, $\bar{Y}_P^d$, $\bar{Y}_{RP}^d$ and $\bar{y}$ under the given conditions K_1, K_3, K_4, and K_5 >0, where $K_1 = C_X^2\left(1-2K_{YX}\right)$, $K_3 = C_X^2 + C_Z^2\left(1-2K_{XZ}\right)$, $K_4 = C_Z^2\left(1-2K_{XZ}+2K_{YZ}\right)$ and $K_5 = C_X^2\left(1+2K_{YX}\right)$.*

V. Bias, MSE and Optimum Value of $\bar{Y}_{RP}^{w(d)}$ in Case II

In this case, we have

$$E\left(e_0\right) = E\left(e_1\right) = E\left(e_2\right) = E\left(e_3\right) = E\left(e_4\right) = 0;$$

$$E\left(e_0^2\right) = \left(\frac{1-f}{n}\right)C_Y^2;\quad E\left(e_1^2\right) = \left(\frac{1-f}{n}\right)C_X^2;\quad E\left(e_3^2\right) = \left(\frac{1-f}{n}\right)C_Z^2;$$

$$E\left(e_2^2\right) = E\left(e_1e_2\right) = \left(\frac{1-f^*}{n}\right)C_X^2;\quad E\left(e_4^2\right) = E\left(e_3e_4\right) = \left(\frac{1-f^*}{n}\right)C_Z^2;$$

$$E\left(e_0e_1\right) = \left(\frac{1-f}{n}\right)\rho_{YX}C_YC_X;\quad E\left(e_0e_3\right) = \left(\frac{1-f}{n}\right)\rho_{YZ}C_YC_Z;$$

$$E\left(e_1e_3\right) = \left(\frac{1-f}{n}\right)\rho_{XZ}C_XC_Z;\quad E\left(e_2e_4\right) = \left(\frac{1-f^*}{n}\right)\rho_{XZ}C_XC_Z;$$

$$E\left(e_0e_2\right) = E\left(e_0e_4\right) = E\left(e_1e_2\right) = E\left(e_1e_4\right) = E\left(e_3e_2\right) = E\left(e_3e_4\right) = 0. \tag{18}$$

Taking expectations in (2) and using the results of (18), we get the bias of $\bar{Y}_{RP}^{w(d)}$ to the first degree of approximation as

Notes

$$B\left(\bar{Y}_{RP}^{w(d)}\right)_{II} = \bar{Y}\left[\alpha^2 N_1 K_3 + \alpha\left\{f''\left(C_X^2 - C_Z^2\right) - f' S_1\right\}\right].$$

where $f' = \frac{1-f}{n}, \quad f'' = \frac{1-f_1}{2n}, \quad N_1 = \frac{1}{2}\left(\frac{1}{n} + \frac{1}{n_1} - \frac{2}{N}\right).$

Squaring and taking expectations in both the sides of (2) and using the results of (18), we obtain the MSE of $\left(\bar{Y}_{RP}^{w(d)}\right)$ to the first degree of approximation as

$$M\left(\bar{Y}_{RP}^{w(d)}\right)_{II} = \bar{Y}^2 f' C_Y^2 + \bar{Y}^2 2\left[\alpha^2 N_1 K_3 - \alpha f' S_1\right]. \tag{19}$$

Minimization of (19) is obtained with optimum value of α as

$$\alpha = \frac{f' S_1}{2 N_1 K_3} = \alpha_{II(opt)}. \tag{20}$$

Substituting the value of α from (20) in (1) gives the AOE of (1) as

$$\left\{\bar{Y}_{RP}^{w(d)}\right\}_{II(opt)} = \bar{y}\left(\frac{\bar{x}_1}{\bar{x}}.\frac{\bar{z}}{\bar{z}_1}\right)^{\alpha_{II(opt)}}. \tag{21}$$

Thus, the resulting bias and MSE of (21) are, respectively given as

$$B\left(\bar{Y}_{RP}^{w(d)}\right)_{II\alpha(opt)} = \bar{Y}\frac{f' S_1}{2N_1 K_3}\left[f'' C_1 - \frac{f' S_1}{2 N_1 K_1}\right]$$

where $C_1 = C_X^2 - C_Z^2$ and

$$M\left(\bar{Y}_{RP}^{w(d)}\right)_{II\alpha(opt)} = \bar{Y}^2 f' C_Y^2 - \bar{Y}^2\frac{f'^2 S_1^2}{2 N_1 K_3}.$$

For simplicity, we assume that the population size N is large enough as compared to the sample sizes n and n_1 so that the finite population correction (FPC) terms $1/N$ and $2/N$ are ignored.

Ignoring the FPC in (19), the MSE of $\left(\bar{Y}_{RP}^{w(d)}\right)_{II}$ reduces to

$$M\left(\bar{Y}_{RP}^{w(d)}\right)_{II} = \bar{Y}\frac{C_Y^2}{n} + \bar{Y}^2\left[\alpha^2\frac{1}{2}\left(\frac{1}{n}+\frac{1}{n_1}\right)K_3 - \alpha\frac{S_1}{n}\right]$$

which is minimized for

$$\alpha = \frac{n_1 S_1}{(n+n_1)\,K_3} = \alpha^*_{II(opt)} \quad \text{(say)} \tag{22}$$

Substituting the value of α from (22) in (1), we obtained AOE of (1) as

$$\left(\bar{Y}_{RP}^{w(d)}\right)^* = \bar{Y}\left[\frac{x}{x_1}.\frac{z_1}{z}\right]^{\alpha^*_{II(opt)}}.$$

Therefore, the resulting MSE of $\left(\bar{Y}_{RP}^{w(d)}\right)^*$ is

$$M\left(\bar{Y}_{RP}^{w(d)}\right)^*_{II\alpha(opt)} = \bar{Y}^2\frac{C_Y^2}{n} - \bar{Y}^2\frac{n_1 S_1^2}{n\,(n+n_1)\,K_3}. \tag{23}$$

Remark 5 *For $\alpha = 1$, the proposed estimator reduces to ratio cum product estimator in double sampling and MSE is given as*

$$M\left(\bar{Y}_{RP}^{w(d)}\right)^*_{II\alpha(opt)} = \bar{Y}^2\frac{C_Y^2}{n} + \bar{Y}^2 2\left[\frac{1}{2}\left(\frac{1}{n}+\frac{1}{n_1}\right)K_3 - \frac{S_1}{n}\right]. \tag{24}$$

Ignoring the FPC, the variance of $\bar{y}$ under SRSWOR is given by

$$V\,(y)_{II} = \bar{Y}^2\frac{C_Y^2}{n}. \tag{25}$$

and the MSE of $\left(\bar{Y}_{Rd}\right)_{II}$ and $\left(\bar{Y}_{Pd}\right)_{II}$ are given by

$$M\left(\bar{Y}_{Rd}\right)_{II} = \bar{Y}^2\frac{C_Y^2}{n} + \bar{Y}^2\left[C_X^2\left(\frac{2}{n}-\frac{1}{n_1}-\frac{2}{n}K_{YX}\right)\right] \tag{26}$$

Notes

and

$$\left(\bar{Y}_{Pd}\right)_{II} = \bar{Y}^2\frac{C_Y^2}{n} + \bar{Y}^2\left[\frac{C_X^2}{n} + \frac{1}{2}\left(\frac{1}{n} - \frac{1}{n_1}\right)C_Z^2 + \frac{2C_{YZ}}{n}\right] \tag{27}$$

respectively.

Notes

VI. Efficiency Copmparisons

Compairison of the optimum proposed estimator $\left(\bar{Y}_{RP}^{w(d)}\right)^*_{II\alpha(opt)}$

a) with sample mean per unit estimator

From (25) and (23), we observed that

$$V\left(\bar{y}\right)_{II} - M\left(\bar{Y}_{RP}^{w(d)}\right)^*_{II\alpha(opt)} = \bar{Y}^2\frac{n_1 S_1^2}{n\left(n+n_1\right)K_3} > 0 \tag{28}$$

if $K_3 > 0$ i.e $K_{XZ} < 1/2$.

b) with ratio estimator in double sampling

From (26) and (23), we observed that

$$M\left(\bar{Y}_{Rd}\right)_{II} - M\left(\bar{Y}_{RP}^{w(d)}\right)^*_{II\alpha(opt)} = \bar{Y}^2\left[\frac{n_1 S_1^2}{n\left(n+n_1\right)K_3} + C_X^2 P\right] > 0 \tag{29}$$

if $K_3 > 0,\ P > 0$ i.e $K_{XZ} < 1/2,\ K_{YX} < 1 - \frac{n}{2n_1},$

where $P = \frac{2}{n}\left(1 - K_{YX}\right) - \frac{1}{n_1}.$

c) with product estimator in double sampling

From (27) and (23), we observed that

$$M\left(\bar{Y}_{Pd}\right)_{II} - M\left(\bar{Y}_{RP}^{w(d)}\right)^*_{II\alpha(opt)} = \bar{Y}^2\left[Q - \frac{C_Z^2}{2n_1} + \frac{n_1 S_1^2}{n\left(n+n_1\right)K_3}\right] > 0 \tag{30}$$

if $K_3 > 0$ i.e $K_{XZ} < 1/2$, where $Q = C_X^2 + \frac{C_Z^2}{2} + 3C_{YX}$.

d) with ratio cum product estimator in double sampling

From (24) and (23), we observed that

$$M\left(\bar{Y}_{RPd}\right)_{II} - M\left(\bar{Y}_{RP}^{w(d)}\right)^{*}_{II\alpha(opt)} = \bar{Y}^{2}\left[\left(\frac{1}{n}+\frac{1}{n_1}\right)K_3 - \frac{S_1}{2n} + \frac{n_1 S_1^2}{n\left(n+n_1\right)K_3}\right] > 0 \tag{31}$$

if $K_3 > 0$ i.e $K_{XZ} < 1/2$.

Notes

VII. Conclusion

We have developed an efficient class of ratio-cum-product estimators in two phase sampling. The comparative study shows that the proposed estimator $\bar{Y}_{RP}^{w(d)}$ established their superiority over sample mean $\bar{Y}$, ratio estimator $\bar{Y_R^d}$, product estimator$\bar{Y_P^d}$ and ratio-cum-product estimator $\bar{Y_{RP}^d}$ in two-phase sampling under the given conditions. Hence from the resulting equation (14), (15), (16) and (17), we conclude that under the given conditions the proposed estimator is consider to be the best estimator.

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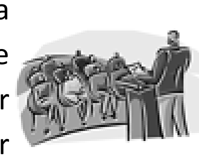

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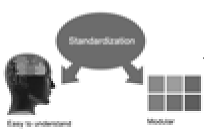

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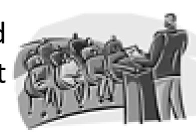

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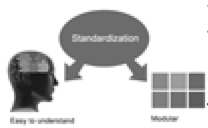

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"

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4. Manuscript’s Category,

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16. Use proper verb tense: Use proper verb tenses in your paper. Use past tense, to present those events that happened. Use present tense to indicate events that are going on. Use future tense to indicate future happening events. Use of improper and wrong tenses will confuse the evaluator. Avoid the sentences that are incomplete.

17. Never use online paper: If you are getting any paper on Internet, then never use it as your research paper because it might be possible that evaluator has already seen it or maybe it is outdated version.

18. Pick a good study spot: To do your research studies always try to pick a spot, which is quiet. Every spot is not for studies. Spot that suits you choose it and proceed further.

19. Know what you know: Always try to know, what you know by making objectives. Else, you will be confused and cannot achieve your target.

20. Use good quality grammar: Always use a good quality grammar and use words that will throw positive impact on evaluator. Use of good quality grammar does not mean to use tough words, that for each word the evaluator has to go through dictionary. Do not start sentence with a conjunction. Do not fragment sentences. Eliminate one-word sentences. Ignore passive voice. Do not ever use a big word when a diminutive one would suffice. Verbs have to be in agreement with their subjects. Prepositions are not expressions to finish sentences with. It is incorrect to ever divide an infinitive. Avoid clichés like the disease. Also, always shun irritating alliteration. Use language that is simple and straight forward. put together a neat summary.

21. Arrangement of information: Each section of the main body should start with an opening sentence and there should be a changeover at the end of the section. Give only valid and powerful arguments to your topic. You may also maintain your arguments with records.

22. Never start in last minute: Always start at right time and give enough time to research work. Leaving everything to the last minute will degrade your paper and spoil your work.

23. Multitasking in research is not good: Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. Think technically: Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page
- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.

- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
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The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

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Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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