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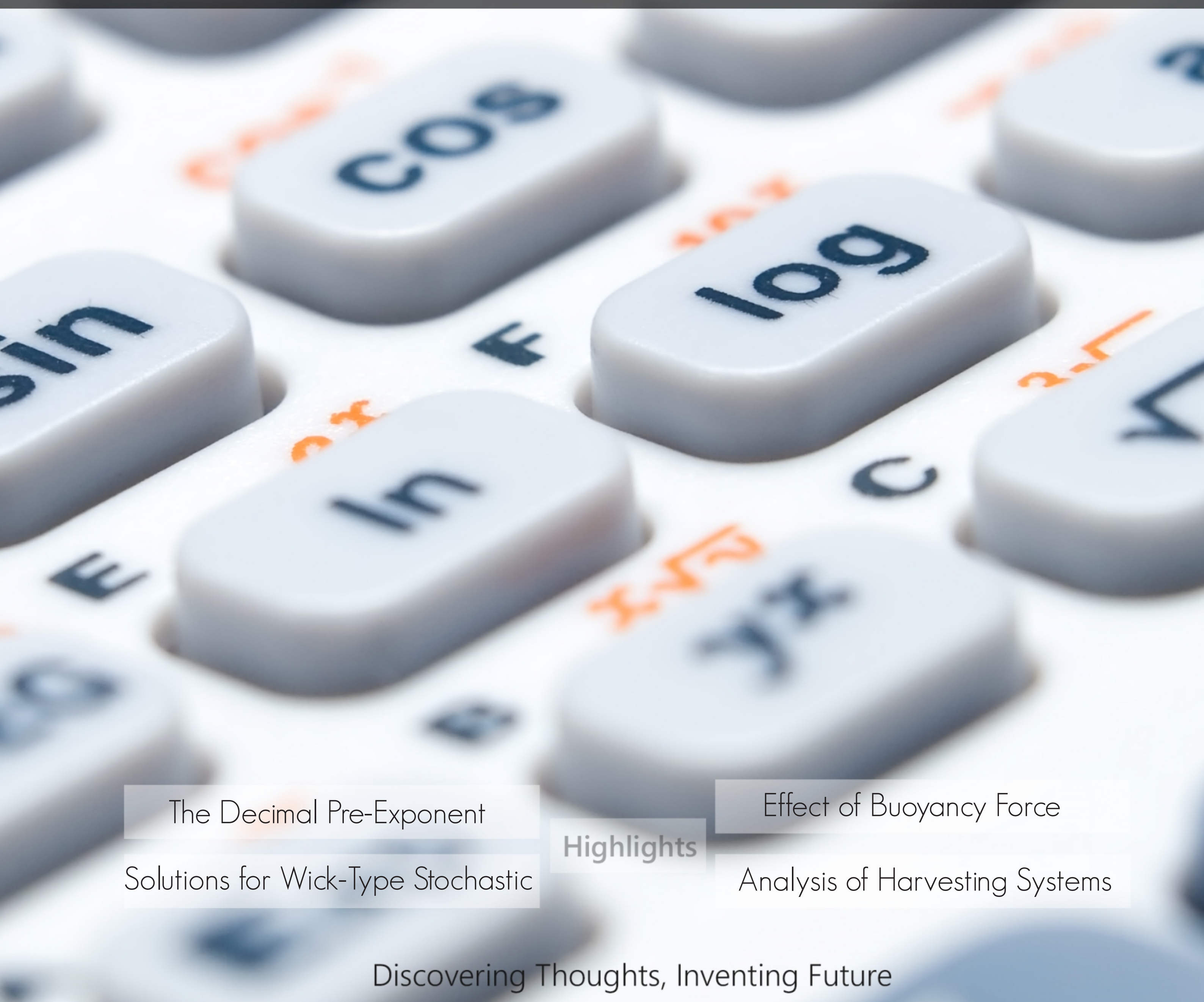

Highlights

The Decimal Pre-Exponent

Solutions for Wick-Type Stochastic

Effect of Buoyancy Force

Analysis of Harvesting Systems

Discovering Thoughts, Inventing Future

VOLUME 14 ISSUE 3 VERSION 1.0

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We Can Do It: Experiential Learning Activities in Mathematics Courses for Liberal Arts Undergraduates

By Gary Stogsdill
Prescott College, United States

Abstract- Mathematics courses are often the last place anyone would expect to find experiential learning. Yet liberal arts undergraduates can benefit greatly from experiential learning activities in their required math course, and it is easy for educators to incorporate such activities into these courses. The author describes his practice of providing an experiential project in his course, Mathematical Explorations.

Keywords: experiential learning, liberal arts mathematics, humanistic mathematics, mathematical explorations, prescott college.

GJSFR-F Classification : *MSC Code : 00A05*

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Keywords: experiential learning, liberal arts mathematics, humanistic mathematics, mathematical explorations, prescott college.

I. Introduction

The mission statement of Prescott College, where I teach, concludes with this: "Our philosophy stresses experiential learning and self-direction within an interdisciplinary curriculum." Many colleges and universities, as well as many individual educators, share Prescott College's respect for experiential learning. We know that opportunities for hands-on application enrich our students' learning process and in some cases provide a depth of learning that is not otherwise possible. Mathematics courses, however, are often the last place anyone would expect to find experiential learning. Yet liberal arts undergraduates can benefit greatly from experiential learning activities in their required math course, and it is easy for educators to incorporate such activities into these courses.

I have taught mathematics for liberal arts students at Prescott College since 1990. In 2004 I was asked to join the College's Limited-Residency Undergraduate Program and specifically to create a math course that would engage students with meaningful learning. I quickly found that most students love doing self-chosen experiential projects on math-related topics. The course that I created is called *Mathematical Explorations*, an online humanistic math course that has four main components: a math therapy exercise (see Stogsdill, 2013a), quantitative reasoning exercises (see Stogsdill, 2014), a meaningful self-chosen experiential project, and self-chosen research into an interdisciplinary math-related topic of vital importance in the human quest to understand the world around us and what it means to be human (see Stogsdill, 2013b).

The experiential project actually addresses all three of the learning mandates in the Prescott College mission statement excerpt that opened this article. Obviously it's experiential,

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it's also self-directed because each student chooses what project to engage in, and it's inherently interdisciplinary because any useful application of math will necessarily venture into the territory of other disciplines.

At the beginning of *Mathematical Explorations*, students are presented with sample ideas for their experiential project along with a scoring rubric so that they understand how to receive full credit. Students are also invited to seek approval for their own unique experiential project that I've not mentioned in the sample ideas. Following are a few of the sample ideas that I provide:

- Design and build something of interest.
- Design and create plans for a substantial architectural project.
- Plan sustainability features for where you live (solar gain, rainwater collection, permaculture designs, etc.) and begin to implement some of these features.
- Calculate an "ecological footprint" for yourself, your family, or your community.
- Design and create a garden or landscape incorporating principles of sacred geometry.
- Explore patterns in nature (golden ratio, fractal, spiral, etc.) and document with photography.
- Create your own original tessellation design art.
- Explore music in terms of its mathematical components.
- Write an original story for children conveying ideas from your math-related research topic.
- Develop a financial business plan.
- Explore and apply the mathematics and physics of rock climbing.

Components of the scoring rubric include criteria for time commitment, quality of product, interest level, and interaction with classmates at the online forum dedicated to the experiential project.

Following are examples of experiential projects that have been completed recently. One student designed and constructed an elaborate tool shed, documenting the extensive use of mathematics at every stage of this process. An adventure education student documented a wide variety of patterns in nature through a 23-page original photo essay. An expressive arts student who was exploring chaos theory and fractals for her research topic purchased a computer fractal program and created several stunning pieces of original fractal art. A student with a creative writing major wrote and illustrated an original story for children on her math-related research topic of cosmology. A sustainability student designed a passive solar home with a permaculture yard, complete with calculations for solar gains, etc. A human development student wrote a substantial business financial plan for her future work as an aromatherapist. A teacher preparation student who is currently an aide in a special education classroom created and taught a series of lesson plans on her research topic of indigenous mathematics.

At the end of *Mathematical Explorations* all students complete a narrative self-evaluation as well as a course questionnaire that invites them to comment on what they found particularly helpful about the course. Following are a few of the comments that students have volunteered specifically about the experiential project, taken from recent offerings of *Mathematical Explorations.*

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Probably my favorite activity was the experiential project. For my project I chose to photograph math [patterns] *in nature, which encompassed both my* [major] *and my* [minor]: *adventure education and photography. I was delighted and surprised to find math in so many everyday things that I would usually walk by.*

The experiential project helped me gain more of my own relationship to math and find my entrance into something I find very interesting and see it from a "mathematical point of view." I can see that the type of thinking required is important to learn and is helpful for life in general.

The self-chosen experiential projects were very practical in nature [and] *showed me how math-related projects such as these help us in our role as human beings to better understand the world around us. I've come to realize that whenever I work on a new project, no matter what it is, I think about how much math is integrated into our lives and the natural world and that we use* [math] *every day of our lives.*

Having the opportunity to do a math project with something I love to do and have a great passion for, was the best experience I have ever had with math.

Mathematics can be found in virtually every aspect of the world around us and in most human endeavors. Experiential learning in math class allows students to discover this for themselves. Such activities are inherently interesting for most students and promote an appreciation for the usefulness of math. A self-chosen experiential project can be a welcome addition to any math course for liberal arts undergraduates.

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Establishment of a Summation Formula Associated to Hypergeometric Function

By Salahuddin & Shakeeluddin
P.D.M College of Engineering, India

Abstract- The main objective of present paper is the development of a summation formulae linked with the Contiguous relation and Hypergeometric function.

Keywords: contiguous relation, recurrence relation, generalized gaussian hy-pergeometric function, prudnikov et al.

GJSFR-F Classification : *MSC 2010: 33C05 , 33C20 , 33C45 , 33C60, 33C70*

ESTABLISHMENTOFASUMMATIONFORMULAASSOCIATEDTOHYPERGEOMETRICFUNCTION

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Establishment of a Summation Formula Associated to Hypergeometric Function

Salahuddin [α] & Shakeeluddin [σ]

Abstract- The main objective of present paper is the development of a summation formulae linked with the Contiguous relation and Hypergeometric function.

Keywords: *Contiguous relation, Recurrence relation, Generalized Gaussian Hy-pergeometric function, Prudnikov et al.*

I. Introduction

Generalized Gaussian Hypergeometric function of one variable :

$$ {}_AF_B\left[\begin{array}{ccc} a_1, a_2, \cdots, a_A & ; & \\ & & z \\ b_1, b_2, \cdots, b_B & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!} \tag{1} $$

or

$$ {}_AF_B\left[\begin{array}{ccc} (a_A) & ; & \\ & & z \\ (b_B) & ; & \end{array}\right] \equiv {}_AF_B\left[\begin{array}{ccc} (a_j)_{j=1}^{A} & ; & \\ & & z \\ (b_j)_{j=1}^{B} & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \tag{2} $$

where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and $A,\ B$ are non-negative integers.

Contiguous Relation :

[Abramowitz p.558(15.2.19)]

$$ (a-b)\,(1-z)\,{}_2F_1\left[\begin{array}{cc} a,\ b\ ; & \\ c\ \ ; & z \end{array}\right] = (c-b)\,{}_2F_1\left[\begin{array}{cc} a,\ b-1\ ; & \\ c\ \ ; & z \end{array}\right] + (a-c)\,{}_2F_1\left[\begin{array}{cc} a-1,\ b\ ; & \\ c\ ; & z \end{array}\right] \tag{3} $$

Recurrence relation :

$$ \Gamma(z+1) = z\,\Gamma(z) \tag{4} $$

Legendre's duplication formula :

$$ \sqrt{\pi}\,\Gamma(2z) = 2^{(2z-1)}\,\Gamma(z)\,\Gamma\left(z+\frac{1}{2}\right) \tag{5} $$

$$ \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} = \frac{2^{(b-1)}\,\Gamma(\frac{b}{2})\,\Gamma(\frac{b+1}{2})}{\Gamma(b)} \tag{6} $$

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$$= \frac{2^{(a-1)}\,\Gamma(\frac{a}{2})\,\Gamma(\frac{a+1}{2})}{\Gamma(a)} \tag{7}$$

In the monograph of Prudnikov et al. , a summation formula is given in the form [Prudnikov,491,equation(7.3.7.8)]

$${}_2F_1\left[\begin{matrix} a, & b & ; \\ \frac{a+b-1}{2} & ; \end{matrix} \quad \frac{1}{2}\right] = \sqrt{\pi}\left[\frac{\Gamma(\frac{a+b+1}{2})}{\Gamma(\frac{a+1}{2})\Gamma(\frac{b+1}{2})} + \frac{2\,\Gamma(\frac{a+b-1}{2})}{\Gamma(a)\Gamma(b)}\right] \tag{8}$$

Now using Legendre's duplication formula and Recurrence relation for Gamma function , the above formula can be written in the form

$${}_2F_1\left[\begin{matrix} a, & b & ; \\ \frac{a+b-1}{2} & ; \end{matrix} \quad \frac{1}{2}\right] = \frac{2^{(b-1)}\,\Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} + \frac{2^{(a-b+1)}\,\Gamma(\frac{a}{2})\,\Gamma(\frac{a+1}{2})}{\{\Gamma(a)\}^2} + \frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\right] \tag{9}$$

It is noted that the above formula [Prudnikov,491,equation(7.3.7.8)], i.e. equation(8) or (9) is not correct.The correct form of equation(8) or (9) is obtained by [Asish et. al(2008), p.337(10)]

$${}_2F_1\left[\begin{matrix} a, & b & ; \\ \frac{a+b-1}{2} & ; \end{matrix} \quad \frac{1}{2}\right] = \frac{2^{(b-1)}\,\Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})}\left\{\frac{(b+a-1)}{(a-1)}\right\} + \frac{2\,\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\right] \tag{10}$$

Involving the derived formula obtained by [Asish et. al(2008), p.337(10)], we establish the main formula.

II. Main Result of Summation Formula

For the result $a \neq b$

$${}_2F_1\left[\begin{matrix} a, & b & ; \\ \frac{a+b-25}{2} & ; \end{matrix} \quad \frac{1}{2}\right] = \frac{2^{(b-1)}\,\Gamma(\frac{a+b-25}{2})}{(a-b)\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-25}{2})}\left\{\frac{(-7905853580625a + 17901641997225a^2)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}} + \right.\right.$$

$$+\frac{(-15467069396610a^3 + 7198061846898a^4 - 2078757113719a^5 + 401014719391a^6)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}} +$$

$$+\frac{(-53845005500a^7 + 5141534684a^8 - 351523887a^9 + 17085783a^{10} - 576290a^{11} + 12818a^{12})}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}} +$$

$$+\frac{(-169a^{13} + a^{14} + 7905853580625b - 24433840638090a^2b + 28220510016972a^3b)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}} +$$

$$+\frac{(-13421680355421a^4b + 4353881703444a^5b - 770676882300a^6b + 116726473656a^7b)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}} +$$

$$+\frac{(-9516460473a^8b + 760534632a^9b - 28610010a^{10}b + 1206972a^{11}b - 16731a^{12}b + 324a^{13}b)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}} +$$

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$$+\frac{(-179016419972225b^2+244338406380 90ab^2-12795899385750a^3b^2+9417308230395a^4b^2)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-2653257473100a^5b^2+642830764680a^6b^2-68974530300a^7b^2+8396447085a^8b^2)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-383418750a^9b^2+25585560a^{10}b^2-410670a^{11}b^2+14625a^{12}b^2+15467069396610b^3)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-28220510016972ab^3+12795899385750a^2b^3-2013943951500a^4b^3+1048337211480a^5b^3)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-174357155700a^6b^3+32962166640a^7b^3-1967793750a^8b^3+196353300a^9b^3-3848130a^{10}b^3)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(215280a^{11}b^3-7198061846898b^4+13421680355421ab^4-9417308230395a^2b^4)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

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$$+\frac{(2653257473100a^2b^5-1048337211480a^3b^5+116257631490a^4b^5-2541111300a^6b^5)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

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$$+\frac{(9516460473ab^8-8396447085a^2b^8+1967793750a^3b^8-629559450a^4b^8+29910465a^5b^8)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-4345965a^6b^8+351523887b^9-760534632ab^9+383418750a^2b^9-196353300a^3b^9)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(16033875a^4b^9-3749460a^5b^9-17085783b^{10}+28610010ab^{10}-25585560a^2b^{10})}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(3848130a^3b^{10}-1332045a^4b^{10}+576290b^{11}-1206972ab^{11}+410670a^2b^{11}-215280a^3b^{11})}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-12818b^{12}+16731ab^{12}-14625a^2b^{12}+169b^{13}-324ab^{13}-b^{14})}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}\Bigg\}+$$

$$+\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-24}{2})}\Bigg\{\frac{(12722110515450a-18824036930640a^2+13489601849988a^3-4546842476656a^4)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(1232640848966a^5-171472666400a^6+23400155704a^7-1531147488a^8+114243558a^9)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(-3420560a^{10}+136708a^{11}-1456a^{12}+26a^{13}-12722110515450b+18472182813972a^2b)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

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$$+\frac{(-6712650500994ab^4+2566312221600a^2b^4-656015223000a^3b^4+28280335860a^5b^4)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

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$$+\frac{(26429760a^5b^7-2674440a^6b^7+1531147488b^8-2655027882ab^8+854334000a^2b^8)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(-383467500a^3b^8+22604400a^4b^8-4601610a^5b^8-114243558b^9+98567040ab^9)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(-77005500a^2b^9+7893600a^3b^9-2466750a^4b^9+3420560b^{10}-6069492ab^{10}+1235520a^2b^{10})}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(-592020a^3b^{10}-136708b^{11}+78624ab^{11}-63180a^2b^{11}+1456b^{12}-2574ab^{12}-26b^{13})}{\prod_{\eta=1}^{12}\{a-2\eta\}}\Bigg\}\Bigg] \tag{11}$$

III. Derivation of Summation Formula

Substituting $c=\frac{a+b-25}{2}$ and $z=\frac{1}{2}$ in equation (3), we get

$$(a-b)\,{}_2F_1\left[\begin{matrix}a,\ b\ ; \\ \frac{a+b-25}{2}\ ;\end{matrix}\ \frac{1}{2}\right]=(a-b-25)\,{}_2F_1\left[\begin{matrix}a,\ b-1\ ; \\ \frac{a+b-25}{2}\ ;\end{matrix}\ \frac{1}{2}\right]+(a-b+25)\,{}_2F_1\left[\begin{matrix}a-1,\ b\ ; \\ \frac{a+b-25}{2};\end{matrix}\ \frac{1}{2}\right]$$

Now involving derived result from (10), we get

$$L.H.S=\frac{2^{(b-1)}\,\Gamma(\frac{a+b-25}{2})}{\Gamma(b)}\Bigg[\frac{(a-b-25)(b-1)}{(a-b+1)}\ \frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-25}{2})}\Bigg\{\frac{(79058535806 25-9679554273375a)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-2821754771550a^2+8156137358850a^3-4793059238825a^4+1486020024775a^5)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-287728912900a^{6}+37194314300a^{7}-3302238225a^{8}+202348575a^{9}-8415550a^{10})}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(226850a^{11}-3575a^{12}+25a^{13}-17901641997225b+29685290993100ab)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-10954559001990a^{2}b-3592186999340a^{3}b+4112199672725a^{4}b-1448281206600a^{5}b)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(297494820380a^{6}b-37999946520a^{7}b+3366572625a^{8}b-191148100a^{9}b+7617610a^{10}b)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-166140a^{11}b+2275a^{12}b+15467069396610b^{2}-28566853405770ab^{2}+16176086383690a^{2}b^{2})}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-2441585948170a^{3}b^{2}-913243080700a^{4}b^{2}+545677186380a^{5}b^{2}-117096855260a^{6}b^{2})}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(16282228540a^{7}b^{2}-1285714950a^{8}b^{2}+76140350a^{9}b^{2}-2203630a^{10}b^{2}+50830a^{11}b^{2})}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-7198061846898b^{3}+13970100703252ab^{3}-9041136026470a^{2}b^{3}+2497465994000a^{3}b^{3})}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-147688140500a^{4}b^{3}-72346451240a^{5}b^{3}+25649183700a^{6}b^{3}-3167274800a^{7}b^{3})}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(298058150a^{8}b^{3}-11511500a^{9}b^{3}+427570a^{10}b^{3}+2078757113719b^{4}-4145753956429ab^{4})}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(2817304348100a^{2}b^{4}-907985381900a^{3}b^{4}+137261041050a^{4}b^{4}-1392487390a^{5}b^{4})}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-1993550300a^{6}b^{4}+448580500a^{7}b^{4}-25138425a^{8}b^{4}+1562275a^{9}b^{4}-401014719391b^{5})}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(798461811640ab^{5}-568209672300a^{2}b^{5}+186672512600a^{3}b^{5}-33928246450a^{4}b^{5})}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(2803879400a^{5}b^{5}+57203300a^{6}b^{5}-16343800a^{7}b^{5}+2414425a^{8}b^{5}+53845005500b^{6})}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

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$$+\frac{(-108967904380ab^6+73253922980a^2b^6-26295630900a^3b^6+4437948900a^4b^6)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-471758980a^5b^6+17681020a^6b^6+742900a^7b^6-51415346684b^7+9939584616ab^7)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-7206053660a^2b^7+2131630800a^3b^7-438662900a^4b^7+32040840a^5b^7-1931540a^6b^7)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(351523887b^8-702055497ab^8+413863450a^2b^8-151787350a^3b^8+17678375a^4b^8)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-2187185a^5b^8-170857783b^9+30147260ab^9-21628750a^2b^9+4275700a^3b^9-904475a^4b^9)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(576290b^{10}-1108250ab^{10}+450450a^2b^{10}-164450a^3b^{10}-12818b^{11}+17732ab^{11})}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-12350a^2b^{11}+169b^{12}-299ab^{12}-b^{13})}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}\Bigg\}+$$

$$+\frac{(a-b-25)}{(a-b+1)}\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-24}{2})}\Bigg\{\frac{(12405876372225-6637991183415a-6934703183742a^2)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(7758765996242a^3-3374923746265a^4+826980359391a^5-133187685316a^6)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(14182039196a^7-1069344705a^8+53304823a^9-1858142a^{10}+37362a^{11}-455a^{12}+a^{13})}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(-31933329617025b+29109857668620ab-1235086611478a^2b-7800325758444a^3b)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(4133160125069a^4b-1097812385480a^5b+176691959996a^6b-19086735512a^7b)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(1363905257a^8b-67092740a^9b+2055482a^{10}b-38844a^{11}b+299a^{12}b+32426508971490b^2)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(-36042012580570ab^2+11400438154410a^2b^2+1320226372070a^3b^2-1786460663100a^4b^2)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(553332436780a^5b^2-92422616860a^6b^2+9931560860a^7b^2-678656550a^8b^2+31123950a^9b^2)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(-807950a^{10}b^2+12350a^{11}b^2-17710198832290b^3+21701551850580ab^3)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(-9177122387830a^2b^3+1292748368400a^3b^3+274928487500a^4b^3-130437209000a^5b^3)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(25589291700a^6b^3-2554775600a^7b^3+180730550a^8b^3-6249100a^9b^3+164450a^{10}b^3)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(5971205695175b^4-7660435075925ab^4+3662292647300a^2b^4-775479934700a^3b^4)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(49625392250a^4b^4+15829012850a^5b^4-3366300700a^6b^4+431486900a^7b^4-20474025a^8b^4)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(904475a^9b^4-1344672714375b^5+1740955966520ab^5-881151572620a^2b^5)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(214647357400a^3b^5-24445050210a^4b^5+597483880a^5b^5+318074820a^6b^5-26132600a^7b^5)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(2187185a^8b^5+206381978620b^6-276634482140ab^6+135560573540a^2b^6-35921756500a^3b^6)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(4653044900a^4b^6-273535780a^5b^6+1040060a^6b^6+1931540a^7b^6-23443530620b^7)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(29006686440ab^7-15427953340a^2b^7+3535477200a^3b^7-538777300a^4b^7+31201800a^5b^7)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(-742900a^6b^7+1777480575b^8-2419163825ab^8+1026870650a^2b^8-288250950a^3b^8)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(27126775a^4b^8-2414425a^5b^8-109570175b^9+116130300ab^9-62984350a^2b^9+9538100a^3b^9)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(-1562275a^4b^9+3893890b^{10}-5352490ab^{10}+1480050a^2b^{10}-427570a^3b^{10}-129090b^{11})}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

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$$+\frac{(91780ab^{11}-50830a^2b^{11}+1625b^{12}-2275ab^{12}-25b^{13})}{\prod_{\eta=1}^{12}\{a-2\eta\}}\Bigg\}\Bigg]+\frac{2^{(b-1)}\ \Gamma(\frac{a+b-25}{2})}{\Gamma(b)}\left[\frac{(a-b+25)}{(a-b-1)}\times\right.$$

$$\times\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-24}{2})}\Bigg\{\frac{(-12405876372225+31933329617025a-32426508971490a^2+17710198832290a^3)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(-5971205695175a^4+1344672714375a^5-206381978620a^6+23443530620a^7-1777480575a^8)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(109570175a^9-3893890a^{10}+129090a^{11}-1625a^{12}+25a^{13}+6637991183415b)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(-29109857668620ab+36042012580570a^2b-21701551850580a^3b+7660435075925a^4b)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(-1740955966520a^5b+276634482140a^6b-29006686440a^7b+2419163825a^8b)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(-116130300a^9b+5352490a^{10}b-91780a^{11}b+2275a^{12}b+6934703183742b^2)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(1235086611478ab^2-11400438154410a^2b^2+9177122387830a^3b^2-3662292647300a^4b^2)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(881151572620a^5b^2-135560573540a^6b^2+15427953340a^7b^2-1026870650a^8b^2)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(62984350a^9b^2-1480050a^{10}b^2+50830a^{11}b^2-7758765996242b^3+7800325758444ab^3)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(-1320226372070a^2b^3-1292748368400a^3b^3+775479934700a^4b^3-214647357400a^5b^3)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(35921756500a^6b^3-3535477200a^7b^3+288250950a^8b^3-9538100a^9b^3+427570a^{10}b^3)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(3374923746265b^4-4133160125069ab^4+1786460663100a^2b^4-274928487500a^3b^4)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(-49625392250a^4b^4+24445050210a^5b^4-4653044900a^6b^4+538777300a^7b^4-27126775a^8b^4)}{\prod_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(1562275a^9b^4-826980359391b^5+1097812385480ab^5-553332436780a^2b^5)}{\prod\limits_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(130437209000a^3b^5-15829012850a^4b^5-597483880a^5b^5+273535780a^6b^5-31201800a^7b^5)}{\prod\limits_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(2414425a^8b^5+1331876853316b^6-176691959996ab^6+92422616860a^2b^6-25589291700a^3b^6)}{\prod\limits_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(3366300700a^4b^6-318074820a^5b^6-1040060a^6b^6+742900a^7b^6-141820391196b^7)}{\prod\limits_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(19086735512ab^7-9931560860a^2b^7+2554775600a^3b^7-431486900a^4b^7+26132600a^5b^7)}{\prod\limits_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(-1931540a^6b^7+10693447705b^8-1363905257ab^8+678656550a^2b^8-180730550a^3b^8)}{\prod\limits_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(20474025a^4b^8-2187185a^5b^8-533048236^9+67092740ab^9-31123950a^2b^9+6249100a^3b^9)}{\prod\limits_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(-904475a^4b^9+1858142b^10-2055482ab^{10}+807950a^2b^{10}-164450a^3b^{10}-37362b^{11})}{\prod\limits_{\eta=1}^{12}\{a-2\eta\}}+$$

$$+\frac{(38844ab^{11}-12350a^2b^{11}+455b^{12}-299ab^{12}-b^{13})}{\prod\limits_{\eta=1}^{12}\{a-2\eta\}}\Bigg\}+\frac{(a-b+25)}{(a-b-1)}\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-25}{2})}\times$$

$$\times\Bigg\{\frac{(-7905853580625+17901641997225a-15467069396610a^2+7198061846898a^3)}{\prod\limits_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}$$

$$+\frac{(-2078757113719a^4+401014719391a^5-53845005500a^6+5141534684a^7-351523887a^8)}{\prod\limits_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(17085783a^9-576290a^{10}+12818a^{11}-169a^{12}+a^{13}+9679554273375b)}{\prod\limits_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-29685290993100ab+28566853405770a^2b-13970100703252a^3b+4145753956429a^4b)}{\prod\limits_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-798461811640a^5b+108967904380a^6b-9939584616a^7b+702055497a^8b-30147260a^9b)}{\prod\limits_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

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$$+\frac{(1108250a^{10}b-17732a^{11}b+299a^{12}b+2821754771550b^2+10954559001990ab^2)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-16176086383690a^2b^2+9041136026470a^3b^2-2817304348100a^4b^2+568209672300a^5b^2)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-73253922980a^6b^2+7206053660a^7b^2-413863450a^8b^2+21628750a^9b^2-450450a^{10}b^2)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(12350a^{11}b^2-8156137358850b^3+3592186999340ab^3+2441585948170a^2b^3)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-2497465994000a^3b^3+907985381900a^4b^3-186672512600a^5b^3+26295630900a^6b^3)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-2131630800a^7b^3+151787350a^8b^3-4275700a^9b^3+164450a^{10}b^3+4793059238825b^4)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-4112199672725ab^4+913243080700a^2b^4+147688140500a^3b^4-137261041050a^4b^4)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(33928246450a^5b^4-4437948900a^6b^4+438662900a^7b^4-17678375a^8b^4+904475a^9b^4)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-1486020024775b^5+1448281206600ab^5-545677186380a^2b^5+72346451240a^3b^5)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(1392487390a^4b^5-2803879400a^5b^5+471758980a^6b^5-32040840a^7b^5+2187185a^8b^5)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(287728912900b^6-297494820380ab^6+117096855260a^2b^6-25649183700a^3b^6)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(1993550300a^4b^6-57203300a^5b^6-17681020a^6b^6+1931540a^7b^6-37194314300b^7)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(37999946520ab^7-16282228540a^2b^7+3167274800a^3b^7-448580500a^4b^7+16343800a^5b^7)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-742900a^6b^7+3302238225b^8-3366572625ab^8+1285714950a^2b^8-298058150a^3b^8)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(251384250a^4b^8-2414425a^5b^8-202348575b^9+191148100ab^9-76140350a^2b^9)}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(11511500a^3b^9-1562275a^4b^9+8415550b^{10}-7617610ab^{10}+2203630a^2b^{10}-427570a^3b^{10})}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}+$$

$$+\frac{(-226850b^{11}+166140ab^{11}-50830a^2b^{11}+3575b^{12}-2275ab^{12}-25b^{13})}{\prod_{\varsigma=1}^{13}\{a-(2\varsigma-1)\}}\Bigg\}\Bigg]$$

On simplification, we get the summation formula.

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Notes

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The *Decimal* Pre-Exponent *"k"* Decimal Counter

By Fernando Mancebo Rodríguez

Introduction- The decimal notation k is born from the necessity of finding a system of decimal metric units of wide spectrum, but as I soon saw, we can also use this as exponential notation in several expressions and mathematical operations.

This way the decimal pre-exponent is a method of double functionality: Firstly as systems of decimal units, and second as decimal exponent for mathematical operations.

GJSFR-F Classification : MSC 2010: 11T23

THEDECIMALPRE-EXPONENTKDECIMALCOUNTER

Strictly as per the compliance and regulations of :

Notes

The *Decimal* Pre-Exponent *"k"* Decimal Counter

Fernando Mancebo Rodríguez

I. Introduction

The decimal notation k is born from the necessity of finding a system of decimal metric units of wide spectrum, but as I soon saw, we can also use this as exponential notation in several expressions and mathematical operations.

This way the decimal pre-exponent is a method of double functionality: Firstly as systems of decimal units, and second as decimal exponent for mathematical operations.

Principle and foundations of the decimal notation "k":

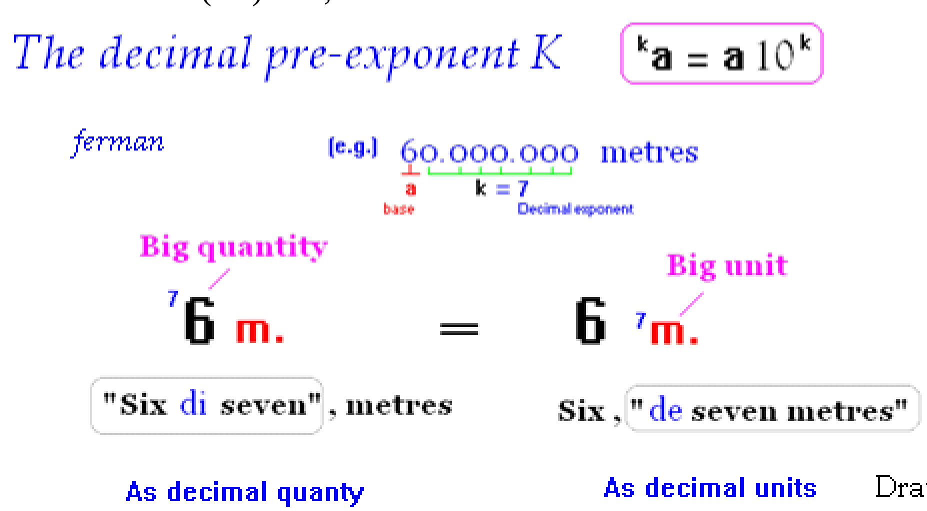
"Any quantity or decimal metric unit can be exposed in simplified or compressed way by means of a dual or bi-parametric expression formed by a base (a) or module of value and a pre-exponent (k) or decimal counter "

$$^{k}a$$

The base **a** contains the extract of the numeric value.

The pre-exponent **k** expresses the number of deduced or compressed decimals from the initial expression.

Example: 2.300.000.000----(^{k}a) = $^{9}2,3$.

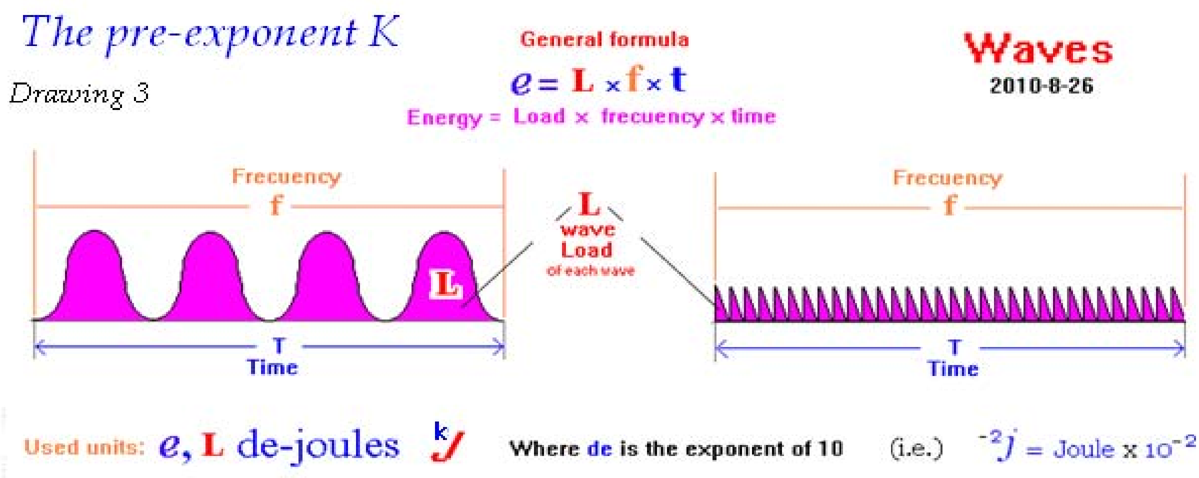


Drawing 2

Author: e-mails: ferman25@hotmail.com, ferman30@yahoo.es

II. Development

The problem arose me in August of 2010 when studying the energy of waves. This way a tsunami is a wave of enormous dimensions and potential energy, while an electromagnetic wave is of minimum energy power.



But how we can measure and relate the energy of both waves with the same energy unit, when for example the joule is insignificant for the tsunami and too big for the electromagnetic wave. And the solution would be:

Applying a method of units of wide spectrum that embraced from the infinitely small things to the infinitely big things.

And this method would be the one of getting a form of indefinite multiples of exponential decimal units, to know, the method of "k" notation.

Let us see:

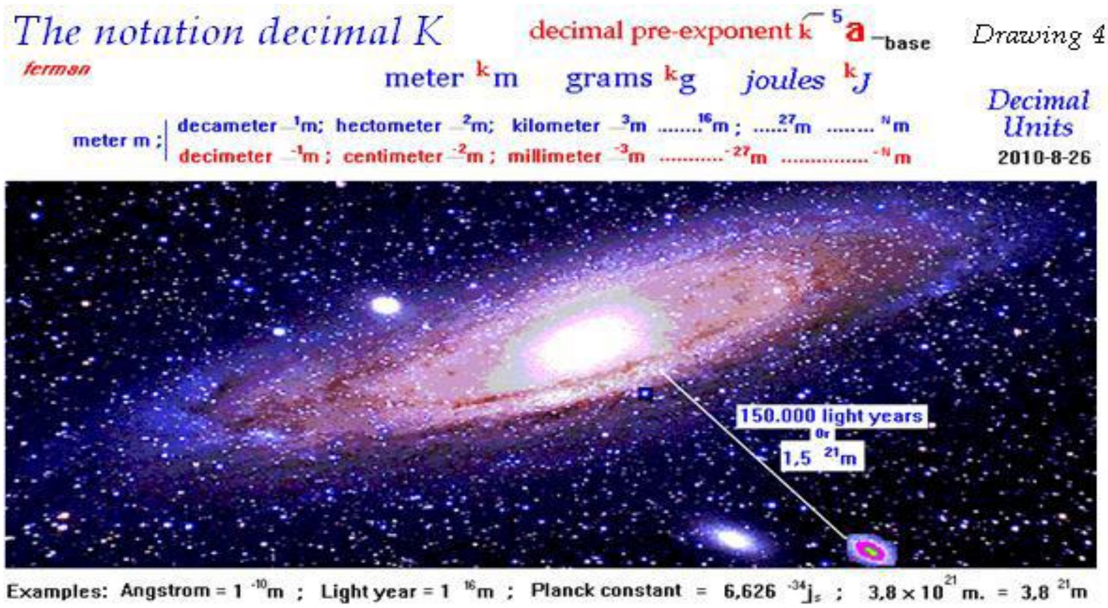
With the unit of longitude, the meter, we make decimal units that are multiples of the same one, such as decameter, hectometer, kilometer, etc.

But this method soon stops to have simple and clear expressions when it arrives to certain values such as: 10^8 meters; 10^{11} meters; 10^{17} meters, etc.

This method consists on applying an exponential pre-index to the symbol of the chosen unit (^{k}m) which values and names to the new resulting unit.

The value of this is the exponent in base 10 that it is applied to this type of chosen unit. For example, Angstrom = 10^{-10} metres is $^{-10}$m; A light-year ---- ^{16}m; 13 x 10^{12}m. ----- 13 ^{12}m.

Notes



Notes

In practice, the "k" notation means the number of decimals that we should apply to the base **a**.

For example:

$^{12}12{,}85 = 12850000000000.$

$^{4}12{,}85 = 128500.$

$^{-4}12{,}85 = 0{,}001285.$

$^{-12}12{,}85 = 0{,}00000000001285.$

As we can see, the "k" notation k allows us any quantity of integer and decimal numbers in the base **a**, as for instance:

$^{12}12{,}85 = 12850000000000.$

$^{13}1{,}285 = 12850000000000.$

$^{14}0{,}1285 = 12850000000000.$

$^{10}1285 = 12850000000000.$

III. Nomenclature

Although it doesn't correspond to me the definition of the pronunciation of this notation method, I would propose the following one:

^{k}m to name the expression "de" or "di" followed by the exponent and of the type of chosen unit. In the case of having chosen the meter, it would be: d-exponent-metre.

For example:

3,6 x 10^{32} metres = 3,6 ^{32}m = three comma six "de thirty two meters."

8 x 10^{20} joules = 8 ^{20}J = eight "de twenty joules."

25 x 10^{-34} joules = 25 $^{-34}$J = twenty-five "de minus thirty four Joules."

As we see, in these cases the definitions "de thirty two metres", "de twenty Joules", "de minus thirty four Joules", etc. they serve as name of the chosen unit, just as if they were decametres, kilometres; decimetres, millimetres, picometres, etc., but with a limitless application ambit.

Notes

Examples of nomenclature: In quantities

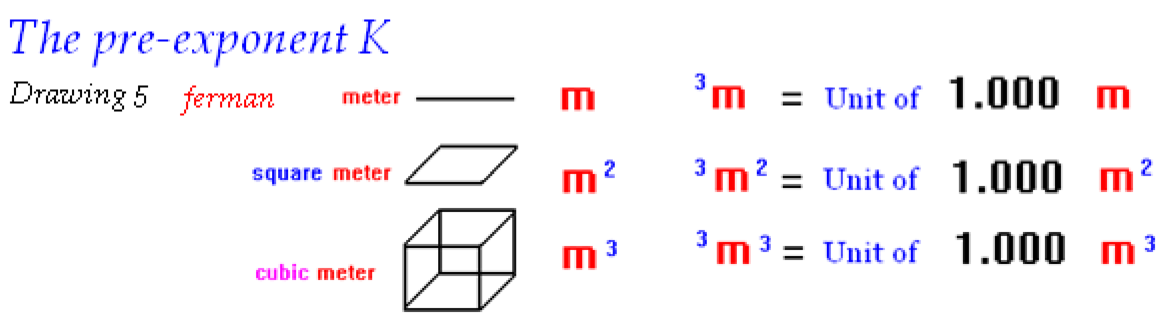
4.000.000.000.000.000.000.000 = $^{21}4$ => "four di twenty-one"
9.000.000.000.000 = $^{12}9$ => "nine di twelve".
0,000.000.000.000.000.7 = $^{-16}7$ => "seven di minus sixteen"

In decimal metric units.

^{7}m => "de seven metres" => **Longitude unit** equivalent to 10.000.000 metres.
^{6}J => "de six joules" => **Energy unit** equivalent to 1.000.000 joules.
$^{-20}$J => "de minus twenty joules" => **Energy unit** equivalent to 0,000.000.000.000.000.000.01 joules.
^{12}g => "de twelve grams" => **Weight unit** equivalent to 1.000.000.000.000 grams.
^{42}Kg => "de forty two Kgs." => **Weight unit** equivalent to 1.000.000.000.000.000.000.000.000.000.000.000.000.000.000 Kgs.

IV. DIMENSIONAL OR PHYSICAL PARAMETERS

In the dimensional o physical parameters (e.g. meter, square meter, cubic meter) the "k" notation follows a logical rule considering the structural reality of the units firstly, to which we can apply later its corresponding multiples.



This way, we firstly look at the consistency of the unit (for example the square meter), and later on we apply its decimal multiples.
^{5}m = Unit of 100.000 metres
$^{5}m^{2}$ = Unit of 100.000 square metres
$^{5}m^{3}$ = Unit of 100.000 cubic metres.
* The flexibility of the decimal notation "k" allows us also the use of any other classic unit as for example:
The kilogram (^{3}kg = Unit of 1.000 kg.); the kilometre (^{3}km = Unit of 1.000 km.); the square kilometre ($^{6}km^{2}$ = Unit of 1.000.000 square kilometres.); the cubic hectometre ($^{4}hm^{3}$ = Unit of 10.000 cubic hectometres), etc.

In Mathematical Operations:

In the mathematical operations, the "k" notation simplifies a lot the expressions and therefore I believe that it could also be very useful sometimes.

Let us do some comparisons with the current method (scientific notation) and with the "de" notation.

$12 \text{ x } 10^{12} \text{ x } 6 \text{ x } 10^{8} \text{ x } 5 \text{ x } 10^{7} = 36 \text{ x } 10^{28}$
${}^{12}12 \text{ x } {}^{8}6 \text{ x } {}^{7}5 = {}^{28}36$

$(12 \text{ x } 10^{12}) : (6 \text{ x } 10^{8}) = 2 \text{ x } 10^{4}$.
${}^{12}12 : {}^{8}6 = {}^{4}2$

Notes

${}^{6}18 \text{ x } 6 = {}^{6}108$
${}^{5}15 \text{ x } {}^{6}9 = {}^{11}135$
${}^{6}8 \text{ x } {}^{5}6 \text{ x } {}^{3}4 \text{ x } {}^{2}5 = {}^{17}96$
${}^{4}6 \text{ x } {}^{2}7 \text{ x } 38 : - {}^{4}3 = {}^{13}112$

As we see the simplification is important, mainly because the "k" notation represents to a simplified number (or compressed number), whereas other notations as $6 \text{x} 10^{3}$; 6^{5}, etc. they are

really operations that we must make previously to take out the number that we are looking for, which leaves some antiquated, obsolete and confused to the scientific notation.

Additions and Subtractions

To add and subtract quantities with "k" notation we can make it by means of the equalization of exponents "k".

For example:

${}^{17}7 + {}^{16}8 + {}^{15}12$

Then we equal them, as for example to "k"= 15

${}^{17}7 = {}^{15}700$
${}^{16}8 = {}^{15}80$
${}^{15}12 = {}^{15}12$

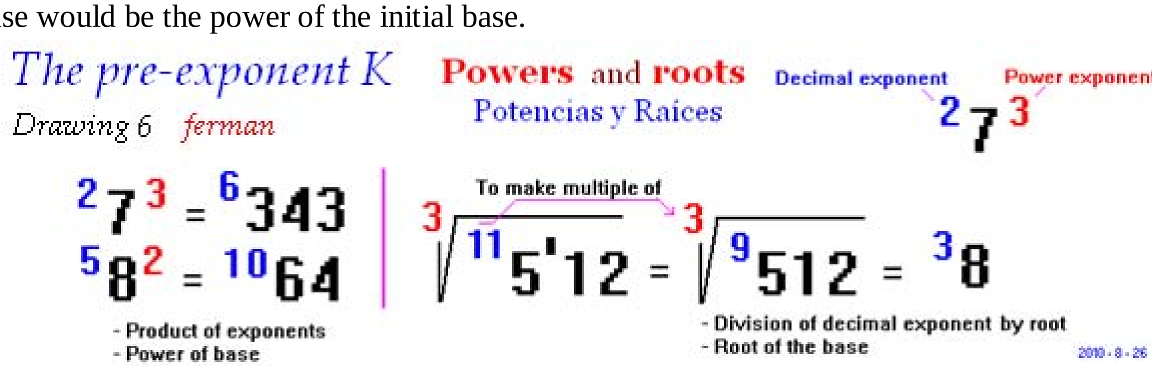
.... ${}^{15}792$

Powers and Roots

Powers with decimal notation (${}^{k}a^{n}$).

The solution or result of a power with decimal notation is an expression with decimal notation and without power exponent, such that:

The decimal exponent is the product of the decimal and power exponents; and the new base would be the power of the initial base.



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Roots of expressions with decimal notation

For the resolution of roots with decimal notation we will follow the next phases:

1.- In the first place we make multiple of the root to the decimal exponent.

2.- Subsequently we solve by means of two steps, which will give us as solution the resulting decimal expression:

A----We divide the decimal exponent by the root and we put it as resulting decimal exponent.

B----We solve the root of the initial base to obtain us the resulting base.

Other possibilities for the use of the decimal notation "k" exist, such as expresion by mean of series, etc.

We can also use this notation in variety of symbols and concepts, as for instance in the mathematical set:

^{2}Ships = Fleet of 100 ships.

^{3}Birds = Goup or flock formed by 1.000 birds.

^{4}Trees = Group of trees or forest formed by 10.000 trees.

^{11}Stars = Group of stars (or galaxy) formed by 100.000.000.000 stars.

Conceptual Meaning:

As general rule, although not strict norm, the decimal notation "k" has some significant differences either when it is applied to numbers and quantities or when it is applied to elements and concepts.

A-- When it is applied to numbers the decimal "k" notation gives quantitative connotation, that is to say, it alone tells us the quantity or numeric value of the expression.

B-- When it is applied to elements or concepts, the "k" notation would indicate us, not alone the value of the expression, but rather the concept and meaning of group, set or structural unit.

Example:

A --- ^{11}3,5 Stars = 350.000.000.000 stars, independently of their organization and situation.

B--- 3,5 ^{11}Stars = **Set or group** of stars (galaxy) formed by 350.000.000.000 stars.

Summarizing, in one hand the notation "k" helps us to express big (or extremely small) numeric quantities in simple or easy way.

And on other hand, it allows us to use a limitless set of units of any type.

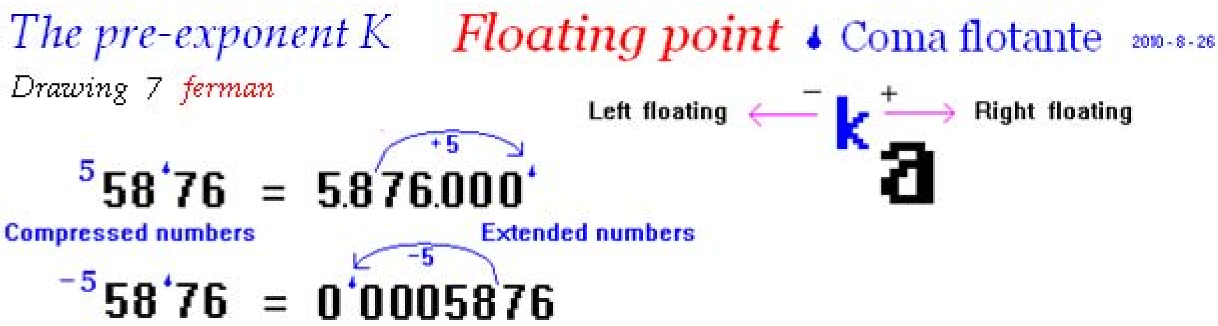
In the practice, if we put the "k" notation in a number, this will take the exponential value that the "k" notation has, and if we apply it to a unit symbol, this will become another unit of the "k" notation level, (for big or small that this is).

We have also seen that sometimes we can apply the "k" notation at the right side of the base for operative reasons, and in this case we should add the letter "d" for not having power confusion.

V. Floating Point

The decimal notation has a direct relation with the method of floating points since, by itself, the decimal exponent (k) marks us the displacement toward the right (+) or toward the left (-) that we must give to the point that separates the integer part from the decimal one in the base (a) to get the initial extended number.

Notes



* To understand us better with the application of the decimal notation, to the initial number we will call it **"extended number"** and to the number with the decimal application will call it **"compressed number"**.

VI. Discussion

The following explanation and discussion is for those who like to know the antecedents and reasons of this proposal.

At the beginning of August of 2010 I was studying and revising the different metric units and their corresponding multiples when I notice that for any unit type, for example the meter, we establish a letter or symbol to designate it and a value-pattern to value it.

Now then, as we need multiples and divisors of any unit-pattern and we use in mathematics the decimal systems, because we conceive the multiples and divisors following this decimal system.

This way, the multiples of the meter would have:

10 decametre, 100 hectometre, 1000 kilometre, 10.000 miriametre, etc.

And the divisors:

1/10 decimetre, 1/100 centimetre, 1/1.000 millimetre, etc.

Then, to designate these multiples we use a relative prefix to their first written letters:

Dm--decametre; hm--hectometre; km--kilometre, etc.

Dcm--decimetre; cm--centimetre; mm--millimetre etc.

And it is here where the first problem arose me, since the applicable letters are scarce and contrarily the numbers, and therefore the multiples, are infinite.

But also in the previous or "classic" form of representation and expression of multiples and divisors of metric units, another problem or complication exists: we should know the numeric value that we have given to each letter.

Therefore we must translate the letters with which we designate to the metric units in numeric values to take conscience of real value of the unit that we are valuing.

And due to these units and their representative letters are diverse, because to remember their real values can be complicated and can induce us to errors.

k--kilo, M--mega, G--giga, T--tera, P--peta, E--exa, Z--zetta, Y--yotta.,

So the logical question arises immediately:

What reason exists to use letters that produce us so much confusion?

Then, we could forget the letters and to put numeric decimal values directly as prefix of the chosen metric unit.

k--kilo, M--mega, G--giga, T--tera, P--peta, E--exa, Z--zetta, Y--yotta.,

---^{3}m-----^{6}m------^{9}m-----^{12}m-----^{15}m-----^{18}m----^{21}m-----^{24}m----^{27}m----^{35}m-----^{45}m-----^{59}m. .... ^{N}m.

This way the number of multiples and divisors it is limitless and there is not confusion possible of value since each number indicates us the value of the unit.

So, ^{9}m "de nine meters" means an unit with value in meters equal to 1 followed by 9 zeros.

Now then, what is good for the decimal metric units, also is good for any other symbolic or numeric concept.

This way if we have the number 14 and we apply it the previous notation (decimal notation "k" ^{k}a) ^{k}14 = ^{6}14 then this number will become a value of 14 followed by 6 zeros, 14.000.000; for k=8 we write ^{8}14 and it value is 1.400.000.000 etc.

As we have seen before, this form of mathematical expression is also very useful and simple to make mathematical operations.

Notes

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Fractional Calculus Pertaining to Generalized *H*-Functions

By Dr. Dinesh Kumar & Jitendra Daiya
Pratap University, India

Abstract- This paper is devoted to study of a pair of unified and extended fractional integral operators involving the multivariable *H*-Function, *I*-Function and general class of polynomials. Mellin transforms of these operators are investigated. Further, some properties of these operators have also been investigated. On account of the general nature of the functions involved herein, a large number of fractional integral operators involving simpler functions can be obtained as special cases of our main results.

Keywords and Phrases: fractional calculus, multivariable H-function, I-function, general class of polynomials, mellin transform.

GJSFR-F Classification : MSC 2010: 26A33, 33C05, 33C40, 33C45, 33C60

FRACTIONALCALCULUSPERTAININGTOGENERALIZED HFUNCTIONS

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Notes

Fractional Calculus Pertaining to Generalized H-Functions

Dr. Dinesh Kumar [α] & Dr. Jitendra Daiya [σ]

Abstract- This paper is devoted to study of a pair of unified and extended fractional integral operators involving the multivariable *H*-Function, *I*-Function and general class of polynomials. Mellin transforms of these operators are investigated. Further, some properties of these operators have also been investigated. On account of the general nature of the functions involved herein, a large number of fractional integral operators involving simpler functions can be obtained as special cases of our main results.

Keywords and Phrases: fractional calculus, multivariable H-function, I-function, general class of polynomials, mellin transform.

I. Introduction and Preliminaries

Fractional calculus is a field of applied mathematics that deals with derivatives and integrals of arbitrary orders. In recent years, it has turned out that many phenomena in engineering, physics, chemistry and other sciences can be described very successfully by models using mathematical tools by models using mathematical tools from fractional calculus. For example, the nonlinear oscillation of earthquake can be modeled with fractional derivatives and the fluid-dynamic traffic model with fractional derivatives can eliminate the deficiency arising from the assumption of continuum traffic flow. Fractional derivatives are also used in modeling of many chemical processes, mathematical biology and many other problems in physics and engineering.

The multivariable H-function has been studied extensively by H.M. Srivastava and R. Panda in their two basic papers on the subject (see [18, pp.119-137] and [19, pp.265-274]). It is defined and represented in the following manner:

$$H\left[x_1,\dots,x_r\right] = H^{0,n:\{m_r,n_r\}}_{p,q:\{p_r,q_r\}}\left[\begin{array}{c|c} x_1 & \left(a_j;\alpha'_j,\dots,\alpha_j^{(r)}\right)_{1,p} : \left\{\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r}\right\} \\ \vdots & \\ x_r & \left(b_j;\beta'_j,\dots,\beta_j^{(r)}\right)_{1,q} : \left\{\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,q_r}\right\} \end{array}\right]$$

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$$= \frac{1}{(2\pi i)^r} \int_{L_1} \cdots \int_{L_r} \phi(\xi_1,\ldots,\xi_r) \prod_{i=1}^{r} \left(\theta_i(\xi_i)\; x_i^{\xi_i}\, d\xi_i \right), \tag{1.1}$$

where
$$\phi(\xi_1,\ldots\xi_r) = \frac{\prod_{j=1}^{n} \Gamma\left(1 - a_j + \sum_{i=1}^{r} \alpha_j^{(i)} \xi_i\right)}{\prod_{j=n+1}^{p} \Gamma\left(a_j - \sum_{i=1}^{r} \alpha_j^{(i)} \xi_i\right) \prod_{j=1}^{q} \Gamma\left(1 - b_j + \sum_{i=1}^{r} \beta_j^{(i)} \xi_i\right)}, \tag{1.2}$$

$$\theta_i(\xi_i) = \frac{\prod_{j=1}^{n_i} \Gamma\left(1 - c_j^{(i)} + \gamma_j^{(i)} \xi_i\right) \prod_{j=1}^{m_i} \Gamma\left(d_j^{(i)} - \delta_j^{(i)} \xi_i\right)}{\prod_{j=n_i+1}^{p_i} \Gamma\left(c_j^{(i)} - \gamma_j^{(i)} \xi_i\right) \prod_{j=m_i+1}^{q_i} \Gamma\left(1 - d_j^{(i)} + \delta_j^{(i)} \xi_i\right)}, \quad \forall i \in \{1,2,\ldots,r\} \tag{1.3}$$

Here, $\{m_r, n_r\}$ stands for $m_1, n_1; \ldots, m_r, n_r$ and $\left\{\left(c_j^{(r)}, \gamma_j^{(r)}\right)_{1,p_r}\right\}$ stand for the sequence of r ordered pairs $\left(c_j', \gamma_j'\right)_{1,p_1}; \ldots; \left(c_j^{(r)}, \gamma_j^{(r)}\right)_{1,p_r}$.

In case $r = 2$, it reduces to the H-function of two variables.

Recently, Ram and Kumar [9] have obtained the images of the product of two H-functions involving Saigo-Maeda operators; Kumar and Daiya [5] obtained the generalized fractional differentiation of the $\overline{H}$-function involving general class of polynomials; Saxena, Ram and Kumar [14] have obtained the generalized fractional integral formulae of the product of Bessel functions of the first kind involving Saigo-Maeda fractional integral operators.

Also, $S_n^m[x]$ occurring in the sequel denotes the general class of polynomials introduced by Srivastava [16]:

$$S_n^m[x] = \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k}\, x^k, \quad n = 0,1,2,\ldots \tag{1.4}$$

where m is an arbitrary positive integer and the coefficient $A_{n,k}\,(n,k \geq 0)$ are arbitrary constants, real or complex. On suitably specialize the coefficients $A_{n,k}$, $S_n^m[x]$ yields a number of known polynomials as its special cases [see Srivastava and Singh [20], pp. 158-161].

The I-function is generalization of Fox's H-function [8], defined and represented as [15]:

$$I_{p_i,q_i;r}^{m,n}[z] = I_{p_i,q_i;r}^{m,n}\left[z \left| \begin{matrix} (a_j,\alpha_j)_{1,n}; (a_{ji},\alpha_{ji})_{n+1,p_i} \\ (b_j,\beta_j)_{1,m}; (b_{ji},\beta_{ji})_{m+1,q_i} \end{matrix} \right. \right] = \frac{1}{2\pi i} \int_L \phi(\xi) z^{\xi} d\xi, \tag{1.5}$$

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where

$$\phi(\xi)=\frac{\prod_{j=1}^{m}\Gamma\left(b_j-\beta_j\xi\right)\prod_{j=1}^{n}\Gamma\left(1-a_j+\alpha_j\xi\right)}{\sum_{i=1}^{r}\left\{\prod_{j=n+1}^{p_i}\Gamma\left(a_{ji}-\alpha_{ji}\xi\right)\prod_{j=m+1}^{q_i}\Gamma\left(1-b_{ji}-\beta_{ji}\xi\right)\right\}}. \tag{1.6}$$

The integration path $L=L_{i\gamma\infty},\gamma\in\Re$ extends from $\gamma-i\infty$ *to* $\gamma+i\infty$, and is such that the poles of $\Gamma\left(1-a_j+\alpha_j\xi\right), j=\overline{1,n}$ (the symbol $\left(\overline{1,n}\right)$ is used for 1,2,…,n) do not coincide with the poles of $\Gamma\left(b_j-\beta_j\xi\right), j=\overline{1,m}$. The parameters p_i,q_i are non-negative integers satisfying the condition $0\le n\le p_i,\ 1\le m\le q_i$ for $i=\overline{1,r}$. The parameters $\alpha_j,\beta_j,\alpha_{ji},\beta_{ji}>0$ *and* $a_j,b_j,a_{ji},b_{ji}\in C$.

The Mellin transform of $f(x)$ will be denoted by $M\left[f(x)\right]$ or $F(s)$. If p and y are real, we write $s=p^{-1}+iy$. If $p\ge1$, $f(x)\in L_p(0,\infty)$, then for $p=1$ we have

$$M\left[f(x)\right]=F(s)=\int_0^\infty x^{s-1}f(x)\,dx, \tag{1.7}$$

and

$$f(x)=\frac{1}{2\pi i}\int_L F(s)\,x^{-s}\,ds. \tag{1.8}$$

For p > 1,

$$M\left[f(x)\right]=F(s)=\ell.i.m.\int_{1/x}^{x} x^{s-1}f(x)\,dx, \tag{1.9}$$

where $\ell.i.m.$ denotes the usual limit in the mean for L_p-spaces.

II. DEFINITIONS

The pair of new extended fractional integral operators are defined by the following equations:

$$D_{\gamma_n}^{\alpha,\beta}\left[f(x)\right]=t\,x^{-\alpha-t\beta-1}\int_0^x y^{\alpha}\left(x^t-y^t\right)\times H\begin{bmatrix}\gamma_1\upsilon\\ \vdots\\ \gamma_n\upsilon\end{bmatrix}$$

$$\times\prod_{j=1}^{k}I_{p_{i'j},q_{i'j};r}^{m_j,n_j}\left[z_j\left(\frac{y^t}{x^t}\right)^{a_j}\left(1-\frac{y^t}{x^t}\right)^{b_j}\left|\begin{matrix}\left(e_{j'j},E_{j'j}\right)_{1,n_j};\left(e_{j'i'j},E_{j'i'j}\right)_{n_j+1,p_{i'j}}\\ \left(f_{j'j},F_{j'j}\right)_{1,m_j};\left(f_{j'i'j},F_{j'i'j}\right)_{m_j+1,q_{i'j}}\end{matrix}\right.\right]$$

$$\times\prod_{i=1}^{r}S_{N_i}^{M_i}\left[z_i\left(\frac{y^t}{x^t}\right)^{g_i}\left(1-\frac{y^t}{x^t}\right)^{h_i}\right]\psi\left(\frac{y^t}{x^t}\right)f(y)\,dy, \tag{2.1}$$

and

$$R_{\gamma_n}^{\delta,\beta}\left[f(x)\right]=t\,x^{\delta}\int_x^{\infty}y^{-\delta-t\beta-1}\left(y^t-x^t\right)^{\beta}\times H\begin{bmatrix}\gamma_1\mu\\ \vdots\\ \gamma_n\mu\end{bmatrix}$$

$$\times\prod_{j=1}^{k}I_{p_{i'j},q_{i'j};r}^{m_j,n_j}\left[z_j\left(\frac{x^t}{y^t}\right)^{a_j}\left(1-\frac{x^t}{y^t}\right)^{b_j}\left|\begin{matrix}\left(e_{j'j},E_{j'j}\right)_{1,n_j};\left(e_{j'i'j},E_{j'i'j}\right)_{n_j+1,p_{i'j}}\\ \left(f_{j'j},F_{j'j}\right)_{1,m_j};\left(f_{j'i'j},F_{j'i'j}\right)_{m_j+1,q_{i'j}}\end{matrix}\right.\right]$$

$$\times\prod_{i=1}^{r}S_{N_i}^{M_i}\left[z_i\left(\frac{x^t}{y^t}\right)^{g_i}\left(1-\frac{x^t}{y^t}\right)^{h_i}\right]\psi\left(\frac{x^t}{y^t}\right)f(y)\,dy\ ,\qquad (2.2)$$

where $\upsilon=\left(\frac{y^t}{x^t}\right)^{u_i}\left(1-\frac{y^t}{x^t}\right)^{v_i}$, $\mu=\left(\frac{x^t}{y^t}\right)^{u_i}\left(1-\frac{x^t}{y^t}\right)^{v_i}$; t,u_i and v_i are positive numbers. The kernels $\psi\left(\frac{y^t}{x^t}\right)$ and $\psi\left(\frac{x^t}{y^t}\right)$ appearing in (2.1) and (2.2) respectively, are assumed to be continuous functions such that the integrals make sense for wide classes of functions $f(x)$.

The conditions for the existence of these operators are as follows:

(i) $f(x)\in L_p(0,\infty)$, (ii) $1\le p,\ q<\infty,\ p^{-1}+q^{-1}=1$,

(iii) $\Re\left(\alpha+t\,a_j\frac{f_{j'i'j}}{F_{j'i'j}}+t\sum_{i=1}^{n}u^{(i)}\frac{c_j^{(i)}}{\psi_j^{(i)}}\right)>-q^{-1}$, (iv) $\Re\left(\beta+t\,b_j\frac{f_{j'i'j}}{F_{j'i'j}}+t\sum_{i=1}^{n}v^{(i)}\frac{c_j^{(i)}}{\psi_j^{(i)}}\right)>-q^{-1}$,

(v) $\Re\left(\delta+t\,a_j\frac{f_{j'i'j}}{F_{j'i'j}}+t\sum_{i=1}^{n}v^{(i)}\frac{c_j^{(i)}}{\psi_j^{(i)}}\right)>-p^{-1}$; where $j=\left(\overline{1,u_r}\right)$, $i'=\left(\overline{1,r}\right)$.

Condition (i) ensures that both operators defined in (2.1) and (2.2) exist and belong to $L_p(0,\infty)$.

These operators are extensions of fractional integral operators defined and studied by several authors like Erdélyi [2], Kober [4], Love [7], Saigo et al. [11], Saxena and Kiryakova [12], Saxena et al. [13], and many more.

III. Main Results

Theorem 3.1. If $f(x)\in L_p(0,\infty)$, $1\le p\le 2$; or $f(x)\in M_p(0,\infty)$, $p>2$, also following conditions satisfies:

$$p^{-1}+q^{-1}=1,\ \Re\left(\alpha+t\,a_j\frac{f_{j'i'j}}{F_{j'i'j}}+t\sum_{i=1}^{n}u^{(i)}\frac{c_j^{(i)}}{\psi_j^{(i)}}\right)>-q^{-1},\ \Re\left(\beta+t\,b_j\frac{f_{j'i'j}}{F_{j'i'j}}+t\sum_{i=1}^{n}v^{(i)}\frac{c_j^{(i)}}{\psi_j^{(i)}}\right)>-q^{-1},$$

Notes

for $j=\left(\overline{1,u_r}\right)$, then we obtain the following relation:

$$M\left\{D_{\gamma_n}^{\alpha,\beta}\left[f\left(x\right)\right]\right\}=M\left\{f\left(x\right)\right\}R_{\lambda_n}^{\alpha-s+1,\beta}\left[1\right]. \tag{3.1}$$

where $M_p\left(0,\infty\right)$ stands for the class of all functions $f\left(x\right)$ of $L_p\left(0,\infty\right)$ with $p>2$, which are inverse Mellin-transforms of the functions of $L_p\left(-\infty,\infty\right)$.

Proof. By taking Mellin transform of (2.1), we get

$$M\left\{D_{\lambda_n}^{\alpha,\beta}\left[f\left(x\right)\right]\right\}=\int_0^\infty x^{s-1}\left\{t\,x^{-\alpha-t\beta-1}\int_0^x y^\alpha\left(x^t-y^t\right)^\beta H\begin{bmatrix}\gamma_1\upsilon\\ \vdots\\ \gamma_n\upsilon\end{bmatrix}\right.$$

$$\times\prod_{j=1}^k I_{p_{i'j},q_{i'j};r}^{m_j,n_j}\left[z_j\left(\frac{y^t}{x^t}\right)^{a_j}\left(1-\frac{y^t}{x^t}\right)^{b_j}\left|\begin{matrix}\left(e_{j'j},E_{j'j}\right)_{1,n_j};\left(e_{j'i'j},E_{j'i'j}\right)_{n_j+1,p_{i'j}}\\ \left(f_{j'j},F_{j'j}\right)_{1,m_j};\left(f_{j'i'j},F_{j'i'j}\right)_{m_j+1,q_{i'j}}\end{matrix}\right.\right]$$

$$\left.\times\prod_{i=1}^r S_{N_i}^{M_i}\left[z_i\left(\frac{y^t}{x^t}\right)^{g_i}\left(1-\frac{y^t}{x^t}\right)^{h_i}\right]\psi\left(\frac{y^t}{x^t}\right)f\left(y\right)dy\right\}dx. \tag{3.2}$$

On interchanging the order of integration, which is permissible under the conditions, the result (3.1) follows easily in view of (2.2).

Theorem 3.2. If $f\left(x\right)\in L_p\left(0,\infty\right)$, $1\le p\le 2$; or $f\left(x\right)\in M_p\left(0,\infty\right)$, $p>2$, and satisfies the following conditions:

$$p^{-1}+q^{-1}=1,\ \Re\left(\delta+t\,a_j\frac{f_{j'i'j}}{F_{j'i'j}}+t\sum_{i=1}^n v^{(i)}\frac{c_j^{(i)}}{\psi_j^{(i)}}\right)>-p^{-1},\ \Re\left(\beta+t\,b_j\frac{f_{j'i'j}}{F_{j'i'j}}+t\sum_{i=1}^n v^{(i)}\frac{c_j^{(i)}}{\psi_j^{(i)}}\right)>-q^{-1},$$

for $j=\left(\overline{1,u_r}\right)$, then we obtain the following relation:

$$M\left\{R_{\gamma_n}^{\delta,\beta}\left[f\left(x\right)\right]\right\}=M\left\{f\left(x\right)\right\}D_{\lambda_n}^{\delta+s-1,\beta}\left[1\right]. \tag{3.3}$$

Proof. By taking Mellin transform of (2.2), we get

$$M\left\{R_{\lambda_n}^{\delta,\beta}\left[f\left(x\right)\right]\right\}=\int_0^\infty x^{s-1}\left\{t\,x^{\delta}\int_x^\infty y^{-\delta-t\beta-1}\left(y^t-x^t\right)^\beta H\begin{bmatrix}\gamma_1\mu\\ \vdots\\ \gamma_n\mu\end{bmatrix}\right.$$

$$\times\prod_{j=1}^k I_{p_{i'j},q_{i'j};r}^{m_j,n_j}\left[z_j\left(\frac{x^t}{y^t}\right)^{a_j}\left(1-\frac{x^t}{y^t}\right)^{b_j}\left|\begin{matrix}\left(e_{j'j},E_{j'j}\right)_{1,n_j};\left(e_{j'i'j},E_{j'i'j}\right)_{n_j+1,p_{i'j}}\\ \left(f_{j'j},F_{j'j}\right)_{1,m_j};\left(f_{j'i'j},F_{j'i'j}\right)_{m_j+1,q_{i'j}}\end{matrix}\right.\right]$$

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$$\times \prod_{i=1}^{r} S_{N_i}^{M_i}\left[z_i \left(\frac{x^t}{y^t}\right)^{g_i} \left(1-\frac{x^t}{y^t}\right)^{h_i} \right] \psi\left(\frac{x^t}{y^t}\right) f(y)\, dy \Bigg\} dx . \qquad (3.4)$$

On interchanging the order of integration, the result (3.3) can easily be obtained with the help of (2.1)

Theorem 3.3. If $f(x) \in L_p(0,\infty)$, $v(x) \in L_p(0,\infty)$ $1 \le p \le 2$; or $f(x) \in M_p(0,\infty)$, $p > 2$, and satisfies the following conditions:

$$p^{-1}+q^{-1}=1,\ \Re\left(\alpha + t\, a_j \frac{f_{j'i'j}}{F_{j'i'j}} + t\sum_{i=1}^{n} u^{(i)} \frac{c_j^{(i)}}{\psi_j^{(i)}}\right) > -q^{-1},\ \Re\left(\beta + t\, b_j \frac{f_{j'i'j}}{F_{j'i'j}} + t\sum_{i=1}^{n} v^{(i)} \frac{c_j^{(i)}}{\psi_j^{(i)}}\right) > -q^{-1},$$

for $j = \left(\overline{1, u_r}\right)$, then we obtain the following relation:

$$\int_0^\infty v(x)\, D_{\gamma_n}^{\alpha,\beta}\left[f(x)\right] dx = \int_0^\infty f(x)\, R_{\gamma_n}^{\alpha,\beta}\left[v(x)\right] dx . \qquad (3.5)$$

Proof. The result (3.5) can easily be obtained in view of equations (2.1) and (2.2).

IV. Inversion Formulae

Theorem 4.1 . If $f(x) \in L_p(0,\infty)$, $1 \le p \le 2$; or $f(x) \in M_p(0,\infty)$, $p > 2$, and satisfies the following conditions:

$$p^{-1}+q^{-1}=1,\ \Re\left(\alpha + t\, a_j \frac{f_{j'i'j}}{F_{j'i'j}} + t\sum_{i=1}^{n} u^{(i)} \frac{c_j^{(i)}}{\psi_j^{(i)}}\right) > -q^{-1},\ \Re\left(\beta + t\, b_j \frac{f_{j'i'j}}{F_{j'i'j}} + t\sum_{i=1}^{n} v^{(i)} \frac{c_j^{(i)}}{\psi_j^{(i)}}\right) > -q^{-1},$$

for $j = \left(\overline{1, u_r}\right)$ and the integrals are absolutely convergent, then we obtain the following relation:

$$D_{\gamma_n}^{\alpha,\beta}\left[f(x)\right] = v_1(x), \qquad (4.1)$$

Then it gives the following result:

$$f(x) = \int_0^\infty y^{-1}\left[v_1(y)\right]\left[h_1\left(x\, y^{-1}\right)\right] dy , \qquad (4.2)$$

where

$$h_1(x) = \frac{1}{2\pi i}\int_L \frac{x^{-s}}{R(s)}\, ds , \qquad (4.3)$$

$$R(s) = R_{\gamma_n}^{\alpha - s+1,\beta}[1]. \qquad (4.4)$$

Proof. On taking Mellin transform of (4.1) and using the relation (3.1), then we arrive at

$$M\{ f(x)\} = \frac{M\{v_1(x)\}}{R(s)},$$

Notes

which on inverting leads to

$$f(x)=\frac{1}{2\pi i}\int_L x^{-s}\frac{M\{v_1(x)\}}{R(s)}ds$$

$$=\frac{1}{2\pi i}\int_L \frac{x^{-s}}{R(s)}\left\{\int_0^\infty y^{s-1}\left[v_1(y)\right]dy\right\}ds.$$

Notes

On interchanging the order of integration and using the (4.3), we get

$$f(x)=\int_0^\infty \frac{v_1(y)}{y}\left\{\frac{1}{2\pi i}\int_L \left(\frac{x}{y}\right)^{-s}\frac{1}{R(s)}\,ds\right\}dy.$$

This completes the proof of (4.2).

Theorem 4.2. If $f(x)\in L_p(0,\infty)$, $1\le p\le 2$; or $f(x)\in M_p(0,\infty)$, $p>2$, and satisfies the following conditions:

$$p^{-1}+q^{-1}=1,\ \Re\left(\delta+t\,a_j\frac{f_{j'i'j}}{F_{j'i'j}}+t\sum_{i=1}^n u^{(i)}\frac{c_j^{(i)}}{\psi_j^{(i)}}\right)>-p^{-1},\ \Re\left(\beta+t\,b_j\frac{f_{j'i'j}}{F_{j'i'j}}+t\sum_{i=1}^n v^{(i)}\frac{c_j^{(i)}}{\psi_j^{(i)}}\right)>-q^{-1},$$

for $j=\left(\overline{1,u_r}\right)$ and the integrals are absolutely convergent, then we obtain the following relation:

$$R_{\gamma_n}^{\delta,\beta}\left[f(x)\right]=v_2(x), \tag{4.5}$$

Then it gives the following result:

$$f(x)=\int_0^\infty y^{-1}\left[v_2(y)\right]\left[h_2\left(x\,y^{-1}\right)\right]dy, \tag{4.6}$$

where

$$h_2(x)=\frac{1}{2\pi i}\int_L \frac{x^{-s}}{D(s)}ds, \tag{4.7}$$

$$D(s)=D_{\gamma_n}^{\delta+s-1,\beta}[1]. \tag{4.8}$$

Proof. By taking Mellin transform of (4.5) and using the relation (3.3), then we get

$$M\{f(x)\}=\frac{M\{v_2(x)\}}{D(s)},$$

which on inverting leads to

$$f(x)=\frac{1}{2\pi i}\int_L x^{-s}\frac{M\{v_2(x)\}}{D(s)}ds$$

$$=\frac{1}{2\pi i}\int_L \frac{x^{-s}}{D(s)}\left\{\int_0^\infty y^{s-1}\left[v_2(y)\right]dy\right\}ds.$$

Further, on interchanging the order of integration and using the (4.7), we get

$$f(x)=\int_0^\infty \frac{v_2(y)}{y}\left\{\frac{1}{2\pi i}\int_L\left(\frac{x}{y}\right)^{-s}\frac{1}{D(s)}\,ds\right\}dy.$$

This completes the proof of (4.6).

Notes

V. General Properties

The relations given in this section can be established with the help of the definitions (2.1) and (2.2).

$$x^{-1}\,D_{\gamma_n}^{\alpha,\beta}\left[\frac{1}{x}f\left(\frac{1}{x}\right)\right]=R_{\gamma_n}^{\alpha,\beta}\left[f(x)\right], \tag{5.1}$$

$$x^{-1}\,R_{\gamma_n}^{\delta,\beta}\left[\frac{1}{x}f\left(\frac{1}{x}\right)\right]=D_{\gamma_n}^{\delta,\beta}\left[f(x)\right], \tag{5.2}$$

$$x^{u}\,D_{\gamma_n}^{\alpha,\beta}\left[f(x)\right]=D_{\gamma_n}^{\alpha-u,\beta}\left[x^{u}f(x)\right], \tag{5.3}$$

$$x^{u}\,R_{\gamma_n}^{\delta,\beta}\left[f(x)\right]=R_{\gamma_n}^{\delta+u,\beta}\left[x^{u}f(x)\right], \tag{5.4}$$

The properties given below express the homogeneity of the operators D and R respectively.

If $D_{\gamma_n}^{\alpha,\beta}\left[f(x)\right]=v_1(x)$, then $D_{\gamma_n}^{\alpha,\beta}\left[f(cx)\right]=v_1(cx)$.

If $R_{\gamma_n}^{\delta,\beta}\left[f(x)\right]=v_2(x)$, then $R_{\gamma_n}^{\delta,\beta}\left[f(cx)\right]=v_2(cx)$.

VI. Special Cases

(i) If we reduce the general class of polynomials to unity and also taking $k=1=r$, the I-function reduces to Fox's H-function [8] and further specifying the parameters appropriately in view of the relationship[3].

$$\frac{t^{rq-\upsilon-1}}{\Gamma(r)}H_{1,2}^{1,1}\left[-at^{q}\left|\begin{matrix}(1-r,1)\\(0,1),(\upsilon-rq+1,q)\end{matrix}\right.\right]=G_{q,\upsilon,r}\left[a,t\right].$$

Then, we obtain the following pair of fractional integral operators in terms of Lorenzo-Hartely G-function [6]:

$$D_{\gamma_n,\varepsilon}^{\alpha,\beta}\left[f(x)\right]=t\,\Gamma(\varepsilon)\,x^{-\alpha-t\left\{\beta-\left(\frac{a}{b}+1\right)(\varepsilon b-\upsilon-1)\right\}-1}\int_0^x y^{-\alpha-\frac{ta}{b}(\varepsilon b-\upsilon-1)}\left(x^t-y^t\right)^{\beta-\varepsilon b+\upsilon+1}$$

$$\times H\begin{bmatrix}\gamma_1\upsilon\\ \vdots\\ \gamma_n\upsilon\end{bmatrix}G_{b,\upsilon,\varepsilon}\left[z,\left(\frac{y^t}{x^t}\right)^{\frac{a}{b}}\left(1-\frac{y^t}{x^t}\right)\right]\psi\left(\frac{y^t}{x^t}\right)f(y)\,dy, \tag{6.1}$$

and

$$R_{\gamma_n,\varepsilon}^{\delta,\beta}\left[f(x)\right]=t\,\Gamma(\varepsilon)\,x^{\delta-\frac{ta}{b}(\varepsilon b-\upsilon-1)}\int_0^x y^{\delta-t\left\{\beta-\left(\frac{a}{b}+1\right)(\varepsilon b-\upsilon-1)\right\}-1}\left(y^t-x^t\right)^{\beta-\varepsilon b+\upsilon+1}$$

$$\times H\begin{bmatrix}\gamma_1\upsilon\\ \vdots\\ \gamma_n\upsilon\end{bmatrix}G_{b,\upsilon,\varepsilon}\left[z,\left(\frac{x^t}{y^t}\right)^{\frac{a}{b}}\left(1-\frac{x^t}{y^t}\right)\right]\psi\left(\frac{x^t}{y^t}\right)f(y)\,dy\,. \tag{6.2}$$

Notes

(ii) Further, taking $\varepsilon=1$ in (6.1) and (6.2), in view of the relationship [3]

$$t^{q-\upsilon-1}H_{1,2}^{1,1}\left[-at^{\upsilon}\left|\begin{matrix}(0,1)\\(0,1),(\upsilon-q+1,q)\end{matrix}\right.\right]=R_{q,\upsilon}\left[a,t\right],$$

then, we get the following pair of fractional integral operators containing Lorentzo-Hartley R-function [6]:

$$D_{\gamma_n,1}^{\alpha,\beta}\left[f(x)\right]=t\;x^{-\alpha-t\left\{\beta-\left(\frac{a}{b}+1\right)(\varepsilon b-\upsilon-1)\right\}-1}\int_0^x y^{-\alpha-\frac{ta}{b}(\varepsilon b-\upsilon-1)}\left(x^t-y^t\right)^{\beta-b+\upsilon+1}$$

$$\times H\begin{bmatrix}\gamma_1\upsilon\\ \vdots\\ \gamma_n\upsilon\end{bmatrix}R_{b,\upsilon}\left[z,\left(\frac{y^t}{x^t}\right)^{\frac{a}{b}}\left(1-\frac{y^t}{x^t}\right)\right]\psi\left(\frac{y^t}{x^t}\right)f(y)\,dy\,, \tag{6.3}$$

and

$$R_{\gamma_n,1}^{\delta,\beta}\left[f(x)\right]=t\;x^{\delta-\frac{ta}{b}(\varepsilon b-\upsilon-1)}\int_0^x y^{\delta-t\left\{\beta-\left(\frac{a}{b}+1\right)(b-\upsilon-1)\right\}-1}\left(y^t-x^t\right)^{\beta-b+\upsilon+1}$$

$$\times H\begin{bmatrix}\gamma_1\upsilon\\ \vdots\\ \gamma_n\upsilon\end{bmatrix}R_{b,\upsilon}\left[z,\left(\frac{x^t}{y^t}\right)^{\frac{a}{b}}\left(1-\frac{x^t}{y^t}\right)\right]\psi\left(\frac{x^t}{y^t}\right)f(y)\,dy\,. \tag{6.4}$$

(iii) If we reduce H-function of several complex variables transforms to the generalized Lauricella function of several complex variables [17] and, we get the following fractional integral operators:

$$X_{\gamma_n}^{\alpha,\beta}\left[f(x)\right]=B\,t\,x^{-\alpha-t\beta-1}\int_0^x y^{\alpha}\left(x^t-y^t\right)^{\beta}$$

$$\times F_{C:D';\ldots;D^{(n)}}^{A:B';\ldots;B^{(n)}}\left[\begin{matrix}\left[1-(a):\theta',\ldots,\theta^{(n)}\right],\left[1-(b'):\phi'\right];\ldots;\left[1-(b)^{(n)}:\phi^{(n)}\right];\\ \left[1-(c):\psi',\ldots,\psi^{(n)}\right],\left[1-(d'):\delta'\right];\ldots;\left[1-(d)^{(n)}:\delta^{(n)}\right];\end{matrix}\;-\gamma_1\upsilon,\ldots,-\gamma_n\upsilon\right]$$

$$\times\prod_{j=1}^{k}I_{p'_{ij},q'_{ij};r}^{m'_j,n'_j}\left[z_j\left(\frac{y^t}{x^t}\right)^{a_j}\left(1-\frac{y^t}{x^t}\right)^{b_j}\left|\begin{matrix}\left(e_{p'_{ij}},E_{p'_{ij}}\right)\\ \left(f_{q'_{ij}},F_{q'_{ij}}\right)\end{matrix}\right.\right]$$

$$\times \prod_{i=1}^{r} S_{N_i}^{M_i}\left[z_i \left(\frac{y^t}{x^t}\right)^{g_i} \left(1-\frac{y^t}{x^t}\right)^{h_i} \right] \psi\left(\frac{y^t}{x^t}\right) f(y)\, dy \;, \tag{6.5}$$

and

$$Y_{\gamma_n}^{\delta,\beta}\left[f(x)\right] = B\, t\, x^{\delta} \int_x^{\infty} y^{-\delta-t\beta-1} \left(y^t - x^t\right)^{\beta}$$

$$\times F_{C:D';\dots;D^{(n)}}^{A:B';\dots;B^{(n)}} \left[\begin{matrix} \left[1-(a):\theta',\dots,\theta^{(n)}\right], \left[1-(b'):\phi'\right];\dots;\left[1-(b)^{(n)}:\phi^{(n)}\right]; \\ \left[1-(c):\psi',\dots,\psi^{(n)}\right], \left[1-(d'):\delta'\right];\dots;\left[1-(d)^{(n)}:\delta^{(n)}\right]; \end{matrix} \; -\gamma_1\mu,\dots,-\gamma_n\mu \right]$$

$$\times \prod_{j=1}^{k} I_{p'_{ij},q'_{ij};r}^{m'_j,n'_j} \left[z_j \left(\frac{x^t}{y^t}\right)^{a_j} \left(1-\frac{x^t}{y^t}\right)^{b_j} \left| \begin{matrix} \left(e_{p'_{ij}}, E_{p'_{ij}}\right) \\ \left(f_{q'_{ij}}, F_{q'_{ij}}\right) \end{matrix} \right. \right]$$

$$\times \prod_{i=1}^{r} S_{N_i}^{M_i}\left[z_i \left(\frac{x^t}{y^t}\right)^{g_i} \left(1-\frac{x^t}{y^t}\right)^{h_i} \right] \psi\left(\frac{x^t}{y^t}\right) f(y)\, dy \;, \tag{6.6}$$

where

$$B = \frac{\prod_{j=1}^{A}\Gamma\left(1-a_j\right)\prod_{j=1}^{B'}\Gamma\left(1-b'_j\right)\dots\prod_{j=1}^{B^{(n)}}\Gamma\left(1-b_j^{(n)}\right)}{\prod_{j=1}^{C}\Gamma\left(1-c_j\right)\prod_{j=1}^{D'}\Gamma\left(1-d'_j\right)\dots\prod_{j=1}^{D^{(n)}}\Gamma\left(1-d_j^{(n)}\right)}\,. \tag{6.7}$$

The operators earlier defined by Saxena et al. [13], Chaurasia and Srivastava [1], can also be easily derived by assigning suitable values to the parameters occurring in (2.1) and (2.2).

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Effect of Buoyancy Force on the flow field in a Triangular Cavity

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Abstract- In this paper, the effect of buoyancy force in a triangular enclosure are studied numerically. The governing differential equations are solved by using finite element method (weighted-residual method). Here the left wall of the triangle is assumed to be adiabatic, the right and horizontal wall are kept at cold and heated respectively. Also all the wall are assumed to be no-slip condition. The effective governing dimensionless parameters for this problem are Rayleigh number, Prandtl number and Hartmann number.

Keywords: *buoyancy force; finite element method; hartmann number; triangular cavity.*

GJSFR-F Classification : *MSC 2010: 57Q15*

EFFECTOFBUOYANCYFORCEONTHEFLOWFIELDINATRIANGULARCAVITY

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I. Introduction

Natural convection heat transfer and fluid flow are widely studied topics in engineering due to their practical importance as reviewed by [1] and [2]. Nearly forty years ago as in [3] and [4] studied the natural convection in enclosures with internal heat generation occurs in nuclear reactors and geothermal heat extraction processes. Based on technological applications of internal heat generation problems in nuclear reactors and geometrical applications, as in [5] and [6] obtained several solutions. Besides regular geometries such as square or rectangle, many studies wavy-walled enclosures with or without internal heat generation. Natural convection in wavy enclosures with volumetric heat sources was investigated by [7]. They found that, both the function of wavy wall and the ratio of internal Rayleigh number Ra_I to external Rayleigh number Ra_E affect the heat transfer and fluid flow significantly. The heat transfer is predicted to be a decreasing function of waviness of the top and bottom walls in case of

$$\frac{Ra_I}{Ra_E} > 1 \text{and} \frac{Ra_I}{Ra_E} < 1$$

Most of the enclosures commonly used in industries are cylindrical, rectangular, trapezoidal and triangular etc. In recent years, triangular enclosures have received a considerable attention because of its applicability in various fields. Finite element analysis of natural convection in triangular enclosure was studied by [8]. They found that at low Rayleigh numbers $(Ra \le 10^4)$, the isotherms are almost parallel near the bottom portion of the triangular enclosure while at $Ra = 10^5$, the isotherms are more distorted. This is because the heat transfer is primarily

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due to conduction for lower values of Rayleigh number. As Rayleigh number increases, there is a change from conduction dominant region to convection dominant region, and the critical Rayleigh number corresponding to on-set of convection is obtained. The main objective of this paper is to study natural convection heat transfer in a triangular shape enclosure. From the above literatures, the aim of present investigation is to investigate the effect of buoyancy force in triangular shape enclosure. The results are presented in terms of streamlines, isotherms, velocity profiles, temperature profiles and local Nusselt number.

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II. Model and Mathematical Formulation

The Figure 1 shows a schematic diagram and the coordinates of a two-dimensional triangular cavity, where the bottom wall is maintained at a uniform temperature T_h and the left wall maintained adiabatic whereas right wall T_c colder. The fluid is permeated by a uniform magnetic field B_0 which is applied normal to the direction of the flow and the gravitational force (g) acts in the vertically downward direction.

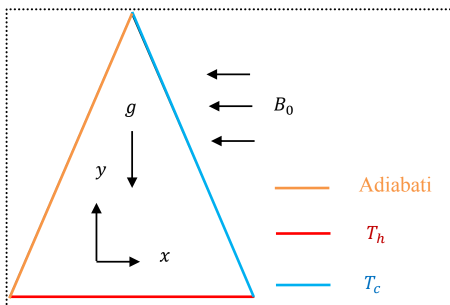

Figure 1 : The flow configuration and coordinate system

The fluid properties, including the electrical conductivity, are considered to be constant, except for the density, so that the Boussinesq approximation is used. Neglecting the radiation mode of the heat transfer and Joule heating, the governing equations for mass, momentum and energy of a steady two-dimensional natural convection flow in a triangular cavity are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\rho g\beta\left(T-T_c\right)-\sigma B_0{}^2 v \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

The governing equations are nondimensionalized using the following dimensionless variables:

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{uL}{\alpha}, \quad V = \frac{vL}{\alpha},$$
$$P = \frac{pL^2}{\rho\alpha^2}, \theta = \frac{T - T_c}{T_h - T_c}, \quad \sigma = \frac{\rho^2\alpha}{L^2}, \quad \alpha = \frac{k}{\rho C_p}$$

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Introducing the above dimensionless variables, the following dimensionless forms of the governing equations are obtained as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{5}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \tag{6}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \Pr\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + \frac{Ra}{\Pr}\theta - Ha^2 \Pr V \tag{7}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \tag{8}$$

Here Pr is the Prandtl number, *Ra* is the Rayleigh number and *Ha* is the Hartmann number, which are defined as:

$$\Pr = \frac{\nu}{\alpha}, \; Ha^2 = \frac{\sigma B_0{}^2 L^2}{\mu}, \; Ra = \frac{g\beta L^3 (T_h - T_c)\Pr}{\nu^2}$$

The corresponding boundary conditions then take the following form:

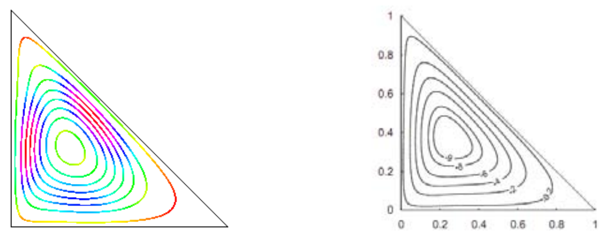
$$U = V = 0, \; \theta = 1 \;(\text{on the bottom wall})$$
$$U = V = 0, \; \theta = 0 \;(\text{on the right wall})$$
$$U = V = 0, \; \frac{\partial \theta}{\partial N} = 0 \;(\text{at left wall})$$
$$P = 0 \left(\begin{array}{c}\text{fluid pressure, at the inside and on the} \\ \text{wall of the enclosure}\end{array}\right)$$

III. Numerical Procedure

The Galerkin weighted residual method of finite element formulation is used to solve the dimensionless governing equations with the boundary conditions. This technique is well described by [9] and [10]. In this method, the solution domain is discretized into finite element meshes and then the nonlinear governing equations are transferred into a system of integral equations by applying the Galerkin weighted residual method. Gauss quadrature method is used to perform the integration involved in each term of these equations. The nonlinear algebraic equations which are obtained are modified by imposition of boundary conditions and Newton's method is used to transform these modified equations into linear algebraic equations, and then these linear equations are solved by applying the triangular factorization method.

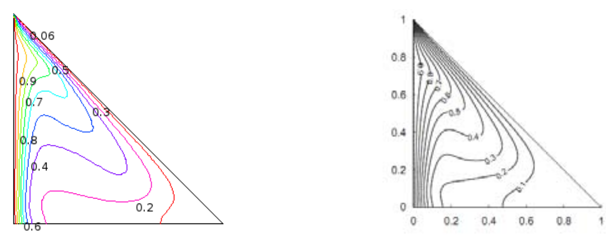
IV. Code Validation

In order to verify the accuracy of the numerical results which are obtained throughout the present study are compared with the previously published results. The present results of streamlines and isotherms are compared with that of [11] while uniformly heated left wall $\theta(0,Y)=1$ and cooled right inclined wall $\theta(X,Y)=0, \forall X+Y=1$ with Pr = 1000 and $Ra=10^5$ and obtained good agreement which is shown in Fig. 2.



Present work Reference [11]

Figure 2 (a) : Obtained results for Streamlines



Present work Reference [11]

Figure 2 (b) : Obtained results Isotherms

V. Results and Discussion

In this section, results of the numerical study on magneto hydrodynamic buoyancy force in a triangular cavity filled with an electric conductive fluid with Pr = 0.71 are presented. The results have been obtained for the Rayleigh number ranging from 10^4 to 10^6 and the three Hartman numbers 0, 50 and 100. The results are presented in terms of streamlines and isotherms inside the cavity, the horizontal velocity component, temperature and the local Nusselt number along the vertical centerline X=0.5 of the cavity. Figures 3 and 4 presents the effects of the Rayleigh number $Ra=10^4$ on the flow field for different Hartmann number (a) Ha=0; (b) Ha=50; (c) Ha=100. They are illustrated by streamlines (Fig. 3) and isotherms (Fig. 4) respectively. The results for $Ra=10^4$ and different Hartmann number given in Fig. 3 and it is found that a triangle shape flow distribution is formed. The main flow moves in clockwise direction and for Ha=0 it is not broken anywhere in the triangle. Also a small cell is formed at the centre of the triangle and in this case flow becomes uniform within the whole enclosure. The flow streamlines fill the whole enclosure geometry except at the corners. Also a large velocity is distributed uniformly within the whole enclosure for Ha=0. As the value of the Hartmann number increases to 0 to 100, the flow velocity decreases due to the insulated edge and the flow started too broken due to the cold edge. Finally for Ha=100 the flow velocity become very weak due to the insulated edge and the last cell of the flow is broken and twisted backward to the

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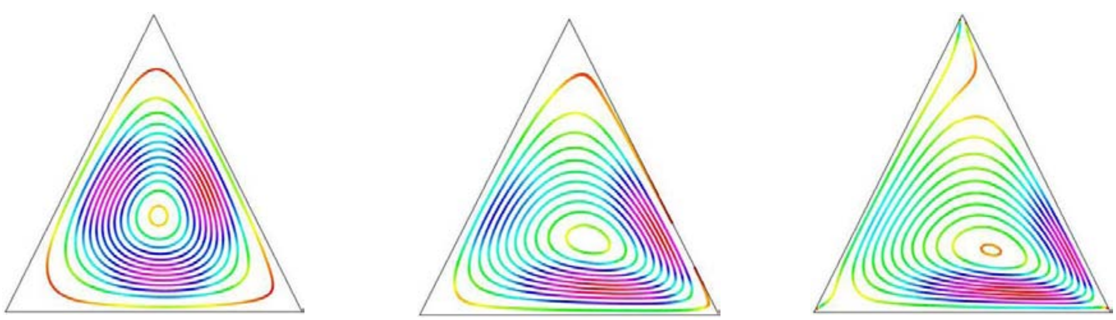
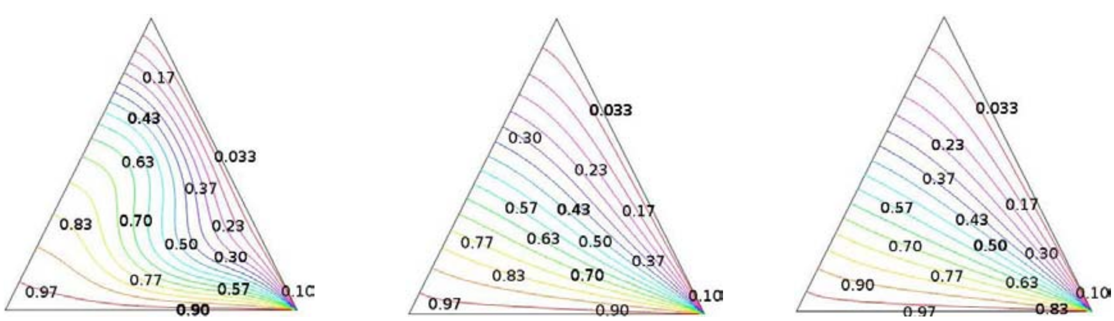
adiabatic edge. The flow becomes motionless at these parts due to the insulated edge. The isotherm lines showed in Fig. 4 indicates similar distribution with a benchmark problem of differentially heated cavity except at the insulated edge. Figure 4(a) shows that, the boundary layer decreases from heated edge to cold edge and isotherms lines are started bending due to insulated edge and become smother near the cold edge for *Ha*=0. As we increase the Hartmann number in this case the lines become smothering but the boundary layer increases. Figures 5 and 6 shows the streamlines and isotherm lines for (a) *Ha*=0, (b) *Ha*=50 and (c) *Ha*=100 while Pr=0.71 and *Ra*=10^5. As we increase Rayleigh number *Ra*=10^5, the changes in streamlines within the triangular enclosure are negligible (same as above) but the isotherm lines are highly bending at the middle of the triangular enclosure and become smother due to the cold edge for *Ha*=0. Also the boundary layer become decreases due to the cold edge for *Ha* =0. But as we increase Hartmann number the bending lines become smother and the boundary layer increases. Finally Fig. 7 and Fig. 8 shows the streamlines and isotherms for (a) *Ha*=0, (b) *Ha*=50 and (c) *Ha*=100 while Pr=0.71 and *Ra*=10^6. For *Ra*=10^6 , the flow velocity become weak and streamlines are bended due to the insulated edge, isotherms lines are highly bended at the middle of the triangular enclosure and become more smother near the cold edge for *Ha*=0. As we increase Hartmann number the isotherm lines becomes smother and the boundary layer increases. Thus, if we increase both Hartmann number and Rayleigh number the flow become weak and boundary layer increases. The local Nusselt number along the vertical centerline X=0.5 of the enclosure is plotted in Figs. 9(a) - 9(c) for different values of Rayleigh number 10^4,10^5 and 10^6. It is also plotted for different values of Hartmann number. The maximum value of the local Nusselt number occur around Y= 0.7 for *Ra*=10^6 and *Ha*=0. Comparison of Fig. 9(b) and 9(c) indicates that the maximum value of local Nusselt number decreases for *Ha*=0 and *Ha*=100 respectively while *Ra*=10^6.



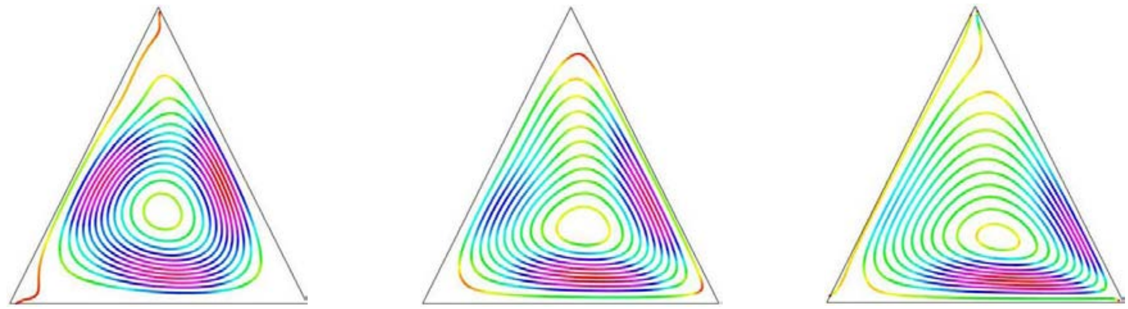
(a) (b) (c)

Figure 3 : Streamlines for (a) *Ha*=0; (b) *Ha*=50; (c) *Ha*=100 while Pr=0.71 and *Ra*=10^4



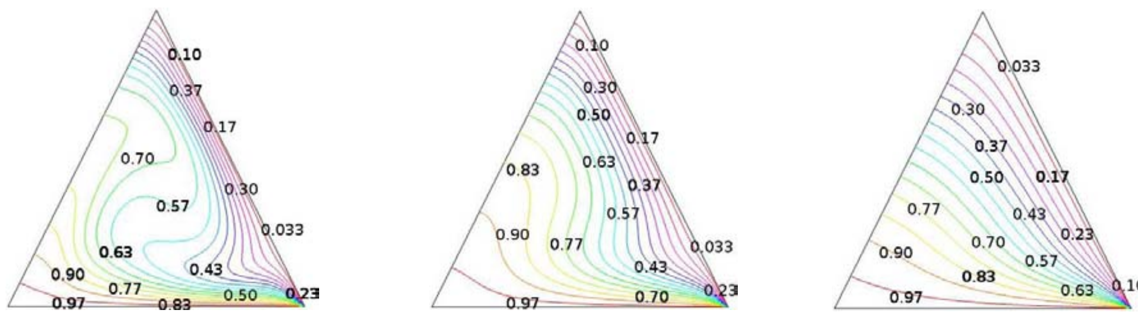
(a) (b) (c)

Figure 4 : Isotherms for (a) *Ha*=0; (b) *Ha*=50; (c) *Ha*=100 while Pr=0.71 and *Ra*=10^4



(a) (b) (c)

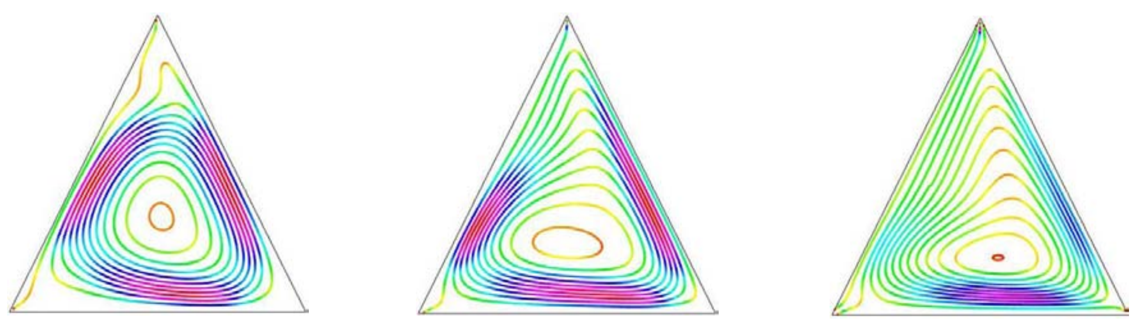
Figure 5 : Streamlines for (a) *Ha*=0; (b) *Ha*=50; (c) *Ha*=100 while Pr=0.71 and $Ra=10^4$



(a) (b) (c)

Figure 6 : Isotherms for (a) *Ha*=0; (b) *Ha*=50; (c) *Ha*=100 while Pr=0.71 and $Ra=10^5$

The profiles of the dimensionless X-component of velocity *U* versus the vertical co-ordinate are plotted at the middle of the enclosure based on the X-axis in Fig. 10(a) to (c). They are plotted for different curved length of the enclosure. It is clearly seen in Fig 10(a), for *Ha*=0 the highest velocity is obtained for $Ra=10^6$. The velocity profiles indicate the most important parameter on the flow field is the shape of the enclosure. Figures 11(a) -11 (c) indicate the temperature profile versus the vertical co-ordinate. It is clear from the above figure that, for $Ra=10^4$ we get the smoothed temperature profile. As we increase Hartmann number the profile become more smother.



(a) (b) (c)

Figure 7 : Streamlines for (a) *Ha*=0; (b) *Ha*=50; (c) *Ha*=100 while Pr=0.71 and $Ra=10^6$

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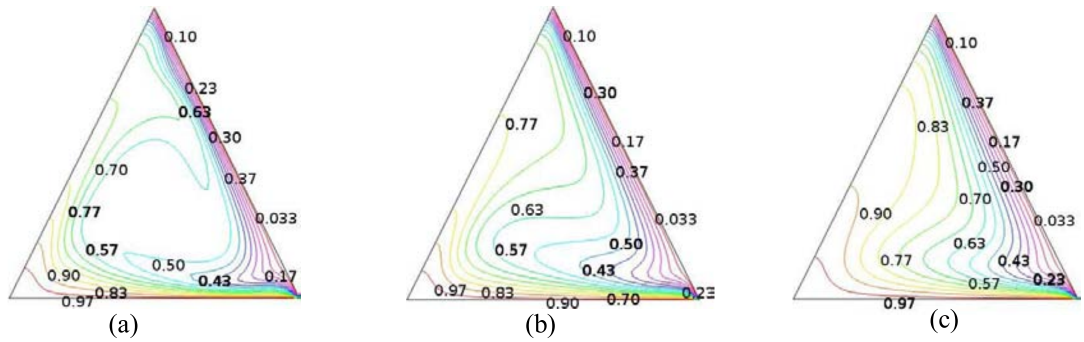

Figure 8 : Isotherms for (a) *Ha*=0; (b) *Ha*=50; (c) *Ha*=100 while Pr=0.71 and *Ra*=10^6

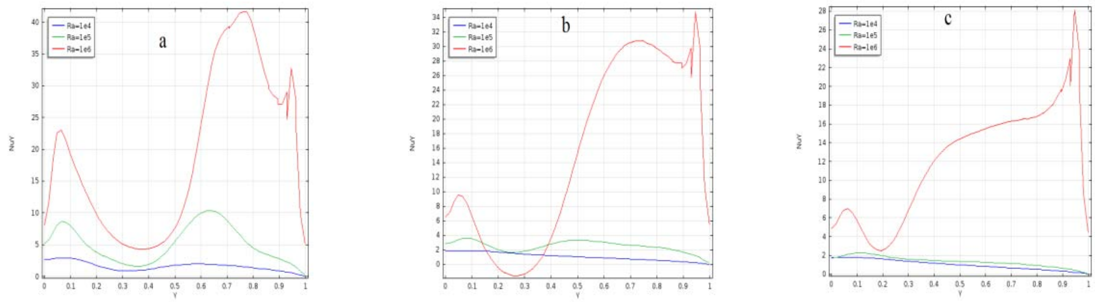

Figure 9 : Variation of local Nusselt number along the vertical centerline X=0.5 for different Rayleigh number with Pr=0.71 for (a) *Ha*=0, (b) *Ha* = 50 and (c) *Ha*=100.

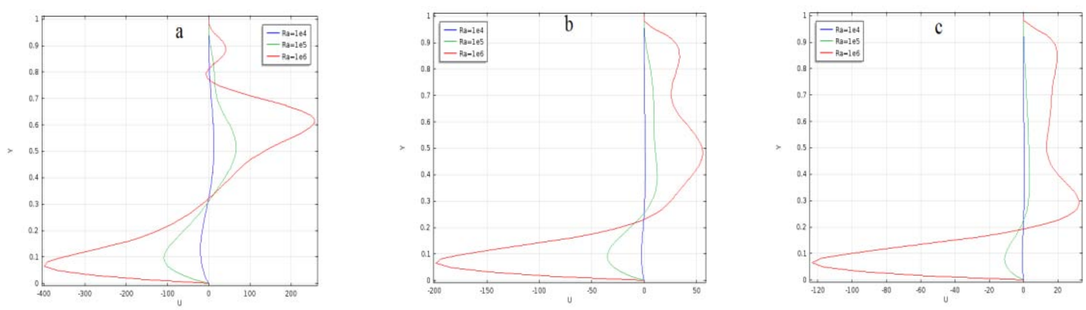

Figure 10 : Variation of velocity profiles along the vertical centerline X=0.5 at different Rayleigh number with Pr=0.71 for a) *Ha*=0, b) *Ha*=50 and c) *Ha*=100.

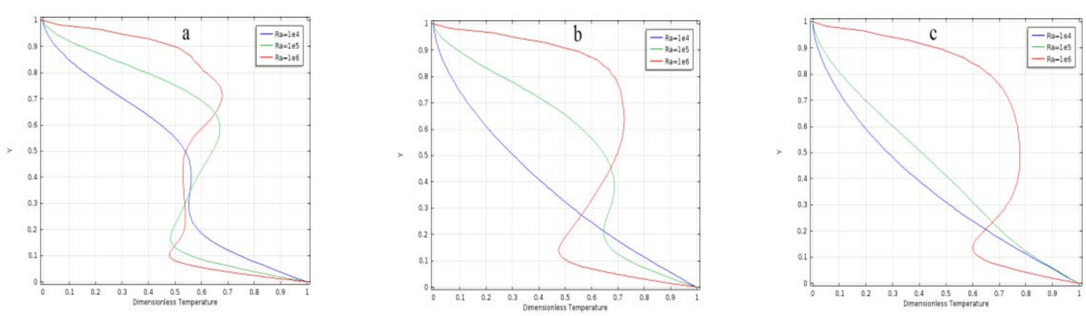

Figure 11 : Variation of temperature profiles along the vertical centerline X=0.5 at different Rayleigh number with Pr=0.71 for a) *Ha*=0, b) *Ha*=50 and c) *Ha*=100.

VI. Conclusion

Effect of buoyancy force on the flow field in a triangular cavity with uniform magnetic field B_0 which is applied normal to the direction of the flow was studied numerically. The governing equations of mass, momentum and energy were solved using the Galerkin weighted residual method of finite element formulation. As indicated above that the governing parameters were the Prandtl number Pr, the Rayleigh number *Ra* and the Hartmann number *Ha*. The effects of Rayleigh number *Ra* with the variations Hartmann number *Ha* on the flow field have been studied in detail while Prandtl number Pr=0.71. From the present investigation the following conclusions may be drawn as: if the Rayleigh number increases, the local Nusselt number increases without the effect of magnetic field whereas the local Nusselt number slowly increases with the increase of Rayleigh number with applying the effect of magnetic field.

Notes

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Issues in Growth of Mathematics Education

By Mr. Jiten Phukan & Dr. Shibu Basak
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Abstract- Mathematics plays an important role in the fast changing educational scenario. It has a wide application in the present growing science and technological world. Mathematical knowledge is the key to understand the science and technology and it can be familiarized through the friendly Mathematical environment among the learners, teachers, parents and other stakeholders from the beginning of schooling. The growth of mathematics education is not satisfactory from the schooling. Due to lack of best practices of the known and unknown factors that to be studied are the hindrances in the growth of mathematics education. This study aims to focus on the issues of the growth of Mathematic education.

Keywords: mathematic education, issues, growth.

GJSFR-F Classification : *MSC 2010: 97A30 , 97M20*

ISSUESINGROWTHOFMATHEMATICSEDUCATION

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Issues in Growth of Mathematics Education

Mr. Jiten Phukan [α] & Dr. Shibu Basak [σ]

Abstract- Mathematics plays an important role in the fast changing educational scenario. It has a wide application in the present growing science and technological world. Mathematical knowledge is the key to understand the science and technology and it can be familiarized through the friendly Mathematical environment among the learners, teachers, parents and other stakeholders from the beginning of schooling. The growth of mathematics education is not satisfactory from the schooling. Due to lack of best practices of the known and unknown factors that to be studied are the hindrances in the growth of mathematics education. This study aims to focus on the issues of the growth of Mathematic education.

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I. Introduction

The development of a country depends on its Science and Technology. Mathematics is the basis of all Science and Technology. It has an interdisciplinary relevance also. Mathematics has a tremendous role to create the potentials of other disciplines. The vision of mathematics teaching is to communicate the mathematics' knowledge to all the students. Mathematics is the subject that has the ability to develop the basic skill such as reasoning and analyzing. Keeping in view of its importance and relevance to modern demands, Kothari commission (1964-66) on its report suggested that "Mathematics should be taught on a compulsory basis to all pupils as a part of general education during first ten years of schooling". But this compulsion is unable to carry a well trend of enrolments from the class '10+' onwards. Present state of mathematics education in schooling is not satisfactory. Most students scored poorly in the subject. One hand the demand of the subject is increasing day by day, on the other hand the enrolments of the students are decreasing. The subject is unable to attract the best students towards it. This restricts the growth of the subject. There is an urgent need to asses critically the basic problems associated with the Mathematics Education.

II. Area of the Study

The domain of the study is covered some selected schools of rural and urban area of Sivasagar district of Assam. Sivasagar is situated between 26.45^0 and 27.15^0 North latitude and 94.25^0 and 95.25^0 East longitudes. It occupies an area of 2,668 square kilometres. It has presently 156 Secondary Schools including 40 in Higher Secondary Standard. These schools can depict the picture of Mathematical Status in schooling of Assam.

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III. Objectives of the Paper

Being a compulsory subject to the class X, it is responsible to create a mathematical environment among the children so that they can feel that the subject "Mathematics" is affordable, understandable and at the same time enjoyable. This paper is an attempt to highlight the issues that have adversely affected the growth of Mathematics Education and to find out the remedial measures to popularize Mathematics. Specifically a comparative study is observed between the Class X Examination and in the Higher Secondary Final Examinations in areas of enrolments of Mathematics, Dropout rate in Mathematics and the Achievements in the subjects.

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IV. Methodology of the Study

The study is based on Primary and Secondary data. Sivasagar district has been selected purposively for the study. To fulfil the aim of the study, the data are collected through Questionnaire. Interactions take place with the Head of the institution and the subject teacher to find out the issues in growth of Mathematics education. The Randomly selection of ten schools of urban and rural areas of the Sivasagar District are taken for the study to reflect the performance in Mathematics in class X Examination and in Higher Secondary Final Examination. This shows the present status of Mathematics in schooling. Therefore, to carry out this study Survey Method and Random Sampling has been followed.

V. Issues in Growth of Mathematics

The factors that affect the growth of the Mathematics Education are observed as follows-

(i) The lack of awareness about the importance of the Role of Mathematics in the present day global perspective among the students, guardians and even in teachers.

(ii) Memorization of the tables in Primary section without techniques, the geometrical theorems without logics is the beginning of dislike mathematics from the schooling.

(iii) Mathematics anxiety for the memorizing and recalling process of examinations rather than knowledge gaining practices.

(iv) The fear of learning Mathematics from the beginning of schooling bring into existence of intense fear which is known as Mathematics Phobia.

(v) Lack of regular practice of the students to learn mathematics thoroughly.

(vi) Teachers-Students poor relationships for conversations and healthy interactions

(vii) Lack of teachers' support in the class rooms to create the confidence in achieving mathematics.

(viii) Very often teachers teach the concepts in the class without discussing its further use or practical utility

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(ix) Lack of proper execution of lesson planning

(x) It is general apprehension among the stakeholders such as parents, guardians and even one portion of teachers that majority of the students have no skills of learning mathematics. They have the myth that the curriculum of mathematics is far off from the real life.

(xi) One of the major lacunae in Mathematics Education is the poor Library for the subject Mathematics in the Schools.

VI. Analysis of Data

We have surveyed the performance of mathematics in H.S.L.C and H.S.S.L.C examinations in ten schools randomly in Sivasagar district which depicts the status of the present scenario of Mathematics in Secondary Schools.

The Enrolments Figures in Mathematics in the sample ten schools are presented in Table-1and 2 respectively for Class X and Class XII.

Table 1

Enrolments in Mathematics in ClassX

Year	Enrolments
2010	827
2011	752
2012	799

Table 2

Enrolments in Mathematics in ClassXII

Year	Enrolments
2010	168
2011	201
2012	204

The above figures shows that the comparison of the enrolments of Mathematics students between Class X and Class XII is the ratio 4.15 to 1 in an average, that is, the dropout rate of Mathematics' student from Class X to Higher Secondary course is 76% which alarms the growth of Mathematics Education.

The Result of "Class X" Examination and "Class XII" Examination of Ten schools in the Sivasagar District are presented below.

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Table 3
School No.1.
Level: Higher Secondary
Status: Rural
H.S.L.C.Examination Results

Year	Students appeared	Students passed	60% above marks in Math.	45% to 60% marks in Math.	Below 45% marks in Math
2010	131	67	Nil	02	65
2011	52	50	Nil	Nil	50
2012	79	79	01	09	69

This rural school has not a quality performance in mathematics. The average pass percentage in mathematics in the three years is 75% in which only 12 students are able to obtain 45% above. No enrolments of Mathematics students appear in this Higher Secondary section in school no.1, which has only Arts stream provision in Higher Secondary.

Table 4
School No.2.
Level: High School
Status: Rural

Year	Students appeared	Students passed	60% above Marks in Math.	45% to 60% marks in Math.	Below 45% Marks in Math.
2010	85	63	03	03	57
2011	82	64	04	03	57
2012	91	83	04	18	61

Considering enrolments together for the three years, out of 258 mathematics' students, the number of pass students is 210. Even the unsuccessful students in mathematics is 20%, this school has to struggle with the quality of mathematics.

Table 5
School No.3.
Level: Higher Secondary
Status: Rural
H.S.L.C.Examination

Year	Students Appeared	Students Passed	60% above Marks in Math.	45% to 60% marks in Math.	Below 45% marks in Math.
2010	41	36	02	03	31
2011	41	39	02	08	29
2012	36	33	01	03	29

Most of the students of this rural school were not able to score better marks in Mathematics. Only 19 students able to score more than 45% marks in mathematics against the 118 students for the three years.

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Table 6

H.S.S.L.C.Examination(Science)

Year	Students Appeared	Students Passed	60% above Marks in Math.	45% to 60% marks in Math.	Below 45% marks in Math.
2010	18	17	01	04	12
2011	22	22	01	05	16
2012	26	25	03	07	15

The achievement in mathematics in Higher Secondary level of the school 'C' is a progressive one. The enrolments of mathematics' students come down by 41% from Class X into Higher Secondary level for the mentioning three years.

Table 7

School No.4.

Level: Higher Secondary

Status: Rural

H.S.L.C.Examination

Year	Students Appeared	Students Passed	60% above Marks in Math.	45% to 60% marks in Math.	Below 45% marks in Math.
2010	96	93	12	21	60
2011	86	80	11	32	37
2012	79	73	13	46	14

This is a rural school in Sivasagar sub-division. The result shows that the school 'D' is able to maintain the quality of mathematics students in Class X Examination. 125 students get more than 45% marks in mathematics in the total of 261 students.

Table 8

H.S.S.L.C.Examination(Arts)

Year	Students appeared in Mathematics	Students passed in Mathematics
2010	03	01
2011	03	02
2012	05	03

It is observed that even this rural school shows good performance in level of matriculation, the enrolments in Mathematics is suddenly fall down in a small one in Higher Secondary stage (Arts).

Table 9

School No.5.

Level: Higher Secondary

Status: Rural

H.S.L.C.Examination

Year	Students Appeared	Students Passed	60% above Marks in Math.	45% to 60% marks in Math.	Below 45% marks in Math.
2010	125	69	04	12	53
2011	206	163	06	36	121
2012	161	133	18	51	64

This rural school shows the developing performance in Mathematics. 127 students are able to score more than 45% marks in mathematics. The number of pass students in the subject is 265 for the appearing of number 492 of the three years.

Table 10

H.S.S.L.C.Examination(Science)

Year	Students Appeared	Students Passed	60% above Marks in Math.	45% to 60% marks in Math.	Below 45% marks in Math.
2010	04	04	Nil	03	01
2011	06	06	Nil	04	02
2012	16	16	04	12	Nil

In Class X level, this rural school has 492 students for the three years. But in the Higher Secondary stage, the enrolments are only 26 students. The dropout rate of mathematics students in Higher Secondary stage is 95%.

Table 11

School No.6.

Level: High School, Private

Status: Rural

H.S.L.C.Examination

Year	Students Appeared	Students Passed	60% above Marks in Math.	45% to 60% marks in Math.	Below 45% marks in Math.
2010	35	35	21	08	06
2011	32	32	14	11	07
2012	42	42	21	15	06

This private school in rural area of Sivasagar Subdivision performs the best result in class X Examination. Within the hundred percentage pass result, 56 students secured 60% above marks, 34students secured 45% above marks and 19 students secured below 45% marks in the subject mathematics in total of 109 students. This private school can be specially studied how to improve the quality in the subject 'mathematics'.

Table 12

School No.7.

Level: Higher Secondary

Status: Urban

H.S.L.C.Examination

Year	Students Appeared	Students Passed	60% above Marks in Math.	45% to 60% marks in Math.	Below 45% marks in Math.
2010	76	70	09	08	53
2011	79	62	04	06	52
2012	100	92	11	13	68

A total number of 255 students have appeared in class X examination from this urban school in the Sivasagar district. 24 students secured 60% above, 27 students secured 45% above and 173 students secured 30% above marks in mathematics which implies that less than half of the students are able to score 45% above.

Notes

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Table 13

H.S.S.L.C.Examination (Science)

Year	Students Appeared	Students Passed	60% above Marks in Math.	45% to 60% marks in Math.	Below 45% marks in Math.
2010	45	30	04	05	21
2011	50	40	09	06	35
2012	39	36	05	07	24

Same trend of 'Class X' result has also been seen for Higher secondary stage of the school 'G'. 36 students were able to score 45% above where as 80 students secured less than 45% marks in mathematics in the total of 134 students.

Table 14

School No.8.

Level: Higher Secondary

Status: Urban

H.S.L.C.Examination

Year	Students Appeared	Students Passed	60% above Marks in Math.	45% to 60% marks in Math.	Below 45% marks in Math.
2010	75	73	Nil	01	72
2011	77	76	10	13	53
2012	63	63	07	29	27

Although the pass percentage in mathematics of this school is a good one, qualitatively it is poor. Gradually this urban school is trying to improve the result. Out of 215 students for the three years, only 60 students secured 45% above marks in mathematics which is in percentage 28%.

Table15

H.S.S.L.C.Examination (Science)

Year	Students Appeared	Students Passed	60% above Marks in Math.	45% to 60% marks in Math.	Below 45% marks in Math.
2010	28	27	Nil	03	24
2011	51	47	02	03	42
2012	53	44	02	02	44

In Higher Secondary level, this urban school is unable to compete qualitatively for better higher education in the field of mathematics. Only 4 students obtained 60% above marks, 8 students secured 45% above marks and majority of students (110) are in the position of below 45% marks in the subject.

Table 16

School No.9.

Level: Higher Secondary

Status: Urban, Govt.

H.S.L.C.Examination

Year	Students Appeared	Students Passed	60% above Marks in Math.	45% to 60% marks in Math.	Below 45% marks in Math.
2010	161	159	34	32	93
2011	134	130	04	12	114
2012	129	129	28	32	69

This Boys Higher Secondary School is situated in the heart of a town. It has a bright future in performing good result in the subject because of guardians' consciousness, teachers' endeavours and students' efforts exist as effective factors of teaching-learning process. Out of 424 students, 142 students secured 45% above marks in mathematics.

Table 17

H.S.S.L.C.Examination

Year	Students Appeared	Students Passed	60% above Marks in Math.	45% to 60% marks in Math.	Below 45% marks in Math.
2010	73	68	20	26	22
2011	71	68	41	18	09
2012	70	70	45	12	13

This urban school is able to carry out the good performance in mathematics. 76% students of toal 214 students secured 45% above marks in the mathematics.

Table 18

School No.10.

Level: High School,Private

Status: Urban

H.S.L.C.Examination

Year	Students Appeared	Students Passed	60% above Marks in Math.	45% to 60% marks in Math.	Below 45% marks in Math.
2010	12	12	02	04	06
2011	13	13	02	06	05
2012	19	19	06	08	05

This new private school is trying to play an important role in the area of teaching- learning of mathematics. But the enrolment is low which is to be increased for value education of mathematics.

Notes

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Table 19

The Result in Mathematics of the total students in H.S.L.C.Exam

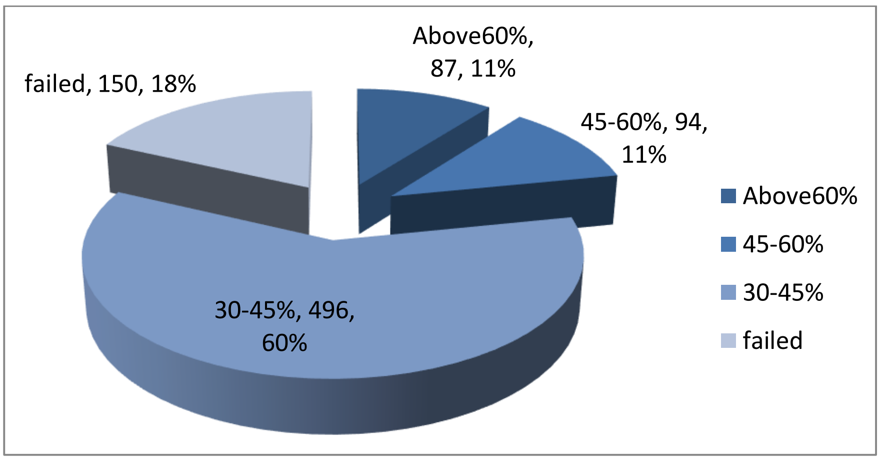
Year	Students Appeared	60% above Marks in Math.	45% to 60% marks in Math.	30% to 45% marks in Math.	Students failed in Math.
2010	827	87	94	496	150
2011	752	57	127	525	43
2012	799	110	224	409	56

Figure 1

Result Pattern in H.S.L.C. Exam.

Year 2010

Total students= 827 nos



Figuer 2 Year 2011

Total Students=752 nos

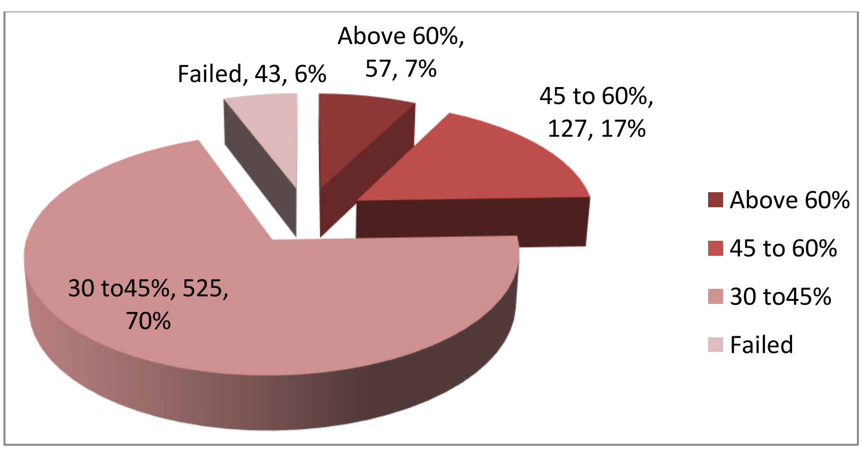


Figure 3
Year 2012
Total Students.799

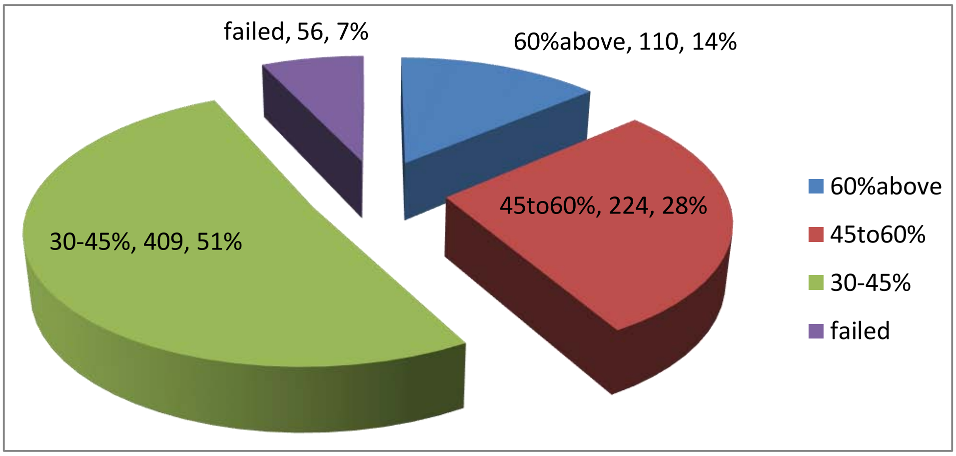
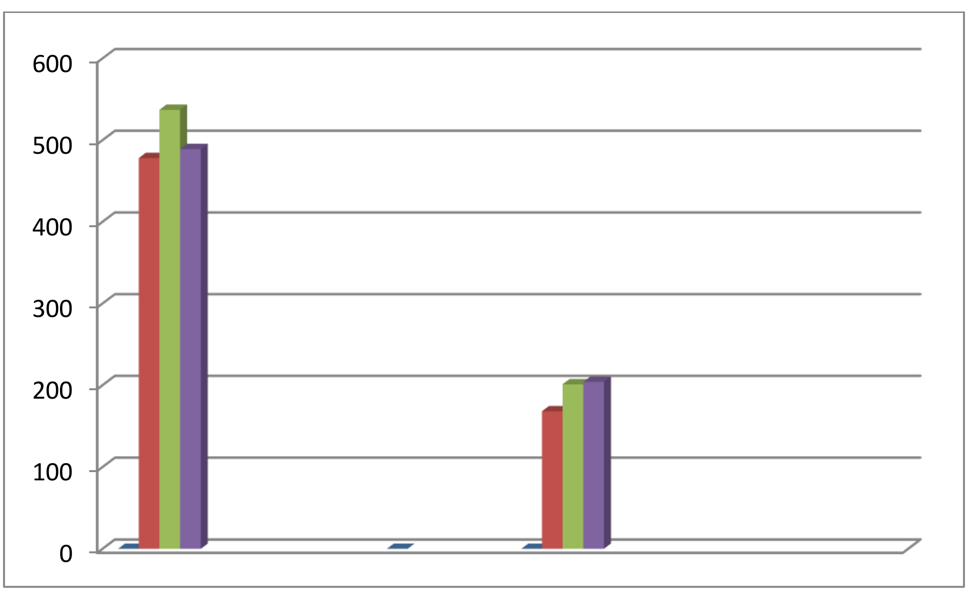


Table 20

Comparison of the Enrolments appeared in H.S.L.C. Exam and H.S.S.L.C. Exam in the Higher Secondary Schools

Year	Total Enrolments in H.S.L.C. Exam	Total Enrolments in H.S.S.L.C. Exam
2010	478	168
2011	537	201
2012	489	204

Enrolments Model in H.S.L.C. and H.S.S.L.C. Exams(Science)



Enrolments in H.S.L.C. Fig.4 Enrolments in H.S.S.L.C

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VII. Findings

The three patterns of the results from the table.2 show that the progress in Mathematics performance from the year 2010 is almost stagnant.

1. The study reveals that majority of the students able to secure in Mathematics the lower range from 30 to 45 percent. The number of the students in this range is 60% in average. This lower achievement indicates the poor performance of the students in Mathematics.
2. The tiny portion 12% of highest achievement in Mathematics shows the quality of mathematics (as we considered generally) is very low.
3. The number of the students who secured in the subject between the ranges of 45 to 60 marks is 20% in average which is to be increased for better quality of Mathematics education.
4. It is observed that the result of privates school is far better than Govt. provincial schools.
5. It is observed from the table.2 that the base of the Mathematics Education is not strong in Schooling.
6. In the table.3, the figure.4 displays the enrolments of science students in Higher Secondary which is one third (approximate) of the enrolments of class X of that particular school. Most of the Higher Secondary schools in the Sivasagar District have no Science section as well as the subject mathematics. This figure alarms the mathematics students rather than should be turning point of better mathematics environment.

VIII. Conclusions

To popularize mathematics and to create a mathematical friendly environment some remedial measure are to be taken such as

(i) The triangular endeavours of teachers, guardians and students can change the mathematics education in positive direction.

(ii) Regular training programmes for teachers to be held to update their knowledge in the present trends of Mathematics Education

(iii) A teacher can create a friendly mathematical environment. He is the friend, philosopher and guide to the students.

(iv) Motivation can play an important role for learning mathematics. If they think, they can win.

(v) Understanding of when and how a mathematical technique is to be used is always important than recalling the technique from memory.

(vi) Students should realize that school is the best place for learning mathematics.

(vii) Logical thinking is a great gift that mathematics can offer us which helps in finding solution of a problem.

(viii) Mathematical fun, games, puzzles and stories can increase the interest of the students in learning mathematics.

(ix) The library with the diversity knowledge of mathematics of a school can motivate the students for learning mathematics.

(x) New initiatives, support of Government and Education policy can strengthen the Mathematics Education in Primary Section, Secondary section and in Higher Education.

(xi) Innovative ideas of the experts for teaching- learning, examinations and research can lead the growth of Mathematics Education.

(xii) Easy access to internet and to E-resources is the key to enhance the quality of teaching.

(xiii) The excellent mathematical education from the children can be expected that all children can learn mathematics and that all children need to learn mathematics. They have more potentialities and abilities for understanding mathematics to challenge the myth that they have not skills in learning mathematics.

Notes

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Exact Solutions for Wick-type Stochastic Coupled KdV Equations

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GJSFR-F Classification : *MSC 2010: 39A50 , 37L55*

EXACTSOLUTIONSFORWICKTYPESTOCHASTICCOUPLEDKDVEQUATIONS

Strictly as per the compliance and regulations of :

Ref

[3] A. de Bouard and A. Debussche. *J. Funct. Anal.*, **154** (1998): 215-251.

Exact Solutions for Wick-type Stochastic Coupled KdV Equations

Hossam A. Ghany [α] & M. Zakarya [σ]

Abstract- Wick-type stochastic coupled KdV equations are researched. By means of Hermite transformation, white noise theory and F-expantion method, three types of exact solutions for Wick-type stochastic coupled KdV equations are explicitly given. These solutions include the white noise functional solutions of Jacobi elibtic function (JEF) type, trigonometric type and hyperbolic type.

Keywords: *coupled KdV equations; F-expantion method; hermite transform; wicktype product; white noise theory.*

PACS No. : 05.40.±a, 02.30.Jr.

I. Introduction

In this paper, we shall explore exact solutions for the following variable coefficients coupled KdV equations.

$$\begin{cases} u_t + h_1(t)uu_x + h_2(t)vv_x + h_3(t)u_{xxx} = 0, \\ v_x + h_4(t)uv_x + h_3(t)v_{xxx} = 0, \end{cases} \quad ,(t,x) \in \mathbb{R}_+ \times \mathbb{R}, \qquad (1.1)$$

where $h_1(t)$, $h_2(t)$, $h_3(t)$ and $h_4(t)$ are bounded measurable or integrable functions on $\mathbb{R}_+$. Random wave is an important subject of stochastic partial differential equations (PDEs). Many authors have studied this subject. Wadati first introduced and studied the stochastic KdV equations and gave the diffusion of soliton of the KdV equation under Gaussian noise in [28, 30] and others [3–6, 23] also researched stochastic KdV-type equations. Xie first introduced Wick-type stochastic KdV equations on white noise space and showed the auto- Backlund transformation and the exact white noise functional solutions in [35]. Furthermore, Xie [36–39], Ghany et al. [12–14, 16–19] researched some Wick-type stochastic wave equations using white noise analysis.

In this paper we use F-expantion method for finding new periodic wave solutions of nonlinear evolution equations in mathematical physics, and we obtain some new periodic wave solution for coupled KdV equations. This method is more powerful and will be used in further works to establish more entirely new solutions for other

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kinds of nonlinear (PDEs) arising in mathematical physics. The effort in finding exact solutions to nonlinear equations is important for the understanding of most nonlinear physical phenomena. For instance, the nonlinear wave phenomena observed in fluid dynamics, plasma, and optical fibers. Many effective methods have been presented, such as variational iteration method [7,8], tanh-function method [9,32,40], homotopy perturbation method [11,27,33], homotopy analysis method [1], tanh-coth method [29,31,32], exp-function method [21,22,34,41,42] , Jacobi elliptic function expantion method [10,24–26], the F-expansion method [43–46]. The main objective of this paper is using the F-expantion method to construct white noise functional solutions for wick-type stochastic coupled KdV equations via hermite transform, wick-type product, white noise theory. If equation (1.1) is considered in a random environment, we can get stochastic coupled KdV equations. In order to give the exact solutions of stochastic coupled KdV equations, we only consider this problem in white noise environment. We shall study the following Wick-type stochastic coupled KdV equations.

$$\begin{cases} U_t + H_1(t) \diamond U \diamond U_x + H_2(t) \diamond V \diamond V_x + H_3(t) \diamond U_{xxx} = 0, \\ V_x + H_4(t) \diamond U \diamond V_x + H_3(t) \diamond V_{xxx} = 0, \end{cases} \tag{1.2}$$

where "$\diamond$" is the Wick product on the Kondratiev distribution space (S_{-1}) which was defined in [20], $H_1(t), H_2(t), H_3(t)$ and $H_4(t)$ are (S_{-1})-valued functions.

II. Description of the F-expantion Method

In order to simultaneously obtain more periodic wave solutions expressed by various Jacobi elliptic functions to nonlinear wave equations, we introduce an F-expansion method which can be thought of as a succinctly over-all generalization of Jacobi elliptic function expansion. We briefly show what is F-expansion method and how to use it to obtain various periodic wave solutions to nonlinear wave equations. Suppose a nonlinear wave equation for u(t,x) is given by.

$$p(u, u_t, u_x, u_{xx}, u_{xxx}, ...) = 0, \tag{2.1}$$

where $u = u(t,x)$ is an unknown function, p is a polynomial in u and its various partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following we give the main steps of a deformation F-expansion method.
Step 1. Look for traveling wave solution of Eq.(2.1)by taking

$$u(t,x) = u(\xi) \ \ ,\xi(t,x) = kx + s\int_0^t \delta(\tau)d\tau + c, \tag{2.2}$$

Hence, under the transformation (2.2). Eq.(2.1) can be transformed into ordinary differential equation (ODE) as following.

$$O(u, s\delta u', ku', k^2u'', k^3u''', ...) = 0, \tag{2.3}$$

Step 2. Suppose that $u(\xi)$ can be expressed by a finite power series of $F(\xi)$ of the form.

$$u(t,x) = u(\xi) = \sum_{i=1}^{N} a_i F^i(\xi), \tag{2.4}$$

Ref

[7] M. Dehghan, J. Manafian and A. Saadatmandi. *Math. Meth. Appl. Sci*, **33** (2010): 1384-1398.

Notes

where $a_0, a_1, ..., a_N$ are constants to be determined later, while $F'(\xi)$ in(2.4) satisfy

$$[F'(\xi)]^2 = PF^4(\xi) + QF^2(\xi) + R, \tag{2.5}$$

and hence holds for $F(\xi)$

$$\begin{cases} F'F'' = 2PF^3F' + QFF', \\ F'' = 2PF^3 + QF, \\ F''' = 6PF^2F' + QF', \\ \dots \end{cases} \tag{2.6}$$

where P, Q, and R are constants.

Step 3. The positive integer N can be determined by considering the homogeneous balance between the highest derivative term and the nonlinear terms appearing in (2.3). Therefore, we can get the value of N in (2.4).

Step 4. Substituting (2.4) into (2.3) with the condition (2.5), we obtain polynomial in $f^i(\xi)[f'(\xi)]^j$, $(i = 0 \pm 1, \pm 2, ..., j = 0, 1)$. Setting each coefficient of this polynomial to be zero yields a set of algebraic equations for $a_0, a_1, ..., a_N, s$ and δ.

Step 5. Solving the algebraic equations with the aid of Maple we have $a_0, a_1, ..., a_N, s$ and δ can be expressed by P, Q and R. Substituting these results into F-expansion (2.4), then a general form of traveling wave solution of Eq. (2.1) can be obtained.

Step 6. Since the general solutions of (2.4) have been well known for us Choose properly (P, Q and R.) in ODE (2.5) such that the corresponding solution $F(\xi)$ of it is one of Jacobi elliptic functions. (See Appendix A, B and C.)

III. New Exact Traveling Wave Solutions of Eq. (1.2)

Taking the Hermite transform, white noise theory, and F-expantion method to explore new exact wave solutions for Eq.(1.2). Applying Hermite transform to Eq.(1.2), we get the deterministic equations:

$$\begin{cases} \tilde{U}_t(t,x,z) + \tilde{H}_1(t,z)\tilde{U}(t,x,z)\tilde{U}_x(t,x,z) + \tilde{H}_2(t,z)\tilde{V}(t,x,z)\tilde{V}_x(t,x,z) \\ +\tilde{H}_3(t,z)\tilde{U}_{xxx}(t,x,z) = 0, \\ \\ \tilde{V}_t(t,x,z) + \tilde{H}_4\tilde{U}(t,x,z)\tilde{V}_x(t,x,z) + \tilde{H}_3\tilde{V}_{xxx}(t,x,z) = 0, \end{cases} \tag{3.1}$$

where $z = (z_1, z_2, ...) \in (\mathbb{C}^{\mathbb{N}})$ is a vector parameter. To look for the travelling wave solution of Eq.(3.1), we make the transformations $\tilde{H}_1(t,z) := h_1(t,z)$, $\tilde{H}_2(t,z) := h_2(t,z)$, $\tilde{H}_3(t,z) := h_3(t,z)$, $\tilde{H}_4(t,z) := h_4(t,z)$, $\tilde{U}(t,x,z) =: u(t,x,z) = u(\xi(t,x,z))$ and $\tilde{V}(t,x,z) =: v(t,x,z) = v(\xi(t,x,z))$ with.

$$\xi(t,x,z) = kx + s\int_0^t \delta(\tau,z)d\tau + c,$$

where k, s and c are arbitrary constants which satisfy $sk \neq 0$, $\delta(\tau)$ is a nonzero function of the indicated variables to be determined later. So, Eq.(3.1) can be transformed into the following (ODE).

$$\begin{cases} s\delta u' + kh_1uu' + kh_2vv' + k^3h_3u''' = 0, \\ s\delta v' + kh_4uv' + k^3h_3v''' = 0, \end{cases} \tag{3.2}$$

where the prime denote to the differential with respect to ξ. In view of F-expantion method, the solution of Eq. (3.1), can be expressed in the form.

$$\begin{cases} u(t,x,z) = u(\xi) = \Sigma_{i=1}^{N}(a_i F^i(\xi)), \\ v(t,x,z) = v(\xi) = \Sigma_{i=1}^{M}(b_i F^i(\xi)), \end{cases} \tag{3.3}$$

where a_i and b_i are constants to be determined later. considering homogeneous balance between $u^{'''}$ and $uu^{'}, vv^{'}$ and the order of $v^{'''}$ and $uv^{'}$ in (3.2), then we can obtain $N = M = 2$, so (3.3) can be rewritten as following.

Notes

$$\begin{cases} u(t,x,z) = a_0 + a_1 F(\xi) + a_2 F^2(\xi), \\ v(t,x,z) = b_0 + b_1 F(\xi) + b_2 F^2(\xi), \end{cases} \tag{3.4}$$

where a_0, a_1, a_2, b_0, b_1 and b_2 are constants to be determined later. Substituting (3.4) with the conditions (2.5),(2.6) into (3.2) and collecting all terms with the same power of $f^i(\xi)[f'(\xi)]^j$, $(i = 0 \pm 1, \pm 2, ..., j = 0, 1)$. as following.

$$\begin{cases} 2k[12k^2 a_2 P h_3 + b_2^2 h_2 + a_2^2 h_1]F^3 F^{'} \\ +3k[2k^2 a_1 P h_3 + a_1 a_2 h_1 + b_1 b_2 h_2]F^2 F^{'} \\ +[2a_2 s\delta + k b_1^2 h_2 + 8k^3 a_2 Q h_3 + 2k a_0 a_2 h_1 + k a_1^2 h_1 + 2k b_0 b_2 h_2]F F^{'} \\ +[s a_1 h_3 + k a_0 a_1 h_1 + k b_0 b_1 h_2 + k^3 a_1 Q h_3]F^{'} = 0, \\ \\ 2k b_2[12k^2 P h_3 + a_2 h_4]F^3 F^{'} \\ +k[6k^2 b_1 P h_3 + 2a_1 b_2 h_4 + a_2 b_1 h_4]F^2 F^{'} \\ +[2s b_2 \delta + 2k a_0 b_2 h_4 + k a_1 b_1 h_4 + 8k^3 b_2 Q h_3]F F^{'} \\ +b_1[s\delta + k a_0 h_4 + k^3 Q h_3]F^{'} = 0. \end{cases} \tag{3.5}$$

Setting each coefficient of $f^i(\xi)[f'(\xi)]^j$ to be zero, we get a system of algebraic equations which can be expressed by.

$$\begin{aligned} & 2k[12k^2 a_2 P h_3 + b_2^2 h_2 + a_2^2 h_1] = 0, \\ & 3k[2k^2 a_1 P h_3 + a_1 a_2 h_1 + b_1 b_2 h_2] = 0, \\ & 2a_2 s\delta + k b_1^2 h_2 + 8k^3 a_2 Q h_3 + 2k a_0 a_2 h_1 + k a_1^2 h_1 + 2k b_0 b_2 h_2 = 0, \\ & s a_1 h_3 + k a_0 a_1 h_1 + k b_0 b_1 h_2 + k^3 a_1 Q h_3 = 0, \\ & 2k b_2[12k^2 P h_3 + a_2 h_4] = 0, \\ & k[6k^2 b_1 P h_3 + 2a_1 b_2 h_4 + a_2 b_1 h_4] = 0, \\ & 2s b_2 \delta + 2k a_0 b_2 h_4 + k a_1 b_1 h_4 + 8k^3 b_2 Q h_3 = 0, \\ & b_1[s\delta + k a_0 h_4 + k^3 Q h_3] = 0. \end{aligned} \tag{3.6}$$

with solving by Maple to get the following coefficient.

$$\begin{cases} a_1 = b_1 = 0, a_0 = \text{arbitrary constant}, \\ a_2 = -\frac{12k^2 P h_3(t,z)}{h_4(t,z)}, \\ b_2 = \pm i \frac{12k^2 P h_3(t,z)}{h_4(t,z)} \sqrt{\frac{h_1(t,z)-h_4(t,z)}{h_2(t,z)}} = \mp i a_2 \sqrt{\frac{h_1(t,z)-h_4(t,z)}{h_2(t,z)}}, \\ b_0 = \pm \frac{[3k^2 Q h_3(t,z) - a_0(h_1(t,z)-h_4(t,z))]}{\sqrt{h_2(t,z)[h_1(t,z)-h_4(t,z)]}}, \\ \delta = \frac{-k[a_0 h_4(t,z) + k^2 Q h_3(t,z)]}{S}. \end{cases} \tag{3.7}$$

Notes

Substituting by coefficient (3.7) into (3.4) yields general form solutions of eq. (1.2).

$$u(t,x,z) = a_0 - [\frac{12k^2 P h_3(t,z)}{h_4(t,z)}]F^2(\xi(t,x,z)), \tag{3.8}$$

$$v(t,x,z) = \pm \frac{[3k^2 Q h_3(t,z) + a_0(h_1(t,z) - h_4(t,z))]}{\sqrt{h_2(t,z)[h_1(t,z) - h_4(t,z)]}} \pm \frac{12ik^2 P h_3(t,z)}{h_4(t,z)} \sqrt{\frac{h_1(t,z) - h_4(t,z)}{h_2(t,z)}} F^2(\xi(t,x,z)), \tag{3.9}$$

with

$$\xi(t,x,z) = k\left[x - \int_0^t \{a_0 h_4(\tau,z) + k^2 Q h_3(\tau,z)\} d\tau\right].$$

From Appendix C, we give the special cases as following:
Case 1. if we take $P = 1, Q = (2 - m^2)$ and $R = (1 - m^2)$, we have $F(\xi) \to cs(\xi)$,

$$u_1(t,x,z) = a_0 - [\frac{12k^2 h_3(t,z)}{h_4(t,z)}]\ cs^2(\xi_1(t,x,z)), \tag{3.10}$$

$$v_1(t,x,z) = \pm \frac{[3k^2 h_3(t,z)(2-m^2) + a_0(h_1(t,z) - h_4(t,z))]}{\sqrt{h_2(t,z)(h_1(t,z) - h_4(t,z))}} \pm \frac{12ik^2 h_3(t,z)}{h_4(t,z)} \sqrt{\frac{h_1(t,z) - h_4(t,z)}{h_2(t,z)}}\ cs^2(\xi_1(t,x,z)), \tag{3.11}$$

with

$$\xi_1(t,x,z) = k\left[x - \int_0^t \{a_0 h_4(\tau,z) + k^2(2-m^2) h_3(\tau,z)\} d\tau\right].$$

In the limit case when $m \to o$ we have $cs(\xi) \to \cot(\xi)$, thus (3.10),(3.11) become.

$$u_2(t,x,z) = a_0 - [\frac{12k^2 h_3(t,z)}{h_4(t,z)}]\ \cot^2(\xi_2(t,x,z)), \tag{3.12}$$

$$v_2(t,x,z) = \pm \frac{[6k^2 h_3(t,z) + a_0(h_1(t,z) - h_4(t,z))]}{\sqrt{h_2(t,z)[h_1(t,z) - h_4(t,z)]}} \pm \frac{12ik^2 h_3(t,z)}{h_4(t,z)} \sqrt{\frac{h_1(t,z) - h_4(t,z)}{h_2(t,z)}}\ \cot^2(\xi_2(t,x,z)), \tag{3.13}$$

with

$$\xi_2(t,x,z) = k\left[x - \int_0^t \{a_0 h_4(\tau,z) + 2k^2 h_3(\tau,z)\} d\tau\right].$$

In the limit case when $m \to 1$ we have $cs(\xi) \to \mathrm{csch}(\xi)$, thus (3.10).(3.11) become.

$$u_3(t,x,z) = a_0 - [\frac{12k^2 h_3(t,z)}{h_4(t,z)}]\ \mathrm{csch}^2(\xi_3(t,x,z)), \tag{3.14}$$

$$v_3(t,x,z) = \pm\frac{[3k^2h_3(t,z)+a_0(h_1(t,z)-h_4(t,z))]}{\sqrt{h_2(t,z)[h_1(t,z)-h_4(t,z)]}} \pm\frac{12ik^2h_3(t,z)}{h_4(t,z)}\sqrt{\frac{h_1(t,z)-h_4(t,z)}{h_2(t,z)}}\ \operatorname{csch}^2(\xi_3(t,x,z)), \tag{3.15}$$

with

$$\xi_3(t,x,z) = k\Big[x - \int_0^t \{a_0h_4(\tau,z) + k^2h_3(\tau,z)\}d\tau\Big].$$

Case 2. if we take $P=1, Q=(2m^2-1)$ and $R=-m^2(1-m^2)$, then $F(\xi)\to ds(\xi)$,

$$u_4(t,x,z) = a_0 - [\frac{12k^2h_3(t,z)}{h_4(t,z)}]\ ds^2(\xi_4(t,x,z)), \tag{3.16}$$

$$v_4(t,x,z) = \pm\frac{[3k^2h_3(t,z)(2m^2-1)+a_0(h_1(t,z)-h_4(t,z))]}{\sqrt{h_2(t,z)[h_1(t,z)-h_4(t,z)]}} \pm\frac{12ik^2h_3(t,z)}{h_4(t,z)}\sqrt{\frac{h_1(t,z)-h_4(t,z)}{h_2(t,z)}}\ ds^2(\xi_4(t,x,z)), \tag{3.17}$$

with

$$\xi_4(t,x,z) = k\Big[x - \int_0^t \{a_0h_4(\tau,z) + k^2(2m^2-1)h_3(\tau,z)\}d\tau\Big].$$

In the limit case when $m\to o$ we have $ds(\xi)\to \csc(\xi)$, thus (3.16),(3.17) become.

$$u_5(t,x,z) = a_0 - [\frac{12k^2h_3(t,z)}{h_4(t,z)}]\ \csc^2(\xi_5(t,x,z)), \tag{3.18}$$

$$v_5(t,x,z) = \pm\frac{[-3k^2h_3(t,z)+a_0(h_1(t,z)-h_4(t,z))]}{\sqrt{h_2(t,z)[h_1(t,z)-h_4(t,z)]}} \pm\frac{12ik^2h_3(t,z)}{h_4(t,z)}\sqrt{\frac{h_1(t,z)-h_4(t,z)}{h_2(t,z)}}\ \csc^2(\xi_5(t,x,z)), \tag{3.19}$$

with

$$\xi_5(t,x,z) = k\Big[x - \int_0^t \{a_0h_4(\tau,z) - k^2h_3(\tau,z)\}d\tau\Big].$$

Remark that. In the limit case when $m\to 1$ we have $ds(\xi)=cs(\xi)\to \operatorname{csch}(\xi)$, thus (3.16),(3.17) become the same solutions in case 1.

Case 3. if we take $P=\frac{1}{4}, Q=\frac{1-2m^2}{2}$ and $R=\frac{1}{4}$, then $F(\xi)\to ns(\xi)\pm cs(\xi)$,

$$u_6(t,x,z) = a_0 - \frac{3k^2h_3(t,z)}{h_4(t,z)}\Big[ns(\xi_6(t,x,z)) \pm cs(\xi_6(t,x,z))\Big]^2, \tag{3.20}$$

$$v_6(t,x,z) = \pm\frac{[3k^2h_3(t,z)(1-2m^2)+2a_0(h_1(t,z)-h_4(t,z))]}{2\sqrt{h_2(t,z)[h_1(t,z)-h_4(t,z)]}} \tag{3.21}$$

Notes

$$\pm\frac{3ik^2h_3(t,z)}{h_4(t,z)}\sqrt{\frac{h_1(t,z)-h_4(t,z)}{h_2(t,z)}}\Big[ns(\xi_6(t,x,z))\pm cs(\xi_6(t,x,z))\Big]^2,$$

with

$$\xi_6(t,x,z)=k\Big[x-\frac{1}{2}\int_0^t\{2a_0h_4(\tau,z)+k^2(1-2m^2)h_3(\tau,z)\}d\tau\Big].$$

Notes

In the limit case when $m\to o$ we have $(ns(\xi)\pm cs(\xi))\to(\csc(\xi)\pm\cot(\xi))$, thus (3.22),(3.23) become.

$$u_7(t,x,z)=a_0-\frac{3k^2h_3(t,z)}{h_4(t,z)}\Big[\csc(\xi_7(t,x,z))\pm\cot(\xi_7(t,x,z))\Big]^2, \tag{3.22}$$

$$v_7(t,x,z)=\pm\frac{[3k^2h_3(t,z)+2a_0(h_1(t,z)-h_4(t,z))]}{2\sqrt{h_2(t,z)[h_1(t,z)-h_4(t,z)]}}$$

$$\pm\frac{3ik^2h_3(t,z)}{h_4(t,z)}\sqrt{\frac{h_1(t,z)-h_4(t,z)}{h_2(t,z)}}\Big[\csc(\xi_7(t,x,z))\pm\cot(\xi_7(t,x,z))\Big]^2, \tag{3.23}$$

with

$$\xi_7(t,x,z)=k\Big[x-\frac{1}{2}\int_0^t\{2a_0h_4(\tau,z)+k^2h_3(\tau,z)\}d\tau\Big].$$

In the limit case when $m\to1$ we have $(ns(\xi)\pm cs(\xi))\to(\coth(\xi)\pm csch(\xi))$, thus (3.22),(3.23) become.

$$u_8(t,x,z)=a_0-\frac{3k^2h_3(t,z)}{h_4(t,z)}\Big[\coth(\xi_8(t,x,z))\pm\operatorname{csch}(\xi_8(t,x,z))\Big]^2, \tag{3.24}$$

$$v_8(t,x,z)=\pm\frac{[-3k^2h_3(t,z)+2a_0(h_1(t,z)-h_4(t,z))]}{2\sqrt{h_2(t,z)[h_1(t,z)-h_4(t,z)]}}$$

$$\pm\frac{3ik^2h_3(t,z)}{h_4(t,z)}\sqrt{\frac{h_1(t,z)-h_4(t,z)}{h_2(t,z)}}\Big[\coth(\xi_8(t,x,z))\pm\operatorname{csch}(\xi_8(t,x,z))\Big]^2, \tag{3.25}$$

with

$$\xi_8(t,x,z)=k\Big[x-\frac{1}{2}\int_0^t\{2a_0h_4(\tau,z)-k^2h_3(\tau,z)\}d\tau\Big].$$

Remark that: there are another solutions for Eq.(1.2). These solutions come from setting different values for the cofficients P,Q and R.(see Appendex C.). The above mentioned cases are just to clarify how far our technique is applicable.

IV. White Noise Functional Solutions of Eq.(1.2)

In this section, we employ the results of the Section 3 by using Hermite transform to obtain exact white noise functional solutions for Wick-type stochastic coupled KdV equations (1.2). The properties of exponential and trigonometric functions yield that there exists a bounded open set $\mathbf{D} \subset \mathbb{R}_+ \times \mathbb{R}$, $\rho < \infty$, $\lambda > 0$ such that the solution $u(t,x,z)$ of Eq.(3.1) and all its partial derivatives which are involved in Eq. (3.1) are uniformly bounded for $(t,x,z) \in \mathbf{D} \times K_\rho(\lambda)$, continuous with respect to $(t,x) \in \mathbf{D}$ for all $z \in K_\rho(\lambda)$ and analytic with respect to $z \in K_\rho(\lambda)$, for all $(t,x) \in \mathbf{D}$. From Theorem 4.1.1 in [20], there exists $U(t,x,z) \in (\mathcal{S})_{-1}$ such that $u(t,x,z) = \tilde{U}(t,x)(z)$ for all $(t,x,z) \in \mathbf{D} \times K_\rho(\lambda)$ and $U(t,x)$ solves Eq.(1.2) in $(\mathcal{S})_{-1}$. Hence, by applying the inverse Hermite transform to the results of Section 3, we get New exact white noise functional solutions of Eq.(1.2) as follows:

- New Wick-type stochastic solutions of (JEF):

$$U_1(t,x) = a_0 - [\frac{12k^2H_3(t)}{H_4(t)}] \diamond cs^{\diamond 2}(\Xi_1(t,x)), \tag{4.1}$$

$$\begin{aligned} V_1(t,x) &= \pm \tfrac{[3k^2H_3(t)(2-m^2)+a_0(H_1(t)-H_4(t))]}{\sqrt{H_2(t)\diamond[H_1(t)-H_4(t)]}} \\ &\pm \tfrac{12ik^2H_3(t)}{H_4(t)} \diamond \sqrt{\tfrac{H_1(t)-H_4(t)}{H_2(t)}} \diamond cs^{\diamond 2}(\Xi_1(t,x)), \end{aligned} \tag{4.2}$$

$$U_2(t,x) = a_0 - [\frac{12k^2H_3(t)}{H_4(t)}] \diamond ds^{\diamond 2}(\Xi_2(t,x)), \tag{4.3}$$

$$\begin{aligned} V_2(t,x) &= \pm \tfrac{[3k^2H_3(t)(2m^2-1)+a_0(H_1(t)-H_4(t))]}{\sqrt{H_2(t)\diamond[H_1(t)-H_4(t)]}} \\ &\pm \tfrac{12ik^2H_3(t)}{H_4(t)} \diamond \sqrt{\tfrac{H_1(t)-H_4(t)}{H_2(t)}} \diamond ds^{\diamond 2}(\Xi_2(t,x)), \end{aligned} \tag{4.4}$$

$$U_3(t,x) = a_0 - \frac{3k^2H_3(t)}{H_4(t)} \diamond \Big[ns^\diamond(\Xi_3(t,x)) \pm cs^\diamond(\Xi_3(t,x))\Big]^{\diamond 2}, \tag{4.5}$$

$$\begin{aligned} V_3(t,x) &= \pm \tfrac{[3k^2H_3(t)(1-2m^2)+2a_0(H_1(t)-H_4(t))]}{2\sqrt{H_2(t)\diamond[H_1(t)-H_4(t)]}} \\ &\pm \tfrac{3ik^2H_3(t)}{H_4(t)} \diamond \sqrt{\tfrac{H_1(t)-H_4(t)}{H_2(t)}} \diamond \Big[ns^\diamond(\Xi_3(t,x)) \pm cs^\diamond(\Xi_3(t,x))\Big]^{\diamond 2}, \end{aligned} \tag{4.6}$$

with

$$\Xi_1(t,x) = k\Big[x - \int_0^t \{a_0H_4(\tau) + k^2(2-m^2)H_3(\tau)\}d\tau\Big],$$

$$\Xi_2(t,x) = k\Big[x - \int_0^t \{a_0H_4(\tau) + k^2(2m^2-1)H_3(\tau)\}d\tau\Big],$$

$$\Xi_3(t,x) = k\Big[x - \frac{1}{2}\int_0^t \{2a_0H_4(\tau) + k^2(1-2m^2)H_3(\tau)\}d\tau\Big].$$

Ref

[20] H. Holden, B. Øsendal, J. U b øe and T. Zhang. *Stochastic partial differential equations*, Bihkäuser: Basel, (1996).

Notes

- New Wick-type stochastic solutions of trigonometric functions:

$$U_4(t,x) = a_0 - \frac{12k^2H_3(t)}{H_4(t)} \diamond \cot^{\diamond 2}(\Xi_4(t,x)), \tag{4.7}$$

$$\begin{aligned} V_4(t,x) &= \pm\frac{[6k^2H_3(t)+a_0(H_1(t)-H_4(t))]}{\sqrt{H_2(t)\diamond(H_1(t)-H_4(t))}} \\ &\pm\frac{12ik^2H_3(t)}{H_4(t)} \diamond \sqrt{\frac{H_1(t)-H_4(t)}{H_2(t)}} \diamond \cot^{\diamond 2}(\Xi_4(t,x)), \end{aligned} \tag{4.8}$$

$$U_5(t,x) = a_0 - [\frac{12k^2H_3(t)}{H_4(t)}] \diamond \csc^{\diamond 2}(\Xi_5(t,x)), \tag{4.9}$$

$$\begin{aligned} V_5(t,x) &= \pm\frac{[-3k^2H_3(t)+a_0(H_1(t)-H_4(t))]}{\sqrt{H_2(t)\diamond[H_1(t)-H_4(t)]}} \pm \frac{12ik^2H_3(t)}{H_4(t)} \\ &\diamond\sqrt{\frac{H_1(t)-H_4(t)}{H_2(t)}} \diamond \csc^{\diamond 2}(\Xi_5(t,x)), \end{aligned} \tag{4.10}$$

$$U_6(t,x) = a_0 - \frac{3k^2H_3(t)}{H_4(t)} \diamond \Big[\csc^{\diamond}(\Xi_6(t,x)) \pm \cot^{\diamond}(\Xi_6(t,x))\Big]^{\diamond 2}, \tag{4.11}$$

$$\begin{aligned} V_6(t,x) &= \pm\frac{[3k^2H_3(t)+2a_0(H_1(t)-H_4(t))]}{2\sqrt{H_2(t)\diamond[H_1(t)-H_4(t)]}} \pm \frac{3ik^2H_3(t)}{H_4(t)} \\ &\diamond\sqrt{\frac{H_1(t)-H_4(t)}{H_2(t)}} \diamond \Big[\csc^{\diamond}(\Xi_6(t,x)) \pm \cot^{\diamond}(\Xi_6(t,x))\Big]^{\diamond 2}, \end{aligned} \tag{4.12}$$

with

$$\Xi_4(t,x) = k\Big[x - \int_0^t \{a_0H_4(\tau) + 2k^2H_3(\tau)\}d\tau\Big],$$

$$\Xi_5(t,x) = k\Big[x - \int_0^t \{a_0H_4(\tau) - k^2H_3(\tau)\}d\tau\Big],$$

$$\Xi_6(t,x) = k\Big[x - \frac{1}{2}\int_0^t \{2a_0H_4(\tau) + k^2H_3(\tau)\}d\tau\Big].$$

- New Wick-type stochastic solutions of hyperbolic functions:

$$U_7(t,x) = a_0 - [\frac{12k^2H_3}{H_4}] \diamond \mathrm{csch}^{\diamond 2}(\Xi_7(t,x)), \tag{4.13}$$

$$\begin{aligned} V_7(t,x) &= \pm\frac{[3k^2H_3(t)+a_0(H_1(t)-H_4(t))]}{\sqrt{H_2(t)\diamond[H_1(t)-H_4(t)]}} \pm \frac{12ik^2H_3(t)}{H_4(t)} \\ &\diamond\sqrt{\frac{H_1(t)-H_4(t)}{H_2(t)}} \diamond \mathrm{csch}^{\diamond 2}\,\Xi_7(t,x), \end{aligned} \tag{4.14}$$

$$U_8(t,x) = a_0 - \frac{3k^2 H_3(t)}{H_4(t)} \diamond \left[\coth^\diamond(\Xi_8(t,x)) \pm \operatorname{csch}^\diamond(\Xi_8(t,x))\right]^{\diamond 2}, \tag{4.15}$$

$$V_8(t,x) = \pm \frac{[-3k^2H_3(t)+2a_0(H_1(t)-H_4(t))]}{2\sqrt{H_2(t)\diamond[H_1(t)-H_4(t)]}} \pm \frac{3ik^2H_3(t)}{H_4(t)} \diamond\sqrt{\frac{H_1(t)-H_4(t)}{H_2(t)}} \diamond \left[\coth^\diamond(\Xi_8(t,x)) \pm \operatorname{csch}^\diamond(\Xi_8(t,x))\right]^{\diamond 2}, \tag{4.16}$$

with

$$\Xi_7(t,x) = k\left[x - \int_0^t \{a_0 H_4(\tau) + k^2 H_3(\tau)\} d\tau\right],$$

$$\Xi_8(t,x) = k\left[x - \frac{1}{2}\int_0^t \{2a_0 H_4(\tau) - k^2 H_3(\tau)\} d\tau\right].$$

We observe that. For different forms of H_1, H_2, H_3 and H_4, we can get different exact white noise functional solutions of Eq.(1.2) from Eqs.(4.1)-(4.16).

V. Example

It is well known that Wick version of function is usually difficult to evaluate. So, in this section, we give non-Wick version of solutions of Eq.(1.2). Let $W_t = \dot{B}_t$ be the Gaussian white noise, where B_t is the Brownian motion. We have the Hermite transform $\tilde{W}_t(z) = \sum_{i=1}^{\infty} z_i \int_0^t \eta_i(s)ds$ [20]. Since $\exp^\diamond(B_t) = \exp(B_t - \frac{t^2}{2})$, we have $\sin^\diamond(B_t) = \sin(B_t - \frac{t^2}{2})$, $\cos^\diamond(B_t) = \cos(B_t - \frac{t^2}{2})$, $\cot^\diamond(B_t) = \cot(B_t - \frac{t^2}{2})$, $\csc^\diamond(B_t) = \csc(B_t - \frac{t^2}{2})$, $\coth^\diamond(B_t) = \coth(B_t - \frac{t^2}{2})$ and $\operatorname{csch}^\diamond(B_t) = \operatorname{csch}(B_t - \frac{t^2}{2})$. Suppose that $H_1(t) = H_2(t) = \lambda_1 H_3(t), H_3(t) = \lambda_2 H_4(t)$ and $H_4(t) = \Gamma(t) + \lambda_3 W_t$ where λ_1, λ_2 and λ_3 are arbitrary constants and $\Gamma(t)$ is integrable or bounded measurable function on $\mathbb{R}_+$. Therefore, for $H_1(t)H_2(t)H_3(t)H_4(t) \neq 0$. thus exact white noise functional solutions of Eq.(1.2) are as follows:

$$U_9(t,x) = 3k^2\left\{\frac{a_0}{3k^2} - 4\lambda_2 \cot^2(\Phi_1(t,x))\right\}, \tag{5.1}$$

$$V_9(t,x) = \pm 6k^2\lambda_2\left\{\frac{\left[1+\frac{a_0}{6k^2\lambda_2(\lambda_1\lambda_2-1)}\right]}{\sqrt{\lambda_1\lambda_2(\lambda_1\lambda_2-1)}} + 2i\sqrt{\frac{(\lambda_1\lambda_2-1)}{\lambda_1\lambda_2}} \cot^2(\Phi_1(t,x))\right\}, \tag{5.2}$$

$$U_{10}(t,x) = 3k^2\left\{\frac{a_0}{3k^2} - 4\lambda_2 \csc^2(\Phi_2(t,x))\right\}, \tag{5.3}$$

$$V_{10}(t,x) = \pm 3k^2\lambda_2\left\{\frac{\left[\frac{a_0}{3k^2\lambda_2(\lambda_1\lambda_2-1)}-1\right]}{\sqrt{\lambda_1\lambda_2(\lambda_1\lambda_2-1)}} + \right.$$

Ref

[20] H. Holden, B. Øsendal, J. U b øe and T. Zhang. *Stochastic partial differential equations*, Bihkäuser: Basel, (1996).

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$$4i\sqrt{\frac{(\lambda_1\lambda_2-1)}{\lambda_1\lambda_2}}\ \csc^2(\Phi_2(t,x))\Bigg\}, \tag{5.4}$$

$$U_{11}(t,x) = a_0 - 3k^2\lambda_2\Bigg[\csc(\Xi(t,x)) \pm \cot(\Phi_3(t,x))\Bigg]^2, \tag{5.5}$$

$$\begin{aligned} V_{11}(t,x) = \pm 3k^2\lambda_2\Bigg\{&\frac{\left[1+\frac{2a_0}{3k^2\lambda_2(\lambda_1\lambda_2-1)}\right]}{2\sqrt{\lambda_1\lambda_2(\lambda_1\lambda_2-1)}} + \\ &i\sqrt{\frac{(\lambda_1\lambda_2-1)}{\lambda_1\lambda_2}}\Bigg[\csc(\Phi(t,x)) \pm \cot(\Phi_3(t,x))\Bigg]^2\Bigg\}, \end{aligned} \tag{5.6}$$

$${}_{12}(t,x) = a_0 - 12k^2\lambda_2\,\mathrm{csch}^2(\Phi_4(t,x)), \tag{5.7}$$

$$\begin{aligned} V_{12}(t,x) = \pm 3k^2\lambda_2\Bigg\{&\frac{\left[1+\frac{a_0}{3k^2\lambda_2(\lambda_1\lambda_2-1)}\right]}{\sqrt{\lambda_1\lambda_2(\lambda_1\lambda_2-1)}} + \\ &4i\sqrt{\frac{(\lambda_1\lambda_2-1)}{\lambda_1\lambda_2}}\,\mathrm{csch}^2(\Phi_4(t,x)\Bigg\}, \end{aligned} \tag{5.8}$$

$$U_{13}(t,x) = a_0 - 3k^2\lambda_2\Bigg[\coth(\Phi_5(t,x)) \pm \mathrm{csch}(\Phi_5(t,x))\Bigg]^2, \tag{5.9}$$

$$\begin{aligned} V_{13}(t,x) = \pm 3k^2\lambda_2\Bigg\{&\frac{\left[\frac{2a_0}{3k^2\lambda_2(\lambda_1\lambda_2-1)}-1\right]}{2\sqrt{\lambda_1\lambda_2(\lambda_1\lambda_2-1)}} + \\ &3i\sqrt{\frac{(\lambda_1\lambda_2-1)}{\lambda_1\lambda_2}}\Bigg[\coth(\Phi_5(t,x)) \pm \mathrm{csch}(\Phi_5(t,x))\Bigg]^{\diamond 2}\Bigg\}, \end{aligned} \tag{5.10}$$

with

$$\Phi_1(t,x) = k\Bigg[x - (a_0 + 2k^2\lambda_2)\Bigg\{\int_0^t \Gamma(\tau)d\tau + \lambda_3[B_t - \frac{t^2}{2}]\Bigg\}\Bigg],$$

$$\Phi_2(t,x) = k\Bigg[x - (a_0 - k^2\lambda_2)\Bigg\{\int_0^t \Gamma(\tau)d\tau + \lambda_3[B_t - \frac{t^2}{2}]\Bigg\}\Bigg],$$

$$\Phi_3(t,x) = k\Bigg[x - \frac{(2a_0 + k^2\lambda_2)}{2}\Bigg\{\int_0^t \Gamma(\tau)d\tau + \lambda_3[B_t - \frac{t^2}{2}]\Bigg\}\Bigg],$$

$$\Phi_4(t,x) = k\Bigg[x - (a_0 + k^2\lambda_2)\Bigg\{\int_0^t \Gamma(\tau)d\tau + \lambda_3[B_t - \frac{t^2}{2}]\Bigg\}\Bigg]$$

and

$$\Phi_5(t,x) = k\Bigg[x - \frac{(2a_0 - k^2\lambda_2)}{2}\Bigg\{\int_0^t \Gamma(\tau)d\tau + \lambda_3[B_t - \frac{t^2}{2}]\Bigg\}\Bigg].$$

VI. Conclusion

We have discussed the solutions of stochastic (PDEs) driven by Gaussian white noise. There is a unitary mapping between the Gaussian white noise space and the Poisson white noise space. This connection was given by Benth and Gjerde [2]. We can see in the section 4.9 [20] clearly. Hence, by the aid of the connection, we can derive some stochastic exact soliton solutions if the coefficients , and are Poisson white noise functions in Eq. (1.2). In this paper, using Hermite transformation, white noise theory and F-expantion method, we study the white noise solutions of the Wick-type stochastic coupled KdV equations. This paper shows that the F-expantion method is sufficient to solve the stochastic nonlinear equations in mathematical physics. The method which we have proposed in this paper is powerful, direct and computerized method, which allows us to do complicated and tedious algebraic calculation. It is shown that the algorithm can be also applied to other NLPDEs in mathematical physics such as modified Hirota-Satsuma coupled KdV, (2+1)-dimensional coupled KdV, KdV-Burgers, schamel KdV, modified KdV Burgers, Sawada-Kotera, Zhiber-Shabat equations and Benjamin-Bona-Mahony equations. Since the equation (1.2) has other solutions if select other values of P, Q and R (see Appendix A, B and C). So there are many other of exact solutions for wick-type stochastic coupled KdV equations.

Appendix A.
the jacobi elliptic functions degenerate into trigonometric functions when $m \to 0$.

$$sn\xi \to \sin\xi, cn\xi \to \cos\xi, dn\xi \to 1, sc\xi \to \tan\xi, sd\xi \to \sin\xi, cd\xi \to \cos\xi,$$
$$ns\xi \to \csc\xi, nc\xi \to \sec\xi, nd\xi \to 1, cs\xi \to \cot\xi, ds\xi \to \csc\xi, dc\xi \to \sec\xi.$$

Appendix B.
the jacobi elliptic functions degenerate into hyperbolic functions when $m \to 1$.

$$sn\xi \to \tan\xi, cn\xi \to \operatorname{sech}\xi, dn\xi \to \operatorname{sech}\xi, sc\xi \to \sinh\xi, sd\xi \to \sinh\xi, cd\xi \to 1,$$
$$ns\xi \to \coth\xi, nc\xi \to \cosh\xi, nd\xi \to \cosh, cs\xi \to \operatorname{csch}\xi, ds\xi \to \operatorname{csch}\xi, dc\xi \to 1.$$

Appendix C. The ODE and Jacobi Elliptic Functions
Relation between values of $(P\ ,\ Q\ ,\ R)$ and corresponding $F(\xi)$ in ODE

$$(F')^2(\xi) = PF^4(\xi) + QF^2(\xi) + R,$$

P	Q	R	$F(\xi)$
m^2	$-1-m^2$	1	$\mathrm{sn}\xi$, $\mathrm{cd}\xi = \frac{cn\xi}{dn\xi}$
$-m^2$	$2m^2-1$	$1-m^2$	$\mathrm{cn}\xi$
-1	$2-m^2$	m^2-1	$\mathrm{dn}\xi$
1	$-1-m^2$	m^2	$\mathrm{ns}\xi = \frac{1}{\mathrm{sn}\xi}$, $\mathrm{dc}\xi = \frac{\mathrm{dn}\xi}{\mathrm{cn}\xi}$
$1-m^2$	$2m^2-1$	$-m^2$	$\mathrm{nc}\xi = \frac{1}{\mathrm{cn}\xi}$
m^2-1	$2-m^2$	-1	$\mathrm{nd}\xi = \frac{1}{\mathrm{dn}\xi}$
$1-m^2$	$2-m^2$	1	$\mathrm{sc}\xi = \frac{\mathrm{sn}\xi}{\mathrm{cn}\xi}$

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$-m^2(1-m^2)$	$2m^2-1$	1	$\mathrm{sd}\xi=\frac{\mathrm{sn}\xi}{\mathrm{dn}\xi}$
1	$2-m^2$	$1-m^2$	$\mathrm{cs}\xi=\frac{\mathrm{cn}\xi}{\mathrm{sn}\xi}$
1	$2m^2-1$	$-m^2(1-m^2)$	$\mathrm{ds}\xi=\frac{\mathrm{dn}\xi}{\mathrm{sn}\xi}$
$\frac{m^4}{4}$	$\frac{m^2-2}{2}$	$\frac{1}{4}$	$\frac{\mathrm{sn}\xi}{1\pm\mathrm{dn}\xi},\ \frac{\mathrm{cn}\xi}{\sqrt{1-m^2}\pm\mathrm{dn}\xi}$
$\frac{m^2}{4}$	$\frac{m^2-2}{2}$	$\frac{m^2}{4}$	$\mathrm{sn}\xi\pm i\mathrm{cn}\xi,\ \frac{\mathrm{dn}\xi}{i\sqrt{1-m^2}\mathrm{sn}\xi\pm\mathrm{cn}\xi},\ \frac{m\,\mathrm{sn}\xi}{1\pm\mathrm{dn}\xi}$
$\frac{1}{4}$	$\frac{1-2m^2}{2}$	$\frac{1}{4}$	$\mathrm{ns}\xi\pm\mathrm{cs}\xi,\ \frac{\mathrm{cn}\xi}{\sqrt{1-m^2}\mathrm{sn}\xi\pm\mathrm{dn}\xi},\ \frac{\mathrm{sn}\xi}{1\pm\mathrm{cn}\xi},$
$\frac{m^2-1}{4}$	$\frac{m^2+1}{2}$	$\frac{m^2-1}{4}$	$\frac{\mathrm{dn}\xi}{1\pm m\mathrm{sn}\xi}$
$\frac{1-m^2}{4}$	$\frac{m^2+1}{2}$	$\frac{1-m^2}{4}$	$\mathrm{nc}\xi\pm i\mathrm{sc}\xi\ \frac{\mathrm{cn}\xi}{1\pm\mathrm{sn}\xi}$
$\frac{-1}{4}$	$\frac{m^2+1}{2}$	$\frac{-(1-m^2)^2}{4}$	$m\mathrm{cn}\xi\pm\mathrm{dn}\xi$
$\frac{1}{4}$	$\frac{m^2-2}{2}$	$\frac{m^2}{4}$	$\mathrm{ns}\xi\pm\mathrm{ds}\xi$

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Reliability and Sensitivity Analysis of Harvesting Systems

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Abstract- The objective of the present paper is to obtain the reliability of a harvesting system having three unit tractor (T), combine (C) and wagon (W) using supplementary variable technique. For successful operation of the system units T and C must be remain operative while when unit W fails system works partially. Two repairmen are involved in repairing of the system. One of the repairmen (the first) is the foreman (boss) and the other an assistant (apprentice). Whenever unit T and C fails repair is undertaken by boss while repair of the wagon is undertaken by the trainee. If the boss is busy in repairing and at the same time other unit fails then the repair is undertaken by apprentice. With the help of Supplementary variable technique, Laplace transformations and copula methodology, the transition state probabilities, asymptotic behavior, reliability, M.T.T.F. and sensitivity analysis of the system have been evaluated.

Keywords: *reliability, sensitivity analysis, harvesting system, gumbel-hougaard copula and M.T.T.F.*

GJSFR-F Classification : MSC 2010: 91B02 , 00A05

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Ashish Kumar [α] & Monika Saini [σ]

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I. Introduction

Machinery dominate all other cost categories in farming and much is to be gained by adapting and balancing resources according to the actual needs arising from farm size, crop plan, etc. In this situation, wheat harvesting is a good example of compromise machinery management, highlighting the inherent complex evaluations. Ismail et al. (2009**)** indicted that the harvesting costs make up 35% of the total machinery costs. This emphasizes the need for developing reliable harvesting equipment. The analysis and prediction of agricultural machinery performance are important aspects of all machinery management efforts **(**Witney, 1995**)**. Abdel-Mageed et al. (1987) mentioned that almost every agricultural operation required for successful crop production must be timely. Untimely completion of any of these operations will cause a substantial loss of yield and quality, which ultimately will affect the farm's income. In view of the above, harvesting systems reliability occupies progressively more significant issue. Maintaining a required level of reliability is often an essential requirement of the systems. On the reliability of harvesting systems not much attention is given by the researcher. Furthermore, repairman is one of the essential parts of harvesting systems, and can affect the economy of the systems, directly or indirectly. Therefore, his action and work forms are vital on improving the reliability of harvesting systems. Singh et al. (2011) developed a reliability model of a three component system with two repairmen. Barak et al.(2012) developed a reliability model for a cold standby system with single server subject to maximum operation and repair time.

In the present study we consider a harvesting system having three unit tractor (T), combine (C) and wagon (W). For successful operation of the system preferred units T and C must be remain operative while when unit W fails system works partially. Two repairmen are involved in repairing of the system. One of the repairmen (the first) is the foreman (boss) and the

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other an assistant (apprentice). Whenever unit T and C fails repair is undertaken by boss while repair of the wagon is undertaken by the trainee. If the boss is busy in repairing and at the same time other unit fails then the repair is undertaken by apprentice. In the present model an important aspect of repairs have been taken, i.e. how to obtain the reliability measures of a system when there are tworepairmen involved in repairing jointly with different repair rates? It is not uncommon to see diverse ranges of performance between repairmen due to high degree of variability that exists in organization providing job as well as the diverse range of training and experience among employees. Keeping this fact in view, i.e. two repairmen, a boss and an apprentice, with the incorporation of human error, the author has tried to study the reliability measures of the harvesting system with the assumptions mentioned in the next section. Whenever both the repairmen are involved in repairing of the harvesting system, the joint probability distribution of the repair is obtained with the help of Gumbel-Hougaard family of copula. Failure rates are assumed to be constant in general whereas the repairs follow general distribution in all the cases.

By using Supplementary variable technique, Laplace transformation and copula following reliability characteristics of the system have been analyzed:

(1) Transition state probabilities of the system.
(2) Steady state behavior of the system using Abel's lemma.
(3) Various measures such as reliability, M.T.T.F and sensitivity analysis of the system.

Some numerical examples have been used to illustrate the model mathematically. Transition diagram of the system is shown in Figure 1.

II. Assumptions

1. Initially all the components are working properly.
2. The system consisting of three components tractor (T), combine (C) and wagon (w), all the units of the system are operative.
3. Each unit is either operative or failed.
4. All the units fails two type of failures either constant failure or human failure.
5. The whole system can fail directly from normal state due to human failure.
6. Repairs are perfect.
7. Joint probability distribution of repair rate, when repair is done by two repairmen followsGumbel-Hougaard family of copula.
8. When one of the preferred units of the system fails, the boss starts its repair while wagon is repaired by the trainee. When the second unit in this state fails, the trainee starts to work on its repair.
9. Failure rate of all the units are constant.

Notes

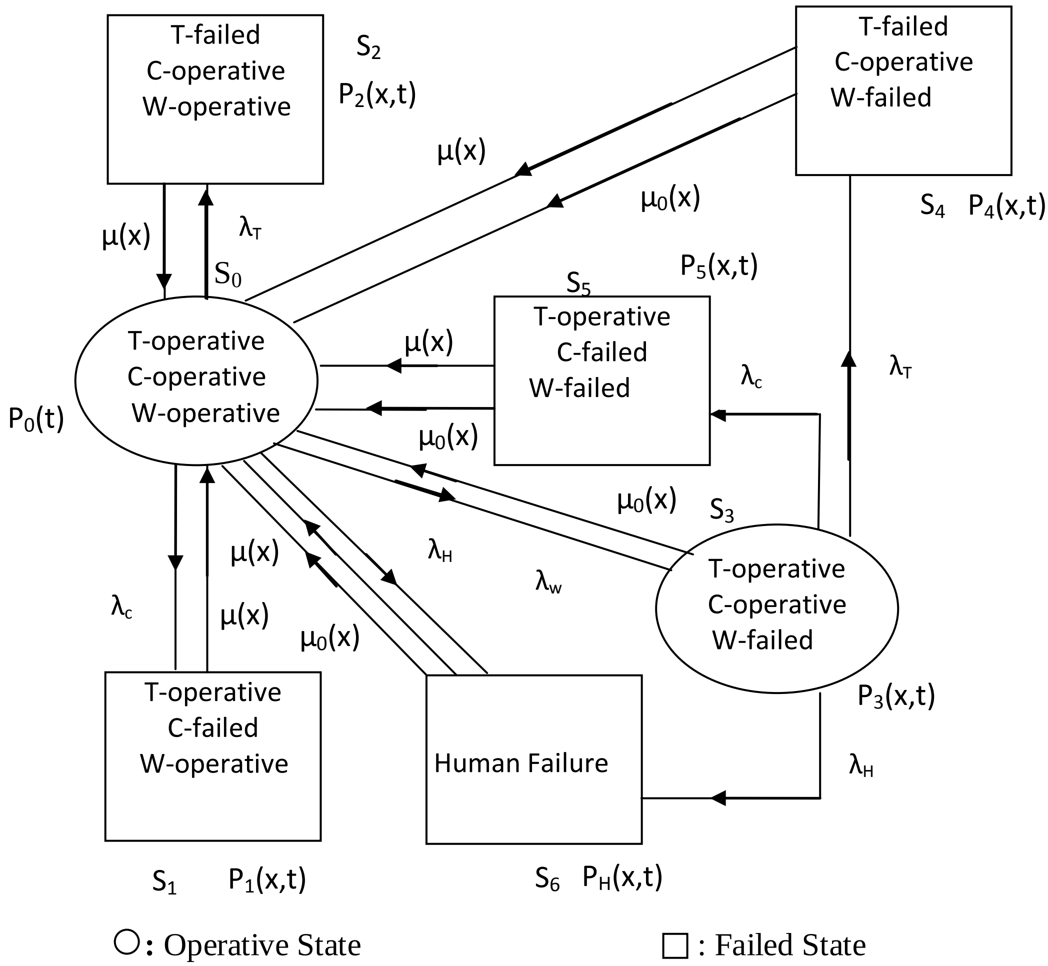

Fig. 1: State Transition Diagram

III. Notations

The following notations are used in this model:

$P_0(t)$: The probability that at time *t*, thesystem is in the state S_0.

$P_i(x,t)$: The pdf, system is in state S_i and isunder repair; elapsed repair time is *x*, *t*,where *i*=1, 2, 3, 4,5.

$P_H(x,t)$: The pdf, system is in state S_6 and isunder repair; elapsed repair time is *x*, *t*.

λ_T : Failure rate of subsystem tractor

λ_C : Failure rate of the combine

λ_W : Failure rate of the wagon

λ_H : Human Failure rate

$\mu_0(x)$: Repair rate when repair is done by trainee

$\mu(x)$: Repair rate when repair is done by boss

$\phi(x)$: Coupled repair rate i.e. repair rate when repair is done by boss and trainee both and it is given by Gumbel Hougaard copula as

$$\phi(x) = \exp\{x^{\theta} + (\log \mu_0(x))^{\theta}\}^{1/\theta}$$

IV. Formulation and Solution of Mathematical Model

By probability considerations and continuity arguments, the following difference-differential equations governing the behavior of the system may seem to be good.

$$(\frac{\partial}{\partial t} + \lambda_C + \lambda_T + \lambda_W + \lambda_H) P_0(t) = \int_0^\infty P_1(x,t)\mu(x)dx + \int_0^\infty P_2(x,t)\mu(x)dx + \int_0^\infty P_3(x,t)\mu(x)dx + \int_0^\infty P_4(x,t)\mu(x)dx + \int_0^\infty P_5(x,t)\mu(x)dx + \int_0^\infty P_H(x,t)\mu(x)dx \quad (1)$$

$$(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu(x)) P_1(x,t) = 0 \quad (2)$$

$$(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu(x)) P_2(x,t) = 0 \quad (3)$$

$$(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_0(x) + \lambda_H + \lambda_T + \lambda_C \;\; P_3 \;\; x \;\; t \;\; = \quad (4)$$

$$(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi(x)) P_4(x,t) = 0 \quad (5)$$

$$(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \phi(x)) P_5(x,t) = 0 \quad (6)$$

$$(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} \phi(x)) P_H(x,t) = 0 \quad (7)$$

Boundary Conditions:

$$P_1(0,t) = \lambda_C P_0(t) \quad (8)$$

$$P_2(0,t) = \lambda_T P_0(t) \quad (9)$$

$$P_3(0,t) = \lambda_W P_0(t) \quad (10)$$

$$P_4(0,t) = \lambda_T P_3(0,t) = \lambda_T \lambda_w P_0(t) \quad (11)$$

$$P_5(0,t) = \lambda_C \lambda_w P_0(t) \quad (12)$$

$$P_H \quad t \;\; = \lambda_H P_3 \quad t \;\; + \lambda_H P_0 \;\; t \;\; = \lambda_H \lambda_w P_0 \;\; t \;\; + \lambda_H P_0 \;\; t \;\; = \lambda_H \quad + \lambda_W \;\; P_0 \;\; t \quad (13)$$

Initial Conditions:

$P_0(0) = 1$ and other probabilities are at t=0 (14)

Solving equations (1-7) through (8-14), we have

$$\overline{P_0(s)} = \frac{1}{T(s)} \quad (15)$$

Transition state probabilities of the system in other states are given by

$$\overline{P_1(s)} = \lambda_c \overline{P_0(s)} \frac{(1 - S_u(s))}{s} \quad (16)$$

$$\overline{P_2(s)} = \lambda_T \overline{P_0(s)} \frac{(1 - S_u(s))}{s} \quad (17)$$

Notes

$$\overline{P_3(s)} = \lambda_W \overline{P_0(s)} \frac{(1 - S_u(\lambda_c + \lambda_T + \lambda_H + s))}{\lambda_c + \lambda_T + \lambda_H + s} \quad (18)$$

$$\overline{P_4(s)} = \lambda_T \lambda_W \overline{P_0(s)} \frac{(1 - S_\phi(s))}{s} \quad (19)$$

Notes

$$\overline{P_5(s)} = \lambda_c \lambda_W \overline{P_0(s)} \frac{(1 - S_\phi(s))}{s} \quad (20)$$

$$\overline{P_H(s)} = \lambda_H (1 + \lambda_W) \overline{P_0(s)} \frac{(1 - S_\phi(s))}{s} \quad (21)$$

Probability that the system is in upstate is obtained as;

$$\overline{P_{up}(s)} = \overline{P_0(s)} + \overline{P_3(s)} \quad (22)$$

$$\overline{P_{up}(s)} = \frac{1}{T(s)}[1 + \frac{\lambda_w (1 - S_u(\lambda_c + \lambda_T + \lambda_H + s))}{\lambda_c + \lambda_T + \lambda_H + s}] \quad (23)$$

Probability that the system is in downstate is obtained as;

$$\overline{P_{down}(s)} = \overline{P_1(s)} + \overline{P_2(s)} + \overline{P_4(s)} + \overline{P_5(s)} + \overline{P_6(s)} \quad (24)$$

$$\overline{P_{down}(s)} = \frac{1}{sT(s)}[\lambda_c \frac{(1 - S_u(s))}{s} + \lambda_T \frac{(1 - S_u(s))}{s} + \lambda_T \lambda_w \frac{(1 - S_\phi(s))}{s} + \lambda_c \lambda_w \frac{(1 - S_\phi(s))}{s} + \lambda_H (1 + \lambda_w) \frac{(1 - S_\phi(s))}{s}] \quad (25)$$

where

$$T(s) = (s + \lambda_c + \lambda_T + \lambda_w + \lambda_H) - [\lambda_c S_u(s) + \lambda_T S_u(s) + \lambda_w S_{u_0}(s) + \lambda_T \lambda_w S_\phi(s) + \lambda_c \lambda_w S_\phi(s) + \lambda_H (1 + \lambda_w) S_\phi(s)] \quad (26)$$

It is worth noticing that

$$p_0(s) + p_1(s) + p_2(s) + p_3(s) + p_4(s) + p_5(s) + p_H(s) = \frac{1}{s} \quad (27)$$

V. Steady State Behavior of the System

Using Abel's lemma, viz.,$\lim_{s \to 0} s[F(s)] = \lim_{t \to \infty} F(t) = F(say)$, Provided the limit R.H.S. exists, in Equations (15) to (21), the time independent probabilities are obtained as follows:

$$\overline{P_0(s)} = \frac{1}{T(0)} \quad (28)$$

and

$$\overline{P_1(s)} = \frac{\lambda_c}{T(0)} \quad (29)$$

$$\overline{P_2(s)} = \frac{\lambda_T}{T(0)} \quad (30)$$

$$\overline{P_3(s)} = \frac{\lambda_w}{T(0)} \tag{31}$$

$$\overline{P_4(s)} = \frac{\lambda_T \lambda_w}{T(0)} \tag{32}$$

$$\overline{P_5(s)} = \frac{\lambda_c \lambda_w}{T(0)} \tag{33}$$

$$\overline{P_6(s)} = \frac{\lambda_H (1+\lambda_w)}{T(0)} \tag{34}$$

Where

$$T(0) = \lambda_w[1 - \frac{\mu_0}{\mu_0 + \lambda_T + \lambda_H + \lambda_c} + (\lambda_T + \lambda_H + \lambda_c)] \tag{35}$$

VI. Particular Cases

Reliability of The System: Assuming all repairs rate zero in (23) reliability of the system becomes

$$\overline{R(s)} = \frac{1}{s + \lambda_c + \lambda_T + \lambda_w + \lambda_H} \tag{36}$$

Taking inverse Laplace transform of (36) the reliability of the system at any time ‘t’ is given by

$$R(t) = e^{-(\lambda_c + \lambda_T + \lambda_w + \lambda_H)t} \tag{37}$$

M.T.T.F. of The System: Taking all repairs zero in (23), Mean-Time-to-Failure (M.T.T.F.) of the system is obtained as

$$\text{M.T.T.F.} = \lim_{s \to 0} \overline{P_{up}(s)} = \frac{1}{\lambda_c + \lambda_T + \lambda_w + \lambda_H} \tag{38}$$

VII. Numerical Computation

Various measures of system effectiveness such as reliability, M.T.T.F. and sensitivity have been analyzed.

Reliability Analysis Let us fix failure rates as $\lambda_c = 0.08$, $\lambda_T = 0.02$, $\lambda_H = 0.05$ and $\lambda_w = 0.07$, repair rates $u = u_0 = \varphi = 0$, $\theta = 1$ and $x = 1$. Also, let the repair follows exponential distribution. Now, by putting all these values in Equation (37) and setting t =0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, one can obtain Table 1and Figure 2 which represent how reliability varies as the time increases.

Notes

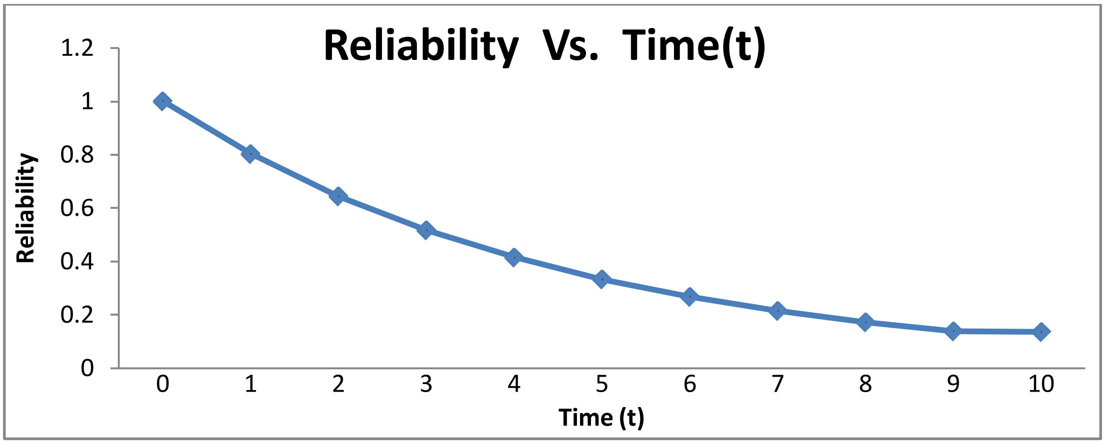

Fig. 2: Reliability Vs. Time

Time	Reliability
0	1
1	0.802519
2	0.644036
3	0.516851
4	0.414783
5	0.332871
6	0.267135
7	0.214381
8	0.172045
9	0.138069
10	0.135335

Table 1: Reliability Vs. Time

M.T.T.F. Analysis: Let us suppose that repair follows exponential distribution then using equation () and fromM.T.T.F. = $\lim_{s\to 0} \overline{P_{up}(s)}$ we have the following four cases:

1. Fixing $\lambda_c = 0.08$, $\lambda_T = 0.02$, $\lambda_H = 0.05$ andvarying the value of λ_w =1,.01,.02,.03,.04,.05,.06,.07,.08,.09,.1, repair rates $u = u_0 = \varphi=0$, $\theta = 1$ and$x = 1$. one can obtain the variation in MTTF with respect to λ_w .
2. Fixing $\lambda_c = 0.08$, $\lambda_T = 0.02$, $\lambda_w = 0.07$ andvarying the value of λ_H =1,.01,.02,.03,.04,.05,.06,.07,.08,.09,.1, repair rates $u = u_0 = \varphi=0$, $\theta = 1$ and$x = 1$. one can obtain the variation in MTTFwith respect to λ_H.
3. Fixing $\lambda_c = 0.08$, $\lambda_w = 0.07$, $\lambda_H = 0.05$ andvarying the value of λ_T =1,.01,.02,.03,.04,.05,.06,.07,.08,.09,.1, repair rates $u = u_0 = \varphi=0$, $\theta = 1$ and$x = 1$. one can obtain the variation in MTTF with respect to λ_T.
4. Fixing $\lambda_W = 0.07$, $\lambda_T = 0.02$, $\lambda_H = 0.05$ andvarying the value of λ_c =1,.01,.02,.03,.04,.05,.06,.07,.08,.09,.1, repair rates $u = u_0 = \varphi=0$, $\theta = 1$ and$x = 1$. one can obtain the variation in MTTFwith respect to λ_c.

With above relations one can obtain Table 2 and Figure 3 which represent how MTTF varies as the failure rate varies..

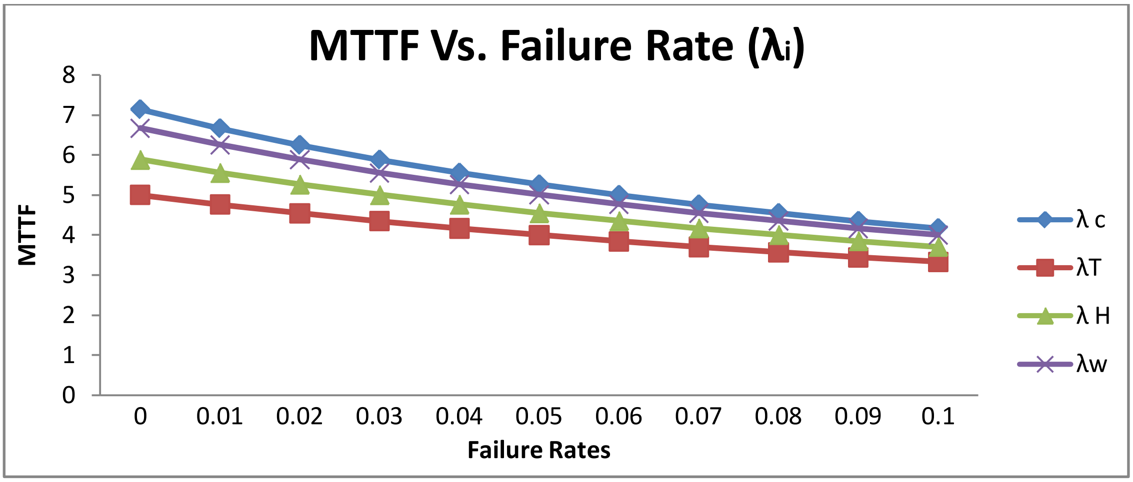

Fig.3: MTTF Vs. Failure Rates (λi)

Variation w.r.t λ c, λT, λw ,λH	λ c	λT	λ H	Λw
0	7.142857	5.000000	5.882353	6.666667
0.01	6.666667	4.761905	5.555556	6.250000
0.02	6.250000	4.545455	5.263158	5.882353
0.03	5.882353	4.347826	5.000000	5.555556
0.04	5.555556	4.166667	4.761905	5.263158
0.05	5.263158	4.000000	4.545455	5.000000
0.06	5.000000	3.846154	4.347826	4.761905
0.07	4.761905	3.703704	4.166667	4.545455
0.08	4.545455	3.571429	4.000000	4.347826
0.09	4.347826	3.448276	3.846154	4.166667
0.1	4.166667	3.333333	3.703704	4.000000

Table.2: MTTF Vs. Failure Rates (λi)

Sensitivity Analysis Assuming that all repair rates follows exponential distribution, we first perform a sensitivity analysis for changes in R(t) resulting from changes in system parameters λ_c, λ_T, λ_H and λ_w . Putting $\lambda_c = 0.08$, $\lambda_T = 0.02$, $\lambda_H = 0.05$ and $\lambda_w = 0.07$, repair rates $u = u_0 = \varphi = 0$, $\theta = 1$ and $x = 1$ in equation (36), and then differentiating w.r.t. λ_c, λ_T, λ_w and λ_H respectively ,we get:

$$\frac{\partial R(s)}{\partial \lambda_c} = \frac{\partial R(s)}{\partial \lambda_T} = \frac{\partial R(s)}{\partial \lambda_w} = \frac{\partial R(s)}{\partial \lambda_H} = -\frac{1}{(s + \lambda_c + \lambda_T + \lambda_w + \lambda_H)^2}$$

After taking inverse Laplace transformation, we get

$$\frac{\partial R(t)}{\partial \lambda_c} = \frac{\partial R(t)}{\partial \lambda_T} = \frac{\partial R(t)}{\partial \lambda_w} = \frac{\partial R(t)}{\partial \lambda_H} = -te^{-(\lambda_c + \lambda_T + \lambda_w + \lambda_H)t}$$

Now, we perform a sensitivity analysis of changes in M.T.T.F. with respect to λ_c, λ_T, λ_w and λ_H . Setting $\lambda_c = 0.08$, $\lambda_T = 0.02$, $\lambda_H = 0.05$ and $\lambda_w = 0.07$, repair rates $u = u_0 = \varphi = 0$, $\theta = 1$ and $x = 1$ in equation (38) and taking $\lim_{s \to 0}$ then differentiating w.r.t. λ_c, λ_T, λ_w and λ_H respectively ,we get:

$$\frac{\partial MTTF}{\partial \lambda_c} = \frac{\partial MTTF}{\partial \lambda_T} = \frac{\partial MTTF}{\partial \lambda_w} = \frac{\partial MTTF}{\partial \lambda_H} = \frac{-1}{(\lambda_c + \lambda_T + \lambda_w + \lambda_H)^2}$$

Notes

Notes

Numerical results of the sensitivity analysis for the system reliability and the M.T.T.F. are presented in Figures 4 - 5 and Tables 3-4.

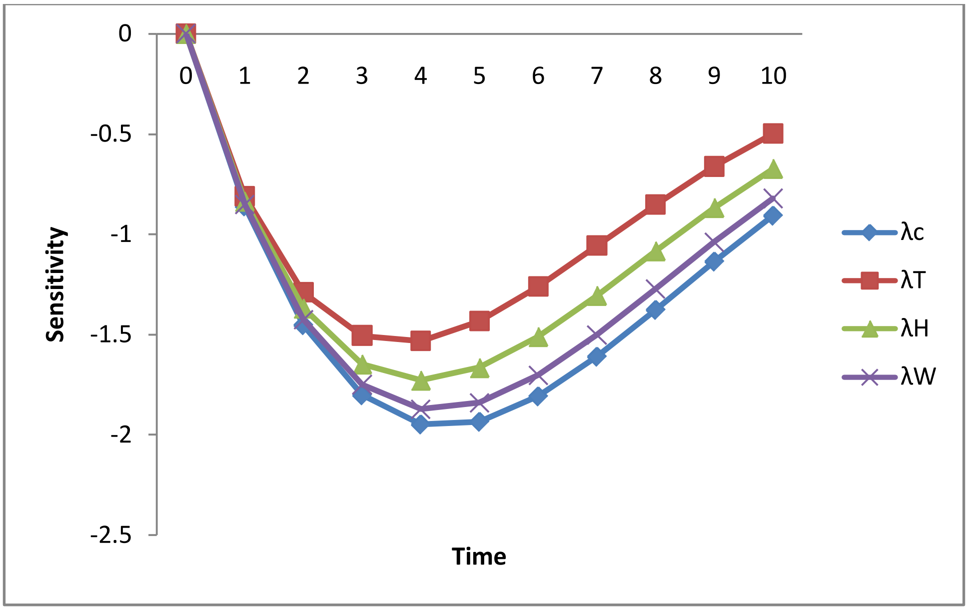

Fig. 4: Sensitivity Analysis w.r.t. failure rates λ_c, λ_T, λ_H and λ_w

Time	λc	λT	λH	λW
0	0	0	0	0
1	-0.86071	-0.81058	-0.83527	-0.85214
2	-1.4523	-1.28807	-1.36772	-1.42354
3	-1.80149	-1.50473	-1.64643	-1.74824
4	-1.94701	-1.53157	-1.72684	-1.87067
5	-1.93371	-1.43252	-1.66436	-1.8394
6	-1.80717	-1.26082	-1.50947	-1.70192
7	-1.60948	-1.0575	-1.30462	-1.50067
8	-1.37636	-0.85167	-1.08268	-1.27054
9	-1.13567	-0.66181	-0.86695	-1.03793
10	-0.90718	-0.49787	-0.67206	-0.82085

Table 3: Sensitivity Analysis of Reliability w.r.t. failure rates λ_c, λ_T, λ_H and λ_w

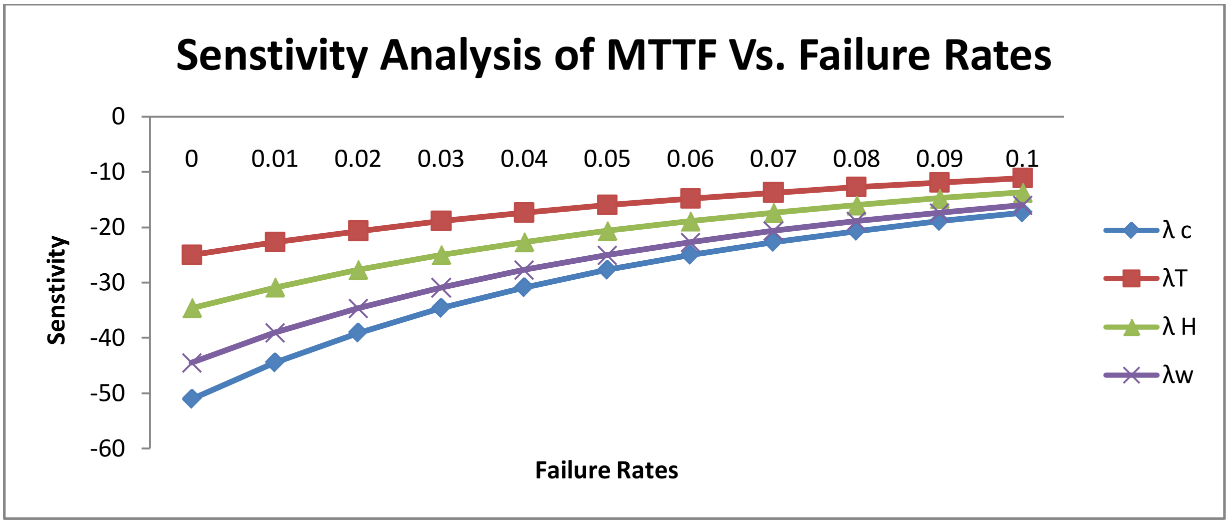

Fig. 5: Sensitivity Analysis of MTTF w.r.t. failure rates λ_c, λ_T, λ_H and λ_w

Variation w.r.t λ c, λT,λw,λH	Sensitivity Analysis of MTTF w.r.t λ c	Sensitivity Analysis of MTTF w.r.t λT	Sensitivity Analysis of MTTF w.r.t λ H	Sensitivity Analysis of MTTF w.r.t λw
0	-51.0204	-25.0000	-34.6021	-44.4444
0.01	-44.4444	-22.6757	-30.8642	-39.0625
0.02	-39.0625	-20.6612	-27.7008	-34.6021
0.03	-34.6021	-18.9036	-25.0000	-30.8642
0.04	-30.8642	-17.3611	-22.6757	-27.7008
0.05	-27.7008	-16.0000	-20.6612	-25.0000
0.06	-25.0000	-14.7929	-18.9036	-22.6757
0.07	-22.6757	-13.7174	-17.3611	-20.6612
0.08	-20.6612	-12.7551	-16.0000	-18.9036
0.09	-18.9036	-11.8906	-14.7929	-17.3611
0.1	-17.3611	-11.1111	-13.7174	-16.0000

Table 4: Sensitivity Analysis of MTTF w.r.t. failure rates λ_c, λ_T, λ_H and λ_w

VIII. Conclusion

In this paper, we analyzed the reliability, MTTF and sensitivity of the harvesting system incorporating different failures. To numerically examine the behavior of reliability and M.T.T.F of the system, the various parameters are fixed as $\lambda_c = 0.08$, $\lambda_T = 0.02$, $\lambda_H = 0.05$ and $\lambda_w = 0.07$, repair rates $u = u0 = \varphi=0$, $\theta = 1$ and $x = 1$. One can easily conclude from Figure 2 and Table 1 that the reliability of thesystem decreases with the increment in time and it attains a value of 0.135 after a long period of time.By critically examining the Figure 3 and table 2 onecan conclude that M.T.T.F. of the system decreasesfrom 7.142857 to 4.166667, from 5.000000 to 3.333333,from 5.882353 to3.703704 andfrom6.666667 to 4.000000 with respect to λ_c , λ_T , λ_H and λ_w respectively in a same manner for the consideredvalues. M.T.T.F. of thesystem has been obtained in the order: M.T.T.F. w. r. t. λ_c> M.T.T.F. w. r. t. λ_w> M.T.T.F. w. r. t.λ_H> M.T.T.F. w. r. t. λ_T. So M.T.T.F. of thesystem is highest with respect to λ_c and lowest withrespect to λ_T.
The sensitivities of the system reliability withrespect to λ_c, λ_T , λ_H and λ_w are shown in Figures 4 and table 3.

It reveals that the sensitivity initiallydecreases and then tends to increase as time passesand attain a value-0.90718, -0.49787, -0.67206 and -0.82085 at t= 10 with respect to λ_c ,

Notes

λ_T , λ_H and $\lambda_w$respectively. It isclear from the graph that system reliability is moresensitive w. r. t. λ_T . It is interesting tonote that the system becomes more sensitive withthe increase in failure rate of tractor (T). So, we can concludethat the system can be made less sensitive bycontrolling its failure rates. Moreover, Figure 5 and table 4 show the sensitivity of M.T.T.F. withrespect to λ_c , λ_T , λ_H and $\lambda_w$which show that itincreases. Criticalobservation of these graphs points out thatM.T.T.F. of the system is more sensitive withrespect toλ_T.

Notes

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Intuitionistic Fuzzy sets in Career Determination

By P. A. Ejegwa, A. J. Akubo & O. M. Joshua
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Abstract- Intuitionistic fuzzy set has proven interesting and useful in providing a flexible framework or model to elaborate uncertainty and vagueness involved in decision making. In this paper, we proposed the application of intuitionistic fuzzy sets (IFS) in career determination. Solution is obtained by looking for the smallest distance between each student and each career.

Keywords: *fuzzy sets, intuitionistic fuzzy sets, career choice, career determination.*

GJSFR-F Classification : *MSC 2010: 94D05*

Ref

[6] S. K. De, R. Biswas, A.R. Roy, An application of Intuitionistic Fuzzy Sets in Medical diagnostic, Fuzzy sets and systems 117 (2) (2001) 209-213.

Intuitionistic Fuzzy sets in Career Determination

P. A. Ejegwa α, A. J. Akubo σ & O. M. Joshua ρ

Abstract- Intuitionistic fuzzy set has proven interesting and useful in providing a flexible framework or model to elaborate uncertainty and vagueness involved in decision making. In this paper, we proposed the application of intuitionistic fuzzy sets (IFS) in career determination. Solution is obtained by looking for the smallest distance between each student and each career.

Keywords: *fuzzy sets, intuitionistic fuzzy sets, career choice, career determination.*

I. Introduction

The theory of fuzzy sets (FS) introduced by Zadeh [14] has showed meaningful applications in many fields of study. The idea of fuzzy set is welcome because it handles uncertainty and vagueness which Cantorian set could not address. In fuzzy set theory, the membership of an element to a fuzzy set is a single value between zero and one. However in reality, it may not always be true that the degree of non-membership of an element in a fuzzy set is equal to 1 minus the membership degree because there may be some hesitation degree. Therefore, a generalization of fuzzy sets was proposed by Atanassov [1, 2] as intuitionistic fuzzy sets (IFS) which incorporate the degree of hesitation called hesitation margin (and is defined as 1 minus the sum of membership and non-membership degrees respectively). The notion of defining intuitionistic fuzzy set as generalized fuzzy set is quite interesting and useful in many application areas. The knowledge and semantic representation of intuitionistic fuzzy set become more meaningful, resourceful and applicable since it includes the degree of belongingness, degree of non-belongingness and the hesitation margin [3, 4]. Szmidt and Kacprzyk [10] showed that intuitionistic fuzzy sets are pretty useful in situations when description of a problem by a linguistic variable given in terms of a membership function only seems too rough. Due to the flexibility of IFS in handling uncertainty, they are tool for a more human consistent reasoning under imperfectly defined facts and imprecise knowledge [11].

De et al. [6] gave an intuitionistic fuzzy sets approach in medical diagnosis using three steps such as; determination of symptoms, formulation of medical knowledge based on intuitionistic fuzzy relations, and determination of diagnosis on the basis of composition of intuitionistic fuzzy relations. Intuitionistic fuzzy set is a tool in modeling real life problems like sale analysis,

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new product marketing, financial services, negotiation process, psychological investigations etc. since there is a fair chance of the existence of a non-null hesitation part at each moment of evaluation of an unknown object [8, 10]. Atanassov [4, 5] carried out rigorous research based on the theory and applications of intuitionistic fuzzy sets. Many applications of IFS are carried out using distance measures approach. Distance measure between intuitionistic fuzzy sets is an important concept in fuzzy mathematics because of its wide applications in real world, such as pattern recognition, machine learning, decision making and market prediction. Many distance measures between intuitionistic fuzzy sets have been proposed and researched in recent years [8, 9, 13] and used by Szmidt and Kacprzyk [10, 11] in medical diagnosis.

We show a novel application of intuitionistic fuzzy set in a more challenging area of decision making (i.e. career choice). An example of career determination will be presented, assuming there is a database (i.e. a description of a set of subjects S, and a set of careers C). We will describe the state of students knowing the results of their performance. The problem description uses the concept of IFS that makes it possible to render two important facts. First, values of each subject performance changes for each student. Second, in a career determination database describing career for different students, it should be taken into account that for different students aiming for the same career, values of the same subject performance can be different. We use the normalized Hamming distance method given in [8, 9, 12] to measure the distance between each student and each career. The smallest obtained value, points out a proper career determination based on academic performance.

II. Concept of Intuitionistic Fuzzy Sets

Definition 1 [14]: Let X be a nonempty set. A fuzzy set A drawn from X is defined as $A = \{\langle x, \mu_A(x)\rangle : x \in X\}$, where $\mu_A(x): X \longrightarrow [0,1]$ is the membership function of the fuzzy set A. Fuzzy set is a collection of objects with graded membership i.e. having degrees of membership.

Definition 2 [4]: Let X be a nonempty set. An intuitionistic fuzzy set A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where the functions $\mu_A(x), \nu_A(x): X \longrightarrow [0,1]$define respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A, which is a subset of X, and for every element $x \in X, 0 \leq \mu_A(x) + \nu_A(x) \leq 1$. Furthermore, we have $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ called the intuitionistic fuzzy set index or hesitation margin of x in A. $\pi_A(x)$ is the degree of indeterminacy of $x \in X$ to the IFS A and $\pi_A(x) \in [0,1]$ i.e.,$\pi_A(x): X \longrightarrow [0,1]$ and $0 \leq \pi_A \leq 1$ for every $x \in X$. $\pi_A(x)$ expresses the lack of knowledge of whether x belongs to IFS A or not.

For example, let A be an intuitionistic fuzzy set with $\mu_A(x) = 0.5$ and $\nu_A(x) = 0.3 \Rightarrow \pi_A(x) = 1 - (0.5 + 0.3) = 0.2$. It can be interpreted as "the degree that the object x belongs to IFS A is 0.5, the degree that the object x does not belong to IFS A is 0.3 and the degree of hesitancy is 0.2". s

Ref

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a) Basic Operations on Intuitionistic Fuzzy Sets [2]

1. [inclusion] $A \subseteq B \leftrightarrow \mu_A(x) \leq \mu_B(x)$ *and* $\nu_A(x) \geq \nu_B(x)$ $i = 1, 2, \dots, n$ $\forall x \in X$.
2. [equality] $A = B \leftrightarrow \mu_A(x) = \mu_B(x)$ *and* $\nu_A(x) = \nu_B(x)$ $i = 1, 2, \dots, n$ $\forall x \in X$.
3. [complement] $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$.
4. [union] $A \cup B = \{\langle x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x))\rangle : x \in X\}$.
5. [intersection] $A \cap B = \{\langle x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x))\rangle : x \in X\}$.
6. [addition] $A \oplus B = \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \nu_A(x)\nu_B(x)\rangle : x \in X\}$.
7. [multiplication] $A \otimes B = \{\langle x, \mu_A(x)\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x)\rangle : x \in X\}$.

Definition 3 [12]: The normalized Hamming distance $d_{n-H}(A, B)$ between two IFSs A and B is defined as $d_{n-H}(A, B) = \frac{1}{2n}\sum_{i=1}^{n}[|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|]$, $X = \{x_1, x_2, \dots, x_n\}$ for $i = 1,2, \dots, n$.

III. Application of Intuitionistic Fuzzy sets in Career Determination

The essence of providing adequate information to students for proper career choice cannot be overemphasized. This is paramount because the numerous problems of lack of proper career guide faced by students are of great consequence on their career choice and efficiency. Therefore, it is expedient that students be given sufficient information on career determination or choice to enhance adequate planning, preparation and proficiency. Among the career determining factors such as academic performance, interest, personality make-up etc; the first mentioned seems to be overriding. We use intuitionistic fuzzy sets as tool since it incorporate the membership degree (i.e. the marks of the questions answered by the student), the non-membership degree (i.e. the marks of the questions the student failed) and the hesitation degree (which is the mark allocated to the questions the student do not attempt).

Let $S = \{s_1, s_2, s_3, s_4\}$ be the set of students, $C = \{$medicine, pharmacy, surgery, anatomy$\}$ be the set of careers and Su ={English Language, Mathematics, Biology, Physics, Chemistry} be the set of subjects related to the careers. We assume the above students sit for examinations (i.e. over 100 marks total) on the above mentioned subjects to determine their career placements and choices. The table below shows careers and related subjects requirements.

Table 1
Careers vs Subjects

	English Language	Mathematics	Biology	Physics	Chemistry
Medicine	(0.8,0.1,0.1)	(0.7,0.2,0.1)	(0.9,0.0,0.1)	(0.6,0.3,0.1)	(0.8,0.1,0.1)
Pharmacy	(0.9,0.1,0.0)	(0.8,0.1,0.1)	(0.8,0.1,0.1)	(0.5,0.3,0.2)	(0.7,0.2,0.1)
Surgery	(0.5,0.3,0.2)	(0.5,0.2,0.3)	(0.9,0.0,0.1)	(0.5,0.4,0.1)	(0.7,0.1,0.2)
Anatomy	(0.7,0.2,0.1)	(0.5,0.4,0.1)	(0.9,0.1,0.0)	(0.6,0.3,01)	(0.8,0.0,0.2)

Each performance is described by three numbers i.e. membership μ, non-membership ν and hesitation margin π. After the various examinations, the students obtained the following marks as shown in the table below.

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[2] K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87-96.

Table 2
Students vs Subjects

	English Language	Mathematics	Biology	Physics	Chemistry
S_1	(0.6,0.3,0.1)	(0.5,0.4,0.1)	(0.6,0.2,0.2)	(0.5,0.3,0.2)	(0.5,0.5,0.0)
S_1	(0.5,0.3,0.2)	(0.6,0.2,0.2)	(0.5,0.3,0.2)	(0.4,0.5,0.1)	(0.7,0.2,0.1)
S_1	(0.7,0.1,0.2)	(0.6,0.3,0.1)	(0.7,0.1,0.2)	(0.5,0.4,0.1)	(0.4,0.5,0.1)
S_1	(0.6,0.4,0.0)	(0.8,0.1,0.1)	(0.6,0.0,0.4)	(0.6,0.3,0.1)	(0.5,0.3,0.2)

Notes

Using Def. 3 above to calculate the distance between each student and each career with reference to the subjects, we get the table below.

Table 3
Students vs Careers

	Medicine	Pharmacy	Surgery	Anatomy
S_1	4.8×10^{-2}	4.4×10^{-2}	4.4×10^{-2}	4.0×10^{-2}
S_2	4.4×10^{-2}	4.4×10^{-2}	2.8×10^{-2}	4.8×10^{-2}
S_3	3.6×10^{-2}	3.6×10^{-2}	4.0×10^{-2}	4.0×10^{-2}
S_4	4.0×10^{-2}	3.6×10^{-2}	4.4×10^{-2}	4.8×10^{-2}

From the above table, the shortest distance gives the proper career determination. S_1 is to read anatomy (anatomist), S_2 is to read surgery (surgeon), S_3 is to read either medicine or pharmacy (doctor or pharmacist), and S_4 is to read pharmacy (pharmacist).

IV. Conclusion

This application of intuitionistic fuzzy sets in career determination is of great significance because it provides accurate and proper career choice based on academic performance. Career choice is a delicate decision making problem since it has a reverberatory effect on efficiency and competency if not properly handled. In the proposed application, we used normalized Hamming distance to calculate the distance of each student from each career in respect to the subjects, to obtained results.

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Mathematical Modeling of Strategic Treatments on Tumor Growth

By U. S. Rana & Jyotsna Baloni
D.A.V College, India

Abstract- We propose to contribute to the emerging body of cancer treatment research by developing and analyzing mathematical models of the treatment of tumor with various strategic treatments. We build on existing models of the immunology that are already successfully developed and then the effects of chemotherapy and interleukin-2 were applied to the model. Thus we build a mathematical model of tumour and effector cell with scheduled chemotherapy.The effect of scheduled il2 dose with chemotherapy and adoptive immunotherapy reduced the tumor growth.

Keywords: mathematical modeling, chemotherapy, effector cell, il-2, adoptive immunotherapy.

GJSFR-F Classification : MSC 2010: 00A71

MATHEMATICALMODELINGOFSTRATEGICTREATMENTSONTUMORGROWTH

Strictly as per the compliance and regulations of :

Mathematical Modeling of Strategic Treatments on Tumor Growth

U. S. Rana [α] & Jyotsna Baloni [σ]

Abstract- We propose to contribute to the emerging body of cancer treatment research by developing and analyzing mathematical models of the treatment of tumor with various strategic treatments. We build on existing models of the immunology that are already successfully developed and then the effects of chemotherapy and interleukin-2 were applied to the model. Thus we build a mathematical model of tumour and effector cell with scheduled chemotherapy.The effect of scheduled il2 dose with chemotherapy and adoptive immunotherapy reduced the tumor growth.

Keywords: mathematical modeling, chemotherapy, effector cell, il-2, adoptive immunotherapy.

I. Introduction

The worldwide burden of cancer (malignant tumor) is a major health problem, with more than 8 million new cases and 5 million deaths per year. Cancer is the second leading cause of death. With growing techniques the survival rate has increased and so it becomes important to contribute even the smallest help in this field favoring the survival rate. Tumor is a mass of tissue formed as the result of abnormal, excessive, uncoordinated, autonomous and purposeless proliferation of cells.

The immune system plays an important role to identify and eliminate tumors. This is called immune surveillance. The theory of immune surveillance says that the immune system continually recognizes the transformed cells of tumors because they express antigens that are not found on normal cells. For the immune system, these antigens appear foreign, and their presence causes immune cells to attack the transformed tumor cells and hence eliminates tumor cells, but when a tumor escapes immune surveillance and grows too large for the immune system to kill, tumor is the result. Tumor antigens are presented on MHC (Major Histocompatibility Complex) class I molecules (present on the surface of tumor cells) in a similar way to viral antigens. This allows killer T cells to recognize the tumor cell as abnormal. NK (Natural Killer) cells also kill tumorous cells in a similar way.Tumor-specific lymphocytes are lymphocytes of immune system which recognises tumor cells and can be found in the blood, draining lymph nodes, and the tumor itself of patients with actively growing tumors.

The main response of the immune system to tumors is to destroy the abnormal cells using killer T cells, which generally require several stimuli for rapid expansion. It has been

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known for long time that interleukin-2 (IL-2) is generally required for expansion of populations of T lymphocytes in vitro [1]. IL-2 is a cytokine which is used in treating advanced cancer taken alone or with combinations with other drugs.

IL-2 causes effector cells to proliferate. However IL-2 can also activate natural killer cells, B lymphocytes, and macrophages and can induce lymphokine-activated killer cells in vitro. High dose intravenous bolus of IL-2 is administered to treat many types of melanomas. But the problem with high dose of IL-2 therapy is that it causes toxicity and many types of side effects to the body and even can cause death. [2]

At present, chemotherapy is the most well established treatment for fighting cancer. Chemotherapy, in its most general sense, is the treatment of disease by chemicals especially by killing micro-organisms or cancerous cells. The basic idea behind chemotherapy is to kill cancerous cells faster than healthy cells. This is accomplished by interrupting cellular division at some phase and thus killing more tumor cells then their slower developing normal cells. The course of therapy depends on the cancer type, the chemotherapy drugs used, the treatment goal and how your body responds. The treatment may be given every day, every week or every month. Their may be breaks between treatments so that the body has a chance to build new healthy cells. [3,4] However some normal cells, for example those that form the stomach lining and immune cells, are also rapidly dividing cells which means that chemotherapy also harms the patient leading to things like a depressed immune system which opens the host to infection [5, 6]. In addition, chemotherapy causes significant side effects in patients, and therefore exploring new forms treatments may prove extremely beneficial.

One of the form of immunotherapy to treat the tumor growth is Adoptive immunotherapy. It is used in treatment of tumor in which lymphocytes are removed from the patients's body to enhance their anti-tumor activity with naturally produced growth factor, cultured in large numbers and then returned to the patient. Adoptive immunotherapy uses immune effector cells as lymphocytes. Lymphokine-activated killer (LAK) cells and Tumor-infiltrating lymphocytes(TILs) are such lymphocytes which are grown in presence of IL-2 an immune stimulator [7].

Despite the existence of anti-tumor immune responses, the tumors still progress and eventually cause disastrous effects on host body. Over the past several decades, investigators have pursued variety of strategies to direct the immune system against many types of tumors. The immune responses to be effective enough must eliminate the tumor cells more rapidly than their rate of proliferation and hence the role of boosting the immune response or immunotherapy. In this article we present a model of immune system against tumor growth. Chemotherapy and immunotherapy are not that success in treatment even high doses of these therapies are effective for tumor eradication but also high doses of these treatments create severe side effects in the body [8].

The growing understanding of the therapy cycles has produced various strategies to increase the effectiveness of cancer treatment [9]. Hence, In this paper a model of tumor growth is presented which describes the treatment of a tumor by chemotherapy over a fixed period of time by the repeated administration of its doses. The model will select that in what interval, at what schedule and what specific doses of IL-2 therapy, chemotherapy and adoptive

Ref

2. Repmann, R., Wagner, S., and Richter, A., "The use of interleukin 2 in the treatment of renal cell cancer and melanoma proved that an immunological treatment is capable, in some cases,of inducing long term regression of metastatic tumours", ***Anticancer Res*.**, Vol. 17 (1997), 2879-2882.

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immunotherapy should be delivered. The model constructs a regimen that both minimizes the tumor population at the end of the treatment and satisfies constraints on the drug toxicity and intermediate tumor size.

II. Mathematical Modeling

The model consists of four ordinary differential equations of immune response to tumor growth which gets affected by treatments chemotherapy, interleukin-2 and adoptive immunotherapy. The following variables are set to the model.

Variables	**x(t)**	**y(t)**	**z(t)**	**λ(t)**	**t**
Description	Effector cell	Tumour cell	IL-2	Chemotherapeutic drug	time

a) *Effector Cell*

Effector cells are lymphocytes which are the relatively short-lived activated cells that have been induced to differentiate into a form capable of mounting a specific immune response. The production of the effector cell was given by Kirschner [10] and shown below that how chemotherapy affects the production of the effector cell.

$$\frac{dx}{dt} = cy - \mu_1 x + p_1 \frac{xz}{g_1 + z} - C_x(1 - e^{-\lambda})x + S_1 \tag{1}$$

In this equation, *c* denotes the antigenicity of tumour. Tumor antigen are main target for cancer immunotherapy. The ideal tumor antigen is immunogenic and expressed exclusively on tumor cells. Higher the value of antigenicity the easier it becomes for the effector cell to detect the tumor. The second term is the mortality rate of effector cell and the third term is the proliferation term due to the presence of the cytokine IL-2. Here p_1 is the maximum proliferation of the effector cell. The fourth term denotes the chemotherapeutic effect on effector cells. The interaction of chemotherapy on sensitive cells is given by saturation term kinetics [11]. At low chemotherapeutic drug doses the death rate of the cells depends on the chemotherapeutic drug λ (t) but at high doses it becomes λ independent. It is assumed that C_x depends on the IL-2 concentration [12]. The fifth term S_1 denotes a treatment called adoptive immunotherapy at constant influx.

Here, $C_x = C_x^{chemo}(1 - e^{-z/z_0})$ where C_x^{chemo} is the death rate of effector cells by chemotherapy, and z_0 is IL-2 z(t) at t=0.

b) *Tumor Cell*

The interaction of immune system with tumor is again based on the Kirschner model.

$$\frac{dy}{dt} = r_1 y(1 - by) - \frac{axy}{g_2 + y} - C_y(1 - e^{-\lambda})y \tag{2}$$

In this equation the first term denotes the production rate of the tumor cells having maximum proliferation rate of r_1 until it reaches to its carrying capacity *b* of tumor [13]. The

Ref

10. Kirschner, D., Panetta, J.C., "Modelling immunotherapy of the tumor-immune interaction", ***Journal of Mathematical Biology***. Vol. 37, (1998), 235-252.

second term shows that the destruction by the effector cell x(t) and *a* is the parameter used to give value of the strength of the immune system against tumor cells [14]. The third term denotes the effect of the chemotherapeutic drug on tumor cells.

Here, $C_y = C_y^{chemo}(1-e^{-z/z_0})$, where C_y^{chemo} is the death rate of tumor cells by chemotherapy.

c) IL-2

The Kirschner model explains the production of the IL-2.

$$\frac{dz}{dt} = \frac{p_3 xy}{(g_3 + y)} - \mu_2 z + S_2 \quad (3)$$

Here the first term is the production of IL-2 which depends on the interaction of the x (t) and y(t) and p_3 is the maximum proliferation of the IL-2. The second term is the decay rate of IL-2. S_2 denotes interleukin therapy at constant influx.

d) Chemotherapeutic Drug

$$\frac{d\lambda}{dt} = V(t) - v\lambda \quad (4)$$

Here, *V(t)* is the time dependent external influx of the chemotherapeutic drug and *v* is the decay rate.

Finally, writing all the equations representing the growth of tumor cells, immune system and chemotherapeutic drug and effects of treatments on tumour growth.

$$\frac{dx}{dt} = cy - \mu_1 x + p_1 \frac{xz}{g_1 + z} - C_x(1-e^{-\lambda})x + S_1$$

$$\frac{dy}{dt} = r_1 y(1-by) - \frac{axy}{g_2 + y} - C_y(1-e^{-\lambda})y$$

$$\frac{dz}{dt} = \frac{p_3 xy}{(g_3 + y)} - \mu_2 z + S_2$$

$$\frac{d\lambda}{dt} = V(t) - v\lambda$$

III. Parameter

The tumor immune dynamics is sensitive to the choice of parameter values [16]. The parameter values changes from patient to patient. The experimental proved values are given below. But some values are studied from the medical history and which are not experimentally proved are fitted to the available data and studied their affects by giving different values on the model. The value of τ_c is estimated to be 10^6 cells as after this value the angiogenic is switched on [17]. The parameter q_1 and q_2 were estimated by experiments presented in [14]. The values of C_x^{chemo} and C_y^{chemo} representing the killing of cells by chemotherapy are estimated from Pilli's model [15]. The decay rate of the chemotherapeutic drug was estimated from the example taken from [18].

Ref

14. Hsieh, C.l., Chen, D.S. and Hwang,L.H., "Tumor-induced immunosuppression: A barrier to immunotherapy of large tumors by cytokine-secreting tumor vaccine", ***Hum. Gene Ther.***, Vol. 11, (2000), 681–692.

Parameter	Description	Value/Units
c	Antigenicity of tumor	0-0.035 $days^{-1}$
g_1	Half saturation constant	2×10^{7} pg /**l**
p_1	Max proliferation rate of effector cell	0.1245$days^{-1}$
r_1	Tumor growth rate	0.18 $days^{-1}$
b	Carrying capacity of tumor	1×10^{-9} cells /ml
a	Tumor cell strength against immune system	1 day^{-1}
g_2	Half saturation constant	1×10^{5} cells /ml
μ_1	Effector cell growth rate	0.03$days^{-1}$
C_x^{chemo}	Effector cell decay by chemotherapy	0.6 day^{-1}
C_y^{chemo}	Tumor cell decay by chemotherapy	0.9 day^{-1}
p_3	Growth rate of IL-2	5 pg /cells×days
g_3	Half saturation constant	1×10^{3} cells /ml
μ_2	Decay rate of IL-2	10 $days^{-1}$
V	External influx of chemotherapeutic drug	1 day^{-1}
v	Decay rate of chemotherapeutic drug	6.4 $days^{-1}$

Initially, x(0)= 1, y(0)= 1, z(0)= 1, λ(0)=0

IV. Results and Discussion

In this section, we present the numerical results to explore the effects of therapies on growth rate of tumor cells. For this purpose we used Equations 1-4 for tumor cell density, effector cell density, il2 level and chemotherapic drug respectively and applied different doses of treatment at different time intervals so as to study their effects on tumor growth. Computer program is developed in software MatLab and run on Pentium IV using ode15s.

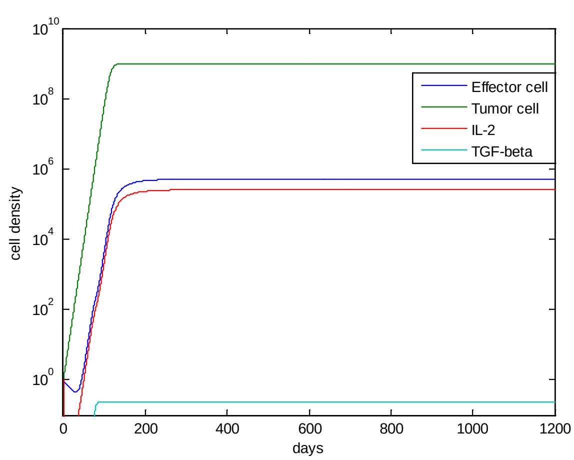

Fig 1(a)

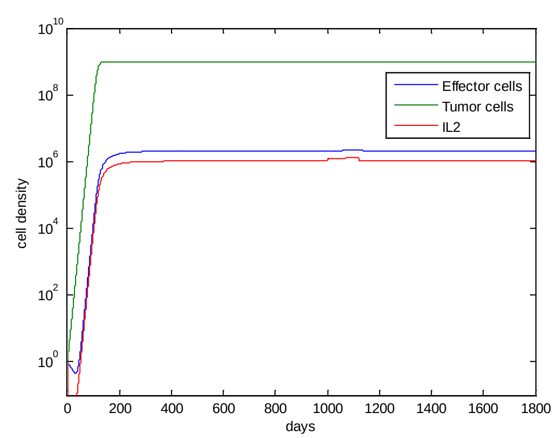

Fig 1(b)

Fig. 1(a) shows the growth of tumor cells, effector cells and il2 growth without giving external therapies like il2 dose, chemotherapy and adoptive immunotherapy. In (b) no chemotherapy and no adoptive immunotherapy are given but a heavy dose of il2, S_2=2000000 started at 1000th day and then given for 120 days. There is not such change seen in cell densities.

a) Effect of chemotherapy

There are a number of strategies in the administration of chemotherapeutic drugs used today. Chemotherapy may be given with a curative intent or it may aim to prolong life or to palliate symptoms [25]. As we know that effector cell density is dependent on il2 level, if il2 level falls, effector cell density also drops. So, we kept chemotherapy schedule not only time dependant but also il2 dependant. Once the chemotherapy starts at some fixed time interval (time dependant) then the therapy is scheduled by il2 level (il2 dependent). This means that chemotherapy schedule is managed such as to keep effector cell density under control. A MatLab code is given such that chemotherapy starts (V =1) when z > required il2 level and chemotherapy stops (V=0) when z < the required il2 level. By keeping such we can know that at what time the chemotherapy should be started and in what intervals it should be given so as not to effect il2 level and effector cells too. Schedules and effects of chemotherapy doses are shown in following figures. The chemotherapy schedule, tumour density, effector cell density and il2 level are well observed from the simulations and then explained.

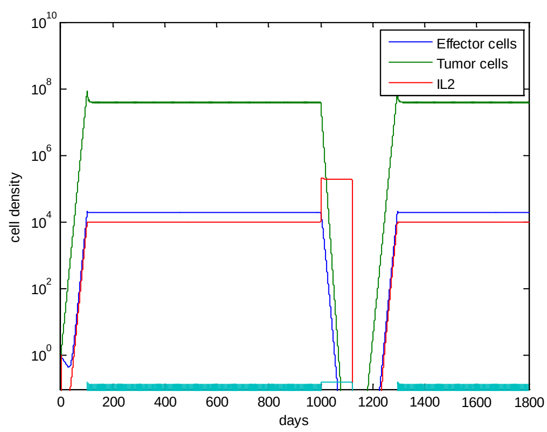

Fig 2(a)

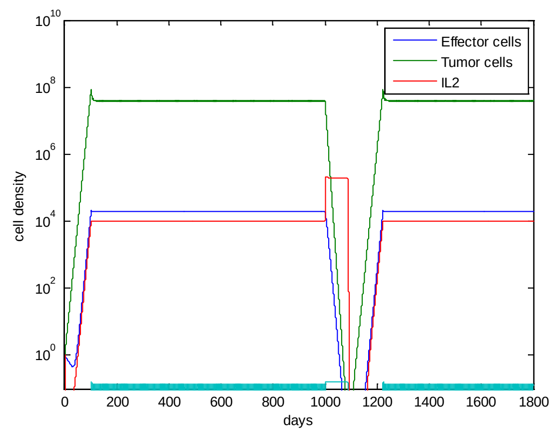

Fig 2(b)

Fig 2. The above figures show that proper and scheduled application of chemotherapy to the patient can be very effective. In fig 2(b) the dose of il2 scheduled without chemotherapy does nothing whereas in fig 2 the same dose of il2 scheduled with chemotherapy brings a big change. As the frequent chemotherapy doses are effective in treatment of tumors, (discussed in *American Cancer Society* [26]) So, here Chemotherapy is started from 108th day and is given after each 21 hours intervals such that il2 $\geq$10000, on the other hand tumour cell density comes below to its carrying capacity to about 39950000 cells. This keeps the effector cells density also stable. The il2 dose of S_2= 200000 (which is less than the standard dose of 600,000 unit and high dose of 720,000 unit [28] [29]) is given from 1000^{th} day for (a) 120 days and (b) 90 days. We see a sudden fall of tumor growth and finally at 1080^{th} day the tumor gets eliminated. As soon as il2 therapy stops the tumour vigorously grows but during this time when the tumour is eradicated there is lot of time for a patient to undergo other treatments including dietary regulations so as to avoid further growth of tumor.

Ref

25. “Chemotherapy Principles-An In-depth Discussion of the Techniques and Its Role in Cancer Treatment”. ***American Cancer Society.*** Retrieved from http://www.cancer.org/acs/groups/cid/documents/webcontent/002995-pdf.pdf

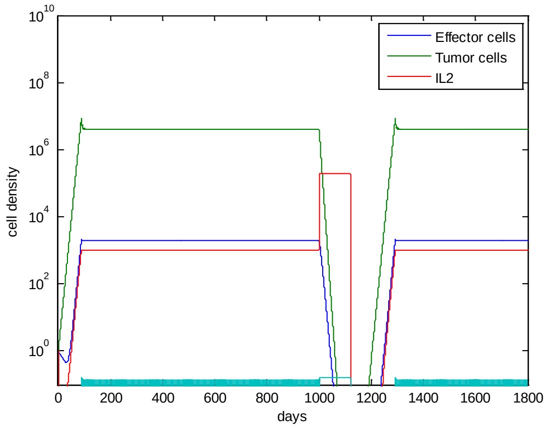

Fig 3(a)

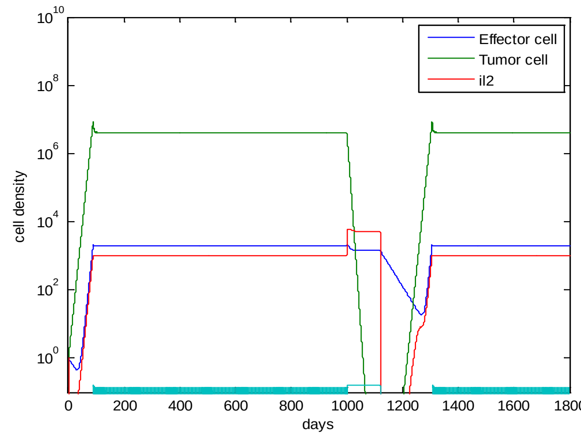

Fig 3(b)

Fig 3. Chemotherapy starts from 90th day. Here we have given chemotherapy after 18 hours intervals such that il2 ≥ 1000 which brings tumour cell density to 4154000 cells. This keeps the effector cells density also stable. The il2 dose of S_2= 200000 is given from 1000th day for 120 days. We see a sudden fall of tumour growth and finally at 1070th day tumour gets eliminated. As soon il2 dose stops the tumour vigorously grows. But this is lot time for other treatment including dietary regulations to stop further tumour growth. Fig (a) shows eradication of tumour cells with effector cells. In (b) il2 dose therapy is companied with adoptive immunotherapy of S_1=500. We see recovery of effector cell growth after a down fall. This is one of the good ways to eliminate tumor cell keeping effector cells alive.

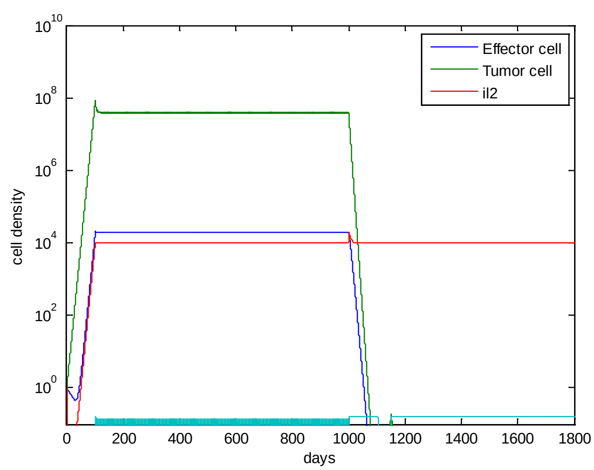

Fig 4

Fig 4. Here all the parameters are kept same as in fig 2 except reduced dose of il2 of S_2= 100000. This shows long term eradication is possible with application of long term adoptive therapy. But this is quite unrealistic.

V. Conclusion and Future Work

In this study various treatments with scheduled time and dose were discussed. It is clear from above discussion that for reducing tumour growth chemotherapy should be started as soon as the tumour is detected and chemotherapy should be scheduled frequently within few hours along with low doses of il2 so that the effector cells and also the host body gets frequent recovery time, which as a result does not harm the body by after effects of chemotherapy and il2 therapy. Also when the effector cell density drops to its half then the therapies should be companied with short term adoptive immunotherapy which may eliminate the tumour. Different types of chemotherapy drugs kill cells at different stages of the cell cycle. Administering the specific drugs when the cancer cells are most sensitive and other cells are less sensitive makes them more effective. Some types of chemotherapy need to be activated in the body before they affect cancer cells; administering them at times when this will happen efficiently also improves effect. The clinicians and nutritionist, has the work to find out various ways to avoid tumor, once eradicated for further evolution.

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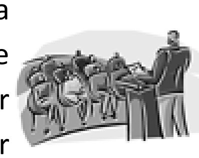

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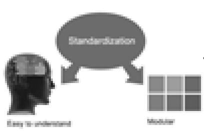

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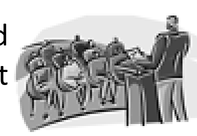
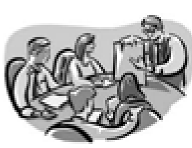
The Institute will be entitled to following benefits:

The IBOARS can initially review research papers of their institute and recommend them to publish with respective journal of Global Journals. It can also review the papers of other institutions after obtaining our consent. The second review will be done by peer reviewer of Global Journals Incorporation (USA) The Board is at liberty to appoint a peer reviewer with the approval of chairperson after consulting us.
The author fees of such paper may be waived off up to 40%.



The Global Journals Incorporation (USA) at its discretion can also refer double blind peer reviewed paper at their end to the board for the verification and to get recommendation for final stage of acceptance of publication.



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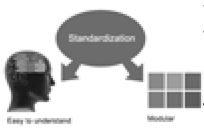
The Board can also play vital role by exploring and giving valuable suggestions regarding the Standards of "Open Association of Research Society, U.S.A (OARS)" so that proper amendment can take place for the benefit of entire research community. We shall provide details of particular standard only on receipt of request from the Board.

The board members can also join us as Individual Fellow with 40% discount on total fees applicable to Individual Fellow. They will be entitled to avail all the benefits as declared. Please visit Individual Fellow-sub menu of GlobalJournals.org to have more relevant details.

We shall provide you intimation regarding launching of e-version of journal of your stream time to time. This may be utilized in your library for the enrichment of knowledge of your students as well as it can also be helpful for the concerned faculty members.



After nomination of your institution as "Institutional Fellow" and constantly functioning successfully for one year, we can consider giving recognition to your institute to function as Regional/Zonal office on our behalf.

The board can also take up the additional allied activities for betterment after our consultation.

The following entitlements are applicable to individual Fellows:

Open Association of Research Society, U.S.A (OARS) By-laws states that an individual Fellow may use the designations as applicable, or the corresponding initials. The Credentials of individual Fellow and Associate designations signify that the individual has gained knowledge of the fundamental concepts. One is magnanimous and proficient in an expertise course covering the professional code of conduct, and follows recognized standards of practice.

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We shall provide print version of 12 issues of any three journals [as per your requirement] out of our 38 journals worth $ 2376 USD.

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- Fellow can also join as paid peer reviewer and earn 15% remuneration of author charges and can also get an opportunity to join as member of the Editorial Board of Global Journals Incorporation (USA)
- • This individual has learned the basic methods of applying those concepts and techniques to common challenging situations. This individual has further demonstrated an in–depth understanding of the application of suitable techniques to a particular area of research practice.

Note :

"

- In future, if the board feels the necessity to change any board member, the same can be done with the consent of the chairperson along with anyone board member without our approval.
- In case, the chairperson needs to be replaced then consent of 2/3rd board members are required and they are also required to jointly pass the resolution copy of which should be sent to us. In such case, it will be compulsory to obtain our approval before replacement.
- In case of "Difference of Opinion [if any]" among the Board members, our decision will be final and binding to everyone.

"

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The Area or field of specialization may or may not be of any category as mentioned in 'Scope of Journal' menu of the GlobalJournals.org website. There are 37 Research Journal categorized with Six parental Journals GJCST, GJMR, GJRE, GJMBR, GJSFR, GJHSS. For Authors should prefer the mentioned categories. There are three widely used systems UDC, DDC and LCC. The details are available as 'Knowledge Abstract' at Home page. The major advantage of this coding is that, the research work will be exposed to and shared with all over the world as we are being abstracted and indexed worldwide.

The paper should be in proper format. The format can be downloaded from first page of 'Author Guideline' Menu. The Author is expected to follow the general rules as mentioned in this menu. The paper should be written in MS-Word Format (*.DOC,*.DOCX).

The Author can submit the paper either online or offline. The authors should prefer online submission.Online Submission: There are three ways to submit your paper:

(A) (I) First, register yourself using top right corner of Home page then Login. If you are already registered, then login using your username and password.

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1. General,

2. Ethical Guidelines,

3. Submission of Manuscripts,

4. Manuscript’s Category,

5. Structure and Format of Manuscript,

6. After Acceptance.

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Research articles: These are handled with small investigation and applications

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The recommended size of original research paper is less than seven thousand words, review papers fewer than seven thousands words also.Preparation of research paper or how to write research paper, are major hurdle, while writing manuscript. The research articles and research letters should be fewer than three thousand words, the structure original research paper; sometime review paper should be as follows:

Papers: These are reports of significant research (typically less than 7000 words equivalent, including tables, figures, references), and comprise:

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(b) A brief Summary, "Abstract" (less than 150 words) containing the major results and conclusions.

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(e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition; sources of information must be given and numerical methods must be specified by reference, unless non-standard.

(f) Results should be presented concisely, by well-designed tables and/or figures; the same data may not be used in both; suitable statistical data should be given. All data must be obtained with attention to numerical detail in the planning stage. As reproduced design has been recognized to be important to experiments for a considerable time, the Editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned un-refereed;

(g) Discussion should cover the implications and consequences, not just recapitulating the results; conclusions should be summarizing.

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It is vital, that authors take care in submitting a manuscript that is written in simple language and adheres to published guidelines.

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Language: The language of publication is UK English. Authors, for whom English is a second language, must have their manuscript efficiently edited by an English-speaking person before submission to make sure that, the English is of high excellence. It is preferable, that manuscripts should be professionally edited.

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References

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23. Multitasking in research is not good: Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. Think technically: Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page
- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.

- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

Administration Rules Listed Before Submitting Your Research Paper to Global Journals Inc. (US)

Please carefully note down following rules and regulation before submitting your Research Paper to Global Journals Inc. (US):

Segment Draft and Final Research Paper: You have to strictly follow the template of research paper. If it is not done your paper may get rejected.

- The **major constraint** is that you must independently make all content, tables, graphs, and facts that are offered in the paper. You must write each part of the paper wholly on your own. The Peer-reviewers need to identify your own perceptive of the concepts in your own terms. NEVER extract straight from any foundation, and never rephrase someone else's analysis.
- Do not give permission to anyone else to "PROOFREAD" your manuscript.
- Methods to avoid Plagiarism is applied by us on every paper, if found guilty, you will be blacklisted by all of our collaborated research groups, your institution will be informed for this and strict legal actions will be taken immediately.)
- To guard yourself and others from possible illegal use please do not permit anyone right to use to your paper and files.

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Topics	Grades		
	A-B	C-D	E-F
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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