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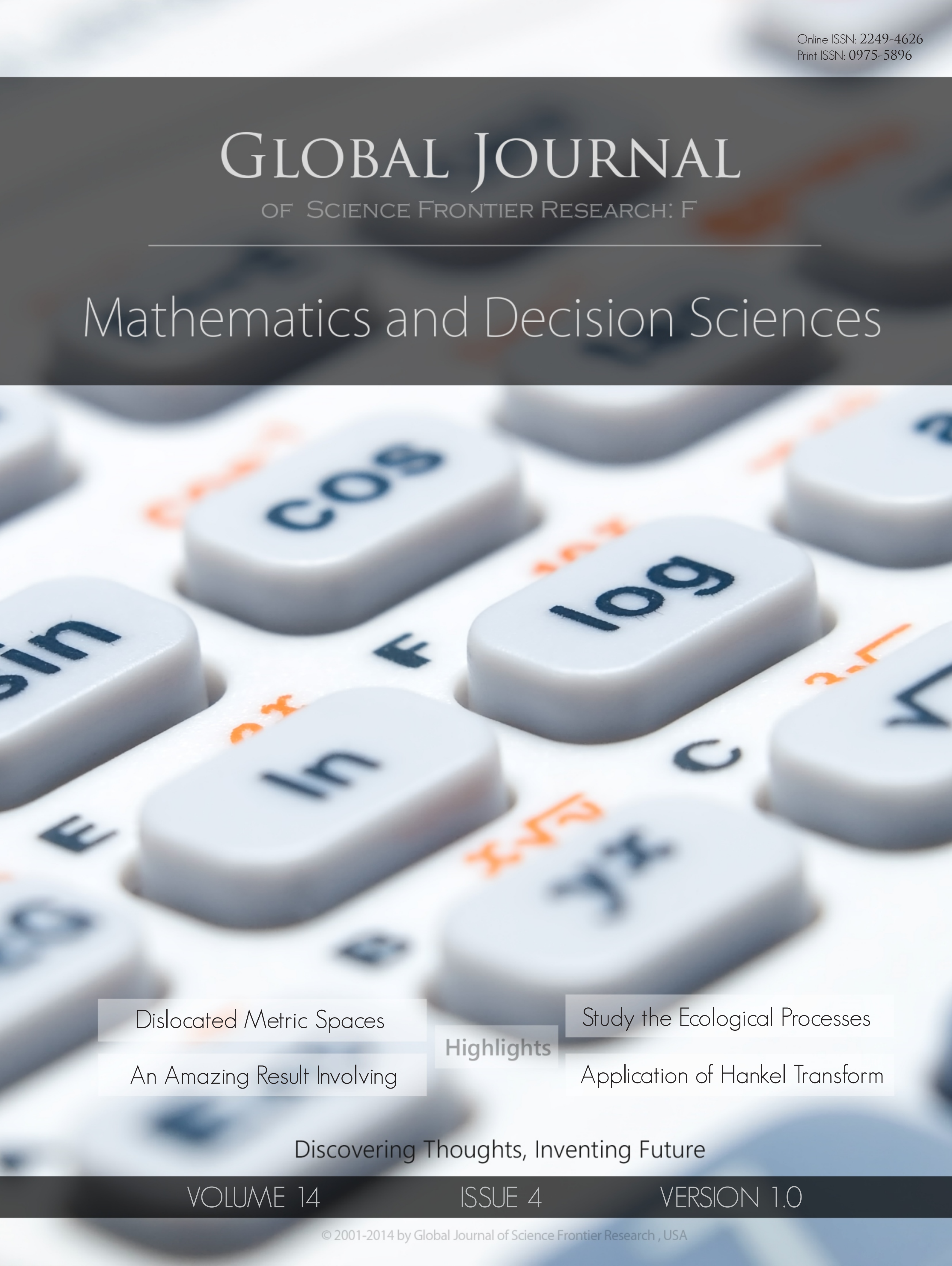

Highlights

Dislocated Metric Spaces

An Amazing Result Involving

Study the Ecological Processes

Application of Hankel Transform

Discovering Thoughts, Inventing Future

VOLUME 14 ISSUE 4 VERSION 1.0

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A Summation Formula in the Light of Special Functions

By Salahuddin, M. P. Chaudhary & Sangeeta Chaudhary

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Abstract- The main objective of the present paper is to derive a summation formula involving certain special functions.

Keywords: *gauss second summation theorem, recurrence relation, prudnikov.*

GJSFR-F Classification : *MSC 2010: 40A25, 11B37*

ASUMMATIONFORMULAINTHELIGHTOFSPECIALFUNCTIONS

Strictly as per the compliance and regulations of :

Notes

A Summation Formula in the Light of Special Functions

Salahuddin [α], M. P. Chaudhary [σ] & Sangeeta Chaudhary [ρ]

Abstract- The main objective of the present paper is to derive a summation formula involving certain special functions.

2010 MSC NO : 33C05 , 33C20 , 33D15 , 33D50 , 33D60

Keywords: gauss second summation theorem, recurrence relation, prudnikov.

I. Introduction

a) *Generalized Hypergeometric Functions*

A generalized hypergeometric function ${}_pF_q(a_1, ..., a_p; b_1, ..., b_q; z)$ is a function which can be defined in the form of a hypergeometric series, i.e., a series for which the ratio of successive terms can be written

$$\frac{c_{k+1}}{c_k} = \frac{P(k)}{Q(k)} = \frac{(k+a_1)(k+a_2)...(k+a_p)}{(k+b_1)(K+b_2)...(k+b_q)(k+1)}\ z. \tag{1}$$

Where $k+1$ in the denominator is present for historical reasons of notation[Koepf p.12(2.9)], and the resulting generalized hypergeometric function is written

$${}_pF_q\left[\begin{array}{ccc} a_1, a_2, \cdots, a_p & ; & \\ & & z \\ b_1, b_2, \cdots, b_q & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_p)_k z^k}{(b_1)_k (b_2)_k \cdots (b_q)_k k!} \tag{2}$$

or

$${}_pF_q\left[\begin{array}{ccc} (a_p) & ; & \\ & & z \\ (b_q) & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{((a_p))_k z^k}{((b_q))_k k!} \tag{3}$$

where the parameters $b_1, b_2, \cdots, b_q$ are positive integers.

The ${}_pF_q$ series converges for all finite z if $p \leq q$, converges for $|\ z\ | < 1$ if $p = q + 1$, diverges for all z , $z \neq 0$ if $p > q + 1$[Luke p.156(3)].

The function ${}_2F_1(a, b; c; z)$ corresponding to $p = 2, q = 1$, is the first hypergeometric function to be studied (and, in general, arises the most frequently in physical problems), and so is frequently known as "the" hypergeometric equation or, more explicitly, Gauss's hypergeometric function

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[Gauss p.123-162]. To confuse matters even more, the term "hypergeometric function" is less commonly used to mean closed form, and "hypergeometric series" is sometimes used to mean hypergeometric function.

The hypergeometric functions are solutions of Gaussian hypergeometric linear differential equation of second order

$$z(1-z)y'' + [c-(a+b+1)z]y' - aby = 0 \tag{4}$$

The solution of this equation is

$$y = A_0\left[1 + \frac{ab}{1!\ c}z + \frac{a(a+1)b(b+1)}{2!\ c(c+1)}z^2 + \cdots\cdots\right] \tag{5}$$

This is the so-called regular solution, denoted

$${}_2F_1(a,b;c;z) = \left[1 + \frac{ab}{1!\ c}z + \frac{a(a+1)b(b+1)}{2!\ c(c+1)}z^2 + \cdots\cdots\right] = \sum_{k=0}^{\infty}\frac{(a)_k\ (b)_k z^k}{(c)_k k!} \tag{6}$$

which converges if c is not a negative integer for all $|\ z\ | < 1$ and on the unit circle $|\ z\ | = 1$ if $R(c-a-b) > 0$.

It is known as Gauss hypergeometric function in terms of Pochhammer symbol $(a)_k$ or generalized factorial function.

Many of the common mathematical functions can be expressed in terms of the hypergeometric function. Some typical examples are

$$(1-z)^{-a} = z\ {}_2F_1(1,1;2;-z) \tag{7}$$

$$\sin^{-1} z = z\ {}_2F_1(\frac{1}{2},\frac{1}{2};\frac{3}{2};z^2) \tag{8}$$

Gauss' Relations for Contiguous Functions:

The six functions $F(a\pm1,b;c;z)$, $F(a,b\pm1;c;z)$, $F(a,b;c\pm1;z)$ are called contiguous to $F(a,b;c;z)$. Relation between $F(a,b;c;z)$ and any two contiguous functions have been given by Gauss. [Abramowitz p.558(15.2.14)]

$$(a-b)\ {}_2F_1\left[\begin{matrix} a,\ b\ ; \\ c\ \ ; \end{matrix}\quad z\right] = a\ {}_2F_1\left[\begin{matrix} a+1,\ b\ \ ; \\ c\ \ \ ; \end{matrix}\quad z\right] - b\ {}_2F_1\left[\begin{matrix} a,\ b+1\ \ ; \\ c\ ; \end{matrix}\quad z\right] \tag{9}$$

Gauss second summation theorem is defined by [Prudnikov., 491(7.3.7.5)]

$${}_2F_1\left[\begin{matrix} a,\ b\ ; \\ \frac{a+b+1}{2}\ ; \end{matrix}\quad \frac{1}{2}\right] = \frac{\Gamma(\frac{a+b+1}{2})\ \Gamma(\frac{1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} \tag{10}$$

$$= \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{a+b+1}{2})}{\Gamma(b)\ \Gamma(\frac{a+1}{2})} \tag{11}$$

In a monograph of Prudnikov et al., a summation theorem is given in the form [Prudnikov., p.491(7.3.7.8)]

$${}_2F_1\left[\begin{matrix} a,\ b\ \ ; \\ \frac{a+b-1}{2}\ ; \end{matrix}\quad \frac{1}{2}\right] = \sqrt{\pi}\left[\frac{\Gamma(\frac{a+b+1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} + \frac{2\ \Gamma(\frac{a+b-1}{2})}{\Gamma(a)\ \Gamma(b)}\right] \tag{12}$$

Notes

Now using Legendre's duplication formula and Recurrence relation for Gamma function, the above theorem can be written in the form

$$
{}_2F_1\left[\begin{matrix} a, \; b & ; & \frac{1}{2} \\ \frac{a+b-1}{2} & ; & \end{matrix}\right] = \frac{2^{(b-1)}\,\Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} + \frac{2^{(a-b+1)}\,\Gamma(\frac{a}{2})\,\Gamma(\frac{a+1}{2})}{\{\Gamma(a)\}^2} + \frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\right] \tag{13}
$$

Notes

Recurrence relation is defined by

$$
\Gamma(\zeta+1) = \zeta\,\Gamma(\zeta) \tag{14}
$$

II. Main Summation Formula

$$
{}_2F_1\left[\begin{matrix} a, \; b \; ; & \frac{1}{2} \\ \frac{a+b+47}{2} \; ; & \end{matrix}\right] = \frac{2^b\,\Gamma(\frac{a+b+47}{2})}{(a-b)\,\Gamma(b)\Big[\prod_{\Xi=1}^{23}\{a-b-(2\Xi-1)\}\Big]\Big[\prod_{\Upsilon=1}^{23}\{a-b+(2\Upsilon-1)\}\Big]}
$$

$$
\Bigg[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+1}{2})}\Big\{4194304(a^{24}+a^{23}(-529+1034b)+23a^{22}(5731-7567b+7050b^2)+253a^{21}(-81305+176908b
$$

$$
-61617b^2+36378b^3)+253a^{20}(8912547-153015555b+150871141b^2-2370633b^3+994332b^4)
$$

$$
+253a^{19}(-729562323+16756420026b-883014850b^2+537100960b^3-47645075b^4+15080702b^5)
$$

$$
+437a^{18}(26795032319-518265029991b+526751872208b^2-13398724750b^3+5687164735b^4
$$

$$
-322080707b^5+79712282b^6)+437a^{17}(-1349398815215+3201026487568b-2051983327545b^2
$$

$$
+1265119609782b^3-188817432225b^4+59627107044b^5-2326985463b^6+459878550b^7)
$$

$$
+7429a^{16}(3224471854274-6691057943825b+6910962796281b^2-2224676333517b^3+944086166793b^4
$$

$$
-92136046035b^5+22451270811b^6-634632399b^7+101173281b^8)+874a^{15}(-907619931345281
$$

$$
+2196439780017778b-1583061524809980b^2+978474032814608b^3-189247186989894b^4
$$

$$
+58979505572724b^5-4023569409660b^6+771418674912b^7-16288898241b^8+2090914474b^9)
$$

$$
+46a^{14}(468566121314958303-1017824993014405123b+1057739894293651278b^2
$$

$$
-392620567074002300b^3+164887334296881130b^4-21145413880972458b^5+5011088841406536b^6
$$

$$
-248923981301100b^7+37710822893535b^8-601137911275b^9+60636519746b^{10})
$$

$$
+46a^{13}(-10480799815306879695+25650191726385965388b-19959681138199929643b^2
$$

$$
+12250081720114143878b^3-2771196216002273550b^4+842260723772929616b^5
$$

$$
-75877233063158814b^6+13859663123661012b^7-509900689579875b^8+59821063101140b^9
$$

$$
-697319977079b^{10}+49611697974b^{11})-46a^{12}(-193076196366405273361+432242928907257686205b
$$

$$
-447986443019768183583b^2+182383159014771602043b^3-74937694261890414058b^4
$$

$$
+11295990811521823810b^5-2557424283239432542b^6+166286552352776310b^7
$$

$$
-23083684967418933b^8+611932257247025b^9-50055447050323b^{10}+322476036831b^{11})
$$

$$
-46a^{11}(2922906805676735359009-7179158161708177901366b+5874213600004333972050b^2
$$

$$
-3541062392770959869904b^3+883384336805408600591b^4-257296997289990731502b^5
$$

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$$+27004281748591247100b^6 - 4532263566070187376b^7 + 2082811939283596695b^8$$
$$-19867377967046610b^9 + 287720286194770b^{10} - 322476036831b^{12} + 49611697974b^{13})$$
$$-506a^{10}(-3289323182409779776423 + 7496159121197134500799b - 7670199805287410414916b^2$$
$$+3305843522579583360570b^3 - 1306390804960229398137b^4 + 215279129671865805533b^5$$
$$-44918367769994723810b^6 + 3246913411866939100b^7 - 367469311611014041b^8$$
$$+9089528264764233b^9 - 26156389654070b^{11} + 4550495186393b^{12} - 633927251189b^{13} + 5512410886b^{14})$$
$$-506a^9(33029344888387937299735 - 80716238290121139577160b + 67908675762513429453765b^2$$
$$-39548924300013894311222b^3 + 10359881175427248692175b^4 - 2790368186666261220084b^5$$
$$+305536664589145308709b^6 - 41928657064513848538b^7 + 1600126853018504525b^8$$
$$-9089528264764233b^{10} + 1806125269731510b^{11} - 55630205204275b^{12} + 5438278463740b^{13}$$
$$-546489010025b^{14} + 36115795546b^{15}) - 23a^8(-5849633469739333659008547$$
$$+13394416074900510289638710b - 13322547414950238766996350b^2$$
$$+5862527517289620649481846b^3 - 2151132892496025940533226b^4 + 355319051526265101386670b^5$$
$$-60814387504205828165478b^6 + 3426198156496859682510b^7 - 35202790766407099550b^9$$
$$+8084324855442308902b^{10} - 416562387856719390b^{11} + 46167369934837866b^{12}$$
$$-1019801379159750b^{13} + 75421645787070b^{14} - 618978133158b^{15} + 32678969763b^{16})$$
$$-23a^7(37134106531137095279787603 - 89101399473246500803885710b$$
$$+74961158813563163662059560b^2 - 40750101410951355053076768b^3$$
$$+10409234708209993517924372b^4 - 2308469208507693358304248b^5$$
$$+189319585005743733067800b^6 - 3426198156496859682510b^8 + 922430455419304667836b^9$$
$$-71432095061072660200b^{10} + 9064527132140374752b^{11} - 332573104705552620b^{12}$$
$$+27719326247322024b^{13} - 497847962602200b^{14} + 29313909646656b^{15} - 204986264877b^{16}$$
$$+8737692450b^{17}) - 23a^6(-181636507550329303114786671 + 409524468899085745553896155b$$
$$-383166283662043789339491690b^2 + 161178272588675143185772280b^3$$
$$-48924876796158281140066500b^4 + 5890553184666820563182180b^5$$
$$-189319585005743733067800b^7 + 60814387504205828165478b^8 - 6721806620961196791598b^9$$
$$+988204090939883923820b^{10} - 54008563497182494200b^{11} + 5114848566478865084b^{12}$$
$$-151754466126317628b^{13} + 10022177682813072b^{14} - 152895637567080b^{15} + 7251760471953b^{16}$$
$$-442127237797b^{17} + 15145333358b^{18}) - 23a^5(662329119693726610533057375$$
$$-1516371020452226580169089924b + 1202265485968967736276728367b^2$$
$$-544218572588615646324600054b^3 + 99386937441957904221828620b^4$$
$$-5890553184666820563182180b^6 + 2308469208507693358304248b^7$$
$$-355319051526265101386670b^8 + 61388100106657746841848b^9 - 4736140852781047721726b^{10}$$
$$+514593994579981463004b^{11} - 22591981623043647620b^{12} + 1684521447545859232b^{13}$$

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$$-42290827761944916b^{14}+2241221211763512b^{15}-297599428693305b^{16}+1132915033836b^{17}$$

$$-6119533433b^{18}+165887722b^{19})-23a^4(-1714166473560132461916610125$$

$$+36186383784320522007506219925b-2849793884317255245197831595b^2$$

$$+845506576871181679688191719b^3-99386937441957904221828620b^5$$

$$+48924876796158281140066500b^6-10409234708209993517924372b^7$$

$$+2151132892496025940533226b^8-227917385859399471227850b^9+28740597709125046759014b^{10}$$

$$-1766768673610817201182b^{11}+149875388523780828116b^{12}-5542392432004547100b^{13}$$

$$+329774668593762260b^{14}-7191393105615972b^{15}+304939831874139b^{16}-3587531212275b^{17}$$

$$+108056129965b^{18}-5240958825b^{19}+10937652b^{20})-23a^3(2910996690958124489461348125$$

$$-5746357719280004177264775750b+3191889854580266346156760950b^2$$

$$-845506576871181679688191719b^4+544218572588615646324600054b^5$$

$$-161178272588675143185772280b^6+40750101410951355053076768b^7$$

$$-5862527517289620649481846b^8+8700763346003056748468884b^9-72728557496750833932540b^{10}$$

$$+7082124785541919739808b^{11}-364766318029543204086b^{12}+24500163440228287756b^{13}$$

$$-785241134148004600b^{14}+37182013246955104b^{15}-718570455725991b^{16}+24037272585858b^{17}$$

$$-254575770250b^{18}+5908110560b^{19}-26076963b^{20}+400158b^{21})$$

$$+a^2(64691533780014247694392179375-96833027583613230625570719375b$$

$$+73413466655346125961605501850b^3-65545259339296870639550126685b^4$$

$$+27652106177286257934364752441b^5-8812824524227007154808308870b^6$$

$$+1724106652711952764227369880b^7-306418590543855491640916050b^8$$

$$+34361789935831795303605090b^9-3881121101475429669947496b^{10}$$

$$+270213825600199362714300b^{11}-20607376378909336444818b^{12}+918145332357196763578b^{13}$$

$$-48656035137507958788b^{14}+1383595772683922520b^{15}-51341542613571549b^{16}$$

$$+896716714137165b^{17}-23019056809896b^{18}+223402757050b^{19}-3817046673b^{20}+15589101b^{21}$$

$$-162150b^{22})+a(-25373791335626257947657609375+96833027583613230625570719375b^2$$

$$-132166227543440096077089842250b^3+83228682703937200617264304275b^4$$

$$-34876533470401211343889068252b^5+9419062784678972147739611565b^6$$

$$-2049332187884669518489371330b^7+308071569722711736661690330b^8$$

$$-40842416574801296626042960b^9+3793056515325750057404294b^{10}$$

$$-330241275438576183462836b^{11}+19883174729733853565430b^{12}-1179908819413754407848b^{13}$$

$$+46819949678662635658b^{14}-1919688367735537972b^{15}+49707869464675925b^{16}$$

$$-1398848575067216b^{17}+22648181807067b^{18}-423937432578b^{19}+3871293415b^{20}-447577724b^{21}$$

$$+174041b^{22}-1034b^{23})-b(-25373791335626257947657609375+64691533780014247694392179375b$$

$$-66952923892036863257611006875b^2+39425828891883046624082032875b^3$$

$$-15233569752955712042260319625b^4+41776396736575739716400934333b^5$$

$$-854084450216153191435114869b^6+134541569804004674157196581b^7$$

$$-16712848513524296273665910b^8+1664397530299348566870038b^9$$

$$-134453713061129826514414b^{10}+8881505032854642574606b^{11}-482116791504116465970b^{12}$$

$$+21554041580488081938b^{13}-793259819995775594b^{14}+23954601405401546b^{15}$$

$$-589687282248955b^{16}+11709429123403b^{17}-184579267719b^{18}+2254874391b^{19}-20570165b^{20}$$

$$+131813b^{21}-529b^{22}+b^{23}))\Big\}-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\Big\{16777216(23a^{23}+23a^{22}(-176+329b)$$

$$+253a^{21}(5215-3008b+2679b^2)+253a^{20}(-452960+879605b-182736b^2+103071b^3)$$

$$+253a^{19}(60311301-52865600b+47979950b^2-5173760b^3+2071525b^4)+437a^{18}(-1866667344$$

$$+3685773417b-1241401600b^2+703813250b^3-46402160b^4+14003509b^5)+437a^{17}(136919748025$$

$$-147120185088b+134215406415b^2-24378110400b^3+9700951575b^4-429551424b^5+101173281b^6)$$

$$+7429a^{16}(-288016551040+5718590666695b-244761992496b^2+138065017179b^3-15710445600b^4$$

$$+4662174165b^5-147161136b^6+27592713b^7)+874a^{15}(110646186438247-134786297605760b$$

$$+122504471439780b^2-28930053259008b^3+11338806793578b^4-879944046720b^5+201426804900b^6$$

$$-4709156352b^7+708212967b^8)+46a^{14}(-52099555819934768+103225068855282101b$$

$$-51223393194057600b^2+28510225961592100b^3-4278910526304480b^4+1236471420894246b^5$$

$$-68930276102400b^6+12367973723700b^7-218534286960b^8+26136430925b^9)$$

$$+46a^{13}(1523275998062398905-2020689131217706688b+1815921091026075341b^2$$

$$-504115076937750400b^3+192797827726790850b^4-19872507249052416b^5+4362411218030418b^6$$

$$-179742611510400b^7+25204793629125b^8-334546315840b^9+30318259873b^{10})$$

$$+26a^{12}(-46738665698539844640+91784298587795117985b-50292357649927039888b^2$$

$$+27376830740023687911b^3-4861870680816380800b^4+1350325066901821930b^5$$

$$-99722984753260512b^6+16721579297625870b^7-508861134117600b^8+53344455517925b^9$$

$$-485092157968b^{10}+24805848987b^{11})+a^{11}(23337757655538605356418-32855399221957698072960b$$

$$+28952254818229910029668b^2-8932261695719974004224b^3+3291278251514442174622b^4$$

$$-400249492825308222720b^5+82302941585946050424b^6-4388608849494744576b^7$$

$$+540688988712851550b^8-11125003186943360b^9+651114949032548b^{10}-644952073662b^{12})$$

$$-22a^{10}(12747810085874924284144-24623395110033406407199b+14414748449268814208640b^2$$

$$-7580198303956038791706b^3+1491441502941101280816b^4-388981529590737600413b^5$$

$$+33102061417630424960b^6-4888486075362902428b^7+173245521378051408b^8$$

$$-11757702406746153b^9+29596134046934b^{11}-573290732144b^{12}+63392725189b^{13})$$

$$-22a^9(-159174616402628497304455+233000874791703045269120b-198962130903196818972645b^2$$

$$+65380182773479633225920b^3-22683573168494062454175b^4+2984544052387419084992b^5$$

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$$
\begin{aligned}
&-5418708301593339957 49b^{6}+29956051044652120960b^{7}-239523653933659 4525b^{8}\\
&+117577024067461 53b^{10}-505681963042880b^{11}+630434474302 75b^{12}-6995059331 20b^{13}\\
&+546489010 25b^{14})-2a^{8}(14135533727749076201623360-2655946442407824474196871 5b\\
&+1612124389809876002742744 0b^{2}-800692167718156613245084 3b^{3}\\
&+16385936016992051901768 00b^{4}-3783592680021002122460 55b^{5}+313399433430451745039 52b^{6}\\
&-30113940382190972886 15b^{7}+26347601932702539 775b^{9}-190570073515856 5488b^{10}\\
&+2703444943564257 75b^{11}-661519474352880 0b^{12}+57971025346987 5b^{13}-502628860008 0b^{14}\\
&+3094890665 79b^{15})+a^{7}(2208793196453847886378068 03-3288394088828426491835609 60b\\
&+2661862282876559483733725 84b^{2}-885738064564245700637475 84b^{3}\\
&+272893414277804823758142 92b^{4}-335967493112619588185472 0b^{5}\\
&+39795844635425001063439 2b^{6}-602278807643819457723 0b^{8}+65903312298234666112 0b^{9}\\
&-10754669365798385341 6b^{10}+4388608849494744457 6b^{11}-43476106173827262 0b^{12}\\
&+826816012947840 0b^{13}-56892679129020 0b^{14}+4115802651 648b^{15}-2049862648 77b^{16})\\
&+a^{6}(-1125939521154262717909258128+2015056862560521162160964955b\\
&-1215154200556285221697944320b^{2}+537848809932380165002052216b^{3}\\
&-100274479066385754433112640b^{4}+15150737223163209692921060b^{5}\\
&-397958446354250010634392b^{7}+62679886686090349007904b^{8}-1192115826350534790647 8b^{9}\\
&+728245351187869349120b^{10}-82302941585946050424b^{11}+2592797603584773312b^{12}\\
&-200670916029399228b^{13}+3170792700710400b^{14}-176047027482600b^{15}+10932600793 44b^{16}\\
&-44212723797b^{17})+a^{5}(506696847852529405565837 0415-7393789471350456781405463232b\\
&+5357661269531597371164089007b^{2}-1593656779593915351419429120b^{3}\\
&+322387545425775413154961100b^{4}-15150737223163209692921060b^{6}\\
&+3359674931126195881854720b^{7}-75671853600420042449211 0b^{8}+65659969152523219869824b^{9}\\
&-8557593650996227209086b^{10}+400249492825308222720b^{11}-35108451739447370180b^{12}\\
&+914135333456411136b^{13}-56877685361135316b^{14}+769071096833280b^{15}-34635291871785b^{16}\\
&+187713972288b^{17}-6119533433b^{18})+a^{4}(-1441167509289801442673939760 0\\
&+2323472231289630313380266536 5b-1235874724062886664113237704 0b^{2}\\
&+360693377625483521374112547 9b^{3}-32238754542577541315496110 0b^{5}\\
&+10027447906638575443311264 0b^{6}-27289341427780482375814292b^{7}\\
&+3277187203398410380353600b^{8}-499038609706869373991850b^{9}+32811713064704228177952b^{10}\\
&-3291278251514442174622b^{11}+126408637701225900800b^{12}-8868700075432379100b^{13}\\
&+196829884210006080b^{14}-9910117137587172b^{15}+116712900362400b^{16}-4239315838275b^{17}\\
&+20277743920b^{18}-524095825b^{19})+a^{3}(31077234776438508516718729725\\
&-3967288204421512256708630304 0b+1906606151437531286390544667 8b^{2}
\end{aligned}
$$

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$$
\begin{aligned}
&-3606933776254835213741125479b^4+15936567795939153514194291 20b^5\\
&-5378488099323801650020522 16b^6+885738064564245700637475 84b^7\\
&-1601384335436313226490168 6b^8+143836402101655193097024 0b^9-1667643626870328534175 32b^{10}\\
&+89322616957199740042 24b^{11}-71179759924061588568 6b^{12}+23189293539136518400b^{13}\\
&-1311470394233236600b^{14}+252848665483729 92b^{15}-1025685012622791b^{16}+10653234244800b^{17}\\
&-307566390250b^{18}+1308961280b^{19}-26076963b^{20})+a^2(-3629224088279214826325689 8000\\
&+391685584588234741575464132 25b-19066061514375312863905446678b^3\\
&+12358747240628866641132377040b^4-53576612695315973711640890 07b^5\\
&+12151542005562852216979443 20b^6-2661862282876559483733725 84b^7\\
&+32242487796197520054854880b^8-437716687987033001739819 0b^9+317124465883913912590080b^{10}\\
&-28952254818229910029668b^{11}+1307601298898103037088b^{12}-83532370187199465686b^{13}\\
&+2356276086926649600b^{14}-107068908038367720b^{15}+1818336842252784b^{16}-58652132603355b^{17}\\
&+542492499200b^{18}-12138927350b^{19}+46232208b^{20}-677787b^{21})+a(202042011581512107782883356 25\\
&-391685584588234741575464132 25b^2+39672882044215122567086303040b^3\\
&-23234722312896303133802665365b^4+739378947135045678140546323 2b^5\\
&-20150568625605211621609649 555b^6+328839408882842649183560960b^7\\
&-5311892884815648948393743 0b^8+5126019245417466995920640b^9-541714692420734940958378b^{10}\\
&+328553992219576980729 60b^{11}-2386391763282673067610b^{12}+92951700036014507648b^{13}\\
&-4748353167342976646b^{14}+117803224107434240b^{15}-4248341006477155b^{16}+64291520883456b^{17}\\
&-1610682983229b^{18}+13374996800b^{19}-222540065b^{20}+761024b^{21}-7567b^{22})\\
&+b(-202042011581512107782883356 25+36292240882792148263256898000b\\
&-31077234776438508516718729725b^2+14411675092898014426739397600b^3\\
&-506696847852529405565837041 5b^4+11259395211542627179092581 28b^5\\
&-22087931964538478863780680 3b^6+282710674554981524032467 20b^7\\
&-3501841560857826940698010b^8+280451821889248334251168b^9-23337757655538605356418b^{10}\\
&+12152053081620359606 40b^{11}-70070695910870349630b^{12}+2396579567716999328b^{13}\\
&-96704766947027878b^{14}+2139674957676160b^{15}-59833929886925b^{16}+815733629328b^{17}\\
&-15258759153b^{18}+114598880b^{19}-1319395b^{20}+4048b^{21}-23b^{22}))\Big\}\Big]
\end{aligned}
\tag{15}
$$

III. Derivation of the Summation Formula

Substituting $c=\frac{a+b+47}{2}$ and $z=\frac{1}{2}$ in equation (9), we get

$$
(a-b)\ {}_2F_1\left[\begin{matrix} a,\ b\ ; \\ \frac{a+b+47}{2}\ ;\end{matrix}\ \ \frac{1}{2}\right]=a\ {}_2F_1\left[\begin{matrix} a+1,\ b\ ; \\ \frac{a+b+47}{2}\ ;\end{matrix}\ \ \frac{1}{2}\right]-b\ {}_2F_1\left[\begin{matrix} a,\ b+1\ ; \\ \frac{a+b+47}{2}\ ;\end{matrix}\ \ \frac{1}{2}\right]
$$

Now involving the derived formula [Salahuddin et. al. p.12-41(8)], the summation formula is obtained.

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Application of Hankel Transform of I-function of one Variable for Solving Axisymmetric Dirichlet Potential Problem

By Reema Tuteja, Shailesh Jaloree & Anil Goyal

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Abstract- In the present paper we have solved the well known Axisymmetric Dirichlet problem for a half-space using the Hankel transform of I-function of one variable. Hankel transform is much effective tool for solving the boundary value problems involving cylindrical coordinates. Here we have considered the Axisymmetric Dirichlet problem for a half-space which is mathematically characterized by

$$u_{xx} + (\tfrac{1}{x})u_x + u_{zz} = 0,\ 0 < x < \infty,\ z > 0$$

Boundary conditions are

$$u(x,0) = f(x),\ 0 < x < \infty$$

$$u(x,z) \to 0 \text{ as } \sqrt{(x^2+z^2)} \to \infty,\ z > 0$$

Our main result is believed to be general and unified in nature. A number of known and new results can be obtained by specializing the coefficients and parameters involved in the kernel.

Keywords and Phrases: potential problem, hankel transform, saxena's I- function of one variable, fox's H-function of one variable.

GJSFR-F Classification : *MSC 2010: 47B35*

APPLICATIONOFHANKELTRANSFORMOFI-FUNCTIONOFONEVARIABLEFORSOLVINGAXISYMMETRICDIRICHLETPOTENTIALPROBLEM

Strictly as per the compliance and regulations of :

Notes

Application of Hankel Transform of I-function of one Variable for Solving Axisymmetric Dirichlet Potential Problem

Reema Tuteja [α], Shailesh Jaloree [σ] & Anil Goyal [ρ]

Abstract- In the present paper we have solved the well known Axisymmetric Dirichlet problem for a half-space using the Hankel transform of I-function of one variable. Hankel transform is much effective tool for solving the boundary value problems involving cylindrical coordinates. Here we have considered the Axisymmetric Dirichlet problem for a half-space which is mathematically characterized by

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I. Introduction

Our aim is to find out the solution of boundary value problem involving cylindrical co-ordinates using Hankel transform of order zero.the most important partial differential equation in mathematicalphysics is the Laplace's equation or potential equation i.e.

$$\nabla^2 u = 0 \tag{1.1}$$

regardless of the co-ordinate system.

Laplace's equation arises in steady state heat conduction problems involving homogeneous solids. this same equation is satisfied by the gravitational potential in free space, the electrostatic potential in a uniform dielectric, the

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magnetic in the steady flow of currents in solid conductors, and the velocity potential of inviscid, irrotational fluids.The mathematical formulation of all potential problems is same despite the physical differences of the applications. Because of this, all solutions of the potential equations are collectively called potential functions, and the study of the many properties associated with these functions forms that branch of mathematics known as potentail theory. Here we have considered Axisymmetric Dirichlet problem for a half-space from the book by Andrews and Shivamoggi[1].

The Hankel transform arise naturally in solving boundary vaue problems formulated in cylindrical co-ordinates. They also occur in other applications such as determining the oscillations of a heavy chain suspended from one end, first treated by D. Bernoulli. This later problem is of some special historical significance since it was in this analysis of Bernoulli in 1703 that the Bessel function of order zero appeared for the first time.

The Hankel transform of order ν of a function $f(x)$ denoted by $H_\nu\{f(x);\rho\}$ is defined as

$$H_\nu\{f(x);\rho\} = \int_0^\infty xJ_\nu(\rho x)f(x)dx = g(\rho), \rho > 0, \nu > -\frac{1}{2} \tag{1.2}$$

The inverse Hankel transform is given by

$$f(x) = \int_0^\infty \rho J_\nu(x\rho)\, g(\rho)d\rho, \Re(\nu) > -1 \tag{1.3}$$

I-function of one variable introduced by Saxena V.P.[6] is defined as

$$I^{m,n}_{p_i,q_i:r} = I^{m,n}_{p_i,q_i:r}\left[x\left|^{\{(a_j,\alpha_j)_{1,n}\},\left\{(a_{ji},\alpha_{ji})_{n+1,p_i}\right\}}_{\{(b_j,\beta_j)_{1,m}\},\left\{(b_{ji},\beta_{ji})_{m+1,q_i}\right\}}\right.\right]$$

$$= \frac{1}{2\pi\omega}\int_L \theta(s)x^s ds \tag{1.4}$$

where $$\theta(s) = \frac{\prod_{j=1}^m \Gamma(b_j-\beta_j s)\prod_{j=1}^n \Gamma(1-a_j-\alpha_j s)}{\sum_{i=1}^r [\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}s)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}+\alpha_{ji}s)]}$$

Here $\omega = \sqrt{-1}$, $p_i(i = 1...r)$, $q_i(i = 1...r)$, m , n are integers satisfying $0 \le n \le p_i$; $0 \le m \le q_i$; r is finite. α_j, β_j, α_{ji}, β_{ji} are real and positive; $a_j, , b_j, a_{ji}, b_{ji}$ are complex numbers. L is the Mellin-Barne's type of contour integral which runs from $-\omega\infty$ to $+\omega\infty$ with indentations.

II. Required Results

In this section we are mentioning the results required for the evaluation of the transform and the solution of the boundary value problem.

a) First Result

Hankel transform of x^μ given in Erdélyi[3] is

$$H_\nu\{x^\mu\} = \int_0^\infty (\rho x)^{\frac{1}{2}} J_\nu(\rho x)x^\mu dx$$

Ref

[1] Andrews Larry C,Shivamoggi Bhimsen K(2009)*Integral Transforms for Engineers*.PHI Learning Pvt Ltd,India,pp 274-285

$$= 2^{(\mu+\frac{1}{2})}\rho^{-\mu-1}\frac{\Gamma(\frac{\mu}{2}+\frac{\nu}{2}+\frac{3}{4})}{\Gamma(\frac{\nu}{2}-\frac{\mu}{2}+\frac{1}{4})} \tag{2.1}$$

where $\rho > 0, -\Re(\nu) - \frac{3}{2} < \Re(\mu) < -\frac{1}{2}$

b) *Second Result*

The following result is taken from Erdélyi[3]

$$M\left\{e^{-ax}J_\nu(\beta x)\right\} = \int_0^\infty x^{s-1}e^{-ax}J_\nu(\beta x)dx$$

$$= \frac{\beta^\nu\Gamma(s+\nu)}{2^\nu a^{s+\nu}\Gamma(1+\nu)}{}_2F_1\left[\frac{s+\nu}{2},\frac{s+\nu+1}{2};\nu+1;\frac{-\beta^2}{a^2}\right] \tag{2.2}$$

where $\Re(\alpha) > |Im(\beta)|, \Re(s) > -\Re(\nu)$

c) *Third Result*

It is a well known result from Rainville[5]

$$(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)} \tag{2.3}$$

III. Hankel Transform of I-function of one Variable

For $\rho, \nu, \alpha \in C, \sigma > 0, \rho > 0, \alpha > 0$ satisfying the condition

$$\Re(\rho) + \Re(\nu) + \sigma_{1\le j\le m}^{min}\left[\frac{\Re(b_j)}{\beta_j}\right] > -\frac{3}{2}$$

and

$$\Re(\rho) + \sigma_{1\le j\le n}^{max}\left[\frac{1-\Re(a_j)}{\alpha_j}\right] < -\frac{1}{2}$$

Then there holds the formula

$$H_\nu\left\{\left(\frac{\rho}{x}\right)^{\frac{1}{2}}I_{p_i,q_i:r}^{m,n}\left[\alpha x^\sigma \left|^{\{(a_j,\alpha_j)_{1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i}\}}_{\{(b_j,\beta_j)_{1,m}\},\{(b_{ji},\beta_{ji}),_{m+1,p_i}\}}\right.\right]\right\}$$

$$= \frac{2^{\frac{1}{2}}}{\rho}I_{p_i+2,q_i:r}^{m,n+1}\left[\alpha\left(\frac{2}{\rho}\right)^\sigma \left|^{(\frac{1}{4}-\frac{\nu}{2},\frac{\sigma}{2}),\{(a_j,\alpha_j)_{1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i}\},(\frac{1}{4}+\frac{\nu}{2},\frac{\sigma}{2})}_{\{(b_j,\beta_j)_{1,m}\},\{(b_{ji},\beta_{ji}),_{m+1,p_i}\}}\right.\right] \tag{3.1}$$

Proof: Applying the definition of Hankel transform from eq.(1.2) to the left-hand side of eq.(3.1) and expressing the I-function in Mellin-Barne's type of contour integral, we get

$$= \int_0^\infty(\rho x)^{\frac{1}{2}}J_\nu(\rho x)\left\{\frac{1}{2\pi\omega}\int_L\theta(s)\alpha^s x^{\sigma s}ds\right\}dx$$

Changing the order of integration permissible under the conditions mentioned and using the result (2.1), we obtain

$$= \frac{1}{2\pi\omega}\int_L\theta(s)\alpha^s 2^{\sigma s+\frac{1}{2}}\rho^{-\sigma s-1}\frac{\Gamma(\frac{\sigma s}{2}+\frac{\nu}{2}+\frac{3}{4})}{\Gamma(\frac{\nu}{2}-\frac{\sigma s}{2}+\frac{1}{4})}$$

Rearranging the terms and expressing the integral in I-function of one variable, we get the right-hand side of eq.(3.1).

Ref

[3] Erdelyi A, Magnus W, Oberhettinger F, Tricomi FG(1954) *Table of integral transforms Vol I,II*.Mc-Graw-Hill,Newyork

Substituting $\nu = 0$, we obtain the Hankel transform of order zero as

$$H_0\left\{(\tfrac{\rho}{x})^{\frac{1}{2}} I^{m,n}_{p_i,q_i:r}\left[\alpha x^{\sigma}\left|^{\{(a_j,\alpha_j)_{1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i}\}}_{\{(b_j,\beta_j)_{1,m}\},\{(b_{ji},\beta_{ji})_{,m+1,p_i}\}}\right.\right]\right\}$$

$$= \frac{2^{\frac{1}{2}}}{\rho} I^{m,n+1}_{p_i+2,q_i:r}\left[\alpha\left(\frac{2}{\rho}\right)^{\sigma}\left|^{(\frac{1}{4},\frac{\sigma}{2}),\{(a_j,\alpha_j)_{1,n}\},\{(a_{ji},\alpha_{ji})_{n+1,p_i}\},(\frac{1}{4},\frac{\sigma}{2})}_{\{(b_j,\beta_j)_{1,m}\},\{(b_{ji},\beta_{ji})_{,m+1,p_i}\}}\right.\right] \tag{3.2}$$

where $\rho, \alpha \in C, \sigma > 0, \rho > 0, \alpha > 0$ satisfying the conditions

$$\Re(\rho) + \sigma^{min}_{1\leq j\leq m}\left[\frac{\Re(b_j)}{\beta_j}\right] > -\frac{3}{2}\text{and } \Re(\rho) + \sigma^{max}_{1\leq j\leq n}\left[\frac{1-\Re(a_j)}{\alpha_j}\right] < -\frac{1}{2}$$

IV. Axisymmetric Dirichlet Potential Problem

Consider the Axisymmetric Dirichlet problem for a half space which is mathematically characterized by

$$u_{xx} + \frac{1}{x}u_x + u_{zz} = 0, 0 < x < \infty, z > 0 \tag{4.1}$$

Boundary conditions are

$$u(x,0) = f(x), 0 < x < \infty$$

$$u(x,z) \to 0 \text{ as } \sqrt{(x^2+z^2)} \to \infty,\ z > 0$$

If we apply Hankel transform of order zero to the variable x in (4.1), we obtain the transformed problem as

$$U_{zz} - \rho^2 U = 0, z > 0 \tag{4.2}$$

Boundary conditions are

$$U(\rho,0) = F(\rho)$$
$$U(\rho,z) \to 0, \text{ as } z \to \infty$$

where

$$H_0\{u(x,z); x \to \rho\} = U(\rho,z) \tag{4.3}$$

$$H_0\{f(x); \rho\} = F(\rho) \tag{4.4}$$

The solution of (4.2) is

$$U(\rho,z) = F(\rho)e^{-\rho z} \tag{4.5}$$

Integrating eq.(4.5) by means of Hankel inversion formula,we have

$$u(x,z) = H_0^{-1}\left[F(\rho)e^{-\rho z}; \rho \to x\right] = \int_0^{\infty} \rho F(\rho)e^{-\rho z}J_0(\rho x)d\rho \tag{4.6}$$

V. Solution of Axisymmetric Dirichlet Potential Problem

For $\rho, \alpha \in C, \sigma > 0, \rho > 0, \alpha > 0, \Re(z) > |Im(x)|$ satisfying the conditions

Notes

$$\sigma^{min}_{1\leq j\leq m}\left[\frac{\Re(b_j)}{\beta_j}\right]>1 \qquad \text{and} \qquad \sigma^{max}_{1\leq j\leq n}\left[\frac{1-\Re(a_j)}{\alpha_j}\right]<-\frac{1}{2}$$

$$u(x,z)=\frac{2^{\frac{1}{2}}}{z}\sum_{k=0}^{\infty}\frac{1}{(n!)^2}\left(-\frac{x^2}{z^2}\right)^n$$

$$\times I^{m+3,n+1}_{p_i+4,q_i+3:r}\left[\alpha(2z)^{\sigma}\left|^{(\frac{1}{4},\frac{\sigma}{2}),\{(a_j,\alpha_j)_{1,n}\},\left\{(a_{ji},\alpha_{ji})_{n+1,p_i}\right\},(\frac{1}{4},\frac{\sigma}{2}),(\frac{1}{2},\frac{\sigma}{2}),(1,\frac{\sigma}{2})}_{(\frac{1}{2}+n,\frac{\sigma}{2}),(1+n,\frac{\sigma}{2}),(1,\sigma),\{(b_j,\beta_j)_{1,m}\},\left\{(b_{ji},\beta_{ji})_{,m+1,p_i}\right\}}\right.\right] \tag{5.1}$$

Notes

Proof: Substitute

$$f(x)=\left(\frac{\rho}{x}\right)^{\frac{1}{2}}I^{m,n}_{p_i,q_i:r}\left[\alpha x^{\sigma}\left|^{\{(a_j,\alpha_j)_{1,n}\},\left\{(a_{ji},\alpha_{ji})_{n+1,p_i}\right\}}_{\{(b_j,\beta_j)_{1,m}\},\left\{(b_{ji},\beta_{ji})_{,m+1,p_i}\right\}}\right.\right] \text{ in eq. (3.1)}$$

Then the Hankel transform of order zero of $f(x)$ from eq. (3.2) is

$$F(\rho)=\frac{2^{\frac{1}{2}}}{\rho}I^{m,n+1}_{p_i+2,q_i:r}\left[\alpha\left(\frac{2}{\rho}\right)^{\sigma}\left|^{(\frac{1}{4},\frac{\sigma}{2}),\{(a_j,\alpha_j)_{1,n}\},\left\{(a_{ji},\alpha_{ji})_{n+1,p_i}\right\},(\frac{1}{4},\frac{\sigma}{2})}_{\{(b_j,\beta_j)_{1,m}\},\left\{(b_{ji},\beta_{ji})_{,m+1,p_i}\right\}}\right.\right]$$

Substituting the value of $F(\rho)$ in eq. (4.4), we get

$$u(x,z)=\int_0^{\infty}\rho e^{-\rho z}J_0(\rho x)$$

$$\times\frac{2^{\frac{1}{2}}}{\rho}I^{m,n+1}_{p_i+2,q_i:r}\left[\alpha\left(\frac{2}{\rho}\right)^{\sigma}\left|^{(\frac{1}{4},\frac{\sigma}{2}),\{(a_j,\alpha_j)_{1,n}\},\left\{(a_{ji},\alpha_{ji})_{n+1,p_i}\right\},(\frac{1}{4},\frac{\sigma}{2})}_{\{(b_j,\beta_j)_{1,m}\},\left\{(b_{ji},\beta_{ji})_{,m+1,p_i}\right\}}\right.\right]d\rho$$

Expressing the I-function in Mellin-Barne's integral and changing the order of integration permissible under the specified conditions, we have

$$=2^{\frac{1}{2}}\frac{1}{2\pi\omega}\int_L\alpha^s\theta(s)2^{\sigma s}\frac{\Gamma(\frac{3}{4}+\frac{\sigma s}{2})}{\Gamma(\frac{1}{4}-\frac{\sigma s}{2})}\left\{\int_0^{\infty}\rho^{-\sigma s}e^{-\rho z}J_0(\rho x)d\rho\right\}ds$$

Using the result(2.2), we obtain

$$=2^{\frac{1}{2}}\frac{1}{2\pi\omega}\int_L\alpha^s\theta(s)2^{\sigma s}\frac{\Gamma(\frac{3}{4}+\frac{\sigma s}{2})}{\Gamma(\frac{1}{4}-\frac{\sigma s}{2})}\left\{\frac{\Gamma(1-\sigma s)}{z^{(1-\sigma s)}}{}_2F_1\left[\frac{(1-\sigma s)}{2},\frac{(2-\sigma s)}{2};1;-\frac{x^2}{z^2}\right]\right\}ds$$

Expanding ${}_2F_1$and using the result(2.3) then rearranging the terms, we obtain the right-hand side of eq.(5.1)

VI. Special Cases

Many special cases can be found by suitably specializing the parameters. One of the special case of our result is mentioned below.

Substituting $r=1$, I-function of one variable reduces to Fox's H-function of one variable assuming $a_{j1},\alpha_{j1},b_{j1},\beta_{j1}$ as a_j,α_j,b_j,β_j respectively.

For $\alpha\in C,\sigma>0,\alpha>0,\Re(z)>|Im(x)|$ satisfying the conditions

$$\sigma^{min}_{1\leq j\leq m}\left[\frac{\Re(b_j)}{\beta_j}\right]>1 \qquad \text{and} \qquad \sigma^{max}_{1\leq j\leq n}\left[\frac{1-\Re(a_j)}{\alpha_j}\right]<-\frac{1}{2}$$

$$u(x,z)=\frac{2^{\frac{1}{2}}}{z}\sum_{k=0}^{\infty}\frac{1}{(n!)^2}\left(-\frac{x^2}{z^2}\right)^n$$

$$\times H^{m+3,n+1}_{p+4,q+3}\left[\alpha(2z)^{\sigma}\left|^{(\frac{1}{4},\frac{\sigma}{2}),\{(a_j,\alpha_j)_{1,p}\},(\frac{1}{4},\frac{\sigma}{2}),(\frac{1}{2},\frac{\sigma}{2}),(1,\frac{\sigma}{2})}_{(\frac{1}{2}+n,\frac{\sigma}{2}),(1+n,\frac{\sigma}{2}),(1,\sigma),\{(b_j,\beta_j)_{1,q}\}}\right.\right] \tag{6.1}$$

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Effect of Magnetic Field on Oscillatory Flow Past Parallel Plates in a Rotating System with Heat and Mass Transfer

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Keywords: MHD, porous medium, incompressible fluid, natural convection, heat and mass transfer.

GJSFR-F Classification : MSC 2010: 00A69

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Strictly as per the compliance and regulations of :

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Pawan Kumar Sharma [α] & Sushil Kumar Saini [σ]

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I. Introduction

Magnetohydrodynamics (MHD) (magneto fluid dynamics or hydromagnetics) is the study of the dynamics of electrically conducting fluids. Examples of such fluids are include plasmas, liquid metals, and salt water or electrolytes. The word magnetohydrodynamics (MHD) is derived from magneto- meaning magnetic field, hydro- meaning liquid, and -dynamics meaning movement. The fundamental concept behind MHD is that magnetic fields can induce currents in a moving conductive fluid, which in turn creates forces on the fluid and also changes the magnetic field itself. The set of equations which describe MHD are a combination of the Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism. MHD applies quite well to astrophysics, content of the universe is made up of plasma, including stars, the interplanetary medium (space between the planets), the interstellar medium (space between the stars), the intergalactic medium, nebulae and jets. Sunspots are caused by the Sun's magnetic fields, the solar wind is also governed by MHD. However, magnetohydrodynamic effects transfer the Sun's angular momentum into the outer solar system, slowing its rotation. MHD is related to engineering problems such as plasma confinement, liquid-metal cooling of nuclear reactors, and electromagnetic casting. The working principle involves electrification of the propellant (gas or water) which can then be directed by a magnetic field, pushing the vehicle in

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the opposite direction. An important task in cancer research is developing more precise methods for delivery of medicine to affected areas. One method involves the binding of medicine to biologically compatible magnetic particles (e.g. ferrofluids), which are guided to the target via careful placement of permanent magnets on the external body.

The flow problems of an electrically conducting fluids under the influence of magnetic field have attracted the interest of many authors in view of its applications to geophysics, astrophysics, engineering, and to the boundary layer control in the field of aerodynamics. On the other hand in view of the increasing technical applications using magnetohydrodynamics effect, it is desirable to extend many of the available viscous hydrodynamic solution to include the effects of magnetic field for those cases when the viscous fluid is electrically conducting. Rossow [1], Greenspan and Carrier [2] have studied extensively the hydromagnetic effects on the flow past a plate with or without injection/suction. The hydromagnetic channel flow and temperature field was investigated by Attia and Kotab [3]. Hossain et al. [4] have studied the MHD free convection flow when the surface kept at oscillating surface heat flux.

In view of applications of the flow through porous medium with the effect of magnetic field, attract attention of a number of scholars. Aldoss et al [5], Helmy [6] and Kim [7] studied the magnetohydrodynamic mixed convection from a vertical plate in a porous medium. Unsteady free convection flow with the combined effect of thermal and mass diffusion in the presence of magnetic field and Hall effect is investigated by Takhar et.al [8]. Ahmed et al [9] studied the thermal diffusion effect on a three-dimensional MHD free convection with mass transfer flow from a porous vertical plate and Chamkha [10] also investigates MHD flow of a uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction. The MHD flow between two parallel horizontal porous plate also investigated by Chaudhary et al [11]. Sharma [12] studied the simultaneous thermal and mass diffusion in three dimensional mixed convection flows in the presence of porous medium. Singh et.al [13] and Lai and Kulacki [14] heve been studied the free convective flow past vertical wall. Nield [15] studied convection flow through porous medium with inclined temperature gradient. Kelleher et al. [16] studied the heat transfer response of laminar free convection boundary layers along vertical heated plates to surface temperature oscillations. Sharma et al [17] studied the unsteady free convection oscillatory flow through porous medium with periodic temperature variation. Also the oscillatory Couette flows in a rotating system have been studied by Jana and Datta [18] Muzumder [19], and Ganapathy [20]. Raptis and Peridikis [21] also studied the oscillatory flow through porous medium in the presence of convection.

Therefore the object of the present paper is to investigate the oscillatory flow through rotating porous vertical channel in the presence of magnetic field with fluctuating thermal and mass diffusion assuming periodic suction velocity at the plate and other plate which is also fluctuating with periodic free stream velocity about a non zero constant mean. The analytical solutions for mean primary, mean secondary velocity, transient velocity, transient temperature and concentration are obtained using regular perturbation technique. The effect of various parameters on flow characteristic are discussed and shown graphically.

II. Mathematical Formulation of the Problem

Consider an oscillatory free convective flow of a conducting viscous incompressible fluid through highly porous medium bounded between two infinite vertical porous plates distance d

Ref

1. V. J. Rossow, On flow of electrically conducting fluids over a flat plate in the presence of a transverse magnetic field, N. A. C. A., Rept., (1958), 1358.

apart. The periodic suction velocity is applied at the stationary plate $z^*=0$ and other plate at $z^* = d$, which is oscillating in its own plane with a velocity U^* about a non zero constant mean velocity U_0. The origin is assumed to be at the plate $z^*=0$ and the channel is oriented vertically upward along the x^*-axis. The channel rotates as a rigid body with uniform angular velocity Ω^* about z^* -axis, which is perpendicular to the vertical plane confined with a viscous fluid occupying the porous region. Since the plates are infinite in extent, all the physical quantities except the pressure, depend only on z^* and t^*. Denoting the velocity components u^*, v^*, w^* in the x^*, y^*, z^* directions, respectively, temperature by T^* and concentration by C^*. The flow in porous medium involves small velocities permitting the neglect of heat due to viscous dissipation in governing equation. A magnetic filed of constant intensity is applied perpendicular to the channel.

The basic equation of magnetofluiddynamics and conventional fluid dynamics are different by only additional force term due to electromagnetic field. The Maxwell's equations have to be satisfied in the entire field. In order to derive the basic equations for the problem under consideration, the following assumptions are made:

1. The flow is steady and laminar and the magnetic field is applied perpendicular to the plate.
2. The fluid under consideration is viscous, incompressible and finitely conducting with constant physical properties.
3. The magnetic Reynolds number is taken to be small enough so that the induced magnetic field is neglected.
4. Hall Effect, electrical and polarization effects are neglected.

The equation expressing the conservation of mass and energy transfer in rotating frame of reference are given by

$$w^* = -w_0(1+\varepsilon\cos\omega^* t^*), \tag{1}$$

$$\begin{aligned}\frac{\partial u^*}{\partial t^*} + w^*\frac{\partial u^*}{\partial z^*} - 2\Omega^* v^* &= -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + g\beta(T^* - T_d^*)\\ &+ g\beta_c(C^* - C_d^*) + \nu\frac{\partial^2 u^*}{\partial z^{*2}} - \frac{\nu u^*}{k^*} - \frac{(\vec{J}\times\vec{B})}{\rho},\end{aligned} \tag{2}$$

$$\frac{\partial v^*}{\partial t^*} + w^*\frac{\partial v^*}{\partial z^*} + 2\Omega^* u^* = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} + \nu\frac{\partial^2 v^*}{\partial z^{*2}} - \frac{\nu v^*}{k^*} - \frac{(\vec{J}\times\vec{B})}{\rho}, \tag{3}$$

where the fourth term on the right hand side of equations (2-3) is the Lorentz force due to magnetic field $\vec{B}$, and is given by

$$\vec{J}\times\vec{B} = \sigma(\vec{v}\times\vec{B})\times\vec{B} \tag{4}$$

Using (4) in equations (2) and (3), we have

$$\begin{aligned}\frac{\partial u^*}{\partial t^*} + w^*\frac{\partial u^*}{\partial z^*} - 2\Omega^* v^* &= -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + g\beta(T^* - T_d^*)\\ &+ g\beta_c(C^* - C_d^*) + \nu\frac{\partial^2 u^*}{\partial z^{*2}} - \frac{\nu u^*}{k^*} - \frac{\sigma B^2}{\rho}u^*\end{aligned} \tag{5}$$

$$\frac{\partial v^*}{\partial t^*} + w^* \frac{\partial v^*}{\partial z^*} + 2\Omega^* u^* = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} + \nu \frac{\partial^2 v^*}{\partial z^{*2}} - \frac{\nu v^*}{k^*} - \frac{\sigma B^2}{\rho} v^* , \quad (6)$$

$$\frac{\partial T^*}{\partial t^*} + w^* \frac{\partial T^*}{\partial z^*} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T^*}{\partial z^{*2}} , \quad (7)$$

Notes

$$\frac{\partial C^*}{\partial t^*} + w^* \frac{\partial C^*}{\partial z^*} = D \frac{\partial^2 C^*}{\partial z^{*2}} . \quad (8)$$

where g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, β_c is the volumetric coefficient of expansion for concentration, T* is the temperature, T^*_d is the temperature in free stream, ν is the kinematic viscosity, Ω^* is the angular velocity, k^* is the permeability, C_p is the specific heat at constant pressure, p* is the pressure, ρ is the density, t* is the time and κ is the thermal conductivity, ω^* frequency of fluctuations.

The boundary conditions of the problem are

$$\left.\begin{aligned} z = 0: u^* = 0,\quad v^* = 0,\quad & T^* = T_0^* + \varepsilon (T_0^* - T_d^*) \cos \omega^* t^* , \\ & C^* = C_0^* + \varepsilon (C_0^* - C_d^*) \cos \omega^* t^* \\ z = d:\ u^* = v^* = U^* = U_0 (1+\varepsilon \cos \omega^* t^2),\ & T^* = T_d , C^* = C_d . \end{aligned}\right\} \quad (9)$$

Considering u + iv = U and eliminating the pressure gradient from (5) and (6), we have

$$\begin{aligned} \frac{\partial U^*}{\partial t^*} + w^* \frac{\partial U^*}{\partial z^*} + 2 i \Omega^* U^* &= g \beta (T^* - T_d^*) \\ + g \beta_c (C^* - C_d^*) + \nu \frac{\partial^2 U^*}{\partial z^{*2}} &- \frac{\nu U^*}{k^*} - \frac{\sigma B^2}{\rho} U^* , \end{aligned} \quad (10)$$

We introduce the following non-dimensional quantities as:

$$z = \frac{z^*}{d} ,\ u = \frac{u^*}{U_0} ,\ v = \frac{v^*}{U_0} ,\quad \omega = \frac{d^2 \omega^*}{\nu} ,\quad \theta = \frac{(T^* - T_d^*)}{(T_0^* - T_d^*)} ,$$

$$k = \frac{k^*}{d^2} ,\quad t = \omega^* t^* ,\quad \lambda\,(\text{Suction parameter}) = \frac{d\, w_0}{\nu},$$

$$\alpha\,(\textit{Thermal diffusivity}) = \frac{\kappa}{\rho C_p},$$

$$M\,(\text{Hartmann Number}) = \sqrt{\frac{\sigma B^2 d^2}{\rho\, \nu\, U_0}},\quad Sc\,(\text{Schmidt Number}) = \frac{\nu}{D},$$

$$Gr\,(\text{Grashof number}) = \frac{g \beta (T_0^* - T_d^*) d^2}{\nu U_0} ,\ Pr\,(\text{Prandtl number}) = \frac{\nu}{\alpha} ,$$

$$C = \frac{(C^* - C_d^*)}{(C_0^* - C_d^*)} ,$$

$$\text{Gc (modified Grashof number)} = \frac{g\beta_c(C_0^* - C_d^*)d^2}{\nu U_0}$$

Substituting these non-dimensional quantities in equations (7), (8) and (10), we get

$$\omega\frac{\partial U}{\partial t} - (1+\varepsilon\cos t)\lambda\frac{\partial U}{\partial z} + 2iRU = Gr\lambda^2\theta + Gc\lambda^2 C + \frac{\partial^2 U}{\partial z^2} - \frac{U}{k} - M^2 U, \tag{11}$$

$$\omega\frac{\partial \theta}{\partial t} - (1+\varepsilon\cos t)\lambda\frac{\partial \theta}{\partial z} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial z^2}, \tag{12}$$

$$\omega\frac{\partial C}{\partial t} - (1+\varepsilon\cos t)\lambda\frac{\partial C}{\partial z} = \frac{1}{Sc}\frac{\partial^2 C}{\partial z^2}, \tag{13}$$

The corresponding boundary conditions (9) become

$$\left.\begin{aligned} z = 0:\ U = 0,\quad \theta = 1+\varepsilon\cos t,\ C = 1+\varepsilon\cos t \\ z = d:\ U = 1+\varepsilon\cos t,\quad \theta = 0,\quad C = 0. \end{aligned}\right\} \tag{14}$$

III. Solution of the Problem

In order to solve the problem, we assume the solutions of the following form because amplitude ε (<< 1) of the variation of temperature is very small

$$\left.\begin{aligned} U(z,t) &= U_0(z) + \varepsilon U_1(z)e^{-it} + \ldots\ldots\ldots\ldots \\ \theta(z,t) &= \theta_0(z) + \varepsilon\theta_1(z)e^{-it} + \ldots\ldots\ldots\ldots \\ C(z,t) &= C_0(z) + \varepsilon C_1(z)e^{-it} + \ldots\ldots\ldots\ldots \end{aligned}\right\} \tag{15}$$

Substituting (15) in equations (11), (12) and (13), and equating the coefficient of identical powers of ε and neglecting those of ε^2, ε^3 etc., we get

$$U_0'' + \lambda U_0' - 2iRU_0 - \frac{U_0}{k} - M^2U_0 = -Gr\lambda^2\theta_0 - Gc\lambda^2 C_0, \tag{16}$$

$$U_1'' + \lambda U_1' - 2iRU_1 + i\omega U_1 - \frac{U_1}{k} - M^2U_1 = -Gr\lambda^2\theta_1 - Gc\lambda^2 C_1 - \lambda U_0', \tag{17}$$

$$\theta_0'' + \lambda Pr\,\theta_0' = 0, \tag{18}$$

$$\theta_1'' + \lambda Pr\,\theta_1' + i\omega Pr\,\theta_1 = -\lambda Pr\,\theta_0' \tag{19}$$

$$C_0'' + \lambda Sc\,C_0' = 0, \tag{20}$$

$$C_1'' + \lambda Sc\,C_1' + i\omega\,Sc\,C_1 = -\lambda Sc\,C_0' \tag{21}$$

The corresponding boundary conditions (14) reduce to

$$\left.\begin{aligned} z = 0:\ U_0 = 0,\ U_1 = 0,\ \theta_0 = 1,\ \theta_1 = 1,\ C_0 = 1,\ C_1 = 1 \\ z = d:\ U_0 = 1,\ U_1 = 1,\ \theta_0 = 0,\ \theta_1 = 0,\ C_0 = 0,\ C_1 = 0. \end{aligned}\right\} \tag{22}$$

Notes

Solving equations (16) to (21) under corresponding boundary conditions (22), we get

$$U_0(z) = n_{17} e^{n_{11} z} + n_{16} e^{n_{12} z} + n_{13} e^{-\lambda \Pr z} + n_{14} e^{-\lambda Sc z} + n_{15} \quad (23)$$

$$U_1(z) = n_{31} e^{n_{18} z} + n_{30} e^{n_{19} z} + n_{20} e^{n_{11} z} + n_{21} e^{n_{12} z} + n_{22} e^{-\lambda \Pr z} + n_{23} e^{-\lambda Sc z} + n_{24} e^{n_2 z} + n_{25} e^{n_1 z} + n_{26} e^{n_7 z} + n_{27} e^{n_6 z} \quad (24)$$

$$\theta_0(z) = \frac{1}{(1-e^{-\lambda \Pr})}(e^{-\lambda \Pr z} - e^{-\lambda \Pr}) \quad (25)$$

$$\theta_1(z) = n_4 e^{n_2 z} + n_5 e^{n_1 z} + n_3 e^{-\lambda \Pr z} \quad (26)$$

$$C_0(z) = \frac{1}{(1-e^{-\Lambda Sc})}(e^{-\lambda Sc z} - e^{-\lambda Sc}) \quad (27)$$

$$C_1(z) = n_9 e^{n_7 z} + n_{10} e^{n_6 z} + n_8 e^{-\lambda Sc z} \quad (28)$$

where

$$n_1 = \frac{1}{2}\left[-\lambda \Pr + \sqrt{\lambda^2 \Pr^2 - 4 i \omega \Pr}\right]$$

$$n_2 = \frac{1}{2}\left[-\lambda \Pr - \sqrt{\lambda^2 \Pr^2 - 4 i \omega \Pr}\right]$$

$$n_3 = \frac{\lambda^2 \Pr}{i(1-e^{-\lambda \Pr})\omega}$$

$$n_4 = \frac{e^{n_1} - n_3(e^{n_1} - e^{-\lambda \Pr})}{(e^{n_1} - e^{n_2})}$$

$$n_5 = -\left[\frac{e^{n_2} - n_3(e^{n_2} - e^{-\lambda \Pr})}{(e^{n_1} - e^{n_2})}\right]$$

$$n_6 = \frac{1}{2}\left[-\lambda Sc + \sqrt{\lambda^2 Sc^2 - 4 i \omega Sc}\right]$$

$$n_7 = \frac{1}{2}\left[-\lambda Sc - \sqrt{\lambda^2 Sc^2 - 4 i \omega Sc}\right]$$

$$n_8 = \frac{\lambda^2 Sc}{i(1-e^{-\lambda Sc})\omega}$$

$$n_9 = \frac{e^{n_6} - n_8(e^{n_6} - e^{-\lambda Sc})}{(e^{n_6} - e^{n_7})}$$

$$n_{10} = -\left[\frac{e^{n_7} - n_3(e^{n_6} - e^{-\lambda Sc})}{(e^{n_6} - e^{n_7})}\right]$$

Notes

Notes

$$n_{11} = \frac{1}{2}\left[-\lambda + \sqrt{\lambda^2 + 4(2iR + \frac{1}{k} + M^2)}\right]$$

$$n_{12} = \frac{1}{2}\left[-\lambda - \sqrt{\lambda^2 + 4(2iR + \frac{1}{k} + M^2)}\right]$$

$$n_{13} = -\frac{Gr\,\lambda^2}{(1 - e^{-\lambda \Pr})[\lambda^2 \Pr^2 - \lambda^2 \Pr - (2iR + \frac{1}{k} + M^2)]}$$

$$n_{14} = -\frac{Gc\,\lambda^2}{(1 - e^{-\lambda Sc})[\lambda^2 Sc^2 - \lambda^2 Sc - (2iR + \frac{1}{k} + M^2)]}$$

$$n_{15} = -\frac{Gr\,\lambda^2 e^{-\lambda \Pr}}{(1 - e^{-\lambda \Pr})\,(2iR + \frac{1}{k} + M^2)} - \frac{Gc\,\lambda^2 e^{-\lambda Sc}}{(1 - e^{-\lambda Sc})\,(2iR + \frac{1}{k} + M^2)}$$

$$n_{16} = \frac{1 + (n_{13} + n_{14} + n_{15})e^{n_{11}} - n_{13}\,e^{-\lambda \Pr} - n_{14}\,e^{-\lambda Sc} - n_{15}}{e^{n_{12}} - e^{n_{11}}}$$

$$n_{17} = -n_{16} - n_{13} - n_{14} - n_{15}$$

$$n_{18} = \frac{1}{2}\left[-\lambda + \sqrt{\lambda^2 - 4(i\omega - 2iR - \frac{1}{k} - M^2)}\right]$$

$$n_{19} = \frac{1}{2}\left[-\lambda - \sqrt{\lambda^2 - 4(i\omega - 2iR - \frac{1}{k} - M^2)}\right]$$

$$n_{20} = -\frac{\lambda\, n_{11}\, n_{17}}{n_{11}^2 + \lambda\, n_{11} + (i\omega - 2iR - \frac{1}{k} - M^2)}$$

$$n_{21} = -\frac{\lambda\, n_{12}\, n_{16}}{n_{12}^2 + \lambda\, n_{12} + (i\omega - 2iR - \frac{1}{k} - M^2)}$$

$$n_{22} = \frac{\lambda^2\, n_{13} \Pr}{\lambda^2 \Pr^2 - \lambda^2 \Pr + (i\omega - 2iR - \frac{1}{k} - M^2)} - \frac{\lambda^2\, n_3\, Gr}{\lambda^2 \Pr^2 - \lambda^2 \Pr + (i\omega - 2iR - \frac{1}{k} - M^2)}$$

$$n_{23} = \frac{\lambda^2\, n_{14}\, Sc}{\lambda^2 Sc^2 - \lambda^2 Sc + (i\omega - 2iR - \frac{1}{k} - M^2)} - \frac{\lambda^2\, n_8\, Gc}{\lambda^2 Sc^2 - \lambda^2 Sc + (i\omega - 2iR - \frac{1}{k} - M^2)}$$

$$n_{24} = -\frac{Gr\,\lambda^2\, n_4}{n_2^2 + \lambda\, n_2 + (i\omega - 2iR - \frac{1}{k} - M^2)}$$

$$n_{25} = -\frac{Gr\,\lambda^2\, n_5}{n_1^2 + \lambda\, n_1 + (i\omega - 2iR - \frac{1}{k} - M^2)}$$

$$n_{26} = -\frac{Gc\,\lambda^2\, n_9}{n_7^2 + \lambda\, n_7 + (i\,\omega - 2\,i\,R - \frac{1}{k} - M^2)}$$

$$n_{27} = -\frac{Gc\,\lambda^2\, n_{10}}{n_6^2 + \lambda\, n_6 + (i\,\omega - 2\,i\,R - \frac{1}{k} - M^2)}$$

$$n_{28} = n_{20} + n_{21} + n_{22} + n_{23} + n_{24} + n_{25} + n_{26} + n_{27}$$

$$n_{29} = 1 - n_{20}\, e^{n_{11}} - n_{21} e^{n_{12}} - n_{22} e^{-\lambda \Pr} - n_{23} e^{-\lambda Sc} - n_{24}\, e^{n_2} - n_{25}\, e^{n_1} - n_{26} e^{n_7} - n_{27} e^{n_6}$$

$$n_{30} = \frac{n_{29} + n_{28}\, e^{n_{18}}}{e^{n_{19}} - e^{n_{18}}}, \quad n_{31} = -n_{30} - n_{28}\,.$$

IV. Discussion and Conclusions

a) Steady Flow

We take $U_0 = u_0 + i\, v_0$ in equation (23) and subsequent comparison of the real and imaginary parts gives the mean primary velocity u_0 and mean secondary velocity v_0. The mean primary velocity is presented in Fig. 1 for fixed values of Gr, Gc and Sc=0.60 (for CO_2) in air (Pr = 0.71). The graph reveals that velocity increases with increasing suction parameter λ and reverse effect is observed for R (rotation parameter) and k (permeability of porous medium). This shows that the porosity and rotation of porous medium exert retarding influence on the primary flow. Fig.2 also shows mean primary velocity for different values of Gr (Grashof Number), Gc(Modified Grashof Number) and Sc(Schmidt Number). It is observed from the figure that the mean primary velocity increases rapidly with increasing either Gr or Gc. The magnitude of velocity is lesser in case of Sc=0.78 (NH_3) than that of Sc=0.60 (CO_2). Furthermore the mean primary velocity increases in the vicinity of the plate. It is interesting to note that if we increase magnetic field parameter M (Hartmann Number) , i.e. medium become conducting then the mean primary velocity become fluctuate sinusoidally.

The mean secondary velocity profiles is shown in Fig. 3 for the fixed values of Gr, Gc Sc and Pr=0.71(air). It is observed that it increases with increasing R while reverse phenomena is observed for λ. It is interesting to note that mean primary velocity increases while mean secondary velocity decreases with R and λ. It is also observed that due to increase in k mean secondary velocity decreases upto middle half of the channel then it increases. Fig.4 also showed the mean secondary velocity for different values of parameters. It is observed that it decreases with increasing either Gr, Gc and Sc. The magnitude is lower in case of NH_3 than that of CO_2. The amount of secondary velocity is much lower for Gc than that of Gr. Also due to increase intensity of magnetic field the mean secondary velocity fluctuating.

The mean temperature and concentration is presented in Fig.9. It is observed that both decreases with increasing λ. The mean temperature and concentration decreases exponentially, the magnitude of concentration is less in case of NH_3 than that of CO_2

Notes

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b) Unsteady Flow

Replacing the unsteady parts

$U_1(z,t)=M_r+iM_i$, $\theta_1(z,t)=T_r+iT_i$, and $C_1(z,t)=C_r+iC_i$ respectively in equations (24), (26) and (28) we get

$$[U(z,t),\theta(z,t),C(z,t)]=[U_0(z),\theta_0(z),C_0(z)] + \varepsilon e^{-it}[(M_r+iM_i),(T_r+iT_i),(C_r+iC_i)] \tag{29}$$

The primary, secondary velocity fields, temperature and concentration in terms of the fluctuating components are

$$u(z,t)=u_0+\varepsilon(M_r\cos t+M_i\sin t) \tag{30}$$

$$v(z,t)=v_0+\varepsilon(M_i\cos t-M_r\sin t) \tag{31}$$

$$\theta(z,t)=\theta_0+\varepsilon(T_r\cos t+T_i\sin t) \tag{32}$$

$$C(z,t)=C_0+\varepsilon(C_r\cos t+C_i\sin t) \tag{33}$$

Taking $t=\dfrac{\pi}{2}$ in equations (30) to (33) we get the expression for transient primary velocity, transient secondary velocity transient temperature and concentration as

$$u\left(z,\frac{\pi}{2}\right)=u_0(z)-\varepsilon M_1(z), \tag{34}$$

$$v\left(z,\frac{\pi}{2}\right)=v_0(z)+\varepsilon M_r(z), \tag{35}$$

$$\theta\left(z,\frac{\pi}{2}\right)=\theta_0(z)-\varepsilon T_i(z) \tag{36}$$

$$C\left(z,\frac{\pi}{2}\right)=C_0(z)-\varepsilon C_i(z) \tag{37}$$

The transient primary velocity component is shown in Fig.5 for fixed values of Pr, Gr, Gc, Sc and ω. It is observed that it decreases with increasing either R and k while transient primary velocity increases with increasing suction parameter λ. It is interesting to note that initially there is decrease in transient primary and than it increasing near the other plate which is fluctuating with free stream velocity. The transient primary velocity shift from positive to negative due to increase in intensity of magnetic field. Fig.6 also shows that due to increase in Gr and Gc the transient primary velocity increases. An increase in ω, the frequency of fluctuation transient velocity behave sinusoidally. The transient primary velocity increases with increasing Sc near the plate upto z<0.6 than it decreases. It is interesting to note that due to increase in M, transient velocity is fluctuating sinusoidally.

The transient secondary velocity profiles is given in Fig.7 for different values of R, k and λ. It is observed that transient secondary velocity increases with increasing either R and k, while it decreasing with increase in λ. The amount of decrease in velocity is much lower due to increase in permeability of the porous medium. Physically this is true because the porous material offers resistance to the flow, so velocity decreases in porous medium. Fig.8 also represented transient secondary velocity for different values of Gr, Gc, Sc and ω. The graph reveals that transient secondary velocity decreases with either Gr and Gc. It is interesting to note that value of transient secondary velocity is greater in case of NH_3 than that of CO_2. Furthermore velocity decreases rapidly in the vicinity of the plate with ω than it increases near the other the plate which is fluctuating with free stream velocity. The transient secondary velocity Fig.7-8, become fluctuating with increasing M.

Transient temperature and transient concentration are given in Fig.10. It is observed that transient temperature and concentration both are decreasing with suction parameter. The temperature and concentration are decreasing exponentially with distance apart vertical channel.

Heat Transfer: In the dynamics of viscous fluid one is not much interested to know all the details of the velocity and temperature fields but would certainly like to know quantity of heat exchange between the body and the fluid. Since at the boundary the heat exchanged between the fluid and the body is only due to conduction, according to Fourier's law, we have

$$q_w^* = -\kappa \left(\frac{\partial T^*}{\partial z^*}\right)_{z^*=0} \tag{38}$$

where z* is the direction of the normal to the surface of the body. We can calculate the dimensionless coefficient of heat transfer in terms of Nusselt Number as follows

$$Nu = -\frac{q_w^* d}{\kappa\,(T_0^* - T_d^*)} = \left(\frac{\partial \theta}{\partial z}\right)_{z=0} = \left(\frac{\partial \theta_0}{\partial z}\right)_{z=0} + \varepsilon e^{-it} \left(\frac{\partial \theta_1}{\partial z}\right)_{z=0} \tag{39}$$

In terms of the amplitude and phase the rate of heat transfer can be written as:

$$Nu = \left(\frac{\partial \theta_0}{\partial z}\right)_{z=0} + \varepsilon |H| \cos(\phi - t) \tag{40}$$

where

$$H = H_r + i H_i = \textit{coefficient of } \varepsilon e^{-it} \textit{ in equation } (40)$$

$$|H| = \sqrt{Hr^2 + Hi^2}\ , \quad \tan\phi = H_i / H_r .$$

Mass Transfer: According to Fick's Law the dimensionless coefficient of mass transfer at the plate in terms of Shearwood Number is given as follows

$$Sh = \left(\frac{\partial C}{\partial z}\right)_{z=0} = \left(\frac{\partial C_0}{\partial z}\right)_{z=0} + \varepsilon e^{-it} \left(\frac{\partial C_1}{\partial z}\right)_{z=0} \tag{41}$$

In terms of the amplitude and phase the rate of mass transfer can be written as:

$$Sh = \left(\frac{\partial C_0}{\partial z}\right)_{z=0} + \varepsilon |S| \cos(\varphi - t) \tag{42}$$

where

$$S = S_r + i S_i = \textit{coefficient of } \varepsilon e^{-it} \textit{ in equation } (42)$$

$$|S| = \sqrt{Sr^2 + Si^2}\ , \quad \tan\varphi = S_i / S_r .$$

The amplitudes of rate of heat and mass transfer in presented in Fig.11. The graph reveals that both are increases with increasing ω the frequency of fluctuations upto λ <0.8 than they decreases for higher values of suction parameter. It is also observed that amplitude of mass transfer is higher in case of NH_3 than that of CO_2.

Notes

Fig.12 shows the phases of rate of heat and mass transfer. It is observed from the figure that phases of heat and mass transfer increases with increasing ω. The phase of mass transfer increases with increase in Sc. The magnitude is higher for NH_3 than that of CO_2. It is also observed from the figure that phases of heat and mass transfer increases for λ<0.5 than they decreases and become negative for small values of ω.

Notes

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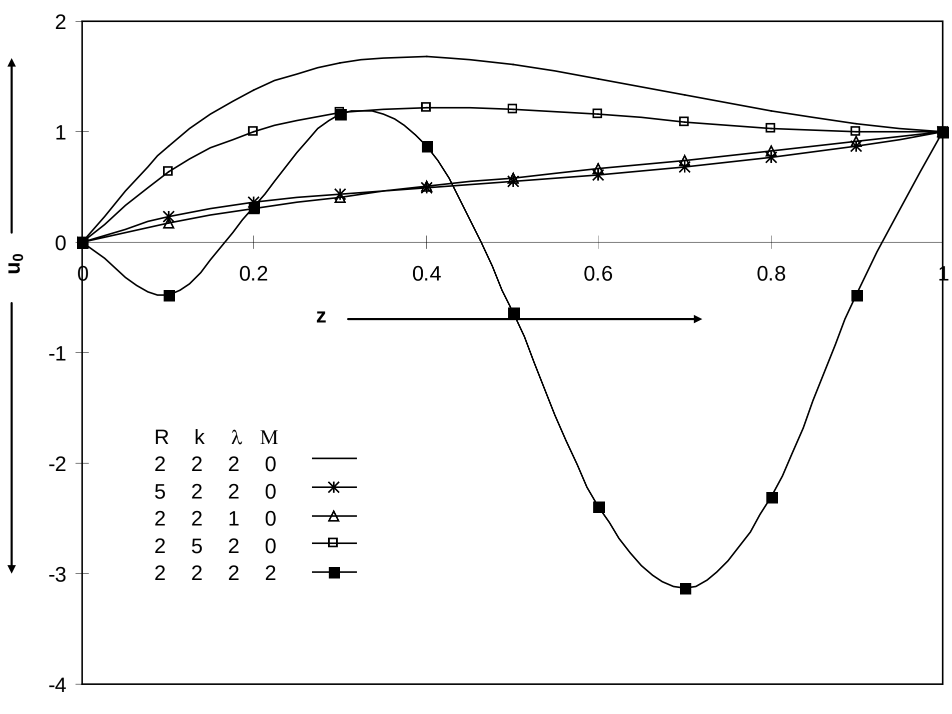
Notes

Fig.1. Mean primary velocity for Pr = 0.71, Gr = 2, Gc = 2 and Sc = 0.60



Notes

Fig.2. Mean primary velocity profiles for Pr = 0.71, R = 2, k = 2 and λ = 2

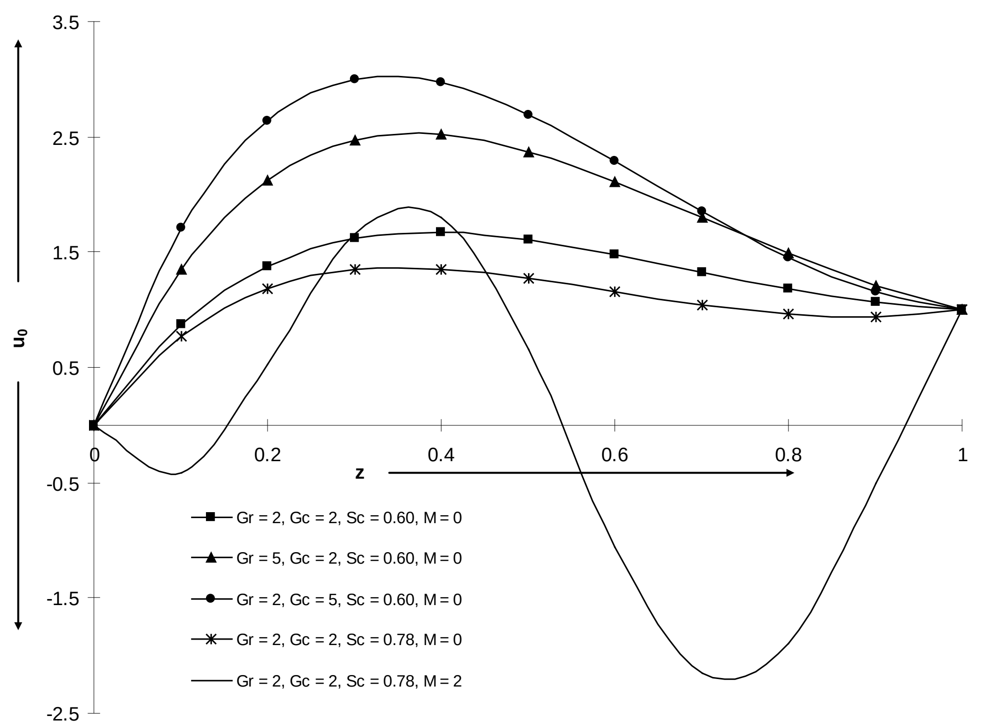


Fig.3. Mean secondary velocity profiles for Pr = 0.71, Gr = 2, Gc = 2 and Sc = 0.60

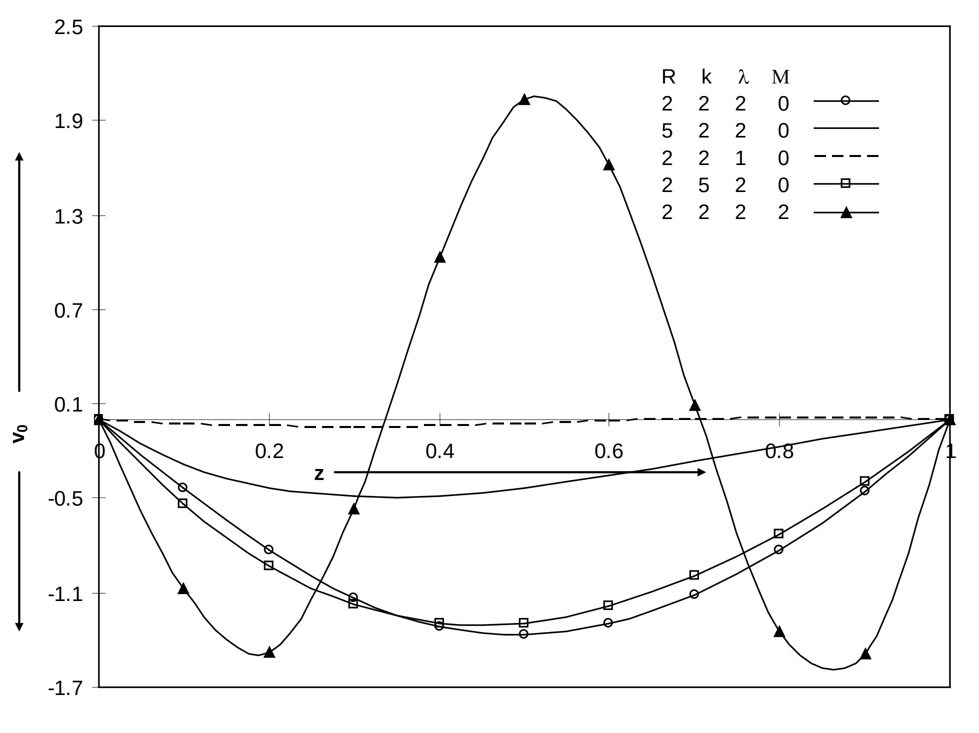


Fig.4. Mean secondary velocity profiles for Pr = 0.71, R = 2, k = 2 and λ = 2

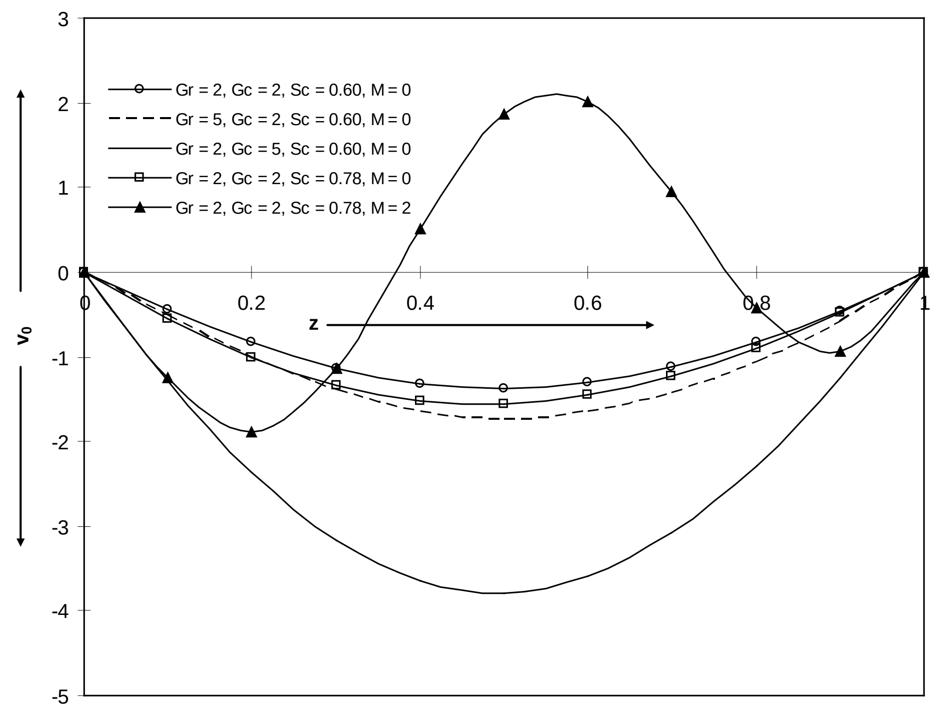
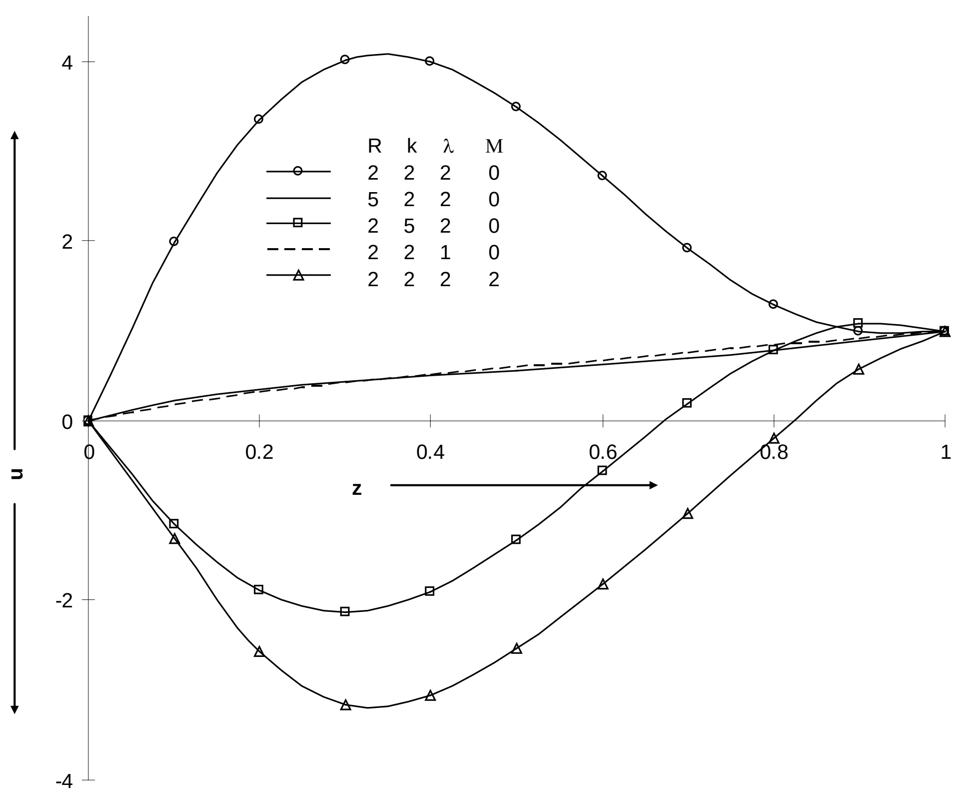


Fig.5. Transient primary velocity profiles fpr Pr = 0.71, Gr = 2, Gc = 2, ε = 0.2, Sc = 0.60, ω = 2 and t = π/2



Notes

Fig.6. Transient primary velocity profiles for Pr = 0.71, R = 2, k = 2, ε = 0.2, λ = 2 and t = π/2

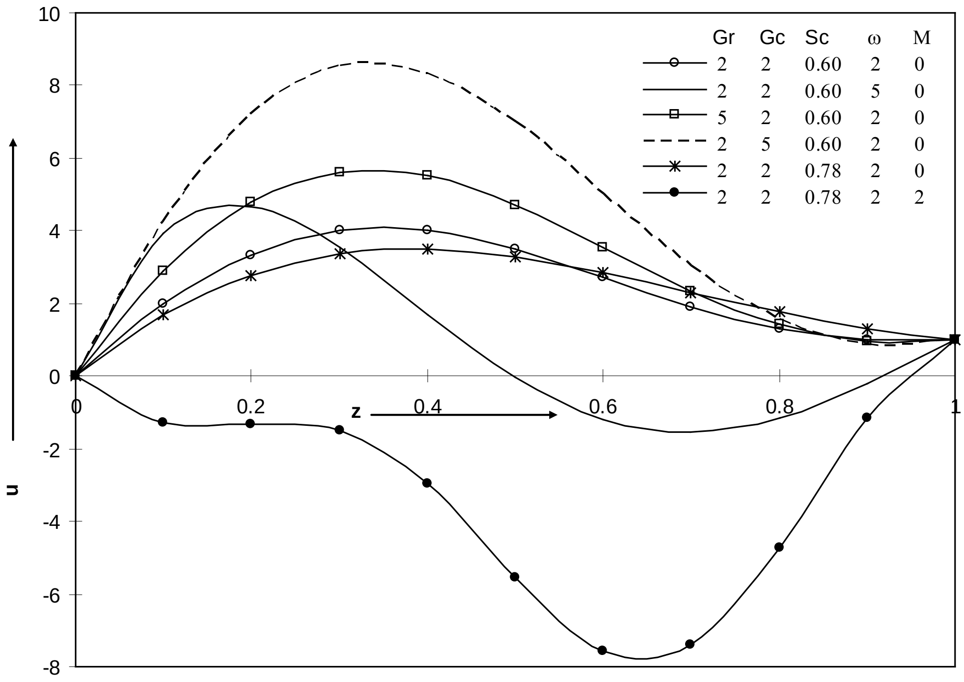


Fig.7. Transient secondary velocity profiles fpr Pr = 0.71, Gr = 2, Gc = 2, ε = 0.2, Sc = 0.60, ω = 2 and t = π/2

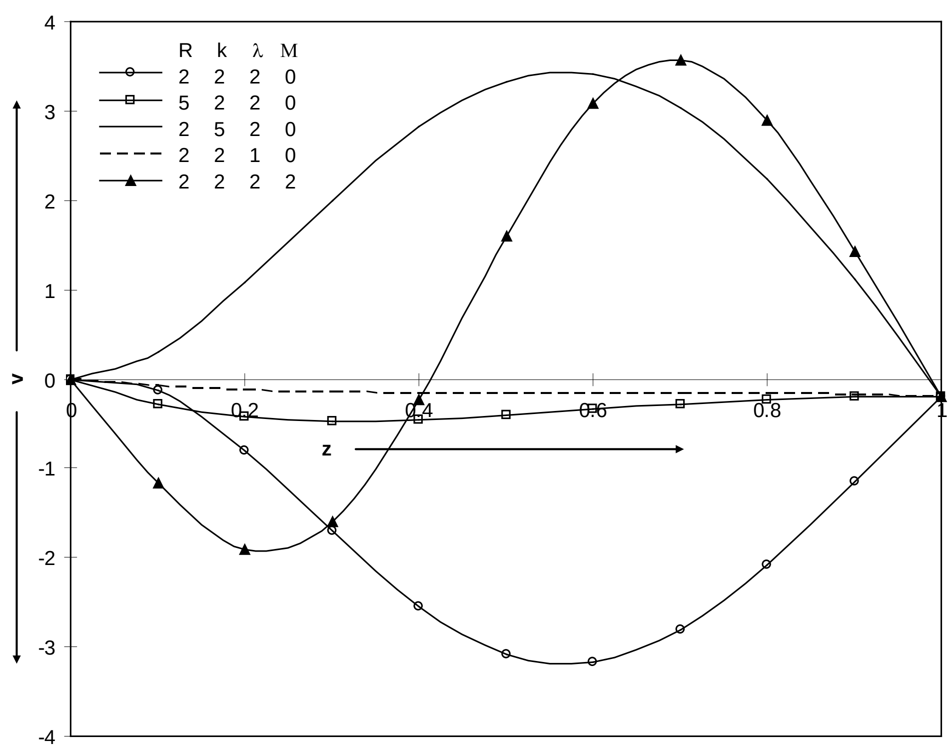


Fig.8. Transient secondary velocity profiles for Pr = 0.71, R = 2, k = 2, ε = 0.2, λ = 2 and t = π/2

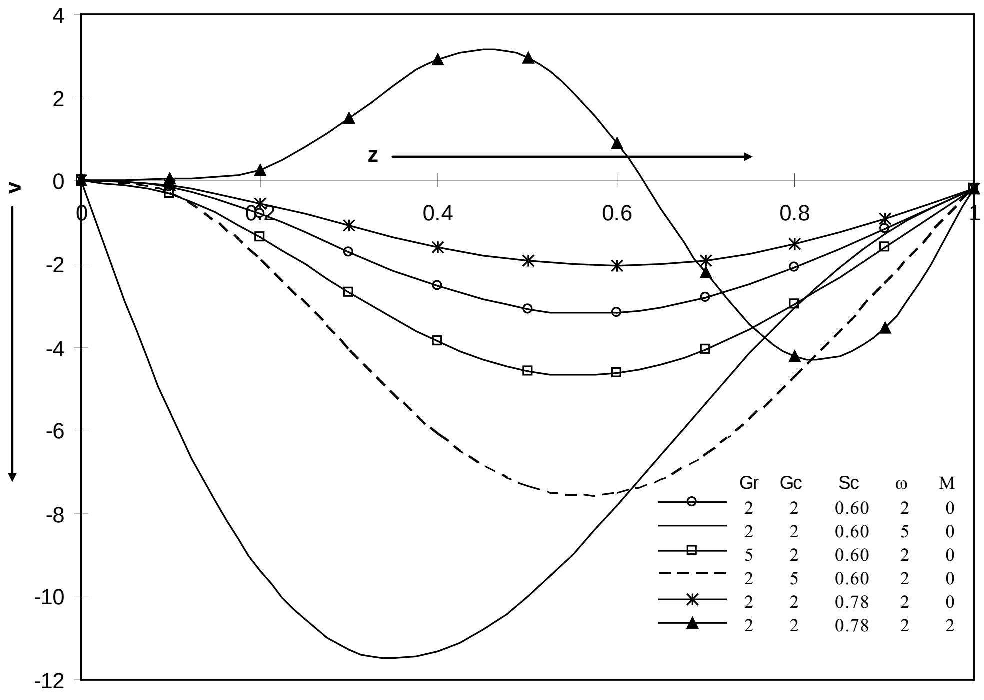


Fig.9. Mean temperature (Pr = 0.71) and concentration profiles

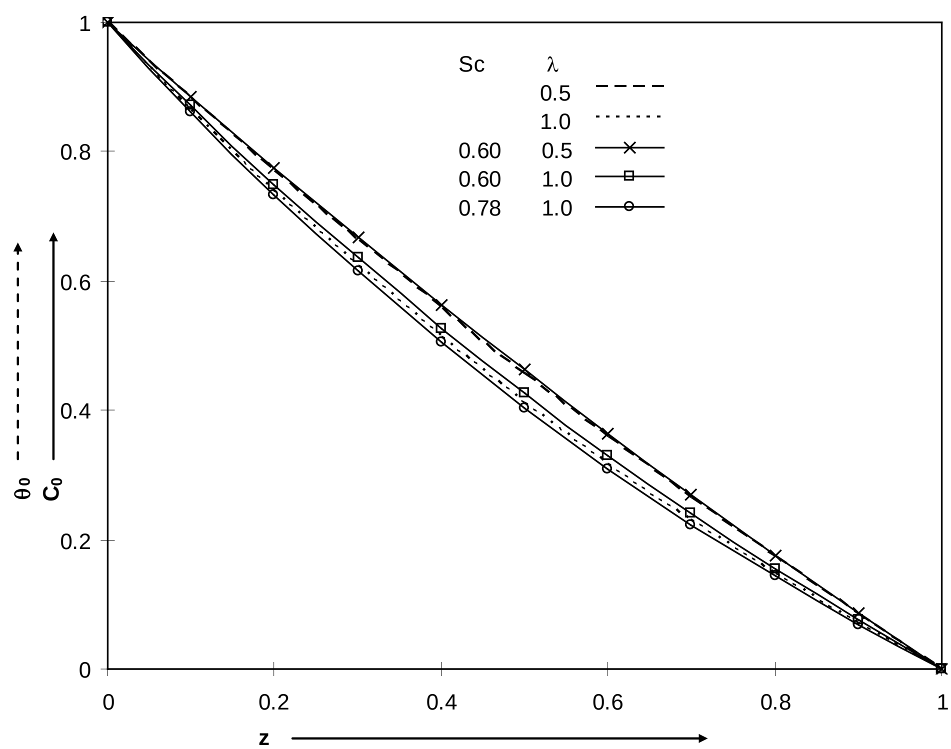


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Fig.10 Transient temperature (Pr = 0.71) and transient concentration (Sc = 0.60) profiles for ε = 0.2, ω = 0.5 and t = π/2

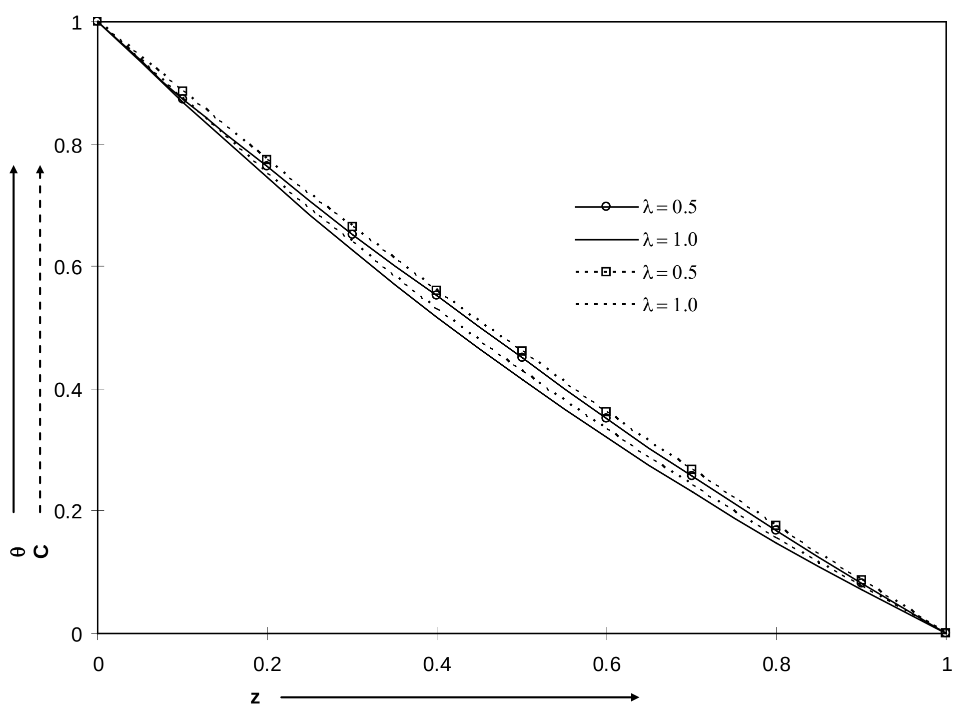


Fig.11 Amplitudes of rate of heat transfer (Pr = 0.71) l H l and rate of mass transfer l S l

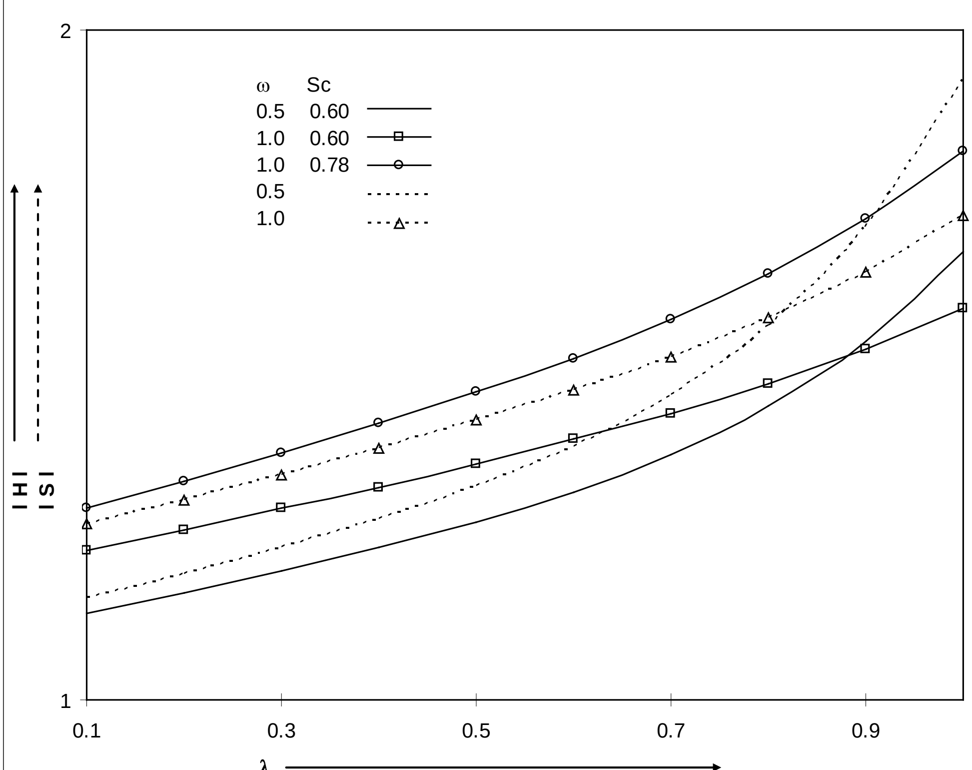
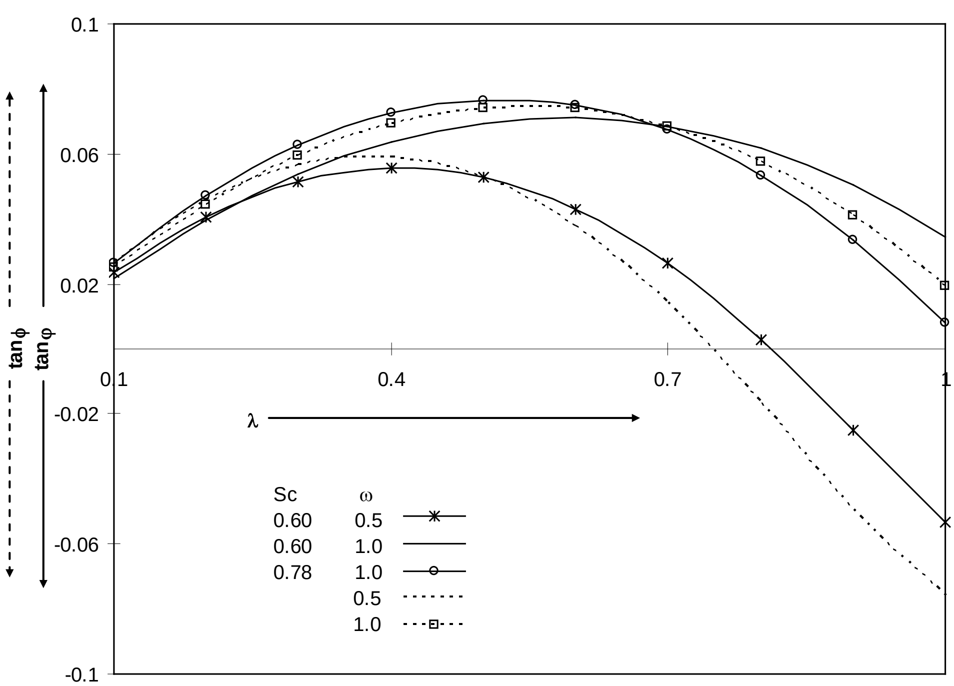


Fig.12 Phases of rate of heat transfer ($\tan\phi$) for Pr = 0.71 and rate of mass transfer ($\tan\varphi$)



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GEOMATIC Techniques to Study the Ecological Processes of Field Data

By Imteaz Husain, Nasiruddin Khan & Syed Shahid Shaukat
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Abstract- Geometic techniques are widely applied in landscape ecology to quantify the spatial patterns exhibited within species and its associated root fungi. Several analyses have been developed and modified to improve the ability to detect and characterize the patterns by the growth of species and population in various scale. Analysis of spatial pattern for disease and pathogen are of interest for the understanding and management plan to control the increase or spread within plant populations due to the numerous shared channels. Mathematical approaches encounter and give quantitative information of infections caused by many soilborne plant pathogens which are generally found in clusters of patches. Reliable approach for the use of spatial data can be established either by the research objective or by the measurement types and sampling designs procedures. We applied several mathematical tools to quantify the quantitative nature of Meloidogyne javanica and its associated soil fungi and soil characteristics in the development of nematode populations in the tomato grown field.

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GJSFR-F Classification : MSC 2010: 92D40

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Imteaz Husain [α], Nasiruddin Khan [σ] & Syed Shahid Shaukat [ρ]

Abstract- Geometic techniques are widely applied in landscape ecology to quantify the spatial patterns exhibited within species and its associated root fungi. Several analyses have been developed and modified to improve the ability to detect and characterize the patterns by the growth of species and population in various scale. Analysis of spatial pattern for disease and pathogen are of interest for the understanding and management plan to control the increase or spread within plant populations due to the numerous shared channels. Mathematical approaches encounter and give quantitative information of infections caused by many soilborne plant pathogens which are generally found in clusters of patches. Reliable approach for the use of spatial data can be established either by the research objective or by the measurement types and sampling designs procedures. We applied several mathematical tools to quantify the quantitative nature of Meloidogyne javanica and its associated soil fungi and soil characteristics in the development of nematode populations in the tomato grown field.

Keywords: GEOMATIC techniques, spatial pattern analysis, canonical correspondence analysis.

I. Introduction

Plant parasitic nematodes are well known parasites worldwide and utilize all parts of the host, and affect virtually every crop within their domain. Root-knot nematode species (*Meloidogyne*), are the most important plant-parasitic nematodes, infecting a wide range of cultivated plants, and are responsible for billions of dollar crop losses annually[1,2]. Since nematodes (saprotrophs or parasites) are more abundant in the rhizosphere, it is not surprising that their natural enemies will also be more numerous in such a habitat. The fungal antagonists of nematodes consist of a wide array of organisms, which include the nematode-trapping or predaceous fungi, endoparasitic fungi, parasites of nematode eggs and fungi, which produce metabolites toxic to nematodes[3]. In view of the long co-evolution of nematodes and fungi, which obviously occurred in the close confines of the soil habitat, it is a great variety of interrelationships have developed between two groups.

There are two important factors whereby environmental forces impact nematode community structure i.e., the community composition of nematode[45], and land management practices[6], mineral fertilizers[7], and toxic substances such as pesticides[8], affect nematode community composition. Most of these factors also influence the survival and proliferation of other micro-inhabitants including fungi in the rhizosphere.

Plant-parasitic nematodes generally have aggregated spatial distribution in an area, such as an agricultural field, with frequency distributions generally described by negative binomial function[9,10]. Taylor's Power Law[11] , which empirically expresses the relation between variance and mean, has also been used to measure the distribution and to develop sampling strategies for nematodes[12,13]. While the soil sampling strategy often required is a systematic sampling over large areas, the studies usually report average values of nematode densities for the entire area field[14]. In the absence of information on spatial variation, site-

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specific management strategies can not be formulated. Vast expanses involved in obtaining and analyzing soil/nematode samples makes it imperative that techniques of assessing spatial variability of root-knot nematode populations be highly efficient. Increased efficiency can be attained through robust sampling designs or by relating root-knot nematode populations to readily measured properties, such as soil pH, soil texture and organic matter that are expected to exhibit some degree of spatial pattern in the field[15].

Mathematical techniques enable us to assess the quantitative relationship of population dynamics of pathogens and has key role for the development of simulation models. It is also used for the design of experimental research[16,17].

Mathematical approach of canonical correspondence analysis (CCA) is a relatively recent technique that exposes the joint structure of populations and the associated environmental data[18]. In terms of a linear model, CCA is based on a weighted multivariate regression of transformed species data on the covariable environmental data set. Our central goal is to determine the causes by which environmental factors within human management practice change the community composition of nematode. In this regard, published data only reported control plot or laboratory experimentation. Assessing the quality of soil at various scales depends on several factors and community composition[19].

Present study applied to check the non-randomness and analyze the spatial pattern of root-knot disease incited by *Meloidogyne javanica* (the root-knot nematode) in tomato grown field, nematode populations (in soil and roots), as well as colonies of the colonization incidence of three soilborne root-infecting fungi, *Fusarium solani*, *Paecilomyces lilacinus* and *Aspergillus* spp. to observe the relationship among nematode population with fungus in the study area.

II. Materials and Methods

A field plot located at Gharo (approximately 50 km from Karachi city) was selected for sampling. The field chosen for the study is a cultivated field where generally tomato (*Lycopersicon esculentum* Mill.) and guar (*Cyamopsis tetragonoloba* L.) are grown in rotation for the last 10 years. For the purpose of sampling, a 14.4 × 14.4 meter square grid matrix was designed in the field. The grid matrix was divided into hundred and forty-four 1.2 m × 1.2m contiguous grids (quadrats). Each grid was sampled for *Meloidogyne javanica* (Treub.) Chitwood and fungi occurs in the soil and the rhizosphere. A number of microfungi were recorded but the analysis was restricted to only the abundant ones in the soil or nematode eggs; these included *Fusarium solani*, *Paecilomyces lilacinus, and Aspergillus* spp. Each grid unit arranged three tomato plants in two rows each row separated 60 cm apart and plant-to-plants distances were kept approximately 40 cm. A steel corer of 13 mm diam. × 189 cm deep is used to collect the six replicate cores near to the roots of selected field plot. The replicate soil samples from each grid were pooled to obtain composite samples. We used different mathematical techniques to analyze our data obtained from the field plot. Data matrix is further divided in four classes for the simplicity to compare the results. In First class (C1), we have taken 1x1 matrix grid (1 block), i.e., 12 rows and 12 columns, total 144 grids. In second class (C2), designed by dividing the whole field plot in terms of 2 x 2 matrix grid (4 blocks), i.e., each matrix consists 6 rows and 6 columns, total 36 grids . Third class (C3), designed by dividing whole field plot in terms of 3x3 matrix grid (3 blocks), i.e., 4 rows and 4 columns, total 16 grids. Fourth class (C4), designed by dividing whole field plot into 4 x 4 matrix grid (4 blocks), i.e., 3 rows and 3 columns, total 9 grids (Table-1).

III. Mathematical Techniques

a) Aggregation Indices

To analyze the spatial pattern of nematode population, rhizosphere and egg parasitic fungi and the associated soil characteristics, an index of pattern detection, which is not affected by the changes in density caused by random thinning, is required[20] . Therefore, aggregation indices including Lloyd's[21] index of patchiness (c) and Morisita[22] index (I_δ) were employed because they are insensitive to changes in density. First mean crowding m* was calculated as:

$$m^{*} = \frac{1}{N}\sum_{i=1}^{Q} X_i(X_i - 1) \qquad [1]$$

Notes

X_i represents nematodes soil population (or colonization frequency by the fungi) in the ith quadrate, whereas, number of quadrats is denoted by Q, with $N = \sum X_i$. While $c = \frac{m}{\lambda}$, such that λ is almost equals to average density in each quadrat. Indices for Lloyd's and Morisita's were determined by the formula:

$$I_d = \frac{Q \sum X_i (X_i - 1)}{N(N-1)} \quad [2]$$

Notes

Morisita's index 1indicate the random distribution whereas, greater than 1 indicate for aggregated pattern of root-knot nematode soil population. The variance/mean ratio[23] for each variable (dependent and independent) was applied to measure the non-randomness. Indices for Lloyd's and Morisita's were also calculated in different blocks such as 1 × 1, 2 × 2, 3 × 3 and 4 × 4 simultaneously.

b) Spatial Autocorrelation

We applied spatial autocorrelation method by using Moran's[24] *I* statistics for continuous variables as opposed to binary variables. As this technique is closely associated to the spatial autocorrelation coefficient. Therefore, a strong positive spatial autocorrelations may determine if Moran's *I* gives a relatively larger value. We can depict spatial information by using Moran's *I* test for content of the field sampling by utilizing information on the location of each sample point[25]. Thus, spatial autocorrelation was employed to determine autocorrelation of the nematode, soil-borne fungi and some soil factors among the quadrates. The statistic was computed as follows:

$$I = \frac{\left(\frac{1}{W}\right) \sum_{i=1}^{Q} \sum_{j=1}^{Q} \left[W_{ij} (x_i - x_m)(x_j - x_m) \right]}{\left[\frac{1}{Q} \sum_{i=1}^{Q} (x_i - x_m)^2 \right]} \quad [3]$$

X_m represent the mean, W_{ij} is the weight assigned to the join between quadrats *i* and *j*, and *W* is the sum of all weights. There is no trend detected if Moran's *I* statistic is approximately equal to zero in the spatial pattern. Spatial autocorrelation is tested against the null hypothesis that sample *I* do not differ from the expected value:

$$E(I) = \frac{-1}{Q-1} \quad [4]$$

Randomization of null hypothesis is used to check the significance of Moran's *I* whereas, a wide range of pattern detection indices techniques were used in research because they measure slightly different aspects of pattern[23].

c) Canonical Correspondence Analysis

The canonical correspondence analysis (CCA)[26] is employed here to expose the correlation structure between the species data set (nematode and fungi) and the environmental data set (i.e., soil variables). CCA has been widely used in community ecology to unravel the interrelationships between species populations and environmental characteristics. In CCA, nematode and fungal species and associated environmental factors are arranged in multidimensional space, with the restriction that the ordination axes must be linear combinations of the specified environmental variables. CANOCO, version 4.5[27] was used to accomplish canonical correspondence analysis. Data on the abundance of fungal species were log-transformed to dampen the influence of dominant species.

IV. Results

Field plot design for analysis are given in Table-I. Results of Lloyd's index (m^*), Morisita's index (I_δ), variance mean ratio and Moran's index for autocorrelation for number of root-knot nematode eggs root and juvenile population density in soil, egg parasitism by *F. solani*, *P. lilacinus* and *Aspergillus* spp are presented in Table-II. Generally, mean crowding (the clumping together of individuals within a quadrat) for nematode eggs per root system increased with the increase in block size. By contrast, mean crowding of nematode juveniles, egg parasitism by fungi and rhizosphere population of fungi decreased if we increased the size of blocks (quadrats). In most cases, variances of Morisita's index were found relatively low as compare to the variance of Lloyd's index. The variance/mean ratios were much higher than unity in every instance, implying aggregation of root-knot nematodes and associated soil-borne fungi. The variances of Lloyd's index for different organisms, in general, increased with increasing of block size. Morisita's index for *F. solani* and *Aspergillus* spp. generally increased with increasing block size but for other organisms it varied. Moran's *I* statistic for spatial autocorrelation was found significantly greater than the expected value *E(I)* for the nematode population in soil ($p<0.05$), and rhizosphere colonization by *F. solani* ($p<0.001$) and *P.lilacinus* ($p<0.01$) and egg-parasitism by fungi ($p<0.001$). In comparison of the results of Lloyd's and Morisita's index, the characteristics of mean crowding values found to be independent of block size (quadrat). In this regard, if the value of Morisita's index varies with the increase or decrease of block size may reflect the scale and pattern intensity. However, index value larger than 1 (expected value for random distribution) in Morisita's index showed significantly at various block sizes with the exception of block size 4×4 for *P. lilacinus* ($p<.001$).

The frequency of eggs produced by *M. javanica* was significantly positively correlated at small block sizes i.e., 1×1 ($p<0.001$) and 2×2 ($p<0.01$). The rhizosphere colonization frequency of the fungi (*F. solani, P.lilacinus* and *Aspergillus* spp.) showed negative correlation with number of eggs produced by *M. javanica* intensity at all block sizes. However, total numbers of eggs were found correlated with parasitism by fungi only at smaller block sizes (1 x 1 and 2 x 2).

A perusal of CCA ordination discloses that the quadrats (grid units) are distributed in almost the entire ordination plane but they exhibit clustering at low values of the first CCA axis and low to middle values of the second canonical axis (Figure-1). These quadrats generally belong to the lower left side of the sampled grid. The total variation of species composition data set was 0.193, and sum of all canonical eigenvalues (total explained variation) was 0.174 (Table-3). The Monte Carlo permutation test (499 permutations) showed that a significant relationship between quadrat to quadrat variation in fungal and nematode populations and the environmental (soil) variables ($F=1.815$, $P=0.0220$). The first two CCA axes were also significant based on the Monte Carlo permutation test with 499 permutations. For the first canonical axis the eigenvalues was 0.097 corresponding to second axis was 0.042 ($p<0.01$) (Table-4).

V. Discussion

Our results showed lowest I_δ value or small plot size (4 x 4) and greater values in plot size (3 x 3). In this regard sampling was consider more reliable if I_δ value could be minimized[22], and selection of appropriate plot size for sampling could have significant bearing on sampling efficiency[17]. Lowest I_δ value in block size (4 x 4) elaborate that highest sampling precision can be achieved. The sampling intensity must be greater enough to measure the populations so that the management decisions can be made at an acceptable level of risk[14]. Not surprisingly, the frequency of eggs produced by *M. javanica* was positively correlated ($p<0.001$) with nematode densities in the soil, particularly at larger block sizes *i.e.*, 1×1 and 2×2. However, such relationships were not recorded at smaller block sizes *i.e.*, 3×3 and 4×4. The colonization frequency of the fungi (*F. solani, P .lilacinus* and *Aspergillus* spp.) indicates significantly positive correlation with egg masses intensity relatively (3 x 3) or (4 x 4) blocks reflecting their interaction at a small spatial scale. Relationship among root-knot nematode and their associated root-infecting fungi is in the cultivation field is very familiar and often ignored to measure their interaction at different scales. After analyzing two data sets, the spatial pattern of nematode population has found to be direct effects of management practices. Ordination of sites and genera for each dataset were performed separately most of them influence in structuring nematode populations, i.e., soil moisture, and pH[28]. Environmental class variables (i.e., Soil moisture, Organic matter, pH, and calcium carbonate, Nitrate, Phosphorus and Potassium) were coded as nominal variables. Log transformation were applied to normalize the data of nematode abundance prior to

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application of CCA[29]. Responses of nematode populations to indirect effects were estimated using partial CCA[26] ordinations. CCA biplots indicate relative importance of a vector is its length; the angle indicates correlation with other vectors and CCA axes and triplot from a CCA of the root-knot nematode and soil fungi of tomato egg plants are shown in Figure-2. Environmental variables are represented by. Eigenvalues for CCA axes indicate the importance of the axes in explaining relationships in the genera-environment data matrices. The total variation of species composition dataset was 0.193, and sum of all canonical eigenvalues (total explained variation) was 0.174 (i.e., 17.4% of variation in species distribution was explained by the seven (environmental) variables. However, results of canonical correlations and corresponding eigenvalues shows the cumulative percentage of variance of species-environmental relations for first axis was 56.2% and for the first two axes nearly 80.1% are indicating high degree of environmental variables control of nematode and fungi populations . Significance of canonical correlation was verified by Monte Carlo test and showed that a significant relationship between quadrat to quadrat variation in fungal and nematode populations and the environmental (soil) variable at P=0.022. The first two CCA axes were also found to be significant, the first canonical axis the eigenvalues was 0.097 corresponding to second axis was 0.042 ($p < 0.01$). The F-ratio for the first canonical axis (eigenvalues=0.097) was 1.024 (p=0.004) while the F-ratio for all canonical axes (all eigenvalues, trace=0.174) was 1.815 (p=0.022).

VI. Conclusion

Plant parasitic nematodes (*Meloidogyne javanica*) largely damaged vegetation of cultivation field used in traditional method to design the plot. However, our mathematical technique to design the field in terms of matrix model in various block sizes (i.e., 1 x 1, 2 x 2, 3 x 3 and 4 x 4) before the vegetation of crops give more reliable results in the prevention of root-knot nematode disease. Moreover, our study also suggested that in small block sizes, disease frequency of root-knot nematode in tomato plants found significantly low, at the same time nematode were controlled by their root infected fungi (*Paecilomyces lilacinus*) as a biocontrol in small blocks.

Table 1 : Arrangements of field plot of the study area, data were arranged in four classes before the analyses.

Class 1 (C1)	Class 2 (C2)	Class 3 (C3)	Class 4 (C4)
1 x 1	2 x 2	3 x 3	4 x 4
144 grids	36 grids	16 grids	9 grids
12R x 12C	6R x 6C	4R x 4C	3R x 3C

Table 2 : Jackknife estimates of Lloyd's index, Morisita's index and Moran's index, variance-mean ratio (V:M) with their autocorrelation for nematode and fungi in soil population.

Variables	Blocks	Mean	V:M	Lloyd's index		Morisita's index		Moran's index	
				value	variance	value	variance	I	P(I = E(I))
No. of eggs	1 x 1	74311	30527	104857	3.2	1.410	0.0030	0.005	n.s.
	2 x 2	-	43874	119349	36.2	1..594	0.030	-	-
	3 x 3	-	30581	145894	58.3	1..275	0.015	-	-
	4 x 4	-	29610	166702	111.9	1.234	0.016	-	-
Nematode soil population	1 x 1	4313	2266	6563.2	5570	1.521	2.97	0.014	<0.05
	2 x 2	-	1846	6154.3	2871	1.411	7.54	-	-
	3 x 3	-	2030	5444.0	1096	1.549	3.28	-	-
	4 x 4	-	1415	5117.0	6660	1.325	1.97	-	-
Fusarium solani	1 x 1	27.6	95.02	122.64	299.7	4.38	0.357	0.051	<0.001
	2 x 2	-	74.28	97.32	425.1	4.52	0.683	-	-

	3 x 3	-	72.67	109.39	3118	6.80	5.157	-	-
	4 x 4	-	29.38	50.43	772.3	6.10	4.105	-	-
Paecilomyces lilacinus	1 x 1	4.08	8.76	11.91	1.73	2.908	0.037	0.031	<0.01
	2 x 2	-	7.09	9.84	11.5	3.102	0.465	-	-
	3 x 3	-	1.383	2..25	1.85	1.50	0.550	-	-
	4 x 4	-	0.252	0.607	0.09	0.483	0.043	-	-
Aspergillus niger	1 x 1	27.6	95.02	122.64	299	4.388	0.357	0.007	n.s.
	2 x 2	-	74.28	97.32	425	4.523	0.683	-	-
	3 x 3	-	17.45	72.67	109	3118	6.80	-	-
	4 x 4	-	29.38	50.43	772	6.10	4.105	-	-
Egg parasitism	1 x 1	10.9	4.68	14.618	0.91	1.334	2.17	0.039	<0.001
	2 x 2	-	3.20	11.668	1.53	1.225	3.54	-	-
	3 x 3	-	2.04	8.3210	1.55	1.133	7.12	-	-
	4 x 4	-	1.39	6.8360	1.29	1.0534	4.08	-	-

Table 3 : Results of canonical correspondence analysis. Eigenvalues, species-environmental correlations, cumulative percentage variance of species data and cumulative percentage variance of species-environment relations.

Axes	1	2	3	4	Total inertia
Eigenvalues	0.097	0.042	0.018	0.016	0.193
Species-environmental correlation	1.000	0.999	0.988	0.997	
Cumulative percentage variance of species data	50.6	72.1	81.3	89.8	
Cumulative percentage variance of species-environmental relation	56.2	80.1	90.3	99.7	
Sum of all Eigenvalues	0.193				
Sum of all canonical eigenvalues	0.174				

Table 4 : Monte Carlo permutation test, test for significance of first canonical axis.

Eigenvalues	0.097
F-ratio	1.024
P-value	0.004
Trace	0.174
F-ratio	1.815
P-value	0.022

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Figure 1 : CCA-ordination diagram of the sampling plots with biological (Δ) and environmental variables (arrows); first axis is horizontal, second axis vertical. The biological variables including root-knot nematode and fungi. The fungi are: A. spp. = Aspergillus spp, Paeci = Paecilomyces lilacinus, Fusar = Fusarium solani. And nema = nematode soil population, eggs = number of eggs, eggpar = egg parasitism.

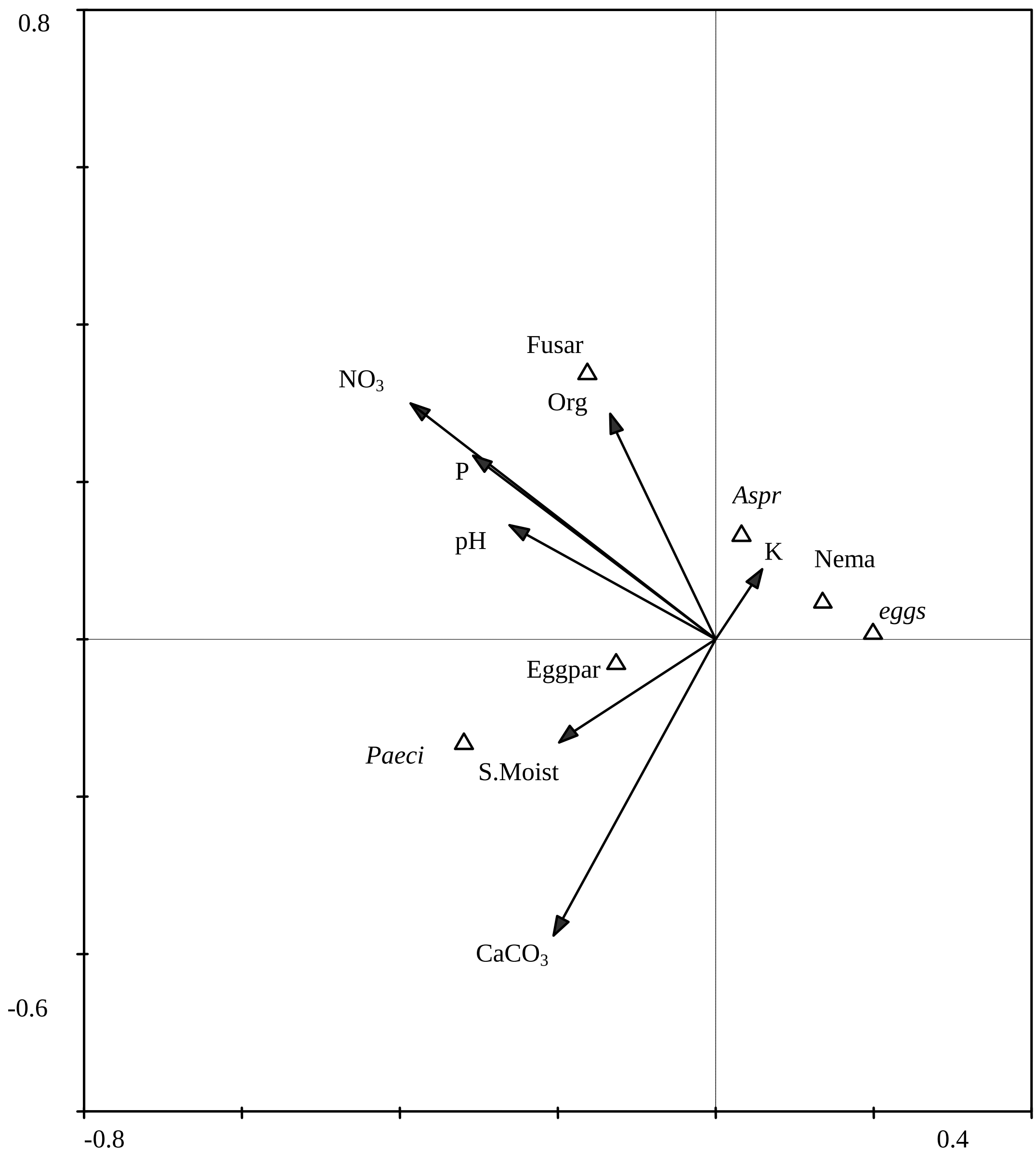
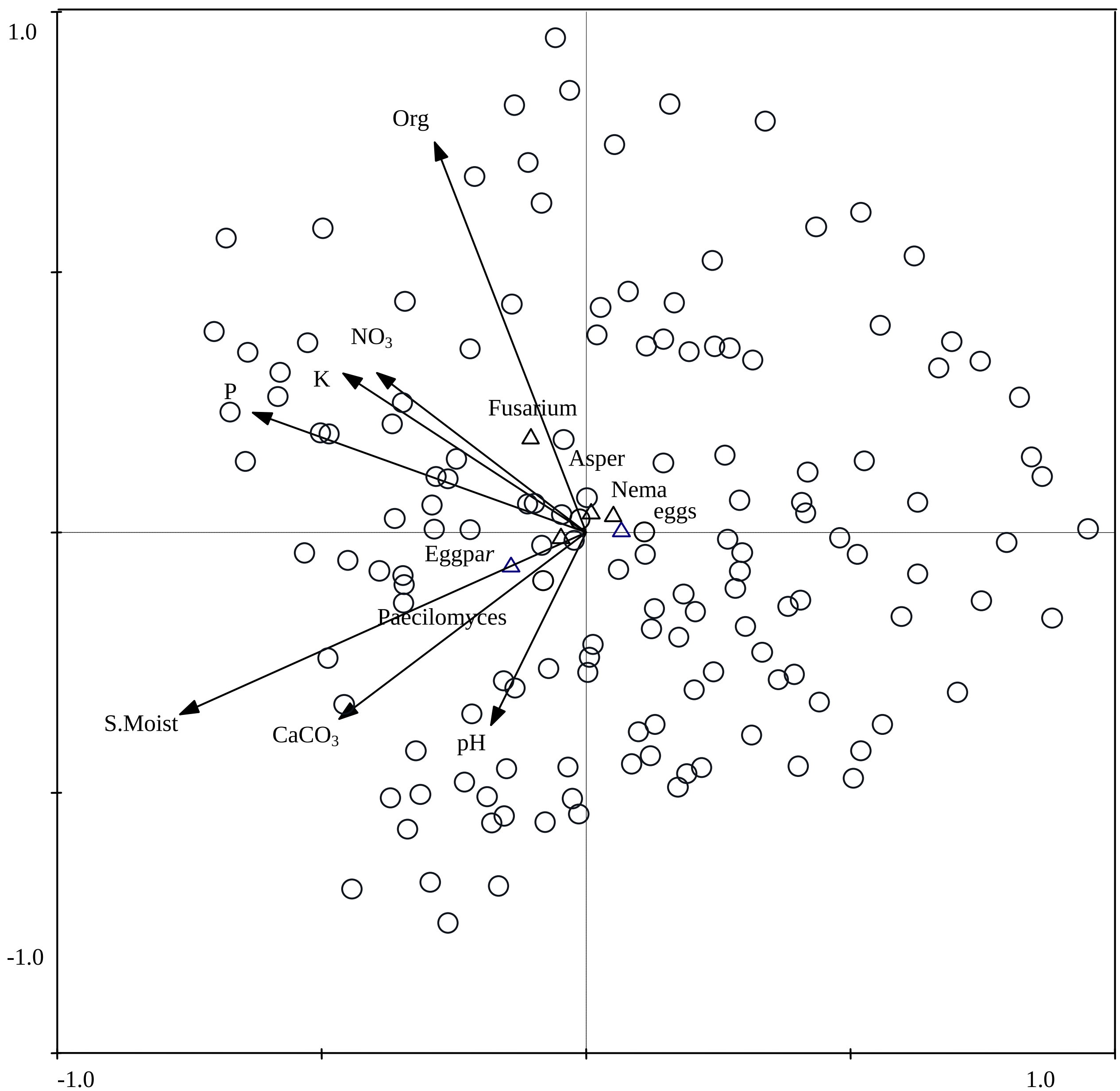


Figure 2 : Triplot from a CCA of the root-knot nematode and soil fungi of tomato egg plants. Environmental variables are represented by arrows, samples (quadrats) by small open circles, and species by triangle (Δ). The species are listed by the initial letters of the genus and the specific epithet, they include: A. fumig. = Aspergillus spp., Paeci = Paecilomyces lilacinus, Fusar = Fusarium solani. And nema = nematode soil population, eggs = number of eggs, eggpar = egg parasitism.



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An Amazing Result Involving Contiguous Relation : A Comutational Approach

By Salahuddin & Vinesh Kumar
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Abstract- The main aim of the present paper is to evaluate an amazing summation formula involving recurrence relation of Gamma function and contiguous relation.

Keywords: *gauss second summation theorem, recurrence relation, prudnikov.*

GJSFR-F Classification : MSC 2010: 33C05, 33C20, 33D15, 33D50, 33D60

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An Amazing Result Involving Contiguous Relation : A Comutational Approach

Salahuddin [α] & Vinesh Kumar [σ]

Abstract- The main aim of the present paper is to evaluate an amazing summation formula involving recurrence relation of Gamma function and contiguous relation.

2010 MSC NO : 33C05 , 33C20 , 33D15 , 33D50 , 33D60

Keywords: gauss second summation theorem, recurrence relation, prudnikov.

I. Introduction

Generalized Gaussian Hypergeometric function of one variable is defined by

$$ {}_AF_B\left[\begin{array}{ccc} a_1, a_2, \cdots, a_A & ; & \\ & & z \\ b_1, b_2, \cdots, b_B & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!} \tag{1} $$

where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and $A,\ B$ are non-negative integers and $\mid z \mid = 1$

Contiguous Relation is defined by

[Andrews p.363(9.16), E. D. p.51(10)]

$$ (a-b)\ {}_2F_1\left[\begin{array}{ccc} a,\ b\ ; & \\ c\ \ ; & z \end{array}\right] = a\ {}_2F_1\left[\begin{array}{ccc} a+1,\ b & ; & \\ c & ; & z \end{array}\right] - b\ {}_2F_1\left[\begin{array}{ccc} a,\ b+1 & ; & \\ c\ ; & & z \end{array}\right] \tag{2} $$

Gauss second summation theorem is defined by [Prudnikov., 491(7.3.7.5)]

$$ {}_2F_1\left[\begin{array}{ccc} a,\ b & ; & \\ \frac{a+b+1}{2} & ; & \frac{1}{2} \end{array}\right] = \frac{\Gamma(\frac{a+b+1}{2})\ \Gamma(\frac{1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} \tag{3} $$

$$ = \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{a+b+1}{2})}{\Gamma(b)\ \Gamma(\frac{a+1}{2})} \tag{4} $$

In a monograph of Prudnikov et al., a summation theorem is given in the form [Prudnikov., p.491(7.3.7.8)]

$$ {}_2F_1\left[\begin{array}{ccc} a,\ b & ; & \\ \frac{a+b-1}{2} & ; & \frac{1}{2} \end{array}\right] = \sqrt{\pi}\left[\frac{\Gamma(\frac{a+b+1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} + \frac{2\ \Gamma(\frac{a+b-1}{2})}{\Gamma(a)\ \Gamma(b)}\right] \tag{5} $$

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Now using Legendre's duplication formula and Recurrence relation for Gamma function, the above theorem can be written in the form

$$ {}_2F_1\left[\begin{matrix} a,\ b & ; & \frac{1}{2} \\ \frac{a+b-1}{2} & ; & \end{matrix}\right] = \frac{2^{(b-1)}\ \Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} + \frac{2^{(a-b+1)}\ \Gamma(\frac{a}{2})\ \Gamma(\frac{a+1}{2})}{\{\Gamma(a)\}^2} + \frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\right] \tag{6} $$

Recurrence relation is defined by

$$ \Gamma(\zeta+1) = \zeta\ \Gamma(\zeta) \tag{7} $$

Notes

II. Main Summation Formula

$$ {}_2F_1\left[\begin{matrix} a,\ b & ; & \frac{1}{2} \\ \frac{a+b+49}{2} & ; & \end{matrix}\right] = \frac{2^b\ \Gamma(\frac{a+b+49}{2})}{(a-b)\ \Gamma(b)\left[\prod_{\mu=1}^{24}\{a-b-(2\mu-1)\}\right]\left[\prod_{\xi=1}^{24}\{a-b+(2\xi-1)\}\right]} $$

$$ \left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+1}{2})}\right\{8388608(a^25+a^24(-576+1127b)+4a^{23}(39169-51744b+48363b^2)+92a^{22}(-290928 $$
$$ +634207b-221088b^2+131271b^3)+3542a^{21}(909563-1563264b+1548462b^2-243648b^3 $$
$$ +103071b^4)+3542a^{20}(-82032288+188926633b-99710688b^2+611091118b^3-5432448b^4+1740081b^5) $$
$$ +92a^{19}(221572333763-429292237920b+4389745190095b^2-111905902080b^3+480216378825b^4 $$
$$ -27284470008b^5+6861719941b^6)+1748a^{18}(-652191333552+1552836538141b-997765722240b^2 $$
$$ +621099121475b^3-930001144480b^4+298139678827b^5-1169334432b^6+236070989b^7) $$
$$ +437a^{17}(118237765837763-245960503305216b+256017026954220b^2-82688287167360b^3 $$
$$ +35577424928010b^4-3489293945088b^5+867694870812b^6-247230708848b^7+40561288811b^8) $$
$$ +437a^{16}(-4391592874255488+10679020370180365b-7723439928047424b^2+4831986595164316b^3 $$
$$ -939198292845120b^4+298335631111518b^5-205124646066144b^6+40429946796612b^7-865307479668b^8 $$
$$ +11567478461b^9)+1288a^{15}(45681097143081937-99574723999503648b+104501659542515343b^2 $$
$$ -38985088377593088b^3+16661320812453658b^4-2153363100178752b^5+523951164774702b^6 $$
$$ -26375984727552b^7+4156630669317b^8-67988444832b^9+7318200659b^{10}) $$
$$ +1288a^{14}(-1160835982753925712+2858915589700984289b-2236018737076611360b^2 $$
$$ +1393738308249650165b^3-317752185259953120b^4+99006207906524274b^5-9037635481929600b^6 $$
$$ +1714993712797890b^7-64723675029840b^8+8093019369325b^9-100363894752b^{10}+8268616329b^{11}) $$
$$ +4a^{13}(7885248558387029118799-17740046200259070839424b+18621420718060507752818b^2 $$
$$ -7640510532582764343360b^3+3212288577782897155725b^4-490605934314569676288b^5 $$
$$ +115216473790726873964b^6-7683233847037257600b^7+1135210887503614425b^8 $$
$$ -32003974007617920b^9+3027437157819074b^{10}-25930380807744b^{11}+1215486600363b^{12}) $$
$$ -4a^{12}(137971112401324506880224-341745755847769091259581b+281805523312710309223584b^2 $$
$$ -173404413877307333549010b^3+43827888254763867124416b^4-13217460237988621117867b^5 $$
$$ +1422491466706905381696b^6-253712233399682275036b^7+12395651957628252768b^8 $$

$$-1365375597501222307b^{9}+2627597019333368 64b^{10}-14010508880184 18b^{11}+1215486600363b^{13})$$

$$-8a^{11}(-9979652555215563016309 37+2290880360805370896208416b$$

$$-2384783868725789458101993b^{2}+1041281209707578516441 8856b^{3}-4251071255373456297 02597b^{4}$$

$$+7182452187364449275 2800b^{5}-1589724846392451112 0437b^{6}+1221363446491517032704b^{7}$$

$$-1593642889524313497 87b^{8}+5235987688270186848b^{9}-321328557128862283b^{10}$$

$$+700525444009209b^{12}-12965190403872b^{13}+1331247228969b^{14})$$

$$-88a^{10}(1078858323154473624051792-2668043597373206882428487b$$

$$+2273448423168555362100672b^{2}-1363876306485811724341981b^{3}+366249849280561645923888b^{4}$$

$$-104419122164168117454239b^{5}+12149335654454468931552b^{6}-1918719572514798578525b^{7}$$

$$+97223011941315724464b^{8}-69646300946524592 29b^{9}+29211687011714753b^{11}$$

$$-1194362281515312b^{12}+137610779900867b^{13}-14689624595 52b^{14}+107111846009b^{15})$$

$$-7a^{9}(-1314350642770923712884991 93+3044705324899863378209126 40b$$

$$-3106772642136253036388782 80b^{2}+1401117841930904892188213 76b^{3}$$

$$-5427455425066842637295 0732b^{4}+9523746494510523997822464b^{5}$$

$$-18719720451893882364077 68b^{6}+1399759836350631784005 12b^{7}-1190875279984040861181 4b^{8}$$

$$+875553497613452017 36b^{10}-5983985929451642112b^{11}+78021462714355560 4b^{12}$$

$$-18287985147210240b^{13}+1489115563955800b^{14}-125098738490 88b^{15}+722141155351b^{16})$$

$$-7a^{8}(1025362604429196125260479168-2506679259033952138613891519b$$

$$+2159784906066357430182552960b^{2}-1235115440333777196206435368b^{3}$$

$$+3350471374498566981873957 12b^{4}-8511830124772857828221562 0b^{5}$$

$$+9261191686825417130273664b^{6}-9563264150266913438766696b^{7}+11908752799840408611814b^{9}$$

$$-1222232150119397678976b^{10}+18213061594563582832 8b^{11}-708322969007328729 6b^{12}$$

$$+648691935716351100b^{13}-11909156205490560b^{14}+764820043154328b^{15}-5401990980288b^{16}$$

$$+253218327201b^{17})-4a^{7}(-1107990220845419349227262 3069+2552200049521438545913453 6608b$$

$$-2498884920995388531015698 1095b^{2}+1115536243631618874577108 5312b^{3}$$

$$-3866575614517011321331687540b^{4}+6173131049168600566149730 56b^{5}$$

$$-7958805301988469036474343 6b^{6}+1673571226296709851784218b^{8}-2449579713613605622008 96b^{9}$$

$$+4221183059532556872755 0b^{10}-24427268929830340654 08b^{11}+253712233399682275036b^{12}$$

$$-7683233847037257600b^{13}+552227975520920580b^{14}-8493067082271744b^{15}+441697168747611b^{16}$$

$$-2700995490144b^{17}+103163022193b^{18})-4a^{6}(528956586037154221773361777 44$$

$$-12550449948451791312048109 6725b+1054246909288081848081182569 92b^{2}$$

$$-542485523600623465721875 75205b^{3}+1312654113224400014927724 7680b^{4}$$

$$-2195627585159893786910327620b^{5}+79588053019884690364743436b^{7}$$

$$-1620708545194447997797891 2b^{8}+3275951079081429413713594b^{9}$$

$$-26728538439799831649414 4b^{10}+31794496927849022240874b^{11}-14224914667069053816 96b^{12}$$

$$+115216473790726873964b^{13}-291011862518133 1200b^{14}+16871227505745404 4b^{15}$$

$$-2240986758221232b^{16}+947956646362 11b^{17}-510999146784b^{18}+15781954643b^{19})$$

$$-2a^5(-377701803640400756305158527625+8400063125970788940576766 20672b$$

$$-746504836196868708716896610562b^2+292982022725550741839778156096b^3$$

$$-671910208246569975921135206 89b^4+439125517031978757382065524 0b^6$$

$$-123462620983372011322994611 2b^7+297914054367050023987754670b^8$$

$$-333331127307868339923786 24b^9+459444137522339716798651 6b^{10}$$

$$-287298087494577971011200b^{11}+264349204759772422357 34b^{12}-981211868629139352576b^{13}$$

$$+637599978918016324 56b^{14}-138676583651511628 8b^{15}+651863353978666 83b^{16}$$

$$-762410727001728b^{17}+260574078807 98b^{18}-1255085623 68b^{19}+3081683451b^{20})$$

$$-2a^4(95998344090527002729473327 6000-2103824541313062802889734547475b$$

$$+1539288905064687878184690274080b^2-527901057497272758919053580818b^3$$

$$+671910208246569975921135206 89b^5-262530822644880002985544953 60b^6$$

$$+77331512290340226426633750 80b^7-117266498107449844365588499 2b^8$$

$$+189960939877339492305327562b^9-16114993368344712420651072b^{10}$$

$$+1700428502149382518810388b^{11}-876557765095277342488 32b^{12}+642457715556579431145 0b^{13}$$

$$-204632407307409809280b^{14}+1072989060322015575 2b^{15}-205214826986658720b^{16}$$

$$+7773667346770185b^{17}-812821000555 20b^{18}+2208995339950b^{19}-9620865408b^{20}+182538741b^{21})$$

$$-28a^3(-11469567702520524360007533937 5+2225153458030773577304777400 00b$$

$$-13354547836357149688938336007 5b^2+377072183926623399227895414 87b^4$$

$$-2092728733753933870284129686 4b^5+77497931942946209388839393 15b^6$$

$$-159362320518802696368158361 6b^7+30877886008344429905160884 2b^8$$

$$-350279460482726223047053 44b^9+428646839181255113364622 6b^{10}$$

$$-297508917059308147554816b^{11}+24772059125329619078430b^{12}-10915015046546806204 80b^{13}$$

$$+641119621794839075 90b^{14}-179331406536928204 8b^{15}+754135050745287 89b^{16}$$

$$-1290527910433440b^{17}+387743308692 25b^{18}-3676908211 20b^{19}+7730303427b^{20}-30821472b^{21}$$

$$+431319b^{22})-28a^2(10949556710701056784228893000 0-1677378798158014617931917618 75b$$

$$+13354547836357149688938336007 5b^3-109949207504620562727477876 720b^4$$

$$+533217740140620506226354721 83b^5-150606701326868835440168938 56b^6$$

$$+35698356014219836157367115 85b^7-53994622651658935754563824 0b^8$$

$$+77669316053406325909719570b^9-71451236156726025666021 12b^{10}$$

$$+6813668196359398451719 98b^{11}-402579319018157584605 12b^{12}+266020295972292967897 4b^{13}$$

$$-102856861905524122560b^{14}+4807076338955705778b^{15}-120540830305597296b^{16}$$

Notes

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$$+3995694313535505b^{17}-622890886598 40b^{18}+14423448484455b^{19}-12613402032b^{20}+195880443b^{21}$$

$$-726432b^{22}+6909b^{23})-b(11925681927744341235399076406 25$$

$$-306587587899629589958409004000 0b+321147895670574682080210950250 0b^{2}$$

$$-191996688181054005458946655200 0b^{3}+755403607280801512610317055250b^{4}$$

$$-2115826344148616887093447109 76b^{5}+4431960883381677396909049227 6b^{6}$$

$$-7177538231004372876823354176b^{7}+920045449939646599019494351b^{8}$$

$$-94939532437593678916557696b^{9}+798372204417245041304749 6b^{10}$$

$$-5518844496052980275208 96b^{11}+31540994233548116475196b^{12}-1495156745787056317056b^{13}$$

$$+58837253120289534856b^{14}-1919126086049648256b^{15}+51669903671102431b^{16}$$

$$-1140030451048896b^{17}+20384654706196b^{18}-290558364096b^{19}+3221672146b^{20}-26765376b^{21}$$

$$+156676b^{22}-576b^{23}+b^{24})-7a(-170366884682062017648558234375$$

$$+670951519263205847172767047500b^{2}-890061383212309430921910960000b^{3}$$

$$+601092726089446515111352727850b^{4}-240001803599165398302193320192b^{5}$$

$$+71716856848295950354560626700b^{6}-14584000282979648833791163776b^{7}$$

$$+2506679259033952138613891519b^{8}-304470532489986337820912640b^{9}$$

$$+33541119509834600807672408b^{10}-2618148983777566738523904b^{11}$$

$$+195283289055868052148332b^{12}-10137169257290897622528b^{13}+526040468504981109176b^{14}$$

$$-18321749215908671232b^{15}+666675985966974215b^{16}-15354962849197056b^{17}$$

$$+387765466952924b^{18}-5642126555520b^{19}+95596876298b^{20}-791011584b^{21}+8335292b^{22}$$

$$-29568b^{23}+161b^{24}))\Big\}-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\Big\{268435456(3a^{2}4+a^{23}(-575+1078b)+23a^{22}(8921-5145b+4606b^{2})$$

$$+253a^{21}(-77095+150388b-31255b^{2}+17766b^{3})+1771a^{20}(1616303-1417115b+1295273b^{2}$$

$$-139825b^{3}+56588b^{4})+161a^{19}(-1046829225+2079583870b-701082550b^{2}+401485280b^{3}$$

$$-265256 25b^{4}+8120378b^{5})+437a^{18}(31431018189-337989625 05b+31115280280b^{2}-5662501250b^{3}$$

$$+22841038885b^{4}-1015047 25b^{5}+243611134b^{6})+437a^{17}(-1247763838705+2496774883556 8b$$

$$-1070526853935b^{2}+611526437130b^{3}-69822872175b^{4}+210967420 44b^{5}-669931185b^{6})$$

$$+1287659 94b^{7}+437a^{16}(63279202335014-77204535727935b+70972718319307b^{2}-16814438930835b^{3}$$

$$+6703353718635b^{4}-523216255485b^{5}+122670490257b^{6}-2897234865b^{7}+450680979b^{8})$$

$$+46a^{15}(-1670772770171612 5+3342798324748628 2b-1663783210911262 0b^{2}+9407919387839952b^{3}$$

$$-1419804931510350b^{4}+419788673696868b^{5}-236344413727 80b^{6}+4382164307808b^{7}$$

$$-788691713 25b^{8}+991498153 8b^{9})+322a^{14}(7929502219489805 9-10547396303966955 5b$$

$$+9614974168385207 0b^{2}-2683390220093990 0b^{3}+1048757469145669 0b^{4}-109143144065025 0b^{5}$$

$$+247315972449000b^{6}-10375779845100b^{7}+1527762530955b^{8}-21077766875b^{9}+2090914474b^{10})$$

$$+14a^{13}(-359501002184716600 65+714741176812498186 12b-393609737979937113 25b^{2}$$

$$+218636654008257351 30b^{3}-391921257901410405 0b^{4}+112219201590076638 4b^{5}$$

$$-843581541921218810b^{6}+1483553578162110 0b^{7}-469070372518125b^{8}+53762807034860b^{9}$$
$$-548865049425b^{10}+38586876202b^{11})-2a^{12}(-55833695107505586300 69$$
$$+789674016716696966 5365b-708770525053954764 2827b^{2}+2206545302765470623411b^{3}$$
$$-836963484922477152786b^{4}+1035712365594097891 70b^{5}-223071498576807289 50b^{6}$$
$$+12353746841594137 98b^{7}-166195491552756393b^{8}+383708959272138 5b^{9}-308337748365647b^{10}$$
$$+1755702867191b^{11})-2a^{11}(7767954491490205996802 5-152468593059178846200422b$$
$$+900326259976548762089 46b^{2}-486513617044728559788 32b^{3}+973739831803463679763 9b^{4}$$
$$-265601624628998309379 0b^{5}+234658508632340285948b^{6}-3778739845058821988 8b^{7}$$
$$+150196611678820952 7b^{8}-139758311120734306b^{9}+1780767039157106b^{10}-1755702867191b^{12}$$
$$+270108133414b^{13})-22a^{10}(-10424701545427185918961 9+15382957440770174714665 5b$$
$$-1346866623313093665911 24b^{2}+450037313111689816689 54b^{3}-163010638280958922001 73b^{4}$$
$$+22258371829550342572 45b^{5}-439971463136072527114b^{6}+27267243691424920092b^{7}$$
$$-2984757035815154509b^{8}+645997980506929 05b^{9}-1618879126506 46b^{11}+28030704396877b^{12}$$
$$-3492777587 25b^{13}+30603384574b^{14})-2a^{9}(1100904826889323264639920 5$$
$$-211491307199022851161842 80b+130458475287305439103053 35b^{2}$$
$$-675036718236365209746121 8b^{3}+143301662536697719066876 5b^{4}-359606374895799331858012b^{5}$$
$$+333770463713285871422 15b^{6}-438340022467236518228 6b^{7}+145377719814313881895b^{8}$$
$$-710597778557621955b^{10}+139758311120734306b^{11}-3837089592721385b^{12}+37633964924402 0b^{13}$$
$$-3393520466875b^{14}+228044575374b^{15})-7a^{8}(-300743046403179040312071 29$$
$$+454576529014104298333960 10b-382342977518027030727929 54b^{2}$$
$$+131897999823219291362660 58b^{3}-4407268569094241056242846b^{4}+607683841814605872882930b^{5}$$
$$-982029030214541229697 14b^{6}+47669351549867450257 14b^{7}-415364913755182519 70b^{9}$$
$$+938066496970477131 4b^{10}-429133176225202722b^{11}+4748442615793039 8b^{12}$$
$$-938140745036250b^{13}+70277076423930b^{14}-518283125850b^{15}+28135369689b^{16})$$
$$-7a^{7}(190235412397699903436626125-3525128676354317853734912 18b$$
$$+220189994397267940717904408b^{2}-105433147752212001296291552b^{3}$$
$$+22000623827493984736569132b^{4}-452570700374199724565901 6b^{5}$$
$$+316238775975782866448424b^{6}-476693515498674502571 4b^{8}+12524000641921043377 96b^{9}$$
$$-856970516016211774 32b^{10}+10796399557310919968b^{11}-352964195474118228b^{12}$$
$$+2967107156324220 0b^{13}-477285872874600b^{14}+28797079737024b^{15}-180870233715b^{16}$$
$$+8038677054b^{17})+a^{6}(777047070580415197112919948 3-1172604234352128024664255783 5b$$
$$+916222376030806638849209323 4b^{2}-304757559717906750436763314 4b^{3}$$
$$+837301129723740812841854612b^{4}-846095506634649671309015 40b^{5}$$
$$+221367143183048006513896 8b^{7}-687420321150178860787998b^{8}+667540927426571742844 30b^{9}$$

$$-967937218899359559650 8b^{10}+4693170172646805718 96b^{11}-446142997153614579 00b^{12}$$
$$+1181014158689705340b^{13}-79635743128578000b^{14}+10871843031478 80b^{15}-536070042423 09b^{16}$$
$$+2927599278 45b^{17}-10645815558b^{18})+a^{5}(-2966570297882889037505281 0935$$
$$+51335791176911784657425859492b-30488813775334767741610658 8895b^{2}$$
$$+120479363570298267458855230 46b^{3}-18074964816800409965750678 20b^{4}$$
$$+846095506634649671309015 40b^{6}-3167994902619398071961 3112b^{7}$$
$$+42537868927022411101805 10b^{8}-7192127497915986637160 24b^{9}+48968418025010753659390b^{10}$$
$$-5312032492579966187580b^{11}+2071424731188195783 40b^{12}-15710688222610729376b^{13}$$
$$+3514409238893805 00b^{14}-19310278990055928b^{15}+2286455036469 45b^{16}-92192762732 28b^{17}$$
$$+443575648 25b^{18}-13073808 58b^{19})+a^{4}(934814741293160291496940 56225$$
$$-132860977892412772958160833 5565b+86088395434515652346309994 951b^{2}$$
$$-2030474793207111788176710 8127b^{3}+18074964816800409965750678 20b^{5}$$
$$-83730112972374081284185461 2b^{6}+15400436679245789315598392 4b^{7}$$
$$-3085087998365968739369992 2b^{8}+28660332507339543813375 30b^{9}$$
$$-358623404218109628403806b^{10}+19474796636069273595278b^{11}-1673926969844954305572b^{12}$$
$$+5486897610619745 6700b^{13}-33769990506490541 80b^{14}+6531102684947 6100b^{15}$$
$$-29293655750434 95b^{16}+30512595140475b^{17}-998153397745b^{18}+42706256 25b^{19}-100217348b^{20})$$
$$+a^{3}(-178746168935420648161428273525+258379684607101604925551403510b$$
$$-107795538320563587651129599382b^{2}+20304747932071117881767108127b^{4}$$
$$-12047936357029826745885523046b^{5}+3047575597179067504367633 1144b^{6}$$
$$-738032034265484009074040864b^{7}+92328599876253503953862406b^{8}$$
$$-13500734364727304194922436b^{9}+990082088845717596716988b^{10}-9730272340894571195 7664b^{11}$$
$$+4413090605530941246822b^{12}-306091315611560291820b^{13}+8640516508702647800b^{14}$$
$$-4327642918406377 92b^{15}+7347909812774895b^{16}-26723705302581 0b^{17}+247451304 6250b^{18}$$
$$-646391300 80b^{19}+24763007 5b^{20}-4494798b^{21})-7a^{2}(-319755545790963647651027 62875$$
$$+317925260868550393400422785 75b-15399362617223369664447085626b^{3}$$
$$+12298342204930807478044284993b^{4}-43555448250478239630872369985b^{5}$$
$$+1308889108615438055498870462b^{6}-2201899943972679407179044 08b^{7}$$
$$+38234297751802703072792954b^{8}-3727385008208726831515810b^{9}+423300938755543723572104b^{10}$$
$$-25723607427901393202556b^{11}+2025058643011299326522b^{12}-78721947595987422650b^{13}$$
$$+442288811745719522 0b^{14}-109334325288454360b^{15}+44307254150767737b^{16}-6683146216708 5b^{17}$$
$$+1942482497480b^{18}-16124898650b^{19}+3277040 69b^{20}-1129645b^{21}+15134b^{22})$$
$$+b(115527957887185081078986770625-223828882053674553355719340125b$$
$$+178746168935420648161428273525b^{2}-93481474129316029149694056225b^{3}$$
$$+29665702978828890375052810935b^{4}-7770470705804151971129199483b^{5}$$

Notes

$$+13316478867838993240563828 75b^6 - 21052013248222532821844 9903b^7$$

$$+2201809653778646529279841 0b^8 - 2293434339993980902171618b^9 + 1553590898298041199360 50b^{10}$$

$$-1116673902150111726013 8b^{11} + 503301403058603240910b^{12} - 25532997146757174998b^{13}$$

$$+768555474278941750b^{14} - 27653011420401118b^{15} + 545272797514085b^{16} - 13735354948593b^{17}$$

$$+1685395052225b^{18} - 2862472613b^{19} + 195050035b^{20} - 205183b^{21} + 575b^{22} - 3b^{23})$$

$$-7a(16503993983883583011283824375 - 31792526086855039340042278575b^2$$

$$+36911383515300229275078771930b^3 - 18980139698916110422594404795b^4$$

$$+7333684453844540665346551356b^5 - 1675148906217325749520365405b^6$$

$$+352512867635431785373491218b^7 - 45457652901410429833396010b^8$$

$$+6042608777114938604624080b^9 - 483464376709919776746630b^{10} + 43562455159765384628692b^{11}$$

$$-2256211476333419904390b^{12} + 142948235362499637224b^{13} - 4851802299824799530b^{14}$$

$$+219669604197766996b^{15} - 4819768873301085b^{16} + 155870089159888b^{17} - 2110020944955b^{18}$$

$$+47830429010b^{19} - 3585300095b^{20} + 5435452b^{21} - 16905b^{22} + 154b^{23}))\Big\}\Bigg] \qquad (8)$$

III. Derivation of the Summation Formula

Putting $c = \frac{a+b+49}{2}$ and $z = \frac{1}{2}$ in equation (2), we get

$$(a-b)\ {}_2F_1\left[\begin{matrix} a,\ b\ ; \\ \frac{a+b+49}{2}\ ; \end{matrix}\ \frac{1}{2}\right] = a\ {}_2F_1\left[\begin{matrix} a+1,\ b\ ; \\ \frac{a+b+49}{2}\ ; \end{matrix}\ \frac{1}{2}\right] - b\ {}_2F_1\left[\begin{matrix} a,\ b+1\ ; \\ \frac{a+b+49}{2}\ ; \end{matrix}\ \frac{1}{2}\right]$$

Now involving the derived formula [Salahuddin et. al. p.12-41(8)], the summation formula is obtained.

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Exact Traveling Wave Solutions for Power law and Kerr law non Linearity Using the Exp($-\varphi$ (ξ))-expansion Method

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Abstract- The exp($-\varphi$ (ξ))-expansion method is used as the first time to investigate the wave solution of the nonlinear Burger equation with power law nonlinearity, the perturbed non-linear Schrodinger equation with kerr law nonlinearity. The proposed method also can be used for many other nonlinear evolution equations.

Keywords: exp($-\varphi(\xi)$)-expansion method, homogeneous balance, travelling wave solutions, solitary wave solutions, the nonlinear burger equation with power law nonlinearity, the perturbed nonlinear schrodinger equation with kerr law nonlinearity.

GJSFR-F Classification : MSC 2010: 35C07

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I. Introduction

The nonlinear equations of mathematical physics are major subjects in physical science [1]. Exact solutions for these equations play an important role in many phenomena in physics such as fluid mechanics, hydrodynamics, Optics, Plasma physics and so on. Recently many new approaches for finding these solutions have been proposed, for example, tanh - sech method [2]-[4], extended tanh - method [5-7], sine - cosine method [8]-[10], homogeneous balance method [11]and [12], Jacobi elliptic function method [13]-[16], F-expansion method [17]-[19], exp-function method [20]-[21], trigonometric function series method [22], $(\frac{G'}{G})-$ expansion method [23]-[26], the modified simple equation method [27]-[32] and so on.

In the present paper, we shall proposed a new method which is called exp-$\varphi(\xi)$-expansion method to seek traveling wave solutions of nonlinear evolution equations. The main ideas of the proposed method are that the traveling wave solutions of nonlinear evolution equation can be expressed by a polynomial in exp-$\varphi(\xi)$.

The paper is organized as follows: In section 2, we give the description of exp-$\varphi(\xi)$-expansion method. In section 3, we use this method to find the exact solutions of the nonlinear evolution equations pointed out above and some figures of our results are drawn. In section 4, conclusion are given.

II. Description of the Exp$(-\varphi(\xi))$-expansion Method

Consider the following nonlinear evolution equation

$$F(u, u_t, u_x, u_{tt}, u_{xx}, ...) = 0, \tag{2.1}$$

where F is a polynomial in $u(x,t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following,we give the main steps of this method

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[1] M. J. Ablowitz, H. Segur, Solitions and Inverse Scattering Transform, SIAM, Philadelphia 1981.

Step 1. We use the wave transformation

$$u(x,t) = u(\xi), \qquad \xi = x - ct, \tag{2.2}$$

where c is a positive constant, to reduce Eq.(1)to the following ODE:

$$P(u, u', u'', u''', ...) = 0, \tag{2.3}$$

where P is a polynomial in $u(\xi)$ and its total derivatives,while $' = \frac{d}{d\xi}'$.
Step 2. Suppose that the solution of ODE(2.3) can be expressed by a polynomial in $exp(-\varphi(\xi))$ as follows

$$u(\xi) = \alpha_m \left(exp(-\varphi(\xi))\right)^m + ..., \quad \alpha_m \neq 0, \tag{2.4}$$

where $\varphi(\xi)$ satisfies the ODE in the form

$$\varphi'(\xi) = exp(-\varphi(\xi)) + \mu exp(\varphi(\xi)) + \lambda, \tag{2.5}$$

the solutions of ODE (2.5) are
when $\lambda^2 - 4\mu > 0, \mu \neq 0$,

$$\varphi(\xi) = ln\left(\frac{-\sqrt{\lambda^2 - 4\mu}\ tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi + C_1)\right) - \lambda}{2\mu}\right), \tag{2.6}$$

when $\lambda^2 - 4\mu > 0, \mu = 0$,

$$\varphi(\xi) = -ln\left(\frac{\lambda}{exp\left(\lambda\left(\xi + C_1\right)\right) - 1}\right), \tag{2.7}$$

when $\lambda^2 - 4\mu = 0, \mu \neq 0, \lambda \neq 0$,

$$\varphi(\xi) = ln\left(-\frac{2\left(\lambda\left(\xi + C_1\right) + 2\right)}{\lambda^2\left(\xi + C_1\right)}\right), \tag{2.8}$$

when $\lambda^2 - 4\mu = 0, \mu = 0, \lambda = 0$,

$$\varphi(\xi) = ln\left(\xi + C_1\right), \tag{2.9}$$

when $\lambda^2 - 4\mu < 0$,

$$\varphi(\xi) = ln\left(\frac{\sqrt{4\mu - \lambda^2}\ tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\ (\xi + C_1)\right) - \lambda}{2\mu}\right), \tag{2.10}$$

where $a_m, ..., \lambda, \mu$ are constants to be determined later,
Step 3. Substitute Eq.(2.4) along Eq.(2.5) into Eq.(2.3) and collecting all the terms of the same power $exp\left(-m\varphi(\xi)\right)$ and equating them to zero, where the positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in ODE(2.3). We obtain a system of algebraic equations, which can be solved by Maple or Mathematica to get the values of α_i.
Step 4. substituting these values and the solutions of Eq.(2.5) into Eq.(2.3) we obtain the exact solutions of Eq.(2.3).

a) Example1. The nonlinear Burger equation with power law nonlinearity.

This equation is well known [33] and has the form:

$$v_t + a(v^n)_x + bv_{xx} = 0, n > 1, \tag{2.11}$$

Ref

[33] M. Hayek, Constructing of exact solutions to the KdV and Burgers equations with power law nonlinearity by the extended $(\frac{G'}{G})$- expansion method, Appl. Math. Comput. 217 (2010) 212-221.

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where a and b are nonzero constants. The solutions of Eq.(2.11) have been discussed, the exact solitary wave solutions, the periodic solutions and the rational function solution are obtaines in [33] by means of the extended $(\frac{G'}{G})$ -expansion method. Let us now solve Eq.(2.11) using the $\exp(-\varphi(\xi))$-expansion method. To this end, we use the wave transformation (2.2) to reduce Eq.(2.11) to the ODE and integrating the equation with zero constant of integration:

$$-cv + av^n + bv' = 0. \tag{2.12}$$

Balancing v' with v^n yields $m = \frac{1}{n-1}, n > 1$. Using the transformation

$$v = u^{\frac{1}{n-1}}, \tag{2.13}$$

to reduce Eq.(12) to the following equation

$$-c(n-1)u + a(n-1)u^2 + bu^{'} = 0, \tag{2.14}$$

where u is a new function of ξ. Balancing u' with u^2 yields $m = 1$. Consequently, Eq.(2.1) has the formal solution

$$u = \alpha_0 + \alpha_1 exp(-\varphi), \tag{2.15}$$

where α_0 and α_1 are constants to be determined, such that $\alpha_1 \neq 0$. It is easy to see that

$$u' = -\alpha_1 exp(-2\varphi) - \mu\alpha_1 - \lambda\alpha_1 exp(-\varphi), \tag{2.16}$$

substituting Eq.(2.15) and its derivatives in Eq.(2.1) and equating the coefficient of different power's of$exp(-\varphi(\xi))$ to zero, we get

$$a(n-1)\alpha_1^2 - b\alpha_1 = 0, \tag{2.17}$$

$$-v(n-1)\alpha_1 + a(n-1)(2\alpha_0\alpha_1) - b\lambda\alpha_1 = 0, \tag{2.18}$$

$$-c\alpha_0(n-1) + a(n-1)\alpha_0^2 - b\mu\alpha_1 = 0, \tag{2.19}$$

Eqs.(2.1)-(2.19) yield

$$\alpha_0 = \frac{b\lambda}{2a(n-1)} + \frac{v}{2a}, \alpha_1 = \frac{b}{a(n-1)}. \tag{2.20}$$

thus the solution is

$$u = \frac{b\lambda}{2a(n-1)} + \frac{v}{2a} + \frac{b}{a(n-1)} exp(-\varphi) \tag{2.21}$$

Let us now discuse the following case:

Case 1. if $\lambda^2 - 4\mu > 0, \mu \neq 0$. then we deduce from Eq.(2.21) that

$$u(\xi) = \frac{b\lambda}{2a(n-1)} + \frac{v}{2a} + \frac{2b\mu}{a(n-1)\left[-\sqrt{\lambda^2-4\mu}tanh\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+c_1) - \lambda\right]} \tag{2.22}$$

Case 2. if $\lambda^2 - 4\mu > 0, \mu = 0$. then we deduce from Eq. that

$$u(\xi) = \frac{b\lambda}{2a(n-1)} + \frac{v}{2a} + \frac{2b\mu}{a(n-1)\ [exp(\lambda\xi + c_1) - 1]}. \tag{2.23}$$

Case 3. if $\lambda^2 - 4\mu = 0, \mu \neq 0, \lambda \neq 0$. then we deduce from Eq. that

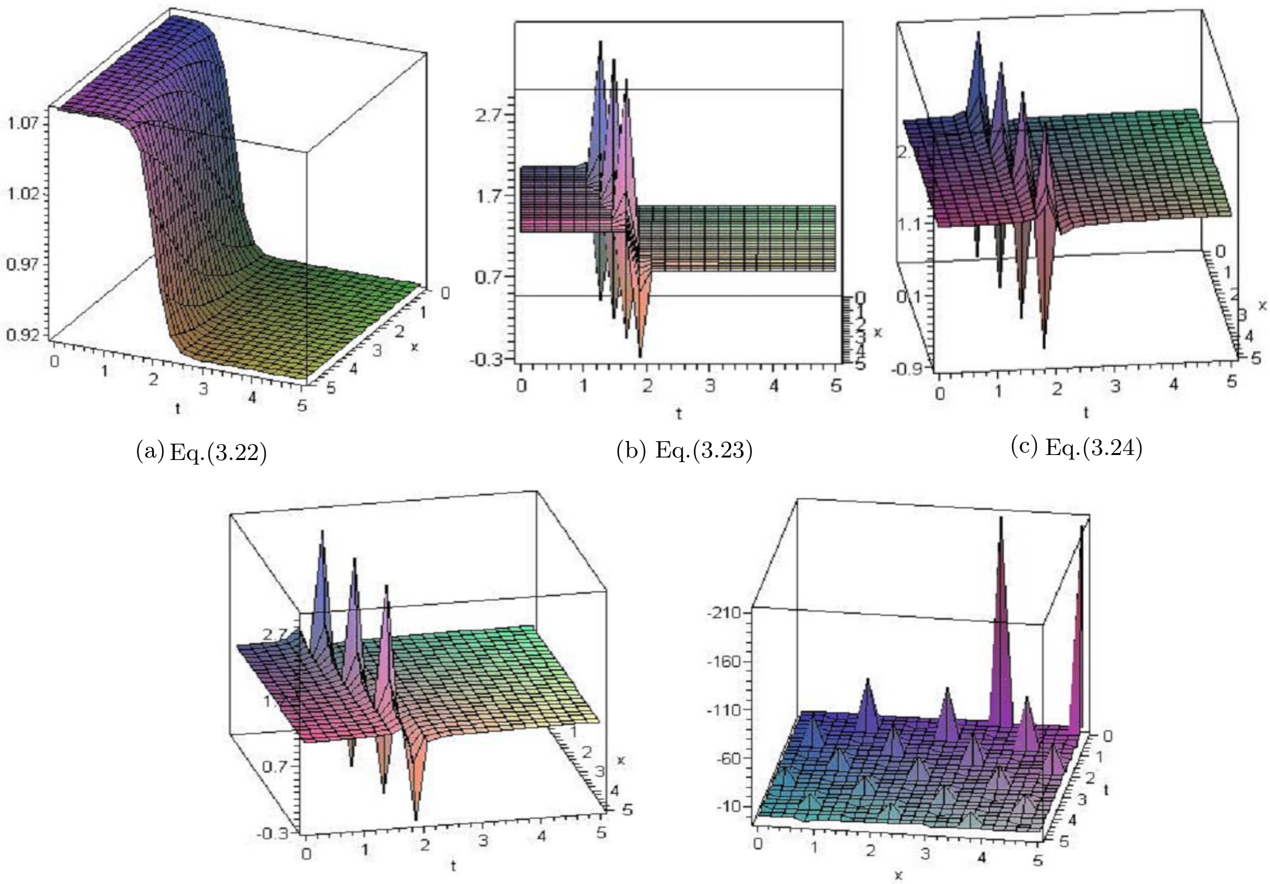
$$u(\xi) = \frac{b\lambda}{2a(n-1)} + \frac{v}{2a} - \frac{b\lambda^2(\xi + c_1)}{2a(n-1)\,[\lambda(\xi + c_1) + 2]}. \tag{2.24}$$

Case 4. if $\lambda^2 - 4\mu = 0, \mu = 0, \lambda = 0$. then we deduce from Eq. that

$$u(\xi) = \frac{b\lambda}{2a(n-1)} + \frac{v}{2a} + \frac{b}{a(n-1)\left[\xi + c_1\right]}. \tag{2.25}$$

Case 5. if $\lambda^2 - 4\mu < 0$, then we deduce from Eq. that

$$u(\xi) = \frac{b\lambda}{2a(n-1)} + \frac{v}{2a} + \frac{2b\mu}{a(n-1)\left[\sqrt{4\mu - \lambda^2}tan(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi + c_1)) - \lambda\right]}. \tag{2.26}$$



(a) Eq.(3.22) (b) Eq.(3.23) (c) Eq.(3.24) (d) Eq.(3.25) (e) Eq.(3.26)

Figure 1 : solution of Eqs.(3.22)-(3.26)

b) Example2. The perturbed nonlinear Schrodinger equation with Kerr law nonlinearity.

This equation is well-known [34],[35] and has the form:

$$iu_t + u_{xx} + \alpha|u|^2u + i\left\{\gamma_1 u_{xxx} + \gamma_2|u|^2u_x + \gamma_3(|u|^2)_x u\right\} = 0, \tag{2.27}$$

where α, γ_1, γ_2, γ_3 are constants such that γ_1 is the third order dispersion, γ_2 is the nonlinear dispersion, while γ_3 is also a version of nonlinear dispersion [36],[37]. Eq.S1 describes the propagation of optical solitons in nonlinear optical fibers that exhibits a Kerr law nonlinearity. Eq.S1 has been discussed in [35] using the first integral method and in [34] using the modified mapping method and its extended. Let us now solve Eq.S1 using the exp($-\varphi(\xi)$)-expansion method. To this end we seek its traveling wave solution of the form [34],[35]:

$$u(x,t) = \phi(\xi)exp[i(kx - \Omega t)], \;\xi = x - ct, \tag{2.28}$$

Ref

[35] H. Moosaei, M. Mirzazadeh and A. Yildirim, Exact solutions to the perturbed nonlinear Scrodinger equation with Kerr law nonlinearity by using the first integral method, Nonlinear Analysis: Modelling and Control, 16 (2011) 332-339.

where k , Ω and c are constants,while $i = \sqrt{-1}$. Substituting S2 into Eq.S1 and equating the real and imaginary parts to zero, we have

$$\gamma_1\phi^{'''} + (2k - c - 3\gamma_1 k^2)\phi^{'} + (\gamma_2 + 2\gamma_3)\phi^2\phi^{'} = 0, \tag{2.29}$$

and

$$(1 - 3\gamma_1 k)\phi^{''} + (\Omega - k^2 + \gamma_1 k^3)\phi + (\alpha - \gamma_2 k)\phi^3 = 0. \tag{2.30}$$

With reference to [34], the two equations (2.29) and (2.30) can be simplified as follows: Integration Eq.(2.29) and vanishing the constant of integration, we have

$$\gamma_1\phi^{''} + (2k - c - 3\gamma_1 k^2)\phi + \frac{1}{3}(\gamma_2 + 2\gamma_3)\phi^3 = 0, \tag{2.31}$$

From Eqs.(2.30) and (2.31) we deduce that

$$\frac{\gamma_1}{1 - 3\gamma_1 k} = \frac{2k - c - 3\gamma_1 k^2}{\Omega - k^2 + \gamma_1 k^3} = \frac{\frac{1}{3}(\gamma_2 + 2\gamma_3)}{\alpha - \gamma_2 k}. \tag{2.32}$$

From Eq.(2.32), we can obtain $k = \frac{\omega - \alpha\gamma_1}{3\omega\gamma_1 - \gamma_1\gamma_2}$, $\Omega = \frac{(1-3\gamma_1 k)(2k-c-3\gamma_1 k^2)}{\omega} + k^2 - \gamma_1 k^3$, where $\omega = \frac{1}{3}\gamma_2 + \frac{2}{3}\gamma_3$. Now, Eq.(2.32) is transformed into the following form:

$$A\phi^{''} + B\phi + \omega\phi^3 = 0, \tag{2.33}$$

where $A = \gamma_1$ and $B = 2k - c - 3\gamma_1 k^2$. Balancing ϕ'' with ϕ^3 yields $m = 1$. Thus, we get the same formulas (2.15). Substituting (2.15) and its derivatives into Eq.(2.33) and equating the coefficients of $exp(-m\varphi)$ to zero, we get

$$2\alpha_1 A + \omega\alpha_1^3 = 0, \tag{2.34}$$

$$3\lambda\alpha_1 A + 3\alpha_0\alpha_1^2\omega = 0, \tag{2.35}$$

$$A\alpha_1(\lambda^2 + 2\mu) + 3\alpha_0^2\alpha_1\omega + B\alpha_1 = 0, \tag{2.36}$$

$$\lambda\mu\alpha_1 A + \omega\alpha_0^3 + B\alpha_0 = 0 \tag{2.37}$$

Eqs.(2.34)-(2.37) yields

$$\alpha_0 = \mp\lambda\sqrt{\frac{-A}{2\omega}}, \alpha_1 = \pm\sqrt{\frac{-2A}{\omega}}. \tag{2.38}$$

thus the solution is

$$u = \mp\lambda\sqrt{\frac{-A}{2\omega}} \pm \sqrt{\frac{-2A}{\omega}} exp(-\varphi) \tag{2.39}$$

Let us now discuse the following case:
Case 1. if $\lambda^2 - 4\mu > 0, \mu \neq 0$. then we deduce from Eq.(2.33) that

$$u(\xi) = \mp\lambda\sqrt{\frac{-A}{2\omega}} \pm \sqrt{\frac{-2A}{\omega}}\left[\frac{2\mu}{-\sqrt{\lambda^2 - 4\mu}\, tanh\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi + c_1) - \lambda}\right], \tag{2.40}$$

Case 2. if $\lambda^2 - 4\mu > 0, \mu = 0$. then we deduce from Eq.(2.33) that

$$u(\xi) = \mp\lambda\sqrt{\frac{-A}{2\omega}} \pm \sqrt{\frac{-2A}{\omega}}\left[\frac{\lambda}{exp(\lambda\xi + c_1) - 1}\right], \tag{2.41}$$

Ref

[34] Z. Y. Zhang, Z. H. Liu, X. J. Miao and Y. Z. Chen, New exact solutions to the perturbed nonlinear Schrodinger equation with Kerr law nonlinearity, Appl. Math. Comput., 216 (2010) 3064-3072.

Case 3. if $\lambda^2 - 4\mu = 0, \mu \neq 0, \lambda \neq 0$. then we deduce from Eq.(2.33) that

$$u(\xi) = \mp\lambda\sqrt{\frac{-A}{2\omega}} \pm \sqrt{\frac{-2A}{\omega}}\left[\frac{\lambda^2(\xi + c_1)}{2(\lambda(\xi + c_1) + 2)}\right]. \tag{2.42}$$

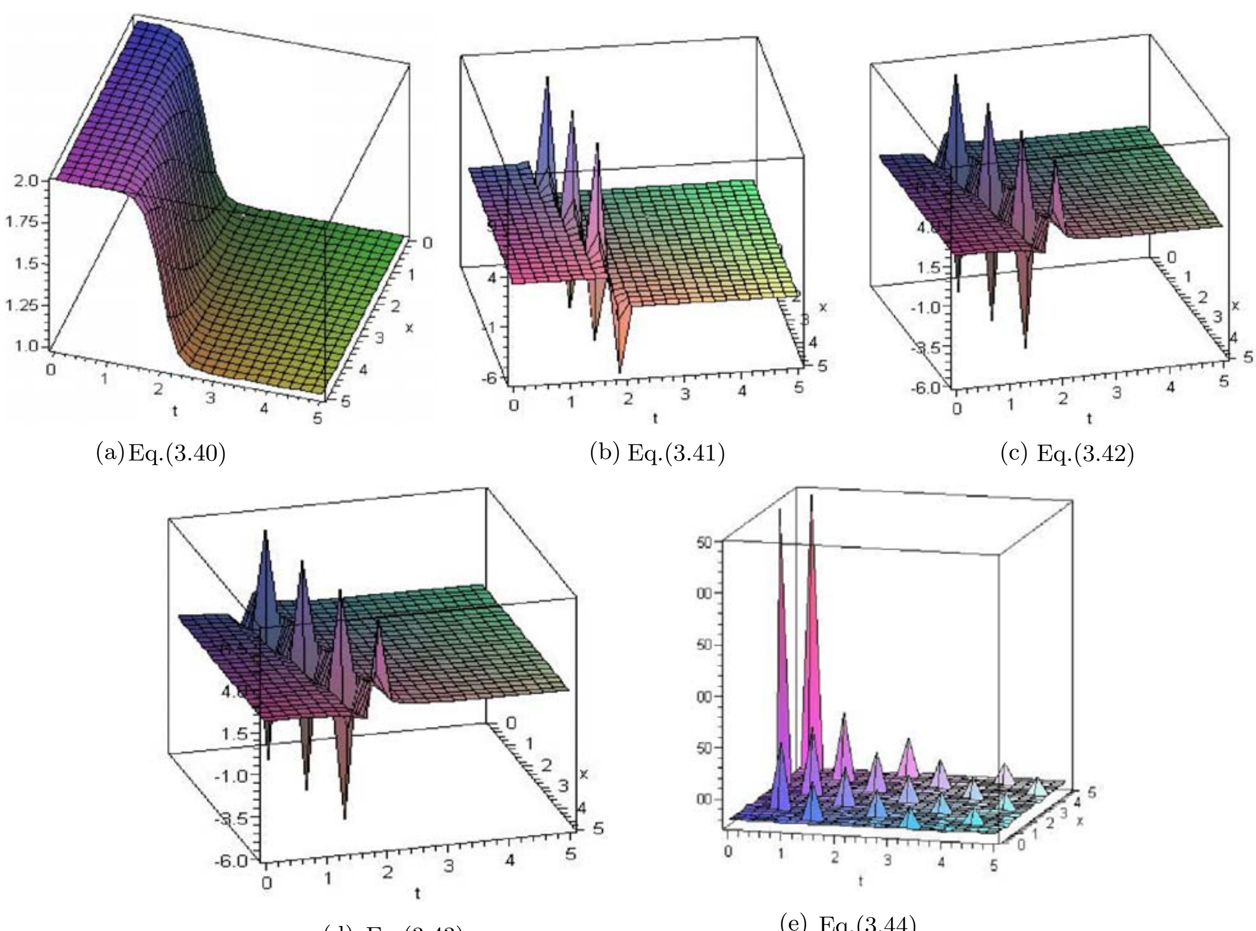
Case 4. if $\lambda^2 - 4\mu = 0, \mu = 0, \lambda = 0$. then we deduce from Eq.(2.33) that

$$u(\xi) = \mp\lambda\sqrt{\frac{-A}{2\omega}} \pm \sqrt{\frac{-2A}{\omega}}\left[\frac{1}{\xi + c_1}\right], \tag{2.43}$$

Notes

Case 5. if $\lambda^2 - 4\mu < 0$, then we deduce from Eq.(2.33) that

$$u(\xi) = \mp\lambda\sqrt{\frac{-A}{2\omega}} + \frac{v}{2a} \pm \sqrt{\frac{-2A}{\omega}}\left[\frac{2\mu}{\sqrt{4\mu - \lambda^2}\, tan(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi + c_1)) - \lambda}\right]. \tag{2.44}$$



(a) Eq.(3.40) (b) Eq.(3.41) (c) Eq.(3.42)

(d) Eq.(3.43) (e) Eq.(3.44)

Figure 2 : solution of Eqs.(3.40)-(3.44)

III. Conclusions

In this paper, it has been shown that the new exp(−(φ))-expansion method is a powerful tool for the nonlinear evolution equations. we can obtained new and more travelling wave solutions for the equations above , such as, the nonlinear Burger equation with power law nonlinearity, the perturbed nonlinear Schrodinger equation with kerr law nonlinearity.. Otherwise, the general solutions of the ODE have been well known for the researchers. Furthermore, the new method can be used for many other nonlinear evolution equations.

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Fixed Points for Cyclic Contractions in Dislocated Metric Spaces

By K. Prudhvi
Osmania University, India

Abstract- In this work, we study fixed point theorems for cyclic contractions in dislocated metric spaces. Our results, extension, improve and generalize some recent results in the literature.

Keywords: *dislocated metric spaces, fixed point, d-convergent, d-cyclic contraction.*

GJSFR-F Classification : MSC 2010: 30L05 , 31E05

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K. Prudhvi

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Keywords: dislocated metric spaces, fixed point, d-convergent, d-cyclic contraction.

I. Introduction and Preliminaries

P.Hitzler et.al.[1] have introduced "the notion of dislocated metric space". Recently, Kirk et.al. [4] have introduced "the notion of cyclic contraction and prove fixed point theorems on this contraction". Later on many authors have proved fixed point theorems on cyclic contractions in dislocated metric spaces (see, [3], [5]). In this paper, we obtain, fixed point theorems for cyclic contractions in dislocated metric spaces which improve,extends and generalises results of [3].

The following definitions are needful to prove main results.

Definition 1.1 [1]. Let X be a non empty set and let d: $X \times X \to [0, \infty)$ be a function called a distance function .Consider the following conditions:

(i) $d(x, x) = 0$, for all $x \in X$.

(ii) $d(x, y) = 0$ then $x = y$, for all $x,y \in X$.

(iii) $d(x, y) = d(y,x)$, for all $x,y \in X$.

(iv) $d(x,y) \leq d(x,z) + d(z,y)$, for all $x,y,z \in X$.

If d satisfies (i) to (iv), then it is called metric. If it satisfies (ii) to (iv), then d is called dislocated metric (or simply d-metric).

Definition 1.2 [1]. A sequence $\{x_n\}$ in a d-metric space(X,d) converges with respect to d , if there exists a point $x \in X$ such that $d(x_n, x)$ converges to 0 as $n \to \infty$.

Proposition 1.1 [1]. Limits in d-metric spaces are unique.

Definition 1.3[1]. A sequence $\{x_n\}$ in a d-metric space(X,d) is called a Cauchy sequence , if for each $\varepsilon > 0$ there exists $n_0 \in N$ such that for all $m,n \geq n_0$, we have $d(x_m, x_n) < \varepsilon$.

Proposition 1.2[1]. Every convergent sequence in d-metric spaces is a Cauchy sequence.

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Definition 1.4[1]. A function f: X → X in a d-metric space is called a contraction if there exists $0 \le \lambda < 1$ such that $d(fx, fy) \le \lambda\, d(x, y)$ for all $x,y \in X$.

Definition 1.5[4]. Let A and B be two non empty subsets of a metric space (X, d) and $T: A \cup B \to A \cup B$, T is called a cyclic map if $T(A) \subseteq B$ and $T(B) \subseteq A$.

Definition 1.6[4]. Let A and B be two non empty subsets of a metric space (X,d) and a cyclic map $T: A \cup B \to A \cup B$ is said to be a cyclic a cyclic contraction if there exists $k \in (0,1)$ such that $d(Tx, Ty) \le \lambda\, d(x, y)$ for all $x \in A$ and $x \in B$.

II. Main Results

In this section, we prove fixed point thoerems for cyclic contractions in dislocated metric spaces. Our results improve, extends and generalises the results of [3].

Theorem 2.1 . Let (X, d) be a complete dislocated metric space. Let A and B be two non empty closed subsets of X and $f : A \cup B \to A \cup B$ be such that

$d(fx,fy) \le \alpha[d(fx, x) + d(fy, y) + d(fy, x)+d(y, fx)]$, where $\alpha \in [0, 1/6)$.

Then f has a unique fixed point in $A \cap B$.

Proof: Let $\{f^n\} \subseteq X$, $\{f^{2n}\} \subseteq A$ and $\{f^{2n-1}\} \subseteq B$. Fix $x \in A$. By above definition there exists $\alpha \in [0, 1/6)$ such that

$$d(f^2x, fx) \le \alpha\, [d(f^2x, fx) + d(fx, x) + d(fx, fx) + d(x, f^2x)],$$

$$d(f^2x, fx) \le \alpha\, [d(f^2x, fx) + d(fx, x) + d(fx,x) + d(x, fx) + d(x,fx)+d(fx, f^2x)],$$

$$\le \alpha\, [2d(f^2x, fx) + 4d(fx, x)],$$

$$(1-2\alpha)\, d(f^2x, fx) \le 4\alpha d(fx, x)$$

$$d(f^2x, fx) \le 4\alpha / 1-2\alpha \;\; d(fx,x).$$

Put $k = 4\alpha / 1-2\alpha < 1$.

$d(f^2x, fx) \le kd(fx, x)$.

By induction we have,

$$d(f^{n+1}x, f^n x) \le k^n d(fx, x).$$

More generally, for $m > n$ we have,

$$d(f^m x, f^n x) \le d(f^m x, f^{m-1} x) + d(f^{m-1} x, f^{m-2} x) + \ldots + d(f^{n+1} x, f^n x)$$
$$\le (k^{m-1} x + k^{m-2} x + \ldots + k^n x) d(fx,x)$$
$$= k^n (1+k+ k^2 + \ldots + k^{m-n-1}) d(fx,x).$$

Since, $k<1$ so as $m,n \to \infty$ we have $k^n (1+k+ k^2 + \ldots + k^{m-n-1}) \to 0$.

Hence, $d(f^m x, f^n x) \to 0$.

Therefore, $\{f^n x\}$ is a Cauchy sequence. Since (X,d) is complete, so $\{f^n x\}$ converges to some point $z \in X$.

Since, $\{f^{2n}\} \subseteq A$ and and $\{f^{2n-1}\} \subseteq B$ so $z \in A \cap B$.

We claim that $fz = z$.

$$d(fz,z) = d(fz, f^{2n} x)$$

Ref

[3] George Reny, R.Rajgopalan and S.Vinayagam, Cyclic contractions and fixed points in dislocated metric spaces, *Int. J. Math.Anal*, 7(9),(2013),403-411.

$$\leq \alpha\,[d(fz, z) + d(f^{2n}x\,,\ f^{2n-1}x) + d(f^{2n}x\,,z) + d(\ f^{2n-1}x, fz)]$$

Letting, as $n \to \infty$ we get that,

$$\begin{aligned} d(fz, z) &\leq \alpha\,[d(fz, z) + d(z\,, z) + d(z, z) + d(z, z)] \\ &\leq \alpha\,[d(fz, z) + 3d(z\,,z)] \\ &\leq \alpha\,[d(fz, z) + 3d(z\,, fz) + 3d(fz\,, z)] \\ &\leq 7\alpha d(fz, z), \quad \text{since } \alpha \in [0, 1/6). \end{aligned}$$

$(1-7\alpha)\, d(fz, z) \leq 0.$

Implies , $d(fz, z) = 0$.

Hence, $fz = z$.

Uniqueness, let u and v be two fixed points of f that is, $fu = u$ and $fv = v$. Then,

$$\begin{aligned} d(u,v) = d(fu,fv) &\leq \alpha\,[d(fu,u) + d(fv,v) + d(fv,u) + d(v,fu)] \\ &\leq \alpha\,[d(u,u) + d(v,v) + d(v,u) + d(v,u)] \\ &\leq \alpha\,[d(u,v) + d(u,v) + d(u,v) + d(u,v) + d(v,u) + d(v,u)] \\ &\leq 6\alpha\, d(u,v). \end{aligned}$$

Implies, $(1-6\alpha)\, d(u,v) \leq 0$. Since, $\alpha \in [0, 1/6)$.

We have, $d(u,v) = 0$. Implies, $u = v$.

Therefore, f has a unique fixed point in $A \cap B$.

Example 2.1. Let $X = R$, $A = [-1, 0]$, $B = [0, 1]$. Define $d(x, y) = |x - y| + 4|x| + 4|y|$ Then d is the dislocated metric.Define $f : A \cup B \to A \cup B$, by $fx = -x/6$, then f is a cyclic mapping. For any two points in A and B, the contractive condition is satisfied and 0 is the unique fixed point of the function f.

Theorem 2.2. Let (X, d) be a complete dislocated metric space. Let A and B be two non empty closed subsets of X and $f : A \cup B \to A \cup B$ be a cyclic mapping satisfying

(i). there exists a number $k \in [0,1/2)$.

(ii) . $d(fx,fy) \leq \max\{ d(x,y), d(x,fx), d(y,fy), d(x, fy), d(y,fx)\}$

for all $x \in A$ and $y \in B$ then f has a unique fixed point in $A \cap B$.

Proof: Let $\{f^n\} \subseteq X$, $\{f^{2n}\} \subseteq A$and $\{f^{2n-1}\} \subseteq B$. Fix $x \in A$ If $f^n x = f^{n+1}x$ for some n, then $f^{n+1}x = f^{n+2}x$ then $\{f^n x\}$ converges to some $z \in X$. So suppose $f^n x \neq f^{n+1}x$. Now by using above contractive condition of the theorem, we have

$$\begin{aligned} d(f^2x, fx) &\leq k \max\{d(fx,x), d(fx, f^2x), d(x, fx), d(fx,fx), d(x, f^2x)\} \\ &= k \max\{d(fx, x), d(fx,fx), d(x, f^2x)\} \\ &\leq k \max\{d(fx,x), d(fx, x) + d(fx, x), d(x,fx) + d(fx, f^2x)\} \\ &\leq k \max\{d(fx,x), 2\, d(fx, x)\} \\ &\leq k\, d(fx,x) \text{ or } 2k\, d(fx, x) \\ &\leq 2k\, d(fx,x) \\ &\leq h d(fx, x) \text{ where, } h = 2k. \end{aligned}$$

Now by induction we have,

$$d(f^2x, fx) \leq h^n x\, d(fx,x).$$

Following the same process as in the above theorem we show that $\{f^n x\}$ is a cauchy sequence. Since (X,d) is complete, so $\{f^n x\}$ converges to some point $z \in X$. Since $\{f^{2n}\} \subseteq A$ and $\{f^{2n-1}\} \subseteq B$ so $z \in A \cap B$. We claim that $fz = z$.

$$d(fz,f^{2n}x) \leq k \max\{d(z, f^{2-1}x), d(fz,z), d(f^{2n}x,\ f^{2n-1}x), d(z, f^{2n}x), d(f^{2n}x,z)\}.$$

Notes

Letting, as $n \to \infty$, we have

$$d(fz, z) \le k \max\{ d(z,z), d(fz, z), d(z, z), d(z, z)\}$$
$$\le k\, d(fz,z), \text{ which is a contradiction.}$$

Hence, $d(fz,z) = 0$.

$\Rightarrow$ $fz = z$.

Therefore, f has a fixed point.

Uniqueness, Let us assume that there exists fixed points u and v, that is $fu = u$ and $fv = v$.

$$d(fu, fv) \le k \max\{ d(u,v), d(u,fu), d(v,fv), d(u,fv), d(v,fu)\}$$
$$\le k \max\{ d(u,v), d(u, u), d(v,v), d(u,v), d(v,u)\}$$
$$= k \max\{ d(u,v), d(u, u), d(v,v), \}$$
$$\le k\, d(u,v) \text{ or } k\, d(u, u) \text{ or } k\, d(v,v)$$

$(1-k)\, d(u,v) \le 0$ or $(1-2k)\, d(u,v) \le 0$.

This implies that, $u = v$.

Therefore, f has a unique fixed point in $A \cap B$.

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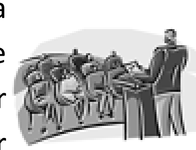

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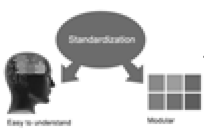

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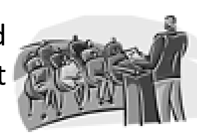

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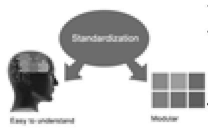

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The recommended size of original research paper is less than seven thousand words, review papers fewer than seven thousands words also.Preparation of research paper or how to write research paper, are major hurdle, while writing manuscript. The research articles and research letters should be fewer than three thousand words, the structure original research paper; sometime review paper should be as follows:

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(c) Up to ten keywords, that precisely identifies the paper's subject, purpose, and focus.

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References

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- To the point depiction of the research
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- Explain materials individually only if the study is so complex that it saves liberty this way.
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What to keep away from

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- Leave out information that is immaterial to a third party.

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The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

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- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
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- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
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Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
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- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
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- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
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Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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