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Highlights

Quadratic Cost Function

Fuzzy Multisets and Sequence

Multi-Objective Geometric

Sen-Dunn Scalar-Tensor Theory

Discovering Thoughts, Inventing Future

VOLUME 14 ISSUE 7 VERSION 1.0

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Mathematics & Decision Sciences

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Bianchi Type-IX Dark Energy Model in Sen-Dunn Scalar-Tensor Theory of Gravitation

By H. R. Ghate & Atish S. Sontakke
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Abstract- Bianchi type-IX cosmological model with variable equation of state (EoS) parameter have been investigated in general scalar-tensor theory of gravitation proposed by Sen and Dunn (J. Math. Phys. 12:578, 1971) when universe is filled with Dark Energy. The field equations have been solved by considering the scale factors in two different forms (i) $R=\left(t^r e^t\right)^{\frac{1}{l}}$, where r and l are positive constants and (ii) $R=\left(\sinh(\xi t)\right)^{\frac{1}{n}}$, where ξ and n are positive constants, which render time dependent deceleration parameter. The physical and geometrical properties of the models are also discussed.

Keywords: Dark Energy, Bianchi Type-Ix, Sen-Dunn Scalar Tensor Theory.

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H. R. Ghate [α] & Atish S. Sontakke [σ]

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Keywords: Dark Energy, Bianchi Type-Ix, Sen-Dunn Scalar Tensor Theory.

I. Introduction

Einstein's theory of general relativity is one of the most beautiful structures of theoretical physics which is also known as the most successful theory of gravitation in terms of geometry. In the last decade, several theories of gravitation have been proposed as alternatives to the theory of general relativity. The most popular amongst them are scalar-tensor theories of gravitation formulated by Brans-Dicke [1], Nordtvedt [2], Sen [3], Sen and Dunn [4], Wagonar [5], Saez-Ballester [6], Barber [7]. Sen and Dunn have proposed scalar tensor theory of gravitation in which a scalar field is characterized by the function $\phi = \phi(x^{i})$, where x^{i} represents co-ordinates in the four-dimensional Lyra manifold and the tensor field is identified with the metric tensor g_{ij} of the manifold. Later on Halford [8], Chatterji and Roy [9], Reddy *et al.* [10, 11] have obtained cosmological models in Sen-Dunn theory of gravitation with different contexts. Recently Venkateswarlu and Satish *et al.* [12, 13] have investigated Kantowski-Sach and Bianchi type-VI string cosmological models in Sen-Dunn theory of gravitation.

In the last decade, one of the most remarkable observational discoveries has shown that our universe is undergoing accelerated expansion [14-22]. The Theory of Dark Energy (DE) with negative pressure is mainly responsible for this scenario. It is estimated that the energy density of our universe consist of about 2/3rd DE and 1/3rd dark matter (DM) of the whole universe [23, 24]. In modern cosmology the origin of cosmology is still mystery.

Many Authors have proposed different DE models such as quintessence [25-31], Chaplygin gas [32], Phantom energy [33], DE in Brane world [34-36]. The existence of DE fluids is the main candidate responsible for the accelerated expansion of the universe and the cosmological models with isotropic pressure give the best fitting for the existence of DE fluids. Thus the DE models have significant importance in study of the expansion of the universe. Many relativists [37-41] have obtained DE cosmological models in various theories of gravitation with different contexts. Recently Ghate and

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Ref

5. Wagoner, R.T.: Physical Review D, 1(12), 3209 (1970).

Sontakke [42-44] have studied DE cosmological models in Saez-Ballester, Lyra and Brans-Dicke theory of gravitation.

The study of Bianchi type models is important in achieving better understanding of anisotropy in the universe. Moreover, the anisotropic universes have greater generality than FRW isotropic models. The simplicity of the field equations made Bianchi type space-times useful. Bianchi type I-IX cosmological models are homogeneous and anisotropic. Bianchi type-IX universe is studied by the number of cosmologists because of familiar solutions like Robertson–Walker Universe, the de-sitter universe, the Taub-Nut solutions etc. Chakraborty [45], Bali and Dave [46], Bali and Yadav [47] studied Bianchi type-IX string as well as viscous fluid models in general relativity. Pradhan et al. [48] have studied some homogeneous Bianchi type-IX viscous fluid cosmological models with varying Λ. Tyagi and Chhajed [49] have obtained Bianchi type-IX string cosmological models for perfect fluid distribution in general relativity. Ghate and Sontakke [50, 51] have studied Bianchi type-IX cosmological models with different context.

To study cosmological models one of the important observational quantity is the deceleration parameter denoted by q and given by $q=-\frac{R\ddot{R}}{\dot{R}^2}$, where R is the mean scale factor of the model. The Deceleration parameter (q) is useful in studying expansion rate of the universe. In any cosmological model, the Hubble constant H_0 and deceleration parameter q play an important role in describing the nature of evolution of the universe. The former one represents the expansion rate of the universe while the latter one characterizes the accelerating $(q<0)$ or decelerating $(q>0)$ nature of the universe. Number of relativists assumes various physical or mathematical conditions to obtain exact solution of the Einstein's field equations. We know that the universe has decelerating expansion if $q>0$, an expansion with constant rate if $q=0$, accelerating power law expansion if $-1<q<0$, exponential expansion (or de-Sitter expansion) if $q=-1$ and super-exponential expansion if $q<-1$. Many relativists have studied the DE cosmological models with different form of deceleration parameters. Pradhan *et al.* [52-54], Yadav and Sharma [55], Rahman and Ansari [56] have studied cosmological models with time dependent deceleration parameters. Singha and Debnath [57], Adhav *et al.* [58, 59] have obtained DE models with special form of deceleration parameter. Adhav *et al.* [60, 61] and Akarsu *et al.* [62, 63] have investigated cosmological models with linearly varying deceleration parameter. Berman [64], Singh *et al.* [65, 66], Adhav *et al.* [67] have studied cosmological models with constant deceleration parameter.

In this paper, Bianchi type-IX space-time has considered when universe is filled with DE in Sen-Dunn scalar-tensor theory of gravitation. This work is organized as follows: In section 2, the model and field equations have been presented. The field equations have been solved in section 3 by choosing two different scale factors which renders time-dependent deceleration parameters. The physical and geometrical behavior of the two models have been discussed in section 3.1 and 3.2. In section 4, concluding remarks have been expressed.

II. Metric and Field Equations

Bianchi type-IX metric is considered in the form,

$$ds^2=-dt^2+a^2dx^2+b^2dy^2+\left(b^2\sin^2 y+a^2\cos^2 y\right)dz^2-2a^2\cos y\,dx\,dz\,, \tag{1}$$

where a, b are scale factors and are functions of cosmic time t.

The field equations given by Sen and Dunn for the combined scalar and tensor fields (in natural units $c=1, G=1$) are

$$R_{ij}-\frac{1}{2}Rg_{ij}=\omega\phi^{-2}\left(\phi_{,i}\phi_{,j}-\frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right)-\phi^{-2}T_{ij}\quad . \tag{2}$$

Ref

48. Pradhan, A., Srivastav, S.K., Yadav, M.K.:Astrophysics and Space Science, 298, 419 (2005)

Notes

Here the function $\omega = \frac{3}{2}$, R_{ij} is the Ricci tensor, R is the Ricci scalar, T_{ij} is the energy stress tensor of the matter. A comma and semicolon denote partial and covariant differentiations and ϕ_i denotes ordinary derivatives with respect to x_i. The scalar field ϕ incorporates the varying nature of Newtonian Gravitational constant.
The energy momentum tensor of the fluid which is taken as

$$T_i^{\ j} = \left[T_0^0, T_1^1, T_2^2, T_3^3 \right] \ . \tag{3}$$

The simplest generalization of EoS parameter of perfect fluid is to determine it separately on each spatial axis by preserving diagonal form of the energy momentum tensor in a consistent way with the considered metric. Hence one can parameterize energy momentum tensor as follows:

$$\begin{aligned} T_i^{\ j} &= \left[-\rho, p_x, p_y, p_z \right], \\ T_i^{\ j} &= \left[-1, w_x, w_y, w_z \right] \rho \ , \\ T_i^{\ j} &= \left[-1, w, w+\delta, w+\gamma \right] \rho . \end{aligned} \tag{4}$$

Here ρ is the energy density of the fluid, p_x, p_y, p_z are the pressures and w_x, w_y and w_z are the directional EoS parameters along *x, y* and *z* axes respectively, w is the deviation free EoS parameter of the fluid.

Now parameterizing the deviation from isotropy by setting $w_x = w$ and then introducing skewness parameters δ and γ which are deviations from w on *y* and *z* axes respectively. Here δ and γ are not necessarily constants and can be functions of the cosmic time t.

In the co-moving coordinate system the field equation (2) for the metric (1) with the help of equation (4) can be written as

$$2\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{1}{b^2} + \frac{\dot{b}^2}{b^2} - \frac{a^2}{4b^4} = \phi^{-2}\rho - \frac{3}{4}\frac{\dot{\phi}^2}{\phi^2} \, , \tag{5}$$

$$2\frac{\ddot{b}}{b} + \frac{1}{b^2} + \frac{\dot{b}^2}{b^2} - \frac{3a^2}{4b^4} = \frac{3}{4}\frac{\dot{\phi}^2}{\phi^2} - \phi^{-2} w\rho \, , \tag{6}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{a^2}{4b^4} = \frac{3}{4}\frac{\dot{\phi}^2}{\phi^2} - \phi^{-2}(w+\delta)\rho \, , \tag{7}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{a^2}{4b^4} = \frac{3}{4}\frac{\dot{\phi}^2}{\phi^2} - \phi^{-2}(w+\gamma)\rho \, , \tag{8}$$

where the overdot $(\dot{\ })$ denotes the differentiation with respect to t.

From equations (7) and (8) we see that, the deviations from w along y and z axes are same i.e. $\gamma = \delta$.

III. Solutions of Field Equations

The field equations (5)–(7) are three independent equations in six unknowns $a, b, \phi, \rho, w, \delta$. We can introduce more conditions either by an assumption corresponding to some physical situation or an arbitrary mathematical supposition. Three additional conditions relating these unknowns may be used to obtain explicit solutions of the systems.

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(i) Firstly, we assume that the expansion θ in the model is proportional to the shear σ. This condition leads to

$$a = b^m, (m \neq 1), \tag{9}$$

where m is proportionality constant.
The motive behind assuming condition is explained with reference to Thorne [68], the observations of the velocity red-shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropic today within ≈ 30 percent [69, 70]. To put more precisely, red-shift studies place the limit $\frac{\sigma}{H} \leq 0.3$ on the ratio of shear σ to Hubble constant H in the neighborhood of our galaxy today. Collin *et al.* [71] have pointed out that for spatially homogeneous metric the normal congruence to the homogeneous expansion satisfies that the condition $\frac{\sigma}{\theta}$ is constant.

(ii) Secondly, we consider the Guage function ϕ [72] as

$$\phi = \phi_0 R^{\alpha} = \phi_0 V^{\frac{\alpha}{3}}. \tag{10}$$

(iii) Riess *et al.* [73], Amendola [74] and Padmanabhan and Roy-Chowdhary [75] investigated that, for a universe which is decelerating in the past and accelerating in at present time, Deceleration parameter must show signature flipping. From the observations of SNe type Ia, Lima *et al.* [76] agree with the results of Riess and Amendola.

To obtain the deterministic solutions of the field equations, we choose two different scale factors which yield time dependent deceleration parameter as follows:

a) Case I: when $R(t) = \left(t^r e^t\right)^{\frac{1}{l}}$

The metric (1) is completely characterized by average scale factor therefore we consider that average scale factor is an integrating function of time [52-56] given by

$$R = (t^r e^t)^{\frac{1}{l}}, \tag{11}$$

where, r and l are positive constants.
The scale factor R is given by

$$R = (a b^2)^{\frac{1}{3}}. \tag{12}$$

Solving field equations (5)-(7) with the help of (1), (11) and (12), we obtain

$$a = (t^r e^t)^{\frac{3m}{(m+2)l}}, \tag{13}$$

$$b = (t^r e^t)^{\frac{3}{(m+2)l}}. \tag{14}$$

With the help of equations (13) and (14), the metric (1) takes the form

$$ds^2 = \begin{pmatrix} -dt^2 + (t^r e^t)^{\frac{6m}{(m+2)l}} dx^2 + (t^r e^t)^{\frac{6}{(m+2)l}} dy^2 + \left((t^r e^t)^{\frac{6}{(m+2)l}} \sin^2 y + (t^r e^t)^{\frac{6m}{(m+2)l}} \cos^2 y \right) dz^2 \\ -2(t^r e^t)^{\frac{6m}{(m+2)l}} \cos y dx dz \end{pmatrix}, \tag{15}$$

Ref
71. Collins, C.B., Glass, E.N., Wilkinson, D.A.: General Relativity and Gravitation, 12(10), 805 (1980)

Notes

Equation (15) represents Bianchi type-IX DE cosmological model in Sen-Dunn scalar tensor theory of gravitation.

i. Some Physical Properties of the Model

For the cosmological model (15), the physical quantities spatial volume V, Gauge function ϕ, Hubble parameter H, expansion scalar θ, mean anisotropy parameter A_m, shear scalar σ^2, energy density ρ, deceleration parameter q, cosmological constant Λ are obtained as follows:

Spatial volume,

$$V = (t^r e^t)^{\frac{3}{l}} . \tag{16}$$

Gauge function,

$$\phi = \phi_0 (t^r e^t)^{\frac{\alpha}{l}} . \tag{17}$$

The rate of expansion H_i,

$$H_x = \frac{3m}{(m+2)l}(rt^{-1}+1), \quad H_y = H_z = \frac{3}{(m+2)l}(rt^{-1}+1). \tag{18}$$

Hubble parameter,

$$H = \frac{1}{l}(rt^{-1}+1) . \tag{19}$$

Expansion scalar,

$$\theta = 3H = \frac{3}{l}(rt^{-1}+1). \tag{20}$$

Mean Anisotropy Parameter,

$$A_m = \frac{2(m-1)^2}{(m+2)^2} = \text{constant } (\neq 0 \text{ for } m \neq 1) . \tag{21}$$

Shear scalar,

$$\sigma^2 = \frac{3(m-1)^2}{l^2(m+2)^2}(rt^{-1}+1)^2 . \tag{22}$$

$$\frac{\sigma^2}{\theta^2} = \frac{(m-1)^2}{3(m+2)^2} = constant\ (\neq 0), \quad \text{for } m \neq 1 . \tag{23}$$

The energy density,

$$\rho = \phi_0^2 (t^r e^t)^{\frac{2\alpha}{l}} \left(\frac{(72m + 36 + 3\alpha^2 m^2 + 12\alpha^2 m + 12\alpha^2)}{4(m+2)^2 l^2}(rt^{-1}+1)^2 + (t^r e^t)^{\frac{-6}{(m+2)l}} - \frac{1}{4}(t^r e^t)^{\frac{6(m-2)}{(m+2)l}} \right) \tag{24}$$

EoS parameter,

$$w=-\frac{\left(\frac{(108-3\alpha^2m^2-12\alpha^2m-12\alpha^2)}{4(m+2)^2l^2}(rt^{-1}+1)^2-\frac{6}{(m+2)l}rt^{-2}+(t^re^t)^{\frac{-6}{(m+2)l}}-\frac{3}{4}(t^re^t)^{\frac{6(m-2)}{(m+2)l}}\right)}{\left(\frac{(72m+36+3\alpha^2m^2+12\alpha^2m+12\alpha^2)}{4(m+2)^2l^2}(rt^{-1}+1)^2+(t^re^t)^{\frac{-6}{(m+2)l}}-\frac{1}{4}(t^re^t)^{\frac{6(m-2)}{(m+2)l}}\right)}. \tag{25}$$

Skewness parameter,

$$\delta=-\frac{\left(\frac{9(m^2-1)}{(m+2)^2l^2}(rt^{-1}+1)^2-\frac{3(m+1)}{(m+2)l}rt^{-2}-(t^re^t)^{\frac{-6}{(m+2)l}}+(t^re^t)^{\frac{6(m-2)}{(m+2)l}}\right)}{\left(\frac{(72m+36+3\alpha^2m^2+12\alpha^2m+12\alpha^2)}{4(m+2)^2l^2}(rt^{-1}+1)^2+(t^re^t)^{\frac{-6}{(m+2)l}}-\frac{1}{4}(t^re^t)^{\frac{6(m-2)}{(m+2)l}}\right)}. \tag{26}$$

Deceleration parameter,

$$q=\frac{l\,r}{(t+r)^2}-1. \tag{27}$$

In absence of any curvature, matter energy density (Ω_m) and dark energy (Ω_Λ) are related by the equation

$$\Omega\ \ +\Omega_\Lambda=1, \tag{28}$$

where $\Omega_m=\frac{\rho}{3H^2}$ and $\Omega_\Lambda=\frac{\Lambda}{3H^2}$.

Then equation (28) reduces to

$$\frac{\rho}{3H^2}+\frac{\Lambda}{3H^2}=1. \tag{29}$$

Using equations (19) and (24), the cosmological constant in equation (29) is obtained as

$$\Lambda=\left[\begin{array}{l}\frac{3}{l^2}(rt^{-1}+1)^2\\ -\phi_0^2(t^re^t)^{\frac{2\alpha}{l}}\left(\frac{(72m+36+3\alpha^2m^2+12\alpha^2m+12\alpha^2)}{4(m+2)^2l^2}(rt^{-1}+1)^2+(t^re^t)^{\frac{-6}{(m+2)l}}-\frac{1}{4}(t^re^t)^{\frac{6(m-2)}{(m+2)l}}\right)\end{array}\right] \tag{30}$$

Notes

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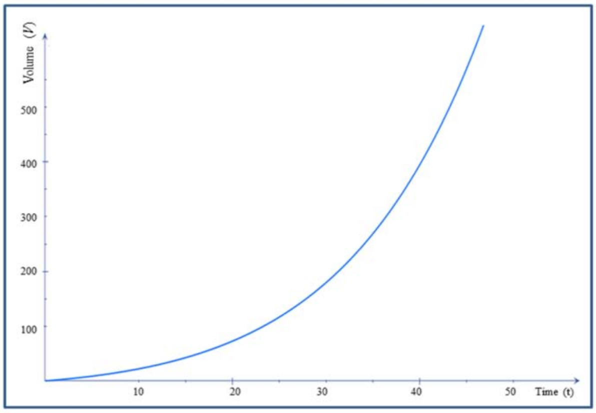

Figure 1. The Plot of Volume verses time (t)

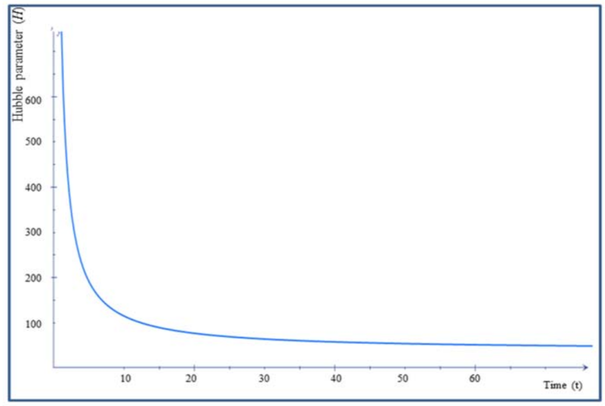

Figure 2. The Plot of Hubble's Parameter (H) verses time (t)

It is observed from equations (13) and (14) that the spatial scale factors are zero at the initial epoch $t=0$, hence the model has a point type singularity [77]. From equations (16) and (20), the spatial volume is zero and expansion scalar is infinite at $t=0$ which show that the universe starts evolving with zero volume at $t=0$ which is big bang scenario. In figures (1) & (2), plots of Spatial volume and Hubble's parameter against time are shown for better understanding.

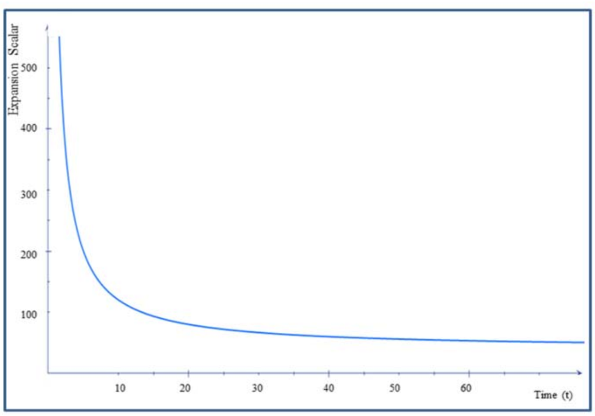

Figure 3. The Plot of Expansion scalar verses time

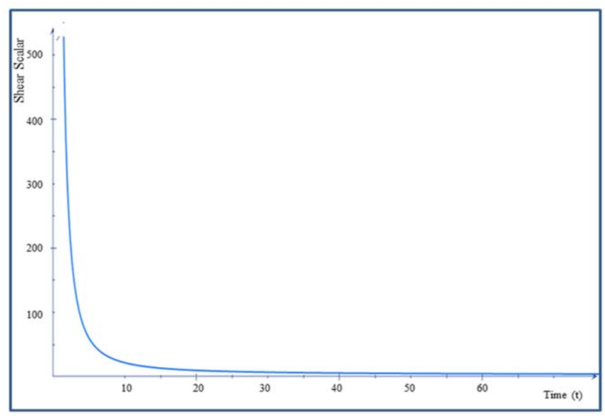

Figure 4. The Plot of Shear scalar verses time

In Fig. 3, the plots of expansion scalar verses time is given, which indicate that the expansion scalar (θ) starts from infinity at $t=0$ and tends to zero for large values of cosmic time t, showing that the universe is expanding with increase of time.

In Fig. 4, the plot of shear scalar (σ^2) verses time (t) is given. This shows that at $t=0$, the Shear scalar (σ^2) tends to infinity *i.e.* $\sigma^2 \to \infty$ and it decreases as time (t) increases. It approaches to zero after infinite time for all cosmological models.

From equations (21) and (23), the mean anisotropy parameter A_m is constant and $\dfrac{\sigma^2}{\theta^2}(\neq 0)$ is also constant, hence the model is anisotropic throughout the evolution of the universe (*i.e.* the model does not approach isotropy) except at $m=1$.

Ref

77. MacCallum, M.H.H.: Communication in Mathematical Physics, 20, 57 (1971)

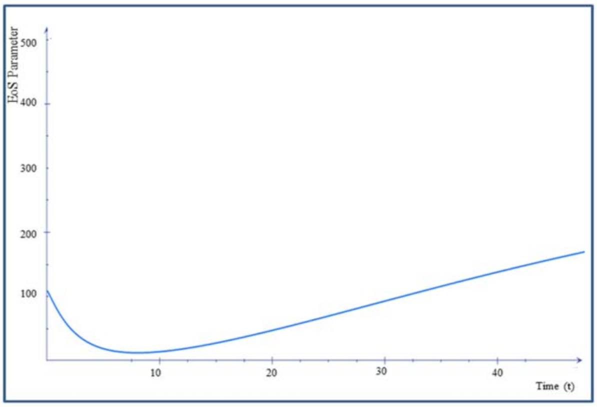

Figure 6. The Plot of Equation of State parameter verses time

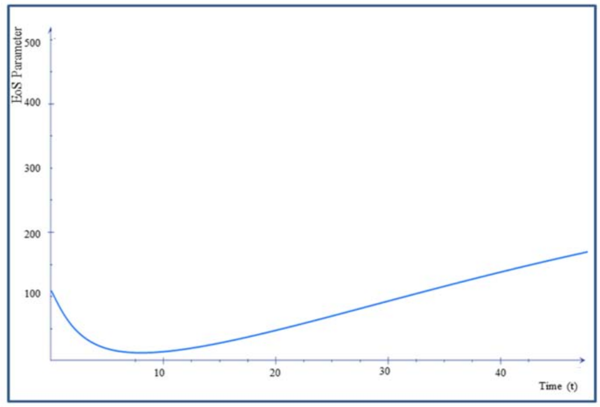

Figure 6. The Plot of Equation of State parameter verses time

From equation (24), we note that the energy density of the fluid $\rho(t)$ is a decreasing function of time. Fig. 5 is the plot of energy density of the fluid versus time. We observed that ρ is positive decreasing function of time and it approaches to zero as $t \to \infty$.

Fig. 6 depicts the variation of EoS parameter (w) versus cosmic time (t) in which we observed that it is positive throughout the evolution of the universe hence the universe is matter dominated.

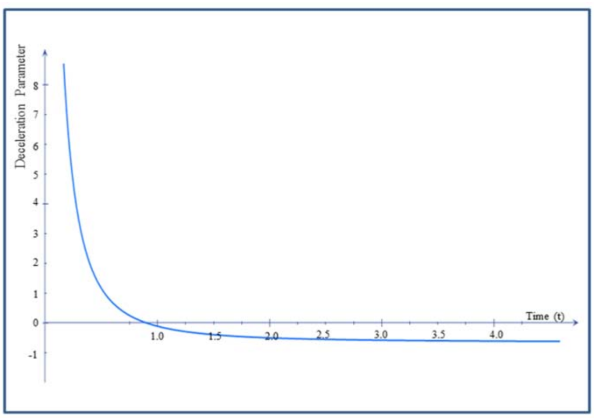

Figure 7. The Plot of Deceleration parameter verses time

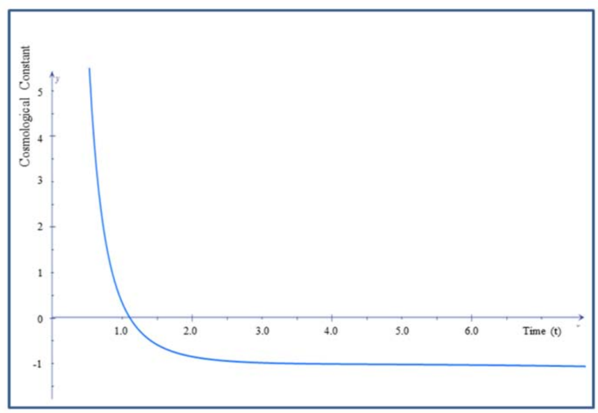

Figure 8. The Plot of Cosmological Constant verses time

We observed that the cosmological model is in accelerating phase when $q > 0$ and it evolves from decelerating phase to accelerating phase when $-1 \leq q < 0$. For the model (15), it is seen that the model is in accelerating phase for $t < \sqrt{lr} - r$ and evolves from decelerating phase to accelerating phase for $t > \sqrt{lr} - r$. Our model is evolving from decelerating phase to accelerating phase for $t \geq 3$ and $r = 1$. For the model (15), one can choose the value of DP consistent with the observations. [*i.e.* $-1 < q < 0$]. Fig. 7 represents the plot of deceleration (q) parameter versus cosmic time which gives the behavior of q as in accelerating phase at present epoch which is consistent with recent observations of Type Ia Supernaovae [14-18, 78, 79].

For illustrative purposes, evolutionary behaviors of some cosmological parameters are shown graphically (Figure 1-8).

Notes

Notes

b) Case II: when $R(t) = (\sinh(\xi t))^{\frac{1}{n}}$

The average scale factor in terms of cosmic time is considered as [80-84]

$$R = (\sinh(\xi t))^{\frac{1}{n}}. \tag{31}$$

Solving field equations (5)-(7) with the help of (1), (12) and (31), we obtain

$$a = (\sinh(\xi t))^{\frac{3m}{(m+2)n}}, \tag{32}$$

$$b = (\sinh(\xi t))^{\frac{3}{(m+2)n}}. \tag{33}$$

With the help ofequations (32) and (33), the metric (1) takes the form

$$ds^2 = \begin{pmatrix} -dt^2 + (\sinh(\xi t))^{\frac{6m}{(m+2)n}} dx^2 + (\sinh(\xi t))^{\frac{6}{(m+2)n}} dy^2 \\ + \left((\sinh(\xi t))^{\frac{6}{(m+2)n}} \sin^2 y + (\sinh(\xi t))^{\frac{6m}{(m+2)n}} \cos^2 y \right) dz^2 - 2(\sinh(\xi t))^{\frac{6m}{(m+2)n}} \cos y dx dz \end{pmatrix}. \tag{34}$$

Equation (34) represents Bianchi type-IX DE cosmological model in Sen-Dunn scalar tensor theory of gravitation.

i. Some Physical Properties of the Model

For the cosmological model (34), the physical quantities spatial volume V ,Gauge function ϕ, Hubble parameter H, expansion scalar θ, mean anisotropy parameter A_m, shear scalar σ^2, energy density ρ ,deceleration parameter q, cosmological constant Λ are obtained as follows:

Spatial volume,

$$V = (\sinh(\xi t))^{\frac{3}{n}}. \tag{35}$$

Gauge function,

$$\phi = \phi_0 (\sinh(\xi t))^{\frac{\alpha}{n}}. \tag{36}$$

The rate of expansion H_i,

$$H_x = \frac{3m\xi}{(m+2)n} \coth(\xi t), \quad H_y = H_z = \frac{3\xi}{(m+2)n} \coth(\xi t). \tag{37}$$

Hubble parameter,

$$H = \frac{\xi}{n} \coth(\xi t). \tag{38}$$

Expansion scalar,

$$\theta = 3H = 3\frac{\xi}{n} \coth(\xi t). \tag{39}$$

Mean Anisotropy Parameter,

$$A_m = \frac{2(m-1)^2}{(m+2)^2} = \text{constant } (\neq 0 \text{ for } m \neq 1). \tag{40}$$

Shear scalar,

$$\sigma^2 = \frac{3\xi^2 (m-1)^2}{(m+2)^2 n^2} (\coth(\xi t))^2. \tag{41}$$

$$\frac{\sigma^2}{\theta^2} = \frac{(m-1)^2}{3(m+2)^2} = constant(\neq 0)\ (m \neq 1). \tag{42}$$

The energy density,

$$\rho = \phi_0^2 (\sinh(\xi t))^{\frac{2\alpha}{n}} \left(\begin{array}{l} \frac{3\xi^2 (m^2\alpha^2 + 4m\alpha^2 + 4\alpha^2 + 24m + 12)}{4(m+2)^2 n^2} (\coth(\xi t))^2 + (\operatorname{cosech}(\xi t))^{\frac{6}{(m+2)n}} \\ -\frac{1}{4} (\sinh(\xi t))^{\frac{6(m-2)}{(m+2)n}} \end{array} \right). \tag{43}$$

EoS parameter,

$$w = -\frac{\left[\begin{array}{l} \frac{3\xi^2 (36 - m^2\alpha^2 - 4m\alpha^2 - 4\alpha^2)}{4(m+2)^2 n^2} (\coth(\xi t))^2 - \frac{6\xi^2}{(m+2)n} (\operatorname{cosech}(\xi t))^2 \\ + (\operatorname{cosech}(\xi t))^{\frac{-6}{(m+2)n}} - \frac{3}{4} (\sinh(\xi t))^{\frac{6(m-2)}{(m+2)n}} \end{array} \right]}{\left[\begin{array}{l} \frac{3\xi^2 (m^2\alpha^2 + 4m\alpha^2 + 4\alpha^2 + 24m + 12)}{4(m+2)^2 n^2} (\coth(\xi t))^2 + (\operatorname{cosech}(\xi t))^{\frac{6}{(m+2)n}} \\ -\frac{1}{4} (\sinh(\xi t))^{\frac{6(m-2)}{(m+2)n}} \end{array} \right]}. \tag{44}$$

Skewness parameter,

$$\delta = -\frac{\left[\begin{array}{l} \frac{9\xi^2 m(m+1)}{(m+2)^2 n^2} (\coth(\xi t))^2 - \frac{3\xi^2 (m+1)}{(m+2)n} (\operatorname{cosech}(\xi t))^2 + (\operatorname{cosech}(\xi t))^{\frac{6}{(m+2)n}} \\ - (\sinh(\xi t))^{\frac{6(m-2)}{(m+2)n}} \end{array} \right]}{\left[\begin{array}{l} \frac{3\xi^2 (m^2\alpha^2 + 4m\alpha^2 + 4\alpha^2 + 24m + 12)}{4(m+2)^2 n^2} (\coth(\xi t))^2 + (\operatorname{cosech}(\xi t))^{\frac{6}{(m+2)n}} \\ -\frac{1}{4} (\sinh(\xi t))^{\frac{6(m-2)}{(m+2)n}} \end{array} \right]}. \tag{45}$$

Deceleration paremeter,

$$q = n\left(1 - \tanh^2(\xi t)\right) - 1 . \tag{46}$$

Using equations (38) and (43), the cosmological constant in equation (29) is obtained as

Notes

$$\Lambda = \frac{3\xi^2}{n^2}(\coth(\xi t))^2 - \phi_0^2(\sinh(\xi t))^{\frac{2\alpha}{n}} \left(\begin{array}{l} \dfrac{3\xi^2(m^2\alpha^2 + 4m\alpha^2 + 4\alpha^2 + 24m + 12)}{4(m+2)^2 n^2}(\coth(\xi t))^2 \\ + (\operatorname{cosech}(\xi t))^{\frac{6}{(m+2)n}} - \dfrac{1}{4}(\sinh(\xi t))^{\frac{6(m-2)}{(m+2)n}} \end{array} \right) . \quad (47)$$

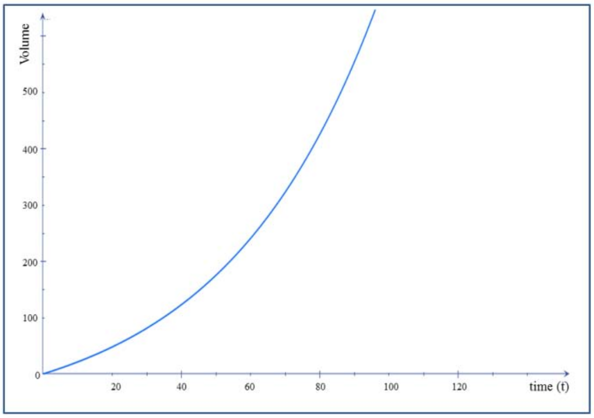

Figure 9. The Plot of Volume verses time

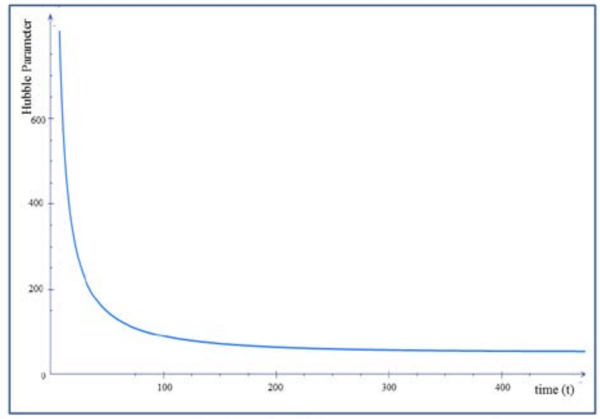

Figure 10. The Plot of Hubble Parameter verses time

It is observed that from equations (32) and (33), the spatial scale factors are zero at the initial epoch $t = 0$, hence the model has a point type singularity [77].

In figures (9) & (10), plots of spatial volume and Hubble's parameter against time are shown for better understanding. From Equation (35) and (39), we observed that the spatial volume is zero at $t = 0$ and the expansion scalar θ is infinite, which show that the universe starts evolving with zero volume at $t = 0$, which is big bang scenario.

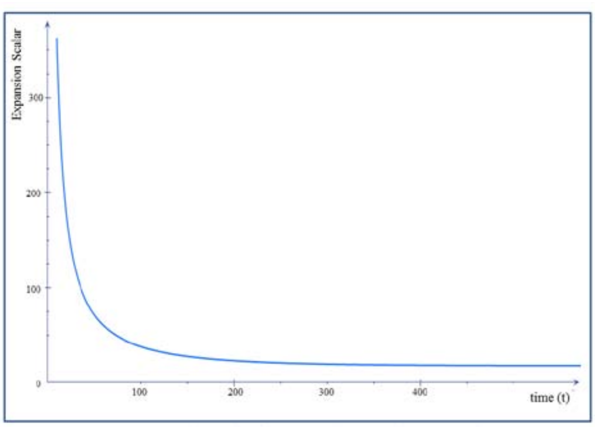

Figure 11. The Plot of Expansion Scalar verses time

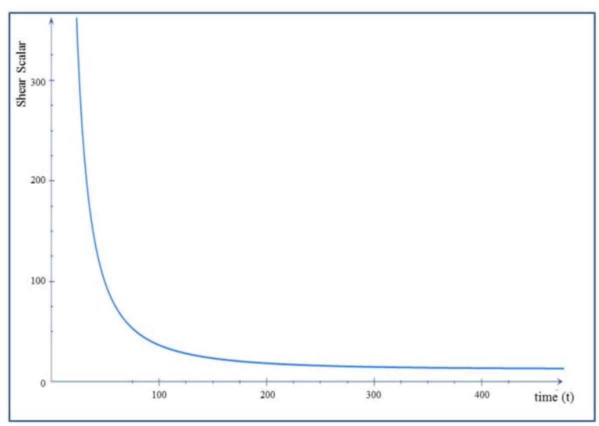

Figure 12. The Plot of Shear Scalar verses time

From equations (39) and (41), the mean anisotropy parameter A_m is constant and $\frac{\sigma^2}{\theta^2}(\neq 0)$ is also constant, hence the model is anisotropic throughout the evolution of the universe (*i.e.* the model does not approach isotropy) except at $m = 1$. The plots of expansion scalar and shear scalar against time are shown in figures (11) & (12).

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Notes

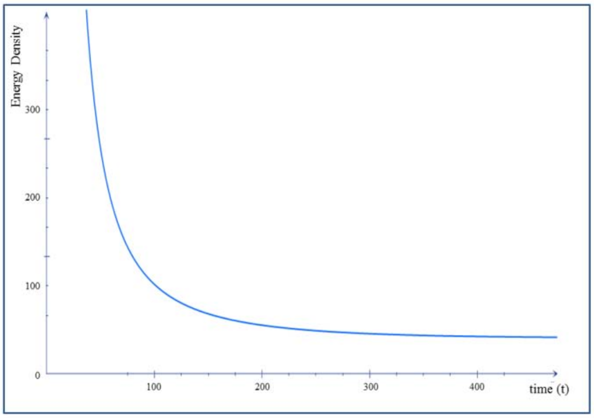

Figure 13. The Plot of Energy Density verses time

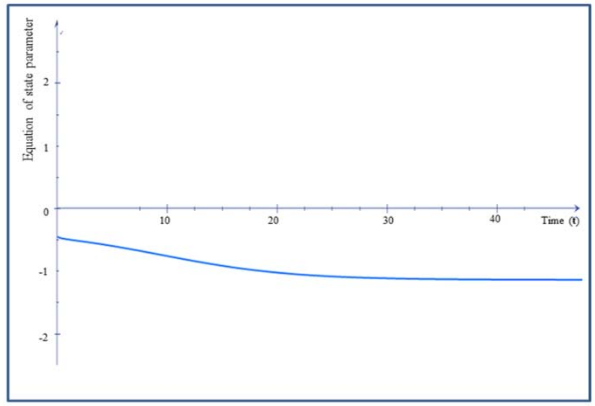

Figure 14. The Plot of Equation of State parameter verses time

From equation (43), we note that the energy density of the fluid $\rho(t)$ is a decreasing function of time. Figure 13 is the plot of energy density of the fluid versus time. We observed that ρ is positive decreasing function of time and it approaches to zero as $t \to \infty$.

Figure 14 depicts the variation of EoS parameter (w) versus cosmic time (t) in which we observed that it is negative throughout the evolution of the universe which shows that the universe is in dark energy era.

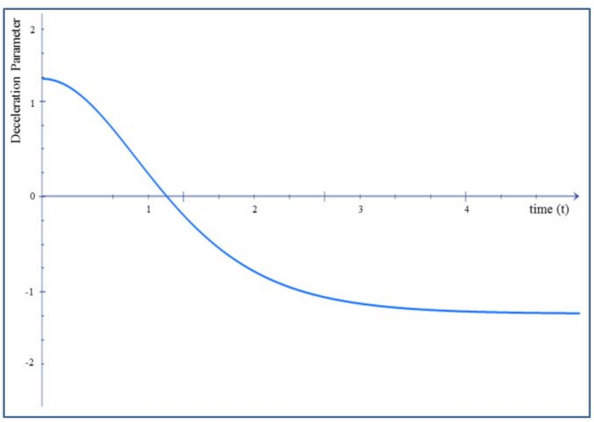

Figure 15. The Plot of Deceleration Parameter verses time

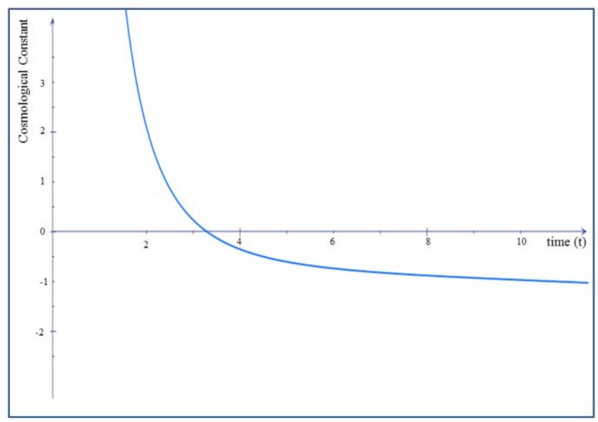

Figure 16. The Plot of Cosmological constant verses time

From equation (46), we observe that the deceleration parameter $q>0$, for $t<\frac{1}{\xi}\tanh^{-1}\left(1-\frac{1}{n}\right)^{\frac{1}{2}}$ and $q<0$, for $t>\frac{1}{\xi}\tanh^{-1}\left(1-\frac{1}{n}\right)^{\frac{1}{2}}$. The cosmological model is in accelerating phase when $q>0$ and it evolves from decelerating phase to accelerating phase when $-1 \le q < 0$. For the model (34), it is seen that the model is in accelerating phase for $t<\frac{1}{\xi}\tanh^{-1}\left(1-\frac{1}{n}\right)^{\frac{1}{2}}$ and evolves from decelerating phase to accelerating phase for $t>\frac{1}{\xi}\tanh^{-1}\left(1-\frac{1}{n}\right)^{\frac{1}{2}}$. For the model (34), one can choose the value of DP consistent with the observations. $[i.e. -1<q<0]$. Figure 15 represents the plot of deceleration parameter (q) versus cosmic time. Also recent observations of SNe Ia, expose that the present universe is accelerating and the value of DP lies to some place in the range

$-1 \le q < 0$. It follows that in our derived model, one can choose the value of DP consistent with the observations [14-18, 78, 79].

For illustrative purposes, evolutionary behaviors of some cosmological parameters are shown graphically (Figure 9-16).

IV. Conclusion

Notes

Bianchi type-IX cosmological models have obtained when universe is filled with DE in Sen-Dunn scalar-tensor theory of gravitation. To find deterministic solution, we have considered two different scale factors which yield time-dependent deceleration parameter.

In Case I, the solution of the field equations has obtained by choosing the scale factor $R = (t^r e^t)^{\frac{1}{l}}$, which yields time dependent deceleration parameter $q = \frac{l\,r}{(t+r)^2} - 1$. It is observed that the universe is matter dominated.

In Case II, we choose the scale factor $R = \left(\sinh(\xi t)\right)^{\frac{1}{n}}$ which yields time dependent deceleration parameter $q = n\left(1 - \tanh^2(\xi t)\right) - 1$. It is observed that the universe is in dark energy era.It is worth to mention that in both cases, the models obtained are point type singular, expanding, shearing. The models obtained are anisotropic throughout evolution of the universe (*i.e.* not isotropic). In an early phase of universe, the deceleration parameter is positive and decreases with increase in cosmic time t. It remains constant ($q = -1$) for large values of t. Hence the universe had a decelerated expansion in the past and has accelerated expansion at present which are in good agreement with the recent SN Ia observations [14-18, 78, 79].We hope that these models will be useful for a better understanding of dark energy in cosmology to study an accelerating expansion of the Universe.

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A Modification of *Bai Bithaman Ajil* Instrument through *Musharakah Mutanaqisah*: Fixation of Robust Optimisation into Rule 78

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Abstract- The development of Islamic financial system in Malaysia had lead to various innovation and establishment of new instruments and concepts whether in debt-based financing or equity-based financing. Though several concepts have been modified to fulfill consumer's need, however, each and every one of the models should abide the law of *Shariah*. For over a few decades, Malaysia has been the lead country in applying the *Bai Bithaman Ajil* instrument in property financing. Until recently, a few legal cases have emerged and its compliancy towards *Shariah* has been debated among the scholars and consumers itself. In spite of the expose issues, a new proxy has been introduced. *Musharakah Mutanaqisah* is introduced as an equity-financing and is proven to be conforming towards *Shariah* as its ownership is shared between the bank and consumer. In contemplation of making the debt-based financing compliance towards *Shariah*, this journal is to schemed a new model in which the original model of *Bai Bithaman Ajil* is embedded with profit sharing ratio exist in *Musharakah Mutanaqisah.*

Keywords: *bai bithaman ajil, musharakah mutanaqisah, profit sharing ratio, rule 78, robust optimisation.*

GJSFR-F Classification : *FOR Code : MSC 2010: 62G35*

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Abd Aziz Arrashid Abd Rajak [α] & Nurfadhlina Abdul Halim [σ]

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I. Introduction

The evolution of Islamic banking and finance as the modern corporate entity for the past few years has brought the industry into several new perspectives [7]. Since its emergence in 1983, various products have been introduced and schemed to meet consumer's demands corresponding to Shariah law. Shariah law is the basis in the innovation of Islamic system instruments and its objectives must conform to the law. The law constitutes from several set of rules and relationship control between human and its creator (ALLAH s.w.t) and relationship in between human which bestow the aspects of life including the financial and banking [5].

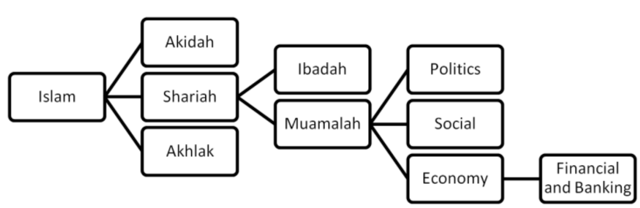

Figure 1 : The law constitutes in Islam

For an instrument to be justified as valid, void or voidable, the issues and the legal effects of a contract rely on the Islamic commercial law which is *fiqh al mu'amalat* [4]. In spite of that, the upbringing of Islamic economy system is mainly emphasises on the benefit of both the corporate entity as well as the customer. The practice of *riba,*

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4. Mohd Daud Bakar dan Engku Rabiah Adawiah Engku Ali. (2008). *Essential Readings in Islamic Finance.* Kuala Lumpur: CERT Publications Sdn Bhd.

gharar and *masyir* is strictly prohibited in Islamic economy system as it is obviously burdening especially towards customer. As been told earlier, the Islamic financial system is bounded by the principle of *fiqh.* It encompasses concepts such as risk sharing in which each participant should share both the risk and return from the transaction, the rights and duties of individuals where no exploitation should exist in the transaction and the sanctity of the contract in which any sinful activities should be prohibited.

Being a growing and developing industries, Islamic financial system also face on several issues and challenges. Various factors such as legal framework, taxation, stamp duty, accounting treatment and other similar considerations have yet to influence the innovation and the enhancement of Islamic financial product. Realistically, these factors pose significant and practical considerations prior to undertaking any product development and enhancement [8]. Despite of being *Shariah* compliant, a financial product must also be legal and compatible to current practice in terms of the mention factors, or otherwise, Islamic financial product will be inferior to the conventional financial product. According to [4], theoretically speaking, Islamic law has introduced variety of alternative when it comes to replacing proxy to conventional banking and financial products. It is precisely of this regard that the Islamic law declare *riba* as unlawful but trading as lawful [9].

II. Bai Bithaman Ajil (BBA)

Unlike equity-based financing, debt-based financing focus solely on the accountability of risk in instruments such as *Bai Bithaman Ajil* (BBA) *Murabahah* and *al-Ijarah Thumma al-Bai* (AITAB). The conformity of debt-based financing towards *Shariah* has always been debated by the scholars as it is working almost similarly to the interest-based financing that has been practice by the conventional financial institute [2]. In terms of property financing, BBA has always been the most popular modes in Malaysia [1]. Basically, BBA is a financing instrument based on deferred instalment.

The instrument BBA can be computed as follows:

$$\beta = \frac{P + U_n}{n} \tag{1}$$

where,

β : Periodical payment of BBA contract

P : Principal

U_n : Profit margin

n : Period

and the selling price can be obtained as follows:

$$\beta n = P + U_n \tag{2}$$

From equation (2), U_n can be computed as:

$$U_n = P \times r \times n \tag{3}$$

where r is the profit rate. The purchasing price, P, is originally a mixed of contribution made by the bank and the customer. Based on the Hire – Purchased Act 1967, customers (lessee) are entitled to contribute at least 10% from its purchasing price, and the remaining 90% will be contributed by the bank (lessor) [5]. Mathematically, this can be simpy put as follow:

$$P = X_t^{pp} + Y_t^{p} \tag{4}$$

Ref

8. Sudin Haron dan Noraffifah Ahmad. (2000). The Islamic banking system in Malaysia: Some issues. *Proceedings of the Fourth Harvard University Forum on Islamic Finance: Islamic Finance: The Task Ahead.* Massachusetts: Center for Middle Eastern Studies, Harvard University.

X_t^{pp} : Lessor's financial contribution

Y_t^{p} : Lessee's financial contribution

In this journal however only consider the contribution made by the lessor.

III. Rule 78

There are two methods that are used by the Islamic financial system in the evaluation of profit for BBA contract that is:

i. Rule 78

ii. Constant Rate of Return (CRR)

This journal will focus only on Rule 78. Rule 78 is the method used to determine the interest charges incurred during the period of annual payments. Rule 78 is basically based on the reduction of interest rate in this case, the profit margin on monthly basis. Eventually, the customer is obligated to pay only the principal with a slight of profit margin.

The profit gained by the bank is simply the selling price minus by the purchasing price. Mathematically, the profit gained can be computed as equation (5):

$$\begin{aligned} U_n &= \beta n - P \\ &= [P + (P \times r \times n)] - P \\ &= P \times r \times n \end{aligned} \tag{5}$$

Then by using Rule 78, the profit amortisation, B_t^{pp} gain by the bank at time t can be computed as in equation (6):

$$B_t^{pp} = \frac{[(n+1)-t] \times U_n}{\frac{n+1}{2} \times n} \tag{6}$$

IV. Musharakah Mutanaqisah (Diminishing Partnership)

Unlike the debt-based financing, equity-based financing emphasised on the profit-loss sharing [4]. The example of instrument which applied the equity-based financing is *Musharakah* and *Mudharabah.* Since the debt-based financing had been debated its compliancy towards *Shariah* for over a decade among the scholars and Islamic economist, an alternative proxy has been introduced to overcome the problems that is known as *Musharakah Mutanaqisah* or simply put as the diminishing partnership [3].

Musharakah Mutanaqisah is a partnership between two parties in which the first party (customer) promise to buy an asset own by the second party (bank) which eventually the ownership of the asset will be owned by the first party. This concept has been developed after it is identified that the bank is not subjected to any risks and liabilities towards the asset in the BBA document even though during the periodic payment the asset is fully own by the bank.

Musharakah Mutanaqisah is said to be compliance towards *Shariah* because of the profit-loss sharing between the customer and the bank [3] [10].

V. Profit Sharing Ratio (PSR): Robust Optimisation

Profit sharing ratio (PSR) can be categorised into two category that is:

i. PSR Robust optimization

ii. PSR Single constraint

This paper only considers the PSR robust optimisation. According to [5], robust optimisation of PSR must fulfil all the constraints. Decision maker should not tolerate on any error or breach regarding the optimisation constraints. By considering where

Ref

5. Nurfadhlina Abdul Halim. (2012). *Pemodelan matematik instrumen sewa-beli Islam alternatif.* Tesis Doktor Falsafah, Pusat Pengajian Sains Matematik, Universiti Kebangsaan Malaysia.

$a = \dfrac{X_0}{n}$ and $u_t(\omega)$ is the Base Financing Rate (BFR) at time t, robust optimisation of PSR can be computed as in equation (7)

$$\{\min\{\vartheta_t^{pp} : \mathrm{P}\{u_t(\omega) : \vartheta_t^{pp} \geq \frac{(X_0 - ta)r_f}{u_t(\omega)P} = 1\} u_t(\omega) \in U\} \tag{7}$$

The optimum PSR through equation (6) is obtained through the highest probability of $u_t(\omega)$ where all the value of $u_t(\omega)$ fulfill the constraints. If there is any $u_t(\omega)$ does not fulfill the constraints, thus the valid solution does not exist and no solution for optimum PSR. In other word, we consider the local optimum for each t.

VI. Modification of Rule 78 by Fixation of Profit Sharing Ratio

BBA improvements occur on the payment to be paid by the customer to the bank after obtaining the assets. If seen from the modus operandi of the BBA, the payment occurs on a monthly basis until settled and then at the end of the payment the assets will be transferred to the customer.

After the approach of *Musharakah Mutanaqisah* is embedded in the BBA model, the execution of BBA is changed and the ownership of assets is transferred gradually from the bank to the customer [6]. Although, given the approach of equity-based financing, the concept is still identified as financing-based debt because the ownership of the assets is still retained by the bank until the last periodical payment is settled in which the customer will become the sole owner of the asset.

The modified of Rule 78 by applying PSR into the model can be computed as in equation (8):

$$\tilde{B}_t^{pp} = \frac{[(n+1)-t] \times (X_t^{pp} \times \omega \times \vartheta_t^{pp})}{\frac{n+1}{2} \times n} \tag{8}$$

Thus, the new rectification of BBA model after been embedded by the *Musharakah Mutanaqisah* model is in equation (9) which is the periodical payment for each period:

$$\beta_t = \frac{P}{n} + \frac{[(n+1)-t] \times (X_t^{pp} \times \omega \times \vartheta_t^{pp})}{\frac{n+1}{2} \times n} \tag{9}$$

And the amount for BBA contract is computed as in equation (10):

$$\beta^* = \sum_{t=1}^{n} \beta_t \tag{10}$$

VII. Empirical Test on the Existing and the Modified Rule 78 and its Comparisons

Consider the following situation, customer A makes an agreement with the bank to obtain an asset by using the BBA contract. The purchasing price of the asset is RM 1,000,000 with profit margin of 10% in 6 years payment. So, the customer will be paying in 72 instalments. The selling price will be:

$$\begin{aligned} \beta n &= P + U_n \\ &= 1{,}000{,}000 + (10\% \times 6 \times 1{,}000{,}000) \end{aligned}$$

Ref

6. Saiful Azhar Rosly. (1999). Al- Bay' Bithaman Ajil Financing: Impacts on Islamic Banking Performance. *Thunderbird International Business Review*, 41 (4-5), 461-480.

$$= \quad 1{,}600{,}000 \tag{11}$$

The periodical payment that will be paid by the customer will be:

$$\beta = \frac{P+U_n}{n}$$

$$= \frac{1{,}000{,}000 + (10\% \times 6 \times 1{,}000{,}000)}{72} \tag{12}$$

$$= 22{,}222.22$$

Notes

The profit gained by the bank will be:

$$U_n = \beta n - P$$

$$= 1{,}600{,}000 - 1{,}000{,}000$$

$$= 600{,}000 \tag{13}$$

By using the existing rule 78, the profit amortisation will be:

$$B_t^{pp} = \frac{[(n+1)-t] \times U_n}{\frac{n+1}{2} \times n}$$

$$= \frac{[(72+1)-1] \times 600{,}000}{\frac{72+1}{2} \times 72} \tag{14}$$

$$= 16{,}438.36$$

Figure 2 : illustrate the execution of existing Rule 78.

Period, t	**Periodical Payment, β**	**Profit amortisation, B_t^{pp}**	**Principal amortisation**	**Remaining principal, P**
0				1,000,000.00
1	22,222.22	16,438.36	5,783.86	994,216.14
2	22,222.22	16,210.05	6,012.17	988,203.96
3	22,222.22	15,981.74	6,240.48	981,963.48
4	22,222.22	15,753.42	6,468.80	975,494.68
5	22,222.22	15,525.11	6,697.11	968,797.58
6	22,222.22	15,296.80	6,925.42	961,872.16
7	22,222.22	15,068.49	7,153.73	954,718.43
8	22,222.22	14,840.18	7,382.04	947,336.40
9	22,222.22	14,611.87	7,610.35	939,726.05
10	22,222.22	14,383.56	7,838.66	931,887.39
11	22,222.22	14,155.25	8,066.97	923,820.42
12	22,222.22	13,926.94	8,295.28	915,525.14
13	22,222.22	13,698.63	8,523.59	907,001.55
14	22,222.22	13,470.32	8,751.90	898,249.65
15	22,222.22	13,242.01	8,980.21	889,269.44
16	22,222.22	13,013.70	9,208.52	880,060.92
17	22,222.22	12,785.39	9,436.83	870,624.09

18	22,222.22	12,557.08	9,665.14	860,958.94
19	22,222.22	12,328.77	9,893.45	851,065.49
20	22,222.22	12,100.46	10,121.76	840,943.73
21	22,222.22	11,872.15	10,350.07	830,593.65
22	22,222.22	11,643.84	10,578.38	820,015.27
23	22,222.22	11,415.53	10,806.69	809,208.57
24	22,222.22	11,187.21	11,035.01	798,173.57
25	22,222.22	10,958.90	11,263.32	786,910.25
26	22,222.22	10,730.59	11,491.63	775,418.63
27	22,222.22	10,502.28	11,719.94	763,698.69
28	22,222.22	10,273.97	11,948.25	751,750.44
29	22,222.22	10,045.66	12,176.56	739,573.88
30	22,222.22	9,817.35	12,404.87	727,169.02
31	22,222.22	9,589.04	12,633.18	714,535.84
32	22,222.22	9,360.73	12,861.49	701,674.35
33	22,222.22	9,132.42	13,089.80	688,584.55
34	22,222.22	8,904.11	13,318.11	675,266.44
35	22,222.22	8,675.80	13,546.42	661,720.02
36	22,222.22	8,447.49	13,774.73	647,945.29
37	22,222.22	8,219.18	14,003.04	633,942.24
38	22,222.22	7,990.87	14,231.35	619,710.89
39	22,222.22	7,762.56	14,459.66	605,251.23
40	22,222.22	7,534.25	14,687.97	590,563.25
41	22,222.22	7,305.94	14,916.28	575,646.97
42	22,222.22	7,077.63	15,144.59	560,502.38
43	22,222.22	6,849.32	15,372.90	545,129.47
44	22,222.22	6,621.00	15,601.22	529,528.26
45	22,222.22	6,392.69	15,829.53	513,698.73
46	22,222.22	6,164.38	16,057.84	497,640.89
47	22,222.22	5,936.07	16,286.15	481,354.75
48	22,222.22	5,707.76	16,514.46	464,840.29
49	22,222.22	5,479.45	16,742.77	448,097.52
50	22,222.22	5,251.14	16,971.08	431,126.44
51	22,222.22	5,022.83	17,199.39	413,927.05
52	22,222.22	4,794.52	17,427.70	396,499.35
53	22,222.22	4,566.21	17,656.01	378,843.34
54	22,222.22	4,337.90	17,884.32	360,959.02
55	22,222.22	4,109.59	18,112.63	342,846.39
56	22,222.22	3,881.28	18,340.94	324,505.45
57	22,222.22	3,652.97	18,569.25	305,936.20
58	22,222.22	3,424.66	18,797.56	287,138.64
59	22,222.22	3,196.35	19,025.87	268,112.76
60	22,222.22	2,968.04	19,254.18	248,858.58
61	22,222.22	2,739.73	19,482.49	229,376.09

Notes

62	22,222.22	2,511.42	19,710.80	209,665.28
63	22,222.22	2,283.11	19,939.11	189,726.17
64	22,222.22	2,054.79	20,167.43	169,558.74
65	22,222.22	1,826.48	20,395.74	149,163.01
66	22,222.22	1,598.17	20,624.05	128,538.96
67	22,222.22	1,369.86	20,852.36	107,686.60
68	22,222.22	1,141.55	21,080.67	86,605.93
69	22,222.22	913.24	21,308.98	65,296.96
70	22,222.22	684.93	21,537.29	43,759.67
71	22,222.22	456.62	21,765.60	21,994.07
72	22,222.38	228.31	21,994.07	0.00

Fig.2. The empirical test on the existing Rule 78.

Notes

Whereas, by considering the same situation, using the modified Rule 78 Figure 3 is obtained. Figure 3 illustrate the execution of modified Rule 78. By considering the principal amortisation for each period is fixed, we will obtain:

$$\begin{aligned} a &= \frac{X_0}{n} \\ &= \frac{1{,}000{,}000}{72} \\ &= 13{,}888.89 \end{aligned} \tag{15}$$

Using the equation (7), (8) and (9), the PSR, profit amortisation and periodical payment for each period is obtained as shown in Figure 3.

Figure 3 : The empirical test on the modified Rule 78.

Period	Profit Sharing Ratio	Term charge	Principal Amortisation	Profit Amortisation	Periodical payment	Remaining Principal
0						1000000
1	0.007043651	6.72	13,888.89	11671.23	25,560.12	986,111.11
2	0.006944444	6.72	13,888.89	11347.03	25,235.92	972,222.22
3	0.006845238	6.72	13,888.89	11027.40	24,916.29	958,333.33
4	0.006746032	6.72	13,888.89	10712.33	24,601.22	944,444.44
5	0.006646825	6.72	13,888.89	10401.83	24,290.72	930,555.56
6	0.006547619	6.72	13,888.89	10095.89	23,984.78	916,666.67
7	0.006448413	6.72	13,888.89	9794.52	23,683.41	902,777.78
8	0.006349206	6.72	13,888.89	9497.72	23,386.61	888,888.89
9	0.00625	6.72	13,888.89	9205.48	23,094.37	875,000.00
10	0.006150794	6.72	13,888.89	8917.81	22,806.70	861,111.11
11	0.006106106	6.66	13,888.89	8634.70	22,523.59	847,222.22
12	0.00617284	6.48	13,888.89	8356.16	22,245.05	833,333.33
13	0.006060606	6.49	13,888.89	8082.19	21,971.08	819,444.44
14	0.006444444	6.00	13,888.89	7812.79	21,701.67	805,555.56
15	0.006797853	5.59	13,888.89	7547.95	21,436.83	791,666.67
16	0.006714628	5.56	13,888.89	7287.67	21,176.56	777,777.78
17	0.006594724	5.56	13,888.89	7031.96	20,920.85	763,888.89

18	0.00647482	5.56	13,888.89	6780.82	20,669.71	750,000.00
19	0.006354916	5.56	13,888.89	6534.25	20,423.14	736,111.11
20	0.006235012	5.56	13,888.89	6292.24	20,181.13	722,222.22
21	0.006115108	5.56	13,888.89	6054.79	19,943.68	708,333.33
22	0.005995204	5.56	13,888.89	5821.92	19,710.81	694,444.44
23	0.0058753	5.56	13,888.89	5593.61	19,482.50	680,555.56
24	0.005755396	5.56	13,888.89	5369.86	19,258.75	666,666.67
25	0.005635492	5.56	13,888.89	5150.68	19,039.57	652,777.78
26	0.005515588	5.56	13,888.89	4936.07	18,824.96	638,888.89
27	0.005154639	5.82	13,888.89	4726.03	18,614.92	625,000.00
28	0.005040092	5.82	13,888.89	4520.55	18,409.44	611,111.11
29	0.00472268	6.07	13,888.89	4319.63	18,208.52	597,222.22
30	0.00461285	6.07	13,888.89	4123.29	18,012.18	583,333.33
31	0.004324895	6.32	13,888.89	3931.51	17,820.40	569,444.44
32	0.004219409	6.32	13,888.89	3744.29	17,633.18	555,555.56
33	0.004113924	6.32	13,888.89	3561.64	17,450.53	541,666.67
34	0.004008439	6.32	13,888.89	3383.56	17,272.45	527,777.78
35	0.003902954	6.32	13,888.89	3210.05	17,098.93	513,888.89
36	0.003797468	6.32	13,888.89	3041.10	16,929.98	500,000.00
37	0.003691983	6.32	13,888.89	2876.71	16,765.60	486,111.11
38	0.003586498	6.32	13,888.89	2716.89	16,605.78	472,222.22
39	0.003481013	6.32	13,888.89	2561.64	16,450.53	458,333.33
40	0.003375527	6.32	13,888.89	2410.96	16,299.85	444,444.44
41	0.003121853	6.62	13,888.89	2264.84	16,153.73	430,555.56
42	0.003021148	6.62	13,888.89	2123.29	16,012.18	416,666.67
43	0.002920443	6.62	13,888.89	1986.30	15,875.19	402,777.78
44	0.002819738	6.62	13,888.89	1853.88	15,742.77	388,888.89
45	0.002719033	6.62	13,888.89	1726.03	15,614.92	375,000.00
46	0.002618328	6.62	13,888.89	1602.74	15,491.63	361,111.11
47	0.002517623	6.62	13,888.89	1484.02	15,372.91	347,222.22
48	0.002416918	6.62	13,888.89	1369.86	15,258.75	333,333.33
49	0.002316213	6.62	13,888.89	1260.27	15,149.16	319,444.44
50	0.002215509	6.62	13,888.89	1155.25	15,044.14	305,555.56
51	0.002114804	6.62	13,888.89	1054.79	14,943.68	291,666.67
52	0.002014099	6.62	13,888.89	958.90	14,847.79	277,777.78
53	0.001913394	6.62	13,888.89	867.58	14,756.47	263,888.89
54	0.001812689	6.62	13,888.89	780.82	14,669.71	250,000.00
55	0.001711984	6.62	13,888.89	698.63	14,587.52	236,111.11
56	0.001611279	6.62	13,888.89	621.00	14,509.89	222,222.22
57	0.001510574	6.62	13,888.89	547.95	14,436.83	208,333.33
58	0.001409869	6.62	13,888.89	479.45	14,368.34	194,444.44
59	0.001309164	6.62	13,888.89	415.53	14,304.41	180,555.56
60	0.001208459	6.62	13,888.89	356.16	14,245.05	166,666.67
61	0.001107754	6.62	13,888.89	301.37	14,190.26	152,777.78

Notes

Notes

62	0.001007049	6.62	13,888.89	251.14	14,140.03	138,888.89
63	0.000906344	6.62	13,888.89	205.48	14,094.37	125,000.00
64	0.000805639	6.62	13,888.89	164.38	14,053.27	111,111.11
65	0.000704935	6.62	13,888.89	127.85	14,016.74	97,222.22
66	0.00060423	6.62	13,888.89	95.89	13,984.78	83,333.33
67	0.000503525	6.62	13,888.89	68.49	13,957.38	69,444.44
68	0.00040282	6.62	13,888.89	45.66	13,934.55	55,555.56
69	0.000302115	6.62	13,888.89	27.40	13,916.29	41,666.67
70	0.00020141	6.62	13,888.89	13.70	13,902.59	27,777.78
71	0.000100705	6.62	13,888.89	4.57	13,893.46	13,888.89
72	0	6.62	13,888.89	0.00	13,888.89	0.00

VIII. Conclusion

We have done an empirical test towards the existing and modified model using Base Financing Rate (BFR) data from the year 2008 until 2013. It is proved that, the establishment of modified Rule 78 can be applied in the real world. The proposed model is able to be the proxy for existing *Bai Bithaman Ajil* (BBA).

The result from these empirical test shows that the amount of BBA contract by using modified Rule 78 is lower than the existing Rule 78. The BBA contract amount by using existing Rule 78 as the computation of profit gained is RM 1,600,000 whereas by applying the modified rule 78 in the BBA contract gives the amount of RM 1,284,000.00. This shows that, customer is obliged to pay lower profit margin by using the modified Rule 78. Besides, the modified Rule 78 is already embedded by profit sharing ratio (PSR) which applies the *Musharakah Mutanaqisah* contract and consider the dual-ratio sharing.

It can be concluded that, this new model is much more compliance towards *Shariah* because it consider the profit-loss sharing (PLS) compared to the existing BBA instrument. The application of dual-ratio sharing by the bank and customers shows that the assets acquired is owned by both parties and the string of payment made by customers will diminished the ownership from bank and increased the customer's ownership. Besides that, the profit margin that needed to be paid is much lower.

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Notes

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The Generalized Goldbach's Conjecture: Symmetry of Prime Number

By Jian Ye
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THEGENERALIZEDGOLDBACHSCONJECTURESYMMETRYOFPRIMENUMBER

Strictly as per the compliance and regulations of :

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I. Introduction

$G(x)$: the number of representatives a large even integer x as a sum of two primes.

$G(x,q,L)$: the number of representatives a large even integer x as a sum of two primes in the former L and tolerance q coprime to arithmetic progression.

p: a prime number.

a: a constant number, $a>0$.

$\mathcal{O}$: mean big O notation describes the limiting behavior of a function when the argument tends towards a particular value or infinity, usually in terms of simpler functions.

$p|k$: p divides k.

$\phi(q)$: $\phi(q)$ is the Euler phi-function.

$Li_2(x)$: express the logarithmic integral function or integral logarithm. $Li_2(x)$ is a special function such as $Li_2(x) = \int_2^x \frac{dt}{\ln^2 t}$.

II. Symmetry of Prime Number

In the former L and tolerance q ($L<q, q\geq 2$) coprime to arithmetic progression, for any one item of this progression such as $\frac{x}{2}$ where $\frac{x}{2} > \phi(q)\cdot q^2$, There are at least a pair of symmetrical primes on $\frac{x}{2}$, namely even integer x can be expressed for a sum of two primes in its series.

Let $G(x,q,L)$ is the number of representatives a large even integer x as a sum of two primes in the former L and tolerance q coprime to arithmetic progression.

1.) If $q=2^m$,

$$G(x,q,L) = \frac{1}{\phi(q)}\cdot 2C\cdot \prod_{p\geq 3}\frac{(p-1)}{(p-2)}\cdot Li_2(x) + \mathcal{O}(x\cdot e^{-a\sqrt{\ln x}}) \qquad (1)$$

where ($p>2$, $p|x$).

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2.) If q is an odd prime number,

$$G(x,q,L) = \frac{1}{q-2} \cdot 2C \cdot \prod_{p \geq 3} \frac{(p-1)}{(p-2)} \cdot Li_2(x) + \mathcal{O}(x \cdot e^{-a\sqrt{\ln x}}) \tag{2}$$

where ($p > 2$, $p|x$).

3.) If q is an odd number, let $\Upsilon(q) = q \cdot \prod(1 - \frac{2}{p})$, $(p > 2, p|q)$,

$$G(x,q,L) = \frac{1}{\Upsilon(q)} \cdot 2C \cdot \prod_{p \geq 3} \frac{(p-1)}{(p-2)} \cdot Li_2(x) + \mathcal{O}(x \cdot e^{-a\sqrt{\ln x}}), \tag{3}$$

where ($p > 2$, $p|x$).

4.) If $q = 2^m \cdot j$ (j is an odd number), let $\Upsilon(j) = j \cdot \prod(1 - \frac{2}{p})$, $(p > 2, p|j)$,

$$G(x,q,L) = \frac{1}{\phi(2^m)} \cdot \frac{1}{\Upsilon(j)} \cdot 2C \cdot \prod_{p \geq 3} \frac{(p-1)}{(p-2)} \cdot Li_2(x) + \mathcal{O}(x \cdot e^{-a\sqrt{\ln x}}), \tag{4}$$

where ($p > 2$, $p|x$).

It can be seen, classify two situations.

While $q = 2$; $q = 3$ or $q = 6$ alternative,

$$G(x) = G(x,q,L) = 2C \cdot \prod_{p \geq 3} \frac{(p-1)}{(p-2)} \cdot Li_2(x) + \mathcal{O}(x \cdot e^{-a\sqrt{\ln x}}), \tag{5}$$

where ($p > 2$, $p|x$, $a > 0$), this formula prompts the expression of Goldbach conjecture.

While $x = 2^n$,

$$G(x) = 2C \cdot Li_2(x) + \mathcal{O}(x \cdot e^{-a\sqrt{\ln x}}), \tag{6}$$

The Goldbach's conjecture expression is same with the twin prime conjecture.
Since

$$C = \prod_{p \geq 3} (1 - \frac{1}{(p-1)^2}), \tag{7}$$

where $p > 2$, p is a prime number.

III. CONCLUSION

In the former L and tolerance q $(L < q, q \geq 2)$ coprime to arithmetic progression, Let $G(x,q,L)$ is the number of representatives a large even integer x as a sum of two primes in its arithmetic progression,

$$G(x,q,L) = \frac{1}{\phi(2^m)} \cdot \frac{1}{\Upsilon(j)} \cdot 2C \cdot \prod_{p \geq 3} \frac{(p-1)}{(p-2)} \cdot Li_2(x) + \mathcal{O}(x \cdot e^{-a\sqrt{\ln x}}).$$

Notes

where $p>2$, $p|x$, $q=2^m\cdot j$, $\Upsilon(j)=j\cdot\prod(1-\frac{2}{p})(p>2,p|j)$,
p is a prime number, j is an odd number.

References

Notes

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Multi-Objective Geometric Programming in Multiple-Response Stratified Sample Surveys with Quadratic Cost Function

By Shafiullah
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Abstract- In this paper, the problem of multiple-response in stratified sample surveys has been formulated as a multi-objective geometric programming problem (MOGPP). The fuzzy programming is described for solving the formulated MOGPP. The formulated MOGPP has been solved by Lingo software and the dual solution is obtained. Subsequently with the help of dual solution of formulated MOGPP and the primal-dual relationship theorem the optimum allocations of multiple-response are obtained. A numerical example is given to illustrate the procedure.

Keywords: *multi-objective, geometric programming, multiple-response, fuzzy programming, primal-dual relationship.*

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MULTIOBJECTIVEGEOMETRICPROGRAMMINGINMULTIPLERESPONSESTRATIFIEDSAMPLESURVEYSWITHQUADRATICCOSTFUNCTION

Notes

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Shafiullah

Abstract- In this paper, the problem of multiple-response in stratified sample surveys has been formulated as a multi-objective geometric programming problem (MOGPP). The fuzzy programming is described for solving the formulated MOGPP. The formulated MOGPP has been solved by Lingo software and the dual solution is obtained. Subsequently with the help of dual solution of formulated MOGPP and the primal-dual relationship theorem the optimum allocations of multiple-response are obtained. A numerical example is given to illustrate the procedure.

Keywords: multi-objective, geometric programming, multiple-response, fuzzy programming, primal-dual relationship.

I. Introduction

Sampling consists of several characteristics that are to be measured on every selected units of the sample. Such type of sampling are called "Multivariate or Multiple Response Sampling". Ghosh (1958) has given a note on stratified random sampling with multiple characters. Kokan and Khan (1967) proposed an optimum allocation in multivariate surveys and obtained an analytical solution. Ahsan and Khan (1977) have obtained an optimum allocation in multivariate stratified random sampling using prior information. Bethel (1985, 1989) has discussed an optimum allocation algorithm and sample allocation for multivariate surveys. Jahan *et al.* (1994, 2001) have discussed generalized compromise allocation and optimum compromise allocation. Jahan and Ahsan (1995) have obtained an optimum allocation using separable programming. Recently many authors have worked in the field of multivariate stratified sample surveys and obtained optimum allocations with the help of different techniques. Some of them are: Khan *et al.* (2003, 2008), Kozak (2006), Díaz-García and Ulloa (2006, 2008), Khan *et al.* (2010), Khowaja *et al.* (2011), Ansari *et al.* (2011), Ghufran *et al.* (2011), Varshney *et al.* (2011), Khan *et al.* (2012), Iftekhar *et al.* (2013), Gupta *et al.* (2013), Raghav *et al.* (2014) and many others have discussed the problem of optimum allocation in multivariate stratified sample surveys as a multi-objective programming problem and suggested techniques for solving problems.

The engineering design problem was firstly solved by Duffin and Zener in the early 1960s with the help of geometric programming (GP) and further extended by Duffin, Peterson and Zener (1967). Geometric programs are not (in general) convex optimization problems, but they can be transformed to convex problems by a change of variables and a transformation of the objective and constraints functions. The convex programming problems occurring in GP are generally represented by an exponential or power function. GP is a mathematical programming technique for optimizing positive polynomials, which are called posynomials. The degree of difficulty (DD) plays a

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significant role for solving a non-linear programming problem by GP method. If the degree of difficulty of primal problem is zero, then unique dual feasible solution exists. If the problem has positive degree of difficulty, then the objective function can be maximized by finding the dual feasible region, and if there is negative degree of difficulty then inconsistency of the dual constraints may occur. GP method was used by many authors such as: Ahmad and Charles (1987), Jitka Dupačová (2010), Maqbool *et al.* (2011), Ghosh and Roy (2013), Shafiullah *et al.* (2013). Multi-objective geometric programming problem was discussed by Ojha and Biswal (2010), Ojha and Das in (2010) and Islam, S. (2010) in different fields. Shafiullah *et al* (2014) have discussed the fuzzy geometric programming in multivariate stratified sample surveys in presence of non-response with quadratic cost function.

Notes

A system with vague and ambiguous information can neither be formulated nor solved effectively by traditional mathematics-based on optimization techniques nor probability-based stochastic optimization approaches. However, fuzzy set theory, which was developed by Zadeh in 1960's and fuzzy programming techniques provide a useful and efficient tool for modeling and optimizing such systems. Zimmermann, H. J. (1978) has discussed fuzzy programming and linear programming. Fuzzy multi-objective programming is given by many authors such as: Sakawa and Yano (1989, 1994), Kanaya (2010), fuzzy non-linear programming is given by many authors such as: Tang and Wang (1997), Tang and Richard (1998), Trappey et al. (1998), Nasseri (2008), Rehana and Mujumdar (2009), Mesbah et al. (2010), Maleki (2002), Kheirfam (2010), Shankar et al. (2010). Nikoo et al. (2013) have described optimal water and waste-load allocations in rivers using a fuzzy transformation technique and many others.

In this paper, we have formulated the problem of multiple-response sample surveys as a multi-objective geometric programming problem (MOGPP). The fuzzy programming approach has described for solving the formulated MOGPP and optimum allocation of sample sizes are obtained. A numerical example is presented to illustrate the procedure.

II. Formulation of The Problem

In stratified sampling the population of N units is first divided into L non-overlapping subpopulation called strata, of sizes $N_1, N_2, ..., N_h, ..., N_L$ with $\sum_{h=1}^{L} N_h = N$ and the respective sample of sizes within strata are drawn to construct the estimators of the unknown parameters which are denoted by $n_1, n_2, ..., n_h, ..., n_L$ with $\sum_{h=1}^{L} n_h = n.$ The total cost C incurred in a sample survey is a function of sample allocations $n_h, h = 1,2,\cdots,L$

The problem of determining sample sizes $n_h, h = 1,2,\cdots,L$ is called the problem of allocation in stratified sampling literature. Usually, the total cost C incurred in a sample survey is a function of sample allocations $n_h, h = 1,2,\cdots,L$. The simplest form of the cost function used in a stratified sample survey is a linear function of sample sizes n_h given as:

$$C = c_0 + \sum_{h=1}^{L} c_h n_h \quad (1)$$

Where $c_h, h = 1,2,\cdots,L$ denote per unit cost of measurement in the h^{th} stratum and c_0 is the overhead cost.

If the cost of travelling between the selected units within a stratum is significant, and then the linear cost function may not be a good approximation to the actual cost incurred. Beardwood *et al.* (1959) suggested that the cost of visiting the c_h selected units in the h^{th} stratum may be taken as $t_h\sqrt{n_h}$, $h=1,2,\cdots,L$, approximately, where t_h is the travel cost per unit in the h^{th} stratum. This conjecture is based on the fact that the distance between k randomly scattered points are proportional to $\sqrt{k}$. Under the above situation, the total cost of a stratified sample survey will be the sum of the overhead cost, the measurement cost and the travel cost.

Notes

The total cost C which is quadratic in $\sqrt{n_h}$ is given as:

$$C = c_0 + \sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} t_h \sqrt{n_h} \tag{2}$$

Ignoring finite population correction (fpc) of the overall population mean $\overline{Y}_j;\, j=1,2,\cdots,p$ of the j^{th} characteristic. $\overline{y}_{jh} = 1/n_h \sum_{k=}^{n_h} y_{jhk}$ is the sample mean from h^{th} stratum for j^{th} characteristic and y_{jhk} is the value of k^{th} selected unit of the sample from h^{th} stratum for the j^{th} characteristic $k=1,2,\cdots,n_h;\, h=1,2,\cdots,L;\, j=1,2,\cdots,p$
The variance will be given as:

$$V\left(\overline{y}_{jst}\right) = \sum_{h=1}^{L}\left(\frac{1}{n_h} - \frac{1}{N_h}\right) W_h^2 S_{jh}^2 \tag{3}$$

The terms in the above eqn. (3) are independent of n_h and therefore it is sufficient to minimize only

$$V\left(\overline{y}_{jst}\right) = \sum_{h=1}^{L} \frac{W_h^2 S_{jh}^2}{n_h},\ j=1,2,\cdots,p \tag{4}$$

Multi-objective nonlinear programming problem (MNLPP) for finding out the optimum compromise allocation for a quadratic cost function is expressed as:

$$\left.\begin{aligned} &Min\ V\left(\overline{y}_{jst}\right) = \sum_{h=1}^{L} \frac{W_h^2 S_{jh}^2}{n_h} \\ &subject\ \ to \\ &\qquad \sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} t_h \sqrt{n_h} \le C_0 \\ &and \qquad n_h \ge 0\ ,\ h=1,2,\cdots,L \end{aligned}\right\},\ j=1,2,\cdots,p \tag{5}$$

where $C_0 = C - c_0$ is the cost available to meet the travel and measurement expenses, $V\left(\overline{y}_{jst}\right)$ is the sampling variance and $S_{jh}^2, h=1,2,\cdots,L$ are the known population variances.

III. Geometric Programming Approach

The following multi-objective nonlinear programming problem (MONLPP) the cost function quadratic in $\sqrt{n_h}$ and significant travel cost are given as follows:

$$\left.\begin{aligned} &Min\ \ V\left(\bar{y}_{jst}\right)=\sum_{h=1}^{L}\frac{a_{hj}}{n_h} \\ &Subject\ to \\ &\qquad \sum_{h=1}^{L}c_h n_h+\sum_{h=1}^{L}t_h\sqrt{n_h}\le C_0, \\ &and \qquad n_h\ge 0\ ,\ \mathrm{h}=1,2,\cdots,\mathrm{L} \end{aligned}\right\} j=1,2,\cdots,P \tag{6}$$

Similarly, the expression (6) can be expressed in the standard primal GPP with cost functionquadratic in $\sqrt{n_h}$ where the travel cost is significant is given as follows:

$$\left.\begin{aligned} &Max \quad f_{0j}(n) \\ &Subject \quad f_q(n)\le 1 \\ &n_h\ge 0,\ \ h=1,2,\cdots,L \end{aligned}\right\} j=1,2,\ldots,p \tag{7}$$

$$where \ \ f_q(n)=\sum_{i\in j[q]} d_i n_1^{p_{i1}}\, n_2^{p_{i2}}\cdots n_L^{p_{iL}}\ ,q=o,1,2,\cdots,k$$

$$or\ \ f_q(n)=\sum_{i\in j[q]} d_i\left[\prod_{h=1}^{L} n_h^{p_{ih}}\right], d_i>0, n_h>0, q=o,1,2,\cdots,k,$$

p_{ih} :arbitrary real numbers, d_i : positive and $f_q(n)$:posinomials

Let for simplicity $a_{hj}=\dfrac{W_h^2 S_{jh}^2}{n_h}$ & $d_i=a_{hj}=\dfrac{c_h}{C_0}=\dfrac{t_h}{C_0}$

The dual form of the primal GPP which is stated in (6) can be given as:

$$\left.\begin{aligned} &Max\ v_{0j}(w)=\prod_{q=0}^{k}\prod_{i\in j[q]}\left\{\left(\frac{d_i}{w_i}\right)^{w_i}\right\}\prod_{q=1}^{k}\left(\sum_{i\in j[q]} w_i\right)^{\sum_{i\in j[q]} w_i} && (i) \\ &Subject\ \ \sum_{i\in[0]} w_i=1 && (ii) \\ &\qquad \sum_{q=0}^{k}\sum_{i\in j[q]} p_{ih}\, w_i=0 && (iii) \\ &\qquad w_i\ge 0, q=o,1,\cdots,k\ \ and\ \ i=1,2,\ldots,m_k && (iv) \end{aligned}\right\} j=1,\ldots,p. \tag{8}$$

The above formulated GPP (8) can be solved in the following two-steps:

Step 1: For the Optimum value of the objective function, the objective function always takes the form:

$$C_0(x^*)=\left(\frac{Coeffi.\ of\ \ first\ term}{w_{01}}\right)^{w_{01}}\times\left(\frac{Coeffi.\ of\ \ Second\ term}{w_{02}}\right)^{w_{02}}$$

$$\times\ldots\ldots\times\left(\frac{Coeffi.\ of\ last\ term}{w_k}\right)^{w_k}\left(\sum w\ 's\ in\ the\ \ first\ constraints\right)^{\sum w\ 's\ in\ \ the\ \ first\ \ constraints}$$

Notes

$\left(\sum w\text{'s in the last constraints}\right)^{\sum w\text{'s in the last constraints}}$

The Multi-Objective objective function for our problem is:

$$\prod_{q=0}^{k}\prod_{i\in j[q]}\left\{\left(\frac{d_i}{w_i}\right)^{w_i}\right\}\prod_{q=1}^{k}\left(\sum_{i\in j[q]} w_i\right)^{\sum_{i\in j[q]} w_i}$$

Notes

Step 2: The equations that can be used for GPP for the weights are given below:

$\sum_{i\in j[o]} w_i$ in the objective function$=1$ (Normality condition)

and for each primal variable n_h and $\sqrt{n_h}$ having m terms.

$\sum_{i=1}^{m_k}\left(w_i \text{ for each term}\right)\times\left(\text{exponent on } n_h \text{ and } \sqrt{n_h'} \text{ in that term}\right)=0$ (Orthogonality condition)

and $w_i \geq 0$ (Positivity condition).

The above problems (8) has been solved with the help of steps (1-2) discussed in section (3) and the corresponding solutions w_{0i}^* is the unique solution to the dual constraints; it will also maximize the objective function for the dual problem. Next, the solution of the primal problem will be obtained using primal-dual relationship theorem which is given below:

a) Primal-dual relationship theorem

If w_{0i}^* is a maximizing point for dual problem (9), each optimal values of the multi-Response model which is the minimizing points $\left(n^*\right)$ for the Primal GPP's (8) satisfies the system of equations:

$$f_{0j}(n)=\begin{cases} w_{0i}^*\, v\left(w^*\right), & i\in J[0], \\ \dfrac{w_{ij}}{v_L\left(w_{0i}^*\right)}, & i\in J[L], \end{cases} \tag{9}$$

where L ranges over all positive integers for which $v_L\left(w_{0i}^*\right)>0$.

The optimal values of the sample sizes of the problems n_h^* can be calculated with the help of the primal - dual relationship theorem (9).

IV. Fuzzy Geometric Programming Approach

The solution procedure to solve the problem (9) consists of the following steps:

Step-1: Solve the MOGPP as a single objective problem using only one objective at a time and ignoring the others. These solutions are known as ideal solution.

Step-2: From the results of step-1, determine the corresponding values for every objective at each solution derived. With the values of all objectives at each ideal solution, pay-off matrix can be formulated as follows:

$$\begin{array}{c} \\ (n^{(1)}) \\ (n^{(2)}) \\ \vdots \\ (n^{(j)}) \\ \vdots \\ (n^{(p)}) \end{array} \begin{array}{c} \begin{array}{cccccc} f_{01}(n) & f_{02}(n) & \cdots & f_{0j}(n) & \cdots & f_{0p}(n) \end{array} \\ \begin{bmatrix} f_{01}^{*}(n^{(1)}) & f_{02}(n^{(1)}) & \cdots & f_{0j}(n^{(1)}) & \cdots & f_{0p}(n^{(1)}) \\ f_{01}(n^{(2)}) & f_{02}^{*}(n^{(2)}) & \cdots & f_{0j}(n^{(2)}) & \cdots & f_{0p}(n^{(2)}) \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ f_{01}(n^{(j)}) & f_{02}(n^{(j)}) & \cdots & f_{0j}^{*}(n^{(j)}) & \cdots & f_{0p}(n^{(j)}) \\ \vdots & \vdots & & \vdots & & \vdots \\ f_{01}(n^{(p)}) & f_{02}(n^{(p)}) & \cdots & .f_{0j}(n^{(p)}) & \cdots & f_{0p}^{*}(n^{(p)}) \end{bmatrix} \end{array}$$

Here $(n^{(1)}),(n^{(2)}),\cdots,(n^{(j)}),\cdots,(n^{(p)})$ are the ideal solutions of the objective functions $f_{01}(n^{(1)}),\ f_{02}(n^{(2)}),\cdots,\ f_{0j}(n^{(j)}),\cdots,\ f_{0p}(n^{(p)})$.

So $U_j = Max\{f_{01}(n^{(1)}),\ f_{02}(n^{(2)}),\ \cdots,\ f_{0p}(n^{(p)})\}$ and $L_j = f_{0j}^{*}(n^{(j)}),\ j=1,2,\ldots,p$.

$[U_j$ and L_j be the upper and lower bonds of the j^{th} objective function $f_{0j}(n),\ j=1,2,.., p.]$

Step 3: The membership function for the given problem can be define as:

$$\mu_j\left(f_{0j}(n)\right)=\begin{cases} 0, & if\ \ f_{0j}(n)\geq U_j \\ \dfrac{U_j(n)-\ f_{0j}(n)}{U_j(n)-L_j(n)}, & if\ L_j \leq f_{0j}(n)\geq U_j,\ \ j=1,2,\ldots,p \\ 1, & if\ \ f_{0j}(n)\leq L_j \end{cases} \tag{10}$$

Here $U_j(n)$ is a strictly monotonic decreasing function with respect to $f_{0j}(n)$.

Following figure 3.1 illustrates the graph of the membership function $\mu_j\left(f_{0j}(n)\right)$.

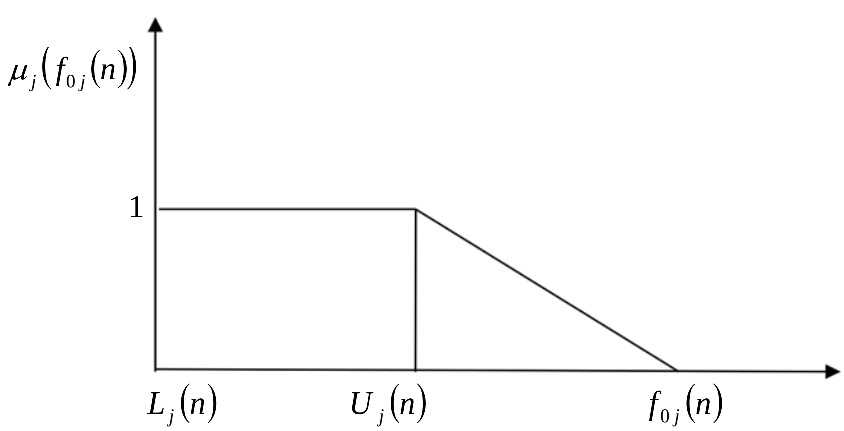

Figure 3.1 : Membership function for minimization variances problem

The membership functions in Eqn. (10)

$$\text{i.e. } \mu_j\left(f_{0j}(n)\right),\ j=1,2,\ldots,p,$$

Notes

Notes

Therefore the general aggregation function can be defined as

$$\mu_{\tilde{D}}(n)=\mu_{\tilde{D}}\{\mu_1(f_{01}(n)),\mu_2(f_{02}(n)),\dots,\mu_p(f_{0p}(n))\}$$

The fuzzy multi-objective formulation of the problem with cost function quadratic in $\sqrt{n_h}$ and significant travel cost can be defined as:

$$\left.\begin{array}{l} Max\ \mu_{\tilde{D}}(n) \\ Subject\ to\ \sum_{h=1}^{L}\frac{c_h}{C_0}n'_h+\sum_{h=1}^{L}\frac{t_h}{C_0}\sqrt{n'_h}\le C-c_0=C_0; \\ n_h\ge 0\ and\ h=1,2,\cdots,L. \end{array}\right\} \tag{11}$$

The problem to find the optimal values of (n^*) for this convex-fuzzy decision based on addition operator (like Tewari *et. al.* (1987)).Therefore the problem (11) is reduced according to max-addition operator as

$$\left.\begin{array}{l} Max\ \mu_D(n^*)=\sum_{j=1}^{p}\mu_j(f_{0j}(n))=\sum_{j=1}^{p}\frac{U_j-(f_{0j}(n))}{U_j-L_j} \\ Subject\ to\ \sum_{h=1}^{L}\frac{c_h}{C_0}n'_h+\sum_{h=1}^{L}\frac{t_h}{C_0}\sqrt{n'_h}\le C_0; \\ 0\le\mu_j(f_{0j}(n))\le 1, \\ n_h\ge 0\ and\ h=1,2,\cdots,L. \end{array}\right\} \tag{12}$$

The problem (12) reduces to

$$\left.\begin{array}{l} Max\ \mu_D(n^*)=\sum_{j=1}^{p}\left\{\frac{U_j}{U_j-L_j}-\frac{(f_{0j}(n))}{U_j-L_j}\right\} \\ Subject\ to \\ f_q(n)\le 1; \\ n_h\ge 0\ \ and\ \ j=1,2,\dots,p. \end{array}\right\} \tag{13}$$

$$\text{where } f_q(n)=\sum_{h=1}^{L}\frac{c_h}{C_0}n'_h+\sum_{h=1}^{L}\frac{t_h}{C_0}\sqrt{n'_h}$$

The problem (13) maximizes if the function $F_{oj}(n)=\left\{\frac{(f_{0j}(n))}{U_j-L_j}\right\}$ attain the minimum values.

The fuzzy multi-objective formulation of the standard primal problem with cost function quadratic in $\sqrt{n_h}$ and significant travel cost can be defined as:

$$\left.\begin{array}{l} Min \quad \sum_{j=0}^{p} F_{oj}(n') \\ Subject\ to \\ \qquad f_q(n') \le 1; \\ and\ \ n_h \ge 0\ ,\ j = 1,2,\ldots,p. \end{array}\right\} \quad (14)$$

$$\text{where } f_q(n) = \sum_{h=1}^{L} \frac{c_h}{C_0} n'_h + \sum_{h=1}^{L} \frac{t_h}{C_0} \sqrt{n'_h}$$

The dual form of the primal GPP which is stated in (14) can be given as:

$$\left.\begin{array}{ll} Max\ v(w) = \prod_{q=0}^{k} \prod_{i \in j[q]} \left\{ \left(\frac{d_i}{w_i} \right)^{w_i} \right\} \prod_{q=1}^{k} \left(\sum_{i \in j[q]} w_i \right)^{\sum_{i \in j[q]} w_i} & (i) \\ Subject\ \sum_{i \in [0]} w_i = 1 & (ii) \\ \sum_{q=0}^{k} \sum_{i \in j[q]} p_{ih}\ w_i = 0 & (iii) \\ w_i \ge 0, q = o,1,\cdots,k\ and\ \ i = 1,2,\ldots,m_k & (iv) \end{array}\right\} \quad (15)$$

The optimal values of the sample sizes of the problems n_h^* can be calculated with the help of the primal-dual relationship theorem (9).

V. Numerical Example

In the table below the stratum sizes, stratum weights, stratum standard deviations, measurement costs, and the travel costs within stratum are given for four different characteristics under study in a population stratified in five strata. The data are mainly from Chatterjee (1968) and rest of data from Ghufran *et al.* (2011).

Table 1 : The Values of N_h, W_h, c_h, t_h *and* S_{jh} for five Strata and four characteristics

H	N_h	W_h	c_h	t_h	S_{jh}			
					S_{1h}	S_{2h}	S_{3h}	S_{4h}
1	1500	0.25	1	0.5	28	206	38	120
2	1920	0.32	1	0.5	24	133	26	184
3	1260	0.21	1.5	1	32	48	44	173
4	480	0.08	1.5	1	54	37	78	92
5	840	0.14	2	1.5	67	9	76	117

The total budget of the survey is assumed to be 1500 units with an overhead cost $c_0 = 300$ units. Thus $C_0 = C - c_0 = 1500 - 300 = 1200$ units are available for measurement and travel within strata for approaching the selected units for measurement.

For solving MOGPP by using fuzzy programming, we shall first solve the four sub-problems:

Notes

Notes

a) *Sub problem1:*

On substituting the table values in sub-problem 1, we have obtained the expressions given below:

$$\left.\begin{aligned}
&Min\ \ f_{01}=\frac{49}{n_1}+\frac{58.9824}{n_2}+\frac{45.1584}{n_3}+\frac{18.6624}{n_4}+\frac{87.9844}{n_5}\\
&Subject\ \ to\\
&\quad 0.0008333n_1+0.0008333n_2+0.00125n_3+0.00125n_4\\
&\quad +0.001667n_5+0.0004167\sqrt{n_1}+0.0004167\sqrt{n_2}\\
&\quad +0.0008333\sqrt{n_3}+0.0008333\sqrt{n_4}+0.00125\sqrt{n_5}\ \le 1\\
&\quad and\qquad n_h\ge 0,h=1,2,\ldots,L
\end{aligned}\right\}\qquad(16)$$

The dual of the problem (15) is obtained as:

$$\left.\begin{aligned}
&Max\ \ v(w_{0i}^{*})=\left((49/w_{01})^{w_{01}}\right)\times\left((58.9824/w_{02})^{w_{02}}\right)\times\left((45.1584/w_{03})^{w_{03}}\right)\\
&\quad\times\left((18.6624/w_{04})^{w_{04}}\right)\times\left((87.9844/w_{05})^{w_{05}}\right)\times\left(\left(\frac{0.0008333}{w_{11}}\right)^{w_{11}}\right)\\
&\quad\times\left(\left(\frac{0.0008333}{w_{12}}\right)^{w_{12}}\right)\times\left(\left(\frac{0.00125}{w_{13}}\right)^{w_{13}}\right)\times\left(\left(\frac{0.00125}{w_{14}}\right)^{w_{14}}\right)\\
&\quad\times\left(\left(\frac{0.001667}{w_{15}}\right)^{w_{15}}\right)\times\left(\left(\frac{0.0004167}{w_{16}}\right)^{w_{16}}\right)\times\left(\left(\frac{0.0004167}{w_{17}}\right)^{w_{17}}\right)\\
&\quad\times\left(\left(\frac{0.0008333}{w_{18}}\right)^{w_{18}}\right)\times\left(\left(\frac{0.0008333}{w_{19}}\right)^{w_{19}}\right)\times\left(\left(\frac{0.00125}{w_{20}}\right)^{w_{20}}\right)\times\\
&\quad((w_{11}+w_{12}+w_{13}+w_{14}+w_{15}+w_{16}+w_{17}+w_{18}+w_{19}+w_{20})\wedge\\
&\quad(w_{11}+w_{12}+w_{13}+w_{14}+w_{15}+w_{16}+w_{17}+w_{18}+w_{19}+w_{20}));\qquad (i)\\
&Subject\ \ to\\
&\qquad w_{01}+w_{02}+w_{03}+w_{04}+w_{05}=1\ \ \text{(normality condition)}\qquad (ii)\\
&\qquad \left.\begin{aligned}
&-w_{01}+w_{11}+(1/2)\,w_{16}=0\\
&-w_{02}+w_{12}+(1/2)\,w_{17}=0\\
&-w_{03}+w_{13}+(1/2)\,w_{18}=0\\
&-w_{04}+w_{14}+(1/2)\,w_{19}=0\\
&-w_{05}+w_{15}+(1/2)\,w_{20}=0
\end{aligned}\right\}\text{(orthogonality condition)}\ \ (iii)\\
&\qquad \left.\begin{aligned}
&w_{01},w_{02},w_{03},w_{04},w_{05}>0\\
&w_{11},w_{12},w_{13},w_{14},w_{15}\ w_{16},\\
&w_{17},w_{18},w_{19},w_{20}\ge 0
\end{aligned}\right\}\ \ \text{(positivity condition)}\qquad (iv)
\end{aligned}\right\}\qquad(17)$$

For orthogonality condition defined in expression 17(*iii*) are evaluated with the help of the payoff matrix which is defined below

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) \end{pmatrix} \begin{pmatrix} w_{01} \\ w_{02} \\ w_{03} \\ w_{04} \\ w_{05} \\ w_{11} \\ w_{12} \\ w_{13} \\ w_{14} \\ w_{15} \\ w_{16} \\ w_{17} \\ w_{18} \\ w_{19} \\ w_{20} \end{pmatrix} = \begin{Bmatrix} -w_{01} + w_{11} + w_{16} = 0 \\ -w_{02} + w_{12} + w_{17} = 0 \\ -w_{03} + w_{13} + w_{18} = 0 \\ -w_{04} + w_{14} + w_{19} = 0 \\ -w_{05} + w_{15} + w_{20} = 0 \end{Bmatrix}$$

Solving the above formulated dual problem (19), we have the corresponding solution as:

$w_{01} = 0.1682412, w_{02} = 0.1845105, w_{03} = 0.1987012, w_{04} = 0.1281561, w_{05} = 0.3203910$

and $v(w^*) = 1.503975$.

Using the primal dual- relationship theorem (9), we have the optimal solution of primal problem: *i.e.*, the optimal sample sizes are computed as follows:

$$f_{0j}(n) = w_{0i}^* \, v(w_{0i}^*)$$

In expression (16), we calculate the values of n as:

$$f_{01}(n) = w_{01}^* \, v(w_{0i}^*)$$
$$\frac{49}{n_1} = 0.1682412 \times 1.503975$$
$$\Rightarrow n_1 = 193.6524$$

$$f_{01}(n) = w_{02}^* \, v(w_{0i}^*)$$
$$\frac{58.9824}{n_2} = 0.1845105 \times 1.503975$$
$$\Rightarrow n_2 \cong 212.5498$$

$$f_{01}(n) = w_{03}^* \, v(w_{0i}^*)$$
$$\frac{45.1584}{n_3} = 0.1987012 \times 1.503975$$
$$\Rightarrow n_3 = 151.1115$$

$$f_{01}(n) = w_{04}^* \, v(w_{0i}^*)$$
$$\frac{18.6624}{n_4} = 0.1281561 \times 1.503975$$
$$\Rightarrow n_4 = 96.8250$$

$$f_{01}(n) = w_{05}^* \, v(w_{0i}^*)$$
$$\frac{87.9844}{n_5} = 0.3203910 \times 1.503975$$
$$\Rightarrow n_5 = 182.5932$$

Notes

Notes

The optimal values and the objective function value are given below:

$n_1^* = 193.6524, n_2^* = 212.5498, n_3^* = 151.1115$, $n_4^* = 96.8250, n_5^* = 182.5932$;
and the objective value of the primal problem is 1.503975.

$$\left.\begin{aligned} &Min\ \ f_{02} = \frac{2652.25}{n_1} + \frac{1811.3536}{n_2} + \frac{101.6064}{n_3} + \frac{8.7616}{n_4} + \frac{1.5876}{n_5} \\ &Subject\ \ to \\ &\quad 0.0008333 n_1 + 0.0008333 n_2 + 0.00125 n_3 + 0.00125 n_4 \\ &\quad + 0.001667 n_5 + 0.0004167\sqrt{n_1} + 0.0004167\sqrt{n_2} \\ &\quad + 0.0008333\sqrt{n_3} + 0.0008333\sqrt{n_4} + 0.00125\sqrt{n_5} \le 1 \\ &and \quad n_h \ge 0\ ,\ h = 1,2,\ldots,L \end{aligned}\right\} \tag{18}$$

$$\left.\begin{aligned} &Max\ \ v(w_{0i}^*) = \left((2652.25/w_{01})^{w_{01}}\right) \times \left((1811.3536/w_{02})^{w_{02}}\right) \times \left((101.6064/w_{03})^{w_{03}}\right) \\ &\quad \times \left((8.7616/w_{04})^{w_{04}}\right) \times \left((1.5876/w_{05})^{w_{05}}\right) \times \left(\left(\frac{0.0008333}{w_{11}}\right)^{w_{11}}\right) \\ &\quad \times \left(\left(\frac{0.0008333}{w_{12}}\right)^{w_{12}}\right) \times \left(\left(\frac{0.00125}{w_{13}}\right)^{w_{13}}\right) \times \left(\left(\frac{0.00125}{w_{14}}\right)^{w_{14}}\right) \\ &\quad \times \left(\left(\frac{0.001667}{w_{15}}\right)^{w_{15}}\right) \times \left(\left(\frac{0.0004167}{w_{16}}\right)^{w_{16}}\right) \times \left(\left(\frac{0.0004167}{w_{17}}\right)^{w_{17}}\right) \\ &\quad \times \left(\left(\frac{0.0008333}{w_{18}}\right)^{w_{18}}\right) \times \left(\left(\frac{0.0008333}{w_{19}}\right)^{w_{19}}\right) \times \left(\left(\frac{0.00125}{w_{20}}\right)^{w_{20}}\right) \times \\ &\quad ((w_{11} + w_{12} + w_{13} + w_{14} + w_{15} + w_{16} + w_{17} + w_{18} + w_{19} + w_{20})\wedge \\ &\quad (w_{11} + w_{12} + w_{13} + w_{14} + w_{15} + w_{16} + w_{17} + w_{18} + w_{19} + w_{20})); \qquad (i) \\ &Subject\ to \\ &\quad w_{01}+w_{02}+w_{03}+w_{04}+w_{05}=1 \ \ \text{(normality condition)} \qquad (ii) \\ &\quad \left.\begin{aligned} &-w_{01} + w_{11} + (1/2)\,w_{16} = 0 \\ &-w_{02} + w_{12} + (1/2)\,w_{17} = 0 \\ &-w_{03} + w_{13} + (1/2)\,w_{18} = 0 \\ &-w_{04} + w_{14} + (1/2)\,w_{19} = 0 \\ &-w_{05} + w_{15} + (1/2)\,w_{20} = 0 \end{aligned}\right\} \text{(orthogonality condition)} \qquad (iii) \\ &\quad \left.\begin{aligned} &w_{01}, w_{02}, w_{03}, w_{04}, w_{05} > 0 \\ &w_{11}, w_{12}, w_{13}, w_{14}, w_{15}\ w_{16}, \\ &w_{17}, w_{18}, w_{19}, w_{20} \ge 0 \end{aligned}\right\} \ \ \text{(positivity condition)} \qquad (iv) \end{aligned}\right\} \tag{19}$$

For orthogonality condition defined in expression 19(iii) are evaluated with the help of the payoff matrix which is defined below

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) \end{pmatrix} \begin{pmatrix} w_{01} \\ w_{02} \\ w_{03} \\ w_{04} \\ w_{05} \\ w_{11} \\ w_{12} \\ w_{13} \\ w_{14} \\ w_{15} \\ w_{16} \\ w_{17} \\ w_{18} \\ w_{19} \\ w_{20} \end{pmatrix} = \begin{Bmatrix} -w_{01} + w_{11} + w_{16} = 0 \\ -w_{02} + w_{12} + w_{17} = 0 \\ -w_{03} + w_{13} + w_{18} = 0 \\ -w_{04} + w_{14} + w_{19} = 0 \\ -w_{05} + w_{15} + w_{20} = 0 \end{Bmatrix}$$

Solving the above formulated dual problems, we have the corresponding solution as:

$w_{01} = 0.4590439, w_{02} = 0.3795607, w_{03} = 0.1114193, w_{04} = 0.3320634, w_{05} = 0.01676970$ and $v(w^*) = 10.78444$.

The optimal values and the objective function value are given below:

$n_1^* = 535.7506, n_2^* = 442.5113, n_3^* = 84.5596, n_4^* = 24.4661, n_5^* = 8.77846;$

and the objective value of the primal problem is 10.78444.

$$\left.\begin{aligned} & Min\ \ f_{03} = \frac{90.25}{n_1} + \frac{69.2224}{n_2} + \frac{85.3776}{n_3} + \frac{38.9376}{n_4} + \frac{113.2096}{n_5} \\ & Subject\ \ to \\ & \qquad 0.0008333 n_1 + 0.0008333 n_2 + 0.00125 n_3 + 0.00125 n_4 \\ & \qquad + 0.001667 n_5 + 0.0004167\sqrt{n_1} + 0.0004167\sqrt{n_2} \\ & \qquad + 0.0008333\sqrt{n_3} + 0.0008333\sqrt{n_4} + 0.00125\sqrt{n_5} \le 1 \\ & and \qquad n_h \ge 0, h = 1,2,\dots,L \end{aligned}\right\} \quad (20)$$

Notes

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Notes

$$\left.\begin{aligned}
&Max\quad v(w_{0i}^{*}) = \left(\left(90.25/w_{01}\right)^{w_{01}}\right)\times\left(\left(69.2224/w_{02}\right)^{w_{02}}\right)\times\left(\left(85.3776/w_{03}\right)^{w_{03}}\right)\\
&\qquad\times\left(\left(38.9376/w_{04}\right)^{w_{04}}\right)\times\left(\left(113.2096/w_{05}\right)^{w_{05}}\right)\times\left(\left(\frac{0.0008333}{w_{11}}\right)^{w_{11}}\right)\\
&\qquad\times\left(\left(\frac{0.0008333}{w_{12}}\right)^{w_{12}}\right)\times\left(\left(\frac{0.00125}{w_{13}}\right)^{w_{13}}\right)\times\left(\left(\frac{0.00125}{w_{14}}\right)^{w_{14}}\right)\\
&\qquad\times\left(\left(\frac{0.001667}{w_{15}}\right)^{w_{15}}\right)\times\left(\left(\frac{0.0004167}{w_{16}}\right)^{w_{16}}\right)\times\left(\left(\frac{0.0004167}{w_{17}}\right)^{w_{17}}\right)\\
&\qquad\times\left(\left(\frac{0.0008333}{w_{18}}\right)^{w_{18}}\right)\times\left(\left(\frac{0.0008333}{w_{19}}\right)^{w_{19}}\right)\times\left(\left(\frac{0.00125}{w_{20}}\right)^{w_{20}}\right)\times\\
&\qquad((w_{11}+w_{12}+w_{13}+w_{14}+w_{15}+w_{16}+w_{17}+w_{18}+w_{19}+w_{20})\wedge\\
&\qquad(w_{11}+w_{12}+w_{13}+w_{14}+w_{15}+w_{16}+w_{17}+w_{18}+w_{19}+w_{20}));\qquad (i)\\
&Subject\ to\\
&\qquad w_{01}+w_{02}+w_{03}+w_{04}+w_{05}=1\quad \text{(normality condition)}\qquad (ii)\\
&\qquad\left.\begin{aligned}
-w_{01}+w_{11}+(1/2)\,w_{16}&=0\\
-w_{02}+w_{12}+(1/2)\,w_{17}&=0\\
-w_{03}+w_{13}+(1/2)\,w_{18}&=0\\
-w_{04}+w_{14}+(1/2)\,w_{19}&=0\\
-w_{05}+w_{15}+(1/2)\,w_{20}&=0
\end{aligned}\right\}\text{(orthogonality condition)}\quad (iii)\\
&\qquad\left.\begin{aligned}
&w_{01},w_{02},w_{03},w_{04},w_{05}>0\\
&w_{11},w_{12},w_{13},w_{14},w_{15}\ w_{16},\\
&w_{17},w_{18},w_{19},w_{20}\geq 0
\end{aligned}\right\}\quad\text{(positivity condition)}\quad (iv)
\end{aligned}\right\}\qquad (21)$$

For orthogonality condition defined in expression 20(*iii*) are evaluated with the help of the payoff matrix which is defined below

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) \end{pmatrix} \begin{pmatrix} w_{01} \\ w_{02} \\ w_{03} \\ w_{04} \\ w_{05} \\ w_{11} \\ w_{12} \\ w_{13} \\ w_{14} \\ w_{15} \\ w_{16} \\ w_{17} \\ w_{18} \\ w_{19} \\ w_{20} \end{pmatrix} = \begin{Bmatrix} -w_{01} + w_{11} + w_{16} = 0 \\ -w_{02} + w_{12} + w_{17} = 0 \\ -w_{03} + w_{13} + w_{18} = 0 \\ -w_{04} + w_{14} + w_{19} = 0 \\ -w_{05} + w_{15} + w_{20} = 0 \end{Bmatrix}$$

Solving the above formulated dual problems, we have the corresponding solution as:

$w_{01} = 0.1826144, w_{02} = 0.1600251, w_{03} = 0.2184575, w_{04} = 0.1479353, w_{05} = 0.2909677$ and $v(w^*) = 2.349672.$

The optimal values and the objective function value are given below:

$n_1^* = 210.3318, n_2^* = 184.0990, n_3^* = 166.3297, n_4^* = 112.0186, n_5^* = 165.5889;$

and the objective value of the primal problem is 2.349672.

$$\left.\begin{aligned} & Min\ \ f_{04} = \frac{900}{n_1} + \frac{3466.8544}{n_2} + \frac{1319.8689}{n_3} + \frac{54.1696}{n_4} + \frac{268.3044}{n_5} \\ & Subject\ \ to \\ & \qquad 0.0008333 n_1 + 0.0008333 n_2 + 0.00125 n_3 + 0.00125 n_4 \\ & \qquad + 0.001667 n_5 + 0.0004167\sqrt{n_1} + 0.0004167\sqrt{n_2} \\ & \qquad + 0.0008333\sqrt{n_3} + 0.0008333\sqrt{n_4} + 0.00125\sqrt{n_5} \le 1 \\ & and \qquad\qquad n_h \ge 0\ \ , h = 1,2,\ldots,L \end{aligned}\right\} \quad (22)$$

Notes

Notes

$$\left.\begin{aligned}
&Max \quad v(w_{0i}^{*}) = \left((900/w_{01})^{w_{01}}\right)\times\left((3466.8544/w_{02})^{w_{02}}\right)\times\left((1319.8689/w_{03})^{w_{03}}\right)\\
&\times\left((54.1696/w_{04})^{w_{04}}\right)\times\left((268.3044/w_{05})^{w_{05}}\right)\times\left(\left(\frac{0.0008333}{w_{11}}\right)^{w_{11}}\right)\\
&\times\left(\left(\frac{0.0008333}{w_{12}}\right)^{w_{12}}\right)\times\left(\left(\frac{0.00125}{w_{13}}\right)^{w_{13}}\right)\times\left(\left(\frac{0.00125}{w_{14}}\right)^{w_{14}}\right)\\
&\times\left(\left(\frac{0.001667}{w_{15}}\right)^{w_{15}}\right)\times\left(\left(\frac{0.0004167}{w_{16}}\right)^{w_{16}}\right)\times\left(\left(\frac{0.0004167}{w_{17}}\right)^{w_{17}}\right)\\
&\times\left(\left(\frac{0.0008333}{w_{18}}\right)^{w_{18}}\right)\times\left(\left(\frac{0.0008333}{w_{19}}\right)^{w_{19}}\right)\times\left(\left(\frac{0.00125}{w_{20}}\right)^{w_{20}}\right)\times\\
&((w_{11}+w_{12}+w_{13}+w_{14}+w_{15}+w_{16}+w_{17}+w_{18}+w_{19}+w_{20})\wedge\\
&(w_{11}+w_{12}+w_{13}+w_{14}+w_{15}+w_{16}+w_{17}+w_{18}+w_{19}+w_{20})); \qquad (i)\\
&\textit{Subject to}\\
&w_{01}+w_{02}+w_{03}+w_{04}+w_{05}=1 \quad \text{(normality condition)} \qquad (ii)\\
&\left.\begin{aligned}
-w_{01}+w_{11}+(1/2)\,w_{16}&=0\\
-w_{02}+w_{12}+(1/2)\,w_{17}&=0\\
-w_{03}+w_{13}+(1/2)\,w_{18}&=0\\
-w_{04}+w_{14}+(1/2)\,w_{19}&=0\\
-w_{05}+w_{15}+(1/2)\,w_{20}&=0
\end{aligned}\right\}\text{(orthogonality condition)} \qquad (iii)\\
&\left.\begin{aligned}
&w_{01},w_{02},w_{03},w_{04},w_{05}>0\\
&w_{11},w_{12},w_{13},w_{14},w_{15}\ w_{16},\\
&w_{17},w_{18},w_{19},w_{20}\geq 0
\end{aligned}\right\}\quad \text{(positivity condition)} \qquad (iv)
\end{aligned}\right\} \qquad (23)$$

For orthogonality condition defined in expression 23(*iii)* are evaluated with the help of the payoff matrix which is defined below

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & (1/2) \end{pmatrix} \begin{pmatrix} w_{01} \\ w_{02} \\ w_{03} \\ w_{04} \\ w_{05} \\ w_{11} \\ w_{12} \\ w_{13} \\ w_{14} \\ w_{15} \\ w_{16} \\ w_{17} \\ w_{18} \\ w_{19} \\ w_{20} \end{pmatrix} = \begin{Bmatrix} -w_{01} + w_{11} + w_{16} = 0 \\ -w_{02} + w_{12} + w_{17} = 0 \\ -w_{03} + w_{13} + w_{18} = 0 \\ -w_{04} + w_{14} + w_{19} = 0 \\ -w_{05} + w_{15} + w_{20} = 0 \end{Bmatrix}$$

Solving the above formulated dual problems, we have the corresponding solution as:

$w_{01} = 0.1807496, w_{02} = 0.3538838, w_{03} = 0.2688674, w_{04} = 0.05522913, w_{05} = 0.1412701$

and $v(w^*) = 23.86496$

The optimal values and the objective function value are given below:

$n_1^* = 208.6433, n_2^* = 410.5009, n_3^* = 205.6989, n_4^* = 41.09857, n_5^* = 79.5823;$

and the objective value of the primal problem is 23.86496.

VI. Conclusions

This paper constitutes a reflective study of fuzzy programming for solving the multi-objective geometric programming problem (MOGPP). The problem of multiple-response in stratified sample survey has been formulated as MOGPP and the dual solution is obtained with the help of Lingo software. The optimum allocations are obtained with the help of primal-dual relationship theorem along with corresponding dual solution. A numerical example is illustrated to ascertain the practical utility of the given method in multiple-response stratified sample surveys.

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Correspondence between Fuzzy Multisets and Sequences

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Abstract- Although fuzzy multisets or fuzzy bags are very interesting in terms of applications, it is less study or explore by researchers. Fuzzy multiset is applicable as a model of information retrieval because it has the mathematical structure or framework which expresses the number and the degree of attribution of an element simultaneously. We present a concise note on fuzzy multisets as the fuzzification or extension of multisets and the generalization of fuzzy sets. We showed the correspondence between fuzzy multisets and sequences. The Bolzano-Weierstrass property of fuzzy multisets is proposed and some theorems stated and proved.

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Strictly as per the compliance and regulations of :

Correspondence between Fuzzy Multisets and Sequences

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Abstract- Although fuzzy multisets or fuzzy bags are very interesting in terms of applications, it is less study or explore by researchers. Fuzzy multiset is applicable as a model of information retrieval because it has the mathematical structure or framework which expresses the number and the degree of attribution of an element simultaneously. We present a concise note on fuzzy multisets as the fuzzification or extension of multisets and the generalization of fuzzy sets. We showed the correspondence between fuzzy multisets and sequences. The Bolzano-Weierstrass property of fuzzy multisets is proposed and some theorems stated and proved.

I. Introduction

The theory of fuzzy multisets or fuzzy bags was introduced by Yager [16] as an attempt to fuzzify multisets proposed by Knuth [2]. In terms of similarity, fuzzy multisets are the generalized fuzzy sets introduced by Zadeh [18]. The theory of fuzzy multisets is a mathematical framework which can represent multiple occurrences of a subject item with degrees of relevance and it has been studied in relation to a variety of information systems including relational database. Fuzzy multiset is a multiset of pairs, where the first part of each pair is an element of and the second part is the degree to which the first part belongs to fuzzy multiset. An element of a fuzzy multiset can occur more than once with possibly the same or different membership values. Miyamoto [6] gave an up-to-date presentation of the theory of fuzzy multisets, which, however, does not differ significantlyfrom [5]. Since Yager [16] proposed fuzzy bags, a number of studies have been done on the theory and applications [3, 4, 11, 12, 17]. In particular, Miyamoto [5, 6, 9, 10] redefined the basic operations such as union and intersection for fuzzy bags. Singh *et al.* [14] gave an outline on the development of the concept of fuzzy multisets. Syropoulos [15] proposed the generalized fuzzy multisets and their application in computation. Bedregal *et al.* [1] generalized Atanassov's operators to higher dimensions using the concept of fuzzy sets which are the special kind of fuzzy multisets, to define a generalization of Atanassov's operator for n-dimensional fuzzy value (called n-dimensional interval).

In this paper, we propose the inter-relationship between fuzzy multisets and sequences, introduce the Bolzano-Weierstrass property of fuzzy multisets, state and prove some important theorems.

II. Precise Note on Fuzzy Multisets

Definition 1: Let X be a nonempty set. A fuzzy multiset(FMS) A drawn from X is characterized by a function, 'count membership' of A denoted by CM_A such that $CM_A : X \to Q$ where Q is the set of all crisp multisets (i.e. non- fuzzy multisets) drawn from the unit interval $[0,1]$. Then for any $x \epsilon X$, the value $CM_A(x)$ is a crisp multiset drawn from $[0,1]$. For each $x \epsilon X$, the membership sequence is defined as a decreasingly

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2. D. E. Knuth, The art of Computer programming, Semi numerical algorithms, Addison-Wesley, Reading, Massachusetts 2 (1969).

ordered sequence of elements in $CM_A(x)$. It is denoted by $\mu_A^1(x), \mu_A^2(x)$,…,$\mu_A^n(x)$ where $\mu_A^1(x) \geq \mu_A^2(x) \geq \cdots \geq \mu_A^n(x)$.

Definition 2(alternative definition): A fuzzy multiset A in X is a set of ordered sequence given as $A = \{\langle x, \mu_1(x), \mu_2(x), \mu_3(x), \ldots, \mu_n(x), \ldots \rangle : x \in X\}$, where $\mu_n(x): X \to [0,1]$is the membership function of A.

If the sequence of the membership functions have only $n-terms$ (finite number of terms), n is called the "*dimension*"of A. The collection of all fuzzy multisets in X of dimension n is denoted by $\mathcal{FM}(X)$. When we define an operation between two fuzzy multisets A and B, the lengths of the membership sequences $\mu_A^1(x), \mu_A^2(x),\ldots,\mu_A^n(x)$, and $\mu_B^1(x), \mu_B^2(x)$,…, $\mu_B^P(x)$, should be equal, if not, we append some zeros if need be.

III. Cardinality of Fuzzy Multisets

Definition 3: The length of an element x in an FMS A is defined as the cardinality of $CM_A(x)$ denoted by $L(x: A)$ i.e. the length of x in A for each $x \in X$ as

$$L(x: A) = \left| CM_A(x) \right|$$

Definition 4: If A and B are FMS drawn from X, then

$L\,(x: A, B) = max\,\{\,L(x: A), L(x: B)\}$ " or

$L(x) = \vee [L(x: A), L(x: B)]$ where $L(x) = L(x: A, B)$ and $\vee$ denotes maximum .

Example1

Consider the fuzzy multisets,

$A = \{(x_1, 0.2), (x_1, 0.3), (x_2, 1), (x_2, 0.5), (x_2, 0.5)$" of $X = \{x_1, x_2, x_3, x_4\}$, which means that A has x_1, with the membership of 0.2 and 0.3, x_2 with membership of 1 and 0.5 twice.

We represent A as

$$A = \{(0.2{,}0.3)/x_1\,, (1, 0.5{,}0.5)/x_2\}",$$

in which the bag of membership $\{0.2, 0.3\}$ corresponds to x_1, and $\{1, 0.5, 0.5\}$ corresponds to x_2.

When we handle a finite number of fuzzy multisets in a finite universal set, the length L of the membership sequences is set to be a constant for all members and for all the concerned fuzzy bags, by appending appropriate numbers of 0 at the end of the membership sequences.

For the above example, we can set $L = 3, \mu_A^1(x_1) = 0.2, \mu_A^2(x_1) = 0.3, \mu_A^3(x_1) = 0,$ $\mu_A^1(x_2) = 1, \mu_A^2(x_2) = \mu_A^3(x_2) = 0.5, \mu_A^1(x_3) = \mu_A^2(x_3) = \mu_A^3(x_3) = \mu_A^1(x_4) = \mu_A^2(x_4) =$ $\mu_A^3(x_4) = 0.$

By appending the zeros, we get

$$A = \{(0.2{,}0.3{,}0)/x_1\,, (1, 0.5{,}0.5)/x_2, (0{,}0{,}0)/x_3\,, (0{,}0{,}0)/x_4\}.$$

Example 2

Consider the set $X = \{x_1, x_2, x_3, x_4\}$ with

$$A = \{(0.3, 0.2{,}0.4, 0.5)/x_1\,, (1, 0.5, 0.5{,}0)/x_2, (0{,}0.5, 0.5)/x_3, (0.1, 0.1)/x_4\,\}$$

$$B = \{(0.3, 0.2{,}0.4,)/x_1\,, (1, 0.5, 0.5)/x_2, (0.5, 0.5)/x_3, (0.1, 0.1{,}0.3)/x_4\,\}$$

Then,

$$L(x_1: A) = 4, L(x_2: A) = 4,\ L(x_3: A) = 3,\ L\,(x_4: A) = 2,\ L(x_1: B) = 3,\ L(x_2: B) = 3, L(x_3: B) = 2,\ L(x_4: B) = 3,\ L(x_1) = 4,\ L(x_2) = 4,\ L(x_3) = 3\,,\ L(x_4) = 3.$$

Notes

Notes

Basic relations and operations on fuzzy multisets

1. Inclusion $A \subseteq B \leftrightarrow \mu_A^i(x) \leq \mu_B^i(x), i = 1, 2, \ldots, n\ \forall x \in X.$
2. Equality $A = B \leftrightarrow \mu_A^i(x) = \mu_B^i(x), i = 1, 2, \ldots, n\ \forall x \in X.$
3. Complement $A' = 1 - A \rightarrow \mu_{A'}^i(x) = 1 - \mu_A^i(x),\ i = 1, 2, \ldots, n\ \forall x \in X.$
4. Union $A \cup B = \{x, \vee(\mu_A^i(x), \mu_B^i(x))\}, i = 1, 2, \ldots, n\ \forall x \in X.$
5. Intersection $A \cap B = \{x, \wedge(\mu_A^i(x), \mu_B^i(x))\}, i = 1, 2, \ldots, n\ \forall x \in X.$
6. Addition $A + B = \{x,\ \mu_A^i(x) + \mu_B^i(x)\}, i = 1, 2, \ldots, n\ \forall x \in X.$

$$A \oplus B = \{x,\ \mu_A^i(x) + \mu_B^i(x) - \mu_A^i(x).\mu_B^i(x)\}, i = 1, 2, \ldots, n\ \forall x \in X.$$

7. Multiplication $A \odot B = \{x,\ \mu_A^i(x).\mu_B^i(x)\}, i = 1, 2, \ldots, n\ \forall x \in X.$
8. Difference $A - B = \{x,\ \mu_A^i(x) - \mu_B^i(x)\}, i = 1, 2, \ldots, n\ \forall x \in X.$
9. Absolute difference $|A - B| = \{x,\ |\mu_A^i(x) - \mu_B^i(x)|\}, i = 1, 2, \ldots, n\ \forall x \in X.$

The following are very important concepts in fuzzy bags or fuzzy multisets:

1. $\mathrm{Supp}(A) = \{x \in X:\ \mu_A^i(x) > 0\}$ where $\mathrm{Supp}(A)$ is the support of A, for $i = 1, 2, \ldots, n\ \forall x \in X$.
2. The crossover point of A is $\{x \in X:\ \mu_A^i(x) = 0.5\}$, for $i = 1, 2, \ldots, n\ \forall x \in X$.
3. $\mathrm{hgt}(A) = \sup \mu_A^i(x)$ where $\mathrm{hgt}(A)$ is the height of A, for $i = 1, 2, \ldots, n\ \forall x \in X$.
4. A is normalized if $\mathrm{hgt}(A) = 1$.

IV. Fuzzy Multisets and Sequences

A sequence is a function, say $\mu_A^i(x)$, from X into some closed unit interval [0, 1]. Then $\mu_A^i(x)$, for $i = 1, 2, \ldots,\ \forall x \in X$, is called the *nth* term of the sequence $\mu_A^1(x), \mu_A^2(x), \ldots$, for $i = 1,2,\ldots$ and this sequence is denoted by $\{\mu_A^i(x)\}$. If A is a fuzzy multiset in X, and every term of $\{\mu_A^i(x)\}$ belongs to A, then $\{\mu_A^i(x)\}$ is said to be a sequence in A or a sequence of elements of A, and we write $\{\mu_A^i(x)\} \subset A\ \forall\ x \in X$ to indicate this.

Definition 5: A sequence $\{\mu_A^i(x)\}$ is said to be finite if $i = 1,2,\ldots,n$ and infinite if $i = 1,2,\ldots$

Definition 6: A sequence $\{\mu_A^i(x)\}$ is called strictly increasing if each term is larger than the one that precedes it, and it is called strictly decreasing if each term is smaller than the one that precedes it.

Definition 7: If $\mu_A^i(x): X \rightarrow [0,1]$ for $x \in X$ and $i = 1,2,\ldots,n$, then $\mu_A^i(x)$ is a sequence in [0, 1], since $A \subset X$.

V. Limit of Fuzzy Multisets

We have established that every fuzzy multiset is a sequence, but the reverse is not true.

Definition 8: Let A be a fuzzy multiset in X s.t. $\mu_A^i(x): X \rightarrow [0,1]$ for $x \in X$ and $i = 1,2,\ldots,n$. Then a number l is called the limit of $\mu_A^1(x), \mu_A^2(x), \ldots$, if for any positive number ϵ, $\exists$ a positive number N depending on ϵ s.t. $|\mu_A^i(x) - l| < \epsilon\ \forall\ i > N$. In such case, we write $\lim_{i\to\infty} \mu_A^i(x) = l$. If l exist, $\{\mu_A^i(x)\}$ is called a convergent sequence, otherwise divergent sequence.

Theorem 1: Let $\lim_{i\to\infty} \mu_A^i(x) = A$ and $\lim_{i\to\infty} \mu_B^i(x) = B$, $i = 1,2,\ldots,$ for FMS A and B in X, then

(a) $\lim_{i\to\infty}(\mu_A^i(x) + \mu_B^i(x)) = \lim_{i\to\infty} \mu_A^i(x) + \lim_{i\to\infty} \mu_B^i(x) = A + B$

(b) $\lim_{i\to\infty}(\mu_A^i(x).\ \mu_B^i(x)) = (\lim_{i\to\infty} \mu_A^i(x))\,(\lim_{i\to\infty} \mu_B^i(x)) = A.B$

(c) $\lim_{i\to\infty}(\mu_A^i(x) - \mu_B^i(x)) = \lim_{i\to\infty} \mu_A^i(x) - \lim_{i\to\infty} \mu_B^i(x) = A - B$

(d) $\lim_{i\to\infty} \mu_A^i(x)' = (\lim_{i\to\infty} \mu_A^i(x))' = A'$

Proof

(a) If we can show that for any given positive number ϵ, $\exists$ a positive number N depending on ϵ s.t. $\left| \mu_A^i(x) + \mu_B^i(x) - (A+B) \right| \leq \epsilon \ \forall\, i > N$, we are done. This implies that,

$$\left| (\mu_A^i(x) - A) + (\mu_B^i(x) - B) \right| = \left| \mu_A^i(x) - A \right| + \left| \left|\mu_B^i(x) - B \right| \leq \epsilon \right. \tag{1}$$

By hypothesis, given $\epsilon > 0, \exists\ N_1 \text{and} N_2$ (where N is the largest) s.t.

$$\left|\mu_A^i(x) - A \right| \leq \frac{\epsilon}{2}\text{and} \left| \mu_B^i(x) - B \right| \leq \frac{\epsilon}{2} \forall\, i > N_1 \text{ and} i > N_2 \tag{2}$$

Substituting 2 into 1, we get; $\left| \mu_A^i(x) - A \right| + \left| \left|\mu_B^i(x) - B \right| =. \right. \left| \mu_A^i(x) + \mu_B^i(x) - (A+B) \right| \leq \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon.$

(b) We must show that for any $\epsilon > 0, \exists$ a positive number N depending on ϵ s.t. $\left| \mu_A^i(x).\ \mu_B^i(x) - (A.B) \right| \leq \epsilon \ \forall\, i > N$. This means,

$$\begin{aligned} &\left| \mu_A^i(x).\ \mu_B^i(x) - \mu_A^i(x)B + \mu_A^i(x)B - (A.B) \right| \\ &= \left|\mu_A^i(x)\big(\mu_B^i(x) - B\big) + B\big(\mu_A^i(x) - A\big)\right| \\ &= \left|\mu_A^i(x)\right| \left|(\mu_B^i(x) - B)\right| \\ &+ \left|B\right| \left|(\mu_A^i(x) - A) \right| \end{aligned} \tag{3}$$

Again, by hypothesis, given $\epsilon > 0, \exists\ N_1 \text{and} N_2$ (where N is the largest) s.t.

$$\left| \mu_A^i(x) - A \right| \leq \frac{\epsilon}{2\left|B\right|}\text{and} \left|\mu_B^i(x) - B \right| \leq \frac{\epsilon}{2\left|\mu_A^i(x)\right|} \forall\, i > N_1, N_2 \tag{4}$$

Substituting 4 into 3, the result follows as

$$\left|\mu_A^i(x).\ \mu_B^i(x) - (A.B) \right| = \frac{\left|\mu_A^i(x)\right|\epsilon}{2\left|\mu_A^i(x)\right|} + \frac{\left|B\right|\epsilon}{2\left|B\right|} \leq \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon.$$

(c) We are done if for any given $\epsilon > 0, \exists$ a positive number N depending on ϵ s.t. $\left| \mu_A^i(x) - \mu_B^i(x) - (A - B) \right| \leq \epsilon \ \forall\, i > N$. Then, $\left| \mu_A^i(x) - \mu_B^i(x) - (A-B) \right| = \left| \mu_A^i(x) - \mu_B^i(x) - A + B \right| = \left|\mu_A^i(x) - A + \big(-\mu_B^i(x) + B\big) \right| = \left|\mu_A^i(x) - A \right| + \left| -(\mu_B^i(x) - B) \right| = \left|\mu_A^i(x) - A \right| + \left|\mu_B^i(x) - B \right|$ and the result follows from (a).

(d)If for any given $\epsilon > 0, \exists$ a positive number N_1(i.e. $N > N_1$) depending on ϵ s.t. $\left| \mu_A^i(x) - A \right| \leq \frac{\epsilon}{2} \ \forall\, i > N$, then the theorem will follow. But $A' = 1 - A \to A' = 1 -$

Notes

$\mu_A^i(x)$ since $\mu_A^i(x)$ is the membership function of A. Then $\lim_{i\to\infty}\mu_A^i(x)' = (\lim_{i\to\infty}\mu_A^i(x))' = A' \to \left|\mu_A^i(x)' - A'\right| \le \epsilon, i.e. \left|1-\mu_A^i(x) - A'\right| = \left|-\mu_A^i(x) + 1 - A'\right| = \left|-\mu_A^i(x) + A\right| = \left|-(\mu_A^i(x) - A)\right| = \left|\mu_A^i(x) - A\right| \le \frac{\epsilon}{2}$, hence the result.

Theorem 2: If $\lim_{i\to\infty}\mu_A^i(x)$ exists, then it must be unique.

Proof: If $lim_{i\to\infty}\mu_A^i(x) = l_1$ *and* $lim_{i\to\infty}\mu_A^i(x) = l_2$, *then* $l_1 = l_2$. *By hypothesis, given any* $\epsilon > 0, \exists$ *a positive number* N *depending on* ϵ *s.t.* $\left|\mu_A^i(x) - l_1\right| \le \frac{\epsilon}{2}$*and* $\left|\mu_A^i(x) - l_2\right| \le \frac{\epsilon}{2}\ \forall\, i > N$. *Then* $\left|l_1 - l_2\right| = \left|l_1 - \mu_A^i(x) + \mu_A^i(x) - l_2\right| \le \left|-\left(-l_1 + \mu_A^i(x)\right)\right| + \left|\mu_A^i(x) - l_2\right| \le \left|\mu_A^i(x) - l_1\right| + \left|\mu_A^i(x) - l_2\right| \le \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon$ *i.e.* $\left|l_1 - l_2\right| < \epsilon \to \left|l_1 - l_2\right| = 0$ *and as such,* $l_1 = l_2$. W5

VI. Bolzano-Weierstrass (B-W)Property of Fuzzy Multisets

Definition 9: Let A be a fuzzy multiset in X. A fuzzy multiset A is compact if every sequence in A has a subsequence that converges to an element x∈X.

Theorem 3: Every fuzzy multiset has a convergent subsequence.

Proof: Every fuzzy multiset is actually a sequence and is contained in the finite interval $[a,b]$ i.e. $a = 0, b = 1$. If we divide the interval into two equal intervals, then at least one of these, say, $[a_1, b_1]$ contains infinitely many points. Dividing $[a_1, b_1]$again, yields $[a_2, b_2]$ which also contains infinitely many points. Continuing the same process again and again, we obtain a set of intervals $[a_i, b_i]$, $i = 1,2,\ldots$, each interval contained in the preceding one and hence;

$$b_1 - a_1 = \frac{(b-a)}{2},\ b_2 - a_2 = \frac{(b_1 - a_1)}{2} = \frac{(b-a)}{2^2}, \ldots, b_i - a_i = \frac{(b-a)}{2^i};$$

from which we see that ,

$$\lim_{i\to\infty}(b_i - a_i) = 0.$$

This set of nested intervals corresponds to [0, 1] which represents a limit point and so proves theorem. Q.E.D.

Corollary: A fuzzy multiset is compact if it has the B-W property.

The proof is straightforward.

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How to Prove the Riemann Hypothesis

By Fayez Fok Al Adeh

Abstract- I have already discovered a simple proof of the Riemann Hypothesis. The hypothesis states that the nontrivial zeros of the Riemann zeta function have real part equal to 0.5 . I assume that any such zero is $s = a + bi$ .I use integral calculus in the first part of the proof. In the second part I employ variational calculus. Through equations (50) to (59) I consider (a) as a fixed exponent , and verify that $a = 0.5$ .From equation (60) onward I view (a) as a parameter ($a < 0.5$) and arrive at a contradiction. At the end of the proof (from equation (73)) and through the assumption that (a) is a parameter, I verify again that $a = 0.5$.

Keywords: definite integrel, indefinite integral, variational calculus.

GJSFR-F Classification : FOR Code : MSC 2010: 11M26

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How to Prove the Riemann Hypothesis

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Abstract- I have already discovered a simple proof of the Riemann Hypothesis. The hypothesis states that the nontrivial zeros of the Riemann zeta function have real part equal to 0.5 . I assume that any such zero is s =a+ bi .I use integral calculus in the first part of the proof. In the second part I employ variational calculus. Through equations (50) to (59) I consider (a) as a fixed exponent , and verify that a = 0.5 .From equation (60) onward I view (a) as a parameter (a <0.5) and arrive at a contradiction. At the end of the proof (from equation (73)) and through the assumption that (a) is a parameter, I verify again that a = 0.5.
Subj-class: functional analysis, complex variables, general mathematics.
Keywords: definite integrel, indefinite integral, variational calculus.

I. Introduction

The Riemann zeta function is the function of the complex variable s = a + bi (i = $\sqrt{-1}$), defined in the half plane a >1 by the absolute convergent series

$$\zeta(s) = \sum_{1}^{\infty} \frac{1}{n^s} \tag{1}$$

and in the whole complex plane by analytic continuation.
The function $\zeta(s)$ has zeros at the negative even integers -2, -4, … and one refers to them as the trivial zeros. The Riemann hypothesis states that the nontrivial zeros of $\zeta(s)$ have real part equal to 0.5.

II. Proof of the Hypothesis

We begin with the equation

$$\zeta(s)=0 \tag{2}$$

And with

$$s= a+bi \tag{3}$$

$$\zeta (a + bi) = 0 \tag{4}$$

It is known that the nontrivial zeros of $\zeta(s)$ are all complex. Their real parts lie between zero and one.
If 0 < a < 1 then

$$\zeta(s) = s \int_0^{\infty} \frac{[x]-x}{x^{s+1}} dx \qquad (0 < a < 1) \tag{5}$$

[x] is the integer function

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Hence

$$\int_0^\infty \frac{[x]-x}{x^{s+1}}\,\mathrm{dx} = 0 \tag{6}$$

Therefore

$$\int_0^\infty ([x]-x)x^{-1-a-bi}dx = 0 \tag{7}$$

$$\int_0^\infty ([x]-x)x^{-1-a}x^{-bi}dx = 0 \tag{8}$$

$$\int_0^\infty x^{-1-a}([x]-x)(\cos(b\log x) - i\sin(b\log x))dx = 0 \tag{9}$$

Separating the real and imaginary parts we get

$$\int_0^\infty x^{-1-a}([x]-x)\cos(b\log x)dx = 0 \tag{10}$$

$$\int_0^\infty x^{-1-a}([x]-x)\sin(b\log x)dx = 0 \tag{11}$$

According to the functional equation, if $\zeta(s)$=0 then $\zeta(1-s)$=0. Hence we get besides equation (11)

$$\int_0^\infty x^{-2+a}([x]-x)\sin(b\log x)dx = 0 \tag{12}$$

In equation (11) replace the dummy variable x by the dummy variable y

$$\int_0^\infty y^{-1-a}([y]-y)\sin(b\log y)dy = 0 \tag{13}$$

We form the product of the integrals (12)and (13).This is justified by the fact that both integrals (12) and (13) are absolutely convergent .As to integral (12) we notice that

$$\int_0^\infty x^{-2+a}([x]-x)\sin(b\log x)dx \le \int_0^\infty \left| x^{-2+a}([\mathrm{x}]\text{-}\mathrm{x})\sin(\mathrm{blog}\ \mathrm{x})\right| \mathrm{dx}$$

$$\le \int_0^\infty x^{-2+a}((x))dx$$

(where ((z)) is the fractional part of z , 0≤ ((z))<1)

$$= \lim(\mathrm{t}\to 0)\int_0^{1-t} x^{-1+a}\,\mathrm{dx} + \lim(\mathrm{t}\to 0)\int_{1+t}^\infty x^{-2+a}((\mathrm{x}))\mathrm{dx}$$

(t is a very small positive number) (since ((x)) =x whenever 0≤x<1)

$$= \frac{1}{a} + \lim(\mathrm{t}\to 0)\int_{1+t}^\infty x^{-2+a}((\mathrm{x}))\mathrm{dx}$$

$$< \frac{1}{a} + \lim(\mathrm{t}\to 0)\int_{1+t}^\infty x^{-2+a}\,\mathrm{dx} = \frac{1}{a} + \frac{1}{a-1}$$

And as to integral (13) $\int_0^\infty y^{-1-a}([y]-y)\sin(b\log y)dy$

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$$\leq \int_0^\infty \left| \, y^{-1-a}([y]-y)\sin(b\log y) \right| \mathrm{dy}$$

$$\leq \int_0^\infty y^{-1-a}((y))dy$$

$$= \lim\,(\mathrm{t}\to 0)\int_0^{1-t} y^{-a}\,\mathrm{dy} + \lim(\mathrm{t}\to 0)\int_{1+t}^\infty y^{-1-a}((y))dy$$

(t is a very small positive number) (since ((y)) =y whenever $0 \leq y<1$)

$$= \frac{1}{1-a} + \lim(\mathrm{t}\to 0)\int_{1+t}^\infty y^{-1-a}((y))dy$$

$$< \frac{1}{1-a} + \int_{1+t}^\infty y^{-1-a}dy = \frac{1}{1-a} + \frac{1}{a}$$

Since the limits of integration do not involve x or y , the product can be expressed as the double integral

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}([x]-x)([y]-y)\sin(b\log y)\sin(b\log x)dxdy = 0 \qquad (14)$$

Thus

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}([x]-x)([y]-y)(\cos(b\log y + b\log x) - \cos(b\log y - b\log x))dxdy = 0 \qquad (15)$$

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}([x]-x)([y]-y)(\cos(b\log xy) - \cos(b\log \frac{y}{x}))dxdy = 0 \qquad (16)$$

That is

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}([x]-x)([y]-y)\cos(b\log xy)dxdy = \qquad (17)$$

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}([x]-x)([y]-y)\cos(b\log \frac{y}{x})dxdy$$

Consider the integral on the right-hand side of equation (17)

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}([x]-x)([y]-y)\cos(b\log \frac{y}{x})dxdy \qquad (18)$$

In this integral make the substitution $\mathrm{x} = \dfrac{1}{z}$ $\qquad \mathrm{dx} = \dfrac{-dz}{z^2}$

The integral becomes

$$\int_0^\infty\int_\infty^0 z^{2-a}y^{-1-a}([\frac{1}{z}]-\frac{1}{z})([y]-y)\cos(b\log zy)\frac{-dz}{z^2}dy \qquad (19)$$

That is

$$-\int_0^\infty\int_\infty^0 z^{-a}y^{-1-a}([\frac{1}{z}]-\frac{1}{z})([y]-y)\cos(b\log zy)dzdy \quad (20)$$

This is equivalent to

$$\int_0^\infty\int_0^\infty z^{-a}y^{-1-a}([\frac{1}{z}]-\frac{1}{z})([y]-y)\cos(b\log zy)dzdy \quad (21)$$

If we replace the dummy variable z by the dummy variable x, the integral takes the form

$$\int_0^\infty\int_0^\infty x^{-a}y^{-1-a}([\frac{1}{x}]-\frac{1}{x})([y]-y)\cos(b\log xy)dxdy \quad (22)$$

Rewrite this integral in the equivalent form

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}(x^{2-2a}[\frac{1}{x}]-\frac{x^{2-2a}}{x})([y]-y)\cos(b\log xy)dxdy \quad (23)$$

Thus equation 17 becomes

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}([x]-x)([y]-y)\cos(b\log xy)dxdy = \quad (24)$$

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}(x^{2-2a}[\frac{1}{x}]-\frac{x^{2-2a}}{x})([y]-y)\cos(b\log xy)dxdy$$

Write the last equation in the form

$$\int_0^\infty\int_0^\infty x^{-2+a}y^{-1-a}([y]-y)\cos(b\log xy)\{(x^{2-2a}[\frac{1}{x}]-\frac{x^{-2a\ 2}}{x})\text{-}([x]\text{-}x)\}dx$$

$$dy=0 \quad (25)$$

Let p >0 be an arbitrary small positive number. We consider the following regions in the x –y plane.

(26) The region of integration I = [0, ∞) ×[0, ∞)
(27) The large region I1 =[p, ∞) × [p, ∞)
(28) The narrow strip I 2 =[p, ∞) × [0,p]
(29) The narrow strip I 3 = [0,p]× [0, ∞)

Note that

$$I = I1 \cup I2 \cup I3 \quad (30)$$

Denote the integrand in the left hand side of equation (25) by

$$F(x,y) = x^{-2+a}y^{-1-a}([y]-y)\cos(b\log xy)\{(x^{2-2a}[\frac{1}{x}]-\frac{x^{2-2a}}{x})\text{-}([x]\text{-}x)\} \quad (31)$$

Let us find the limit of F (x,y) as x $\to\infty$ and y $\to\infty$. This limit is given by

$$\text{Lim } x^{-a}y^{-1-a}[-((y))]\cos(b\log xy)[-((\frac{1}{x}))+((x))\ x^{2a-2}] \quad (32)$$

Notes

((z)) is the fractional part of the number z ,$0 \le ((z)) < 1$

The above limit vanishes ,since all the functions [- ((y))] ,cos (blog xy), - (($\frac{1}{x}$)), and ((x)) remain bounded as $x \to \infty$ and $y \to \infty$

Note that the function F (x,y) is defined and bounded in the region I 1. We can prove that the integral

$$\iint_{I1} F(x,y)\, dx\, dy \text{ is bounded as follows} \tag{33}$$

$$\iint_{I1} F(x,y)\, dx\, dy = \iint_{I1} x^{-a} y^{-1-a} [- ((y))] \cos(b \log xy) [- ((\tfrac{1}{x})) + ((x)) \tag{34}$$

$$x^{2a-2}]\, dx\, dy$$

$$\le \left| \iint_{I1} x^{-a} y^{-1-a} [- ((y))] \cos(b \log xy) [- ((\tfrac{1}{x})) + ((x))\, x^{2a-2}]\, dx\, dy \right|$$

$$= \left| \int_p^\infty \left(\int_p^\infty x^{-a} \cos(b \log xy) [- ((\tfrac{1}{x})) + ((x))\, x^{2a-2}]\, dx \right) y^{-1-a} [- ((y))] dy \right|$$

$$\le \int_p^\infty \left| \left(\int_p^\infty x^{-a} \cos(b \log xy) [- ((\tfrac{1}{x})) + ((x))\, x^{2a-2}]\, dx \right) \right| \left| y^{-1-a} [- ((y))] \right| dy$$

$$\le \int_p^\infty \left(\int_p^\infty x^{-a} \left| \cos(b \log xy) \right| \left| [- ((\tfrac{1}{x})) + ((x))\, x^{2a-2}] \right| dx \right) \left| y^{-1-a} [- ((y))] \right| dy$$

$$< \int_P^\infty x^{-a} [((\tfrac{1}{x})) + ((x))\, x^{2a-2}]\, dx \int_P^\infty y^{-1-a}$$

$$= \frac{1}{ap^a} \int_P^\infty x^{-a} [((\tfrac{1}{x})) + ((x))\, x^{2a-2}]\, dx$$

$$= \frac{1}{ap^a} \{ \lim(t \to 0) \int_P^{1-t} x^{-a} [((\tfrac{1}{x})) + ((x))\, x^{2a-2}]\, dx + \lim(t \to 0)$$

$$\int_{1+t}^\infty x^{-a} [((\tfrac{1}{x})) + ((x))\, x^{2a-2}]\, dx \}$$

where t is a very small arbitrary positive. number.Since the integral

$\lim(t \to 0) \int_P^{1-t} x^{-a} [((\frac{1}{x})) + ((x))\, x^{2a-2}]\, dx$ is bounded, it remains to

show that $\lim(t \to 0)$

$\int_{1+t}^\infty x^{-a} [((\frac{1}{x})) + ((x))\, x^{2a-2}]\, dx$ is bounded.

Since x >1 ,then $((\frac{1}{x})) = \frac{1}{x}$ and we have $\lim(t \to 0) \int_{1+t}^\infty x^{-a} [((\frac{1}{x})) + ((x)) x^{2a-2}]\, dx$

$$= \lim(t \to 0) \int_{1+t}^{\infty} x^{-a} \left[\frac{1}{x} + ((x))\, x^{2a-2} \right] dx$$

$$= \lim(t \to 0) \int_{1+t}^{\infty} \left[x^{-a-1} + ((x))\, x^{a-2} \right] dx$$

$$< \lim(t \to 0) \int_{1+t}^{\infty} \left[x^{-a-1} + x^{a-2} \right] dx$$

$$= \frac{1}{a(1-a)}$$

Hence the boundedness of the integral $\iint_{I1}$ F(x,y) dx dy is proved.

Consider the region

$$I4 = I2 \cup I3 \tag{35}$$

We know that

$$0 = \iint_{I} F(x,y)\, dx\, dy = \iint_{I1} F(x,y)\, dx\, dy + \iint_{I4} F(x,y)\, dx\, dy \tag{36}$$

and that

$$\iint_{I1} F(x,y)\, dx\, dy \text{ is bounded} \tag{37}$$

From which we deduce that the integral

$$\iint_{I4} F(x,y)\, dx\, dy \text{ is bounded} \tag{38}$$

Remember that

$$\iint_{I4} F(x,y)\, dx\, dy = \iint_{I2} F(x,y)\, dx\, dy + \iint_{I3} F(x,y)\, dx\, dy \tag{39}$$

Consider the integral

$$\iint_{I2} F(x,y)\, dx\, dy \le \left| \iint_{I2} F(x,y)\, dx\, dy \right| \tag{40}$$

$$= \left| \int_{0}^{p} \left(\int_{p}^{\infty} x^{-a} \left\{ ((\frac{1}{x})) - ((x))\, x^{2a-2} \right\} \cos(b \log xy)\, dx \right) \frac{1}{y^{a}}\, dy \right|$$

$$\le \int_{0}^{p} \left| \int_{p}^{\infty} \left(x^{-a} \left\{ ((\frac{1}{x})) - ((x))\, x^{2a-2} \right\} \cos(b \log xy)\, dx \right) \right| \frac{1}{y^{a}}\, dy$$

$$\le \int_{0}^{p} \left(\int_{p}^{\infty} \left| x^{-a} \left\{ ((\frac{1}{x})) - ((x))\, x^{2a-2} \right\} \right| \left| \cos(b \log xy) \right| dx \right) \frac{1}{y^{a}}\, dy$$

$$\le \int_{P}^{\infty} \left| x^{-a} \left\{ ((\frac{1}{x})) - ((x))\, x^{2a-2} \right\} \right| dx \times \int_{0}^{P} \frac{1}{y^{a}}\, dy$$

Notes

Notes

(This is because in this region ((y)) = y). It is evident that the integral $\int_{P}^{\infty} \left| x^{-a}\{((\frac{1}{x}))-((x))\, x^{2a-2}\}\right| dx$ is bounded,this was proved in the course of proving that the integral $\iint_{I1} F(x,y)\, dx\, dy$ is bounded .Also it is evident that the integral

$$\int_{0}^{P} \frac{1}{y^{a}}\, dy$$

is bounded. Thus we deduce that the integral (40) $\iint_{I2} F(x,y)\, dx\, dy$ is bounded

Hence ,according to equation(39),the integral $\iint_{I3} F(x,y)\, dx\, dy$ is bounded.

Now consider the integral

$$\iint_{I3} F(x,y)\, dx\, dy \tag{41}$$

We write it in the form

$$\iint_{I3} F(x,y)\, dx\, dy = \int_{0}^{p}\left(\int_{0}^{\infty} y^{-1-a}\, ((y)) \cos(b \log xy)\, dy\right) \frac{\{((\frac{1}{x})) - x^{2a-1}\}}{x^{a}}\, dx \tag{42}$$

(This is because in this region ((x)) = x)

$$\leq \left| \int_{0}^{p}\left(\int_{0}^{\infty} y^{-1-a}\, ((y)) \cos(b \log xy)\, dy\right) \frac{\{((\frac{1}{x})) - x^{2a-1}\}}{x^{a}}\, dx \right|$$

$$\leq \int_{0}^{p} \left| \left(\int_{0}^{\infty} y^{-1-a}\, ((y)) \cos(b \log xy)\, dy\right) \right| \left| \frac{\{((\frac{1}{x})) - x^{2a-1}\}}{x^{a}} \right| dx$$

$$\leq \int_{0}^{p}\left(\int_{0}^{\infty} y^{-1-a}\, ((y))\, dy\right) \left| \frac{\{((\frac{1}{x})) - x^{2a-1}\}}{x^{a}} \right| dx$$

Now we consider the integral with respect to y

$$\int_{0}^{\infty} y^{-1-a}\, ((y))\, dy \tag{43}$$

$$= (\lim t \to 0) \int_{0}^{1-t} y^{-1-a} \times y\, dy + (\lim t \to 0) \int_{1+t}^{\infty} y^{-1-a}\, ((y))\, dy$$

(where t is a very small arbitrary positive number) .(Note that ((y))=y whenever $0 \le y < 1$).

Thus we have $(\lim t \to 0) \int_{1+t}^{\infty} y^{-1-a}((y))\, dy < (\lim t \to 0) \int_{1+t}^{\infty} y^{-1-a}\, dy = \frac{1}{a}$

and $(\lim t \to 0) \int_{0}^{1-t} y^{-1-a} \times y\, dy = \frac{1}{1-a}$

Hence the integral (43) $\int_{0}^{\infty} y^{-1-a}((y))\, dy$ is bounded.

Since $\left| \int_{0}^{\infty} y^{-1-a}((y)) \cos(b \log xy) dy \right| \le \int_{0}^{\infty} y^{-1-a}((y))\, dy$, we conclude that the integral $\left| \int_{0}^{\infty} y^{-1-a}((y)) \cos(b \log xy) dy \right|$ is a bounded function of x . Let this function be H(x) . Thus we have

$$\left| \int_{0}^{\infty} y^{-1-a}((y)) \cos(b \log xy) dy \right| = H(x) \le K \text{ (K is a positive number)} \tag{44}$$

Now equation (44) gives us

$$-K \le \int_{0}^{\infty} y^{-1-a}((y)) \cos(b \log xy) dy \le K \tag{45}$$

According to equation (42) we have

$$\iint_{I3} F(x,y)\, dx\, dy = \int_0^p \left(\int_0^{\infty} y^{-1-a}((y)) \cos(b \log xy)\, dy \right) \frac{\{((\frac{1}{x})) - x^{2a-1}\}}{x^a} dx \tag{46}$$

$$\ge \int_0^p (-K) \frac{\{((\frac{1}{x})) - x^{2a-1}\}}{x^a} dx = K \int_p^0 \frac{\{((\frac{1}{x})) - x^{2a-1}\}}{x^a} dx$$

Since $\iint_{I3} F(x,y)\, dx\, dy$ is bounded, then $\int_p^0 \frac{\{((\frac{1}{x})) - x^{2a-1}\}}{x^a} dx$ is also bounded. Therefore the integral

$$G = \int_0^p \frac{\{((\frac{1}{x})) - x^{2a-1}\}}{x^a} dx \text{ is bounded} \tag{47}$$

We denote the integrand of (47) by

$$F = \frac{1}{x^a} \{ ((\frac{1}{x})) - x^{2a-1} \} \tag{48}$$

Let δ G [F] be the variation of the integral G due to the variation of the integrand δ F. Since

$$G[F] = \int F\, dx \text{ (the integral (49) is indefinite)} \tag{49}$$

Notes

Notes

(here we do not consider a as a parameter, rather we consider it as a given exponent)

We deduce that $\frac{\delta G[F]}{\delta F(x)} = 1$

that is

$$\delta \, G \, [F] = \delta \, F \, (x) \tag{50}$$

But we have

$$\delta \, G[F] = \int dx \, \frac{\delta G[F]}{\delta F(x)} \, \delta F(x) \text{(the integral (51) is indefinite)} \tag{51}$$

Using equation (50) we deduce that

$$\delta \, G[F] = \int dx \, \delta F(x) \text{(the integral (52) is indefinite)} \tag{52}$$

Since G[F] is bounded across the elementary interval [0,p] , we must have that

$$\delta \, G[F] \text{ is bounded across this interval} \tag{53}$$

From (52) we conclude that

$$\delta G = \int_0^P dx \, \delta F(x) = \int_0^P dx \frac{dF}{dx} \, \delta x = [\, F \, \delta x \,] \text{ (at } x = p) - [\, F \, \delta x \,] \text{ (at } x = 0) \tag{54}$$

Since the value of $[\, F \, \delta x \,]$ (at x = p)is bounded, we deduce from equation (54) that

$$\lim \, (x \to 0) \, F \, \delta x \text{ must remain bounded.} \tag{55}$$

Thus we must have that

$$(\lim x \to 0) \, [\, \delta x \, \frac{1}{x^a} \{ ((\frac{1}{x})) - x^{2a-1} \}] \text{ is bounded .} \tag{56}$$

First we compute

$$(\lim x \to 0) \, \frac{\delta x}{x^a} \tag{57}$$

Applying L 'Hospital ' rule we get

$$(\lim x \to 0) \, \frac{\delta x}{x^a} = (\lim x \to 0) \frac{1}{a} \times x^{1-a} \times \frac{d(\delta x)}{dx} = 0 \tag{58}$$

We conclude from (56) that the product

$$0 \times (\lim x \to 0 \,) \, \{ ((\frac{1}{x})) - x^{2a-1} \} \text{must remain bounded.} \tag{59}$$

Assume that a =0.5 .(remember that we considered a as a given exponent)This value a =0.5 will guarantee that the quantity $\{ ((\frac{1}{x})) - x^{2a-1} \}$

will remain bounded in the limit as $(x \to 0)$ .Therefore , in this case (a=0.5) (56) will approach zero as $(x \to 0)$ and hence remain bounded .

Now suppose that a< 0.5 .In this case we consider a as a parameter.Hence we have

$$G_a \, [x] = \int dx \frac{F(x,a)}{x} x \text{ (the integral (60) is indefinite)} \tag{60}$$

Thus

$$\frac{\delta G_a [\, x \,]}{\delta x} = \frac{F(x,a)}{x} \tag{61}$$

But we have that

$$\delta G_a[x] = \int dx \frac{\delta G_a[x]}{\delta x} \delta x \quad \text{(the integral (62) is indefinite)} \tag{62}$$

Substituting from (61) we get

$$\delta G_a[x] = \int dx \frac{F(x,a)}{x} \delta x \text{ (the integral (63) is indefinite)} \tag{63}$$

Notes

We return to equation (49) and write

$$\mathrm{G} = \lim (\mathrm{t} \to 0) \int_t^p F dx \text{ (t is a very small positive number 0<t<p)} \tag{64}$$

$$= \{ \text{ F x(at p)} - \lim (\mathrm{t} \to 0) \text{Fx (at t) } \} - \lim (\mathrm{t} \to 0) \int_t^p \text{ x dF}$$

Let us compute

$$\lim (\mathrm{t} \to 0) \text{ Fx (at t)} = \lim (\mathrm{t} \to 0) \ \mathrm{t}^{1-a} ((\frac{1}{t})) - \mathrm{t}^{a} = 0 \tag{65}$$

Thus equation (64) reduces to

$$\mathrm{G} - \text{Fx (at p)} = - \lim (\mathrm{t} \to 0) \int_t^p \text{ x dF} \tag{66}$$

Note that the left – hand side of equation (66) is bounded. Equation (63) gives us

$$\delta \mathrm{G}_a = \lim (\mathrm{t} \to 0) \int_t^p \mathrm{dx} \frac{F}{x} \delta x \tag{67}$$

(t is the same small positive number 0<t<p)

We can easily prove that the two integrals $\int_t^p$ x dF and $\int_t^p$ dx $\frac{F}{x}\delta x$ are absolutely convergent .Since the limits of integration do not involve any variable , we form the product of **(66)** and **(67)**

$$\mathrm{K} = \lim(\mathrm{t} \to 0) \int_t^p \int_t^p \text{ xdF} \times \text{dx} \frac{F}{x} \delta x = \lim(\mathrm{t} \to 0) \int_t^p \text{ FdF } \times \int_t^p \delta x dx \tag{68}$$

(K is a bounded quantity)
That is

$$\mathrm{K} = \lim(\mathrm{t} \to 0) \left[\frac{F^2}{2} \text{(at p)} - \frac{F^2}{2} \text{(at t) } \right] \times \left[\delta \text{ x (at p)} - \delta \text{ x (at t) } \right] \tag{69}$$

We conclude from this equation that

$$\{ \left[\frac{F^2}{2} \text{(at p)} - \lim(\mathrm{t} \to 0) \frac{F^2}{2} \text{(at t) } \right] \times \left[\delta \text{ x (at p) } \right] \} \text{ is bounded .} \tag{70}$$

(since $\lim(\mathrm{x} \to 0) \delta \mathrm{x} = 0$, which is the same thing as $\lim(\mathrm{t} \to 0) \ \delta \mathrm{x} = 0$)

Since $\frac{F^2}{2}$(at p) is bounded , we deduce at once that $\frac{F^2}{2}$ must remain bounded in the limit as $(\mathrm{t} \to 0)$, which is the same thing as saying that F must remain bounded in the limit as $(\mathrm{x} \to 0)$.
Therefore .

Notes

$$\lim\,(x \to 0)\ \frac{((\frac{1}{x})) - x^{2a-1}}{x^{a}} \text{ must remain bounded} \tag{71}$$

But

$$\lim\,(x \to 0)\ \frac{((\frac{1}{x})) - x^{2a-1}}{x^{a}} = \lim(x \to 0)\ \frac{x^{1-2a}}{x^{1-2a}} \times \frac{((\frac{1}{x})) - x^{2a-1}}{x^{a}} \tag{72}$$

$$= \lim(x \to 0)\ \frac{x^{1-2a}((\frac{1}{x})) - 1}{x^{1-a}} = \lim(x \to 0)\ \frac{-1}{x^{1-a}}$$

It is evident that this last limit is unbounded. This contradicts our conclusion (71) that

$$\lim\,(x \to 0)\ \frac{((\frac{1}{x})) - x^{2a-1}}{x^{a}} \text{ must remain bounded (for } a< 0.5\text{)}$$

Therefore the case a<0.5 is rejected .We verify here that ,for a = 0.5 (71)remains bounded as $(x \to 0)$.

We have that

$$((\frac{1}{x})) - x^{2a-1} < 1\text{-}x^{2a-1} \tag{73}$$

Therefore

$$\lim(a \to 0.5)\ (x \to 0)\ \frac{((\frac{1}{x})) - x^{2a-1}}{x^{a}} < \lim(a \to 0.5)\ (x \to 0)\ \frac{1 - x^{2a-1}}{x^{a}} \tag{74}$$

We consider the limit

$$\lim(a \to 0.5)\ (x \to 0)\ \frac{1 - x^{2a-1}}{x^{a}} \tag{75}$$

We write

$$a = (\lim x \to 0)\ (\ 0.5 + x\ ) \tag{76}$$

Hence we get

$$\lim(a \to 0.5)\ (x \to 0)\ \ x^{2a-1} = \lim\,(x \to 0)\ \ x^{2(0.5+x)-1} = \lim\,(x \to 0)\,x^{2x} = 1 \tag{77}$$

$$(\text{Since } \lim(x \to 0)\ \ x^{x} = 1)$$

Therefore we must apply L 'Hospital ' rule with respect to x in the limiting process (75)

$$\lim(a \to 0.5)\ (x \to 0)\ \frac{1 - x^{2a-1}}{x^{a}} = \lim(a \to 0.5)\ (x \to 0)\ \frac{-(2a-1)x^{2a-2}}{ax^{a-1}} \tag{78}$$

$$= \lim(a \to 0.5)\ (x \to 0)\ \frac{(\frac{1}{a} - 2)}{x^{1-a}}$$

Now we write again

$$a = (\lim x \to 0)\ (\ 0.5 + x\ ) \tag{79}$$

Thus the limit (78) becomes

$$\lim(a \to 0.5)\ (x \to 0)\ \frac{(\frac{1}{a}-2)}{x^{1-a}} = \lim\ (x \to 0)\ \frac{(0.5+x)^{-1}-2}{x^{0.5-x}} = \lim\ (x \to 0)\ \frac{(0.5+x)^{-1}-2}{x^{0.5}\times x^{-x}}$$

$$= \lim\ (x \to 0)\ \frac{(0.5+x)^{-1}-2}{x^{0.5}}\ (\text{ Since } \lim\ (x \to 0)\, x^{-x} = 1\ ) \tag{80}$$

We must apply L 'Hospital ' rule

$$\lim\ (x \to 0)\ \frac{(0.5+x)^{-1}-2}{x^{0.5}} = \lim\ (x \to 0)\ \frac{-(0.5+x)^{-2}}{0.5x^{-0.5}} = \lim\ (x \to 0)\ \frac{-2\times x^{0.5}}{(0.5+x\ )^{2}} = 0 \tag{81}$$

Thus we have verified here that ,for a = 0.5 (71) approaches zero as ($x \to 0$) and hence remains bounded.

We consider the case a >0.5 . This case is also rejected, since according to the functional equation, if ($\zeta(s)$=0) (s = a+ bi) has a root with a>0.5 ,then it must have another root with another value of a < 0.5 . But we have already rejected this last case with a<0.5
Thus we are left with the only possible value of a which is a = 0.5
Therefore a = 0.5
This proves the Riemann Hypothesis .

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Notes

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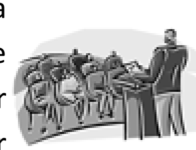

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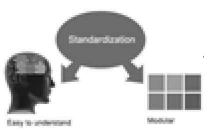

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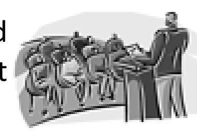

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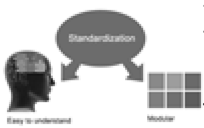

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Structure

All manuscripts submitted to Global Journals Inc. (US), ought to include:

Title: The title page must carry an instructive title that reflects the content, a running title (less than 45 characters together with spaces), names of the authors and co-authors, and the place(s) wherever the work was carried out. The full postal address in addition with the e-mail address of related author must be given. Up to eleven keywords or very brief phrases have to be given to help data retrieval, mining and indexing.

Abstract, used in Original Papers and Reviews:

Optimizing Abstract for Search Engines

Many researchers searching for information online will use search engines such as Google, Yahoo or similar. By optimizing your paper for search engines, you will amplify the chance of someone finding it. This in turn will make it more likely to be viewed and/or cited in a further work. Global Journals Inc. (US) have compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

Key Words

A major linchpin in research work for the writing research paper is the keyword search, which one will employ to find both library and Internet resources.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy and planning a list of possible keywords and phrases to try.

Search engines for most searches, use Boolean searching, which is somewhat different from Internet searches. The Boolean search uses "operators," words (and, or, not, and near) that enable you to expand or narrow your affords. Tips for research paper while preparing research paper are very helpful guideline of research paper.

Choice of key words is first tool of tips to write research paper. Research paper writing is an art.A few tips for deciding as strategically as possible about keyword search:

- One should start brainstorming lists of possible keywords before even begin searching. Think about the most important concepts related to research work. Ask, "What words would a source have to include to be truly valuable in research paper?" Then consider synonyms for the important words.
- It may take the discovery of only one relevant paper to let steer in the right keyword direction because in most databases, the keywords under which a research paper is abstracted are listed with the paper.
- One should avoid outdated words.

Keywords are the key that opens a door to research work sources. Keyword searching is an art in which researcher's skills are bound to improve with experience and time.

Numerical Methods: Numerical methods used should be clear and, where appropriate, supported by references.

Acknowledgements: Please make these as concise as possible.

References

References follow the Harvard scheme of referencing. References in the text should cite the authors' names followed by the time of their publication, unless there are three or more authors when simply the first author's name is quoted followed by et al. unpublished work has to only be cited where necessary, and only in the text. Copies of references in press in other journals have to be supplied with submitted typescripts. It is necessary that all citations and references be carefully checked before submission, as mistakes or omissions will cause delays.

References to information on the World Wide Web can be given, but only if the information is available without charge to readers on an official site. Wikipedia and Similar websites are not allowed where anyone can change the information. Authors will be asked to make available electronic copies of the cited information for inclusion on the Global Journals Inc. (US) homepage at the judgment of the Editorial Board.

The Editorial Board and Global Journals Inc. (US) recommend that, citation of online-published papers and other material should be done via a DOI (digital object identifier). If an author cites anything, which does not have a DOI, they run the risk of the cited material not being noticeable.

The Editorial Board and Global Journals Inc. (US) recommend the use of a tool such as Reference Manager for reference management and formatting.

Tables, Figures and Figure Legends

Tables: Tables should be few in number, cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g. Table 4, a self-explanatory caption and be on a separate sheet. Vertical lines should not be used.

Figures: Figures are supposed to be submitted as separate files. Always take in a citation in the text for each figure using Arabic numbers, e.g. Fig. 4. Artwork must be submitted online in electronic form by e-mailing them.

Preparation of Electronic Figures for Publication

Even though low quality images are sufficient for review purposes, print publication requires high quality images to prevent the final product being blurred or fuzzy. Submit (or e-mail) EPS (line art) or TIFF (halftone/photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Do not use pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings) in relation to the imitation size. Please give the data for figures in black and white or submit a Color Work Agreement Form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

For scanned images, the scanning resolution (at final image size) ought to be as follows to ensure good reproduction: line art: >650 dpi; halftones (including gel photographs) : >350 dpi; figures containing both halftone and line images: >650 dpi.

Color Charges: It is the rule of the Global Journals Inc. (US) for authors to pay the full cost for the reproduction of their color artwork. Hence, please note that, if there is color artwork in your manuscript when it is accepted for publication, we would require you to complete and return a color work agreement form before your paper can be published.

Figure Legends: Self-explanatory legends of all figures should be incorporated separately under the heading 'Legends to Figures'. In the full-text online edition of the journal, figure legends may possibly be truncated in abbreviated links to the full screen version. Therefore, the first 100 characters of any legend should notify the reader, about the key aspects of the figure.

6. AFTER ACCEPTANCE

Upon approval of a paper for publication, the manuscript will be forwarded to the dean, who is responsible for the publication of the Global Journals Inc. (US).

6.1 Proof Corrections

The corresponding author will receive an e-mail alert containing a link to a website or will be attached. A working e-mail address must therefore be provided for the related author.

Acrobat Reader will be required in order to read this file. This software can be downloaded

(Free of charge) from the following website:

www.adobe.com/products/acrobat/readstep2.html. This will facilitate the file to be opened, read on screen, and printed out in order for any corrections to be added. Further instructions will be sent with the proof.

Proofs must be returned to the dean at dean@globaljournals.org within three days of receipt.

As changes to proofs are costly, we inquire that you only correct typesetting errors. All illustrations are retained by the publisher. Please note that the authors are responsible for all statements made in their work, including changes made by the copy editor.

6.2 Early View of Global Journals Inc. (US) (Publication Prior to Print)

The Global Journals Inc. (US) are enclosed by our publishing's Early View service. Early View articles are complete full-text articles sent in advance of their publication. Early View articles are absolute and final. They have been completely reviewed, revised and edited for publication, and the authors' final corrections have been incorporated. Because they are in final form, no changes can be made after sending them. The nature of Early View articles means that they do not yet have volume, issue or page numbers, so Early View articles cannot be cited in the conventional way.

6.3 Author Services

Online production tracking is available for your article through Author Services. Author Services enables authors to track their article - once it has been accepted - through the production process to publication online and in print. Authors can check the status of their articles online and choose to receive automated e-mails at key stages of production. The authors will receive an e-mail with a unique link that enables them to register and have their article automatically added to the system. Please ensure that a complete e-mail address is provided when submitting the manuscript.

6.4 Author Material Archive Policy

Please note that if not specifically requested, publisher will dispose off hardcopy & electronic information submitted, after the two months of publication. If you require the return of any information submitted, please inform the Editorial Board or dean as soon as possible.

6.5 Offprint and Extra Copies

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Before start writing a good quality Computer Science Research Paper, let us first understand what is Computer Science Research Paper? So, Computer Science Research Paper is the paper which is written by professionals or scientists who are associated to Computer Science and Information Technology, or doing research study in these areas. If you are novel to this field then you can consult about this field from your supervisor or guide.

TECHNIQUES FOR WRITING A GOOD QUALITY RESEARCH PAPER:

1. Choosing the topic: In most cases, the topic is searched by the interest of author but it can be also suggested by the guides. You can have several topics and then you can judge that in which topic or subject you are finding yourself most comfortable. This can be done by asking several questions to yourself, like Will I be able to carry our search in this area? Will I find all necessary recourses to accomplish the search? Will I be able to find all information in this field area? If the answer of these types of questions will be "Yes" then you can choose that topic. In most of the cases, you may have to conduct the surveys and have to visit several places because this field is related to Computer Science and Information Technology. Also, you may have to do a lot of work to find all rise and falls regarding the various data of that subject. Sometimes, detailed information plays a vital role, instead of short information.

2. Evaluators are human: First thing to remember that evaluators are also human being. They are not only meant for rejecting a paper. They are here to evaluate your paper. So, present your Best.

3. Think Like Evaluators: If you are in a confusion or getting demotivated that your paper will be accepted by evaluators or not, then think and try to evaluate your paper like an Evaluator. Try to understand that what an evaluator wants in your research paper and automatically you will have your answer.

4. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

5. Ask your Guides: If you are having any difficulty in your research, then do not hesitate to share your difficulty to your guide (if you have any). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work then ask the supervisor to help you with the alternative. He might also provide you the list of essential readings.

6. Use of computer is recommended: As you are doing research in the field of Computer Science, then this point is quite obvious.

7. Use right software: Always use good quality software packages. If you are not capable to judge good software then you can lose quality of your paper unknowingly. There are various software programs available to help you, which you can get through Internet.

8. Use the Internet for help: An excellent start for your paper can be by using the Google. It is an excellent search engine, where you can have your doubts resolved. You may also read some answers for the frequent question how to write my research paper or find model research paper. From the internet library you can download books. If you have all required books make important reading selecting and analyzing the specified information. Then put together research paper sketch out.

9. Use and get big pictures: Always use encyclopedias, Wikipedia to get pictures so that you can go into the depth.

10. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right! It is a good habit, which helps to not to lose your continuity. You should always use bookmarks while searching on Internet also, which will make your search easier.

11. Revise what you wrote: When you write anything, always read it, summarize it and then finalize it.

12. Make all efforts: Make all efforts to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in introduction, that what is the need of a particular research paper. Polish your work by good skill of writing and always give an evaluator, what he wants.

13. Have backups: When you are going to do any important thing like making research paper, you should always have backup copies of it either in your computer or in paper. This will help you to not to lose any of your important.

14. Produce good diagrams of your own: Always try to include good charts or diagrams in your paper to improve quality. Using several and unnecessary diagrams will degrade the quality of your paper by creating "hotchpotch." So always, try to make and include those diagrams, which are made by your own to improve readability and understandability of your paper.

15. Use of direct quotes: When you do research relevant to literature, history or current affairs then use of quotes become essential but if study is relevant to science then use of quotes is not preferable.

16. Use proper verb tense: Use proper verb tenses in your paper. Use past tense, to present those events that happened. Use present tense to indicate events that are going on. Use future tense to indicate future happening events. Use of improper and wrong tenses will confuse the evaluator. Avoid the sentences that are incomplete.

17. Never use online paper: If you are getting any paper on Internet, then never use it as your research paper because it might be possible that evaluator has already seen it or maybe it is outdated version.

18. Pick a good study spot: To do your research studies always try to pick a spot, which is quiet. Every spot is not for studies. Spot that suits you choose it and proceed further.

19. Know what you know: Always try to know, what you know by making objectives. Else, you will be confused and cannot achieve your target.

20. Use good quality grammar: Always use a good quality grammar and use words that will throw positive impact on evaluator. Use of good quality grammar does not mean to use tough words, that for each word the evaluator has to go through dictionary. Do not start sentence with a conjunction. Do not fragment sentences. Eliminate one-word sentences. Ignore passive voice. Do not ever use a big word when a diminutive one would suffice. Verbs have to be in agreement with their subjects. Prepositions are not expressions to finish sentences with. It is incorrect to ever divide an infinitive. Avoid clichés like the disease. Also, always shun irritating alliteration. Use language that is simple and straight forward. put together a neat summary.

21. Arrangement of information: Each section of the main body should start with an opening sentence and there should be a changeover at the end of the section. Give only valid and powerful arguments to your topic. You may also maintain your arguments with records.

22. Never start in last minute: Always start at right time and give enough time to research work. Leaving everything to the last minute will degrade your paper and spoil your work.

23. Multitasking in research is not good: Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. Think technically: Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page
- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.

- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

Please carefully note down following rules and regulation before submitting your Research Paper to Global Journals Inc. (US):

Segment Draft and Final Research Paper: You have to strictly follow the template of research paper. If it is not done your paper may get rejected.

- The **major constraint** is that you must independently make all content, tables, graphs, and facts that are offered in the paper. You must write each part of the paper wholly on your own. The Peer-reviewers need to identify your own perceptive of the concepts in your own terms. NEVER extract straight from any foundation, and never rephrase someone else's analysis.

- Do not give permission to anyone else to "PROOFREAD" your manuscript.

- Methods to avoid Plagiarism is applied by us on every paper, if found guilty, you will be blacklisted by all of our collaborated research groups, your institution will be informed for this and strict legal actions will be taken immediately.)
- To guard yourself and others from possible illegal use please do not permit anyone right to use to your paper and files.

Criterion for Grading a Research Paper (Compilation) by Global Journals Inc. (US)

Please note that following table is only a Grading of "Paper Compilation" and not on "Performed/Stated Research" whose grading solely depends on Individual Assigned Peer Reviewer and Editorial Board Member. These can be available only on request and after decision of Paper. This report will be the property of Global Journals Inc. (US).

Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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