

GLOBAL JOURNAL OF SCIENCE FRONTIER RESEARCH: A PHYSICS AND SPACE SCIENCE Volume 15 Issue 1 Version 1.0 Year 2015 Type : Double Blind Peer Reviewed International Research Journal Publisher: Global Journals Inc. (USA) Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Some Dust Cosmological Models with Time Dependent $\Lambda(t)$ in Creation Field Cosmology

By H. R. Ghate & Sanjay A. Salve

Jijamata Mahavidyalaya Buldana, India

Abstract- We have studied the Hoyle-Narlikar's Creation-field cosmology for LRS Bianchi type-II, LRS Bianchi type-VI₀, Plane Symmetric and Kantowski-Sachs universes with time dependent cosmological constant $\Lambda(t)$, when the universe is filled with dust distribution. To get deterministic model of the universe, a relation between shear (σ) and expansion (θ) is assumed. The physical aspects of the models are also discussed.

Keywords: Creation-field cosmology, Cosmological constant $\Lambda(t)$.

GJSFR-A Classification : FOR Code: 020109

SOME DUST COSMO LOGICALMO DE LOWITHTIME DE PENDENTINCREATIONFIEL DO SMO LOGY

Strictly as per the compliance and regulations of :



© 2015. H. R. Ghate & Sanjay A. Salve. This is a research/review paper, distributed under the terms of the Creative Commons Attribution-Noncommercial 3.0 Unported License http://creativecommons.org/licenses/by-nc/3.0/), permitting all non commercial use, distribution, and reproduction in any medium, provided the original work is properly cited.

2015

Some Dust Cosmological Models with Time Dependent $\Lambda(t)$ in Creation Field Cosmology

H. R. Ghate ^a & Sanjay A. Salve ^o

Abstract- We have studied the Hoyle-Narlikar's Creation-field cosmology for LRS Bianchi type-II, LRS Bianchi type-VI₀, Plane Symmetric and Kantowski-Sachs universes with time dependent cosmological constant $\Lambda(t)$, when the universe is filled with dust distribution. To get deterministic model of the universe, a relation between shear (σ) and expansion (θ) is assumed. The physical aspects of the models are also discussed.

Keywords: Creation-field cosmology, Cosmological constant $\Lambda(t)$.

I. INTRODUCTION

osmology is the scientific study of origin, evolution, large scale structures & dynamics of the universe. It involves the formation of theories or hypothesis about the universe which makes specific predictions for phenomenon. These predictions can be tested with observations. Einstein's theory of general relativity is a very successful gravitational theory in describing the gravitational phenomena which also served as a basis for the models of the universe. All the investigations dealing with physical process are successfully explained by Einstein's field equations based big-bang model. The phenomenon of expanding universe, Primordial nucleo synthesis & the observed isotropy of Comic Microwave Background Radiations (CMBR) are important observations in astronomy which were successfully explained the big-bang model. This model is described by FRW line element and a matter density source which obeys equation of state $\rho = 3p$, where p and ρ are the fluid pressure and matter density respectively. But this model has various problems like; singularity in the past and may possibly in the future; no remarkable predictions in the big-bang that explain the origin, model evolution and characteristic of structures in the universe ; the conservation of the energy is violated; the flatness and horizon problem explanations given from big-bang model of the universe.

However Smoot *et al.* [1] revealed that the astronomical predictions of the FRW type of models do not always exactly meet our expectations. The

Author α: Department of Mathematics, Jijamata Mahavidyalaya, Buldana (M.S.) India. e-mail: hrghate@gmail.com Author σ: Department of Mathematics, Shri Shivaji Science and Arts College, Chikhli, Dist. Buldana (M.S.) India. e-mail: salvesanjaya@gmail.com theoretical explanations given from big -bang type model were contradicted by some puzzling results regarding the red-shifts from extra-galactic objects. The gravitational collapse of massive objects is an unavoidable consequence of general relativity [2-3]. Also CMBR discovery did not prove it to be an outcome of big-bang theory. Therefore alternative theories of gravitation were proposed time to time to overcome the drawbacks of big-bang model. Bondi & Gold [4] proposed a steady state theory in which the universe does not have any singular beginning or an end on the cosmic time scale where the matter density is throughout constant. Also they state that the statistical properties of the large scale features of the universe do not change. Further the constancy of mass density has been accounted by continuous creation of matter going on in contrast to the one time infinite and explosive creation of matter at t = 0 as in earlier standard model. But the principle of conservation of matter was violated in this formalism. This difficulty was overcome by Hoyle & Narlikar [5-7] by adopting a field theoretical approach and introducing a massless & chargeless scalar field C in the Einstein-Hilbert action to explain creation of matter. In Hoyle-Narlikar C-field theory there is no bigbang type singularity as in steady state theory by Bondi & Gold. Narlikar [8] has shown that the matter creation is accomplished at the expense of negative energy Cfield in which he solves horizon and flatness problem faced by big-bang model. Narlikar & Padmanabhan [9] have obtained a solution of Einstein field equations admitting radiation with a negative energy massless scalar field C. In fact Narlikar et al. [10] have proved the possibilities of non-relic interpretation of CMBR. Chatterjee & Banerjee [11] have investigated higher dimensional cosmology in C-field theory. Singh & Chaubey [12] have studied Bianchi type I, III, V, VI_o and Kantowski-Sachs universes in C-field Cosmology. Adhav et al. [13-14] have investigated higher dimensional Bianchi Type VI₀ and Bianchi type-I string cosmological models in Creation field cosmology. Katore [15] has studied plane symmetric universe in C-field cosmology. Recently Patil et al. [16] have obtained Bianchi type-IX dust filled universe with ideal fluid distribution in Creation field theory.

Einstein's basic cosmological model was a static, homogeneous with spherical geometry. Since at that time universe was not known to be expanding, it is considered as the gravitational effect of matter caused acceleration in the model. In 1917, Einstein introduced a cosmological constant Λ in his equation as the universal repulsion or anti-gravity effect to make the universe static in accordance with generally accepted picture of that time. Hubble showed that the universe was expanding by his study about nearby galaxies. Then Einstein regretted modifying his elegant theory and viewed the cosmological constant as his greatest mistake. Recent cosmological observations by the High-Z Supernovae Team and Supernovae Cosmological Project [17-25] suggest the existence of a positive cosmological constant Λ with the magnitude $\Lambda(G\hbar/c^3) \approx 10^{-123}$. Zel'dovich [26] has tried to visualize the meaning of cosmological constant from the theory of elementary particles. Bergmann [27] has interpreted the cosmological constant Λ in terms of Higgs scalar field. In quantum field theory, the cosmological constant is considered as the vacuum energy density. Linde [28] had shown that the cosmological term arises from spontaneous symmetry breaking and also suggested that the cosmological term is not a constant but a function of temperature. Dolgo has [29-30] focused that cosmological constant remains constant in the absence of any interaction with matter and radiation. Pavon [31] have studied model with cosmological constant problem as the discrepancy between the negligible value for the present universe. Glashow-Salam-Weinberg model [33] expected 10^{50} larger value whereas grand unified theory [34] expected

 $10^{107} \, \text{larger}$ value whereas of cosmological constant. Bertolami [35] was the first who consider cosmological models with a variable cosmological constant of the form $\Lambda \sim t^{-2}$. Berman and Som [36] have shown that $\Lambda \sim t^{-2}$ plays a very important role in cosmology. Chen and Wu [37] have also solved the problem by considering $\Lambda \sim R^{-2}$, where R is the scale factor in the Robertson-Walker space time. Carvalho et al. [38] generalized the proposed model of Chen and Wu by including a term proportional to H^2 on the time dependence of Λ . Overduin [39] considered the assumptions $\Lambda \sim t^{-2}$ and $\Lambda \sim H^{-2}$ in FRW models and shows there compability with various observations. At the same time cosmologist viz. Olson et al. [40], Gasperini [41], Pebbles & Ratra [42], Abdel Rahman [43], Maia et al. [44], Silveira et al. [45], Moffat [46], Torres et al. [47], Abdussattar & Vishwakarma [48], Hoyle et al. [49], Podariu et al. [50] and many other authors had shown that cosmological term decays with time. Carmeli & Kuzmenko [51] have shown that cosmological relativistic theory predicts the value of cosmological constant $\Lambda = 1.934 \times 10^{-35} s^{-2}$. Recent observations indicate that $\Lambda \sim 10^{-56} cm^{-2}$ but the

theory of physics of elementary particles predicts that the value of Λ must have been $10^{120}\,$ times larger in the past. It is worth noting that cosmological models based on Einstein field equations with a time-dependent cosmological constant Λ had been the subject of numerous papers in recent years. Several attempts have made by many researchers viz. Lui & Wesson [52], Cunha et al. [53], Carneiro et al. [54], Pradhan et al. [55], Singh et al. [56], Katore et al. [57], Tiwari et al. [58], Dwivedi [59] and Amirhashchi & Mohamadian [60] in the favor of time dependent $\Lambda \sim t^{-2}$ in different contexts. Bali et al. [61-62] have investigated Bianchi type III & FRW cosmological models with varying Λ in Creation field cosmology. Rahman & Ansari [63] obtained some new LRS Bianchi type-I bulk viscous cosmological models with decaying Λ -term. Recently Ghate et al. [64-66] have studied cosmological models with varying $\Lambda(t)$ in creation field theory of gravitation.

In this paper, we have investigated LRS Bianchi type-II (LRS B-II), LRS Bianchi type-VI₀ (LRS B-VI₀), Plane Symmetric (PS) and Kantowsaki-Sachs (K-S) spacetimes with varying cosmological constant $\Lambda(t)$ in creation field theory of gravitation. To obtain solution, a relation between shear (σ) and expansion (θ) is assumed. This work is organized as follows. In Section 2, Hoyle-Narlikar creation field theory is briefly discussed. An exact solution of field equations for LRS B-II, LRS B-VI₀, PS & K-S spaces-times have been obtained in section 3, 4, 5 & 6 respectively. The physical aspects of the anisotropic models have been discussed in section 7, while in Section 8, concluding remarks have been expressed.

II. HOYLE-NARLIKAR THEORY

Hoyle and Narlikar [5-7] have modified Einstein field equations through the introduction of a massless scalar field usually called Creation field *viz. C*-field. The modified field equations are

$$R_{i}^{j} - \frac{1}{2} R g_{i}^{j} = -8\pi G \left[T_{i}^{j} + T_{i}^{j} \\ {}_{(m) \quad (c)} \right] - \Lambda(t) g_{i}^{j}, \qquad (2.1)$$

where T_i^j is a matter tensor for perfect fluid of $m^{(m)}$

Einstein's theory given by

$$T_{i}^{\ j} = (\rho + p)v_{i}v^{j} - pg_{i}^{\ j}$$
(2.2)

and $T_i^{\ j}$ is a matter tensor due to $\ _C$ -field given by

$$T_{i}^{\ j} = -f\left(C_{i}C^{\ j} - \frac{1}{2}g_{i}^{\ j}C^{\alpha}C_{\alpha}\right).$$
(2.3)

Here ρ is the energy density of massive particles and p is the pressure. v_i are co-moving four

velocities which obeys the relation $v_i v^j = 1$, $v_{\alpha} = 0$, $\alpha = 1, 2, 3$. f > 0 is the coupling constant between matter and creation field and $C_i = \frac{dC}{dx^i}$.

As T^{00} has negative value (*i.e.* $T^{00} < 0$), the *C*-field has negative energy density producing repulsive gravitational field which causes the expansion of the universe. Thus the energy conservation law reduces to

$${}^{(m)}T^{ij}_{;j} = -{}^{(c)}T^{ij}_{;j} = fC^iC^j_{;j}, \qquad (2.4)$$

i.e. the matter creation through a non-zero left hand side is possible while conserving the overall energy and momentum.

The above equation is identical with

$$mg_{ij}\frac{dx^i}{ds} - C_j = 0, \qquad (2.5)$$

which indicates the 4-momentum of the created particle is compensated by 4-momentum of the C-field. In order to maintain the balance, the C-field must have negative energy.

Further the C-field satisfies the source equation

$$fC^{i}_{;i} = J^{i}_{;i}$$
 and $J^{i} = \rho \frac{dx^{i}}{ds} = \rho v^{i}$, (2.6)

where ρ is the homogeneous mass density.

The conservation equation for C-field is given by

$$\left(8\pi GT_{i}^{j} + \Lambda g_{i}^{j}\right)_{j,j} = 0$$
. (2.7)

The physical quantities in cosmology are the mean anisotropy parameter (Δ) and the deceleration parameter (*q*) are defined as

$$\Delta = \frac{1}{3} \sum_{i=1}^{3} \left(\frac{H_i - H}{H} \right)^2 , \qquad (2.8)$$

$$q = -\frac{R\ddot{R}}{\dot{R}^2},$$
 (2.9)

where H is Hubble parameter.

III. LRS BIANCHI TYPE-II MODEL

We consider homogeneous LRS Bianchi-II metric in the form of

$$ds^{2} = dt^{2} - A^{2}dx^{2} - B^{2}dy^{2} - 2B^{2}xdydz - (B^{2}x^{2} + A^{2})dz^{2}$$
(3.1)

where metric potentials *A* , *B* are functions of time *t* only and $\sqrt{-g} = A^2 B$.

It is assumed that Creation field C is a function of time t only *i.e.*

$$C(x,t) = C(t)$$
 and ${}^{m}T_{j}^{i} = diag(\rho,-p,-p,-p)$ (3.2)

The Hoyle-Narlikar field equations (2.1) with the help of equations (2.2) and (2.3) for the metric (3.1) given by

$$\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} - \frac{1}{4}\frac{B^2}{A^4} = 8\pi G \left(\rho - \frac{1}{2}f\dot{C}^2\right) + \Lambda \quad , \qquad (3.3)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{1}{4}\frac{B^2}{A^4} = 8\pi G \left(-p + \frac{1}{2}f\dot{C}^2 \right) + \Lambda \quad , \quad (3.4)$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{3}{4}\frac{B^2}{A^4} = 8\pi G \left(-p + \frac{1}{2}f\dot{C}^2\right) + \Lambda \quad , \qquad (3.5)$$

where overhead dot ($^{\circ}$) denotes differentiation with respect to time t.

The conservation equation (2.7) leads to

$$\frac{d}{dt}\dot{C}^2 + 2\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\dot{C}^2 = \frac{2\dot{\rho}}{f} + \frac{2\rho}{f}\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) + \frac{\dot{\Lambda}}{4\pi G f} (3.6)$$

where p being isotropic pressure.

Following Hoyle and Narlikar, we have taken p = 0, for the dust distribution. The source equation of *C*-field $C_{;i}^{j} = \frac{n}{f}$ leads to C = t for large r. Thus $\dot{C} = 1$.

Using p = 0 & $\dot{C} = 1$, equations (3.3)-(3.5) lead to

$$\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} - \frac{1}{4}\frac{B^2}{A^4} = 8\pi G\rho - 4\pi Gf + \Lambda , \qquad (3.7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{1}{4}\frac{B^2}{A^4} = 4\pi G f + \Lambda$$
, (3.8)

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{3}{4}\frac{B^2}{A^4} = 4\pi Gf + \Lambda .$$
(3.9)

The field equations (3.7)-(3.9) is a system of three equations with four unknown parameters A, B, ρ and Λ . To find the deterministic solution of the field equations, we need one extra condition. We assume that the shear scalar (σ) is proportional to expansion scalar (θ) which leads to (Collins *et al.* [67])

$$B = A^n , \qquad (3.10)$$

where *n* is arbitrary constant.

To get the deterministic value of A, Let $\dot{A} = F(A)$.

which implies
$$\ddot{A} = FF'$$
, where $F' = \frac{dF}{dA}$. (3.13)

With the help of equation (3.13), equation (3.12) reduces to

$$\frac{dF^2}{dA} + \frac{2(n+1)}{A}F^2 = \frac{2}{(1-n)}A^{2n-3}.$$
 (3.14)

Integrating equation (3.14), we get

$$F^{2} = \frac{1}{2n(1-n)} A^{2(n-1)} = \left(\frac{dA}{dt}\right)^{2}.$$
 (3.15)

The constant of integration has taken to be zero for simplicity.

Using A = F(A) and solving equation (3.15), we get

$$A^{2-n} = \frac{(2-n)}{\sqrt{2n(1-n)}}t + (2-n)C_2, \qquad (3.16)$$

where C_2 is a constant of integration. Simplifying equation (3.16), we get

$$A = (at+b)^{\frac{1}{2-n}},$$
 (3.17)

where
$$a = \frac{(2-n)}{\sqrt{2n(1-n)}}$$
, $b = (2-n)C_2$ and $0 < n < 1$.

Using equation (3.17), equation (3.10) leads to

$$B = (at+b)^{\frac{n}{2-n}}.$$
 (3.18)

Substituting equations (3.17) & (3.18) in equation (3.9), we have

$$\Lambda = \frac{-4a^2 + 8na^2 - 3n^2 + 12n - 12}{4(2-n)^2(at+b)^2} - k \quad , \tag{3.19}$$

where $4\pi Gf = k$.

Using equations (3.17), (3.18) and (3.19) in equation (3.7), we get

$$8\pi G\rho = \frac{4a^2 + n^2 - 4n + 4}{2(2 - n)(at + b)^2} + 2k \quad , \tag{3.20}$$

as
$$\frac{1}{a^2} = 2n \left(\frac{(1-n)}{(2-n)^2} \right)$$

Thus using equations (3.17) & (3.18) in metric (3.1), the cosmological model is given by

$$ds^{2} = dt^{2} - (at+b)^{\frac{2}{2-n}} dx^{2} - (at+b)^{\frac{2n}{2-n}} dy^{2} -$$
(3.21)

$$2(at+b)^{\frac{2n}{2-n}}xdydz - \left[(at+b)^{\frac{2n}{2-n}}x^2 + (at+b)^{\frac{2}{2-n}}\right]dz^2.$$

Using equations (3.17), (3.18), (3.19) and (3.20) in equation (3.6), we have

$$\frac{d}{dt}\dot{C}^2 + \left[2\left(\frac{2+n}{2-n}\right)\left(\frac{a}{at+b}\right)\right]\dot{C}^2 = \left[2\left(\frac{2+n}{2-n}\right)\left(\frac{a}{at+b}\right)\right] (3.22)$$

On integration, equation (3.22) reduces to

$$\dot{C} = 1$$
, (3.23)

which leads to

$$C = t + b_1$$
, (3.24)

where b_1 is a constant of integration.

The value $\dot{C} = 1$, So obtained agrees with the value used in the source equation. Thus Creation field *C* is proportional to time *t*.

The physical parameters such as the spatial volume (V), the mean anisotropic parameter (Δ) and the deceleration parameter (q) are defined as

$$V = (at+b)^{\frac{2+n}{2-n}},$$
 (3.25)

$$\Delta = 2\left(\frac{1-n}{2+n}\right)^2 = \text{Constant}, \quad (3.26)$$

$$q = -4 \frac{(n-1)}{(n+2)}$$
 = Constant, where $0 < n < 1$. (3.27)

Here q > 0, for 0 < n < 1.

IV. LRS BIANCHI TYPE-VI₀ Model

We consider the LRS Bianchi type VI_{o} metric in the form of

$$ds^{2} = dt^{2} - A^{2}dx^{2} - B^{2}\left(e^{-2x}dy^{2} + e^{2x}dz^{2}\right)$$
(4.1)

where metric potentials *A*, *B* are functions of time *t* only and $\sqrt{-g} = AB^2$.

It is assumed that Creation field C is a function of time t only

i.e.
$$C(x,t) = C(t)$$
 and ${}^{m}T_{j}^{i} = diag(\rho,-p,-p,-p)$ (4.2)

(3.11)

(3.12)

The Hoyle-Narlikar field equations (2.1), with the help of equations (2.2) and (2.3) for the metric (4.1) given by

$$\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = 8\pi G \left(\rho - \frac{1}{2}f\dot{C}^2\right) + \Lambda , \qquad (4.3)$$

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{A^2} = 8\pi G \left(-p + \frac{1}{2}f\dot{C}^2 \right) + \Lambda, \qquad (4.4)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = 8\pi G \left(-p + \frac{1}{2}f\dot{C}^2 \right) + \Lambda , \qquad (4.5)$$

where overhead do $t(\cdot)$ denotes differentiation with respect to time t.

The conservation equation (2.7) leads to

$$\frac{d}{dt}\dot{C}^2 + 2\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\dot{C}^2 = \frac{2\dot{\rho}}{f} + \frac{2\rho}{f}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) + \frac{\dot{\Lambda}}{4\pi G f} (4.6)$$

where p being isotropic pressure.

Following Hoyle and Narlikar, we have taken p = 0, for the dust distribution. The source equation of *C*-field $C_{;i}^{j} = \frac{n}{f}$ leads to C = t for large r. Thus $\dot{C} = 1$.

Using p = 0 & $\dot{C} = 1$, equations (4.3)-(4.5) lead to

$$\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = 8\pi G\rho - 4\pi Gf + \Lambda , \qquad (4.7)$$

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{A^2} = 4\pi G f + \Lambda , \qquad (4.8)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = 4\pi G f + \Lambda .$$
(4.9)

The field equations (4.7)-(4.9) is a system of three equations with four unknown parameters A, B, ρ and Λ . To find the deterministic solution of the field equations, we need one extra condition. We assume that the shear scalar (σ) is proportional to expansion scalar (θ) which leads to (Collins *et al.* [67])

$$A = B^n , \qquad (4.10)$$

where n is arbitrary constant. From equations (4.8) and (4.9), we get

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}^2}{B^2} - \frac{2}{A^2} = 0.$$
(4.11)

Using equations (4.10) and (4.11), we have

$$2\ddot{B} + 2(n+1)\frac{\dot{B}^2}{B} = \frac{4}{(n-1)}B^{1-2n}.$$
 (4.12)

To get the deterministic value of B , Let $\dot{B} = F(B)$, ... dF

which implies
$$\ddot{B} = FF'$$
, where $F' = \frac{dF}{dB}$. (4.13)

With the help of equation (4.13), equation (4.12) reduces to

$$\frac{dF^2}{dB} + \frac{2(n+1)}{B}F^2 = \frac{4}{(n-1)}B^{1-2n}.$$
(4.14)

Integrating equation (4.14), we get

$$F^{2} = \frac{1}{(n-1)} B^{2(1-n)} = \left(\frac{dB}{dt}\right)^{2}.$$
 (4.15)

The constant of integration has taken to be zero for simplicity.

Using $\dot{B} = F(B)$ and solving equation (4.15), we get

$$B^{n} = \frac{n}{\sqrt{n-1}}t + nC_{2}, \qquad (4.16)$$

where C_2 is constant of integration. Simplifying equation (4.16), we get

$$A = (at+b), \qquad (4.17)$$

where
$$a = \frac{n}{\sqrt{n-1}}$$
 , $b = nC_2$ and $n > 1$.

Using equation (4.17), equation (4.10) leads to

$$B = (at+b)^{\frac{1}{n}}.$$
 (4.18)

Substituting equations (4.17) & (4.18) in equation (4.8), we have

$$\Lambda = \frac{3a^2 - 2na^2 + n^2}{n^2(at+b)^2} - k , \qquad (4.19)$$

where $4\pi Gf = k$.

Using equations (4.17), (4.18) and (4.19) in equation (4.7), we get

$$8\pi G\rho = \frac{-2a^2 + 4na^2 - 2n^2}{n^2(at+b)^2} + 2k . \qquad (4.20)$$

as
$$\frac{1}{a^2} = \frac{1}{n} - \frac{1}{n^2}$$
.

Thus using equations (4.17) & (4.18) in metric (4.1), the cosmological model is given by

$$ds^{2} = dt^{2} - (at+b)^{2} dx^{2} - (at+b)^{\frac{2}{n}} \left(e^{-2x} dy^{2} + e^{2x} dz^{2} \right).$$
(4.21)

Using equations (4.17), (4.18), (4.19) and (4.20) in equation (4.6), we have

$$\frac{d}{dt}\dot{C}^2 + \left[2\left(1+\frac{2}{n}\right)\left(\frac{a}{at+b}\right)\right]\dot{C}^2 = \left[2\left(1+\frac{2}{n}\right)\left(\frac{a}{at+b}\right)\right].$$
 (4.22)

On integration of equation (4.22), we get

$$\dot{C} = 1$$
, (4.23)

which leads to

$$C = t + b_2$$
, (4.24)

where b_2 is a constant of integration.

The value $\dot{C} = 1$, So obtained agrees with the value used in the source equation. Thus creation field C is proportional to time *t*.

The physical parameters such as the spatial volume (V), the mean anisotropic parameter (Δ) and the deceleration parameter (q) are defined as

$$V = (at+b)^{1+\frac{2}{n}},$$
 (4.25)

$$\Delta = 2 \left(\frac{n-1}{n+2}\right)^2, \qquad (4.26)$$

$$q = 2 \frac{(n-1)}{(n+2)}$$
, where $n > 1$. (4.27)

Here q > 0, for n > 1.

V. Plane Symmetric Universe

We consider the plane symmetric metric in the form of

$$ds^{2} = dt^{2} - A^{2} (dx^{2} + dy^{2}) - B^{2} dz^{2}, \qquad (5.1)$$

where metric potentials *A* , *B* are functions of time *t* only and $\sqrt{-g} = A^2 B$.

It is assumed that creation field C is a function of time t only *i.e.* C(x,t) = C(t) and

$${}^{t}T_{i}^{t} = diag(\rho, -p, -p, -p)$$
 (5.2)

The Hoyle-Narlikar field equations (2.1), with the help of equations (2.2) and (2.3) for the metric (5.1) given by

$$\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} = 8\pi g \left(\rho - \frac{1}{2}f\dot{C}^2\right) + \Lambda , \qquad (5.3)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = 8\pi G \left(-p + \frac{1}{2}f\dot{C}^2 \right) + \Lambda , \qquad (5.4)$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} = 8\pi G \left(-p + \frac{1}{2}f\dot{C}^2 \right) + \Lambda , \qquad (5.5)$$

where overhead dot () denotes differentiation with respect to time t.

The conservation equation (2.7) leads to

$$\frac{d}{dt}\dot{C}^2 + 2\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\dot{C}^2 = \frac{2\dot{\rho}}{f} + \frac{2\rho}{f}\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) + \frac{\dot{\Lambda}}{4\pi G f} ,(5.6)$$

where p being isotropic pressure.

Following Hoyle and Narlikar, we have taken p = 0, for the dust distribution. The source equation of *C*-field $C_{,i}^{j} = \frac{n}{f}$ leads to C = t for large r. Thus $\dot{C} = 1$.

Using p = 0 & C = 1, equations (5.3)-(5.5) lead to

$$\frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} = 8\pi G\rho - 4\pi Gf + \Lambda \quad , \tag{5.7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = 4\pi G f + \Lambda , \qquad (5.8)$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}}{A^2} = 4\pi G f + \Lambda .$$
(5.9)

The field equations (5.7)-(5.9) is a system of three equations with four unknown parameters A, B, ρ and Λ . To find the deterministic solution of the field equations, we need one extra condition. We assume that the shear scalar (σ) is proportional to expansion scalar (θ) which leads to (Collins *et al.* [67])

$$B = A^n , \qquad (5.10)$$

where n is a arbitrary constant. From equations (5.8) and (5.9), we get

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{A}^2}{A^2} = 0.$$
 (5.11)

Using equations (5.10) and (5.11), we have

$$2\ddot{A} + 2(n+1)\frac{\dot{A}^2}{A} = 0.$$
 (5.12)

To get the deterministic value of A , Let A = F(A) ,

which implies
$$\ddot{A} = FF'$$
, where $F' = \frac{dF}{dA}$. (5.13)

With the help of equation (5.13), equation (5.12) reduces to

$$\frac{dF^2}{dA} + \frac{2(n+1)}{A}F^2 = 0.$$
 (5.14)

Integrating equation (5.14), we get

$$F^{2} = \frac{k_{1}^{2}}{A^{2(n+1)}} = \left(\frac{dA}{dt}\right)^{2},$$
 (5.15)

where k_1^2 is the constant of integration.

Using A = F(A) and solving equation (5.15), we get

$$A = \left[(n+2)(k_1t + k_2) \right]_{n+2}^{1} , \qquad (5.16)$$

where k_2 is constant of integration. Simplifying equation (5.16), we get

$$A = (at+b)^{\frac{1}{n+2}},$$
 (5.17)

where $a = (n+2)k_1$, $b = (n+2)k_2$ and n > 0 & $n \neq 1$.

Using equation (5.17), equation (5.10) leads to

$$B = (at+b)^{\frac{n}{n+2}}.$$
 (5.18)

Substituting equations (5.17) & (5.18) in equation (5.9), we have

$$\Lambda = -\frac{(2n+1)a^2}{(n+2)^2(at+b)^2} - k , \qquad (5.19)$$

where $4\pi Gf = k$.

Using equations (5.17), (5.18) and (5.19) in equation (5.7), we get

$$8\pi G\rho = \frac{(2n+1)(2a^2)}{(n+2)^2(at+b)^2} + 2k.$$
 (5.20)

Thus, using equations (5.17) & (5.18) in metric (5.1), the cosmological model is given by

$$ds^{2} = dt^{2} - \left(at+b\right)^{\frac{2}{n+2}} \left(dx^{2} + dy^{2}\right) - \left(at+b\right)^{\frac{2n}{n+2}} dz^{2}.$$
 (5.21)

Using equations (5.17), (5.18), (5.19) and (5.20) in equation (5.6), we have

$$\frac{d}{dt}\dot{C}^2 + \left[2\left(\frac{a}{at+b}\right)\right]\dot{C}^2 = \left[2\left(\frac{a}{at+b}\right)\right].$$
 (5.22)

On integration, equation (5.22) reduces to

$$\dot{C} = 1$$
, (5.23)

which leads to

$$C = t + b_3$$
, (5.24)

where b_3 is a constant of integration.

The value $\dot{C} = 1$, So obtained agrees with the value used in the source equation. Thus creation field *C* is proportional to time *t*.

The physical parameters such as the spatial volume (V), the mean anisotropic parameter (Δ) and the deceleration parameter (q) are defined as

$$V = (at+b)$$
, (5.25)

$$\Delta = 2\left(\frac{n-1}{n+2}\right)^2 = \text{Constant}, \text{ where } n > 0 \& n \neq 1 (5.26)$$

$$q = 2$$
 . (5.27)

VI. KANTOSKI-SACHS UNIVERSE

We consider the Kantowski-Sachs metric in the form of

$$ds^{2} = dt^{2} - A^{2}dr^{2} - B^{2}\left(d\theta^{2} + \sin^{2}\theta d\phi^{2}\right), \qquad (6.1)$$

where metric potentials *A*, *B* are functions of time *t* only and $\sqrt{-g} = AB^2 \sin \theta$.

It is assumed that creation field C is a function of time t only

i.e.
$$C(r,t) = C(t)$$
 and ${}^{m}T_{j}^{i} = diag(\rho,-p,-p,-p)$ (6.2)

The Hoyle-Narlikar field equations (2.1) with the help of equations (2.2) and (2.3) for the metric (6.1) given by

$$\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{1}{B^2} = 8\pi G \left(\rho - \frac{1}{2}f\dot{C}^2\right) + \Lambda , \qquad (6.3)$$

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} = 8\pi G \left(-p + \frac{1}{2}f\dot{C}^2 \right) + \Lambda, \qquad (6.4)$$

$$\frac{\dot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = 8\pi G \left(-p + \frac{1}{2}f\dot{C}^2 \right) + \Lambda , \qquad (6.5)$$

where overhead dot (`) denotes differentiation with respect to time t.

The conservation equation (2.7) leads to

$$\frac{d}{dt}\dot{C}^2 + 2\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\dot{C}^2 = \frac{2\dot{\rho}}{f} + \frac{2\rho}{f}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) + \frac{\dot{\Lambda}}{4\pi G f} (6.6)$$

where p being iotropic pressure.

Following Hoyle and Narlikar, we have taken p = 0, for the dust distribution. The source equation of

C-field
$$C_{j}^{j} = \frac{n}{f}$$
 leads to $C = t$ for large r. Thus $\dot{C} = 1$.

Using p = 0 & $\dot{C} = 1$, equations (6.3)-(6.5) lead to

$$\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{1}{B^2} = 8\pi G\rho - 4\pi Gf + \Lambda , \qquad (6.7)$$

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} = 4\pi G f + \Lambda , \qquad (6.8)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = 4\pi G f + \Lambda .$$
(6.9)

The field equations (6.7)-(6.9) is a system of three equations with four unknown parameters A, B, ρ and Λ . To find the deterministic solution of the field equations, we need one extra condition. We assume that the shear scalar (σ) is proportional to expansion scalar (θ) which leads to (Collins *et al.* [67])

$$A = B^n \quad , \tag{6.10}$$

where n is arbitrary constant.

From equations (6.8) and (6.9), we get

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}^2}{B^2} - \frac{1}{B^2} = 0 \quad . \tag{6.11}$$

Using equations (6.10) and (6.11), we have

$$2\ddot{B} + 2(n+1)\frac{\dot{B}^2}{B} = \frac{2}{(n-1)B}.$$
 (6.12)

To get the deterministic value of B , Let $\dot{B} = F(B)$,

which implies $\ddot{B} = FF'$, where $F' = \frac{dF}{dB}$. (6.13)

With the help of equation (6.13), equation (6.12) reduces to

$$\frac{dF^2}{dB} + \frac{2(n+1)}{B}F^2 = \frac{2}{(n-1)B}.$$
(6.14)

Integrating equation (6.14), we get

$$F^{2} = \frac{1}{n^{2} - 1} = \left(\frac{dB}{dt}\right)^{2}.$$
 (6.15)

The constant of integration has taken to be zero for simplicity.

Using $\dot{B} = F(B)$ and solving equation (6.15), we get

$$B = \frac{1}{\sqrt{n^2 - 1}} t + b \quad , \tag{6.16}$$

where b is a constant of integration. Simplifying equation (6.16), we get

$$B = (at + b)$$
, (6.17)

Where
$$a = \frac{1}{\sqrt{n^2 - 1}}$$
 and $n > 1$.

A

Using equation (6.17), equation (6.10) leads to

$$= (at+b)^n \,. \tag{6.18}$$

Substituting equations (6.17) & (6.18) in equation (6.8), we have

$$\Lambda = \frac{a^2 + 1}{(at+b)^2} - k , \qquad (6.19)$$

where $4\pi Gf = k$.

Using equations (6.17), (6.18) and (6.19) in equation (6.7), we get

$$8\pi G\rho = \frac{2na^2}{(at+b)^2} + 2k , \qquad (6.20)$$

as $\frac{1}{a^2} = n^2 - 1$.

Thus using equations (6.17) & (6.18) in metric (6.1), the cosmological model is given by

$$ds^{2} = dt^{2} - (at+b)^{2n} dr^{2} - (at+b)^{2} \left(d\theta^{2} + \sin^{2} \theta d\phi^{2} \right).$$
(6.21)

Using equations (6.17), (6.18), (6.19) and (6.20) in equation (6.6), we have

$$\frac{d}{dt}\dot{C}^2 + \left[2(2+n)\left(\frac{a}{at+b}\right)\right]\dot{C}^2 = \left[2(2+n)\left(\frac{a}{at+b}\right)\right] \quad (6.22)$$

On integration, equation (6.22) reduces to

$$\dot{C} = 1$$
, (6.23)

which leads to

$$C = t + b_4$$
, (6.24)

where b_4 is a constant of integration.

The value C = 1, So obtained agrees with the value used in the source equation. Thus creation field C is proportional to time t.

The physical parameters such as the spatial volume (V), the mean anisotropic parameter (Δ) and the deceleration parameter (q) are defined as

$$V = (at+b)^{n+2}, (6.25)$$

$$\Delta = 2 \left(\frac{n-1}{n+2} \right)^2, \tag{6.26}$$

$$q = -\frac{(n-1)}{(n+2)}$$
, where $n > 1$. (6.27)

Here q < 0, for n > 1.

VII. DISCUSSIONS

a) Spatial volume(V)

In Fig. 1, the plots of spatial volume (V) versus time (t) are given for LRS B-II, LRS B-VI₀, PS & K-S

Global

cosmological models which indicates that the models start evolving with finite volume at t = 0 and expand infinitely with the increase in cosmic time t.



Figure 1: The plots of spatial volume (V) verses time (t).

b) Energy density (ρ)

In Fig. 2, the dynamics of energy density (ρ) for cosmological models LRS B-II, LRS B-VI₀, PS & K-S models are given. We observed that all models start with big-bang having infinite density and are decreasing functions of time which tends to positive finite value for large values of time (*t*) representing steady state.



Figure 2 : The plots of energy density (ho) verses time (t).

c) Cosmological constant(Λ)

In Fig. 3, the plots of cosmological constants (Λ) versus cosmic time(t) for LRS B-II, LRS B-VI₀, PS & K-S cosmological models are given. We observed that the value of cosmological constant(Λ) is initially infinite



for all models. For K-S model, it is decreasing function of time and approaches to small positive value almost

closer to zero at late times which matches with the

recent CMBR observations.

Figure 3 : The plots of cosmological constant (Λ) verses time (t)

For LRS B-II, LRS B-VI₀ & PS models, Cosmological constant (Λ) is negative at initial stage but increases very rapidly with time and approaches to small negative value almost closer to zero at late times. We observed that LRS B-II, LRS B-VI₀ & PS models with negative cosmological constant follow the study by authors' *viz.* Monerat *et al.* [68], Pedram *et al.* [69], Pradhan *et al.* [70] and Yadav [71] on cosmological models with negative cosmological constant.

d) Deceleration parameter(q)

For LRS B-II, LRS B-VI₀ & PS models, the value of deceleration parameter q > 0 indicating that the models are decelerating and for K-S model, the value of deceleration parameter q < 0 indicating that the model is accelerating throughout the evolution of the universe.

e) Mean Anisotropy Parameter(Δ)

It is also observed that for all cosmological models the mean anisotropy parameter (Δ) is constant. Hence the models are anisotropic throughout the evolution of the universe except at n = 1 (*i.e.* the model does not approach isotropy).

VIII. Conclusion

LRS Bianchi type-II, LRS Bianchi type-VI_o, Plane symmetric and Kantowsaki-Sachs cosmological models have been investigated in Hoyle-Narlikar's creation field theory of gravitation. The source for energy momentum tensor is dust filled universe in the presence of time

dependent cosmological constant $\Lambda(t)$. We conclude that all the models are expanding and anisotropic but LRS B-II, LRS B-VI₀ & PS models are decelerating while K-S model is accelerating throughout the evolution of the universe. The creation field C is directly proportional to time t. Hence the creation of matter increases as time increases which follows the results as obtained by Hoyle and Narlikar.

References Références Referencias

- 1. Smoot, G. F. et al., Astrophys. J., **396**(1), L1,(1992).
- 2. Penrose, R., Phys. Rev. Lett., 14, 57,(1965).
- 3. Hawking, S. W., Phys. Rev. Lett., 15, 689,(1965).
- 4. Bondi, H., Gold, T., Mon. Not. Roy. Astro. Soc., **108**, 252,(1948).
- Hoyle, F., Narlikar, J. V., Proc. Roy. Soc. of Lon., Ser. A, Math. Phys. Sci., 278(1375), 465,(1964 a).
- Hoyle, F., Narlikar, J. V., Proc. Roy. Soc. of Lon., Ser. A, Math. Phys. Sci., 282(1389), 178,(1964 b).
- Hoyle, F., Narlikar, J. V., Proc. Roy. Soc. of Lon., Ser. A, Math. Phys. Sci., 282(1389), 191,(1964 c).
- 8. Narlikar, J. V., Nat. Phys. Sci., 242, 135,(1973).
- Narlikar, J. V., Padmanabhan, T., The Ame. Phys. Soc., Phys. Rev. D, 32(8), 1928, (1985).
- 10. Narlikar et al., Astrophys. J., 585, 1,(2003).
- 11. Chatterjee, S., Banerjee, A., Gen. Rel. Grav., **36**, 2, 303,(2004).
- 12. Singh, T., Chaubey, R., Astrophys. Space Sci., **321**, 5,(2009).
- Adhav, K. S., Katore, S. D, Bansod, A. S., Gadodia, P. S., Bulg. J. Phys., **37**, 90,(2010).
- Adhav, K. S., Gadodia, P. S., Bansod, A. S., Int. J. Theor. Phys., **50**(9), 2720,(2011).
- Katore, S. D., The Afr. Rev. Phys., 8:0024, 157, (2013).
- Patil, V. R., Bolke, P. A., Bayaskar, N. S., Int. J. Theor. Phys., DOI: 10.1007/s10773-014-2175-9,(2014).
- 17. Perlmutter, S., *et al.*, Astrophys J., **483**(2), 565,(1997).
- Perlmutter, S., *et al.*, Nature, **391**, 51,(1998).(astroph/9712212).
- 19. Perlmutter, S. *et al.*, Astrophys. J., **517**(2), 565,(1999).
- 20. Garnavich, P. M. *et al.*, Astrophys. J. Lett., **493**(2), L53,(1998 a).
- 21. Garnavich, P. M. *et al.*, Astrophys. J., **509**(1), 74, (1998 b).
- Schmidt, B. P., *et al.*, Astrophysical Jour., **507**(1), 46, (1998).(astro-ph/9805200).
- 23. Riess, R.G. *et al.*, The Astronomical J., **116**(3), 1009, (1998).
- 24. Riess, R.G. *et al.*, Pub. Astronomical Soc. Pacific (PASP), **112**(776), 1284,(2000).

- 25. Riess, R. G. *et al.*, Astrophysics J., **607**(2), 665,(2004).
- 26. Zel'dovich, Ya. B., Sov. Phys. Usp., **11**(3), 381,(1968).
- 27. Bergman, P. G., Int. J. Theo. Phys., **1**(1), 25,(1968). Doi:10.1007/BF00668828.
- 28. Linde, A. D., JETP Lett., 19, 183,(1974).
- 29. Dolgov, A, D., (eds. Gibbons, G. W., Hawking, S. W. and Siklos, S. T. C.), Cambridge University Press, Cambridge, 449,(1983).
- 30. Dolgov, A. D., Sazhin, M. V., Zeldovich, Ya, B., Editions Frontieres,(1990).
- 31. Pavon, D., Physical Rev. D., 43, 375,(1991).
- 32. Weinberg, S., Rev. Mod. Phys., 61(1), 1,(1989).
- 33. Abers, E. S., Lee, B. W., Phys. Rep., 9(1), 1,(1973).
- 34. Langacker, P., Phys. Rep., 72, 185,(1981).
- 35. Bertolami, O., IL Nuovo Cimento B, Series 11, 93(1), 36,(1986).
- 36. Berman, M. S., Som, M. M., Int. J. Theor. Phys., 29, 1411,(1990).
- 37. Chen, W., Wu, Y. S., Phys. Rev. D, **41**(2), 695,(1990).
- Carvalho, J. C., Lima, J. A. S., Waga, I., Phys. Rev. D, 46(6), 2404,(1992).
- 39. Overduin, J. M., Astrophys. J., 517(1), L1, (1999).
- 40. Olson, T. S., Jordan, T. F., Phys. Rev. D., **35**(10), 3258,(1987).
- 41. Gasperini, M., Class. Quant. Grav., 5, 521,(1988).
- 42. Pebbles, P. J. E., Ratra, B., Astrophys. J. Lett., **325**, L17,(1988).
- 43. Abdel-Rahman, A. M. M., Phys. Rev. D, **45**, 3497,(1992).
- 44. Maia, M. D., Silva, G. S., Phys. Rev. D, **50**, 7233,(1994).
- 45. Silveira, V., Waga, I., Phys. Rev. D., 50, 4890,(1994).
- 46. Moffat, J. W., Phys. Lett. B, 357, 526,(1995).
- Torres, L. F. B., Waga, I., Mon. Not. R. Astron. Soc., 279, 712,(1996).
- 48. Abdussattar, Vishwakarma, R. G., Pramana J. Phys., **47**, 41,(1996).
- 49. Hoyle, F., Burbidge, G., Narlikar, J. V., Mon. Not. R. Astron. Soc., **286**, 173,(1997).
- 50. Podariu, S., Ratra, B., Astrophys. J., **532**, 109,(2000).
- 51. Carmeli, M., Kuzmenko, T., Int. J. Theor. Phys., **41**(1), 131,(2002).
- 52. Liu, H., Wesson, P. S., The Astrophysical J., **562**, 1, (2001).
- 53. Cunha, J. V., Santos, R. C., Int. J. Mod. Phys. D, **13**, 1321,(2004).
- 54. Carneiro, S., Lima, J. A., Int. J. Mod. Phys. A, 20, 2465,(2005).
- 55. Pradhan, A., Yadav, A. K., Yadav, L., Czech. J. Phys., **55**(4), 503,(2005).
- 56. Singh C. P., Suresh Kumar, Int. J. Theor. Phys., **47**(12), 3171,(2008).

2015

- 57. Katore, S. D., Rane, R. S., Wankhede, K. S., Int. J. Theo. Phys., **49**(9), 2105,(2010).
- 58. Tiwari, R. K., Sharma, S., Chin. Phys. Lett., **28**(2), 020401-(1-3),(2011).
- 59. Dwivedi, U. K., Int. J. of Adv. Res. in Eng. & App. Sci. (IJAREAS), **1**(1), 31,(2012).
- Amirhashchi, H., Mohamadian, H., Pramana- J. of Physics, 78(4), 651,(2012).
- Bali, R., Saraf, S., Proc. Natl. Acad. Sci., India, Sec. A, Phys. Sci., 83(1), 29,(2013 a).
- 62. Bali, R., Saraf, S., Prespacetime J.,4(5),545,(2013 b).
- Rahman, M. A., Ansari, M., Prespacetime J., 4(9), 871,(2013).
- Ghate, H. R., Salve, S. A., Res. J. Recent Sci, 3(ISC-2013), 53,(2014 a).
- Ghate, H. R., Salve, S. A., Prespacetime J., 5(3), 198,(2014 b).
- Ghate, H. R., Salve, S. A., Int. J. Sci. & Eng. Res., 5(6), 254,(2014 c).
- Collins, C. B., Glass, E. N., Wilkinson, D. A., Gen. Rel. Grav., **12**, 805,(1980).
- Monerat, G. A., Correea Silva, E. V., Oliveira-Neto, G., Ferreira Filho, L. G., Lemos, N. A., Phy. Rev. D, 73(4), 044022,(2006). Doi: 10.1103/PhyRevD. 73.044022.
- Pedram, P., Mirzaei, M., Jalalzadeh, S., Gousheh, S. S., Gen. Rel. Grav., 40, 1663, (2008).
- Pradhan, A., Kumhar, S. S., Yadav, P., Jotania, K., Int. J. Math. Eng. & Sci. (IJMES), 1(2), 74, (2012).
- 71. Yadav, A. K., Elec. J. Theor. Phys., **10**, 169, (2013). arXiv:0911.0177[gr-qc].

This page is intentionally left blank