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A Unified Integral Associated with the Aleph Function

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Abstract- In this note we obtain a unified new integral whose integrand contains product of Aleph function and generalized multivariable polynomials having general arguments. Several integrals containing many simpler functions follow as special cases of this integral.

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10. Srivastava H.M., Gupta K.C., and Goyal S.P., The H functions of one and two variables with applications (New Delhi and Madras: South Asian publ.)1982 p.11, 18-19.

A Unified Integral Associated with the Aleph Function

Harshita Garg [α] & Ashok Singh Shekhawat [σ]

Abstract- In this note we obtain a unified new integral whose integrand contains product of Aleph function and generalized multivariable polynomials having general arguments. Several integrals containing many simpler functions follow as special cases of this integral.

Keywords: aleph function, generalized polynomials, hypergeometric function, fox's H- function.

I. Introduction

The Aleph function introduced by Südland et al [10] is defined as Mellin-Barnes type contour integrals as following:

$$\aleph(x)=\aleph^{e,f}_{p_i,q_i,c_i;r}\left[x\left|\begin{matrix}(a_j,A_j)_{1,f},[c_i(a_{ji},A_{ji})]_{f+1,p_i;r}\\(b_j,B_j)_{1,e},[c_i(b_{ji},B_{ji})]_{e+1,q_i;r}\end{matrix}\right.\right]$$

$$=\frac{1}{2\pi i}\int_L \Omega^{e,f}_{p_i,q_i,c_i;r}(\xi)\,x^{-\xi}\,d\xi \qquad \ldots (1.1)$$

For all x ≠ 0, where $i=\sqrt{-1}$ and

$$\Omega^{e,f}_{p_i,q_i,c_i;r}(\xi)=\frac{\prod_{j=1}^{e}\Gamma(b_j+B_j\xi)\prod_{j=1}^{f}\Gamma(1-a_j-A_j\xi)}{\sum_{i=1}^{r}c_i\prod_{j=f+1}^{p_i}\Gamma(a_{ji}+A_{ji}\xi)\prod_{j=e+1}^{q_i}\Gamma(1-b_j-B_{ji}\xi)} \qquad \ldots (1.2)$$

The $L=L_{i\gamma\infty}$ is a suitable contour of the Mellin-Barnes type which runs from γ−i∞ to γ+i∞ with γ ∈ R, the integers e, f, p_i , q_i satisfy the inequality $0 \le f \le p_i$, $1 \le e \le q_i$, c_i ¿ 0; i = 1,…,r. The parameters A_j, B_j, A_{ji}, B_{ji} are positive real numbers and a_j, b_j, a_{ji}, b_{ji} are complex numbers, such that the poles of $\Gamma(b_j+B_j\xi)$, j = 1,2,.....,e separating from those of $\Gamma(1-a_j-A_j\xi)$, j = 1,…,f. All the poles of the integrand (1.2) are supposed to be easy and empty products are considered as unity. The existence conditions [4] for the Aleph function (1.2) are given below:

$$\psi_k>0,|\arg(x)|<\frac{\pi}{2}\psi_k;k=1,...,r, \qquad \ldots (1.3)$$

$$\psi_k\ge 0,|\arg(x)|<\frac{\pi}{2}\psi_k \text{ and } R\{\Lambda_k\}+1<0 \qquad \ldots (1.4)$$

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Where

$$\psi_k = \sum_{j=1}^{f} A_j + \sum_{j=1}^{e} B_j - C_k \left(\sum_{j=f+1}^{p_k} A_{jk} + \sum_{j=e+1}^{q_k} B_{jk} \right) \qquad \ldots (1.5)$$

$$\Lambda_k = \sum_{j=1}^{e} b_j - \sum_{j=1}^{f} a_k + C_k \left(\sum_{j=1}^{q_k} b_{jk} - \sum_{j=f+1}^{p_k} a_{jk} \right) + \frac{1}{2}(p_k - q_k) \qquad \ldots (1.6)$$

The generalized polynomial defined by Srivastava [5] is as follows:

$$S_{f_1,\ldots,f_s}^{e_1,\ldots,e_s}[z_1,\ldots,z_s] = \sum_{\beta_1=0}^{[f_1/e_1]} \ldots \sum_{\beta_s=0}^{[f_s/e_s]} \frac{(-f_1)_{e_1\beta_1}}{\beta_1!} \ldots \frac{(-f_s)_{e_s\beta_s}}{\beta_s!} \, . \, A[f_1,\beta_1;\ldots;f_s,\beta_s] z_1^{\beta_1} \ldots z_s^{\beta_s} \qquad \ldots (1.7)$$

Where $f_i = 0, 1, 2\ldots \ \forall i = (1,\ldots,s)$, $e_1,\ldots,e_s$ are arbitrary positive integers and the coefficients $[f_1,\beta_1;\ldots;f_s,\beta_s]$ are arbitrary constants, real or complex.

II. The Main Integral

We derive the following result:

$$\int_0^\infty z^{\delta-1} \left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}} \right]^{-\mu} . \aleph_{p_i,q_i,c_i;r}^{e,f} \left[t \left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}} \right]^{-\lambda} \right]$$

$$S_{f_1,\ldots,f_s}^{e_1,\ldots,e_s} \left[x_1 \left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}} \right]^{-a_1} \ldots x_2 \left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}} \right]^{-a_s} \right] dz$$

$$= 2\Gamma(2\delta) \left(\frac{\alpha}{2} \right)^{\delta} \sum_{\beta_1=0}^{[f_1/e_1]} \ldots \sum_{\beta_s=0}^{[f_s/e_s]} \frac{(-f_1)_{e_1\beta_1}}{\beta_1!} \ldots \frac{(-f_s)_{e_s\beta_s}}{\beta_s!} A[f_1,\beta_1;\ldots;f_s,\beta_s] x_1^{\beta_1} \ldots x_s^{\beta_s} \alpha^{\left(-\mu-\sum_{i=1}^{s} a_i\beta_i\right)}$$

$$\aleph_{p_i+2,q_i+2,c_i;r}^{e,f+2} \left[t\alpha^{-\lambda} \left| \begin{matrix} \left(-\mu-\sum_{i=1}^{s} a_i\beta_i;\lambda\right), \left(1+\delta-\mu-\sum_{i=1}^{s} a_i\beta_i;\lambda\right) (a_j,A_j)_{1,f}, [C_i(a_{ji},A_{ji})]_{f+1,p_i;r} \\ (b_j,B_j)_{1,e}, [C_i(b_{ji},B_{ji})]_{e+1,q_i;r} \left(-\mu-\sum_{i=1}^{s} a_i\beta_i - \delta;\lambda\right), \left(1-\mu-\sum_{i=1}^{s} a_i\beta_i;\lambda\right) \end{matrix} \right. \right] \qquad \ldots (2.1)$$

Where

(i) Where $\lambda > 0$, Re $(\delta, \mu, a) > 0$

(ii) $\mathrm{Re}(\delta) - \mathrm{Re}(\mu) - \lambda \min_{1 \le j \le e} \mathrm{Re}\left(\frac{b_j}{\beta_j} \right) < 0$ and

(iii) $e_1,\ldots,e_s$ are arbitrary positive integers and the coefficients $[f_1,\beta_1;\ldots;f_s,\beta_s]$ are arbitrary constants, real or complex.

PROOF: The integral in (2.1) can be obtained by using the Aleph function in terms of Mellin-Barnes contour integral given by (1.1) and the definition of a generalized polynomials given by (1.7), then interchanging the order of summation and integration (which is permissible under the conditions stated with (2.1)) and evaluating the inner integral by using a result given by Oberthettinger F. [3] and we get the desired result.

III. Special Cases

(1) Taking general class of polynomials in our main integral(2.1), we have

Ref

3. Oberthettinger F, Tables of Mellin transforms (Berlin, Heidelberg, New York; Springer-verlag) p.22, 1974.

$$\int_0^\infty z^{\delta-1}\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\mu} .\aleph^{e,f}_{p_i,q_i,c_i;r}\left[t\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\lambda}\right]$$

$$S^{e_1,\ldots,e_s}_{f'}.\left[x_1\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-a_1}\ldots.x_2\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-a_s}\right]dz$$

$$=2\Gamma(2\delta)\left(\frac{\alpha}{2}\right)^{\delta}\sum_{\beta_1,\ldots\ldots,\beta_s=0}^{e_1\beta_1+\ldots..+e_s\beta_s\le f}\frac{(-f')_{e_1\beta_1+\ldots\ldots+e_s\beta_s}}{\beta_1!\ldots\ldots.\beta_s!}A[f';\beta_1;\ldots;\beta_s]\;x_1^{\beta_1}\ldots x_s^{\beta_s}\;\alpha^{\left(-\mu-\sum_{i=1}^{s}a_i\beta_i\right)}$$

$$.\aleph^{e,f+2}_{p_i+2,q_i+2,c_i;r}\left[t\alpha^{-\lambda}\left|\begin{matrix}\left(-\mu-\sum_{i=1}^{s}a_i\beta_i;\lambda\right),\left(1+\delta-\mu-\sum_{i=1}^{s}a_i\beta_i;\lambda\right)(a_j,A_j)_{1,f},[C_i(a_{ji},A_{ji})]_{f+1,p_i;r}\\(b_j,B_j)_{1,e},[C_i(b_{ji},B_{ji})]_{e+1,q_i;r}\left(-\mu-\sum_{i=1}^{s}a_i\beta_i\text{-}\delta;\lambda\right),\left(1-\mu-\sum_{i=1}^{s}a_i\beta_i;\lambda\right)\end{matrix}\right.\right] \quad \ldots (3.1)$$

Where $e_1,\ldots,e_s$ are arbitrary positive integers and the coefficients $[f',\beta_1;\ldots;\beta_s]$ are arbitrary constants, real or complex and valid sufficient conditions (i), (ii) and of (2.1).

(2) If we take $s\to 1$, $e_1 = 2$, $A_{f_1,\beta_1}=(-1)^{\beta_1}$ then by applying our results given in (2.1) to the case of Hermite polynomial [7] and [12] and by taking

$$S^2_{f_1}(x)\to x^{f_1/2}H_{f_1}\left[\frac{1}{2\sqrt{x}}\right]$$

We have the following result

$$\int_0^\infty z^{\delta-1}\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\mu}\left[x_1\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-a_1}\right]^{\frac{f_1}{2}}$$

$$.H_{f_1}\left[\frac{1}{2\sqrt{x_1\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-a_1}}}\right]\aleph^{e,f}_{p_i,q_i,c_i;r}\left[t\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\lambda}\right]dz$$

$$=2\Gamma(2\delta)\left(\frac{\alpha}{2}\right)^{\delta}\sum_{\beta_1=0}^{[f_1/2]}\frac{(-f_1)_{2\beta_1}}{\beta_1!}(-1)^{\beta_1}x_1^{\beta_1}\alpha^{(-\mu-a_1\beta_1)}$$

$$.\aleph^{e,f+2}_{p_i+2,q_i+2,c_i;r}\left[t\alpha^{-\lambda}\left|\begin{matrix}(-\mu-a_1\beta_1;\lambda),(1+\delta-\mu-a_1\beta_1;\lambda)(a_j,A_j)_{1,f},[C_i(a_{ji},A_{ji})]_{f+1,p_i;r}\\(b_j,B_j)_{1,e},[C_i(b_{ji},B_{ji})]_{e+1,q_i;r}(-\mu-a_1\beta_1\text{-}\delta;\lambda),(1-\mu-a_1\beta_1;\lambda)\end{matrix}\right.\right] \quad \ldots (3.2)$$

Valid under the set of sufficient conditions (i) and (ii) of (2.1)

(3) For the Laguerre polynomials ([7] and [12]) setting $s\to 1, S'_{f_1}(x)\to L^{(\alpha')}_{f_1}(x)$ in which case $e_1 = 1$, $A_{f_1,\beta_1}=\binom{f_1+\alpha'}{f_1}\frac{1}{(\alpha'+1)_{\beta_1'}}$ the results (2.1) reduce to the following formulae:

Ref

7. Srivastava H.M. and Daoust M.C., Certain generalized Neumann expansions associated with the Kampé de Fériet function, Nederal. Akad. Wetensch. Indag. Math. 31 (1969), 449-457.

$$\int_0^{\infty} z^{\delta-1}\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\mu} L_{f_1}^{(\alpha')}\left(x_1\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-a_1}\right)$$

$$.\aleph_{p_i,q_i,c_i;r}^{e,f}\left[t\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\lambda}\right]dz$$

$$=2\Gamma(2\delta)\left(\frac{\alpha}{2}\right)^{\delta}\sum_{\beta_1=0}^{[f_1/2]}\frac{(-f_1)_{2\beta_1}}{\beta_1!}\binom{f_1+\alpha'}{f_1}\frac{1}{(\alpha'+1)_{\beta_1'}}\; x_1^{\beta_1}\,\alpha^{(-\mu-a_1\beta_1)}$$

$$.\aleph_{p_i+2,q_i+2,c_i;r}^{e,f+2}\left[t\alpha^{-\lambda}\left|\begin{matrix}(-\mu-a_1\beta_1;\lambda),(1+\delta-\mu-a_1\beta_1;\lambda)(a_j,A_j)_{1,f},[C_i(a_{ji},A_{ji})]_{f+1,p_i;r}\\(b_j,B_j)_{1,e},[C_i(b_{ji},B_{ji})]_{e+1,q_i;r}\,(-\mu-a_1\beta_1\text{-}\delta;\lambda),(1-\mu-a_1\beta_1;\lambda)\end{matrix}\right.\right] \quad \ldots (3.3)$$

Valid under the set of sufficient conditions (i) and (ii) of (2.1)

(4) Taking $c_i \to 1$, Aleph function reduces to I-function given by Saxena [5], then our main integral (2.1) reduces to the following form:

$$\int_0^{\infty} z^{\delta-1}\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\mu} .I_{p_i,q_i;r}^{e,f}\left[t\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\lambda}\right]$$

$$.S_{f_1,\ldots,f_s}^{e_1,\ldots,e_s}\left[x_1\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-a_1}\ldots .x_2\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-a_s}\right]dz$$

$$=2\Gamma(2\delta)\left(\frac{\alpha}{2}\right)^{\delta}\sum_{\beta_1=0}^{[f_1/e_1]}\cdots\sum_{\beta_s=0}^{[f_s/e_s]}\frac{(-f_1)_{e_1\beta_1}}{\beta_1!}\cdots\frac{(-f_s)_{e_s\beta_s}}{\beta_s!}\;A[f_1,\beta_1;\ldots;f_s,\beta_s]\,x_1^{\beta_1}\ldots x_s^{\beta_s}$$

$$.\alpha^{\left(-\mu-\sum_{i=1}^{s}a_i\beta_i\right)}\;.I_{p_i+2,q_i+2;r}^{e,f+2}\left[t\alpha^{-\lambda}\left|\begin{matrix}\left(-\mu-\sum_{i=1}^{s}a_i\beta_i;\lambda\right),\left(1+\delta-\mu-\sum_{i=1}^{s}a_i\beta_i;\lambda\right)(a_j,A_j)_{1,f},[a_{ji},A_{ji}]_{f+1,p_i;r}\\(b_j,B_j)_{1,e},[b_{ji},B_{ji}]_{e+1,q_i;r}\left(-\mu-\sum_{i=1}^{s}a_i\beta_i\text{-}\delta;\lambda\right),\left(1-\mu-\sum_{i=1}^{s}a_i\beta_i;\lambda\right)\end{matrix}\right.\right] \quad \ldots (3.4)$$

Valid under the set of sufficient conditions (i), (ii) and (iii) of (2.1)

(5) Taking $c_i \to 1$ and r=1 Aleph function reduces to Fox's H-function[1], then our main integral (2.1) reduces to the following form:

$$\int_0^{\infty} z^{\delta-1}\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\mu} .H_{p,q}^{e,f}\left[t\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\lambda}\right]$$

$$.S_{f_1,\ldots,f_s}^{e_1,\ldots,e_s}\left[x_1\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-a_1}\ldots .x_2\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-a_s}\right]dz$$

$$=2\Gamma(2\delta)\left(\frac{\alpha}{2}\right)^{\delta}\sum_{\beta_1=0}^{[f_1/e_1]}\cdots\sum_{\beta_s=0}^{[f_s/e_s]}\frac{(-f_1)_{e_1\beta_1}}{\beta_1!}\cdots\frac{(-f_s)_{e_s\beta_s}}{\beta_s!}\;A[f_1,\beta_1;\ldots;f_s,\beta_s]\,x_1^{\beta_1}\ldots x_s^{\beta_s}$$

Ref

5. Saxena V.P., The I-function, *Anamaya Publishers*, New Delhi, (2008).

$$.\alpha^{\left(-\mu-\sum_{i=1}^{s} a_i\beta_i\right)} .H_{p+2,q+2}^{e,f+2}\left[t\alpha^{-\lambda}\left|\begin{matrix}\left(-\mu-\sum_{i=1}^{s} a_i\beta_i;\lambda\right),\left(1+\delta-\mu-\sum_{i=1}^{s} a_i\beta_i;\lambda\right)(a_j,A_j)_{1,p}\\(b_j,B_j)_{1,q}\left(-\mu-\sum_{i=1}^{s} a_i\beta_i-\delta;\lambda\right),\left(1-\mu-\sum_{i=1}^{s} a_i\beta_i;\lambda\right)\end{matrix}\right.\right] \quad \ldots (3.5)$$

Valid under the set of sufficient conditions (i), (ii) and (iii) of (2.1)

(6) If we take $c_i \to 1$, r=1and $e_1,\ldots,e_s \to e$ and $f_1,\ldots,f_s \to f$ i.s. (1,..,s→1) in the integral (2.1), we arrive at the following result which is obtained by Garg and Mittal [2] .

$$\int_0^\infty z^{\delta-1}\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\mu} .H_{p,q}^{e,f}\left[t\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\lambda}\right]$$

$$S_f^e\left[x\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-a}\right]dz$$

$$=2\Gamma(2\delta)\left(\frac{\alpha}{2}\right)^{\delta}\sum_{\beta=0}^{[f/e]}\frac{(-f)_{e\beta}}{\beta!}A[f,\beta]x^{\beta}\alpha^{(-\mu-a\beta)}$$

$$.H_{p+2,q+2}^{e,f+2}\left[t\alpha^{-\lambda}\left|\begin{matrix}(-\mu-a\beta;\lambda),(1+\delta-\mu-a\beta;\lambda),(a_1,A_1),(a_p,A_p)\\(b_1,B_1),(b_q,B_q),(-\mu-a\beta-\delta;\lambda),(1-\mu-a\beta;\lambda)\end{matrix}\right.\right] \quad \ldots (3.6)$$

Where

(i) $\lambda>0$, Re $(\delta, \mu, a) > 0$

(ii) Re (δ) - Re (μ) $-\lambda \min_{1\le j\le e} \mathrm{Re}\left(\frac{b_j}{\beta_j}\right) < 0$ and

(iii) e is are arbitrary positive integers and the coefficients $[f,\beta]$ is arbitrary constant, real or complex.

(7) If we take $S_{f_1,\ldots,f_s}^{e_1,\ldots,e_s} \to 1$ and reduce Aleph function to Gauss hypergeometric function [9] in the integral (2.1), we arrive at the following result after some simplifications:

$$\int_0^\infty z^{\delta-1}\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\mu} {}_2F_1(l,m;n;t\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-1})dx$$

$$=2^{1-\delta}\mu\Gamma(2\delta)(\alpha)^{\delta-\mu}\frac{\Gamma(\mu-\delta)}{\Gamma(\mu+\delta+1)}\,{}_4F_3(\ell,m,\mu-\delta,\mu+1;n,\mu,\mu+\delta+1;\frac{t}{\alpha}) \quad \ldots (3.7)$$

Where 0<Re (δ) < Re (μ); $|t|<|\alpha|$

IV. Conclusion

The result so established may be found useful in several interesting situation appearing in the literature on mathematical analysis. The result (3.1) not only gives the value of the integral but also 'augments' the coefficients in the series in the integrand to give a ${}_4F_3$ series as the integrated series.

Ref

2. Garg M. and Mittal S. 'On a new unified integral' proc. Indian. acad. sci. (Math sci.) vol. 114 no.2, 2004 pp 99-101.

References Références Referencias

1. Fox C, The G and H functions as symmetrical Fourier kernels, Trans. Am. Math. Soc. 98 (1961) 395-429.
2. Garg M. and Mittal S. 'On a new unified integral' proc. Indian. acad. sci. (Math sci.) vol. 114 no.2, 2004 pp 99-101.
3. Oberthettinger F, Tables of Mellin transforms (Berlin, Heidelberg, New York; Springer-verlag) p.22, 1974.
4. Saxena, R.K. and Pogany, T.K. : Mathieu-type series for the Aleph-function occurring in Fokker-Planck equation, Eur. J. Pure Appl. Math., 3(6), 958-979 (2010).
5. Saxena V.P., The I-function, *Anamaya Publishers*, New Delhi, (2008).
6. Srivastava, H.M., A multilinear generating function for the Konhauser sets of bi-orthogonal polynomials suggested by the Laguerre polynomials, Pacific J. Math. 117, 183-191 (1985).
7. Srivastava H.M. and Daoust M.C., Certain generalized Neumann expansions associated with the Kampé de Fériet function, Nederal. Akad. Wetensch. Indag. Math. 31 (1969), 449-457.
8. Srivastava H.M. and Panda R., some bilateral generating function for a class of generalized hyper geometric polynomials J. Raine Angew. Math., 283/284 (1996), 265-274.
9. Srivastava H.M. and Singh N.P., The integration of certain products of the multivariable H-function with a general class of polynomials, Rend. Circ. Mat. Palermo 2(32) (1983), 157-187.
10. Srivastava H.M., Gupta K.C., and Goyal S.P., The H functions of one and two variables with applications (New Delhi and Madras: South Asian publ.)1982 p.11, 18-19.
11. Südland, N., Baumann, B. and Nonnenmacher, T.F., Who knows about the Aleph (ℵ)-function? Fract. Calc. Appl. Anal., 1(4), 401-402, (1998).
12. Szego, C., Orthogonal polynomials, Amer. Math. Soc. Colloq. Publ.23 Fourth edition, Amer. Math. Soc. Providence, Rhode Island (1975).

Notes