



GLOBAL JOURNAL OF SCIENCE FRONTIER RESEARCH: F  
MATHEMATICS AND DECISION SCIENCES  
Volume 15 Issue 10 Version 1.0 Year 2015  
Type : Double Blind Peer Reviewed International Research Journal  
Publisher: Global Journals Inc. (USA)  
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

## Importance of Mathematical Communication and Discourse in Secondary Classrooms

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**GJSFR-F Classification :** *MSC 2010: 00A05*



IMPORTANCE OF MATHEMATICAL COMMUNICATION AND DISCOURSE IN SECONDARY CLASSROOMS

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# Importance of Mathematical Communication and Discourse in Secondary Classrooms

Roland G. Pourdavood <sup>α</sup> & Patrick Wachira <sup>σ</sup>

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## I. INTRODUCTION

Across the US teachers are implementing the Common Core State Standards for Mathematics (CCSSM). These are a single set of standards for Kindergarten to twelfth grade (K-12) in mathematics which outlines what a student should know and be able to do at the end of each grade. In addition to content coverage, the standards also specify the mathematical ways of thinking students should develop while learning mathematics content. These process standards are described as eight Common Core Practices for Mathematics (CCSSM 2010). One of these is the ability to construct viable arguments and critique the reasoning of others. Engaging in discourse to make conjectures, justify and defend one answer in a collaborative exchange of ideas about a mathematics concept provides students with the ideal opportunity to construct viable arguments and critique the reasoning of others, which is key to achieving mathematical understanding.

Another key component necessary for success in mathematics highlighted in the common core standards is the need to attend to precision which explicitly calls for students to attend to precision both calculation and language. The use of discourse is so important that it fits into the National Council of Teachers of Mathematics (NCTM, 2000) communication standard which calls for Instructional programs to enable all students to communicate their mathematical thinking coherently and clearly to peers, teachers, and others; to analyze and evaluate the mathematical thinking and strategies

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of others and to use the language of mathematics to express mathematical ideas precisely.

This paper is based on the NCTM's communication standard and the Common core standards for mathematics practices and focuses on how a secondary mathematics teacher orchestrated classroom discourse with an emphasis on the importance of students' mathematical communication, both verbal and written as they engaged in problem solving, reasoning and proofs. The paper focuses on how the teacher's emphasis on discourse and specifically on the promotion of formal mathematics language influenced students' thinking and beliefs about learning.

## II. MATHEMATICS DISCOURSE

Reform-based mathematics invites students to investigate mathematical problems and encourages students to use mathematical discourse in order to develop a deeper conceptual understanding called for in the Common Core mathematics standards. Mathematical discourse means that students are able to make conjectures, talk, question, and agree or disagree about problems in order to develop important mathematical concepts (Stein, 2007). Pirie & Schwarzenberger, 1988 as cited in Truxaw, Gorgievski & DeFranco, 2008, p. 58) defined mathematical discourse as "Purposeful talk on a mathematical subject in which there are genuine contributions and interactions". Mathematical discourse contributes to deeper analyses of mathematics on the part of teachers as well as students (Manouchehri, 2007; Manouchehri & St. John, 2006).

According to Schwols & Dempsey (2012b), there are several components to high quality mathematics discourse. One of these is facilitation of conversation. The level and effectiveness of classroom discourse depends heavily on the facilitation skills of the teacher. First, classroom norms should be set up so everyone knows their role. Teachers need to provide a safe and appropriate learning environment that encourages students to participate including building on the response of others, support students throughout their conversations, and show students that they value conceptual understanding, rather than simply focusing on arriving at the right answer (Stein, 2007). During mathematics discourse, teachers need to pose questions that challenge students thinking. Questioning should challenge students to be inquisitive and help them extend their existing mathematics knowledge. Divergent questions which elicit a broader response can help foster students' problem solving and increase conceptual understanding. Teachers need to listen very carefully and monitor students' understanding. The role of the student includes listening and responding to the teacher and to other students. In addition students ought to be comfortable to make arguments of particular concepts and procedures and be able to communicate clearly their reasoning.

Another indispensable component of mathematical discourse is formal mathematical language. The quality of classroom discourse depends on the ability of students to process language in order to build on the ideas of others. The ability to process language promotes mathematical thinking (Kabasakalian, 2007). Students need to know the meaning of mathematics vocabulary words, whether written or spoken, in order to better understand and communicate mathematical ideas (Gay, 2008). Teachers need to provide learning opportunities that encourage students to use mathematical language so that students better grasp the underlying mathematical meaning of

concepts (Adler, 1999; Kotsopoulos, 2007). Likewise, teachers must remain mindful of their own use of mathematical language because a teacher's choice of words directly contributes to a student's understanding or misunderstanding of concepts (Gay, 2008).

#### a) *Problem and significance of study*

Communication and discourse are most effective in the context of inquiry-based instruction, however despite the power and benefits of inquiry-based instruction teachers and students resist to use these viable strategies. Students may not engage in instructional conversations because they may not know how. Often students' contributions relate more to procedure rather than deeper level of mathematical concepts. Further in attempting classroom discourse, students' experience interference when they borrow language from their everyday lives to use in their mathematical world; their inability to minimize this interference could potentially undermine their ability to learn (Kotsopoulos, 2007). Thus implementing and managing classroom communication and discourse can be difficult.

This paper focuses on how a high school mathematics teachers orchestrated classroom discourse with an emphasis on the importance of students' mathematical communication, both verbal and written as they engaged in problem solving, reasoning and proofs and how this influenced their thinking. Because discourse is most effective in the context of inquiry-based instruction this study is important to research and report educators' and students' experiences when they change from a predominantly teacher-centered to a more student-centered and inquiry-based learning environment.

### III. THE STUDY

#### a) *Participants and methods*

The study was conducted in an Introduction to Calculus (IC) course in a suburb of a large Midwestern city in the USA. Participants of this study were twelfth grade students enrolled in the *Introduction to Calculus* (IC) course that focused on inductive treatments of functions, limits, differential calculus, and integral calculus. The class consisted of students who dropped out of the prerequisite track for the advanced level calculus or those who transferred into the course during the first semester of their 12<sup>th</sup> grade year in lieu of receiving a poor grade in the advanced level calculus courses. This qualitative and descriptive study is grounded in constructivist inquiry (Guba & Lincoln, 1989, 1994; Lincoln & Guba, 1985; McCracken, 1988). Further, this study uses the social constructivist theory (Cobb, 1994) to explain and interpret the mathematical discourse of students as they engaged in solving non-routine problems and communicated their solutions. From a socio-constructivist perspective, individuals build learning and knowing within the social and cultural milieu. The teacher's role becomes that of a facilitator to students; learning, guiding and supporting students construction of viable mathematical ideas. Data sources include transcripts of audio tape recordings of classroom discussions, students written responses to various problems, interviews with the classroom teacher, and the field notes of the researchers. In the analysis of data the researchers had focus group discussions after every observation about their understanding and interpretation of the data. The recursive relationship between student voice and classroom problem solving discourse is pivotal for understanding and explaining the mathematical understanding of students.

### b) *Facilitating discourse*

The classroom teacher incorporated four instructional strategies inspired by Hafferd-Ackles, Fuson, & Sherin (2004), to engage his students in mathematical discourse. The four instructional strategies that helped transform the informal classroom mathematics discussions into productive dialogue were inspired by the four levels of mathematics talk learning community framework as explained by Hafferd-Ackles et al, (2004). For this study, the levels were renamed and defined as; Establishing expectations, Mathematics language, Mathematics community, and Establishing Formal Discourse. These four strategies were interactive and emerging throughout the year and not used in a linear fashion. During any lesson two or more of the strategies could be present, but one was always dominant.

Four lessons are presented below to highlight these strategies. The mathematical content of each lesson focused on the line that is tangent to a graph for a given function at a given point on a graph. For each lesson a worksheet stated the pedagogical strategy for conducting the lesson, identified the mathematical tasks, provided a blank grid for drawing graphs, and allowed space for indicating methods, explaining thinking, and justifying responses. All four lessons revisited the same four tasks and relied on the same given function, the same tangent line, and the same graphs. However, the pedagogical strategy and content varied in that each lesson employed a more sophisticated strategy and included an additional task that extended the lesson.

### c) *Strategy 1: Establishing Expectations*

In the first lesson the teacher led and dominated the lesson by speaking in a purposeful manner, by demonstrating every detail on the board, by explaining his thinking, justifying his methods and by telling students what to write. The teacher purposefully used this strategy in communicating his expectations regarding how to talk and write about mathematics and also to connect with students' prior experiences in which they were accustomed to listen and maintain silence as they watched the teacher and copied notes. In this first lesson it was important for the students to observe how the language changed from informal language into formal language using standard mathematical vocabulary including graphing calculator syntax to clarify directives.

Teacher: Key it in. By *it*, I mean the expression  $4x^2 + 5$  and by "in", I mean  $Y_2$  on the "Y=" screen. I'll say *it* again without pronouns. Go to the function screen on your calculator by pressing the "Y=" key. On the function screen, enter the expression  $4x^2 + 5$  to define  $Y_2$ .

From lesson one, students became aware of the important differences between informal language riddled with pronouns and formal language clarified with appropriate mathematical vocabulary.

Teacher:[pointing to a graph on the board] See how it touches right here? See how it, this line, touches this graph right here, at this point? See how this line, Line  $l$ , touches the graph of function  $f$  right here, at the point  $(1, 8)$  where  $x$  is one and  $f$  of one is  $8$ ? Line  $l$  is tangent to the graph for function  $f$  at the point where  $x$  equals  $1$ .

At the end of the lesson, the teacher entertained questions from students. Students' questions were short and vague and avoided standard mathematical language and vocabulary in which case the teacher had to guess the meaning.

### d) *Mathematics language*

The second lesson which the teacher titled *Say "It" in Words*, directed students to complete the same four tasks, indicate their methods and justify their responses and



in addition to write step-by-step instructions. In this lesson, the students were to experiment with the formal language. They were to reflect and contribute responses by formulating a brief but meaningful response before responding aloud. The teacher posed questions, challenged students with follow up questions and helped students to clarify their responses. He also ensured classroom norms were followed where classmates did not distract others who were formulating responses.

Teacher: I ask a question; you answer. Use vocabulary words; not pronouns. No *its*. And don't interrupt the speaker.

Students persisted in requesting yet another teacher dominated lesson that avoided standard mathematical vocabulary and notation but the teacher used the lesson for demonstrating the inadequacy of informal mathematics language.

Teacher: Okay, okay. Apparently, yesterday didn't make sense. Maybe I did all the talking and maybe I was too formal. Okay, I'll explain it again. I'll use informal everyday English instead of standard mathematical vocabulary. Okay?

Group: [chatter and agitation subside]

Teacher: We will talk about the complete graph for the given function  $f$  and the line  $l$  that is tangent to the graph for function  $f$  at the point where  $x = 1$ .

Student 6: *English?*

Teacher: Okay, okay, not formal, informal. In *English*.

Teacher: [mimicking and exaggerating informal language patterns] I will talk about it and this one. Plug it into the calculator and it's approximately eight. It's  $f$ , so it's  $4x^2 + 5$ . When it's zero, it's five and when it's one, it's nine. Plug it into the original problem, work it out, it's exactly eight. Plug it into the problem, work it out, it's  $y = 8x + 1$ . So it's  $l$ . See?

Group: [chatter and agitation]

Teacher: The problem is: Too many pronouns. Each *it* refers to something different. You must say *it* in words. Clarify; use antecedents. Don't use pronouns. Don't say *it*.

Group: [silence]

In this lesson, the teacher encouraged students to use adequate mathematics language for communication by shifting from informal to more formal mathematical language. Using a list of five powerful rules of engagement for classroom discourse developed by the teacher, each student was helped to formulate at least one meaningful response during the lesson. These rules were; calling upon each student individually by name; Posing a question that the student should be able to understand; providing a reasonable length of time for the student to formulate a response; Verifying that the response is meaningful, delving and prompting to clarify if needed and finally validating the significance of the response before engaging the next student. The second rule often involved more than a quick answer therefore some students hesitated as they responded. The teacher responded by asking everyone to treat others as high achievers thus establishing a supportive climate. As the lesson proceeded, the teacher called on a different student each time. Students generally gave vague responses. The teacher continued to prompt and delve until responses made sense, for example:

Teacher: So, (Student 9),  $f$  of  $x$  is  $4x^2 + 5$ . What is  $f$  of zero?

Student 9: It's five.

Teacher: What *it* is five?

Student 9: Zero is five.

Teacher: Zero and five are two different numbers. Sorry, but zero is not five.

Student 9: It's still five, you know.

Teacher: Yes. *It* is still five. What *it* is still five?

Student 9: You know. I just can't say it.

Teacher: When you can't say *it* as a fact, ask *it* as a question. We'll give you a minute to collect your thoughts. What *it* can't you say?

Student 9: You know. How do you say what the *f* of the zero is? Oh! *f* of zero is five.

Teacher: Sounds good. Spoken with authority, like a college professor: *f* of zero is five.

With prompting from the teacher, students began to learn to speak with authority, justifying their answers. They depended less and less on the teacher. The students were also learning to combine speaking with writing in order to clarify their thinking. They attempted to write step-by-step instructions in general for finding the specified equation. The teacher offered several suggestions to help them.

1. Pick any step that makes sense to you and say it to yourself; use pronouns.
2. Write the step exactly the way you said it to yourself including pronouns.
3. Replace pronouns with standard mathematical vocabulary or notation.
4. Revise the step until it makes sense to you and to the person next to you.
5. Repeat the process and arrange the steps in an order that makes sense.

Conversations developed in the classroom as students attempted to write and revise the steps. The teacher moved around the room prompting students in order to help them transition their language from informal to formal. At this point most of the students wrote a partial list of vague steps while some were able to write more complete lists that included more than one precise, clearly worded step. Generally, students accepted this second strategy and valued its potential but they were not completely happy with it. The students felt that the teacher consumed too much time talking with individuals rather than to the whole class thus leading to loss of focus. Students also did not like a lesson that involved academic risks and being on the spotlight. They also did not like the inherent lack of closure for open-ended lessons. The teacher was however encouraged because this lesson provided for multiple levels of questioning and provided for differentiated levels of thinking, speaking and writing mathematically.

#### e) *Mathematics Community*

In Lesson 3 the teacher directed students to complete the same four tasks, indicate their methods and justify their responses. In addition the students were to work collaboratively with a partner to develop an analytical method for finding an equation for the line that is tangent to a graph for a given function at a given point. Each student was to make sure they understood their method thoroughly. The role of the students in this lesson was to attempt using formal discourse. The role of the teacher was the same as in Lesson 2 including enforcing the social/behavioral norms to provide a supportive environment. In this lesson, work varied among the pairs of partners who used mathematical vocabulary and standard mathematical notation in various ways to explain their thinking. The analytical method varied among the groups. Regardless, nearly all students could use at least one form with confidence to determine an equation for the tangent line. More importantly, students acquired communication skills, which in turn helped clarify their explanations and justifications. The teacher helped the class become aware of its collective intelligence by connecting Lesson 3 to previous lessons. Their own work clearly had value for students but their collective

work appeared to have added value. The teacher further motivated the students by awarding every student points not for writing correct answers but for connecting formal mathematical language with important mathematical ideas. As students became more collaborative and less competitive, the teacher felt reassured, noting that students not only valued their points but also valued their work.

f) *Formal Discourse*

Lesson 4 – The directions for lesson 4 were similar to the previous lessons. In addition the teacher directed the students to;

Draw a graph in general that represents any function  $g$  in the family –  $(x, y): x \in \mathbb{R}$  and  $0 \leq x \leq 2$  and  $n \in \mathbb{J}^+$  and  $y = 4xn + 5$  ; draw a line that is tangent to the graph for function  $g$  at the point where  $x = t$  ; state the coordinates in general for the point of tangency and for the  $y$ -intercept of the line.

In this lesson, the student were to take turns adding steps to problems at the board while analyzing and thinking mathematically, by speaking loudly, and by writing clearly. The role of the teacher was to monitor and to assist in order to help students connect their own responses in earlier lessons to possible contributions in this lesson. An important feature of this lesson was the manner in which students were able to distinguish between meaningful contributions and less-than-meaningful contributions. Statements that either explained a method or justified a response were meaningful. Likewise, statements that either validated or challenged a contribution were meaningful. Even incorrect statements could be meaningful provided that they initiated an exchange of mathematical ideas.

Teacher: Simply saying, “I don’t know,” is not a meaningful contribution.

Student 29: What about “I don’t know blank.” and you say something mathematical. Is that meaningful enough?

Teacher: Yes, those are all meaningful enough if you fill in the blank with something specific.

Another important feature of this lesson was the way in which students maintained the formality of the class discussion. The teacher directed that only one person speak at a time with points taken off for interrupting the speaker. Students took turns at the board to contribute pieces and parts of the complete response that they learned in the previous lessons. Some students spoke as they wrote; others wrote in silence and then read the contribution aloud. While their mathematical language had some flaws, students spoke with authority. At times students’ contributions reverted to old habits with comments and language that avoided standard mathematical language. Sometimes informal questions were directed at the teacher rather than authoritative statements directed at the class. The teacher monitored the discussion and interpreted any lapse into old habits as an indicator of stress. Instead of reverting to Strategy 1, a teacher dominated lesson, the teacher assisted by reminding students to transform their questions into statements.

Student 33: [plotting and labeling the point at  $(0,5)$ ] I’ll do zero, five.

Student 34: [directed at teacher] How did she know that?

Teacher: We have a challenger; (Student 33) made a statement without justification and (Student 34) wants justification. Don’t ask me, ask her.

Student 34: [directed at (Student 33)] How did you know that?

Student 33: It says *it* on the paper. Wait, I can’t say *it*.

Teacher: That’s terrific. You know what not to say. That’s progress. Now look for



words.

Student 33:[pausing to formulate a response] It says  $x$  is a real number and  $x$  is greater than or equal to zero. Can I just say that? What about  $y$ ?

Teacher: Turn your questions into a statement. Try to use the word *because*.

Student 33: [pausing to formulate a response] The endpoint must be  $(0, 5)$  because the smallest  $x$  is zero ... and  $0^n$  is always zero, so  $g(0)$  is five.

Teacher: That makes sense to me; you plotted and labeled an endpoint, that's one of the essential features of a complete graph.

In lesson 4 students made significant progress. They produced a generalized graph for function  $g$ , plotted and labeled endpoints, plotted and labeled the point of tangency, drew the tangent line and began to discuss an equation for the tangent line. While most students still did not speak with authority or clarity, a few students, however, made somewhat formal summary statements. For example;

Student 37: [Writing  $y - (4t^n + 5) = (4nt^{n-1})(x - t)$  on the chalk board and read it]I

did more with  $g$  prime. I worked *it* all out and *it's*.. [Writing  $\lim_{h \rightarrow 0} \frac{g(t+h) - g(t)}{(t+h) - (t)} = 4nt^{n-1}$  on the chalkboard] Does *it* always work this way?

Teacher: That's the kind of question that mathematicians ask. We'll see. Can you tell us what *it* you are talking about?

The student justified his contribution step by step, referring to his conclusion as a *short cut* for his analytical work. With minor adjustments, of course, his *short cut* would be the Power Rule; a significant accomplishment for a high school student.

Student 37: Can I use the *short cut* on the test instead of writing out all these steps?

Teacher: Logical *short cuts*, like yours, are what mathematicians call theorems. So, if you state your short cut clearly and explicitly (by) using standard mathematical vocabulary and notation, then you'll have a theorem. You may always use theorems on tests.

As the lesson proceeded, students determined the  $y$ -intercept for the tangent line in general. Some students, feeling overwhelmed by the breadth and depth of the discussion, lost their focus. The teacher assisted by reassuring students that the important mathematical ideas would make sense in due time and by reassuring the class as a whole that they were making commendable progress even if they did not understand every detail yet. A few students seemed to become empowered and convinced about the meaningfulness of their classroom discourses. Most others remained either passive resistors or active opponents; participation points were a small reward for their discomfort.

#### IV. DISCUSSION

Students' verbal and written communication and discourse should not be underestimated. Communication and classroom discourse fulfill three broad and interlocking goals for learning, teaching and assessment. First, as students communicate their mathematical thinking and reasoning they become observers of themselves. They make invisible mathematical solutions more clear and visible to themselves and to their peers. That is called metacognition (i.e. thinking about thinking). In addition, as they explain their thinking and problem solving to their peers, they become teachers in the

classroom. They become more confident in their abilities to do significant mathematics. In this sense, they become more empowered mathematically (NCTM, 2000).

Second, students' verbal and written communication helps their classroom teachers to understand students' understanding. Therefore, students' communication and classroom discourse not only enhances student learning but also it inform teacher's instructional decision making. Third, classroom communication and discourse are powerful tools for Teacher to assess students' learning and can create a safe environment for risk taking, exploring ideas, and genuine dialogue. Furthermore, it may involve parents regarding their children's education build a stronger communication between the classroom teacher and parents(Tsuruda, 1994).

This study examined four pedagogical strategies that a secondary mathematics classroom teacher used to engage students in mathematical discourse as part of an inquiry-based mathematics classroom. The findings of this study suggest that mathematical discourse can promote mathematical understanding among secondary school students. First, the study finds that the discursive teaching strategies used by the classroom teacher are mainly responsible for transforming mathematics discourse. While the lessons seemed time consuming and repetitive, students not only learned about tangent lines but also how to use formal language skills to clarify their thinking and communicate important mathematical ideas. Most students came to prefer Strategies 2 and 3 thus losing their dependency on Strategy 1 in which the teacher played a dominant role during the discursive process. As their confidence grew, their resistance to Strategy 4 diminished slowly but did not dissipate completely as the students still needed time to accept a classroom culture that differed significantly from their prior experiences. Over time, the teacher can minimally engage in the discourse by opting for more of a student-led discourse and acting solely as a guardian and facilitator of the process.

This study also found that students' attitudes towards discourse can change over time. At the onset, all of the participating students were more comfortable with conventional instruction, most in which the teacher demonstrates mathematical procedures and students practice the procedures (Kawanaka & Stigler, 1999). After being immersed in the classroom setting for the better part of a school year, most of the participating students in this study become more open to inquiry-based instruction as indicated by one of the students Dan;

"I learned that there's more than one way to learn math. In the past..., we've gone over a section and then you'd do the homework for that section that night and the process repeats everyday without any change. In this new way that I've been learning, it is not with homework but with group discussions and taking notes. It varies. That's a good way to learn it because it keeps you interested more than it would if you just did the same thing over and over again."

Students, over time, show increasing appreciation for mathematical discourse.As noted by another student;

"I get frustrated when I can't do math, when I don't know what to say. But when I am able to communicate my thinking using mathematical vocabulary, I think that's cool. That's an accomplishment. So, I won't say that this class was a waste of time. Ask me next year, after college [laughing]. Because, maybe I'll go to college and found out I learned a lot. I don't see that happening but it's possible".

One of the challenges of creating a meaningful classroom communication and discourse is change in epistemology by teachers as well as students. Teacher's view on how students learn mathematics and their role in the classroom is crucial for creating learning opportunities for all students. Most teachers teach the way they were taught (i.e. direct instruction). In the U.S. most teachers see their role as dispenser of knowledge and students' role as passive recipients of the knowledge that imposed to them. Another challenge is students' view of learning and knowing and their role in mathematics classroom. Most students have not experienced learning as active construction of meaning. Discrepancy between teacher's view and students' view on learning may create a dilemma in the classroom. These epistemological differences may impact students' participation in and contribution to the classroom activities.

g) *Concluding remarks*

"[T]he most critical shift in education in the past 20 years has been a move away from a conception of 'learner as sponge' toward an image of 'learner as active constructor of meaning'" (Wilson & Peterson, 2006, p. 2). For several decades educational practices were significantly influenced by behaviorist perspective. American classrooms still largely exemplify the behaviorist approach. As such, lessons are very procedure-oriented. During a typical U.S. mathematics lesson, students can expect to spend half of the class reviewing content they have already learned. The remainder of class time is normally split between introducing new material and practicing it (Hiebert et al, 2003; Kilpatrick, Martin, & Schifter, 2003).

This behavioral perspective has been challenged by current research on how students learn mathematics. It is a shift that pulls away from dominant educational practices rooted in behaviorism toward practices reflective of constructivism and socio cultural perspectives. These perspectives are largely accepted by mathematics education community such as NCTM. According to constructivist view, knowing and learning is constructed by individuals as they participate in and contribute to the classroom activities (Cobb, 1994). Ways of knowing and understanding differs from individual to individual even in the same sociocultural situation. Constructivism sees the role of teacher as a facilitator of the process of learning. It argues that in the classroom mathematical activity, as the individual student is involved in problem solving, he/she may be faced with conflicting situations. The resolution of this perturbation helps the student to reorganize his/her thought (cognition).

Another dimension of learning is the notion culture. Mathematics learning is influenced by social and cultural situations. One of the important roles of a teacher is to mediate between students' personal mathematical meaning and wider sociocultural norms of the society. The coordination of constructivism and sociocultural perspectives are pivotal for creating learning opportunities for all students.

I contend that the two perspectives [constructivism and sociocultural] are complementary...I argue that the sociocultural perspective informs theories of the conditions for the possibility of learning, whereas theories develop from the constructivist perspective focus on what students learn and the processes by which they do so, (Cobb, 1994, p.13)

Although the coordinating perspective has focused on understanding student's mathematics learning. It may offer insights as to how teachers may collect authentic data and analyze data relative to students' understanding of mathematical concept. In

particular, the coordinating perspective may provide teacher a viable window to look at how students make sense of their mathematical activities.

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