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Robustness of the Sequential Test for the Scale Parameter of Nakagami Distribution

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Keywords: *nakagami distribution, SPRT, OC and ASN functions, robustness.*

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Robustness of the Sequential Test for the Scale Parameter of Nakagami Distribution

Surinder Kumar [α] & Mayank Vaish [σ]

Abstract- In the present study, Sequential Probability Ratio Test (SPRT) is developed for the scale parameter of Nakagami Distribution and the robustness of scale parameter is studied when the shape parameter has undergone a change, for testing the hypothesis regarding the parameter of Nakagami Distribution. The expression for the Operating Characteristic (OC) and Average Sample Number (ASN) functions are derived and the results are presented through Graphs and Tables.

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I. Introduction

Nakagami distribution is a lifetime distribution, given by M. Nakagami (1960) has the probability density function (pdf)

$$f(x;\lambda,\beta)=\frac{2\lambda^{\lambda}}{\Gamma\lambda\,\beta^{\lambda}}x^{2\lambda-1}e^{-\frac{\lambda}{\beta}x^{2}}\;;\qquad x>0,\ \lambda,\beta>0\,, \qquad \ldots(1.1)$$

where λ is a shape parameter and β is scale parameter. Nakagami distribution is related to Rayleigh distribution and one-sided Gaussian distribution when $\lambda=1$, and $\lambda=1/2$, respectively.

In this paper, we have developed the SPRT for scale parameter of Nakagami distribution and studied the robustness of the scale parameter when there is change in the shape parameter. The robustness of SPRT has been studied by several authors. For a brief review, one may refer to Epstein and Sobel (1955), Johnson(1966), Barlow and Proschan (1967), Phatarfod (1971), Harter and Moore (1976), Montagne and Singpurwalla (1985), Joshi and Shah(1990), Chaturvedi, kumar and Kumar (1998), Chaturvedi, Kumar and Kumar (2000), Chaturvedi, Tiwari and Tomer (2002), Surinder and Naresh (2009).

In section 2, we state the problem, and develop SPRT for testing the simple null hypothesis against the simple alternative hypothesis. The expressions for OC and ASN functions are obtained in section 3. In section 4, robustness of the SPRT is studied and the results are discussed in section 5. Finally, the conclusions are given in section 6.

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II. Set-up of the Problem

Let the random variable X follows the Nakagami distribution given by the probability density function (pdf)

$$f(x;\lambda,\beta)=\frac{2\lambda^{\lambda}}{\Gamma\lambda\,\beta^{\lambda}}x^{2\lambda-1}e^{-\frac{\lambda}{\beta}x^{2}}\quad;\qquad x>0,\ \lambda,\beta>0 \qquad ...(2.1)$$

Given a sequence of observations x_1, x_2, x_3,...from (2.1), suppose one wish to test the simple null hypothesis $H_0:\beta=\beta_0$ against the simple alternative $H_1:\beta=\beta_1(\beta_1>\beta_0)$. The expression for OC and ASN function is obtained and their behaviour is studied by plotting graph.

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III. SPRT for Testing the Hypothesis Regarding 'β'

The SPRT for testing the null hypothesis $H_0:\beta=\beta_0$ against the simple alternative $H_1:\beta=\beta_1$ is defined as

$$z_i=\ln\frac{f(x_i;\lambda,\beta_1)}{f(x_i;\lambda,\beta_0)} \qquad ...(3.1)$$

Or,

$$z_i=\lambda\ln\left(\frac{\beta_0}{\beta_1}\right)-x^2\lambda\left(\frac{1}{\beta_1}-\frac{1}{\beta_0}\right) \qquad ...(3.2)$$

Or,

$$e^{z_i}=\left(\frac{\beta_0}{\beta_1}\right)^{\lambda}e^{-\lambda x^2\left(\frac{1}{\beta_1}-\frac{1}{\beta_0}\right)} \qquad ...(3.3)$$

Now, we choose two numbers A and B such that $0<B<1<A$. At the n^{th} stage, accept H_0, if $\sum_{i=1}^{n}z_i\le\ln B$, reject H_0 if $\sum_{i=1}^{n}z_i\le\ln A$, otherwise continue sampling by taking the $(n+1)^{th}$ observation.

If $\alpha\in(0,1)$ and $\beta\in(0,1)$ are TYPE I and TYPE II errors respectively, then according to Wald (1947), A and B are approximately given by

$$A\approx\frac{1-\beta}{\alpha}\ \text{and}\ B\approx\frac{\beta}{1-\alpha} \qquad ...(3.4)$$

The Operating Characteristic (OC) function $L(\theta)$ is given by

$$L(\theta)=\frac{A^{h}-1}{A^{h}-B^{h}}, \qquad ...(3.5)$$

where h is the non-zero solution of

$$\mathrm{E}\left[e^{hz}\right]=1 \qquad ...(3.6)$$

Or,

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$$\int_0^\infty \left[\frac{f(x_i;\lambda,\beta_1)}{f(x_i;\lambda,\beta_0)}\right]^h f(x_i;\lambda,\beta)\,dx = 1 \qquad ...(3.7)$$

From (2.1) and (3.3), since

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$$E\left[e^z\right]^h = \frac{\left(\frac{\beta_0}{\beta_1}\right)^{\lambda h}}{\left[1+\beta h\left(\frac{1}{\beta_1}-\frac{1}{\beta_0}\right)\right]^{\lambda}}, \qquad ...(3.8)$$

we get from (3.6) that

$$\beta = \frac{1-\left(\frac{\beta_0}{\beta_1}\right)^h}{h\left(\frac{1}{\beta_1}-\frac{1}{\beta_0}\right)} \qquad ...(3.9)$$

The ASN function is approximately given by

$$E(N\,|\,\theta) = \frac{L(\theta)\ln B + \{1-L(\theta)\}\ln A}{E(Z)}\;, \qquad ...(3.10)$$

Provided $E(Z)\neq 0$, where

$$E(Z) = \lambda\left[\ln\left(\frac{\beta_0}{\beta_1}\right) - \beta\left(\frac{1}{\beta_1}-\frac{1}{\beta_0}\right)\right] \qquad ...(3.11)$$

From (3.11) ASN function under H_0 and H_1 are given by

$$E_0(N) = \frac{(1-\alpha)\ln B + \alpha \ln A}{\lambda\left[\ln\left(\frac{\beta_0}{\beta_1}\right) - \beta\left(\frac{1}{\beta_1}-\frac{1}{\beta_0}\right)\right]} \qquad ...(3.12)$$

and

$$E_1(N) = \frac{\beta\ln B + (1-\beta)\ln A}{\lambda\left[\ln\left(\frac{\beta_0}{\beta_1}\right) - \beta\left(\frac{1}{\beta_1}-\frac{1}{\beta_0}\right)\right]} \qquad ...(3.13)$$

IV. Robustness of SPRT for Parameter of Nakagami Distribution

Let us suppose that the parameter 'λ' has undergone a change then the probability distribution in (2.1) becomes $f(x;\lambda^*,\beta)$. To study the robustness of SPRT developed in section 3 with respect to OC function, consider 'h' as the solution of the equation

$$E_{\lambda^*}\left[e^z\right]^h = 1 \qquad ...(4.1)$$

Or,
$$\int_0^\infty \left[\frac{f(x_i;\lambda,\beta_1)}{f(x_i;\lambda,\beta_0)}\right]^h f(x_i;\lambda^*,\beta)\,dx = 1 .$$

After solving, we get

$$\beta = \frac{1-\left(\frac{\beta_0}{\beta_1}\right)^{\left(\frac{\lambda}{\lambda^*}\right)h}}{h\frac{\lambda}{\lambda^*}\left(\frac{1}{\beta_1}-\frac{1}{\beta_0}\right)} \qquad ...(4.2)$$

For different values of β, h is evaluated and the OC function is obtained. The Robustness of SPRT with respect to ASN can be studied by replacing denominator of (3.10) by

$$E_{\lambda^*}(z) = \int_0^\infty z\, f(x;\lambda^*,\beta)\,dx$$

$$= \lambda\left[\ln\left(\frac{\beta_0}{\beta_1}\right) - \beta\left(\frac{1}{\beta_1}-\frac{1}{\beta_0}\right)\right] \qquad ...(4.3)$$

We consider the cases $\lambda > \lambda^*$ and $\lambda < \lambda^*$ to study the robustness of SPRT.

V. Results and Discussions

Consider the equation (4.2) and taking the logarithms of both sides, expanding and retaining the terms up to third degree in 'h', we get

$$\left\{\beta^3 P^3\left(\frac{1}{\beta_1}-\frac{1}{\beta_0}\right)^3\right\}\frac{h^2}{3} - \left\{\beta^2 P^2\left(\frac{1}{\beta_1}-\frac{1}{\beta_0}\right)^2\right\}\frac{h}{2} + \left\{\beta\left(\frac{1}{\beta_1}-\frac{1}{\beta_0}\right) - \ln\left(\frac{\beta_0}{\beta_1}\right)\right\}P = 0 \qquad ...(5.1)$$

where $P = \frac{\lambda}{\lambda^*}$,

For testing $H_0:\beta = 13$ verses $H_1:\beta = 15$ and taking $\alpha = \beta = 0.05$, the real roots of 'h' are obtained by using (5.1) for the different values of β. The OC and ASN functions are evaluated by using the equations (3.5) and (3.10) by considering the cases $\lambda > \lambda^*$, $\lambda = \lambda^*$ and $\lambda < \lambda^*$ respectively. The results are presented in *Table 1.* and *Table 2,* respectively. The graph for OC and ASN functions are plotted in *Fig.1* and *Fig.2*, respectively.

Table1 : Values of OC Function for Scale Parameter of Nakagami Distribution

	L(β)		
β	**P=0.5**	**P=1**	**P=1.5**
12.8	0.999316	0.97451	0.919019
12.9	0.998652	0.964562	0.900481
13	**0.997371**	**0.951168**	**0.878632**
13.1	0.994931	0.933375	0.853186
13.2	0.990337	0.910101	0.823936
13.3	0.98182	0.880222	0.790789
13.4	0.966332	0.842704	0.753801
13.5	0.938946	0.796814	0.713205
13.6	0.892517	0.742377	0.669422
13.7	0.818716	0.680014	0.62306

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13.8	0.71206	0.611282	0.574883
13.9	0.57676	0.538609	0.525768
14	0.430317	0.464988	0.476637
14.1	0.296256	0.393507	0.428397
14.2	0.190791	0.326856	0.381868
14.3	0.117107	0.266968	0.337742
14.4	0.069694	0.214889	0.296546
14.5	0.040728	0.170848	0.258633
14.6	0.023563	0.134457	0.224191
14.7	0.013561	0.104946	0.193261
14.8	0.007784	0.081368	0.165766
14.9	0.004462	0.062748	0.141541
15	**0.002555**	**0.048174**	**0.120358**
15.1	0.001461	0.036846	0.101959
15.2	0.000834	0.028087	0.086067

Figure 1 : Graph of OC Function for Nakagami Distribution

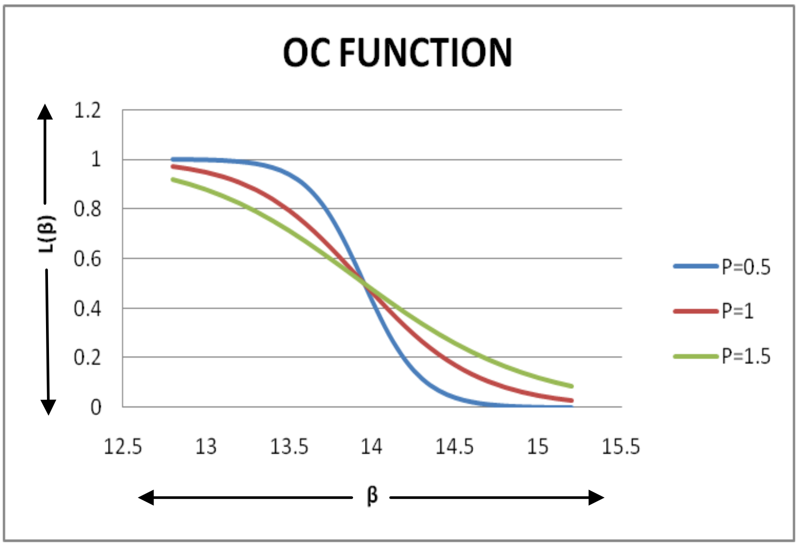
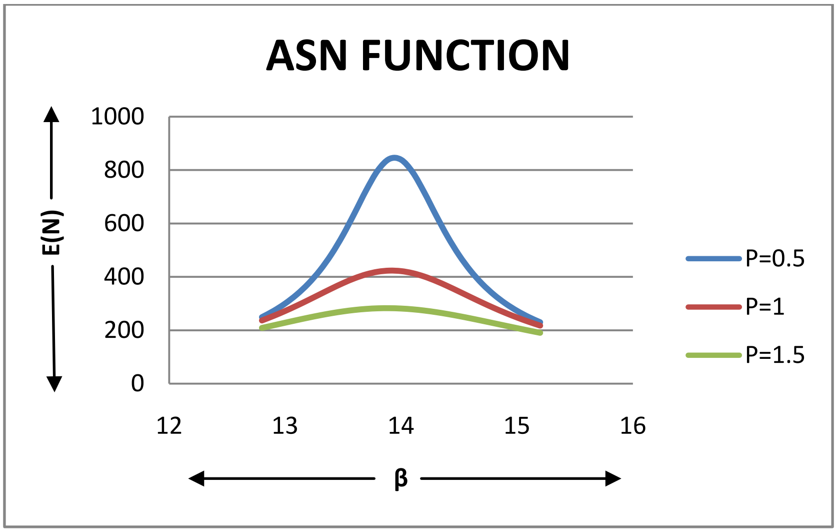


Table 2 : Values of ASN Function for Scale Parameter of Nakagami Distribution

	E(N)		
β	**P=0.5**	**P=1**	**P=1.5**
12.8	248.791298	236.431136	208.782162
12.9	272.070729	253.470573	218.507593
13	**299.867491**	**272.011541**	**228.279256**
13.1	333.40532	291.938749	237.9207
13.2	374.215811	312.981253	247.221902
13.3	424.084893	334.661168	255.944863
13.4	484.766036	356.250404	263.833837
13.5	557.173848	376.759353	270.630404
13.6	639.65168	394.980317	276.092553
13.7	725.216594	409.609487	280.01483
13.8	799.289775	419.441269	282.247663
13.9	842.175927	423.598729	282.7132
14	839.340213	421.72793	281.407782
14.1	792.201008	414.068829	278.409715
14.2	716.835926	401.397756	273.86376
14.3	632.339526	384.846574	267.964957
14.4	551.898184	365.674585	260.944299
14.5	481.492999	345.077459	253.045105
14.6	422.367225	324.058467	244.508307
14.7	373.556718	303.37858	235.557736
14.8	333.40091	283.559294	226.392386

14.9	300.232638	264.918963	217.180598
15	**272.620552**	**247.619408**	**208.059374**
15.1	249.413412	231.71082	199.13543
15.2	229.711533	217.169974	190.488238

Figure 2 : Graph of ASN Function for Nakagami Distribution



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VI. Conclusions

The values of OC and ASN functions for the cases $\lambda<\lambda^*$, $\lambda=\lambda^*$ and $\lambda>\lambda^*$ are plotted in *Fig.1* and *Fig.2,* respectively. From the *Fig.1,* we observe that for $\lambda<\lambda^*$ ($\lambda>\lambda^*$), the OC curve shifts to the right side (left side) of the curve when $\lambda=\lambda^*$. From the *Fig.1,* it is clear that SPRT is non-robust for $\lambda^*=\lambda\pm0.5$ as the deviation in OC function is significant. Again, from *Fig.2,* we observe that for $\lambda<\lambda^*$ ($\lambda>\lambda^*$), the ASN curve shifts above (below) of the curve when $\lambda=\lambda^*$. Both the curves are highly sensitive for the changes in λ. Thus we conclude that for the present model, the SPRT for testing the hypothesis regarding β, is highly non-robust for changes in λ.

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