Generalized Class of Exponential Chain Ratio-Cum-Chain Product Type Estimator for Finite Population Mean under Double Sampling Scheme in Presence of Non-Response

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Keywords: exponential, chain, estimator, mean, bias, mean square error.

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Yater Tato [α] & B. K. Singh [σ]

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I. Introduction

Using auxiliary information in proposing selection procedure and the estimators for the population parameters was initiated by Bowely (1926), Neyman (1934, 1938), Watson (1937), Cochran (1940, 1942), Hansen et al. (1953) and Robson (1957).If the population mean $\overline{X}$ of the auxiliary variable x is not known but the population mean $\overline{Z}$ of an additional auxiliary variable z is known which is less correlated to the study variable y in comparison to the main auxiliary variable x (i.e. $\rho_{yx} > \rho_{yz}, \rho_{yx}, \rho_{yx} > 0$), then in such case Chand (1975), Kiregyera (1980, 1984) and Srivastava et al. (1990) proposed chain ratio type estimators using additional auxiliary variable with its known population mean.

Sometimes, it may not be possible to collect the complete information for all the units selected in the sample due to non-response. The missing observations due to non-response may occur during the investigation, which may be at random, and the ignorance of such missing observations may lead to biased estimator, though the amount of the bias may be very negligible. If the missing observation due to non-response is not at random then the amount of bias in the estimator will be large and may increase the error in the estimation, and the sampling error will also increase.

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Little and Rubin (2002) suggested to ignore the missing data completely if the percentage of incomplete cases are very low. This practice will reduce the sample size and may increase the bias and the variance of the estimator when the incomplete cases are large. However, some imputation techniques to replace the missing observation are considered by Rao and Toutenburg (1995) and Toutenburg and Srivastava (1998, 2003). Estimation of the population mean $\overline{Y}$ in sample surveys when some observations are missing due to non-response not at random was considered by Hansen and Hurwitz (1946), Rao (1986, 1987), Khare and Srivastava (1993, 1995).

Let $\overline{Y}$, $\overline{X}$ and $\overline{Z}$ be the population means of study character y, auxiliary character x and additional auxiliary character z. Let a finite population of size N is divided into N_1 responding units and N_2 not responding units and $W_1 = \dfrac{N_1}{N}$, $W_2 = \dfrac{N_2}{N}$ are their corresponding weights. According to Hansen and Hurwitz a sample of size n is taken from population of size N by using simple random sampling without replacement (SRSWOR) scheme of sampling and it has been observed that n_1 units respond and n_2 units do not respond. Again from the n_2 non-respondents, a sub sample of size $m = \left(n_2 f^{-1} \right)$ is drawn from n_2 non-responding units and information is collected on m units for study character y. Hence, the estimator for $\overline{Y}$ based on $(n_1 + m)$ units on study character y is given by:

$$\overline{y}^* = w_1 \overline{y}_1 + w_2 \overline{y}_{2m}$$

where $w_1 = \dfrac{n_1}{n}$, $w_2 = \dfrac{n_2}{n}$; $\overline{y}_1$ and $\overline{y}_{2m}$ denote the sample means of variable y based on n_1 and m units respectively. The estimators $\overline{y}^*$ is unbiased and has variance

$$V(\overline{y}^*) = \lambda S_y^2 + \lambda' S_{2y}^2$$

(1)

where $\lambda = \left(\dfrac{1}{n} - \dfrac{1}{N} \right)$, $\lambda' = \dfrac{W_2(k-1)}{n}$, $W_2 = \dfrac{N_2}{N}$;

$$S_y^2 = \frac{1}{N-1} \sum_{i=1}^{N} (y_i - \overline{Y})^2 \quad \text{and} \quad S_{2y}^2 = \frac{1}{N_2-1} \sum_{i=1}^{N_2} (y_i - \overline{Y}_2)^2 \quad \text{are population mean squares}$$

of y for entire population and non-responding part of the population.

Similarly, the estimator $\overline{x}^*$ for population mean $\overline{X}$ in the presence of non-response based on corresponding $(n_1 + m)$ units is given by

$$\overline{x}^* = w_1 \overline{x}_1 + w_2 \overline{x}_{2m}$$

where $\overline{x}_1$ and $\overline{x}_{2m}$ denote the sample means of variable x based on n_1 and m units respectively. We have

$$V(\overline{x}^*) = \lambda S_x^2 + \lambda' S_{2x}^2$$

where

$$S_x^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \overline{X})^2 \quad \text{and} \quad S_{2x}^2 = \frac{1}{N_2-1} \sum_{i=1}^{N_2} (x_i - \overline{X})^2 \quad \text{are population mean squares of } x$$

for the entire population and non-responding part of the population.

In case, when population mean $\bar{X}$ is not known, then, it is estimated by taking a preliminary sample of size $n'\,(n' < N)$ from the population of size N by using simple random sampling without replacement (SRSWOR) method of sampling. In this situation, Khare and Srivastava (1995) proposed conventional T_1 and alternative T_2 two phase sampling ratio estimators for population mean $\bar{Y}$ in the two different cases of non-response, i.e. when there is non-response on both the study variable y as well as on the auxiliary variable x and when there is non-response in the study variable y only, which are given as follows:

$$T_1 = \bar{y}^* \frac{\bar{x}'}{\bar{x}^*}$$

$$T_2 = \bar{y}^* \frac{\bar{x}'}{\bar{x}}$$

where $\bar{x} = \dfrac{1}{n}\sum_{i=1}^{n} x_i$, $\bar{x}' = \dfrac{1}{n'}\sum_{i=1}^{n'} x_i$.

The bias and MSE of T_1 and T_2 are given respectively as

$$B(T_1) = \frac{\lambda}{\bar{X}}(RS_x^2 - S_{xy}) + \frac{\lambda'}{\bar{X}}(RS_{2x}^2 - S_{2xy}) \qquad MSE(T_1) = \lambda S_d^2 + \lambda' S_{2d}^2 ,$$

$$B(T_2) = \frac{\lambda}{\bar{X}}(RS_x^2 - S_{xy})$$

$$MSE(T_2) = \lambda S_d^2 + \lambda' S_{2y}^2 .$$

where $R = \dfrac{\bar{Y}}{\bar{X}}$ is the population ratio of $\bar{Y}$ to $\bar{X}$,

$$S_d^2 = S_y^2 - 2RS_{xy} + R^2 S_x^2 ,$$

$$S_{2d}^2 = S_{2y}^2 - 2RS_{2xy} + R^2 S_{2x}^2$$

S_{xy} , S_{2xy} are the covariances for the whole population and the population of non-respondents respectively.

Now, when population mean $\bar{X}$ is not known, but the population mean $\bar{Z}$ of the additional auxiliary variable z closely related to x but compared to x remotely related to y i.e. $\rho_{yx} > \rho_{yz}$ is known, then we take a preliminary sample of size $n'\,(n' < N)$ from the population of size N with SRSWOR scheme and estimate the population mean $\bar{X}$ by using the sample means $\bar{x}'$ and $\bar{z}'$ based on n' units and the known additional population mean $\bar{Z}$. We observe that $\bar{X} = \dfrac{\bar{x}'}{\bar{z}'}\bar{Z}$ is more precise than preliminary sample mean $\bar{x}'$ if $\rho_{xz} > \dfrac{1}{2}\dfrac{C_x}{C_z}$, where $\bar{z}' = \dfrac{1}{n'}\sum_{i=1}^{n'} z_i$. Using available information on two auxiliary variables x

and z, Khare et al (2012) proposed chain ratio type estimators in the presence of non-response given as follows:

$$t_1 = \frac{\bar{y}^*}{\bar{x}^*} \frac{\bar{x}'}{\bar{z}'} \bar{Z}$$

$$t_2 = \frac{\bar{y}^*}{\bar{x}} \frac{\bar{x}'}{\bar{z}'} \bar{Z}$$

The MSE of t_1 is given by

$$MSE(t_1) = \bar{Y}^2 \left[\lambda C_y^2 + \lambda' C_{2y}^2 + B' - 2A' \right] \tag{2}$$

Where

$$A' = \left(\frac{1}{n} - \frac{1}{n'} \right) k_{yx} C_x^2 + \left(\frac{1}{n'} - \frac{1}{N} \right) k_{yz} C_z^2 + \lambda' k_{2yx} C_{2x}^2$$

and

$$B' = \left(\frac{1}{n} - \frac{1}{n'} \right) C_x^2 + \left(\frac{1}{n'} - \frac{1}{N} \right) C_z^2 + \lambda' C_{2x}^2$$

$$k_{yx} = \rho_{yx} C_y / C_x , \quad k_{yz} = \rho_{yz} C_y / C_z ,$$

$$k_{2yx} = \rho_{2yx} C_{2y} / C_{2x} \text{ and } k_{2yz} = \rho_{2yz} C_{2y} / C_{2z}$$

The MSE of t_2 is given by,

$$MSE(t_2) = \bar{Y}^2 \left[\lambda C_y^2 + \lambda' C_{2y}^2 + B - 2A \right] \tag{3}$$

where

$$A = \left(\frac{1}{n} - \frac{1}{n'} \right) k_{yx} C_x^2 + \left(\frac{1}{n'} - \frac{1}{N} \right) k_{yz} C_z^2 ,$$

$$B = \left(\frac{1}{n} - \frac{1}{n'} \right) C_x^2 + \left(\frac{1}{n'} - \frac{1}{N} \right) C_z^2 ,$$

Bahl and Tuteja (1991) introduced an exponential ratio-type and exponential product-type estimators for population mean as

$$\bar{y}_{er} = \bar{y} \exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right]$$

$$\bar{y}_{ep} = \bar{y} \exp\left[\frac{\bar{x} - \bar{X}}{\bar{X} + \bar{x}} \right]$$

Singh and Choudhury (2012) suggested the exponential chain ratio and product type estimators for $\bar{Y}$ in double sampling respectively as

Notes

$$\overline{y}_{eR}^{dc} = \overline{y}\exp\left(\frac{\overline{x}'\dfrac{\overline{Z}}{\overline{z}'} - \overline{x}}{\overline{x}'\dfrac{\overline{Z}}{\overline{z}'} + \overline{x}}\right) \tag{4}$$

and

$$\overline{y}_{eP}^{dc} = \overline{y}\,\exp\left(\frac{\overline{x} - \overline{x}'\dfrac{\overline{Z}}{\overline{z}'}}{\overline{x} + \overline{x}'\dfrac{\overline{Z}}{\overline{z}'}}\right) \tag{5}$$

Motivated by Singh and Choudhury (2012) and Khare et al (2012), we suggest a generalized class of exponential chain ratio-cum-chain product type estimator in presence of non-response. The expressions for bias and mean square error of the proposed estimator have been obtained. Comparative studies of the proposed estimator have been made with the other relevant estimators and an empirical study has been given to illustrate its efficiency.

II. The Proposed Estimator

Utilizing information on the auxiliary variables x and z, using a scalar quantity α, we suggest a generalized class of exponential chain ratio-cum-chain product type estimator for the population mean $\overline{Y}$ in two different cases of non-response, which is given as follows.

$$\overline{y}_{eRP}^{dc} = \overline{y}\left[\alpha\exp\left(\frac{\overline{x}'\dfrac{\overline{Z}}{\overline{z}'} - \overline{x}}{\overline{x}'\dfrac{\overline{Z}}{\overline{z}'} + \overline{x}}\right)\right.$$

$$\left.+ (1-\alpha)\exp\left(\frac{\overline{x} - \overline{x}'\dfrac{\overline{Z}}{\overline{z}'}}{\overline{x} + \overline{x}'\dfrac{\overline{Z}}{\overline{z}'}}\right)\right] \tag{6}$$

The proposed estimator will be studied in two cases of non-response.
Case I: When non-response occurs only on y.
Case II: When non-response occurs both on y and x.

III. Case I: Non Response only on y

The estimator is

$$\overline{y}_{eRP}^{dc*} = \overline{y}^*\left[\alpha_1\exp\left(\frac{\overline{x}'\dfrac{\overline{Z}}{\overline{z}'} - \overline{x}}{\overline{x}'\dfrac{\overline{Z}}{\overline{z}'} + \overline{x}}\right)\right.$$

$$+\left(1-\alpha_1\right)\exp\left[\frac{\bar{x}-\bar{x}'\dfrac{\overline{Z}}{\overline{z}'}}{\bar{x}+\bar{x}'\dfrac{\overline{Z}}{\overline{z}'}}\right] \tag{7}$$

where α_1 is a scalar constant.

In order to obtain the expressions for the bias and MSE of $\overline{y}_{eRP}^{dc*}$, let

$$\overline{y}^* = \overline{Y}\left(1+e_0^*\right), \quad \bar{x} = \overline{X}\left(1+e_1\right),$$

$\bar{x}' = \overline{X}\left(1+e_2\right)$ and $\overline{z}' = \overline{Z}\left(1+e_3\right)$ such that

$$E(e_0^*) = E(e_1) = E(e_2) = E(e_3) = 0.$$

$$E(e_0^{*2}) = \frac{V\left(\overline{y}^*\right)}{\overline{Y}^2}, = \frac{1}{\overline{Y}^2}\left(\lambda S_y^2 + \lambda' S_{2y}^2\right)$$

$$E(e_0^* e_1) = \frac{\mathrm{cov}\left(\overline{y}^*, \bar{x}\right)}{\overline{YX}} = \frac{\lambda S_{xy}}{\overline{YX}}.$$

$$E(e_1^2) = \frac{V\left(\bar{x}\right)}{\overline{X}^2} = \frac{\lambda S_x^2}{\overline{X}^2}, \tag{8}$$

Expressing $\overline{y}_{eRP}^{dc*}$ in terms of e's, we obtain

$$\overline{y}_{eRP}^{dc*} = \overline{Y}\left(1+e_0^*\right)\Big[\alpha_1 \exp\Big\{\left(1+e_2\right)\left(1+e_3\right)^{-1}$$

$$-\left(1+e_1\right)\Big\{\left(1+e_2\right)\left(1+e_3\right)^{-1}+\left(1+e_1\right)\Big\}^{-1}\Big\}$$

$$+\left(1-\alpha_1\right)\exp\Big\{\left(1+e_1\right)-\left(1+e_2\right)\left(1+e_3\right)^{-1}$$

$$\Big\{\left(1+e_1\right)+\left(1+e_2\right)\left(1+e_3\right)^{-1}\Big\}^{-1}\Big\}\Big]$$

Expanding the right hand side of the above equation and retaining terms of e's up to second degree, we get

$$\overline{y}_{eRP}^{dc*} - \overline{Y} = \overline{Y}\bigg[e_0^* + \frac{1}{2}\Big\{e_1 - e_2 + e_3 + e_0^* e_1 - e_0^* e_2 + e_0^* e_3$$

$$-\frac{e_1^2}{4} + \frac{3e_2^2}{4} - \frac{e_3^2}{4} - \frac{e_1 e_2}{2} - \frac{e_2 e_3}{4} + \frac{e_1 e_3}{4}\Big\}$$

$$+\alpha_1\Big\{-e_1 + e_2 - e_3 - e_0^* e_1 + e_0^* e_2 - e_0^* e_3$$

$$+\frac{e_1^2}{2} - \frac{e_2^2}{2} + \frac{e_3^2}{2}\Big\}\bigg] \tag{9}$$

The bias of the estimator $\overline{y}_{eRP}^{dc*}$ can be obtained by using the results of (8) in equation (9) as

$$B(\overline{y}_{eRP}^{dc*}) = \overline{Y}\left[\frac{A}{2} - \frac{B}{8} + \alpha_1\left(\frac{B}{2} - A\right)\right] \qquad (10)$$

Squaring both the sides of equation (9), taking expectations and using the results of (8) we get the MSE of $\overline{y}_{eRP}^{dc*}$ to the first degree of approximation as

$$MSE\left(\overline{y}_{eRP}^{dc*}\right) = \overline{Y}^2\left[\lambda C_y^2 + \lambda' C_{2y}^2 + A + \frac{B}{4}\right.$$

$$\left. -\alpha_1(2A + B) + \alpha_1^2 B\right] \qquad (11)$$

Minimization of (11) with respect to α_1 yields its optimum as

$$\alpha_1 = \frac{1}{2} + \frac{A}{B} = \alpha_{1(opt)} \qquad \text{(say)} \qquad (12)$$

Substituting the value of α_1 from (12) in (7) gives the asymptotically optimum estimator (AOE) as

$$\overline{y}_{eRP(opt)}^{dc*} = \overline{y}^*\left[\alpha_{1(opt)} \exp\left(\frac{\overline{x}'\dfrac{\overline{Z}}{\overline{z}'} - \overline{x}}{\overline{x}'\dfrac{\overline{Z}}{\overline{z}'} + \overline{x}}\right)\right.$$

$$\left. +(1 - \alpha_{1(opt)})\exp\left(\frac{\overline{x} - \overline{x}'\dfrac{\overline{Z}}{\overline{z}'}}{\overline{x} + \overline{x}'\dfrac{\overline{Z}}{\overline{z}'}}\right)\right]$$

Thus, the resulting MSE of $\overline{y}_{eRP(opt)}^{dc*}$ is given as

$$MSE(\overline{y}_{eRP(opt)}^{dc*}) = \overline{Y}^2\left[\lambda C_y^2 + \lambda' C_{2y}^2 - \frac{A^2}{B}\right] \qquad (13)$$

Remarks:

1. When $\alpha_1 = 1$, the proposed estimator $\overline{y}_{eRP}^{dc*}$ in (7) reduces to exponential chain ratio estimator $\overline{y}_{eR}^{dc*}$ when non-response occurs on y. The MSE of $\overline{y}_{eR}^{dc*}$ is obtained by putting $\alpha_1 = 1$ in (11) as

$$MSE(\overline{y}_{eR}^{dc*}) = \overline{Y}^2\left[\lambda C_y^2 + \lambda' C_{2y}^2 + \frac{B}{4} - A\right] \qquad (14)$$

2. When $\alpha_1 = 0$ the proposed estimator $\overline{y}_{eRP}^{dc*}$ reduces to the exponential chain product estimator $\overline{y}_{eP}^{dc*}$ when non-response occurs on y. The MSE of $\overline{y}_{eP}^{dc*}$ is obtained by putting $\alpha_1 = 0$ in (11) as

$$MSE(\overline{y}_{eP}^{dc*}) = \overline{Y}^2 \left[\lambda C_y^2 + \lambda' C_{2y}^2 + \frac{B}{4} + A \right] \tag{15}$$

IV. Efficiency Comparisons in Case I

a) Comparison with mean per unit estimator $\overline{y}^*$

From (1) and (13), we get

$$V\left(\overline{y}^*\right) - MSE(\overline{y}_{eRP(opt)}^{dc*}) = \overline{Y}^2 \left[\frac{A^2}{B} \right] > 0 \tag{16}$$

b) Comparison with chain ratio estimator in double sampling t_2

From (3) and (13), we get

$$MSE(t_2) - MSE\left(\overline{y}_{eRP(opt)}^{dc*}\right) = \overline{Y}^2 \left[\frac{A}{\sqrt{B}} - \sqrt{B} \right]^2 > 0 \tag{17}$$

c) Comparison with exponential chain ratio estimator in double sampling $\overline{y}_{eR}^{dc*}$

From (14) and (13), we get

$$MSE(\overline{y}_{eR}^{dc*}) - MSE\left(\overline{y}_{eRP(opt)}^{dc*}\right) = \overline{Y}^2 \left[\frac{A}{\sqrt{B}} - \frac{\sqrt{B}}{2} \right]^2 > 0 \tag{18}$$

d) Comparison with exponential chain product estimator in double sampling $\overline{y}_{eP}^{dc*}$

From (15) and (13), we get

$$MSE(\overline{y}_{eP}^{dc*}) - MSE\left(\overline{y}_{eRP(opt)}^{dc*}\right) = \overline{Y}^2 \left[\frac{A}{\sqrt{B}} + \frac{\sqrt{B}}{2} \right]^2 > 0 \tag{19}$$

V. Case II: Non Response on both y and x

Assuming that there is non-response on study variable y and auxiliary variable x. The proposed estimator is given as

$$\overline{y}_{eRP}^{dc**} = \overline{y}^* \left[\alpha_2 \exp\left(\frac{\overline{x}' \dfrac{\overline{Z}}{\overline{z}'} - \overline{x}^*}{\overline{x}' \dfrac{\overline{Z}}{\overline{z}'} + \overline{x}^*} \right) \right.$$

$$\left. + (1 - \alpha_2) \exp\left(\frac{\overline{x}^* - \overline{x}' \dfrac{\overline{Z}}{\overline{z}'}}{\overline{x}^* + \overline{x}' \dfrac{\overline{Z}}{\overline{z}'}} \right) \right] \tag{20}$$

where α_2 is a scalar constant and let $\bar{x}^* = \bar{X}\left(1+e_1^*\right)$.

In this case to obtain the bias (B) and MSE of $\bar{y}_{eRP}^{dc**}$, we have

$$E\left(e_1^{*2}\right) = \frac{V\left(\bar{x}^*\right)}{\bar{X}^2} = \frac{1}{\bar{X}^2}\left(\lambda S_x^2 + \lambda' S_{2x}^2\right)$$

$$E(e_0^* e_1^*) = \frac{\text{cov}\left(\bar{y}^*, \bar{x}^*\right)}{\bar{Y}\bar{X}} = \frac{\left(\lambda S_{xy} + \lambda' S_{2xy}\right)}{\bar{Y}\bar{X}}$$

Using the above results and following the procedure in case I, the bias and MSE of $\bar{y}_{eRP}^{dc**}$ are given as

$$B(\bar{y}_{eRP}^{dc**}) = \bar{Y}\left[\frac{A'}{2} - \frac{B'}{8} + \alpha_2\left(\frac{B'}{2} - A'\right)\right] \tag{21}$$

$$MSE\left(\bar{y}_{eRP}^{dc**}\right) = \bar{Y}^2\left[\lambda C_y^2 + \lambda' C_{2y}^2 + A' + \frac{B'}{4}\right.$$

$$\left. -\alpha_2\left(2A' + B'\right) + \alpha_2^2 B'\right] \tag{22}$$

Differentiating the equation (22) with respect to α_2 and equating it to zero, we get the optimum value of α_2 as

$$\alpha_2 = \frac{1}{2} + \frac{A'}{B'} = \alpha_{2(opt)} \tag{23}$$

Substitution of α_2 from (23) in (22) gives the optimum MSE of $\bar{y}_{eRP}^{dc**}$ as

$$MSE(\bar{y}_{eRP(opt)}^{dc**}) = \bar{Y}^2\left[\lambda C_y^2 + \lambda' C_{2y}^2 - \frac{A'^2}{B'}\right] \tag{24}$$

Remarks:

1. When $\alpha_2 = 1$, $\bar{y}_{eRP}^{dc**}$ in (20) reduces to Singh and Choudhury exponential chain ratio estimator $\bar{y}_{eR}^{dc**}$ when non-response is on both y and x. The MSE of (4) is obtained by putting $\alpha_2 = 1$ in (22) and is expressed as

$$MSE(\bar{y}_{eR}^{dc**}) = \bar{Y}^2\left[\lambda C_y^2 + \lambda' C_{2y}^2 + \frac{B'}{4} - A'\right] \tag{25}$$

2. When $\alpha_2 = 0$, the proposed estimator $\bar{y}_{eRP}^{dc**}$ reduces to Singh and Choudhury exponential chain product estimator $\bar{y}_{eP}^{dc**}$. The MSE of (5) is obtained by putting $\alpha_2 = 0$ in (22) and is given by

$$MSE(\bar{y}_{eP}^{dc**}) = \bar{Y}^2\left[\lambda C_y^2 + \lambda' C_{2y}^2 + \frac{B'}{4} + A'\right] \tag{26}$$

Notes

VI. Efficiency Comparisons in Case II

a) Comparison with mean per unit estimator $\overline{y}^*$

From (1) and (24), we get

$$V\left(\overline{y}^*\right) - MSE(\overline{y}_{eRP(opt)}^{dc**}) = \overline{Y}^2\left[\frac{A'^2}{B'}\right] > 0 \tag{27}$$

b) Comparison with chain ratio estimator in double sampling t_1

From (2) and (24), we get

$$MSE(t_1) - MSE\left(\overline{y}_{eRP(opt)}^{dc**}\right) = \overline{Y}^2\left[\frac{A'}{\sqrt{B'}} - \sqrt{B'}\right]^2 > 0 \tag{28}$$

c) Comparison with exponential chain ratio estimator in double sampling $\overline{y}_{eR}^{dc**}$

From (25) and (24), we get

$$MSE(\overline{y}_{eR}^{dc**}) - MSE\left(\overline{y}_{eRP(opt)}^{dc**}\right) = \overline{Y}^2\left[\frac{A'}{\sqrt{B'}} - \frac{\sqrt{B'}}{2}\right]^2 > 0 \tag{29}$$

d) Comparison with exponential chain product estimator in double sampling $\overline{y}_{eP}^{dc**}$

From (26) and (24), we get

$$MSE(\overline{y}_{eP}^{dc**}) - MSE\left(\overline{y}_{eRP(opt)}^{dc**}\right) = \overline{Y}^2\left[\frac{A'}{\sqrt{B'}} + \frac{\sqrt{B'}}{2}\right]^2 > 0 \tag{30}$$

VII. Empirical Study

To illustrate the performances of the different estimators we have considered the data used by Khare and Sinha (2009). The description of the population is given below:

96 village wise population of rural area under Police-station-Singur, District-Hooghly, West Bengal has been taken under the study from the District Census Handbook 1981. The 25% villages (i.e. 24 villages) whose area is greater than 160 hectares have been considered as non-response group of the population. The number of agricultural labourers in the village is taken as study character (y) while the area (in hectares) of the village, the number of cultivators in the village and the total population of the village are taken as auxiliary characters x and z respectively. The values of the parameters of the population under study are as follows:

$$N = 96,\ n = 24,\ n' = 60,\ W_2 = 0.25$$

$$\overline{Y} = 137.9271,\ \overline{X} = 144.8720,\ \overline{Z} = 185.2188,\ S_y = 182.5012,\ S_{2y} = 287.4202,$$

$$C_x = 0.8115, C_{2x} = 0.9408,$$

$$C_z = 1.0529, C_{2z} = 1.4876,$$

$$\rho_{yx} = 0.773, \rho_{2yx} = 0.724,$$

$$\rho_{yz} = 0.786, \rho_{2yz} = 0.787,$$

$$\rho_{xz} = 0.819, \rho_{2xz} = 0.724\,.$$

Notes

Table 1 : Percentage relative efficiencies of different estimators with respect to $\overline{y}$

W_2	f	$\overline{y}^*$	t_2	$\overline{y}_{eR}^{dc*}$	$\overline{y}_{eP}^{dc*}$	$\overline{y}_{eRP}^{dc*}$
0.1	1.5	100	199.48	151.76	65.59	204.30
	2.0	100	177.53	142.59	68.52	180.86
	2.5	100	163.52	136.18	70.99	166.02
	3.0	100	153.79	131.44	73.10	155.78
0.2	1.5	100	177.53	142.59	68.52	180.86
	2.0	100	153.79	131.44	73.10	155.78
	2.5	100	141.18	124.92	76.52	142.58
	3.0	100	133.36	120.64	79.16	134.43
0.3	1.5	100	163.52	136.18	70.99	166.02
	2.0	100	141.18	124.92	76.52	142.58
	2.5	100	130.47	119.01	80.27	131.42
	3.0	100	124.18	115.36	82.99	124.90

Table 2 : Percentage relative efficiencies of different estimators with respect to $\overline{y}$

W_2	f	$\overline{y}^*$	t_1	$\overline{y}_{eR}^{dc**}$	$\overline{y}_{eP}^{dc**}$	$\overline{y}_{eRP}^{dc**}$
0.1	1.5	100	228.59	161.35	63.36	238.14
	2.0	100	221.50	158.04	64.38	232.28
	2.5	100	216.28	155.56	65.19	228.13
	3.0	100	212.27	153.62	65.86	225.06
0.2	1.5	100	221.50	158.04	64.38	232.28
	2.0	100	212.27	153.62	65.86	225.06
	2.5	100	206.52	150.81	66.88	220.90
	3.0	100	202.59	148.86	67.64	218.26
0.3	1.5	100	216.28	155.56	65.19	228.13
	2.0	100	206.52	150.81	66.88	220.90
	2.5	100	201.07	148.09	67.95	217.29
	3.0	100	197.58	146.33	68.67	215.22

VIII. Conclusion

The PRE of the suggested estimator has been compared with the usual unbiased estimator $\overline{y}^*$, estimators $(t_2, \overline{y}_{eR}^{dc*}, \overline{y}_{eP}^{dc*}, \overline{y}_{eRP}^{dc*})$ in Case I; and $\overline{y}^*$, estimators $(t_1, \overline{y}_{Re}^{dc**}, \overline{y}_{Pe}^{dc**}, \overline{y}_{eRP}^{dc**})$ in Case II.

From tables 1 and 2, it is concluded that the proposed estimator in its optimality is performing better than the estimators taken for comparisons. Also, it has been observed that the percent relative efficiency of the suggested estimator decreases when non-response rate increases in its both the cases.

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