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On the New Solitary Wave Solution of the Generalized Hirota-Satsuma Couple KdV System

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I. Introduction

No one can deny the important role which played by the nonlinear partial differential equations in the description of many and a wide variety of phenomena not only in physical phenomena, but also in plasma, fluid mechanics, optical fibers, solid state physics, chemical kinetics and geochemistry phenomena. So that, during the past five decades, a lot of method was discovered by a diverse group of scientists to solve the nonlinear partial differential equations. For examples tanh - sech method [12],[16] and [18], extended tanh - method [13], [6] and [20], sine – cosine method [19], [17] and [22], homogeneous balance method [4], the $\exp(-\varphi(\xi))$ expansion Method [11], Jacobi elliptic function method [3], [5], [14] and [24], F-expansion method [2], [21] and [9], exp-function method [8] and [7], trigonometric function series method [32], $(\frac{G'}{G})$ expansion method [10], [15], [29] and [26], the modi_ed simple equation method [1], [27], [30], [28], [31] and [25] and so on.

The objective of this article is to apply the $\left(\frac{G'}{G}\right)$ expansion method for finding the exact traveling wave solution of the generalized Hirota-Satsuma couple KdV system [23], which play an important role in mathematical physics.

The rest of this paper is organized as follows: In section 2, we give the description of the modified simple equation method. In section 3, we use this method to find the exact solutions of the nonlinear evolution equations pointed out above. In section 5, conclusions are given.

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II. Description of Method

Consider the following nonlinear evolution equation

$$F(u, u_t, u_x, u_{tt}, u_{xx}, ...) = 0, \qquad (2.1)$$

where F is a polynomial in u(x; t) and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method :

Step 1. We use the wave transformation

$$u(x,t) = u(\xi), \qquad \xi = x - ct, \qquad (2.2)$$

where c is a constant, to reduce Eq.(2.1) to the following ODE:

$$P(u, u', u'', u''', ...) = 0, \qquad (2.3)$$

where P is a polynomial in $u(\xi)$and its total derivatives, while $\left\{u' = \frac{du}{d\xi}\right\}$.

Step 2. Suppose that the solution of Eq.(2.3) has the form:

$$U(\xi) = a_0 + \sum_{j=1}^{m} a_j \left(\frac{G'}{G}\right)^j, \quad a_m \neq 0, \quad (2.4)$$

where a_0 and a_j, for $(j = 1, 2, 3, ..., m)$, are constants to be determined later, $G(\xi)$ satisfies a second order linear ordinary differential equation (LODE):

$$G'' + \lambda G' + \mu G = 0, \qquad (2.5)$$

where λ and μ are arbitrary constants. The positive integer M can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in Eq.(2.3). Moreover precisely, we define the degree of $u(\xi)$ as $D(u(\xi)) = m$, which gives rise to degree of other expression as follows:

$$D\left(\frac{d^q u}{d\xi^q}\right)=m+q,\ D\left(u^p\left(\frac{d^q u}{d\xi^q}\right)^s\right)=mp+s\left(m+q\right)$$

Therefore, we can find the value of m in Eq.(2.4)

Step 3. Substitute Eq.(2.4) along Eq.(2.5) into Eq.(2.3) and collecting all the terms of the same power $\left(\frac{G'}{G}\right)^j$ $j=0,1,2,3,....$ and equating them to zero, we obtain a system of algebraic equations, which can be solved by Maple or Mathematica to get the values of a_j, since $a_m \neq 0$. The solution of Eq.(2.5) depending on whether $\lambda^2-4\mu>0,\ \lambda^2-4\mu<0,\ \lambda^2-4\mu=0$ are given as

Case 1. When $\lambda^2-4\mu>0$

$$\left(\frac{G'}{G}\right)=\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{A_1 sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu})\xi\ +A_2 cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu})\xi}{A_1 cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu})\xi\ +A_2 sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu})\xi}\right)-\frac{\lambda}{2}. \tag{2.6}$$

Case 2. When $\lambda^2-4\mu<0$

$$\left(\frac{G'}{G}\right)=\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-A_1 sin(\frac{1}{2}\sqrt{4\mu-\lambda^2})\xi\ +A_2 cos(\frac{1}{2}\sqrt{4\mu-\lambda^2})\xi}{A_1 cos(\frac{1}{2}\sqrt{4\mu-\lambda^2})\xi\ +A_2 sin(\frac{1}{2}\sqrt{4\mu-\lambda^2})\xi}\right)-\frac{\lambda}{2}. \tag{2.7}$$

Case 3. When $\lambda^2-4\mu=0$

$$\left(\frac{G'}{G}\right)=\frac{A_2}{A_1+A_2\xi}-\frac{\lambda}{2}. \tag{2.8}$$

The above results can be written in simplified forms as
Case 1. When $\lambda^2-4\mu>0$

$$\left(\frac{G'}{G}\right)=\frac{\sqrt{\lambda^2-4\mu}}{2}tanh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi+\xi_0)-\frac{\lambda}{2},\quad tanh\,\xi_0=\frac{A_1}{A_2},\quad \left|\frac{A_1}{A_2}\right|>1. \tag{2.9}$$

Case 2. When $\lambda^2-4\mu<0$

$$\left(\frac{G'}{G}\right)=\frac{\sqrt{\lambda^2-4\mu}}{2}coth(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi+\xi_0)-\frac{\lambda}{2},\quad coth\,\xi_0=\frac{A_1}{A_2},\quad \left|\frac{A_1}{A_2}\right|<1. \tag{2.10}$$

Case 3. When $\lambda^2-4\mu=0$

$$\left(\frac{G'}{G}\right)=\frac{\sqrt{\lambda^2-4\mu}}{2}cot\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\xi+\xi_0\right)-\frac{\lambda}{2},\quad cot\,\xi_0=\frac{A_2}{A_1}. \tag{2.11}$$

Step 4. substituting these values and the solutions of Eq.(2.5) into Eq.(2.3) we obtain the exact solutions of Eq.(2.1).

III. Application

Here, we will apply the $\left(\frac{G'}{G}\right)$-expansion method described in Sec.2 to find the exact traveling wave solutions and the solitary wave solutions of the generalized Hirota-Satsuma couple KdV system[23]. We consider the generalized Hirota-Satsuma couple KdV system

$$\begin{cases} u_t=\frac{1}{4}u_{xxx}+3uu_x+3\left(-v^2+w\right)_x, \\ v_t=-\frac{1}{2}v_{xxx}-3uv_x, \\ w_t=-\frac{1}{2}w_{xxx}-3uw_x. \end{cases} \tag{3.1}$$

When $w=0$, Eq.(3.1) reduce to be the well known Hirota-Satsuma couple KdV equation. Using the wave transformation $u(x,t) = u(\xi), v(x,t) = v(\xi), w(x,t) = w(\xi)$, $\xi = k(x - \lambda_1 t)$ carries the partial differential equation (3.1) into the ordinary differential equation

$$\begin{cases} -\lambda_1 k u' = \frac{1}{4}k^3 u''' + 3 k u u' + 3k\left(-v^2 + w\right)', \\ -\lambda_1 k v' = -\frac{1}{2}k^3 v''' - 3 k u v', \\ -\lambda_1 k w' = -\frac{1}{2}k^3 w''' - 3 k u w'. \end{cases} \tag{3.2}$$

Suppose we have the relations between (u *and* v) and (w *and* v) $\Rightarrow \left(u = \alpha v^2 + \beta v + \gamma\right)$ and $(w = Av + B)$ where α, β, γ, A and B are arbitrary constants. Substituting this relations into second and third equations of Eq.(3.2) and integrating them , we get the same equation and integrate it once again we obtain

$$k^2 v'^2 = -2\alpha v^4 - 2\beta v^3 + 2\left(\lambda_1 - 3\gamma\right) v^2 + 2c_1 v + c_2, \tag{3.3}$$

where c_1 and c_2 is the arbitrary constants of integration, and hence, we obtain

$$\begin{aligned} k^2 u'' &= 2\alpha k^2 v'^2 + k^2\left(2\alpha v + \beta\right) v'' \\ &= 2\alpha\left[-\alpha v^4 - 2\beta v^3 + 2\left(\lambda_1 - 3\gamma\right) v^2 + 2c_1 v + c_2\right] \\ &+ \left(2\alpha v + \beta\right)\left[-2\alpha v^3 - 3\beta v^2 + 2\left(\lambda_1 - 3\gamma\right) v + c_1\right]. \end{aligned} \tag{3.4}$$

So that, we have

$$P'' + lP - mP^3 = 0. \tag{3.5}$$

Where

$$c_1 = \frac{1}{2\alpha^2\left(\beta^2 + 2\lambda_1\alpha\beta - 6\alpha\beta\gamma\right)}, \quad v(\xi) = aP(\xi) - \frac{\beta}{2\alpha}, \quad \alpha = \frac{\beta^2 - 4}{4\left(\gamma - \lambda_1\right)}, \quad A = \frac{4\beta\left(\lambda_1 - \gamma\right)}{\beta^2 - 4},$$

$$\begin{aligned} B = &\frac{1}{6\left(-\gamma + \lambda_1\right)\left(\beta^2 - 4\right)^2}(16c_3\lambda_1\beta^2 - 2c_3\lambda_1\beta^4 - 16c_3\gamma\beta^2 + 3c_3\gamma\beta^4 + 56\lambda_1^2\gamma\beta^2 \\ &- 48\gamma^2\lambda_1\beta^2 - 16c_2 + c_2\beta^6 - 12c_2\beta^4 + 12c_2\beta^2 - 16\gamma^2\lambda_1 - 32\lambda_1^2\gamma - 8\lambda_1^3\beta^2 + \beta^4\gamma^3 \\ &- 2\beta^4\lambda_1^3 + 32c_3\gamma - 32c_3\lambda_1 + 48\gamma^3 + \beta^4\gamma^2\lambda_1), \end{aligned}$$

$$l = \frac{-a}{k^2}\left(\frac{3\beta^2}{2\alpha} + 2\lambda_1 - 6\gamma\right), \quad m = \frac{-2\alpha a^3}{k^2}.$$

Balancing between the highest order derivatives and nonlinear terms appearing in P'' and $P^3 \Rightarrow (N + 2 = 3N) \Rightarrow (N = 1)$ So that, by using Eq.(2.4) we get the formal solution of Eq.(3.5)

$$P(\xi) = a_0 + a_1\left(\frac{G'}{G}\right). \tag{3.6}$$

Substituting Eq.(3.6) and its derivative into Eq.(3.5) and collecting all term with the same power of $\left(\frac{G'}{G}\right)^3, \left(\frac{G'}{G}\right)^2, \left(\frac{G'}{G}\right)^1, \left(\frac{G'}{G}\right)^0$ we obtained:

$$\left(\frac{G'}{G}\right)^3 : 2a_1 + m{a_1}^3 = 0, \tag{3.7}$$

$$\left(\frac{G'}{G}\right)^2 : 3a_1\lambda + 3ma_0{a_1}^2 = 0, \tag{3.8}$$

$$\left(\frac{G'}{G}\right)^1 : a_1\lambda^2 + 2a_1\mu + la_1 + 3m{a_0}^2 a_1 = 0, \tag{3.9}$$

$$\left(\frac{G'}{G}\right)^0 : a_1\lambda\mu + la_0 + m{a_0}^3 = 0 \tag{3.10}$$

Solving above system of algebraic equations by using Maple program, we obtain

$$l = \frac{1}{2}\lambda^2 - 2\mu,\ a_1 = \pm\sqrt{\frac{-2}{m}},\ a_0 = \pm\lambda\sqrt{\frac{-1}{2m}}\ since\ (m < 0).$$

So that, the exact traveling wave solution

$$P(\xi) = \pm\lambda\sqrt{\frac{-1}{2m}} \pm \sqrt{\frac{-2}{m}}\left(\frac{G'}{G}\right). \tag{3.11}$$

here, we discuss the three cases:
Case 1. When $\lambda^2 - 4\mu > 0$

$$P(\xi) = \pm\lambda\sqrt{\frac{-1}{2m}}\pm\sqrt{\frac{-2}{m}}\left[\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{A_1 sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu})\xi + A_2 cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu})\xi}{A_1 cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu})\xi + A_2 sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu})\xi}\right) - \frac{\lambda}{2}\right] \tag{3.12}$$

Case 2. When $\lambda^2 - 4\mu < 0$

$$P(\xi) = \pm\lambda\sqrt{\frac{-1}{2m}}\pm\sqrt{\frac{-2}{m}}\left[\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-A_1 sin(\frac{1}{2}\sqrt{4\mu-\lambda^2})\xi + A_2 cos(\frac{1}{2}\sqrt{4\mu-\lambda^2})\xi}{A_1 cos(\frac{1}{2}\sqrt{4\mu-\lambda^2})\xi + A_2 sin(\frac{1}{2}\sqrt{4\mu-\lambda^2})\xi}\right) - \frac{\lambda}{2}\right]. \tag{3.13}$$

Case 3. When $\lambda^2 - 4\mu = 0$

$$P(\xi) = \pm\lambda\sqrt{\frac{-1}{2m}} \pm \sqrt{\frac{-2}{m}}\frac{A_2}{A_1 + A_2\xi} - \frac{\lambda}{2}. \tag{3.14}$$

The above results can be written in simplified forms as
Case 1. When $\lambda^2 - 4\mu > 0$

$$P(\xi) = \pm\lambda\sqrt{\frac{-1}{2m}} \pm \sqrt{\frac{-2}{m}}\frac{\sqrt{\lambda^2-4\mu}}{2}\left[tanh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi + \xi_0) - \frac{\lambda}{2}\right]. \tag{3.15}$$

Case 2. When

$$P(\xi) = \pm\lambda\sqrt{\frac{-1}{2m}} \pm \sqrt{\frac{-2}{m}}\frac{\sqrt{\lambda^2-4\mu}}{2}\left[coth(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi + \xi_0) - \frac{\lambda}{2}\right]. \tag{3.16}$$

Case 3. When $\lambda^2 - 4\mu = 0$

$$P(\xi) = \pm\lambda\sqrt{\frac{-1}{2m}} \pm \sqrt{\frac{-2}{m}}\frac{\sqrt{\lambda^2-4\mu}}{2}\left[cot\ \ \frac{\sqrt{4\mu-\lambda^2}}{2}\xi + \xi_0\right) - \frac{\lambda}{2}\right]. \tag{3.17}$$

Note that:

All the obtained results have been checked with Maple 16 by putting them back into the original equation and found correct.

IV. Physical Interpretations of the Solutions

In this section, we depict the graph and signify the obtained solutions to the generalized Hirota-Satsuma couple KdV system. Now, we will discuss all possible physical significances for parameter.

case 1.
when $m = \frac{-1}{2}$, $\lambda = 3$, $\mu = 2$, $A_1 = 4$, $A_2 = 5$, $k = 1$, $\lambda_{-2} \Rightarrow$ we obtain the hyperbolic function solution and represent singular kink type solitary wave solution.

case 2.
when $m = \frac{-1}{2}$, $\lambda = 2$, $\mu = 3$, $A_1 = 4$, $A_2 = 5$, $k = 1$, $\lambda_{-2} \Rightarrow$ we obtain the trigonometric function solution and represent bell solitary wave solution.

case 1.
when $m = \frac{-1}{2}$, $\lambda = 2$, $\mu = 1$, $A_1 = 4$, $A_2 = 5$, $k = 1$, $\lambda_{-2} \Rightarrow$ we obtain the rational function solution and represent singular kink type solitary wave solution.

V. Conclusion

The $\left(\frac{G'}{G}\right)$- expansion method has been successfully used to find the exact traveling wave solutions of some nonlinear evolution equations. As an application, the traveling wave solutions for the generalized Hirota-Satsuma couple KdV system which have been constructed using the $\left(\frac{G'}{G}\right)$ -expansion method. Let us compare between our results obtained in the present article with the well-known results obtained by other authors using different methods as follows: Our results of the generalized Hirota-Satsuma couple KdV system are new and different from those obtained in [23] It can be concluded that this method is reliable and propose a variety of exact solutions NPDEs. The performance of this method is effective and can be applied to many other nonlinear evolution equations. The solutions represent the solitary traveling wave solution for the generalized Hirota-Satsuma couple KdV system.

Competing Interests

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors. The author did not have any competing interests in this research.

Author's Contributions

All parts contained in the research carried out by the researcher through hard work and a review of the various references and contributions in the field of mathematics and the physical Applied.

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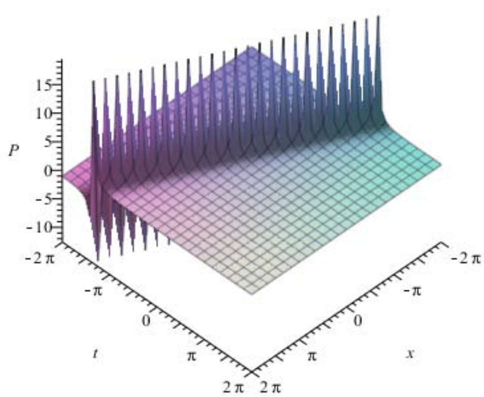
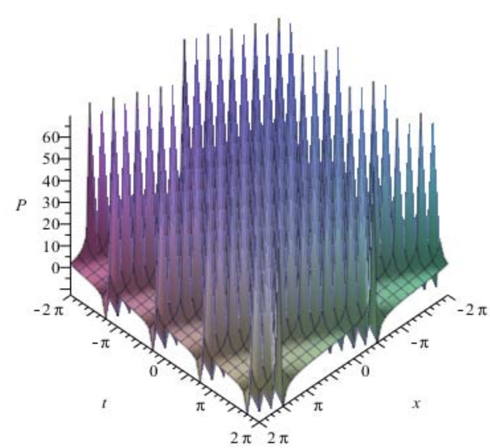
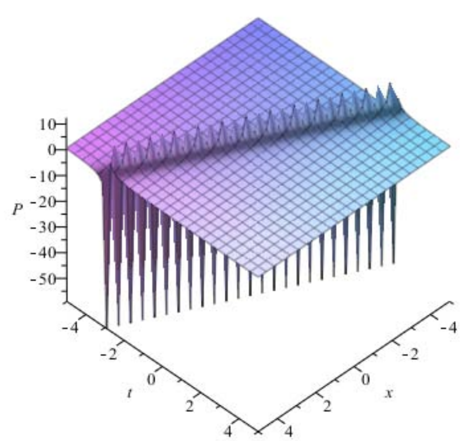


(a)
Eq.(3.12)



(b)
Eq.(3.13)



(c)
Eq.(3.14)

Figure 1 : The Solitary wave solution of Eqs.(3.12),(3.13) and (3.14)

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