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Special Pythagorean Triangles and 10 Digit Dhuruva Numbers

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SPECIALPYTHAGOREANTRIANGLESAND10DIGITDHURUVANUMBERS

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Special Pythagorean Triangles and 10 Digit Dhuruva Numbers

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Abstract- Pythagorean triangles, each with a leg represented by 10-digit Dhuruva numbers are obtained. A few interesting results are given.

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I. Introduction

The fascinating branch of mathematics is the theory of numbers where in Pythagorean triangles have been a matter of interest to various mathematicians and to the lovers of mathematics, because it is a treasure house in which the search for many hidden connection is a treasure hunt. For a rich variety of fascinating problems one may refer [1-17].A careful observer of patterns may note that there is a one to one correspondence between the polygonal numbers and the number of sides of the polygon. Apart from the above patterns we have some more fascinating patterns of numbers namely Jarasandha numbers, Nasty numbers and Dhuruva numbers. These numbers have been presented in [18-21].

In [22-25], special Pythagorean triangles connected with polygonal numbers and Nasty numbers are obtained. Recently in [26], special Pythagorean triangles in connection with Hardy Ramanujan number 1729 are exhibited. Thus the objective of this paper is to find out the special Pythagorean triangles in connection with 10 digit Dhuruvanumber 9753086421.

In this communication we have presented Pythagorean triangles each with a leg represented by 10 digit Dhuruva numbers 9753086421,9975084201 and 6333176664. Also, a few special Pythagorean triangles in connection with these three numbers are obtained.

II. Basic Definitons

a) *Definition 2.1*

The ternary quadratic Diophantine equation given by $x^2 + y^2 = z^2$is known as Pythagorean equation where x, y, z are natural numbers. The above equation is also referred to as Pythagorean triangle and denote it by $T(x,y,z)$.

Also, in Pythagorean triangle $T(x,y,z)$: $x^2 + y^2 = z^2$, x and y are called its legs and z its hypotenuse.

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b) Definition 2.2

Most cited solution of the Pythagorean equation is $x = m^2 - n^2; y = 2mn ; z = m^2 + n^2$, where $m > n > 0$. This solution is called primitive, if *m, n* are of opposite parity and gcd(*m, n*)=1.

c) Definition 2.3: Dhuruva numbers

The numbers which do not change when we perform a single operation or a sequence of operations are known as Dhuruva numbers.

Notes

III. METHOD OF ANALYSIS

In this section, we exhibit Pythagorean triangles, each with a leg represented by the 10-digit Dhuruva number 9753086421 and denote this number by N_1.

To start with, it is noted that the leg *y* cannot be represented by N_1 as *y* is even and N_1 is odd. Also *z* cannot be written as sum of two squares since a positive integer P can be written as a sum of two integer squares iff the canonical prime factorization $P = p_1^{r_1}.p_2^{r_2} \dots p_r^{r_r}$, (where p_i are distinct primes) satisfies the condition if $p_i \equiv 3 \ (mod\ 4)$ then r_i is even. A prime $p \equiv 1 (mod\ 4)$ can be written as $p = a^2 + b^2$.

Now, consider $x = N_1 \Rightarrow x = m^2 - n^2$ which is a binary quadratic Diophantine equation. Solving the above equation for *m,n* we get 15 integer solutions and thus, we have 15 Pythagorean triangles, each having the leg *x* to be represented by the ten digit Dhuruva number $N_1 = 9753086421$ as shown in table 1 below:

Table.1

No.	*m*	*n*	*x*	*y*	*z*
1	1625514405	1625514402	9753086421	5284594151971921620	5284594151971921629
2	541838139	541838130	9753086421	587177127996880140	587177127996880221
3	18062725	18062698	9753086421	652523093464100	652523093464829
4	157307861	257307830	9753086421	4949166438109260	49491516511704221
5	52435995	52435902	9753086421	5499057390184980	5499057390193629
6	17478789	17478510	9753086421	611006376648780	611006376726621
7	375118715	375118702	9753086421	281428090933415860	281428090933416029
8	12100805	12100402	9753086421	292849210047220	292849210209629
9	5826635	5825798	9753086421	67889597059460	67889597760029
10	453611	442730	9753086421	401654396060	401772792221
11	13893461	13893110	9753086421	386046763907420	38604764030621
12	41679915	41679798	9753086421	3474420875714340	3474420875728029
13	4034139	4032930	9753086421	32538800394540	32538801856221
14	1346325	1342698	9753086421	3615415769700	3615428924829
15	4876543211	4876543210	9753086421	47561347367747294620	47561347367747294621

Note that there are 8 primitive and 7 non-primitive triangles. Also the relation $\frac{4A}{P} - y + z$ represents the 10-digit Dhuruva number 9753086421 for each of the above Pythagorean triangles, where A and P represents the area and perimeter of the Pythagorean triangle.

Notes

In a similar manner, it is seen that there are 30 Pythagorean triangles wherein, each of the following expressions $\frac{2A}{P}, \frac{1}{2}(y + x - z)$represent 9753086421 as shown in table 2 below:

Table 2

No	m	n	x	y	z
1	9753086422	1	95122694735494589240	19506172842	95122694735494589242
2	3251028810	3	10569188323450016091	19506172860	10569188323450016109
3	1083676278	9	1174354275499933203	19506173004	1174354275499933365
4	361225450	27	130483825727701771	19506174300	130483825727703229
5	314615722	31	98983052529580323	19506174764	98983052529582245
6	104871990	93	10998134286551451	19506190140	10998134286568749
7	34957578	279	1222032259548243	19506328524	1222032259703925
8	750237430	13	562856201373004731	19506173180	562856201373005069
9	24201610	403	585717926429691	19506497660	585717926754509
10	11653270	837	135798700992331	19506172842	135798702393469
11	907222	10881	822933361123	19742965164	823170153445
12	27786922	351	772113034110883	19506419244	772113034357285
13	83359830	117	6948861257615211	19506200220	6948861257642589
14	8068278	1209	65097108423603	19509096204	65097111346965
15	2692650	3627	7250350867371	19532483100	7250377177629
16	9753086422	9753086421	19506172843	190245389490495351324	190245389490495351325
17	3251028810	3251028807	19506172851	21138376627393859340	21138376627393859349
18	1083676278	1083676269	19506172923	2348708531493693564	2348708531493693645
19	361225450	361225423	19506173571	260967631949230700	260967631949231429
20	314615722	314615691	19506173803	197966085552987804	197966085552988765
21	104871990	104871897	19506181491	21996249066930060	21996249066938709
22	34957578	34957299	19506250683	2444045012923644	2444045013001485
23	750237430	750237417	19506173011	1125712383239836620	1125712383239836789
24	24201610	24201207	19506335251	1171416346686540	1171416346848949
25	11653270	11652433	19506873411	271577895811820	271577896512389
26	907222	896341	19624569003	1626360549404	1626478945565
27	27786922	27786571	19506296043	1544206562048924	1544206562172125
28	83359830	83359713	19506186531	13897703009057580	13897703009071269
29	8068278	8067069	19507634523	130174710674364	130174712136045
30	2692650	2689023	19519327971	14481195561900	14481208717029

Note that there are 15 primitive and 15 non-primitive triangles.

Also,it is observed that there are 15 Pythagorean triangles wherein each of the expressions $x - \frac{2A}{P}, \frac{1}{2}(z + x - y)$ is represented by 9753086421 as shown in table 3 below:

Table 3

No.	*m*	*n*	*x*	*y*	*z*
1	3251028807	3251028804	19506172833	21138765883815136560	21138765883815136650
2	1083676269	1083676260	19506172761	2348708492481347880	2348708492481347961
3	361225423	361225396	19506172113	260967592936885016	260967592936885745
4	314615691	314615660	19506171881	197966046540642120	197966046540643081
5	104871897	104871804	19506164193	21996210054584376	21996210054593025
6	34957299	34957020	19506095001	2444006000577960	2444006000655801
7	750237417	750237404	19506172673	1125712344227490936	1125712344227491105
8	24201207	24200804	19506010433	1171377334340856	1171377334503265
9	11652433	11651596	19505472273	271538883466136	271538884166705
10	896341	885460	19387776681	1587348203720	1587466599881
11	27786571	27786220	19506049641	1544167549703240	1544167549826441
12	83359713	83359596	19506159153	13897663996711896	13897663996725585
13	8067069	8065860	19504711161	130135698328680	130135699790361
14	2689023	2685396	19493017713	14442183216216	14442196371345
15	9753086421	9753086420	19506172841	190245389451483005640	190245389451483005641

Note that there are 8 primitive and 7 non-primitive triangles.

For simplicity, in table 4, we exhibit the connections between special Pythagorean triangles and the other ten digit Dhuruva numbers N_2 and N_3 in the following table respectively:

Table 4

Dhuruva Number	Expressions	No.of Triangles	Remark
$N_2 = 9975084201$	$\frac{4A}{P} - y + z$; x	4	2-Primitive, 2-Non-primitive
	$x - \frac{2A}{P}$ and $\frac{1}{2}(z + x - y)$	4	2-Primitive, 2-Non-primitive
	$\frac{2A}{P}$; $\frac{1}{2}(y + x - z)$	8	5-Primitive, 3-Non-primitive
$N_3 = 6333176664$	$\frac{4A}{P} - y + z$; x	6	6-Non-primitive
	y	9	4- Primitive 5-Non-primitive
	$x - \frac{2A}{P}$ and $\frac{1}{2}(z + x - y)$	12	2-Primitive 10-Non-primitive
	$\frac{2A}{P}$; $\frac{1}{2}(y + x - z)$	24	6-Primitive 18-Non-primitive

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Notes

IV. Conclusion

One may search for the connections between Pythagorean triangles and other Dhuruva numbers.

Notes

References Références Referencias

1. W. Sierpinski, Pythagorean triangles, Dover publications, INC, New York, 2003.
2. M. A. Gopalan and G. Janaki, "Pythagorean triangle with area/perimeter as a special polygonal number", Bulletin of Pure and Applied Science, Vol.27E (No.2), 393-402,2008.
3. M. A. Gopalan and A. Vijayasankar, "Observations on a Pythagorean problem", Acta Ciencia Indica, Vol.XXXVI M, No 4, 517-520, 2010.
4. M. A. Gopalan and S. Leelavathi, "Pythagorean triangle with area/perimeter as a square integer", International Journal of Mathemtics, Computer sciences and Information Technology, Vol.1, No.2, 199-204, 2008.
5. M. A. Gopalan and A. Gnanam, "Pairs of Pythagorean triangles with equal perimeters", Impact J. Sci. Tech., Vol 1(2), 67-70, 2007.
6. M. A. Gopalan and S. Leelavathi, "Pythagorean triangle with 2 area/perimeter as a cubic integer", Bulletin of Pure and Applied Science, Vol. 26E(No.2), 197-200,2007.
7. M. A. Gopalan and A. Gnanam, "A special Pythagorean problem", Acta Ciencia Indica, Vol.XXXIII M, No 4, 1435-1439,2007.
8. M. A. Gopalan, A. Gnanam and G. Janaki, "A Remarkable Pythagorean problem", Acta Ciencia Indica, Vol.XXXIII M, No 4, 1429-1434,2007.
9. M. A. Gopalan, and S. Devibala, "On a Pythagorean problem", Acta Ciencia Indica, Vol.XXXII M, No 4, 1451-1452,2006.
10. M. A. Gopalan and B. Sivakami, "Special Pythagorean triangles generated through the integral solutions of the equation $y^2 = (k^2 + 2k)x^2 + 1$", Diophantus J. Math., Vol 2(1), 25-30, 2013.
11. M. A. Gopalan and A. Gnanam, "Pythagorean triangles and Polygonal numbers", International Journal of Mathematical Sciences, Vol 9, No. 1-2,211-215,2010.
12. K. Meena, S. Vidhyalakshmi, B. Geetha, A. Vijayasankar and M. A. Gopalan,"Relations between special polygonal numbers generated through the solutions of Pythagorean equation", IJISM, Vol 5(2), 15-18, 2008.
13. M. A. Gopalan and G. Janaki, "Pythagorean triangle with perimeter as Pentagonal number", AntarticaJ.Math., Vol 5(2), 15-18,2008.
14. M. A. Gopalan and G. Sangeetha, "Pythagorean triangle with perimeter as triangular number", GJ-AMMS, Vol. 3, No 1-2, 93-97,2010.
15. M. A. Gopalan, Manjusomanath and K. Geetha, "Pythagorean triangle with area/perimeter as a special polygonal number", IOSR-JM, Vol.7(3),52-62,2013.
16. M. A. Gopalan and V. Geetha, "Pythagorean triangle with area/perimeter as a special polygonal number", IRJES, Vol.2(7), 28-34,2013.
17. M. A. Gopalan and B. Sivakami, "Pythagorean triangle with hypotenuse minus 2(area/perimeter) as a square integer", Archimedes J. Math., Vol 2(2), 153-166,2012.
18. J. N. Kapur, Dhuruva numbers, Fascinating world of Mathematics and Mathematical sciences, Trust society, Vol 17,1997.

19. Bert Miller, Nasty numbers, The mathematics teacher, No.9, Vol 73,649,1980.
20. Charles Bown. K, Nasties are primitives, The mathematics teacher, No.9, Vol 74,502-504,1981.
21. P.S.N. Sastry, Jarasandha numbers, The mathematics teacher, No.9,Vol 37,issues 3 and 4,2001.
22. M. A. Gopalan V. Sangeetha and Manjusomanath, "Pythagorean triangle and Polygonal number", Cayley J. Math., Vol 2(2),151-156,2013.
23. M. A. Gopalan and G. Janaki, "pythagorean triangle with nasty number as a leg", Journal of applied Mathematical Analysis and Applications, Vol 4,No 1-2,13-17,2008.
24. M. A. Gopalan and S. Devibala, "Pythagorean triangle with triangular number as a leg", Impact J. Sci. Tech., Vol 2(4), 195-199,2008.
25. M. A. Gopalan S. Vidhyalakshmi, E. Premalatha and R. Presenna , "Special Pythagorean triangles and 5 digit dhuruva numbers ", IRJMEIT., Vol 1(4),29-33,2014.
26. Dr. Mita Darbari, A connection between Hardy-Ramanujan number and special Pythagorean triangle," Bulletin of society for Mathematical services and standards, Vol 3, No.2, 71-73, 2014.

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