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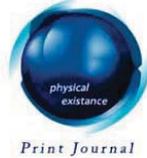
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Gaussian Elimination Method-A Study of Applications

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Abstract- In linear algebra Gaussian elimination method is the most ancient and widely used method. In this paper we discuss the applications of Gaussian Elimination method, as it can be performed over any field. The method is being used in channel decoding algorithm as it's very resourceful moreover we have presented a Successive Gaussian elimination method that is used for solution of parallel linear equations. Successive Gaussian Elimination method is observed to be more rapid, efficient and accurate than that of Gaussian elimination method. Another important application of Gaussian elimination is Robust Fingerprint Image Enhancement. Gaussian filter is used to enhance the image. The SGE method is also appropriate for solving linear equations on mesh-connected processors. The Gaussian method is also used in scheduling algorithms.

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I. INTRODUCTION

Solving linear equation is the important application of linear algebra and it also plays an important role in many fields of science i.e. physics, statistics, computer science etc. There are many methods to solve linear equations but the main focus of this paper is on Gaussian elimination method. For many scientific computations it is necessary to solve linear equation so good option is to solve it by algorithm of Gaussian elimination method. The series of operations that are performed on the matrix of coefficients for reduction of matrix is called Gaussian elimination method.

The main aim of this paper is to discuss the uses and applications of Gaussian elimination method. This method is applicable “to find the rank of a matrix, to calculate the determinant of a matrix, and to calculate the inverse of an invertible square matrix”. With Gaussian elimination method being the ancient and most widely used method, the method is not only restricted for figures and numbers but can be used in various fields. The main concern of whether it is more complex and if more new and innovative methods have been developed. According to modern analysts, to those that prefer complexity rather than solution, there are many less feasible methods still more appealing but in the long run, Gaussian method still takes the crown until further development is done. Gauss elimination method is used in enhancing image. The technique is called “Directional Gaussian Filter”.

Fingerprint Image using Directional Gaussian Filtering and Non-Subsample Contour let Transform method is used along with other techniques to clarify and attain

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better images from thumb impressions. During the finger print identification process, the most important phase is minutiae where clear and quality image is basis of further advancement. Many techniques are there to enhance the image but the level of clarity due to filters is a major shortcoming. So a new method based on directional Gaussian filter has been presented [3]. In this method, the image is deconstructed into a set of images, after using Gaussian filter to remove noise. Then it is constructed block by block using "NSCT" for maximum clarity. The main purpose in internet and wireless transmission is to pass over the information error free and to discern the data that is being sent and received. To avoid such error different error correction techniques are being used. It is known that Gaussian elimination method for solving simultaneous linear equations is not best approach, but still it is useful technique for solving systems. [2] The main reason of using forward error correction (FEC) is to raise efficiency of a channel by addition of some designed information to the data that is being transmitted through that channel [1]. This process is known as channel coding. There are two types of channel coding:

1. Convolutional coding
2. Block coding

Convolutional codes work on serial data. Block codes work on relatively large message blocks. A method is used to find the generator polynomial and the code rate of the convolution encoded data. The received values are then encoded by Convolution (n, k, and m) codes and then its generator polynomial is detected by using the Gaussian Elimination Method [1].

The problem of solving a system of N linear equations on a mesh-connected multiprocessor structure is considered. The solution to the problem is obtained by using a Gaussian-elimination-based algorithm called 'successive Gaussian elimination'. The new algorithm does not contain a separate back substitution phase. This scheme eliminates the use of two processor structures in conjunction. Most importantly, the new algorithm supports pair wise pivoting to assure numerical stability.

Consider a graph theoretical model and study a parallel implementation of the well-known Gaussian elimination method on parallel distributed memory architectures, where the communication delay for the transmission of an elementary data is higher than the computation time of an elementary instruction. We propose and analyze two low-complexity algorithms for scheduling the tasks of the parallel Gaussian elimination on an unbounded number of completely connected processors. We compare these two algorithms with a higher-complexity general-purpose scheduling algorithm.

Many Channel decoding algorithm uses Gaussian elimination method as a major operation as it provides large progress of frame error and is efficient then present algorithms[4]. First of all state-of-art architectures are implemented then secondly we apply a new hardware architecture Gaussian Elimination over GF(2) that utilizes less resource and more throughputs. SMITH architecture and Systolic arrays according to their hardware implementation on FPGA have been analyzed and the Gaussian Elimination over GF(2) method is implemented methodically and result is observed that this new implementation consumes less FPGA resources and is more efficient then the state of art architectures The Method proposed is actually the improved version for channel decoding algorithm[4].

Gaussian Elimination method is a solution for matrix of the form $Ax=b$. Successive Gaussian Elimination commonly called SGE is a method for solution of linear algebraic equations is an alternative of Gaussian elimination method. [5] SGE does not contain a separate back substitution phase as Back substitution phase requires $O(N)$ or

$O(\log N)$ steps either using $O(N)$ processor or $O(N^3)$ processor respectively. In spite of using back substitution SGE uses one step division and partial pivoting. In Successive Gaussian Elimination the pivot column and non-pivot columns are executed in parallel that allows efficient multiprocessing [5]. The solution achieved by Successive Gaussian Elimination method is observed to be more rapid, efficient and exact than that of Gaussian elimination method.

Section 2 describes the related work and analysis done on the applications of Gaussian elimination method. As discussed earlier the method is not only applicable in numeric's field but it has vast applications in all other areas.

II. RELATED WORK

a) Gaussian-elimination-based algorithm for mesh-connected processors

Mesh-connected multiprocessor structure is considered for the problem of solving a system of N linear equations. By using the new algorithm, which is called the successive Gaussian elimination (SGE) algorithm that is used to solve the problem of system of N linear equation and it is represented in the form of a binary tree. In this algorithm SGE no separate back substitution part occurs." A two-dimensional array of $N \times (N + 1)$ processors is used to obtain the solution in $(5N - \log N - 4)$ time steps without requiring any intermediate I/O operations". For the production of complete solution using the existing Gaussian elimination algorithm this System reduces the use of two processor structures one for back substitution, and other for triangulation. For the assurance of numerical stability the new algorithm SGE supports pair wise pivoting.

b) Scheduling Algorithms for Parallel Gaussian Elimination

Most important problems in parallel processing are scheduling the tasks of weighted Directed Acyclic Graphs (DAGs). This problem is solved by the use of Gaussian elimination method.[6] Cosnard et al. proposed a complete classification of different versions of Gaussian Elimination, leading to two important classes of DAGs: the two-step graph and the greedy one.

Thus, the problem of efficiently implementing Gaussian elimination is reduced to the problem of finding an algorithm for scheduling the tasks of the two-step or the greedy graph that minimizes the schedule length.

c) Recover Generator Polynomials of Convolutional Codes

To avoid the errors for transmission of data ,error correction technique is used known as channel coding. Convolution encoding method is mostly used among channel coding methods. In this paper a technique is proposed to identify the generator polynomial and the code rate of the convolution encoded data. The data received is encoded by using Gaussian Elimination Method. Variables are eliminated step by step. This removal of variables is different because it is done using GF (2) [1]. If the received encoded data contains up to 7 bits of error then there are chances of more than 60% ,that the given technique correctly identify the generator polynomial. It can identify the correct polynomial with an error free coded data with the matrix size of 20×20 . And the adequate matrix size calculated from various calculations and simulations is 64×64 . [1]

d) Some Improvements of the Gaussian Elimination Method

Gaussian elimination is a technique mostly used for solving simultaneous linear equations, it is not optimal but still preferable to be used for solving problems of moderate size. New methods are being proposed to expedite this technique. Eventual symmetry of the system is given, which increases the calculation. Then it is presented

that by doing slightly changes in the arrangement of the calculated values ,it can help to expedite the calculation[2].This method of accelerating the Gaussian elimination method can be useful in many fields.

e) *A New Gaussian Elimination-based Algorithm for Parallel Solution of Linear Equations*

Advancements in the field have been made and a new method Successive Gaussian elimination method is developed. The SGE method is does not contain a separate back substitution phase as Back substitution phase requires $O(N)$ or $O(\log N)$ steps either using $O(N)$ processor or $O(N^3)$ processor respectively. In spite of using back substitution SGE uses one step division and partial pivoting. In Successive Gaussian Elimination the pivot column and non-pivot columns are executed in parallel that allows efficient multiprocessing.

f) *Performance Comparison of Gauss Elimination and Gauss-Jordan Elimination*

One of the most important applications of linear algebra is solving linear equations. Many problems of engineering, economics biology etc can be easily solved through linear equations to find the faster method of solving linear equation comparison between Gauss elimination and Gauss-Jordan elimination method a experiment is purposed. Experiment is performed with the help of java programming language. According to the result of this experiment Gauss elimination method performance is faster than Gauss-Jordan method.

III. CONCLUSION

Gaussian Elimination method has many applications which are being discussed above. The method is not only restricted for figures and numbers but can be used in various fields. In the field of Computers and Mathematics the method is efficient for algorithms, enhancement of images and networks.

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