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Kaluza-Klein Barotropic Cosmological Models with Varying Gravitational Constant G in Creation Field Theory of Gravitation

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I. Introduction

From the Astronomical observations in the late eighties, it is concluded that the predictions of FRW type models do not always meet our requirements as was believed earlier (Smooth *et al.* (1992)). Hence alternatives theories were proposed of which the most well-known theory was steady state theory by Bondi and Gold (1948). The main approach of this theory is the universe does not have any singular beginning nor an end on the cosmic time scale where the matter density is constant throughout. They have considered a very slow but continuous creation of the matter in contrast to explosive creation of standard model for maintaining constancy of matter density. But the theory was discarded for not giving any physical justification about continuous creation of matter. To overcome this difficulty Hoyle and Narlikar (1964 a, b, c) adopted a field theoretic approach introducing a massless and charge-less scalar field in the Einstein Hilbert action to account for a matter creation. Bali and Saraf (2013a, b, c) have investigated cosmological models with varying (λ) in Creation field theory of gravitation. Narlikar (1973) stated that introduction of negative energy C-field solved the horizon and flatness problem faced by big bang model. Chatterjee and Banerjee (2004) have extended the study of Hoyle Narlikar theory in higher dimensional space times. Singh and Chaubey (2009) have studied Kantowski-Sachs and Bianchi type universes in creation field theory. Adhav *et al.* (2010) have obtained Bianchi type-I universe with cosmological models in Creation field theory of gravitation with different contexts. Recently Ghate *et al.* (2014 a) have investigated LRS Bianchi type-V dust filled universe with varying $\Lambda(t)$ in creation field theory of gravitation.

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In Einstein's general theory of relativity, the gravitational constant G plays the role of coupling constant between geometry and matter. The concept of gravitational constant G was first proposed by Dirac (1937). Pochoda and Schwarzschild (1963) and Gamow (1967) have studied the solar evolution in the presence of a time varying gravitational constant. Barrow (1978) assumed that $G\alpha t^{-n}$ and obtained from Helium abundance for $5.9\times10^{-13} < n < 7\times10^{-13}$, $\left|\frac{\dot{G}}{G}\right| < (2\pm.93)\times10^{-12}\,yr^{-1}$, by assuming a flat Universe.

Subsequently alternative theories of gravity especially Brans and Dicke (1961), Canuto *et al.* (1977) were developed to generalized Einstein's general theory of relativity by including variable G and satisfying conservation equation. Bali and Kumawat (2011, 12) have investigated cosmological models with variable G in C-field cosmology.

The Kaluza-Klein theory was introduced to unify Maxwell's theory of electromagnetism and Einstein's gravity theory by adding the fifth dimension. Kaluza (1921) has demonstrated that GR when interpreted as a vacuum 5D theory contains four-dimensional GR in the presence of electromagnetic field, together with Maxwell's electromagnetism. To do so, Kaluza supposed that - (i) model should maintain Einstein's vision that nature is purely geometric (ii) GR mathematics is not modified but just extended to five dimensions and (iii) there is no physical dependence on the fifth dimension. Klein (1926) suggested the compactification of fifth dimension. Number of relativists have studied Kaluza-Klein cosmological models with different contexts [Leon (1988), Chi (1990), Fukui (1993), Coley (1994), Liu & Wesson (1994)]. Alvarez *et al.* (1983), Ranjibar– Daemi *et al.,* (1984) and Marciano (1984) suggested that the experimental detection of time variation of fundamental constants could provide strong evidence for the existence of extra dimensions. Recently Ghate *et al.* (2014 b, c) have investigated Kaluza-Klein dust filled universe with time dependent Λ in creation field cosmology.

In this paper, Kaluza-Klein cosmological models with variable G in Hoyle Narlikar's creation field theory of gravitation for barotropic fluid distribution have been investigated. The solution have been obtained by assuming that $G = A^l$, where A is a scale factor and l is a constant $(l=-1)$. The physical properties of the model are studied.

II. Hoyle-Narlikar Theory

The Einstein field equations are modified by Hoyle and Narlikar [3-5] through the introduction of a massless scalar field usually called Creation field *viz.* C-field. The modified field equations are

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi G\left[\underset{(m)}{T_i^j} + \underset{(c)}{T_i^j}\right], \tag{1}$$

where $\underset{(m)}{T_i^j}$ is a matter tensor for perfect fluid of Einstein's theory given by

$$\underset{(m)}{T_i^j} = (\rho + p) v_i v^j - p g_i^j \tag{2}$$

and $\underset{(c)}{T_i^j}$ is a matter tensor due to C-field given by

Ref

3. Alvarez, E. and Gavelo, M. B.: Phys. Rev. Lett., **51**, 931 (1983).

Notes

$$\underset{(c)}{T_i^{\,j}} = -f\left(C_i C^j - \frac{1}{2} g_i^{\,j} C^\alpha C_\alpha\right). \tag{3}$$

Here ρ is the energy density of massive particles and p is the pressure. v_i are co-moving four velocities which obeys the relation $v_i v^j = 1$, $v_\alpha = 0$, $\alpha = 1, 2, 3$. $f > 0$ is the coupling constant between matter and creation field and $C_i = \frac{dC}{dx^i}$.

As T^{00} has negative value (i.e. $T^{00} < 0$), the *C*-field has negative energy density producing repulsive gravitational field which causes the expansion of the universe. Thus the energy conservation law reduces to

$$^{(m)}T^{ij}{}_{;j} = -{}^{(c)}T^{ij}{}_{;j} = fC^i C^j{}_{;j}\,, \tag{4}$$

i.e. the matter creation through a non-zero left hand side is possible while conserving the overall energy and momentum.
The above equation is identical with

$$mg_{ij}\frac{dx^i}{ds} - C_j = 0\,, \tag{5}$$

which indicates the 4-momentum of the created particle is compensated by 4-momentum of the *C*-field. In order to maintain the balance, the *C*-field must have negative energy.
Further, the *C*-field satisfies the source equation

$$f\,C^i{}_{;i} = J^i{}_{;i} \quad \text{and} \quad J^i = \rho\frac{dx^i}{ds} = \rho v^i\,, \tag{6}$$

where ρ is the homogeneous mass density.
The conservation equation for *C*-field is given by

$$\left(8\pi G T_i^{\,j}\right)_{;j} = 0\,. \tag{7}$$

The physical quantities in cosmology are the expansion scalar θ, the mean anisotropy parameter Δ, the shear scalar σ^2 and the deceleration parameter q defined as

$$\theta = 3H\,, \tag{8}$$

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2, \tag{9}$$

$$\sigma^2 = \frac{1}{2}\left(\sum H_i^{\,2} - 3H^2\right) = \frac{3}{2}\Delta H^2, \tag{10}$$

$$q = -\frac{\ddot{R}/R}{\dot{R}^2/R^2}\,, \tag{11}$$

where H is a Hubble parameter.

III. Metric and Field Equations

Kaluza-Klein metric is considered in the form

$$ds^2 = dt^2 - A^2\left[dx^2 + dy^2 + dz^2\right] - B^2 du^2\,, \tag{12}$$

where A, B are scale factors and are functions of cosmic time t and u is a space-time co-ordinate.

It is assumed that creation field C is a function of time only $i.e.\ C(x,t) = C(t)$ and $\underset{(m)}{T_i^j} = (\rho, -p, -p, -p, -p)$.

The field equations (1) for metric (12) with the help of equations (2) and (3) are given by

$$3\frac{\dot{A}^2}{A^2} + 3\frac{\dot{A}\dot{B}}{AB} = 8\pi G\left(\rho - \frac{1}{2} f\,\dot{C}^2\right) \tag{13}$$

and

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + \frac{\ddot{B}}{B} + 2 = 8\pi G\left(\frac{1}{2} f\,\dot{C}^2 - p\right), \tag{14}$$

where the overdot $\left(\dot{\ }\right)$ denotes partial differentiation with respect to t.

The conservation equation (7) for metric (12) is

$$8\pi\dot{G}\left(\rho - \frac{1}{2} f\dot{C}^2\right) + 8\pi G\left[\dot{\rho} - f\dot{C}\ddot{C} + \rho\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) - f\dot{C}^2\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) + p\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\right] = 0. \tag{15}$$

IV. Solutions of Field Equations

The field equations (13) and (14) are two independent equations in five unknowns A, B, p, ρ and G. Hence two additional conditions may be used to obtain the solution.

We assume that the expansion θ is proportional to shear scalar σ. This condition leads to

$$B = A^n, \quad n \neq 1 \tag{16}$$

where n is a proportionality constant.

The motive behind assuming condition is explained with reference to Thorne (1967), the observations of the velocity red-shift relation for extra galactic sources suggest that Hubble expansion of the universe is isotropic today within ≈ 30 percent (Kantowski and Sachs (1966), Kristian and Sachs (1966)). To put more precisely, red shift place the limit $\frac{\sigma}{\theta} \leq 0.3$ on the ratio of shear σ to Hubble constant H in the neighborhood of our galaxy today. Collin *et al.* (1980) have pointed that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies that the condition $\frac{\sigma}{\theta}$ is constant.

With the help of (16), field equations (13), (14) and (15) take the form

$$3(n+1)\frac{\dot{A}^2}{A^2} = 8\pi G\left(\rho - \frac{1}{2} f\,\dot{C}^2\right), \tag{17}$$

$$(n+2)\frac{\ddot{A}}{A} + \left(n^2 + n + 1\right)\frac{\dot{A}^2}{A^2} = 8\pi G\left(\frac{1}{2} f\,\dot{C}^2 - p\right), \tag{18}$$

and

$$8\pi\dot{G}\left(\rho - \frac{1}{2} f\dot{C}^2\right) + 8\pi G\left(\dot{\rho} - f\dot{C}\ddot{C} + \rho(n+3)\frac{\dot{A}}{A} - f\dot{C}^2(n+3)\frac{\dot{A}}{A} + p(n+3)\frac{\dot{A}}{A}\right) = 0. \tag{19}$$

Notes

Notes

Following Hoyle and Narlikar, the source equation of C-field: $C^i_{;i}=0$ leads to $C=t$ for large r. Thus $\dot{C}=1$.
Using $\dot{C}=1$, equation (17) leads to

$$8\pi G\rho=(3n+1)\frac{\dot{A}^2}{A^2}+4\pi Gf\ . \tag{20}$$

Using $\dot{C}=1$ and barotropic condition $p=\gamma\rho$ in (18), we have

$$(n+2)\frac{\ddot{A}}{A}+(n^2+n+1)\frac{\dot{A}^2}{A^2}=4\pi Gf-8\pi G\gamma\rho\ , \tag{21}$$

where $0\le\gamma\le1$.
Multiplying equation (20) by γ and adding (21) gives

$$(n+2)\frac{\ddot{A}}{A}+\left[(n^2+n+1)+3(n+1)\gamma\right]\frac{\dot{A}^2}{A^2}=(1-\gamma)4\pi Gf\ . \tag{22}$$

To obtain the deterministic solution, we assume

$$G=A^l\ , \tag{23}$$

where l is a constant and A is the scale factor.
Using equation (23) in equation (22), we get

$$2\ddot{A}+2\left[\frac{n^2+(1+3\gamma)n+(1+3\gamma)}{n+2}\right]\frac{\dot{A}^2}{A^2}=\frac{2(1-\gamma)4\pi f}{(n+2)}A^{l+1}\ . \tag{24}$$

Let $\dot{A}=F(A)$,

Thus $\ddot{A}=FF'$ with $F'=\dfrac{dF}{dA}$.

Using this in equation (24), it reduces to

$$\frac{dF^2}{dA}+2\left[\frac{n^2+n(1+3\gamma)+(1+3\gamma)}{n+2}\right]\frac{F^2}{A}=\frac{2(1-\gamma)4\pi f}{(n+2)}A^{l+1}\ , \tag{25}$$

which on simplification gives

$$F^2=2\frac{4\pi f(1-\gamma)}{(n+2)}.\frac{A^{l+2}}{l+2+2\left[\frac{n^2+n(1+3\gamma)+(1+3\gamma)}{n+2}\right]}\ . \tag{26}$$

The integration constant has been taken zero for simplicity.
Equation (26) leads to

$$\frac{dA}{\sqrt{A^{l+2}}}=\sqrt{\frac{2(1-\gamma)4\pi f}{(n+2)\left[l+2+2\left(\frac{n^2+n(1+3\gamma)+(1+3\gamma)}{n+2}\right)\right]}}\,dt\ . \tag{27}$$

To obtain the determinate value of A in terms of cosmic time t, let $l=-1$.
Putting $l=-1$ in (27), we have

$$\frac{dA}{\sqrt{A}}=\sqrt{\frac{2(1-\gamma)4\pi f}{(n+2)\left[1+2\left(\frac{n^2+n(1+3\gamma)+(1+3\gamma)}{n+2}\right)\right]}}\,dt\ . \tag{28}$$

On integration equation (28) gives

$$A=(at+b)^2, \tag{29}$$

where $$a=\frac{1}{2}\sqrt{\frac{2(1-\gamma)4\pi f}{(n+2)\left[1+2\left(\frac{n^2+n(1+3\gamma)+(1+3\gamma)}{n+2}\right)\right]}}. \tag{30}$$

and $$b=\frac{N}{2}. \tag{31}$$

Here N is a constant of integration.

Thus we have $$G=A^{-1}=(at+b)^{-2}. \tag{32}$$

Using equations (29) and (32), equation (20) simplifies to

$$8\pi\rho=12a^2(n+1)+4\pi f. \tag{33}$$

With the help of (29), the metric (12) leads to

$$ds^2=dt^2-(at+b)^4[dx^2+dy^2+dz^2]-(at+b)^{4n}du^2. \tag{34}$$

Using $p=\gamma\rho$, equation (19) leads to

$$8\pi(G\dot{\rho}+\dot{G}\rho)-4\pi\dot{G}f\dot{C}^2-8\pi Gf\dot{C}\ddot{C}-8\pi G(n+3)f\dot{C}^2\frac{\dot{A}}{A}+8\pi G(n+3)\rho\frac{\dot{A}}{A}(1+\gamma)=0. \tag{35}$$

With the help of equations (29) and (32), equation (33) yields

$$\frac{d\dot{C}^2}{dt}+(2n+5)\frac{2a}{(at+b)}\dot{C}^2=\frac{a[(2n+4)+(2n+6)\gamma]}{(at+b)}\left[\frac{3a^2(n+1)}{\pi f}+1\right]. \tag{36}$$

To reach the deterministic value of $\dot{C}$, we assume $a=1$ and $b=0$.
Thus (36) leads to

$$\frac{d\dot{C}^2}{dt}+\frac{2(2n+5)}{t}\dot{C}^2=\frac{[(2n+4)+(2n+6)\gamma]}{t}\left[\frac{3(n+1)}{\pi f}+1\right]. \tag{37}$$

On integration equation (37) reduces to

$$\dot{C}^2t^{4n+10}=[(2n+4)+(2n+6)\gamma]\left[\frac{3(n+1)}{\pi f}+1\right]\int\frac{1}{t}t^{4n+10}dt. \tag{38}$$

Simplifying equation (38), we get

$$\dot{C}=\sqrt{\frac{(n+2)+(n+3)\gamma}{4(2n+5)}\left[\frac{3(n+1)}{\pi f}+1\right]}. \tag{39}$$

On integration of (39), we get

$$C=t\sqrt{\frac{(n+2)+(n+3)\gamma}{4(2n+5)}\left[\frac{3(n+1)}{\pi f}+1\right]}. \tag{40}$$

Notes

Notes

Taking $\pi f = \left[\dfrac{3(n+1)}{\dfrac{4(2n+5)}{(n+2)+(n+3)\gamma} - 1} \right]$, we find $\dot{C} = 1$, which agrees with the value used in the source equation. Thus creation field C is proportional to time t and the metric (12) for the constraints mentioned above, leads to

$$ds^2 = dt^2 - t^4\left(dx^2 + dy^2 + dz^2\right) - t^{4n} du^2 . \tag{41}$$

For the model (41), the physical parameters homogeneous mass density (ρ), Gravitational constant (G), the scale factor (A) and the deceleration parameter (q) are given by

Energy density (ρ),

$$8\pi\rho = 12(n+1)\left[1 + \frac{(n+2)+(n+3)\gamma}{(7n+18)-(n+3)\gamma}\right]. \tag{42}$$

Gravitational constant G,

$$G = t^{-2}, \tag{43}$$

Scale factor A,

$$A = t^2 , \tag{44}$$

Spatial Volume,

$$V = t^{2n+6}$$

Expansion Scalar (θ),

$$\theta = \frac{2(n+3)}{t}, \tag{45}$$

Anisotropic parameter (Δ),

$$\Delta = \frac{3\left[(n+2)^2 + 3\right]}{4(n+3)^2}, \tag{46}$$

Shear Scalar (σ),

$$\sigma^2 = \frac{6\left[(n+2)^2 + 3\right]}{t^2}, \tag{47}$$

Deceleration parameter (q),

$$q = -\frac{1}{2}. \tag{48}$$

Physical Behavior of the Model:

From equations (16) and (29), the spatial scale factors are finite at the initial epoch $t = 0$, hence the model has a point type singularity (MacCallum (1971)). From equations (45) and (46), we observe that the spatial volume is zero and expansion scalar is infinite at $t = 0$ which show that the universe starts evolving with zero volume at $t = 0$ which is big bang scenario.

Notes

In fig. 1, the plot of Gravitational constant versus cosmic time t has shown. We observed that, the gravitational constant G is initially infinite for the model. The Gravitational constant G is a decreasing function of time and approaches to zero for large values of t. In most variable G cosmologies (Weinberg (1972), Norman (1986)) G is a decreasing function of time. But the possibility of an increasing G has also been suggested by several authors (Abdel-Rahaman, A. M. M. (1990), Pradhan *et al.* (2007), Singh *et al.* (2008), Singh and Kale (2009), Singh *et al.* (1998)).

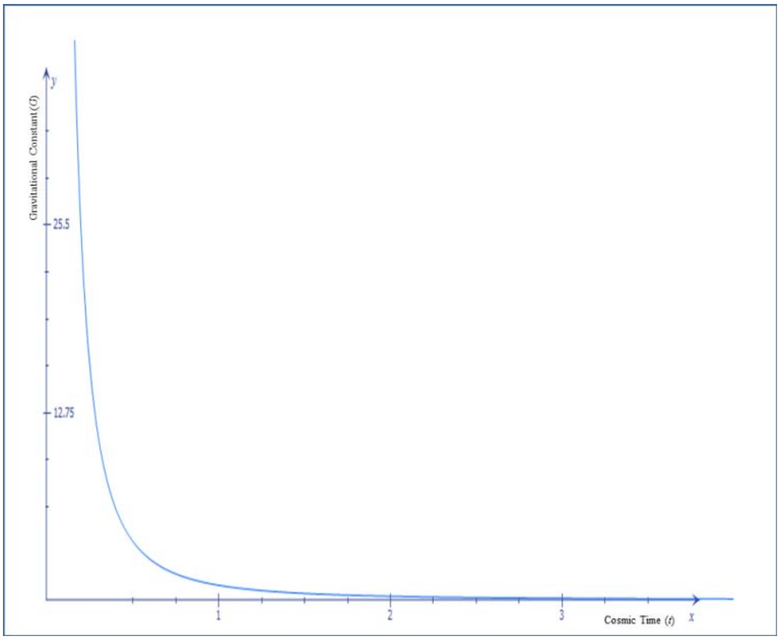

Figure 1 : Plot of Gravitational Constant (G) versus Cosmic Time (t)

From equations (46) and (48), the mean anisotropy parameter Δ is constant and $\frac{\sigma^2}{\theta^2} = constant\ (\neq 0)$ is also constant, hence the model is anisotropic throughout the evolution of the universe (*i.e.* the model does not approach isotropy).

From equation (42), it is observed that the energy density is constant throughout the evolution of the universe hence the model represents steady state universe.

From equation (48), the value of deceleration parameter is $q = -\frac{1}{2}$, which lies between -1 and 0, hence the model is accelerating throughout the evolution.

V. Conclusion

Kaluza-Klein cosmological models have been investigated in Hoyle Narlikar's Creation field theory of gravitation. The source of energy momentum tensor is considered as barotropic fluid with varying gravitational constant G. It is worth to mention that the model obtained is point type singular, expanding, shearing. The mean anisotropic parameter is constant and $\frac{\sigma^2}{\theta^2} = \text{constant}\ (\neq 0)$, hence the model is anisotropic throughout the evolution of the universe. (i.e. the model is not isotropic for the complete evolution). The deceleration parameter is constant $\left(q = -\frac{1}{2}\right)$, which lies between -1 and 0, hence the model is accelerating throughout the evolution.

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