Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 15 Issue 1 Version 1.0 Year 2015
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Special Pairs of Pythagorean Triangles and Dhuruva Number

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GJSFR-F Classification : FOR Code : MSC 2010: 12D15

SPECIALPAIRSOFPYTHAGOREANTRIANGLESANDDHURUVANUMBER

Strictly as per the compliance and regulations of :

Global Journal of Science Frontier Research (F) Volume XV Issue I Version I Year 2015

Ref

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Special Pairs of Pythagorean Triangles and Dhuruva Number

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Abstract- We present pairs of Pythagorean triangles, such that in each pair, the difference between their perimeters is two times the Dhuruva number. Also we present the number of pairs of primitive and non-primitive Pythagorean triangles.

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I. Introduction

The fascinating branch of mathematics is the theory of numbers where in Pythagorean triangles have been a matter of interest to various mathematicians and to the lovers of mathematics, because it is a treasure house in which the search for many hidden connection is a treasure hunt. For a rich variety of fascinating problems one may refer [1-17].A careful observer of patterns may note that there is a one to one correspondence between the polygonal numbers and the number of sides of the polygon. Apart from the above patterns we have some more fascinating patterns of numbers namely Jarasandha numbers, Nasty numbers and Dhuruva numbers. These numbers have been presented in [18-21].

In [22-24], special Pythagorean triangles connected with polygonal numbers and Nasty numbers are obtained. Recently in [25], special Pythagorean triangles in connection with Hardy Ramanujan number 1729 are exhibited. In [26], Pythagorean triangles in connections with 5-digit Dhuruva numbers are presented.

In this communication, we search for pairs of Pythagorean triangles, such that in each pair, the difference between their perimeters is two times the Dhuruva number.

II. Basic Definitons

Definition 2.1

The ternary quadratic Diophantine equation given by $x^2+y^2=z^2$ is known as Pythagorean equation where x, y, z are natural numbers. The above equation is also referred to as Pythagorean triangle and denote it by T(x,y,z).

Also, in Pythagorean triangle T(x,y,z) : $x^2+y^2=z^2$, x and y are called its legs and z its hypotenuse.

Definition 2.2

Most cited solution of the Pythagorean equation is $x=m^2-n^2, y=2mn, z=m^2+n^2$, where $m>n>0$. This solution is called primitive, if m,n are of opposite parity and gcd(m,n)=1.

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Definition 2.3: Dhuruva numbers

The numbers which do not change when we perform a single operation or a sequence of operations are known as Dhuruva numbers.

III. Method of Analysis

Let PT_1, PT_2 be two distinct Pythagorean triangles with generators m, q (m > q > 0), and p, q (p > q > 0) respectively.Let P_1, P_2 be the perimeters of PT_1, PT_2 such that P_1 - P_2 =2 times the 3-digit Dhuruva numer 495.

The above relation leads to the equation

$$(2m+q)^2 - (2p+q)^2 = 1980 \quad (1)$$

After performing numerical computations, it is noted that there are 82 distinct values for m, p and q satisfying (1). For simplicity and clear understanding, we have presented below in table1 the values of m, p, q, P_1 and P_2.

S.no	m	q	p	P_1	P_2	$(P_1-P_2)/2$
1	166	164	165	109560	108570	495
2	167	162	166	109886	108896	495
3	168	160	167	110208	109218	495
4	169	158	168	110526	109536	495
5	170	156	169	110840	109850	495
6	171	154	170	111150	110160	495
7	172	152	171	111456	110466	495
8	173	150	172	111758	110768	495
9	174	148	173	112056	111066	495
10	175	146	174	112350	111360	495
11	176	144	175	112640	111650	495
12	177	142	176	112926	111936	495
13	178	140	177	113208	112218	495
14	179	138	178	113486	112496	495
15	180	136	179	113760	112770	495
16	181	134	180	114030	113040	495
17	182	132	181	114296	113306	495
18	183	130	182	114558	113568	495
19	184	128	183	114816	113826	495
20	185	126	184	115070	114080	495
21	186	124	185	115320	114330	495
22	187	122	186	115566	114576	495
23	188	120	187	115808	114818	495
24	189	118	188	116046	115056	495
25	190	116	189	116280	115290	495
26	191	114	190	116510	115520	495
27	192	112	191	116736	115746	495
28	193	110	192	116958	115968	495
29	194	108	193	117176	116186	495
30	195	106	194	117390	116400	495

Notes

Notes

31	196	104	195	117600	116610	495
32	197	102	196	117806	116816	495
33	198	100	197	118008	117018	495
34	199	98	198	118206	117216	495
35	200	96	199	118400	117410	495
36	201	94	200	118590	117600	495
37	202	92	201	118776	117786	495
38	203	90	202	118958	117968	495
39	204	88	203	119136	118146	495
40	205	86	204	119310	118320	495
41	206	84	205	119480	118490	495
42	207	82	206	119646	118656	495
43	208	80	207	119808	118818	495
44	209	78	208	119966	118976	495
45	210	76	209	120120	119130	495
46	211	74	210	120270	119280	495
47	212	72	211	120416	119426	495
48	213	70	212	120558	119568	495
49	214	68	213	120696	119706	495
50	215	66	214	120830	119840	495
51	216	64	215	120960	119970	495
52	217	62	216	121086	120096	495
53	218	60	217	121208	120218	495
54	219	58	218	121326	120336	495
55	220	56	219	121440	120450	495
56	221	54	220	121550	120560	495
57	222	52	221	121656	120666	495
58	223	50	222	121758	120768	495
59	224	48	223	121856	120866	495
60	225	46	224	121950	120960	495
61	226	44	225	122040	121050	495
62	227	42	226	122126	121136	495
63	228	40	227	122208	121218	495
64	229	38	228	122286	121296	495
65	230	36	229	122360	121370	495
66	231	34	230	122430	121440	495
67	232	32	231	122496	121506	495
68	233	30	232	122558	121568	495
69	234	28	233	122616	121626	495
70	235	26	234	122670	121680	495
71	236	24	235	122720	121730	495
72	237	22	236	122766	121776	495
73	238	20	237	122808	121818	495
74	239	18	238	122846	121856	495

75	240	16	239	122880	121890	495
76	241	14	240	122910	121920	495
77	242	12	241	122936	121946	495
78	243	10	242	122958	121968	495
79	244	8	243	122976	121986	495
80	245	6	244	122990	122000	495
81	246	4	245	123000	122010	495
82	247	2	246	123006	122016	495

Notes

Thus it is seen that there are 82 pairs of Pythagorean triangles such that for each pair the difference in the perimeters is twice the 3- digit Dhuruva number 495.

Out of these 82 pairs of Pythagorean triangles 6-pairs are non-primitive and in each of the remaining pairs, one of the triangles is primitive and the other is non-primitive triangle.

A similar observation, regarding 5- digit and 6- digit dhuruva numbers are exhibited in the table2 below.

Dhuruva number	**pairs of Pythagorean triangles**	**pairs of non-primitve Pythagorean triangles**	**pairs of primitve and non-primitve Pythagorean triangles**
53955	8992	908	8084
59995	9998	2111	7887
549945	91657	1	91656

IV. Conclusion

One may search for the connections between the pairs of Pythagorean triangles and other Dhuruva numbers of higher order.

V. Acknowledgement

The financial support from the UGC, New Delhi (F-MRP-5122/14(SERO/UGC) dated march 2014) for a part of this work is gratefully acknowledged.

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