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The Modified Simple Equation Method and its Applications in Mathematical Physics and Biology

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2. W. Maliet, Solitary wave solutions of nonlinear wave equation, Am. J. Phys., 60 (1992) 650-654.

The Modified Simple Equation Method and its Applications in Mathematical Physics and Biology

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I. Introduction

No one can deny the important role which played by the nonlinear partial differential equations in the description of many and a wide variety of phenomena not only in physical phenomena, but also in plasma, fluid mechanics, optical fibers, solid state physics, chemical kinetics and geochemistry phenomena. So that, during the past five decades, a lot of method was discovered by a diverse group of scientists to solve the nonlinear partial differential equations. For example, tanh - sech method [2]-[4], extended tanh - method [5]-[7], sine - cosine method [8]-[10], homogeneous balance method [11], the $\exp(\varphi(\xi))$-expansion Method [12], Jacobi elliptic function method [13]-[16], F-expansion method [17]-[19], exp-function method [20] and [21], trigonometric function series method [22], $(\frac{G'}{G})-$expansion method [23]-[26], the modified simple equation method [27]-[32] and so on.

The objective of this article is to apply the modified simple equation method for finding the exact traveling wave solution of some nonlinear partial differential equations, namely the system of shallow water wave equations [33], modified Benjamin-Bona-Mahony equation [34] and nonlinear dynamics of microtubules-A new model [35], which play an important role in mathematical physics.

The rest of this paper is organized as follows: In section 2, we give the description of the modified simple equation method. In section 3, we use this method to find the exact solutions of the nonlinear evolution equations pointed out above. In section 5, conclusions are given.

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II. Description of the Modified Simple Equation Method

Consider the following nonlinear evolution equation

$$F(u, u_t, u_x, u_y, u_{tt}, u_{xx}, u_{yy}....) = 0, \tag{2.1}$$

where F is a polynomial in $u(x,t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method [27]-[32]:

Step 1. We use the wave transformation

$$u(x, y, t) = u(\xi), \qquad \xi = (x + y - ct), \tag{2.2}$$

where c is a nonzero constant, to reduce Eq.(2.1) to the following ODE:

$$P(u, u', u'', u''', ...) = 0, \tag{2.3}$$

where P is a polynomial in $u(\xi)$ and its total derivatives, while $' = \frac{d}{d\xi}$.

Step 2. Suppose that the solution of Eq.(2.3) has the formal solution:

$$u(\xi) = \sum_{k=0}^{N} A_k \left[\frac{{}'(\xi)}{(\xi)}\right]^k, \tag{2.4}$$

where A_k are arbitrary constants to be determined, such that $A_N \neq 0$, while the function $\psi(\xi)$ is an unknown function to be determined later, such that $\psi' \neq 0$.

Step 3. Determined the positive integer N in Eq.(2.4) by considering the homogenous balance between the highest order derivatives and the nonlinear terms in Eq.(2.3). Moreover precisely, we define the degree of $u(\xi)$ as $D(u(\xi)) = m$, which gives rise to degree of other expression as follows:

$$D\left(\frac{d^q u}{d\xi^q}\right) = n + q, \; D\left(u^p \left(\frac{d^q u}{d\xi^q}\right)^s\right) = np + s\,(n+q)\,.$$

Step 4. Substitute Eq.(2.4) into Eq.(2.3), we calculate all the necessary derivative $u', u'', ...$ of the function $u(\xi)$ and we account the function $\psi(\xi)$. As a result of this substitution, we get a polynomial of $\psi^{-j}(j = 0, 1, 2, ...)$. In this polynomial, we gather all terms of the same power of $\psi^{-j}(j = 0, 1, 2, ...)$, and we equate with zero all coe˙cient of this polynomial. This operation yields a system of equations which can be solved to find A_k and $\psi(\xi)$. Consequently, we can get the exact solution of Eq.(2.1).

III. Application

Here, we will apply the modified simple equation method described in sec.2 to find the exact traveling wave solutions and then the solitary wave solutions for the following nonlinear systems of evolution equations.

a) Example 1: The system of shallow water wave equations

We first consider the system of the shallow water wave equation[33]

$$\begin{cases} u_t + (uv)_x + v_{xxx} = 0, \\ v_t + u_x + vv_x = 0. \end{cases} \tag{3.1}$$

Using the wave transformation $u(x,t) = u(\xi), \xi = (x - ct)$ carries the partial differential equation (3.1) into the ordinary differential equaion:

$$\begin{cases} -cu' + vu' + uv' + v''' = 0, \\ u' - cv' + vv' = 0, \end{cases} \tag{3.2}$$

Ref

27. A. J. M. Jawad, M. D. Petkovic and A. Biswas, Modified simple equation method for nonlinear evolution equations, Appl. Math. Comput., 217 (2010) 869-877.

Integrating once the second ordinary differential equation with zero constant of integration, we get

$$u = cv - \frac{v^2}{2}. \tag{3.3}$$

Substituting Eq.(3.3) into the ˙rst equation of Eq.(3.2) we obtain

$$v''' + (3cv - \frac{3v^2}{2} - c^2)v' = 0. \tag{3.4}$$

Notes

Integrating Eq.(3.4) and neglecting the constant of integration, we obtain

$$v'' + \frac{3}{2}cv^2 - \frac{1}{2}v^3 - c^2 v = 0. \tag{3.5}$$

Balancing v'' and v^3 in Eq.(3.5) yields, $(N + 2 = 3N) \Longrightarrow (N = 1)$. So that, by using Eq.(2.4) we get the formal solution of Eq.(3.5)

$$v = A_0 + A_1\left(\frac{\psi'}{\psi}\right). \tag{3.6}$$

Substituting Eq.(3.6) and its derivative into Eq.(3.5) and collecting all term with the same power of ψ^{-3}, ψ^{-2}, ψ^{-1}, ψ^{0} we get:

$$\psi^{-3}:\ A_1\psi'^3\left(1 - \frac{1}{2}A_1^2\right) = 0, \tag{3.7}$$

$$\psi^{-2}:\ A_1\psi'\left[-3\psi'' + \frac{3}{2}A_1\psi'(c - A_0)\right] = 0, \tag{3.8}$$

$$\psi^{-1}:\ A_1\left[\psi''' + \psi'\left(3cA_0 - \frac{3}{2}A_0^2 - c^2\right)\right] = 0, \tag{3.9}$$

$$\psi^{0}:\ A_0\left[\frac{-1}{2}A_0^2 + \frac{3}{2}cA_0 - c^2\right] = 0. \tag{3.10}$$

From Eqs.(3.7) and (3.10), we deduce that

$$A_1 = \pm 2,\ A_0 = 0,\ A_0 = c \ and\ A_0 = 2c.$$

Let us discuss the following cases.

Case 1. If $A_0 \neq 0$.

In this case, we deduce from Eqs.(3.8) and (3.9) that :

$$\psi' = \frac{1}{c^2 + \frac{3}{2}A_0^2 - 3cA_0}\psi''', \tag{3.11}$$

and

$$\psi' = \frac{1}{\frac{c}{2}A_1 - \frac{1}{2}A_0A_1}\psi''. \tag{3.12}$$

Eqs.(3.11) and (3.12) yield

$$\frac{\psi'''}{\psi''} = E_0, \tag{3.13}$$

where $\left(E_0 = \frac{c^2+\frac{3}{2}A_0^2-3cA_0}{\frac{c}{2}A_1-\frac{1}{2}A_0A_1} \neq 0\right) \Rightarrow$ consequently, when $(A_0 = c)$ is rejected since it make $(E_0 = 0)$ whilst, when $(A_0 = 2c)$ it make $(E_0 = \mp c \neq 0)$ Integrating (3.13) and using (3.12), we deduce that

$$\psi' = c_2 exp(\mp c\,\xi), \tag{3.14}$$

where $c_2 = \frac{c_1}{\frac{c}{2}A_1 - \frac{1}{2}A_0 A_1} = \frac{c_1}{\mp c}$, and consequently, we get

$$\psi = \frac{c_2}{\mp c} exp\left(\mp c\,\xi\right) + c_3, \tag{3.15}$$

where c_1, c_2 and c_3 are arbitrary constants.
Substituting (3.14) and (3.15) into Eq.(3.6), we have the exact traveling wave solution:

$$v = 2c \mp 2c\left[\frac{exp\left(\mp c\,\xi\right)}{exp\left(\mp c\,\xi\right) + c_3}\right], \tag{3.16}$$

Notes

when $c_1 = 1$, we obtain $c_2 = E0$. So that we get the solitary wave solutions

- If $c_3 = 1$ and $c > 0$

$$v_{(1,2)} = 2c \mp c\left[1 \mp tanh\left(\frac{c}{2}\xi\right)\right], \tag{3.17}$$

- while, if $c_3 = 1$ and $c < 0$

$$v_{(3,4)} = 2c \mp c\left[1 \pm tanh\left(\frac{c}{2}\xi\right)\right], \tag{3.18}$$

- If $c_3 = -1$, and $c > 0$

$$v_{(5,6)} = 2c \mp c\left[1 \mp coth\left(\frac{c}{2}\xi\right)\right], \tag{3.19}$$

- while, if $c_3 = -1$, and c < 0

$$v_{(7,8)} = 2c \mp c\left[1 \pm coth\left(\frac{c}{2}\xi\right)\right]. \tag{3.20}$$

Case 2. If $A_0 = 0$.
In this case, we deduce from Eqs.(3.8) and (3.9) that :

$$\psi' = \frac{1}{c^2}\,\psi''', \tag{3.21}$$

and

$$\psi' = \frac{2}{cA_1}\psi''. \tag{3.22}$$

Eqs.(3.21) and (3.22) yield

$$\frac{\psi'''}{\psi''} = E_1, \tag{3.23}$$

where $\left(E_1 = \frac{2c}{A_1} = \pm c \neq 0\right)$ integrating (3.23) and using (3.22), we deduce that

$$\psi' = c_5 exp\left(\pm c\,\xi\right), \tag{3.24}$$

where $c_5 = \frac{2c_4}{cA_1} = \frac{\pm c_4}{c}$, and consequently, we get

$$\psi = \frac{\pm c_2}{c} exp\left(\pm c\,\xi\right) + c_6, \tag{3.25}$$

where c_4, c_5 and c_6 are arbitrary constants.
Substituting (3.24) and (3.25) into Eq.(3.6), we have the exact traveling wave solution:

$$v = \pm 2c\left[\frac{exp\left(\pm c\,\xi\right)}{exp\left(\pm c\,\xi\right) + c_6}\right], \tag{3.26}$$

when $c_4 = c^2$, we obtain $c_5 = E_1$. So that we get the solitary wave solutions

- If $c_6 = 1$, $c > 0$, we get

$$v_{(9,10)} = \pm c\left[1 \pm tanh\left(\frac{c}{2}\xi\right)\right], \tag{3.27}$$

Notes

- If $c_6 = 1,\ c < 0$, we get

$$v_{(11,12)} = \pm c\left[1 \mp tanh\left(\frac{c}{2}\xi\right)\right]. \tag{3.28}$$

- If $c_6 = 1,\ c > 0$

$$v_{(13,14)} = \pm c\left[1 \pm coth\left(\frac{c}{2}\xi\right)\right], \tag{3.29}$$

- While, if $c_6 = 1,\ c < 0$

$$v_{(15,16)} = \pm c\left[1 \mp coth\left(\frac{c}{2}\xi\right)\right]. \tag{3.30}$$

b) Example 2: Modified Benjamin-Bona-Mahony equation

The modified Benjamin-Bona-Mahony(MBBM)equation [34] is in the form,

$$u_t + u_x + au^2u_x + bu_{xxt} = 0, \tag{3.31}$$

where a and b are positive constants. Using the transformation $u(x,t) = u(\xi)$; $(\xi = x + kt)$ to reduce Eq.(3.31) to the following ordinary di˙erential equation

$$ku' + u' + au^2u' + bku''' = 0, \tag{3.32}$$

Integrating Eq.(3.32) with zero constant of integration we obtain

$$(k+1)u + \frac{a}{3}u^3 + bku'' = 0, \tag{3.33}$$

hence, Eq.(3.33) take the form:

$$u - \alpha u^3 + \beta u''' = 0, \tag{3.34}$$

when, $\left(\alpha = \frac{-a}{3(k+1)}\right)$ and $\left(\beta = \frac{bk}{(k+1)}\right)$. Balancing u'' with u^3 in Eq.(3.34) yield, $(N+2 = 3N)$ $\Rightarrow (N = 1)$. So that, we have the same formal solution of Eq.(3.5). Substituting Eq.(3.6) and its derivative into Eq.(3.34) and collecting all term with the same power of $\psi^{-3}, \psi^{-2}, \psi^{-1}, \psi^{0}$ we get:

$$\psi^{-3}: \ A_1\psi'^3\left[-\alpha A_1^2 + 2\beta\right] = 0, \tag{3.35}$$

$$\psi^{-2}: \ -3A_1\psi'\left[\alpha A_0 A_1\psi' + \beta\psi''\right] = 0, \tag{3.36}$$

$$\psi^{-1}: \ A_1\left[\psi'\left(1 - 3\alpha A_0^2\right) + \beta\psi'''\right] = 0, \tag{3.37}$$

$$\psi^{0}: \ A_0\left[1 - \alpha A_0^2\right] = 0. \tag{3.38}$$

From Eqs.(3.35) and (3.38), we deduce that

$$A_1 = \pm\sqrt{\frac{2\beta}{\alpha}},\ A_0 = \pm\sqrt{\frac{1}{\alpha}},\ and\ A_0 = 0.$$

Let us discuss the following cases.

case 1. when $A_0 \neq 0$.

in this case, we deduce from Eqs.(3.36) and (3.37) that

$$\psi' = \frac{-\beta}{\alpha A_0 A_1}\psi'', \tag{3.39}$$

and

$$\psi' = \frac{\beta}{3\alpha A_0^2 - 1}, \tag{3.40}$$

Eqs.(3.39) and (3.40) yield.

$$\frac{\psi'''}{\psi''} = E_2, \tag{3.41}$$

where $\left(E_2 = \frac{1-3\alpha A_0^2}{\alpha A_0 A_1} = \frac{-2}{\sqrt{2\beta}} \neq 0\right)$ and (β >0). Integrating Eq.(3.41) and using Eq.(3.39), We deduce that

$$\psi' = c_8 exp\left(\frac{-2}{\sqrt{2\beta}}\xi\right), \tag{3.42}$$

where $\left(c_8 = \frac{-\beta c_7}{\alpha A_0 A_1} = \frac{-c_7\sqrt{2\beta}}{2}\right)$ and consequently, we get

$$\psi = \frac{-c_8\sqrt{2\beta}}{2} exp\left(\frac{-2}{\sqrt{2\beta}}\xi\right) + c_9, \tag{3.43}$$

where c_7, c_8 and c_9 are arbitrary constant of integration.
Substituting Eq.(3.42) and (3.43) into Eq.(3.6) we have the exact traveling wave solution

$$u = \pm\sqrt{\frac{2\beta}{\alpha}} \mp \frac{2}{\sqrt{2\alpha\beta}}\left[\frac{exp\left(\frac{-2}{\sqrt{2\beta}}\xi\right)}{exp\left(\frac{-2}{\sqrt{2\beta}}\xi\right) + c_9}\right], \tag{3.44}$$

when $c_7 = \frac{2}{\beta}$, we obtain $c_8 = E_2$. So that we get the solitary wave solutions

- If $c_9 = 1$,

$$u_{(1,2)} = \pm\sqrt{\frac{2\beta}{\alpha}} \mp \frac{1}{\sqrt{2\alpha\beta}}\left[1 - tanh\left(\frac{1}{\sqrt{2\beta}}\xi\right)\right], \tag{3.45}$$

- If $c_9 = -1$,

$$u_{(3,4)} = \pm\sqrt{\frac{2\beta}{\alpha}} \mp \frac{1}{\sqrt{2\alpha\beta}}\left[1 - coth\left(\frac{1}{\sqrt{2\beta}}\xi\right)\right], \tag{3.46}$$

case 2. when $A_0 = 0$
In this case, we deduce from Eqs.(3.36) and (3.37) that $\psi' = 0$, and hence this case will be rejected.

c) Example 3: Nonlinear dynamics of microtubules- A new model
We consider the nonlinear dynamical equation of motion [35]

$$m\frac{\partial^2 z}{\partial t^2} - kl^2\frac{\partial^2 z}{\partial x^2} - qE - Az + Bz^3 + \gamma\frac{\partial z}{\partial t} = 0, \tag{3.47}$$

where m, k, l, q, A, B and γ are arbitrary constants to be determined later. It is well known that, for a given wave equation, a traveling wave $z(\xi)$ is a solution which depends upon x and t only through a unified variable $(\xi = \kappa x - \omega t)$, where κ and ω are constants. This allows us to obtain the final dimensionless ordinary differential equation

$$\alpha u'' - \rho u' - u + u^3 - \sigma = 0, \tag{3.48}$$

where

$$u' = \frac{du}{d\xi}, \alpha = \frac{m\omega^2 - kl^2\kappa^2}{A}, z = \sqrt{\frac{A}{B}}u\,, \rho = \frac{\gamma\omega}{A}\, and\, \sigma = \frac{qE}{A\sqrt{\frac{A}{B}}}.$$

Balancing between u'' and u^3 yield, $(N + 2 = 3N) \Rightarrow (N = 1)$. So that, we have the same formal solution of Eq.(3.5). Substituting Eq.(3.6) and its derivative into Eq.(3.48) and collecting all of the term with the same power of ψ^{-3}, ψ^{-2}, ψ^{-1}, ψ^0 we get:

Notes

$$\psi^{-3}: \ A_1 \psi'^3 \left[2\alpha + A_1^2\right] = 0, \tag{3.49}$$

$$\psi^{-2}: \ A_1 \psi' \left[-3\alpha \psi'' + \psi' \left(\rho + 3A_0 A_1\right)\right] = 0, \tag{3.50}$$

$$\psi^{-1}: \ A_1 \left[\alpha \psi''' - \rho \psi'' - \psi' \left(1 - 3A_0^2\right)\right] = 0, \tag{3.51}$$

$$\psi^{0}: \ A_0^3 - A_0 - \sigma = 0. \tag{3.52}$$

Notes

From Eqs. (3.49) and (3.52), we deduce that

$$A_1 = \pm\sqrt{-2\alpha}, \ \ where \ \alpha < 0,$$

$$A_0 = \frac{1}{6}\sqrt[3]{108\,\sigma + 12\sqrt{-12 + 81\,\sigma^2}} + 2\,\frac{1}{\sqrt[3]{108\,\sigma + 12\sqrt{-12 + 81\,\sigma^2}}}$$

and

$$A_0 = \frac{-1}{12}\sqrt[3]{108\,\sigma + 12\sqrt{-12 + 81\,\sigma^2}} - \frac{1}{\sqrt[3]{108\,\sigma + 12\sqrt{-12 + 81\,\sigma^2}}}$$

$$\pm\frac{\sqrt{3}}{2}\left(\frac{1}{6}\sqrt[3]{108\,\sigma + 12\sqrt{-12 + 81\,\sigma^2}} - 2\,\frac{1}{\sqrt[3]{108\,\sigma + 12\sqrt{-12 + 81\,\sigma^2}}}\right).$$

where $\left(-12 + 81\,\sigma^2\right) > 0$.

So that, we deduce from Eqs.(3.50) and (3.51) that

$$\psi' = \frac{3\alpha}{\rho + A_0 A_1}\psi'', \tag{3.53}$$

and

$$\alpha\psi''' - \rho\psi'' - \left(1 - 3A_0^2\right)\psi' = 0, \tag{3.54}$$

and hence, Eqs. (3.53) and (3.54) yield

$$\frac{\psi'''}{\psi''} = E_3, \tag{3.55}$$

where $E_3 = \left(\frac{\rho}{\alpha} + \frac{3\left(1 - 3A_0^2\right)}{\rho + 3A_0 A_1}\right)$.

Integrating Eq.(3.55) and using (3.53), we deduce that

$$\psi' = c_{11} exp\left(E_3 \xi\right), \tag{3.56}$$

where $c_{11} = \frac{3\alpha c_{10}}{\rho + 3A_0 A_1}$, and consequently, we get

$$\psi = \frac{c_{11}}{E_3} exp\left(E_3 \xi\right) + c_{12} \tag{3.57}$$

where c_{10}, c_{11} and c_{12} are arbitrary constants of integration.

Substituting Eqs.(3.56) and (3.57) into (3.6), we have the exact solution:

$$u = A_0 \pm E_3\sqrt{-2\alpha}\left[\frac{exp\left(E_3\xi\right)}{exp\left(E_3\xi\right) + c_{12}}\right], \tag{3.58}$$

when $c_{11} = E_3$, we have the solitary wave solutions.

- If $c_{12} = 1, \ E_3 > 0$

$$u_1 = A_0 \pm \frac{E_3\sqrt{-2\alpha}}{2}\left[1 + tanh\left(\frac{E_3}{2}\xi\right)\right]. \tag{3.59}$$

- While, if $c_{12} = 1,\ E_3 < 0$

$$u_2 = A_0 \pm \frac{E_3\sqrt{-2\alpha}}{2}\left[1 - tanh\left(\frac{E_3}{2}\xi\right)\right]. \tag{3.60}$$

- If $c_{12} = -1,\ E_3 > 0$

$$u_3 = A_0 \pm \frac{E_3\sqrt{-2\alpha}}{2}\left[1 + coth\left(\frac{E_3}{2}\xi\right)\right]. \tag{3.61}$$

- ˙While, if $c_{12} = -1,\ E_3 < 0$

$$u_4 = A_0 \pm \frac{E_3\sqrt{-2\alpha}}{2}\left[1 - coth\left(\frac{E_3}{2}\xi\right)\right]. \tag{3.62}$$

- Note that:

All the obtained results have been checked with Maple 16 by putting them back into the original equation and found correct.

IV. Physical Interpretations of the Solutions

In this section, we depict the graph and signify the obtained solutions to each of the system of shallow water wave equations, modified Benjamin-Bona-Mahony equation and nonlinear dynamics o microtubules-a new model. Now, we will discuss all possible physical signi˙cances for parameter. For this value of parameter c > 0 the solution $v_{(1)}$ and $v_{(2)}$ in Eq.(3.17) represent kink shape soliton solution also when parameter c < 0 the solution $v_{(3)}$ and $v_{(4)}$ in Eq.(3.18) represent kink shape soliton solution, when parameter c > 0the solution $v_{(5)}$ and $v_{(6)}$ in Eq.(3.19) represent singular soliton solution while, when parameter c < 0the solution $v_{(7)}$ and $v_{(8)}$ in Eq.(3.20) represent dark singular soliton solution, when parameter c > 0the solution $v_{(9)}$ and $v_{(10)}$ in Eq.(3.27) represent kink shape soliton solution also when parameter c < 0the solution $v_{(11)}$ and $v_{(12)}$ in Eq.(3.28) represent kink shape soliton solution, when parameter c > 0the solution $v_{(13)}$ and $v_{(14)}$ in Eq.(3.29) represent singular soliton solution also when parameter c < 0the solution $v_{(15)}$ and $v_{(16)}$ in Eq.(3.30) represent dark singular soliton solution, when parameter $\beta = 2, \alpha = 4\,, k = 1$ the solution $u_{(1)}$ and $u_{(2)}$ in Eq.(3.45) represent kink shape soliton solution, when parameter $\beta = 2,\ \alpha = 4\,, k = 1$ the solution $u_{(3)}$ in Eq.(3.46) represent dark singular soliton solution and $u_{(4)}$ in Eq.(3.46) represent bell singular soliton solution, when parameter $\alpha = -2,\ A_1 = 2,\ \sigma = 1,\ A_0 = 1.32,\ \rho = -10,\ E_3 = 10.87$ the solution $u_{(1)}$ and $u_{(2)}$ in Eq.(3.59) represent kink shape soliton solution, when parameter $\alpha = -2,\ A_1 = 2,\ \sigma = 1,\ A_0 = 1.32,\ \rho = 4,\ E_3 = -3.07$ the solution $u_{(3)}$ and u(4) in Eq.(3.60) represent kink shape soliton solution, when parameter $\alpha = -2,\ A_1 = 2,$ $\sigma = 1,\ A_0 = 1.32,\ \rho = -10,\ E_3 = 10.87$ the solution $u_{(5)}$ in Eq.(3.61) represent dark singular shape soliton solution and $u_{(6)}$ in Eq.(3.61) represent singular bell shape soliton solution, and when parameter $\alpha = -2,\ A_1 = 2,\ \sigma = 1,\ A_0 = 1.32\quad = 4,\ E_3 = -3.07$ the solution $u_{(7)}$ in Eq.(3.62) represent dark singular shape soliton solution and $u_{(8)}$ in Eq.(3.62) represent bell singular shape soliton solution.

Notes

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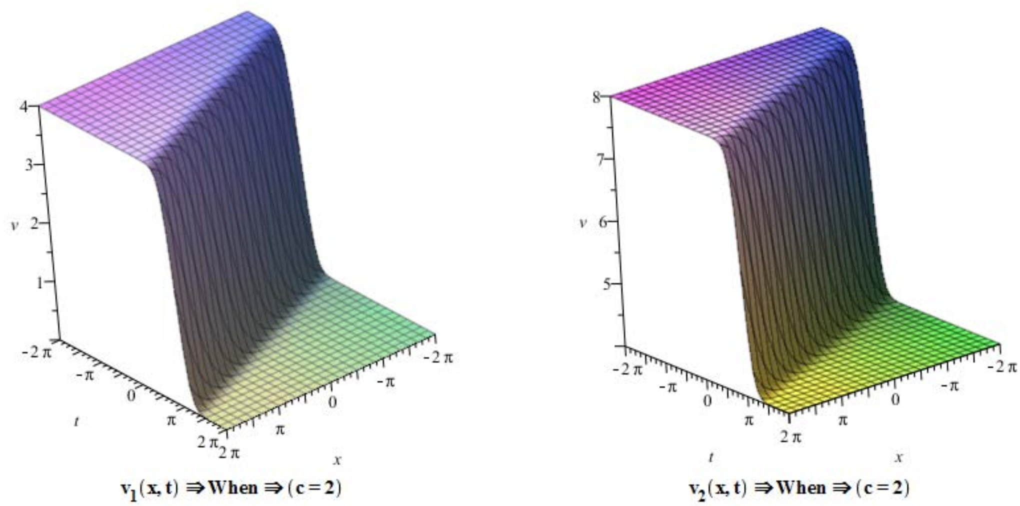
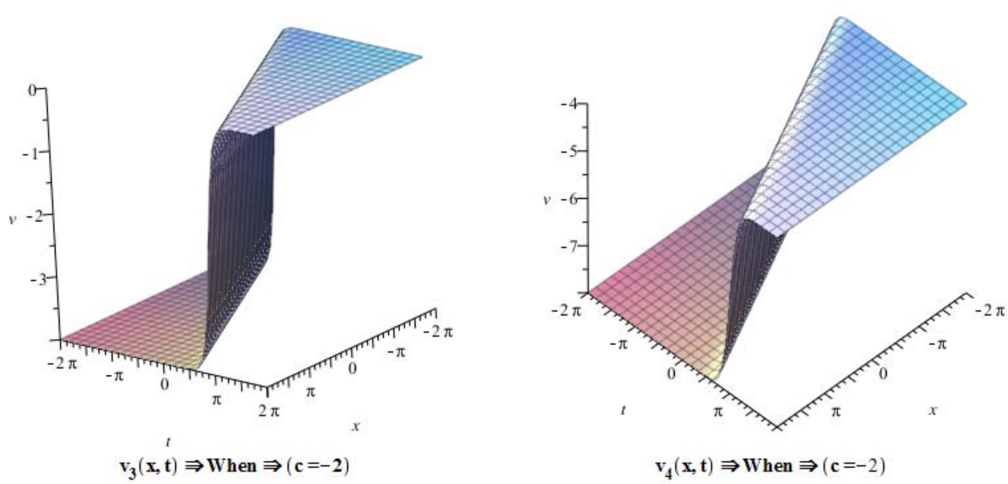


(a) Eq.(3.17)

(b) Eq.(3.17)

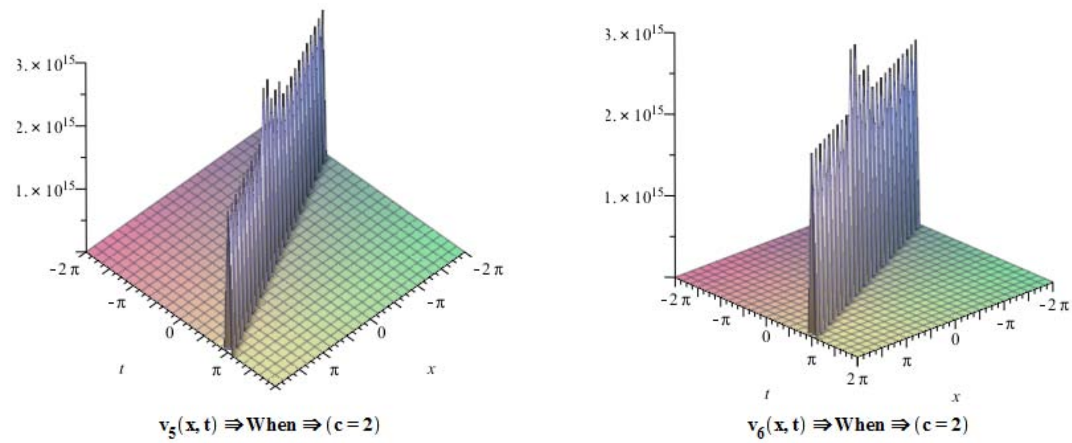
Figure 1 : The Solitary wave solution of Eqs.(3.17)



(a) Eq.(3.18)

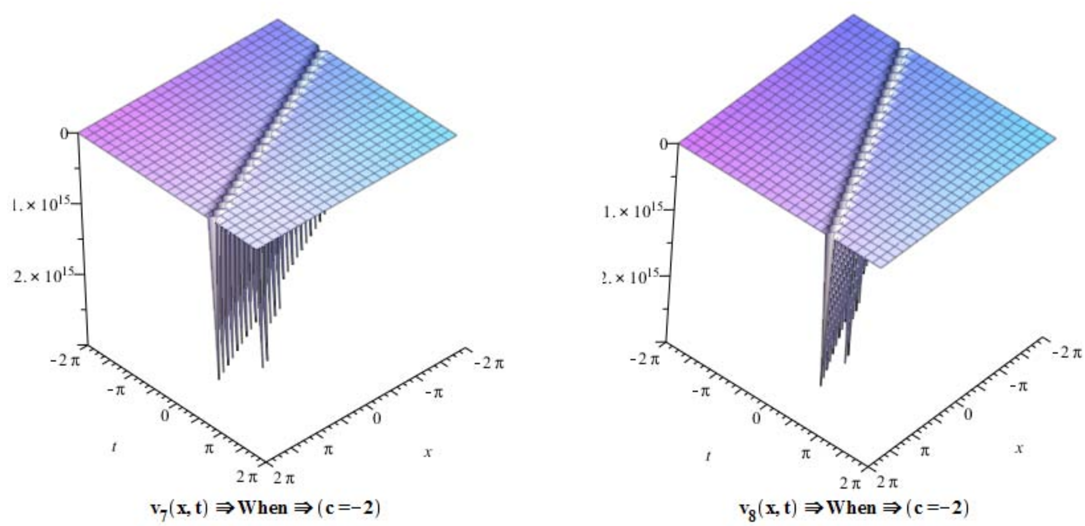
(b) Eq.(3.18)

Figure 2 : The Solitary wave solution of Eqs.(3.18)



(a) Eq.(3.19) (b) Eq.(3.19)

Figure 3 : The Solitary wave solution of Eqs.(3.19)



(a) Eq.(3.20) (b) Eq.(3.20)

Figure 4 : The Solitary wave solution of Eqs.(3.20)

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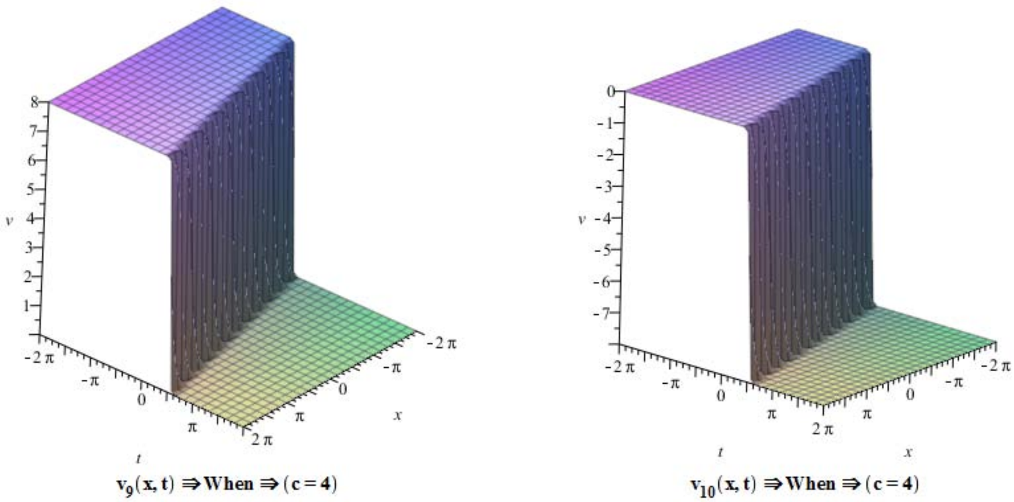
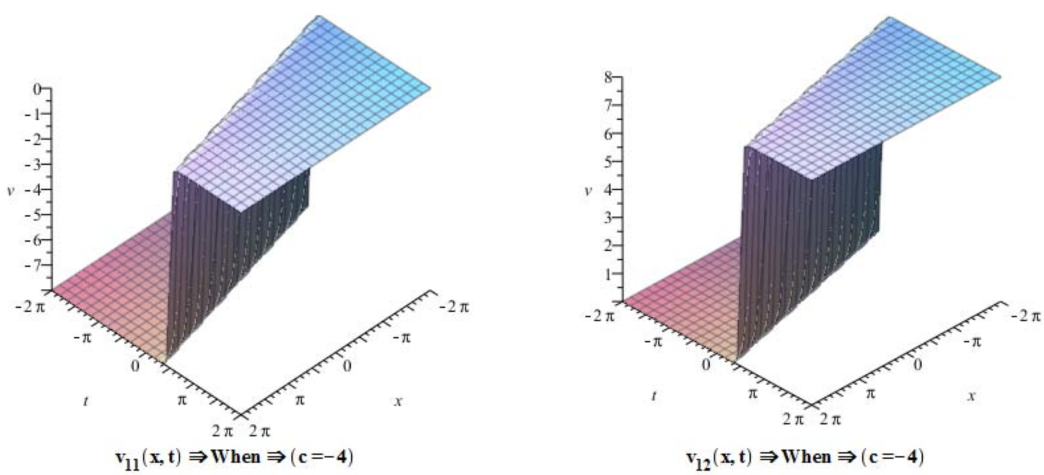


(a) Eq.(3.27)

(b) Eq.(3.27)

Figure 5 : The Solitary wave solution of Eqs.(3.27)



(a) Eq.(3.28)

(b) Eq.(3.28)

Figure 6 : The Solitary wave solution of Eqs.(3.28)

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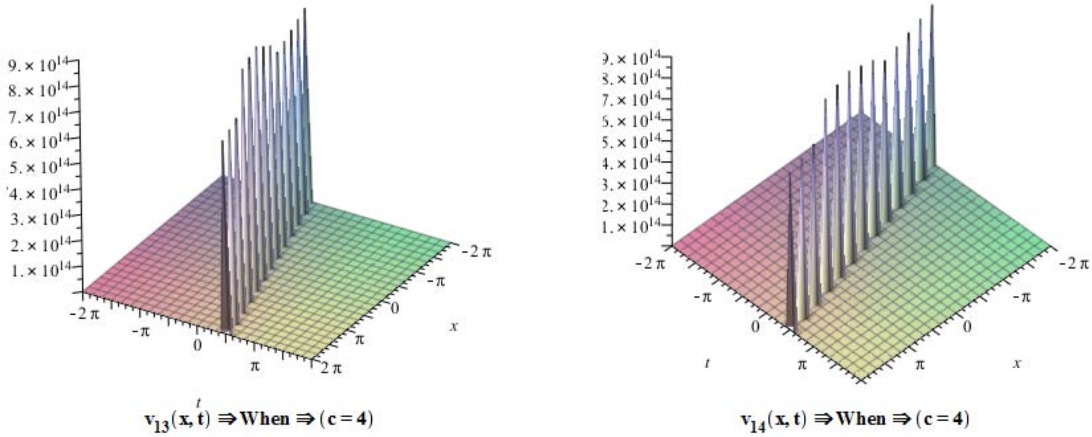
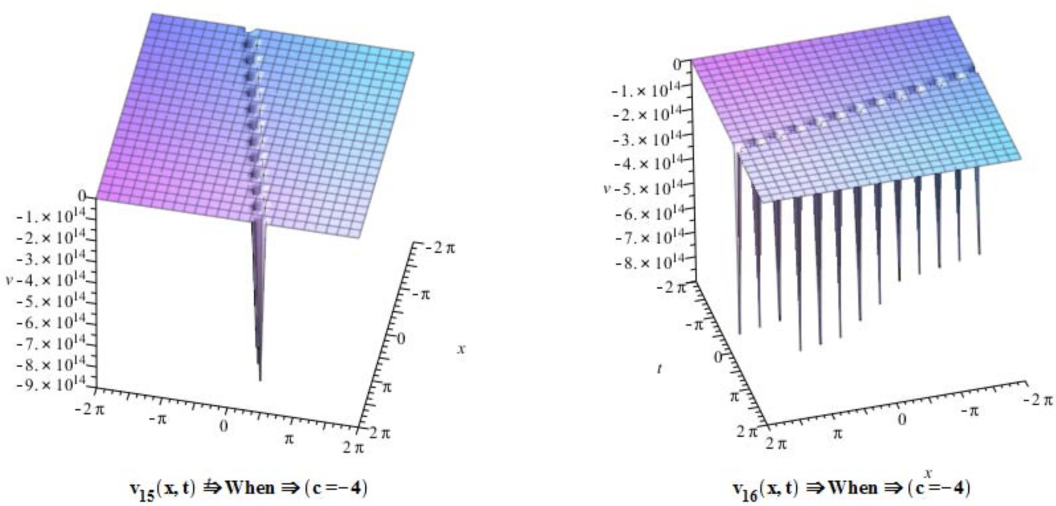


(a) Eq.(3.29)

(b) Eq.(3.29)

Figure 7 : The Solitary wave solution of Eqs.(3.29)



(a) Eq.(3.30)

(b) Eq.(3.30)

Figure 8 : The Solitary wave solution of Eqs.(3.30)

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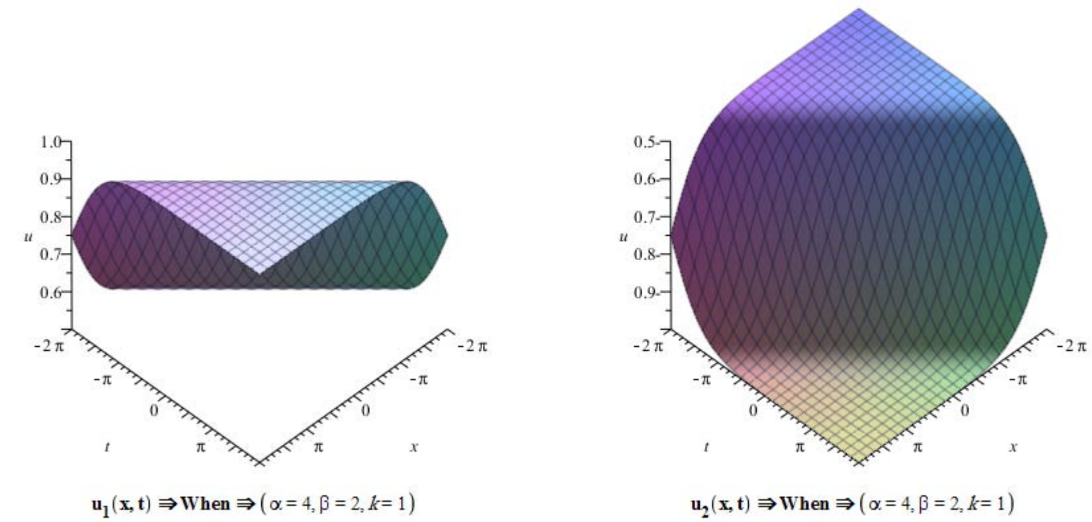
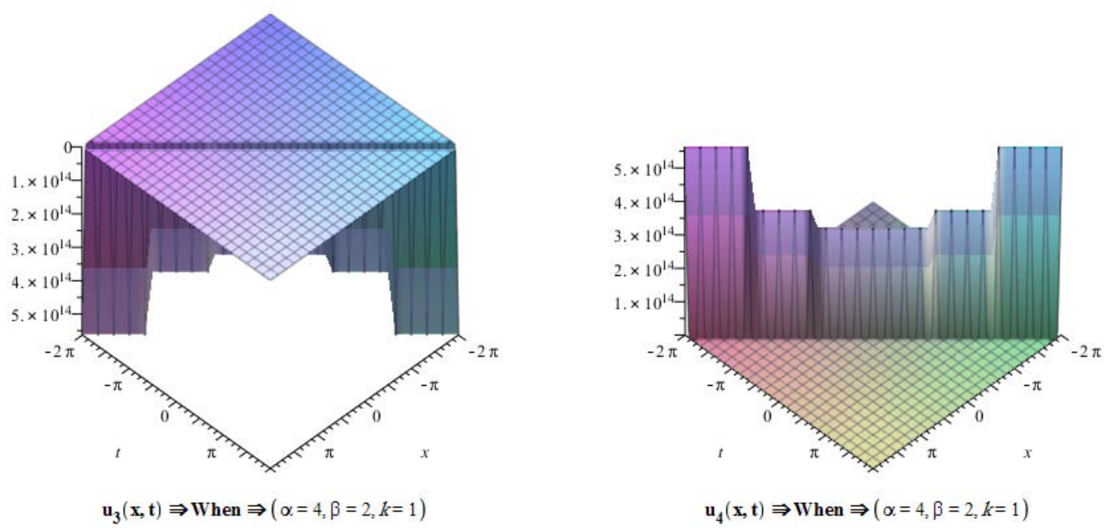


(a) Eq.(3.45)

(b) Eq.(3.45)

Figure 9 : The Solitary wave solution of Eqs.(3.45)



(a) Eq.(3.46)

(b) Eq.(3.46)

Figure 10 : The Solitary wave solution of Eqs.(3.46)

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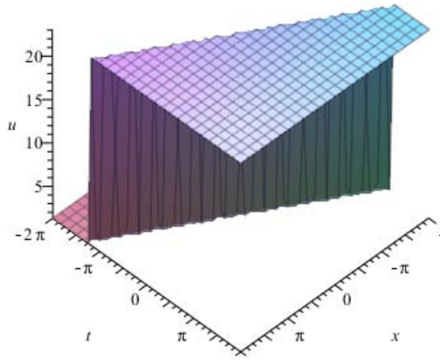
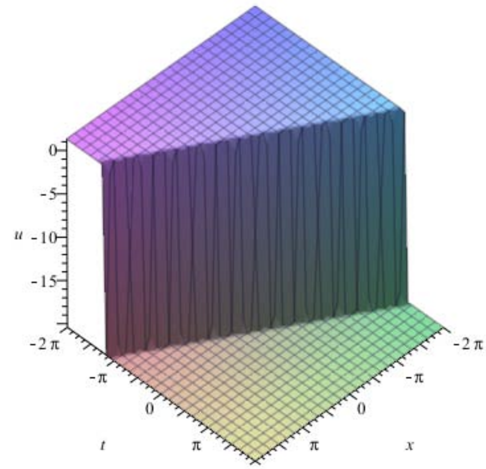


$\mathbf{u_1(x,t)} \Rightarrow \textbf{When} \Rightarrow \left(\alpha=-2, A_1=2, \sigma=1, A_0=1.32, \rho=-10\right)$

(a)

Eq.(3.59)

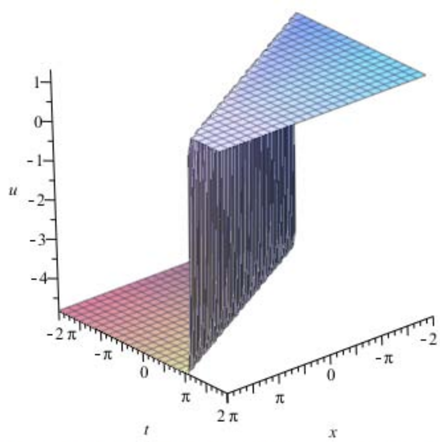


$\mathbf{u_2(x,t)} \Rightarrow \textbf{When} \Rightarrow \left(\alpha=-2, A_1=2, \sigma=1, A_0=1.32, \rho=-10\right)$

(b)

Eq.(3.59)

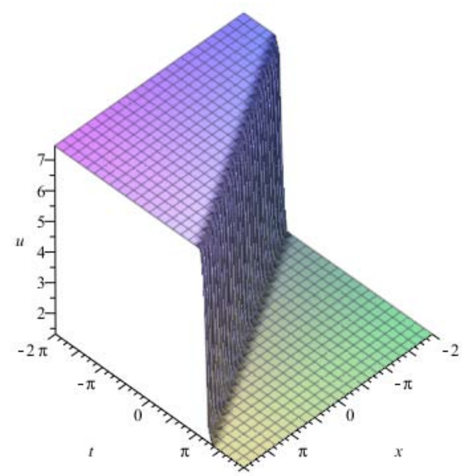
Figure 11 : The Solitary wave solution of Eqs.(3.59)



$\mathbf{u_3(x,t)} \Rightarrow \textbf{When} \Rightarrow \left(\alpha=-2, A_1=2, \sigma=1, A_0=1.32, \rho=4\right)$

(a)

Eq.(3.60)



$\mathbf{u_4(x,t)} \Rightarrow \textbf{When} \Rightarrow \left(\alpha=-2, A_1=2, \sigma=1, A_0=1.32, \rho=4\right)$

(b)

Eq.(3.60)

Figure 12 : The Solitary wave solution of Eqs.(3.60)

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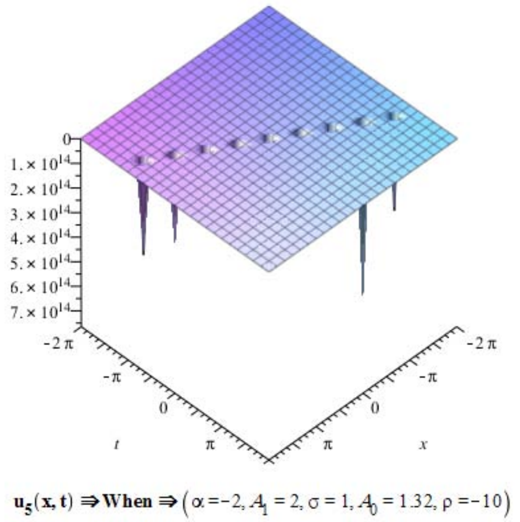
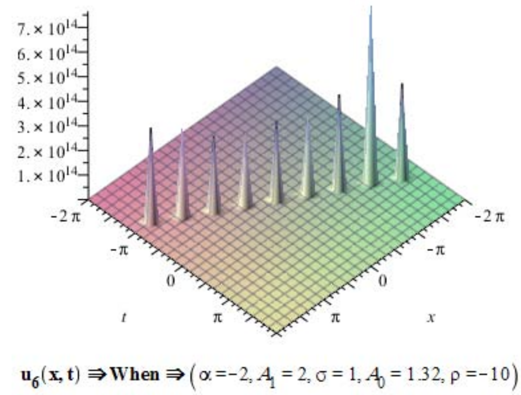


(a)

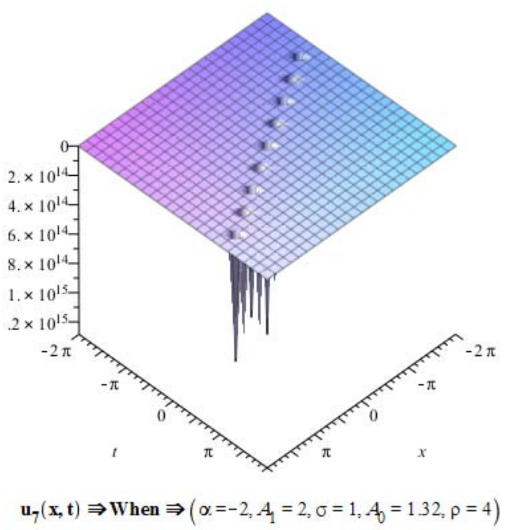
Eq.(3.61)



(b)

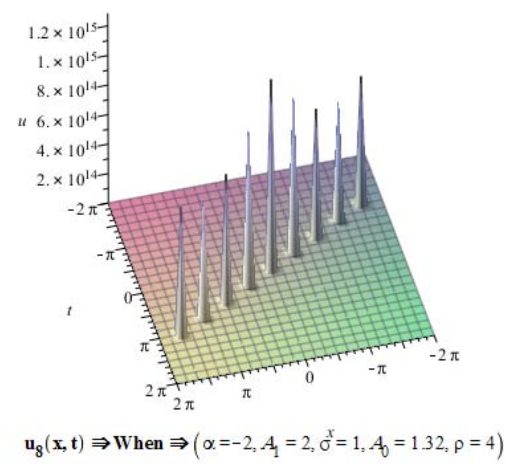
Eq.(3.61)

Figure 13 : The Solitary wave solution of Eqs.(3.61)



(a)

Eq.(3.62)



(b)

Eq.(3.62)

Figure 14 : The Solitary wave solution of Eqs.(3.62)

V. Conclusion

The modified simple equation method has been successfully used to find the exact traveling wave solutions of some nonlinear evolution equations. As an application, the traveling wave solutions for the system of shallow water wave equations, modified Benjamin-Bona-Mahony equation and nonlinear dynamics of microtubules-a new model which have been constructed using the modified simple equation method. Let us compare between our results obtained in the present article with the well-known results obtained by other authors using different methods as follows: Our results of the system of shallow water wave equations, modified Benjamin-Bona-Mahony equation and nonlinear dynamics of microtubules-A new model are new and different from those obtained in [[33]; [36] and [37]], [[34]; [38] and [39]] and [[35] and [40]]. and also we can see [39] which is considered a special case of modified Benjamin-Bona-Mahony equation when $a = 1$. It can be concluded that this method is reliable and propose a variety of exact solutions NPDEs. The performance of this method is effective and can be applied to many other nonlinear evolution equations. Figs.[1−14] represent the solitary traveling wave solution for the system of shallow water wave equations, modified Benjamin-Bona-Mahony equation and nonlinear dynamics of microtubules A new model.

Competing interests

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors. The author did not have any competing interests in this research.

Author's contributions

All parts contained in the research carried out by the researcher through hard work and a review of the various references and contributions in the field of mathematics and the physical Applied.

VI. Acknowledgment

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