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The Univalence of A Generalized Integral Operator

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Abstract- For analytic function $f_i, j=\overline{1,n}$, in the open disk U, an integral operator $K_{\alpha_1},\cdots\cdots_{\alpha_n},_{\beta_1}\cdots\cdots_{\beta_n}$ is introduced. In this paper we obtain the conditions of the univalence for the integral operator $K_{\alpha_1},\cdots\cdots_{\alpha_n},_{\beta_1}\cdots\cdots_{\beta_n}$.

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THEUNIVALENCEOFAGENERALIZEDINTEGRALOPERATOR

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The Univalence of a Generalized Integral Operator

Dr. Poonam Dixit [α] & Puneet Shukla [σ]

Abstract- For analytic function $f_i, j = \overline{1,n}$ in the open disk U, an integral operator $K_{\alpha_1,\ldots\alpha_n,\beta_1\ldots\beta_n}$ is introduced. In this paper we obtain the conditions of the univalence for the integral operator $K_{\alpha_1,\ldots\alpha_n,\beta_1\ldots\beta_n}$.

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I. Introduction

Let A be the class of functions f of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k$$

which are analytic in the open disk $U = \{z \in C : |z| < 1\}$ with $f(0) = f'(0) - 1 = 0$. Let S denote the subclass of A consisting of the functions $f \in A$, which are univalent in U .We denote by P the class of functions p which are analytic in U, $p(0) = 1$ and $Rep(z) > 0$, for all $z \in U$. In this work, we introduce a new integral operator, which is given by

$$K_{\alpha_1,\ldots\alpha_n,\beta_1\ldots\beta_n}(z) = \int_0^z \prod_{j=1}^{n} \left(\frac{D^m f_i(u)}{u}\right)^{\alpha_j} \left[(D^n f_j(u))'\right]^{\beta_j} du \qquad (1)$$

for α_j, β_j be complex numbers, $f_i \in A, f_j' \in P, \quad j = \overline{1,n}$.

For $m = 1 \quad \beta_j = 0 \quad j = \overline{1,n}$ we obtain the integral operator, which is defined in [4].

For $m = 1 \quad \alpha_j = 0 \quad j = \overline{1,n}$ we have the integral operator, which is defined in [5].

II. Preliminary Results

In order to prove our main results we will need the following lemmas .

Lemma 2.1 [1] If the function f is analytic in U and

$$(1 - |z|)^2 \left|\frac{zf''(z)}{f'(z)} - 1\right| \le 1 \qquad (2)$$

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for all $z \in U$, then the function f is univalent in U.

Lemma 2.2 [3] Let γ be a complex number $Re\gamma > 0$ and $f \in A$. If

$$\frac{1-|z|^{2Re\ \gamma}}{Re\ \gamma}\left|\frac{zf''(z)}{f'(z)}\right| \leq 1 \tag{3}$$

for all $z \in U$, then for any complex number $\delta, Re\ \delta \geq Re\ \gamma$, the function

$$f_\delta(z) = \left[\delta\int_0^z u^{\delta-1}f'(u)du\right]^{1/\delta} \tag{4}$$

is regular and univalent in U .

Lemma 2.3 (Schwarz [2]) Let f be the function regular in the disk $U_R = \{z \in C : |z| < R\}$ with $|f(z)| < M, M$ fixed .

If f has in $z = 0$ one zero multiply $\geq m$ then

$$|f(z)| \leq \frac{M}{R^m}|z|^m \qquad (z \in U_R) \tag{5}$$

the equality (in the inequality $(5) z \neq 0$) can hold only if .

$$f(z) = e^{i\theta}\frac{M}{R^m}z^m$$

where θ is constant.

III. Main Results

Theorem 3.1: Let α_j, β_j be the complex numbers M_j, L_j positive real numbers, $j = \overline{1,n}$ and the functions

$f_j \in A, f_j' \in P, f_j(z) = z + a_{2j}z^2 + a_{3j}z^3 + ...j = \overline{1,n}$ if .

$$\left|z\frac{D^{m+1}f_j(z)}{D^m f_j(z)} - 1\right| \leq M_j \qquad (j = \overline{1,n} \quad : \ z \in U) \tag{6}$$

$$\left|z\frac{[D^n f_j(z)]''}{[D^n f_j(z)]'}\right| \leq L_j \qquad (j = \overline{1,n} \quad : \ z \in U) \tag{7}$$

and

$$\sum_{j=1}^{n}[|\alpha_j|\, M_j + |\beta_j|\, L_j] \leq \frac{3\sqrt{3}}{2} \tag{8}$$

Then the integral operator $K_{\alpha_1,.....\alpha_n,\beta_1.....\beta_n}$ defined by (1) is in the class S .

Proof : The function $K_{\alpha_1,.....\alpha_n,\beta_1.....\beta_n}(z)$ is regular in U and

$K_{\alpha_1,.....\alpha_n,\beta_1.....\beta_n}(0) = K'_{\alpha_1,.....\alpha_n,\beta_1.....\beta_n}(0) - 1 = 0$

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2. O. Mayer, The functions theory of one variable complex, Bucuresti. 1981.

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we have

$$\frac{zK''_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ldots\ldots\beta_n}(z)}{K'_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ldots\ldots\beta_n}(z)}=\sum_{j=1}^{n}\left[\alpha_j\left(\frac{D^{m+1}f_j(z)}{D^m f_j(z)}-1\right)\right]+\sum_{j=1}^{n}\left[\beta_j z\frac{[D^n f_j(z)]''}{[D^n f_j(z)]'}\right] \tag{9}$$

and hence we get

$$(1-|z|^2)\left|\frac{zK''_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ldots\ldots\beta_n}(z)}{K'_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ldots\ldots\beta_n}(z)}\right|\leq(1-|z|^2)\sum_{j=1}^{n}\left[|\alpha_j|\left|z\frac{D^{m+1}f_j(z)}{D^m f_j(z)}-1\right|+|\beta_j|\left|z\frac{[D^n f_j(z)]''}{[D^n f_j(z)]'}\right|\right] \tag{10}$$

for all $z\in U$.

By (6), (7) and Lemma 2.3, we obtain

$$\left|z\frac{D^{m+1}f_j(z)}{D^m f_j(z)}-1\right|\leq M_j\,|z| \qquad (j=\overline{1,n}\quad : \ z\in U) \tag{11}$$

$$\left|z\frac{[D^n f_j(z)]''}{[D^n f_j(z)]'}\right|\leq L_j\,|z| \qquad (j=\overline{1,n}\quad : \ z\in U) \tag{12}$$

and from (10) we have

$$(1-|z|^2)\left|\frac{zK''_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ldots\ldots\beta_n}(z)}{K'_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ldots\ldots\beta_n}(z)}\right|\leq(1-|z|^2)\,|z|\left\{\sum_{j=1}^{n}[|\alpha_j|\,M_j+|\beta_j|\,L_j]\right\} \tag{13}$$

for all $z\in U$.

Since

$$max_{|z|<1}[(1-|z|^2)\,|z|]=\frac{2}{3\sqrt{3}}$$

from (8) and (13) we get

$$(1-|z|^2)\left|\frac{zK''_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ldots\ldots\beta_n}(z)}{K'_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ldots\ldots\beta_n}(z)}\right|\leq 1, \qquad (z\in U)$$

and by Lemma 2.1, it results that the integral operator $K_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ldots\ldots\beta_n}$ is in the class S.

Theorem 3.2 : Let α_j,β_j,γ be the complex numbers $j=\overline{1,n}$, $0<Re\ \gamma\leq 1$ and the functions

$$f_j\in A, f_j'\in P, f_j(z)=z+a_{2j}z^2+a_{3j}z^3+\ldots.j=\overline{1,n}$$

if

$$\left|z\frac{D^{m+1}f_j(z)}{D^m f_j(z)}-1\right|\leq\frac{(2Re\ \gamma+1)^{\frac{2Re\ \gamma+1}{2Re\ \gamma}}}{2} \qquad (j=\overline{1,n}\quad : \ z\in U) \tag{14}$$

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$$\left| z\frac{[D^n f_j(z)]''}{[D^n f_j(z)]'} \right| \leq \frac{(2Re\ \gamma+1)^{\frac{2Re\ \gamma+1}{2Re\ \gamma}}}{2} \qquad (j=\overline{1,n} \quad : \ z \in U) \tag{15}$$

and

$$\sum_{j=1}^{n}[|\alpha_j|+|\beta_j|] \leq 1 \tag{16}$$

then the integral operator $K_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ldots\ldots\beta_n}$ defined by (1) belong to the class S .

Proof : From (9) we obtain

$$\frac{zK''_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ldots\ldots\beta_n}(z)}{K'_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ldots\ldots\beta_n}(z)} = \sum_{j=1}^{n}\left[\alpha_j\left(\frac{zD^{m+1}f_j(z)}{D^m f_j(z)}-1\right)\right] + \sum_{j=1}^{n}\left[\beta_j\frac{z[D^n f_j(z)]''}{[D^n f_j(z)]'}\right]$$

and hence we get

$$\frac{1-|z|^{2Re\gamma}}{Re\ \gamma}\left|\frac{zK''_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ldots\ldots\beta_n}(z)}{K'_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ldots\ldots\beta_n}(z)}\right| \leq \frac{1-|z|^{2Re\gamma}}{Re\ \gamma}\sum_{j=1}^{n}\left[|\alpha_j|\left|z\frac{D^{m+1}f_j(z)}{D^m f_j(z)}-1\right|+|\beta_j|\left|z\frac{[D^n f_j(z)]''}{[D^n f_j(z)]'}\right|\right] \tag{17}$$

for all $z \in U$ by (14), (15) and Lemma 2.3 we have

$$\left| z\frac{D^{m+1}f_j(z)}{D^m f_j(z)}-1\right| \leq \frac{(2Re\ \gamma+1)^{\frac{2Re\gamma+1}{2Re\ \gamma}}}{2}|z| \qquad (j=\overline{1,n} \quad : \ z \in U) \tag{18}$$

$$\left| z\frac{[D^n f_j(z)]''}{[D^n f_j(z)]'} \right| \leq \frac{(2Re\ \gamma+1)^{\frac{2Re\gamma+1}{2Re\ \gamma}}}{2}|z| \qquad (j=\overline{1,n} \quad : \ z \in U) \tag{19}$$

and hence by (17) we get

$$\frac{1-|z|^{2Re\gamma}}{Re\ \gamma}\left|\frac{zK''_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ldots\ldots\beta_n}(z)}{K'_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ldots\ldots\beta_n}(z)}\right| \leq \frac{1-|z|^{2Re\gamma}}{Re\ \gamma}|z|\frac{(2Re\ \gamma+1)^{\frac{2Re\gamma+1}{2Re\ \gamma}}}{2}\sum_{j=1}^{n}[|\alpha_j|+|\beta_j|] \tag{20}$$

for all $z \in U$.

$$max_{|z|\leq 1}\left[\frac{1-|z|^{2Re\gamma}}{Re\ \gamma}|z|\right] = \frac{2}{(2Re\ \gamma+1)^{\frac{2Re\gamma+1}{2Re\ \gamma}}}$$

From (16) and (20) we obtain that

$$\frac{1-|z|^{2Re\gamma}}{Re\ \gamma}\left|\frac{zK''_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ldots\ldots\beta_n}(z)}{K'_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ldots\ldots\beta_n}(z)}\right| \leq 1 \tag{21}$$

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for all $z \in U$ and by Lemma 2.2 for $\delta = 1$ and $f = K_{\alpha_1, ...\alpha_n, \beta_1 .....\beta_n}$ it results that the integral operator $K_{\alpha_1, ...\alpha_n, \beta_1 .....\beta_n}$ defined by (1) belongs to the class S.

IV. Corollaries

Notes

Corollary 4.1: Let α_j be the complex numbers M_j positive real numbers, $j = \overline{1, n}$ and the functions

$$f_j \in A, f_j' \in P, f_j(z) = z + a_{2j}z^2 + a_{3j}z^3 + ...j = \overline{1, n} \text{ if } .$$

$$\left| z \frac{D^{m+1} f_j(z)}{D^m f_j(z)} - 1 \right| \leq M_j \qquad (j = \overline{1, n} \quad : \quad z \in U) \tag{22}$$

and

$$\sum_{j=1}^{n} [|\alpha_j| \, M_j] \leq \frac{3\sqrt{3}}{2} \tag{23}$$

then the function

$$G_{\alpha_1, ...\alpha_n}(z) = \int_0^z \prod_{j=1}^{n} \left(\frac{D^m f_i(u)}{u} \right)^{\alpha_j} du$$

is in the class S.

Corollary 4.2 : Let β_j be the complex numbers L_j positive real numbers, $j = \overline{1, n}$ and the functions

$$f_j \in A, f_j' \in P, f_j(z) = z + a_{2j}z^2 + a_{3j}z^3 + ...j = \overline{1, n}$$

and

$$\left| z \frac{[D^n f_j(z)]''}{[D^n f_j(z)]'} \right| \leq L_j \qquad (j = \overline{1, n} \quad : \quad z \in U) \tag{24}$$

and

$$\sum_{j=1}^{n} [|\beta_j| \, L_j] \leq \frac{3\sqrt{3}}{2} \tag{25}$$

then the function

$$H_{\beta_1, ...\beta_n}(z) = \int_0^z \prod_{j=1}^{n} \left[(D^n f_i(u))' \right]^{\beta_j} du$$

belongs to the class S.

References Références Referencias

1. J. Becker, L Lownersche differentialgleichung and quasikonforrn fortsetzbare schlichte functionen, J. Reine Angew. Math., 255, 1972,23-43.
2. O. Mayer, The functions theory of one variable complex, Bucuresti. 1981.
3. N. N. Pascu, An improvement of Becker's univalence criterion, proceedings of the com-memorative session stoilow, University of Brasov. 1987, 43-48.
4. V. Pescar and V. D. Breaz. The univalence of integral operators, Monograph, Academic Publishing House, Sofia 2008.
5. V. Pescar, On the univalence of an integral operator, General Math. 19 (4) (2011), 69-74.

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