Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 15 Issue 1 Version 1.0 Year 2015
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Dissemination Sinusoidal Waves in of A Viscoelastic Strip

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Abstract- In this paper we consider the spectral problem for the wave propagation in extended plates of variable thickness. Describes how to solve problems and numerical results of wave propagation in infinitely large plates of variable thickness. Viscous properties of the material are taken into account by means of an integral operator Voltaire. The study is part of the spatial theory of visco elastic. The technique is based on the separation of spatial variables and formulating boundary eigenvalues problem to be solved by the method of orthogonal sweep Godunov. Numerical values obtained for the real and imaginary parts of phase velocity as a function of wave number. When this coincidence numerical results obtained with the known data.

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GJSFR-F Classification : FOR Code : MSC 2010: 15B10

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Strictly as per the compliance and regulations of :

Ref

1. I.I. Safarov,. Z.F.Dzhumaev, Z.I.Boltaev. Harmonic waves in an infinite cylinder with radial crack in view of the damping ability of the material. Problem of Mechanics. 2011. p.20-25.

Dissemination Sinusoidal Waves in of A Viscoelastic Strip

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Annotation- In this paper we consider the spectral problem for the wave propagation in extended plates of variable thickness. Describes how to solve problems and numerical results of wave propagation in infinitely large plates of variable thickness. Viscous properties of the material are taken into account by means of an integral operator Voltaire. The study is part of the spatial theory of visco elastic. The technique is based on the separation of spatial variables and formulating boundary eigenvalues problem to be solved by the method of orthogonal sweep Godunov. Numerical values obtained for the real and imaginary parts of phase velocity as a function of wave number. When this coincidence numerical results obtained with the known data.

Keywords: plate, spectral problem, frequency, variable thickness, orthogonal sweep.

I. Introduction

Known [1,2] that in normal wave deformable layer (Lamb wave) is not orthogonal thickness, i.e. the integral of the scalar product of vectors of displacements of two different waves, considered as functions of the coordinate perpendicular to the surface layer is not zero. They also are not orthogonal conjugate waves is obtained by considering the dual problem. This introduces additional difficulties in solving practical problems [3,4,8]. In this paper, we present spectral problem formulation and methods of its tasks.

II. Statement of the Wave Problem and the Basic Relations for the Plate Kirchhoff - Love Variable Thickness

Derive the fundamental relationships of the classical theory of plates with variable thickness on the basis of the principle of virtual displacements. In the three-dimensional formulation of the elasticity problem reduces to the solution of the variation equation, which has the form:

$$\delta A_F + \delta A_I = 0 \tag{1}$$

For virtual work (δA_F) internal forces, we have:

$$\delta A_F = -\delta \Pi = -\int_V \sigma_{ij} \delta\varepsilon_{ij} dV \tag{2}$$

where Π – potential energy; σ_{ij} – components of the stress tensor; ε_{ij} – components the deformation tensor; V – the volume occupied by the body.

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The physical properties of the plastic material describes the relationship

$$\sigma_{ij} = \bar{\lambda}\varepsilon_{kk}\delta_{ij} + 2\bar{\mu}\varepsilon_{ij} \quad (i, j, k = 1,2,3) \tag{3}$$

Where $\sigma_{ij}, \varepsilon_{ij}$ - components of the stress and strain tensors.

$$\lambda = \frac{E\nu(1+i\eta)}{(1+\nu)(1-2\nu)}; \qquad \mu = \frac{E(1+i\eta)}{2(1+\nu)} \tag{4}$$

Integrated in the case the Young's modulus of the viscoelastic material $E^* = E' + iE'' = E'(1+i\eta_e)$ an analogue of the classical Young modulus [85]. Using a complex representation for the elastic modulus (Young's modulus) for the polymeric material can be written as

$$E^*(\omega) = E(\omega)[1 + i\eta\omega] \tag{5}$$

Where two functions of vibration frequency $E(\omega)$ and $\eta(\omega)$ may be represented by analytical variety of ways [1,2].

For virtual work of inertial forces (δA_I) we can write the following relation:

$$\delta A_I = -\int_V \rho \ddot{u}_i \delta u_i dV , \tag{3}$$

where ρ - body density; u_i – displacement components; $\ddot{u}_i = \partial^2 ui / \partial t^2$; t - time. Here and below, summation over repeated indices. Consider the wedge plate shown in Fig. 1, along the axis of an infinite x_2. In accordance with the hypotheses of Kirchhoff - Love have:

$$\sigma_{13} = \sigma_{23} = \sigma_{33} = 0;$$

$$u_i = -x_3 \frac{\ddot{a}W}{\ddot{a}x_i}; \tag{4}$$

$$W(x_3) \equiv W ,$$

where W – deflection of the middle plane of the plate.

Neglecting in (3) members to take account of the inertia of rotation normal to the median plane we obtain:

$$-\int_s ds \int_{-h/2}^{h/2} (\sigma_{11}\delta\varepsilon_{11} + 2\sigma_{12}\delta\varepsilon_{12} + \sigma_{22}\delta\varepsilon_{22}) dx_3 -$$

$$-\int_s ds \int_{-h/2}^{h/2} \rho \frac{\partial^2 W}{\partial t^2} \delta W dz = 0 \tag{5}$$

The expressions for the components of strain and stress tensors are determined from the geometric relationships and relations generalized Hooke's law, which, taking into account the kinematic hypotheses (4) takes the form:

Ref

2. Il Safarov, Z.I.Boltaev. Propagation of harmonic waves in a plate of variable thickness. Math. Institutions of higher education. Volga region. Series: Phys. -mat. Sciences, №4, 2011 p. 31-39.

Notes

$$\begin{cases} \varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - x_3 \frac{\partial^2 W}{\partial x_i \partial x_j}; \qquad i, j = 1,2 \\ \sigma_{11} = \frac{E}{1-\nu}\left(\varepsilon_{11} + \nu\varepsilon_{22}\right)\Gamma_\kappa; \\ \sigma_{22} = \frac{E}{1-\nu}\left(\varepsilon_{22} + \nu\varepsilon_{11}\right)\Gamma_\kappa \\ \sigma_{12} = \frac{E}{1-\nu}\varepsilon_{12}\Gamma_\kappa, \end{cases} \tag{6}$$

Where E - Young's modulus; ν - Poisson's ratio of the plate material. Introducing the following notation:

$$M_{11} = -D\left(\frac{\partial^2 W}{\partial x_1^2} + \nu \frac{\partial^2 W}{\partial x_2^2}\right);$$

$$M_{22} = -D\left(\frac{\partial^2 W}{\partial x_2^2} + \nu \frac{\partial^2 W}{\partial x_1^2}\right); \tag{7}$$

$$M_{12} = -D(1-\nu)\frac{\partial^2 W}{\partial x_1 \partial x_2};$$

$$\overline{D} = \frac{\overline{E}h^3}{12(1-\nu^2)} = D_1\Gamma_k; \quad D_1 = \frac{Eh^3}{12(1-\nu^2)}; \quad \Gamma_\kappa = 1 + i\eta(\omega)$$

and integrating over the thickness of the plate, let (2.5) to the following form

$$\int_s \left(M_{11}\frac{\partial^2 \delta W}{\partial x_1^2} + 2M_{12}\frac{\partial^2 \delta W}{\partial x_1 \partial x_2} + M_{22}\frac{\partial^2 \delta W}{\partial x_2^2}\right) dS - \tag{8}$$

$$-\int_s \rho h \frac{\partial^2 W}{\partial t^2}\delta W ds = 0$$

Converting the first integral (8) twice by parts and equating to zero the coefficients of variation δW inside the body and on its borders obtain the following differential equation:

$$\frac{\partial^2 M_{11}}{\partial x_1^2} + 2\frac{\partial^2 M_{12}}{\partial x_1 \partial x_2} + \frac{\partial^2 M_{22}}{\partial x_2^2} - \rho h \frac{\partial^2 W}{\partial t^2} = 0 \tag{9}$$

with natural boundary conditions

$$M_{11}(0, l_1) = 0;$$

$$\frac{\partial M_{11}}{\partial x_1} + 2\frac{\partial M_{12}}{\partial x_2} = 0, \qquad x_1 = 0, l_1$$

the main alternative, which will be the following:

$$\begin{cases} \dfrac{\partial W}{\partial x_1} = 0, \\ W = 0, \;\; x_1 = 0, \;\; l_1 \end{cases},$$

Introducing new variables

$$W, \varphi_1 = \frac{\partial W}{\partial x_1}, \;\; M_{11}, Q_1 = \frac{\partial M_{11}}{\partial x_1} + 2\frac{\partial M_{12}}{\partial x_2}$$

and express through them M_{22} with the help of (2.7). Then

$$M_{22} = -D\frac{\partial^2 W}{\partial x_2^2} + vM_{11} + v^2 D\frac{\partial^2 W}{\partial x_2^2},$$

or

$$M_{22} = -\frac{Eh^3}{12}\frac{\partial^2 W}{\partial x_2^2} + vM_{11} \tag{10}$$

We note that M_{11} and M_{22} are bending moments, at M12 the torque.

Thus, we arrive at the following system of equations:

$$\begin{cases} \dfrac{\partial W}{\partial x_1} = \varphi_1; \\ \dfrac{\partial \varphi}{\partial x_1} = -\dfrac{M_{11}}{D} - v\dfrac{\partial^2 W}{\partial x_2^2}; \\ \dfrac{\partial M_{11}}{\partial x_1} = Q_1 + \dfrac{\bar{E}h^3}{6(1+v)}\dfrac{\partial^2 \varphi}{\partial x_2^2}; \\ \dfrac{\partial Q_1}{\partial x_1} = -v\dfrac{\partial^2 M_{11}}{\partial x_2^2} + \dfrac{\bar{E}h^3}{12}\dfrac{\partial^2 W}{\partial x_2^2} + \rho h\dfrac{\partial^2 W}{\partial t^2}, \end{cases} \tag{11}$$

Or

$$\begin{cases} \dfrac{\partial W}{\partial x_1} = \varphi_1; \\ \dfrac{\partial \varphi}{\partial x_1} = -\dfrac{6(1-v)}{h^3}\dfrac{M_{11}\cdot 2(1-v)}{\bar{E}} - v\dfrac{\partial^2 W}{\partial x_2^2}; \\ \dfrac{2(1+v)}{\bar{E}}\dfrac{\partial M_{11}}{\partial x_1} = \dfrac{2(1+v)}{\bar{E}}Q_1 + \dfrac{h^3}{3}\dfrac{\partial \varphi}{\partial x_2}; \\ \dfrac{2(1+v)}{\bar{E}}\dfrac{\partial Q_1}{\partial x_1} = -v\dfrac{2(1+v)}{\bar{E}}\dfrac{\partial^2 M_{11}}{\partial x_2^2} + \dfrac{(1+v)h^3}{6}\dfrac{\partial^4 W}{\partial x_2^4} + \dfrac{2(1+v)}{\bar{E}}\rho h\dfrac{\partial^2 W}{\partial t^2}, \end{cases}$$

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Or

$$\begin{cases} \dfrac{\partial y_1}{\partial x_1} = y_2; \\ \dfrac{\partial y_2}{\partial x_1} = -\dfrac{6(1-v)}{h^3} y_3 - v\dfrac{\partial^2 y_1}{\partial x_2^2}; \\ \dfrac{\partial y_3}{\partial x_1} = y_4 + \dfrac{h^3}{3}\dfrac{\partial y_2}{\partial x_2}; \\ \dfrac{\partial y_4}{\partial x_1} = -v\dfrac{\partial^2 y_3}{\partial x_2^2} + \dfrac{(1+v)h^3}{6}\dfrac{\partial^4 y_1}{\partial x_2^4} + \dfrac{h}{C_s^2}\dfrac{\partial^2 y_1}{\partial t^2}, \end{cases} \tag{12}$$

Where $y_1=W$, $y_2=\varphi_1$, $y_3=\dfrac{2(1+v)}{E}M_{11}$, $y_4=\dfrac{2(1+v)}{E}Q$, $\tilde{N}_s^2 = \dfrac{E}{2(1+v)\rho}$, C_s – shear wave Velocity

Among the many solutions of (12) we choose those that describe harmonic plane waves propagating along the axis x_2

$$y_i = z_i(x_1)e^{i(\hat{e}\tilde{o}_2-\omega t)} \tag{13}$$

Substituting the solution (13) in the system of differential equations (12), we obtain a system of ordinary differential equations of the first order, solved for the derivative:

$$\begin{cases} z_1' = z_2; \\ z_2' = -\dfrac{6(1-v)}{h^3} z_3 + v\kappa^2 z_1; \\ z_3' = z_4 - \dfrac{h^3\Gamma_k}{3}\kappa^2 z_2; \\ z_4' = v\kappa^2 z_3 + \dfrac{(1+v)h}{6}\kappa^4 z_1 - h\left(\dfrac{\omega}{C_s}\right)^2 \Gamma_k z_1; \end{cases} \tag{14}$$

The boundary conditions for this system can be written as follows:

а) free left edge of the plate:

$$z_3(0) = z_4(0) = 0 \tag{15}$$

б) free right edge of the plate:

$$z_3(l_1) = z_4(l_1) = 0 \tag{16,a}$$

в) pinched right edge of the plate:

$$z_1(l_1) = z_2(l_1) = 0 \tag{16,б}$$

T Thus formed the spectral problem (14-16) in the parameter ω, describing the propagation of flexural waves in a flat edge plate Kirchhoff-Love.

III. Basic Relations for Timoshenko Plates of Variable Thickness. Statement of the Wave Problem

Applying the principle of virtual displacements (1-3), replacing the Kirchhoff-Love hypotheses (2.4) on the hypothesis Timoshenko:

$$\sigma_{33} = 0;\ \sigma_{3i} = \frac{\chi \overline{E}}{2(1+\nu)}\left(\frac{\partial W}{\partial x_i} - \theta_i\right); \qquad (17)$$

$$u_i^{(x_3)} = x_3\theta_i;\ W^{(x_3)} = W; \qquad i = 1,2,$$

where $\theta_i -$ normal rotation angles (Fig. 2) $\chi -$ correction factor that takes into account the distribution of shear stresses across the thickness.

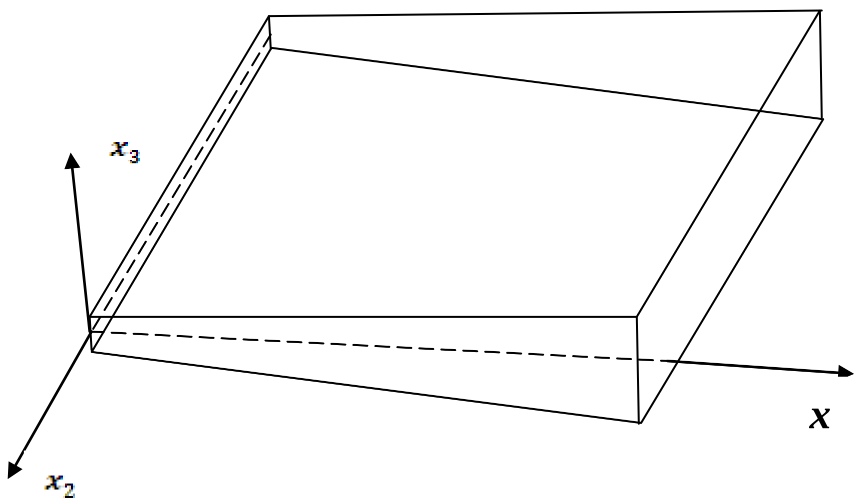

Figure 1 : Design scheme

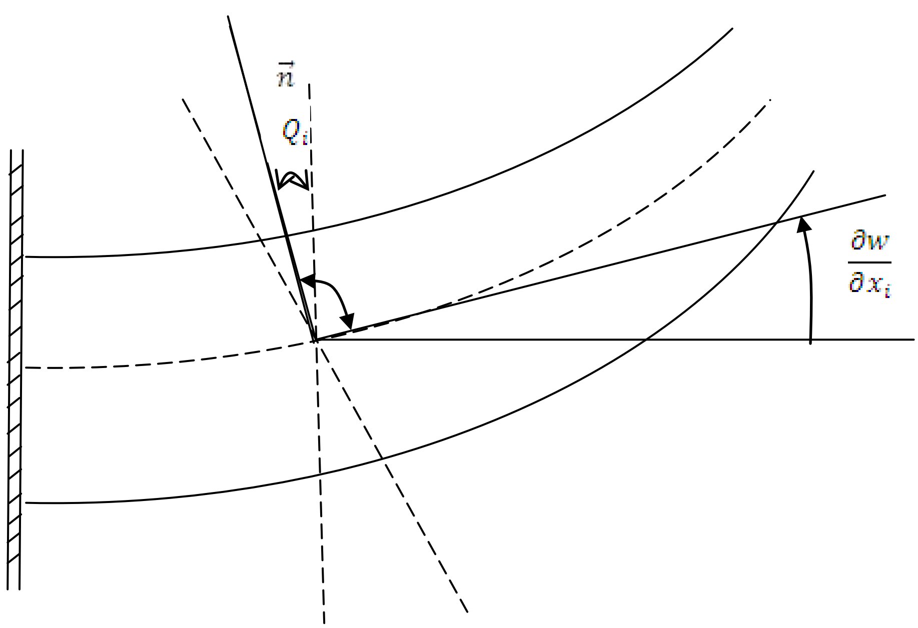

Figure 2 : shows the angle of rotation of the normal

In this case, the tensor components of strain and stress take the form:

$$\varepsilon_{ij} = -\frac{1}{2}x_3\left(\frac{\partial \theta_i}{\partial x_j} + \frac{\partial \theta_j}{\partial x_i}\right);$$

Notes

$$\varepsilon_{3i} = \frac{1}{2}\left(\frac{\partial W}{\partial x_i} - \theta_i\right);$$

$$\sigma_{11} = -\frac{E\Gamma_k}{1-\nu^2} x_3 \left(\frac{\partial \theta_1}{\partial x_1} + \nu \frac{\partial \theta_2}{\partial x_2}\right); \quad (18)$$

$$\sigma_{22} = -\frac{E\Gamma_k}{1-\nu^2} x_3 \left(\frac{\partial \theta_2}{\partial x_2} + \nu \frac{\partial \theta_1}{\partial x_1}\right);$$

$$\sigma_{12} = -\frac{E\Gamma_k}{2(1+\nu)} x_3 \left(\frac{\partial \theta_1}{\partial x_2} + \nu \frac{\partial \theta_2}{\partial x_1}\right);$$

$$\sigma_{3i} = \frac{\chi E\Gamma_k}{2(1+\nu)}\left(\frac{\partial W}{\partial x_i} - \theta_i\right), \qquad i,\ j=1,\ 2$$

Substitute the expression for the work on virtual displacements, we obtain:

$$\delta A = \int_{-h/s}^{h/2} \int_s \left[-\sigma_{ij} \frac{x^3}{2}\left(\frac{\partial \delta\theta_i}{\partial x_j} + \frac{\partial \delta\theta_j}{\partial x_i}\right) + \sigma_{3i}\left(\frac{\partial \delta W}{\partial x_i} - \delta\theta_i\right) + \right.$$

$$\left. + \rho \ddot{W} \delta W + \rho x_3^2 \ddot{\theta}_i \delta\theta_i \right] dS dx_3 = 0 \quad (19)$$

Or by introducing a notation for the corresponding moments:

$$\overline{M}_{11} = D_1 \Gamma_k \left(\frac{\partial \theta_1}{\partial x_1} + \nu \frac{\partial \theta_2}{\partial x_2}\right) = \Gamma_k M_{11};$$

$$\overline{M}_{22} = D_1 \Gamma_k \left(\frac{\partial \theta_2}{\partial x_2} + \nu \frac{\partial \theta_1}{\partial x_1}\right) = \Gamma_k M_{22}; \quad (20)$$

$$\overline{M}_{12} = D_2 \Gamma_k \left(\frac{\partial \theta_1}{\partial x_2} + \frac{\partial \theta_2}{\partial x_1}\right) = \Gamma_k M_{12}$$

where $D_2 = \frac{1}{2} D_1$

$$M_{22} = -D_1 \left(\frac{\partial \theta_2}{\partial x_2} + \nu \frac{\partial \theta_1}{\partial x_1}\right)$$

$$M_{11} = -D_1 \left(\frac{\partial \theta_1}{\partial x_1} + \nu \frac{\partial \theta_2}{\partial x_2}\right)$$

$$M_{12} = D_2\left(\frac{\partial\theta_1}{\partial x_2} + \frac{\partial\theta_2}{\partial x_1}\right)$$

and integrating over x_3 we have

$$\delta A = -\int_s \left[-\frac{\partial}{\partial x_j}\left(\overline{M}_{ij}\delta\theta_i\right) + \frac{\partial}{\partial x_j}\left(h\delta_{3j}\delta W\right)\right] dS +$$

$$+\int_s \left(-\frac{\partial\overline{M}_{ij}}{\partial x_j}\delta\theta_i + \frac{\partial\left(h\overline{\sigma}_{3j}\right)}{\partial x_j}\delta W + h\overline{\sigma}_{3i}\delta\theta_i - \right.$$

$$\left. -\rho h\ddot{W}\delta W - \frac{\rho h^3}{12}\ddot{\theta}_i\delta\theta_i\right) dS = 0 \qquad (21)$$

Integrating (21) by parts and equating to zero the coefficients of variation δW and $\delta\theta_i$ inside the body and on its borders obtain the following system of differential equations

$$\begin{cases} -\dfrac{\partial M_{12}}{\partial x_2} - \dfrac{\partial M_{11}}{\partial x_1} + h\sigma_{31} - \dfrac{\rho h^3}{12\Gamma_k}\ddot{\theta}_1 = 0; \\ -\dfrac{\partial M_{22}}{\partial x_2} - \dfrac{\partial M_{12}}{\partial x_1} + h\sigma_{32} - \dfrac{\rho h^3}{12\Gamma_k}\ddot{\theta}_2 = 0; \\ \dfrac{\partial\left(h\sigma_{32}\right)}{\partial x_2} + \dfrac{\partial\left(h\sigma_{31}\right)}{\partial x_1} - \dfrac{\rho h\ddot{W}}{\Gamma_k} = 0 \end{cases} \qquad (22)$$

With natural boundary conditions:

$$\begin{cases} M_{12} = 0; \\ M_{11} = 0; \\ h\sigma_{31} = 0, \; x_1 = 0, \; l_1 \end{cases}$$

The main alternative, which will be the following:

$$\begin{cases} \theta_1 = 0; \\ \theta_2 = 0; \\ W = 0, \; x_1 = 0, \; l_1 \end{cases}$$

Equation (22) is a differential complex coefficients, it is possible to write in the following form

$$\begin{pmatrix} -\dfrac{\partial M_{12}}{\partial x_2} - \dfrac{\partial M_{11}}{\partial x_1} + h\tau_{31} - \dfrac{sh^3}{12\Gamma_{KR}}\theta_1'' \\ \dfrac{-\partial M_{22}}{\partial x_2} - \dfrac{\partial M_{12}}{\partial x_1} + h\tau_{32} - \dfrac{sh^3}{12\Gamma_{KR}}\theta'' \\ \dfrac{\partial\left(h\tau_{32}\right)}{\partial x_2} + \dfrac{\partial\left(h\tau_{31}\right)}{\partial x_1} - \dfrac{sh^3}{\Gamma_{K12}}\ddot{W} \end{pmatrix} + i\Gamma_{KI}\begin{pmatrix} -\dfrac{\partial M_{12}}{\partial x_2} - \dfrac{\partial M_{11}}{\partial x_1} + h\tau_{31} \\ \dfrac{-\partial M_{22}}{\partial x_2} - \dfrac{\partial M_{12}}{\partial x_1} + h\tau_{32} \\ \dfrac{\partial\left(h\tau_{32}\right)}{\partial x_2} + \dfrac{\partial\left(h\tau_{31}\right)}{\partial x_1} \end{pmatrix} = 0$$

Notes

The main variables in this system, we assume: W_1, θ_1, θ_2, M_{12}, M_{11}, $Q_1=h\sigma_{31}$. Out of the equation variables M_{22} and Q_2.

$$M_{22}=-\frac{Eh^3}{12}\frac{\partial\theta_2}{\partial x_2}+\nu M_{11};\ Q_2=h\sigma_{32}=\frac{\chi Eh}{2(1+\nu)}\left(\frac{\partial W}{\partial x_2}-\theta_2\right).$$

Notes

Thus we arrive at the following system of equations:

$$\begin{cases}\dfrac{\partial W}{\partial x_1}=\theta_1+\dfrac{2(1+\nu)}{\chi Eh}Q_1;\\ \dfrac{\partial\theta_2}{\partial x_1}=-\dfrac{\partial\theta_1}{\partial x_2}-\dfrac{24(1+\nu)}{Eh^3}M_{12};\\ \dfrac{\partial\theta_1}{\partial x_1}=-\nu\dfrac{\partial\theta_2}{\partial x_2}-\dfrac{12(1-\nu^2)}{Eh^2}M_{12};\\ \dfrac{\partial M_{11}}{\partial x_1}=-\dfrac{\partial M_{12}}{\partial x_2}+Q_1-\dfrac{ph^3}{12\Gamma_k}\ddot{\theta}_1;\\ \dfrac{\partial M_{22}}{\partial x_1}=-\dfrac{Eh^3}{12}\dfrac{\partial^2\theta_2}{\partial x_2^2}-\nu\dfrac{\partial M_{11}}{\partial x_2}+\dfrac{\chi Eh}{2(1+\nu)}\left(\dfrac{\partial W}{\partial x_2}-\theta_2\right)-\dfrac{ph^3}{12\Gamma_k}\ddot{\theta}_2;\\ \dfrac{\partial Q_1}{\partial x_1}=-\dfrac{\chi Eh}{2(1+\nu)}\left(\dfrac{\partial^2 W}{\partial x_2^2}-\dfrac{\partial\theta_2}{\partial x_2}\right)+\dfrac{\rho h\ddot{W}}{\Gamma_k}.\end{cases}\quad(23)$$

or

$$\frac{\partial y_1}{\partial x_1}=y_2+\frac{y_4}{\chi h};\qquad \frac{\partial y_2}{\partial x_1}=-\nu\frac{\partial y_3}{\partial x_2}-\frac{6(1-\nu)}{h^3}y_5;$$

$$\frac{\partial y_3}{\partial x_1}=-\frac{\partial y_2}{\partial x_2}-\frac{12}{h^3}y_6;$$

$$\frac{\partial y_4}{\partial x_1}=\chi h\frac{\partial}{\partial x_2}\left(y_3-\frac{\partial y_1}{\partial x_2}\right)+\frac{h}{\Gamma_k}\frac{\partial^2 y_2}{\partial\tilde{t}^2};\qquad(24)$$

$$\frac{\partial y_5}{\partial x_1}=-\frac{\partial y_6}{\partial x_2}+y_4-\frac{h^3}{12\Gamma_k}\frac{\partial^2 y_2}{\partial\tilde{t}^2};$$

$$\frac{\partial y_6}{\partial x_1}=\frac{\partial}{\partial x_2}\left(\frac{(1+\nu)h^3}{6}\cdot\frac{\partial y_3}{\partial x_2}-\nu y_5\right)+\chi h\left(\frac{\partial y_1}{\partial x_2}-y_3\right)-\frac{h^3}{12\Gamma_k}\frac{\partial^2 y_3}{\partial\tilde{t}^2}.$$

Where

$$y_1=W;\quad y_2=\theta_2;\quad y_3=\theta/\nu;\quad y_4=\frac{2(1+\nu)}{E}Q_1;$$

$$y_5=\frac{4(1+\nu)}{1-\nu}M_{12};\quad y_6=\frac{h(1-\nu^2)}{E\nu}M_{12}$$

$$M_{22}=-D\left(\frac{\partial\theta_2}{\partial x_2}+\nu\frac{\partial\theta_1}{\partial x_1}\right)+\nu M_{11}-\nu M_{11}=$$

$$= -D\left(1-\nu^2\right)\frac{\partial\theta_2}{\partial x_2} + \nu M_{11} = -\frac{Eh^3}{12\left(1-\nu^2\right)}\left(1-\nu^2\right)\frac{\partial\theta_2}{\partial x_2} + \nu M_{11} =$$

$$= -\frac{Eh^3}{12}\frac{\partial\theta_2}{\partial x_2} + \nu M_{11}$$

Finding, as before, the solutions described by a plane harmonic waves propagating along the axis x_1, we seek a solution of (24) in the form

$$\begin{cases} y_1 = z_1(x_1)\cos(\kappa x_2 - \omega t); \\ y_2 = z_2(x_1)\cos(\kappa x_2 - \omega t); \\ y_3 = z_3(x_1)\sin(\kappa x_2 - \omega t); \\ y_4 = z_4(x_1)\cos(\kappa x_2 - \omega t); \\ y_5 = z_5(x_1)\cos(\kappa x_2 - \omega t); \\ y_6 = z_6(x_1)\sin(\kappa x_2 - \omega t). \end{cases} \quad (25)$$

Substituting relation (25) in the system of differential equations (24) we obtain a system of ordinary differential equations of the first order, solved for the derivative:

$$\begin{cases} z_1' = z_2 + \dfrac{z_n}{\chi h}; \\ z_2' = -\nu\kappa z_3 - \dfrac{6(1-\nu)}{3} z_5; \\ z_3' = \kappa z_2 - \dfrac{12}{h^3} z_6; \\ z_4' = \chi h \kappa z_3 + \kappa^2\left(\chi h - \dfrac{hc^2}{\Gamma_л}\right) z_1; \\ z_5 = -\kappa z_6 + z_4 + \dfrac{h^3}{12\Gamma_л}\omega^2 z_2; \\ z_6' = -\chi h \kappa z_1 - \left[\chi h + \dfrac{\kappa^2 h^3}{12\Gamma_л}\left(2(1+\nu) - \dfrac{c^2}{\Gamma_л}\right)\right] z_3 + \nu\kappa z_5. \end{cases} \quad (26)$$

The boundary conditions for this system can be written as follows:

а) free left edge of the plate:

$$z_4 = z_5 = z_6 = 0, \qquad x_1 = 0; \quad (27)$$

б) free right edge of the plate:

$$z_4 = z_5 = z_6 = 0, \qquad x_1 = l_1; \quad (28,a)$$

в) pinched right edge of the plate:

$$z_1 = z_2 = z_3 = 0, \qquad x_1 = l_1; \quad (28, б)$$

Thus formulated spectral problem (26-28) in the parameter ω, describing the propagation of flexural waves in a flat edge plate Timoshenko.

Notes

IV. Numerical Analysis of the Dispersion of the Edge Waves in the Wedge-Shaped Plates

The decision stated above spectral boundary-value problems (14), (15), (16) and (26), (27), (28) was performed by the method of orthogonal sweep Godunov [4]. Numerical implementation of this method was carried out on a computer using software package MAPLE. To test the method and the program was designed version of the album with the boundary conditions can be solved analytically in terms of trigonometric functions.

For resolving the system of equations (14) Kirchhoff-Love plate, these boundary conditions of the form:

$$X_1=0,1; \qquad z_2=z_4=0 \tag{29}$$

Here and below we use the dimensionless system of units in which the bandwidth *l*, shear modulus *G* and bulk density equal to unity.
In this case, the waveform is given by the expression W

$$z_1 = z_o \cos 2\pi\, n\, x_1 \tag{30}$$

$$z = -(2\pi n)^2 z_1 = A_2 z_1$$

$$z_3 = \frac{\left(\nu K^2 + (2\pi\, n)^2\right)h^3}{6(1-\nu)} z = A_3 z_1$$

$$z_4 = \left[\nu k^2 \frac{\left(\nu k^2 + (2\pi n)^2\right)h^3}{6(1-\nu)} + \frac{(1+\nu)}{6}k^4 - h\left(\frac{\omega}{g\,*}\right)^2\right] z_1 = A_4 z_1$$

$$z_3^1 = -\frac{\left(\nu k^2 + (2\pi n)^2\right)h^3}{6(1-\nu)}(2\pi n) z_1 \qquad z_2^1 = (2\pi n) z_2$$

$$z_4^1 = -A_4 (2\pi n) z_1 = -(2\pi n) z_4$$

Where z_o – arbitrary constant; c_n - The real part of the complex frequency; successively substituting the expression (30) into equation (26) we obtain the dispersion equation

$$\begin{vmatrix} 2\pi n & 1 & 0 & 0 \\ \nu k^2 & -(2\pi n) & -\frac{6(1-\nu)}{n^3} & 0 \\ 0 & -\frac{n^3}{3}k^2 & -(2\pi n) & 1 \\ B_1 & 0 & \nu k^2 & 2\pi n \end{vmatrix} = 0, \tag{31}$$

Where $B_1 = \frac{(1+\nu)h}{6}k^4 - n\left(\frac{\omega}{C_K + iC_I}\right)^2$

Ref

4. Grinchenko V.T., V.V. Myaleshka Harmonic oscillations and waves in elastic bodies, K .: Science Dumka, 1981, -283 p.

Similarly, choosing the boundary conditions for the resolution of the system (22) in the form of plates Timoshenko

$$x=0,1; \quad z_4=z_5=z_6=0 \tag{32}$$

Find the expression for the wave form

$$\begin{aligned} z_1 &= A_1 \cos 2\pi n x_2; & z_4 &= A_4 \sin 2\pi n x_2; \\ z_2 &= A_2 \cos 2\pi n x_2; & z_5 &= A_5 \sin 2\pi n x_2; \\ z_3 &= A_3 \sin 2\pi n x_2; & z_6 &= A_6 \cos 2\pi n x_2. \end{aligned} \tag{33}$$

Notes

In (33) permanent A_i $(i=1,2, 3,4,5 , 6)$ are determined by solving the system of equations

$$\begin{cases} A_3 + \dfrac{A_4}{\chi h} = 0; \\ \kappa A_3 - \dfrac{12}{h^3} A_5 = 0; \\ -\nu\kappa\kappa_2 - \dfrac{6(1-\nu)}{h^3} A_6 = 0; \\ \chi h\kappa\kappa_2 + \kappa^2(\chi h - hc^2)A_1 = 0; \\ -\chi h\kappa\kappa_1 - \left[\dfrac{(1-\nu)h^3}{6}\kappa^2 + \chi h - \dfrac{h^3}{12}\omega^2\right]A_2 + \nu\kappa\kappa_6 = 0; \\ -\kappa A_5 + A_4 + \dfrac{h^3}{12}\omega^2 A_3 = 0. \end{cases} \tag{34}$$

The system of equations (34) is obtained by substituting (33) in the resolution of the system of differential equations (22). Condition vanishing of the determinant of the system (34) is the dispersion equation boundary value problem (22), (33). The values of the phase velocities found from the above dispersion equations and solving the corresponding test problems (14). (29). (26) and (32) coincide with each other up to the fourth decimal place in the wave number range from 0.1 to 15 for the first two modes (n = 0.1). For the Kirchhoff-Love plates of variable thickness were investigated first five modes with minimum phase velocity of the complexes. Where $C = C_R + iC_I$, C_R - the phase velocity of wave propagation; C_I - speed damping. Figure 3a shows the dispersion curves of the first mode, depending on the thickness varies linearly. Here we assume that the two edges of the plate are free. The straight line I corresponds to a constant thickness $h_1=h_2=0,1$. In this case, the plate varies as a rod. Curve II - variant $h_1=h_2/2=0,05$; curve III - variant $h_1=h_2/100=0,001$, curve IV $h_1 = h_2/1000 = 0,0001$ and $E_{\min} = 6,9\cdot 10^6\ \kappa/м^2$, $E_{\max s} = 6,9\cdot 10^8\ \kappa/м^2$, $\beta =\ 10^{-4}$. Found that $\kappa > 9$ speed damping increase depending on k. For plastics constant thickness C_Γ on the segment $10^{-4} < C < 70$ decreases in a straight line. It can be seen, the dependence of the damping of the wave number starts on the wave number 3-6. With enthusiasm wave number damping factor tends to reduce hand. It can be seen that for a plate of constant thickness, the phase velocity tends to infinity, and for acute wedge plate there is a finite limit as $\kappa \to \infty$, i.e. the bending edge wave length sufficiently small (compared with the width of the plate) are distributed without dispersion. This fact is evident physical, since the edge of the wedge is no characteristic linear dimension. Land without dispersive waveguide movement begins with a wave of 3-9, which corresponds to the length of the waves, is less than 1.

It should be noted one fundamental point. Strictly speaking, this study did not consider the case of theoretical $h_1=0$ or $\kappa \to \infty$. All the numerical results obtained by the simulation of wave processes on a computer that can not operate with an infinitely small and infinitely large numbers. However, the numerical stability can check the result in sufficiently large range of parameters h_1 or κ. Despite the lack of theoretical basis, this verification sufficiently suggests that a known controlled precision found the limit value of any quantity at $h_1 \to 0$ or $\kappa \to \infty$. Physically, it is obvious that the parameters h_1 and κ must be coordinated so that the wavelength was substantially greater than the width edge h_1.

The numerical experiments show that the maximum dimensionless phase velocity (the real part of the complex frequency) during the first mode $K \to \infty$ largest coincides with the dimensionless thickness h_2. In dimensional terms, this corresponds to the following changes in the law (the actual number of complex frequency) limit the phase velocity C_{Ro} the angle of the wedge φ_o.

$$C_{Ro} = C_s tg \frac{\varphi_o}{2} \tag{35}$$

Coincides with the results of $(C_{Ro} = C_o)$ [6]. Numerical experiment also showed that the family of the dispersion curves with different angles at the vertex of the wedge has a certain similarity property, namely: the ratio of the phase velocity to the speed C_o (35) does not depend on the angle φ_o. For constant thickness form varies only slightly, while the wedge-shaped plate with increasing K, observed near the localization own form an acute angle. Figure 4 shows the dispersion curves and the second oscillation mode, depending on wavelength in distinguishing values of the thickness of the plate. When $K = 0$, the phase velocity is finite. Localization waveform and a limited range of the phase velocity with valid for this mode. Figure 4 b shows the imaginary part of the complex, depending on the speed of the wave hours for different thicknesses. It is seen that the rate of 3-4 imaginary second mode at $K > 5$ does not tolerate dispersion. Figure 4 b shows the evolution of the dispersion curves as a function of the wedge angle and thickness h_2. For small to form close to the line that corresponds to the torsion vibration at large to the observed localization. In contrast to the first mode is available hotspot. Fig. 5 a, b shows the dispersion curves and mode shapes for III and IV of fashion. With integrated small wave numbers phase velocity tends to infinity, and for large - to a finite limit. Also observed localization forms. The number of nodal points two and three, respectively modes (Figure 5).

Figure 6 shows the dependence of the real part of the phase velocities of the first four vibration modes acute wedge plate with different Poisson's ratio. As can be seen from the figure, the maximum phase velocity C_o the first mode is virtually independent of Poisson's ratio. In the last phase velocity modes C_o increases with increasing v, where the effect of Poisson's ratio is more pronounced at the higher-order modes (real part of complex velocity). Fig. 7 shows the dispersion curves of phase velocities of the four vibration modes for two variants of the legal termination edge of the plate: the free edge (dashed) and fixed (solid). Unlike these options significantly at small wave numbers and virtually absent at large, that is, as one would expect, the maximum phase velocity is independent of the conditions of securing the plate away from the edge of the wedge.

In [7], the distribution of the bending edge waves in the wedge-shaped waveguides in the framework of the linear theory of elasticity. We used the finite element method, based on which the empirical relation for the phase velocities of the normal modes of oscillation depending on the angle of the wedge φ:

$$C_o = C_r \sin (m\varphi); \quad m = 1,2,\ldots, \quad m\varphi¡90^{\circ}, \tag{36}$$

Where: C_r – Rayleigh wave speed for a half; m – mode number. It is easy to see that the relations (35) and (36) do not agree with each other at small angles φ. It is therefore of interest to find out what the limiting phase velocities obtained in the

Ref

7. Sunchaliev R.M., Filatov A. On some methods for the study of nonlinear problems in the theory of viscoelasticity // Dokl, 1972.206, №1. p. 201-203.

framework of a more general theory of plates Timoshenko. The spectral problem (26-28), which describes the distribution of edge waves plate Timoshenko was solved numerically orthogonal sweep method of Godunov. To control the numerical convergence of the method, the number of points equal to the orthogonalization taken from 10 to 100. In parallel redundant calculations were carried out in double precision. The result is considered satisfactory if the doubling of the number of points did not change the orthogonalization four significant digits in the phase velocity. Limiting the phase velocity of the first mode for thickness $h_2=0, 2$ equal to 0.1945 and is independent of Poisson's ratio. Compared with the same result obtained in the theory of Kirchhoff-Love, in this case the difference is less than 3%. Figure 8 shows the first three modes indeed part of the complex phase velocity of the plate Timoshenko (b) compared to the corresponding modes of Kirchhoff-Love plate (a) in the Poisson's ratio of 0.25. In the case of the Kirchhoff-Love plate limit above the phase II and III modes, and with increasing mode number increases contrast. The comparative analysis of the propagation of the edge waves on the basis of these theories plates shows a satisfactory agreement for the first vibration mode. The resulting discrepancy with the results in [5] indicates the need for more detailed research into the general theory of elasticity. Overall, however, conducted a numerical analysis of edge waves in the Kirchhoff-Love plates and Timoshenko suggests that the Kirchhoff-Love hypotheses are justified in the calculation of wave processes in the wedge-shaped plates, including frequencies with a wavelength of the order of the thickness of the plate. This discrepancy with the classical results of the theory of Kirchhoff-Love plates of constant thickness above phenomenon is explained by established localization waveforms with increasing frequency, which occurs only in the plates of variable thickness. At the same time, the relative simplicity of the mathematical apparatus of the theory of Kirchhoff-Love plates, allows us to investigate the dispersion characteristics of the waveguides with a more complex configuration section, which is very difficult to build as part of three-dimensional theory. Consider a plate, whose Thickness varies in accordance

$$h(x_1) = h_o /x_1/, \qquad -b \le x_1 \le b.$$

It is clear that such a plate vibrations are reduced to fluctuations in the wedge plate with boundary conditions at $x_1=0$, corresponds to the case of symmetry

$$\varphi = 0, \qquad Q = 0 \tag{37}$$

and of ant symmetry

$$W = 0, \qquad M = 0 \tag{38}$$

Figure 9.a. and 9.b. (solid lines) shows the dispersion curves of phase velocities of the first three modes in the Kirchhoff-Love plate with a linear variation of thickness.

$$h(x_1) = h_0 x_1^p, \qquad o < x_1 \le b,$$

where the parameter p taken equal to 1.5; 2; 2.5; 3 in accordance with designations of curves 1, 2, 3 and 4. For comparison, the dashed lines indicate similar curves discussed above relating to the wedge plate with a thickness $h\ (1) = h_o = 0,\ 2$. Note the qualitative difference in the behavior of solid and dotted lines. When p = 1, as mentioned above, the phase velocities approaching asymptoticity nonzero limits, the curve of the first mode increases monotonically. For p>1, the curve of the first mode is not monotonic and has a characteristic maximum in the medium range. Starting with a certain wave number of the phase velocities of all modes decrease monotonically without entering the asymptote nonzero. With increasing p the maximum curve of the first mode is shifted to lower frequencies, and shortwave phase velocities decrease more rapidly. Thus, summarizing the results obtained earlier in the event of a non-linear law of variation of the thickness of the plate, it can be argued that the phase velocity of the first mode in the wedge plate at high frequencies is determined by the rate of change of the thickness in the vicinity of the sharp edge.

Ref

5. Koltunov M.A. Creep and relaxation. - Publishing Moscow, 1976.- 276 p.

Notes

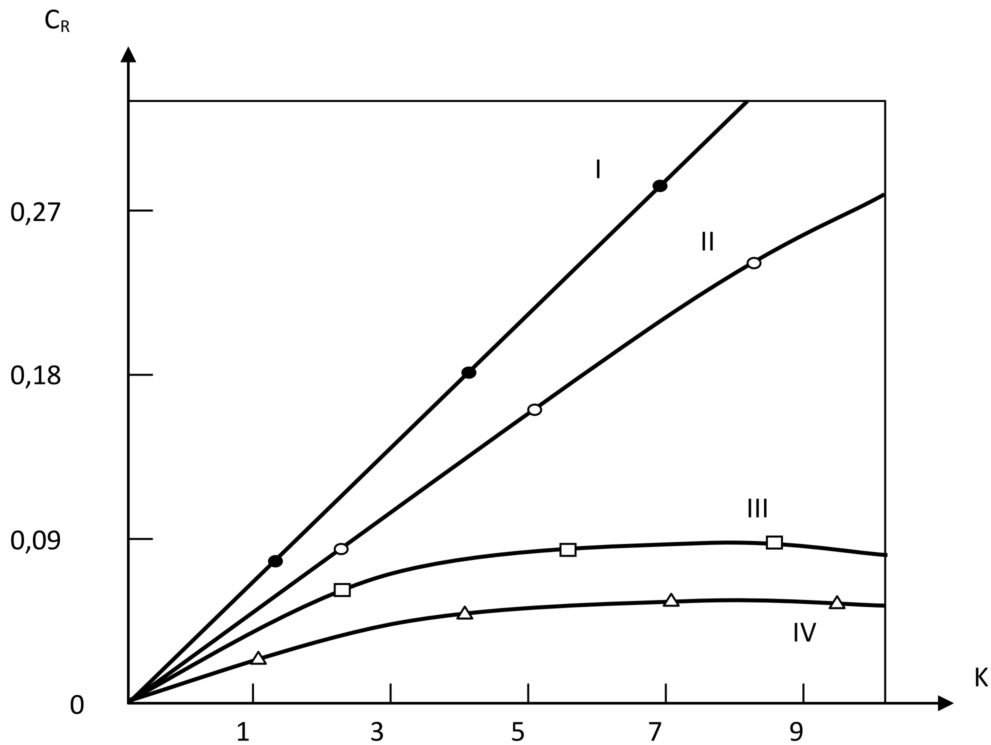

Figure 3 : The dispersion curves of the first mode

I. $h_1=h_2=0{,}1$; *II.* $h_1=h_{2/2}=0{,}05$; *III.* $h_{2/100}=0{,}001$; *IV.* $h_1=h_{2/1000}=0{,}001$

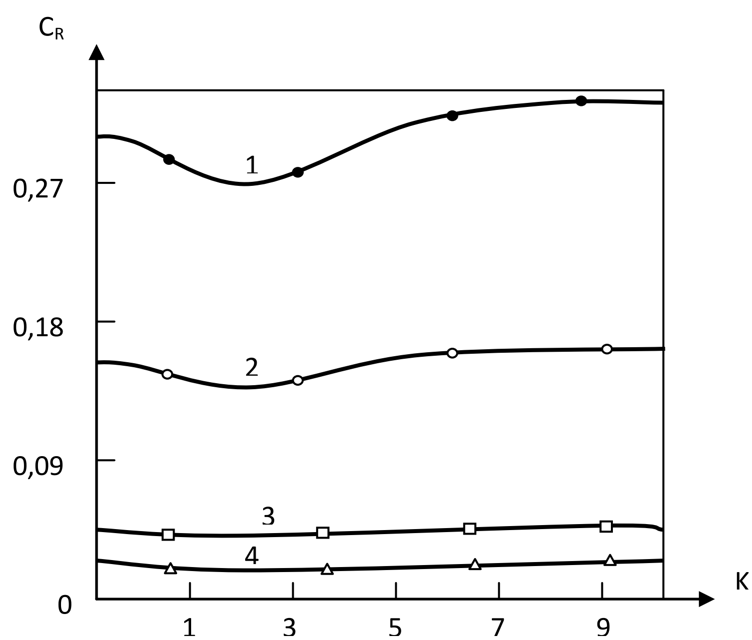

Figure 4a : The dispersion curves of the second mode

1. $h_2=0{,}002$, $h_2=0{,}2$; *2.* $h_1=0{,}001$, $h_2=0{,}1$; *3.* $h_1=0{,}0002$, $h_2=0{,}02$

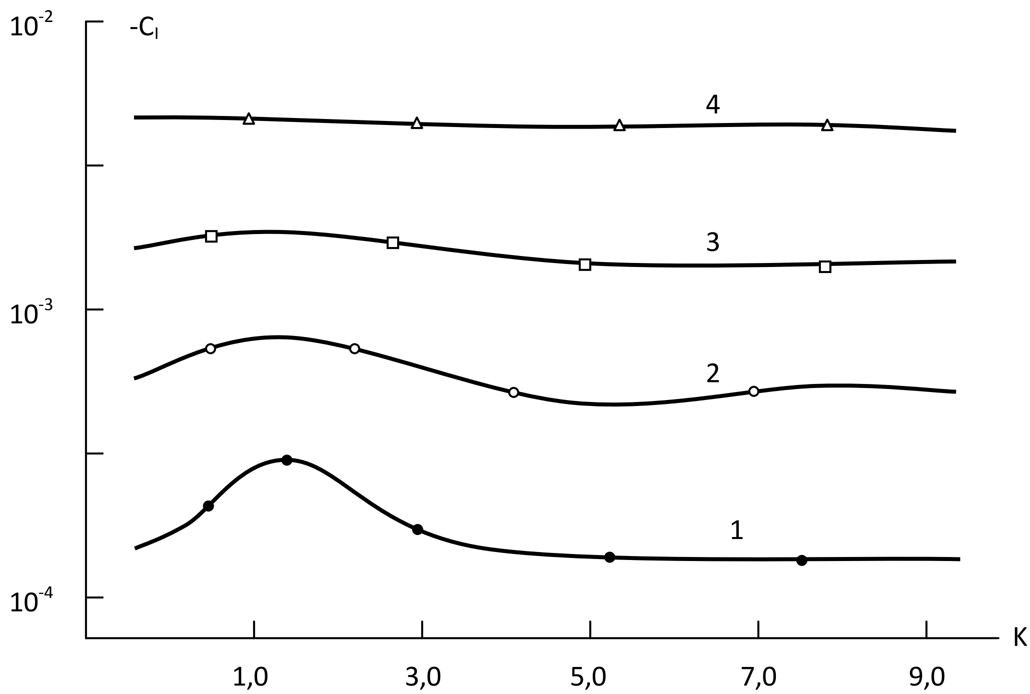

Figure 4 b : The dependence of the damping rate κ

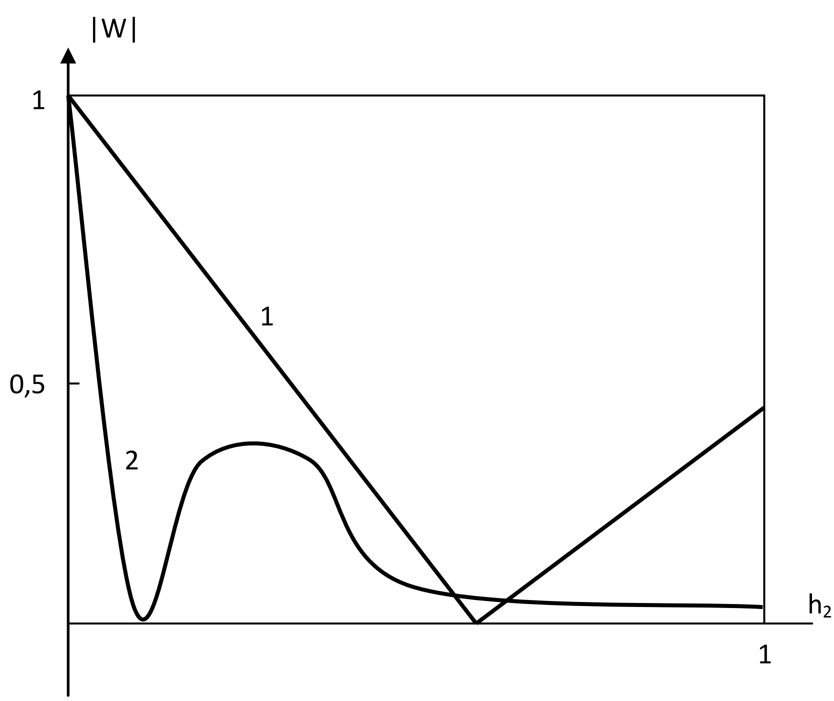

Figure 4 v. : Wave form corresponding dispersion curves of the second mode *1.K=1; 2.K=10*

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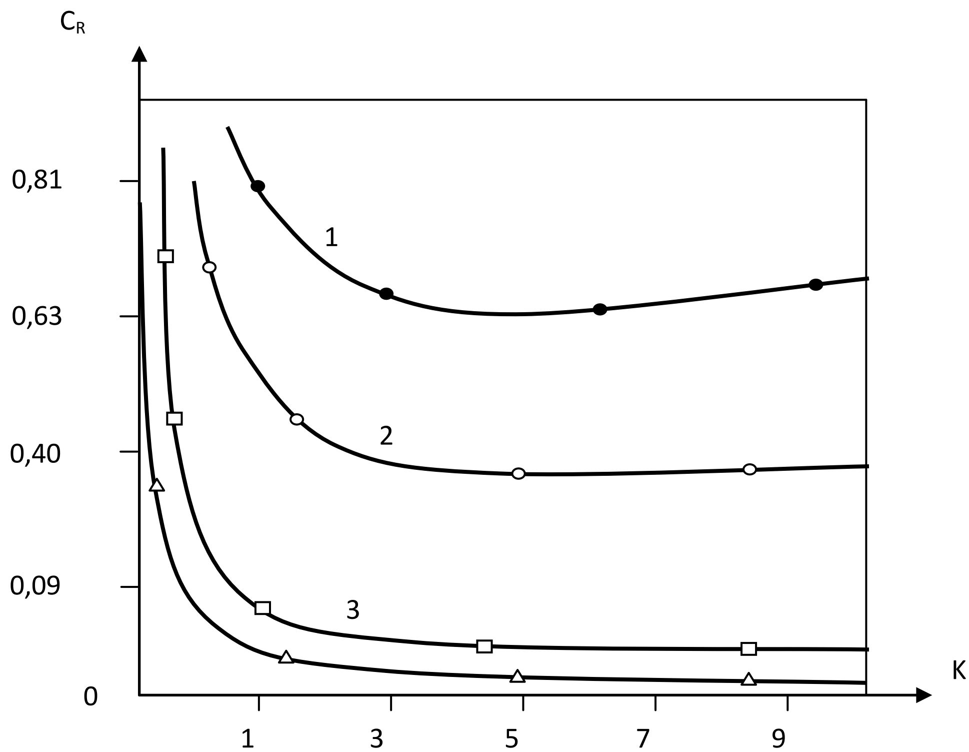

Figure 5a : The dispersion curves of the third mode

h_2=0,002, h_2=0,2; 2. h_1=0,001, h_2=0,1; 3. h_1=0,0002, h_2=0,02

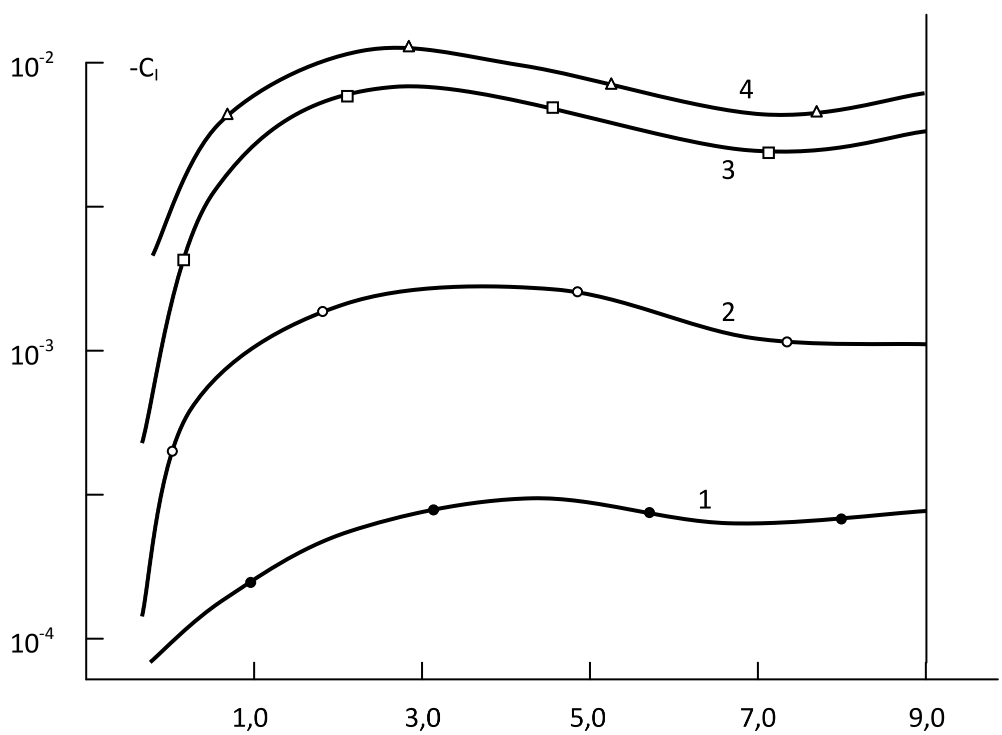

Figure 5 b : The dependence of the damping rate κ

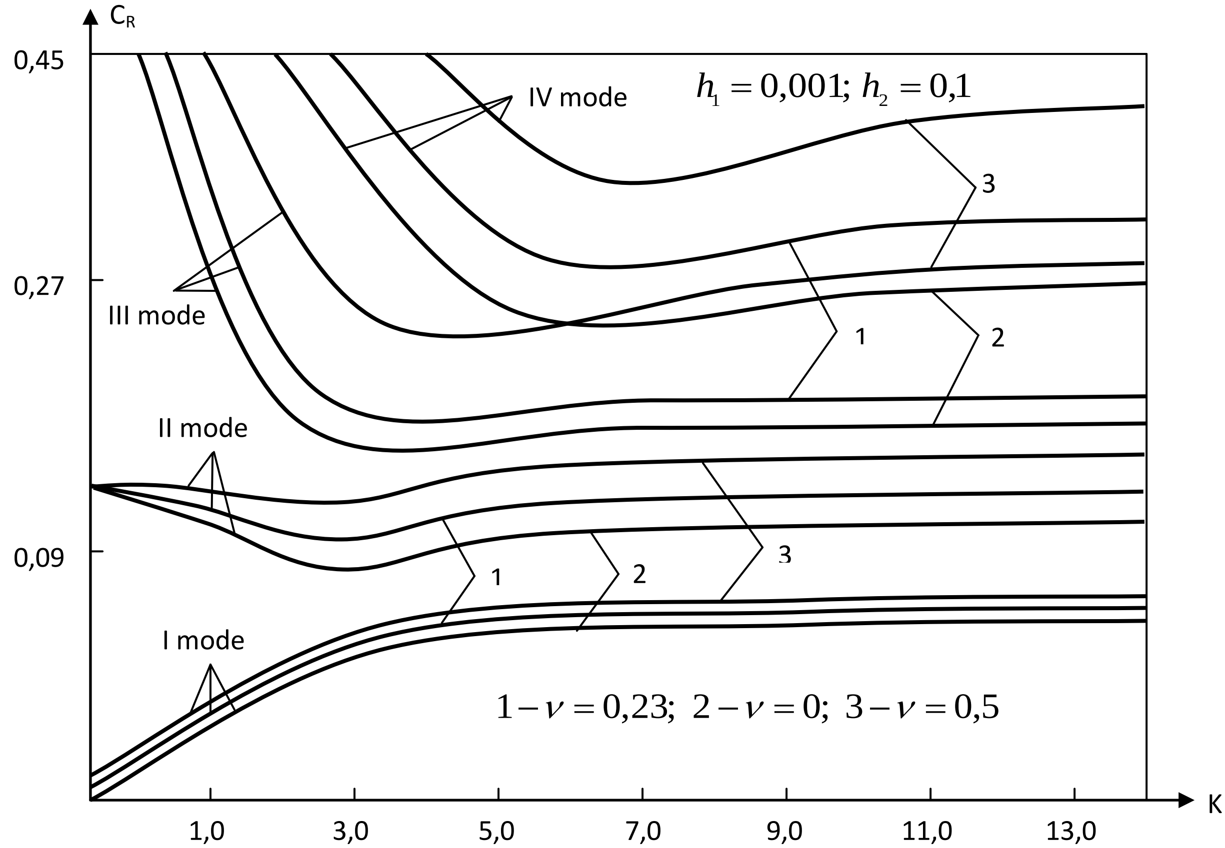

Figure 6 : The dependence of the phase velocity of the first four modal wedge plates with different Poisson's ratios

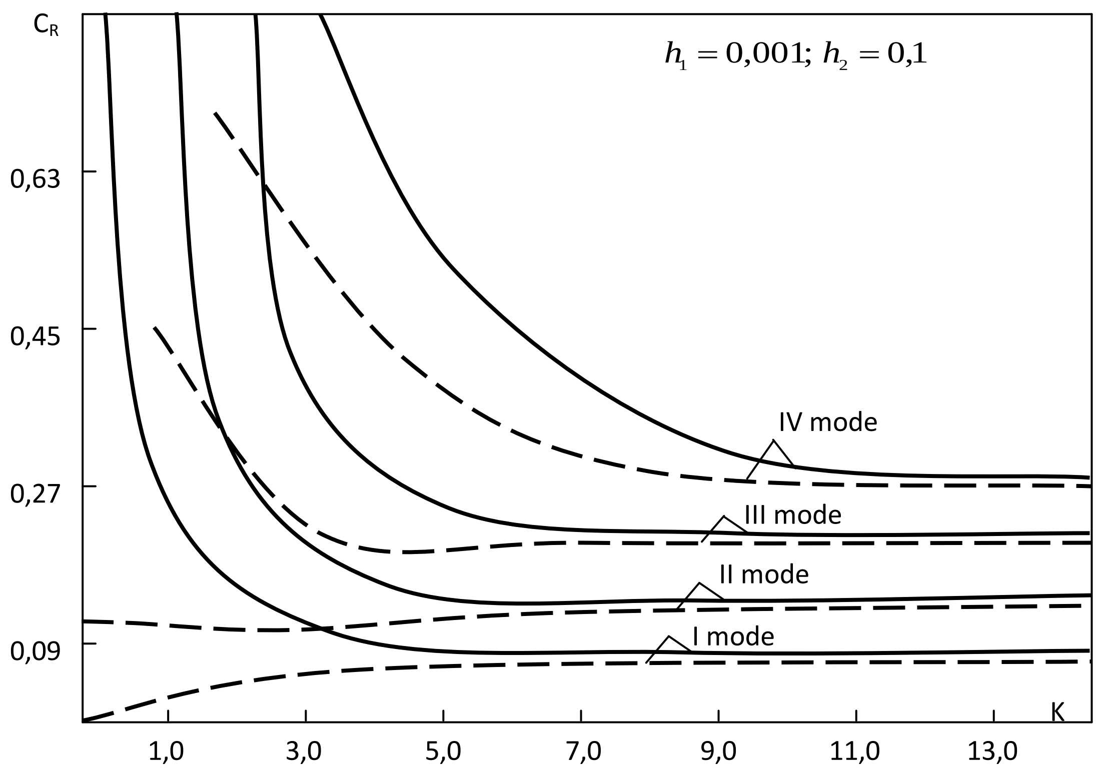

Figure 7 : The dispersion curves of phase velocities of the four modes for the two variants of the legal termination edge of the plate

——— free edge, -------- rigidly fixed

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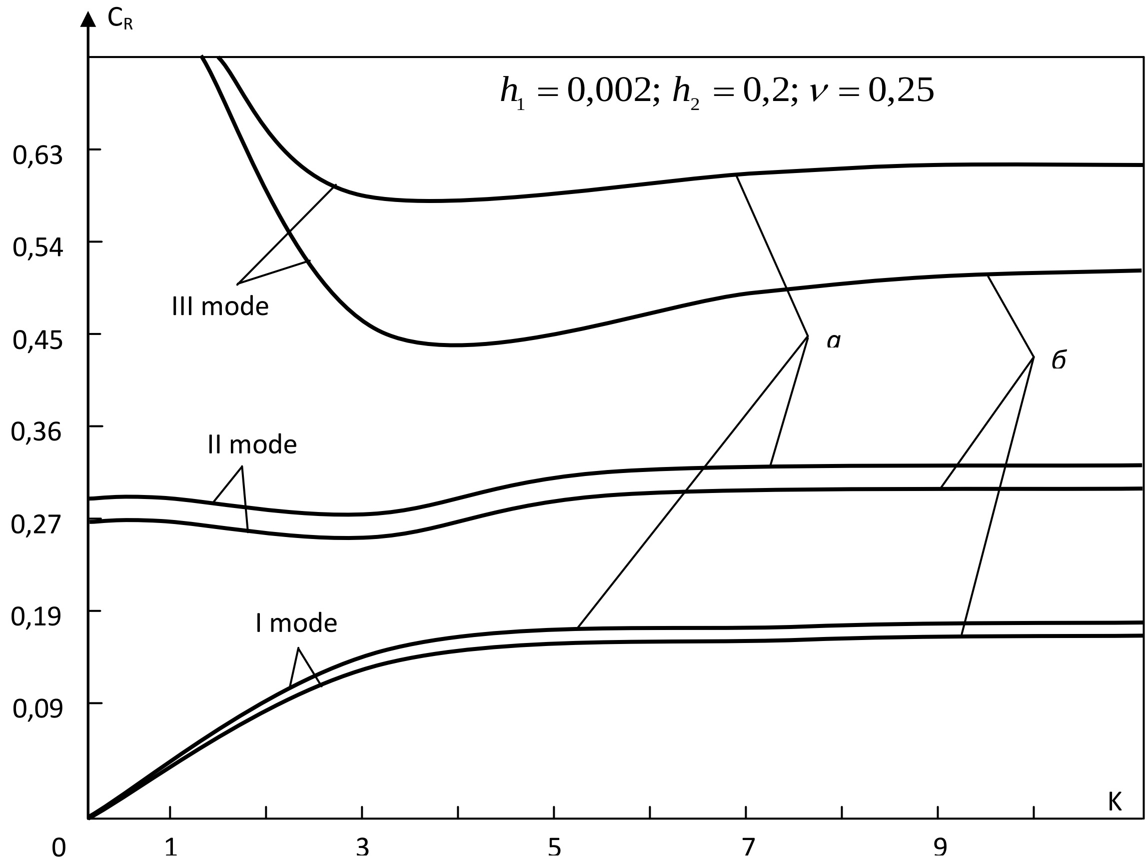

Figure 8 : The dispersion curves of phase velocities a Kirchhoff - Love; **б** - Timoshenko

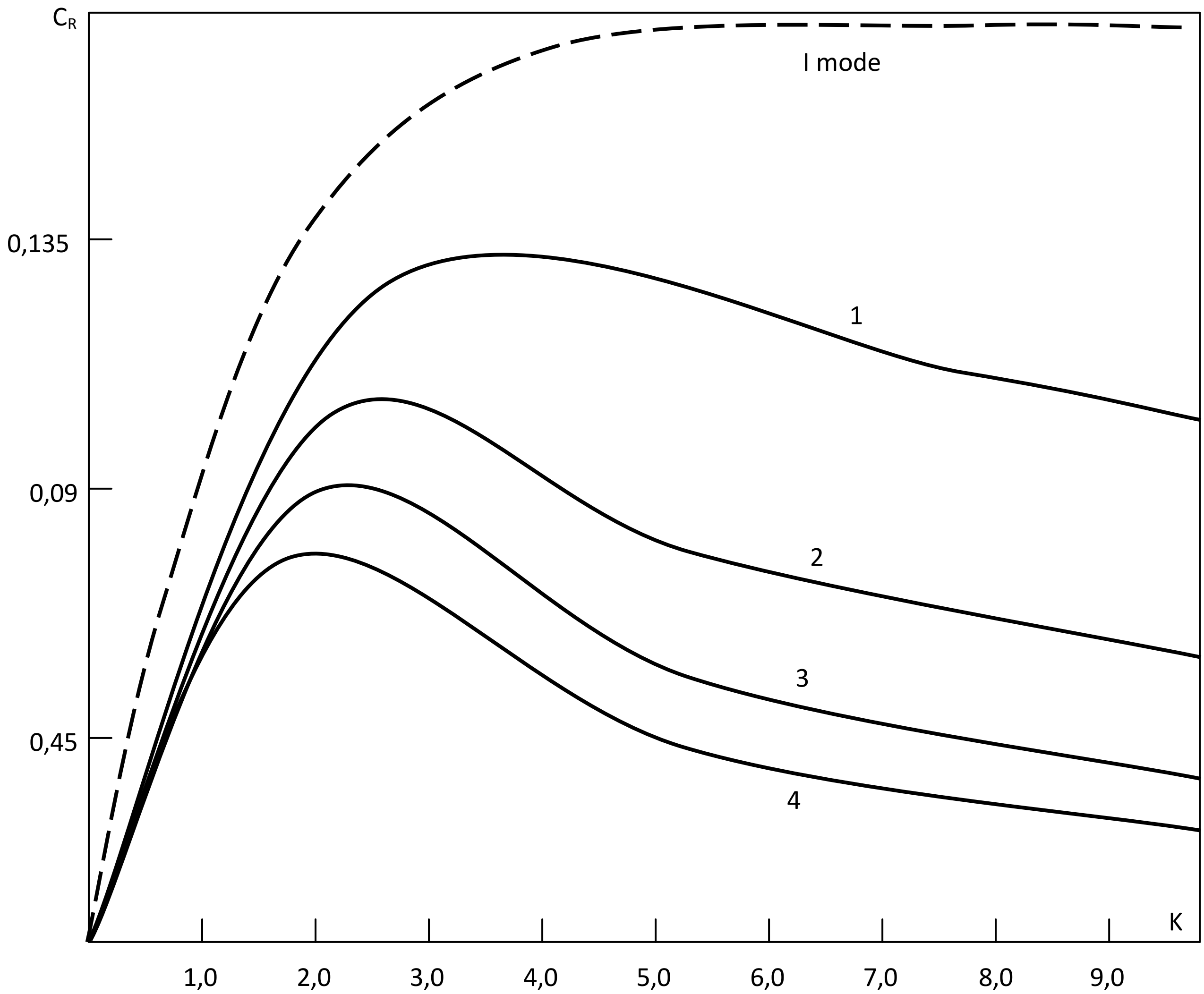

Figure 9a : The dispersion curves of phase velocities at different values P

1. P=1,5; 2. P=2; 3. P=2,5; ;. P=3

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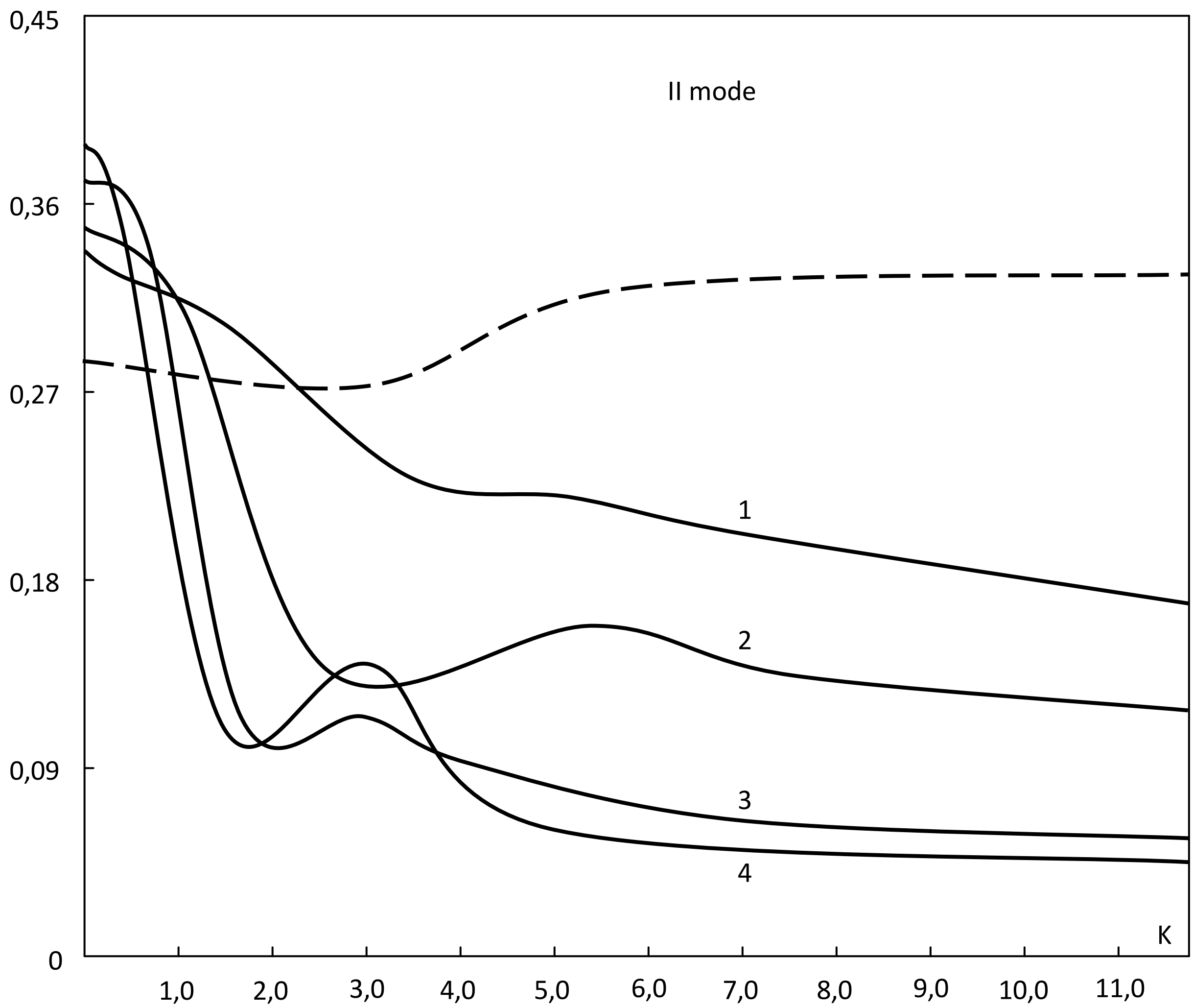

Figure 9.v. : The dispersion curves of phase velocities of the second mode at different values P.

1. P=1,5; 2. P=2; 3. P=2,5; ;. P=3

On the basis of these results the following conclusions:

- With increasing wave number of the velocity of propagation is real and the imaginary part of the normal modes in a wedge-shaped (plate) band Kirchhoff-Love and Timoshenko tend to constant values. At the same time there is the localization movement near the sharp edge of the waveguide.

- For small wedge angles comparison of the results obtained by the Kirchhoff-Love theory and Timoshenko, shows satisfactory agreement.

- Valid and imaginary parts of the complex phase velocity of the first mode in the wedge plate practically does not depend on the Poisson ratio (change within 0.5%).

- In the short-range limit value is valid and the imaginary part of the phase velocity of the first mode in the tapered waveguide is determined by the rate of change of the thickness in the vicinity of the sharp edge.

- In wedge-shaped plates with a small angle at the apex of a no dispersive waves propagate with a length not exceeding bandwidth.

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