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Numerical Investigation of the Stability of Equilibrium Points for the Model of Three Mutually Competing, Symmetric and Continuous Time Reproducing Organism in a Fairly Stable Ecological Environment

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Abstract- In this paper, we examined the stability of the equilibrium points for the model of three mutually competing species within a stable ecosystem. For the stable ecosystem, we proposed that there are combinations of initial population densities and efficiency parameters that can balance the coexistence of these species for a very long time. This desirable biological property of a stable non-extinction equilibrium point for the ecosystem was obtained for all cases considered in this research work.

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Numerical Investigation of the Stability of Equilibrium Points for the Model of Three Mutually Competing, Symmetric and Continuous Time Reproducing Organism in a Fairly Stable Ecological Environment

A. A. Obayomi [α] & M. O. Oke [σ]

Abstract- In this paper, we examined the stability of the equilibrium points for the model of three mutually competing species within a stable ecosystem. For the stable ecosystem, we proposed that there are combinations of initial population densities and efficiency parameters that can balance the coexistence of these species for a very long time. This desirable biological property of a stable non-extinction equilibrium point for the ecosystem was obtained for all cases considered in this research work.

Keywords: stability, equilibrium points, ecosystem, population density.

I. Introduction

In Obayomi and Oke (2015), we investigated the equilibrium state for the kind of system considered in this paper. We also confirmed the analytic equilibrium points and showed that all the numerical equilibrium points coincide with at least one of the analytic ones. This implies that the schemes may not possess numerical instabilities, Mickens (1994) and Mickens (2000). In this paper, we shall investigate the stability of these equilibrium points based on various combinations of parameters and initial values. Consider a general first order ordinary differential equation (ODE) of the form

$$y' = f(t,y)\,,\ \ y(t_0) = y_0 \qquad (1)$$

Suppose equation (1) possesses the properties of existence and uniqueness of solution in its domain U, then the following definitions holds.

Definition 1: The zeros of the function f in equation (1) is a critical point. A point $\underline{y} \in \mathrm{R}$ is called a fixed point or equilibrium point of the dynamical system defined by (1) if $f(\underline{y}) = 0$. If c is any critical point of f, then $y(x) = c$ is a constant solution of the differential equation.

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Remark 1: A fixed point y is stable if all nearby solutions stay nearby. It is asymptotically stable if all nearby solutions stay nearby and also tend to y or are attracted by y.

Definition 2: (Stability of equilibrium point)

Let $U \in R^m$ be the domain of definition of f in equation (1), then any equilibrium point y of (1) is said to be stable if and only if, for every neighborhood **Ω** of y in U, there is an open neighborhood V of y such that any orbit starting globally in V remains in **Ω** for all $t \geq 0$. If V = U then y is globally stable. If y is stable and there exists a domain V_0 such that any orbit originally in V_0 tends to y as t tends to infinity, then y is asymptotically stable, Beltrami (1986).

II. Materials and Methods

a) Stability of Non-standard Finite Difference Schemes

We derived our standard reference for the qualitative property of non-standard schemes from the work of Anguelov and Lubuma (2003) and the references therein.

It is a common fact to write the functional dependence y_{n+1} on the quantities x_n, y_n and h in the form

$$y_{k+1} = y_k + h\varphi(x_k, y_k; h) \tag{2}$$

where $\varphi(x_k, y_k; h)$ is called the increment function.

Definition 3:

Let us denote (2) by the sequence

$$y_k = F(h; y_k) \tag{3}$$

and let us assume that the solution of equation (1) satisfies some property $\mathcal{P}$. The numerical scheme (3) is said to be qualitatively stable with respect to property $\mathcal{P}$ or $\mathcal{P}$-stable, if for every value $h > 0$, the set of solutions of (3) satisfies $\mathcal{P}$, Anguelov and Lubuma(2003).

Definition 4: A set $G(\Omega)$ of real-valued functions defined on a subset Ω of $[t_0, \infty)$ monotonically depend on the initial value (t_0) if for every two functions $y, z \in G(\Omega)$, we have $y(t_0) \leq z(t_0)$ implies that $y(t) \leq z(t)$, for $t \in \Omega$, Anguelov and Lubuma(2003).

Definition 5: The finite difference scheme (3) is stable with respect to the property of monotonicity of solutions if for every $y_0 \in \mathbb{R}$, the solution y_k of (3) is an increasing or a decreasing sequence just as the $y(t)$ of equation (1) is increasing or decreasing.

Definition 6: Any fixed point $\bar{y}$ of (1) is called hyperbolic fixed point if it satisfies the relationship $j \equiv f'(\bar{y}) \neq 0$, Anguelov and Lubuma(2003).

Remark 2: The asymptotic behavior of solutions of (1) with initial data near y may be reduced to the behavior of linear equation of the form

$$\epsilon' = J\epsilon. \tag{4}$$

Definition 7: A hyperbolic fixed point is called linearly-stable provided that the solution ϵ of equation (1) corresponding to any small initial data $\epsilon(0)$, for $|\epsilon(0)| << 1$ say,

Notes

satisfies $\lim_{t\to\infty} \epsilon(t) = 0$. Otherwise the fixed point is linearly unstable. The discrete analogue of (4) is given by

$$\epsilon_{k+1} = J_h \epsilon_k \quad \text{where} \quad J_h = \frac{\partial F}{\partial y}(h;\bar{y}) \qquad (5)$$

Notes

Definition 8: Assume that a hyperbolic fixed point $\bar{y}$ of the differential equation (1) is a solution to the discrete method (3). We say that the constant solution $\bar{y}$ is linearly stable provided that the solution ϵ_k of the equation (1) corresponding to any initial data $\epsilon(0)$, for $|\epsilon(0)| << 1$ say, satisfies $\lim_{k\to\infty} \epsilon_k = 0$. This is equivalent to saying that $|J_h|<1$ in (5). Otherwise the fixed point is linearly unstable, Anguelov and Lubuma (2003).

Definition 9: The finite difference method (3) is called elementary stable if for any value of the step size h , its only fixed points $\bar{y}$ are those of the differential equation (1), the linear stability property of each $\bar{y}$ being the same for both the differential equation and the discrete method.

Theorems and conditions supporting these stability properties above may be found in Anguelov and Lubuma (2003).

Numerical experiments have been carried out in which several set of different initial values have been used to test the stability of the equilibrium points while fixing the efficiency parameters. The equilibrium points tested for stability are those that were obtained in Obayomi and Oke (2015).

III. Numerical Experiments

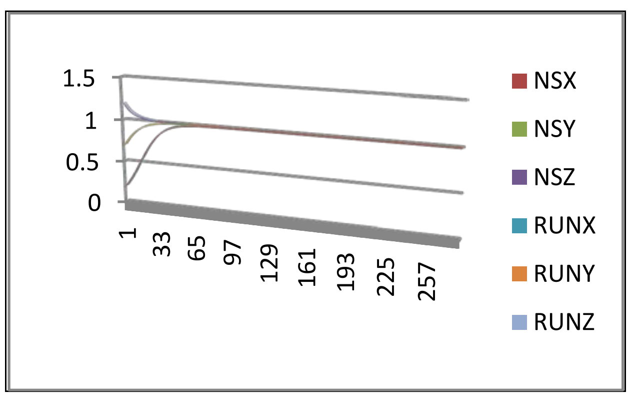

Figure 1 : Orbit of the schemes for $\propto = 0\,, \beta = 0$

The equilibrium state is (1, 1, 1) for any non-zero initial values

Note: NS = Non-standard and RUN= Runge Kutta

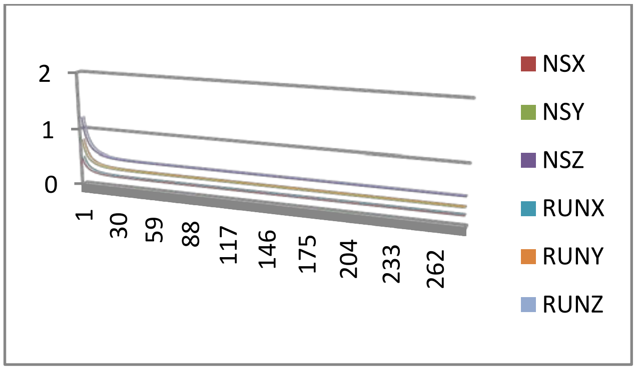

Figure 2 : Orbit of the schemes when $\propto = 1, \beta = 1$

The equilibrium state is (0.195, 0.317, 0.488) for initial value (0.5, 0.8, 1.2) which is in the form (X, Y, Z) such that $X + Y + Z = 1$

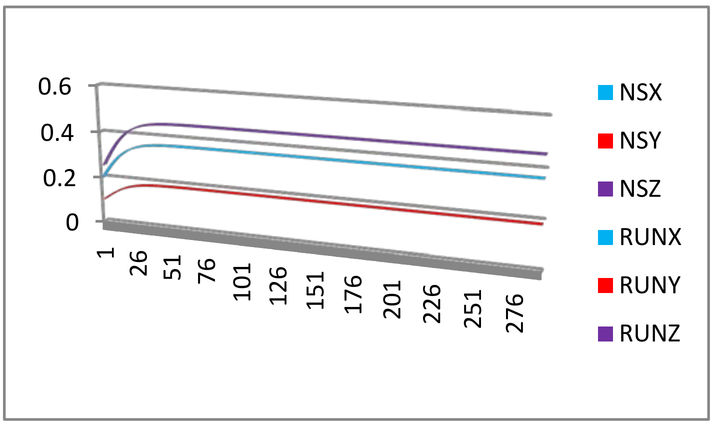

Figure 3 : Orbit of the schemes when $\propto = 1, \beta = 1$

The equilibrium state is (0.364, 0.183, 0.453) for initial value (0.2, 0.1, 0.25) which is in the form (X, Y, Z) such that $X + Y + Z = 1$

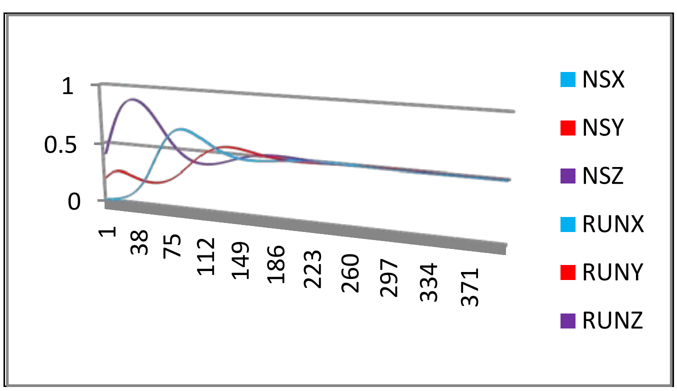

Figure 4 : Orbit of the schemes for $\propto = 1, \beta = 0$ or $\propto = 0, \beta = 1$

The equilibrium state is (0.5 ,0.5, 0.5) for any non-zero initial value

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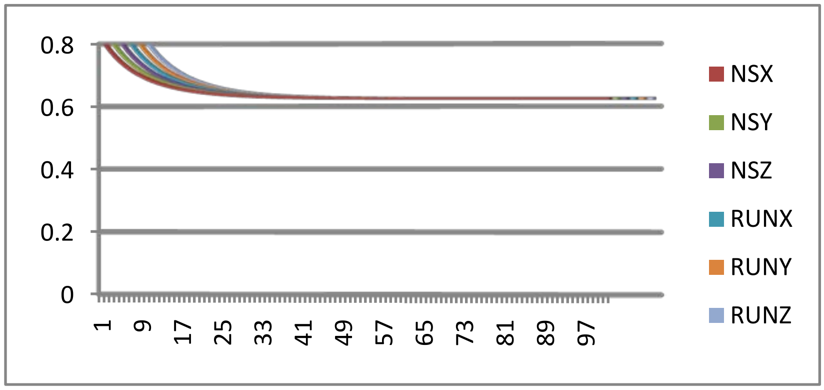

Figure 5 : Orbit of the schemes for $\alpha, \beta \epsilon (0,1)$ *and* $\alpha\, \beta < 1$

The equilibrium point is (0.625, 0.625, 0.625) when $(\alpha = 0.25, \beta = 0.35)$ with initial value (0.8, 0.8,0.8) which confirms (k, k, k) where $k = \frac{1}{1+\alpha+\beta}$.

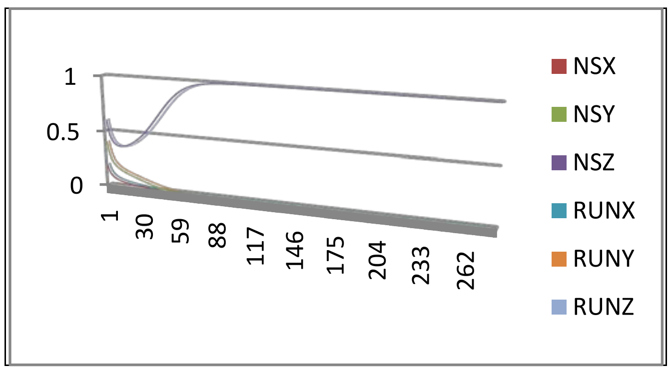

Figure 6 : Orbit of the schemes for $\alpha, \beta > 1$ i.e $\alpha\, \beta > 1$

The equilibrium point is (0, 0, 1) for $(\alpha = \beta = 3)$ with initial value (0.2, 0.4, 0.6).

Note: The equilibrium point is (1,0,0), (0,1,0) or (0,0,1) depending on the initial values. The specie with the highest initial value will wipe out the other two species on the long run

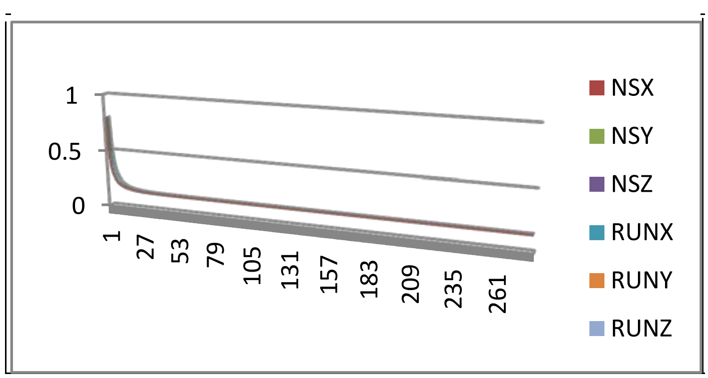

Figure 7 : Orbit of the schemes for $\alpha, \beta > 1$ i.e $\alpha\, \beta > 1$

The equilibrium point is (0.14286, 0.14286, 0.14286) for $(\alpha = \beta = 3)$ with initial value (0.8, 0.8, 0.8) which confirms (k, k, k) where $k = \frac{1}{1+\alpha+\beta}$

Note: The equilibrium point is (k, k, k) where $k = \frac{1}{1+\alpha+\beta}$ if the initial values are the same.

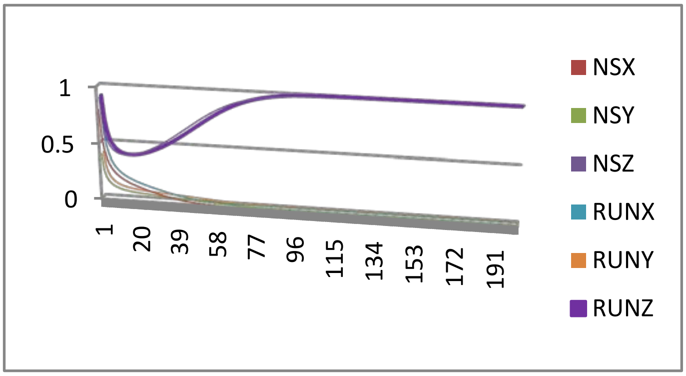

Figure 8 : Orbit of the schemes for $\alpha, \beta > 1, \alpha \neq \beta$ i.e $\alpha\,\beta > 1$

The equilibrium point is (0, 1, 0) for $\alpha = 2, \beta = 3$

The equilibrium point is reduced to one of (1, 0, 0), (0, 1, 0) or (0, 0, 1) in relation to the density at the initial time

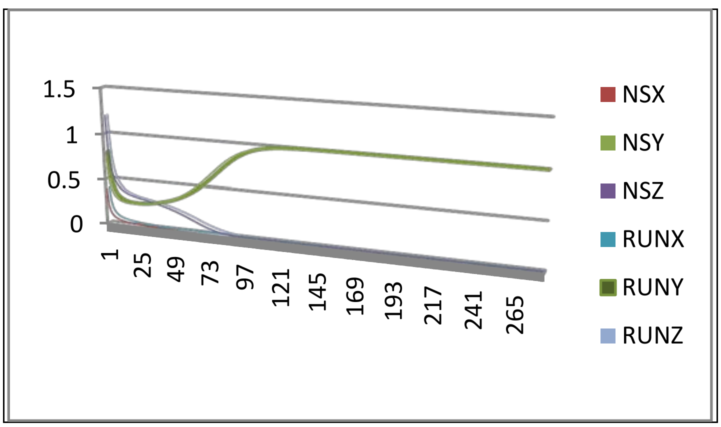

Figure 9 : Orbit of the schemes for $\alpha, \beta > 1, \alpha \neq \beta$ i.e $\alpha\,\beta > 1$

The equilibrium point is (0,0,1) for $\alpha = 3, \beta = 2$, initial value is (0.4, 0.8, 1.2).

The equilibrium point is reduced to one of (1,0,0),(0,1,0) or (0,0,1) in relation to the initial density.

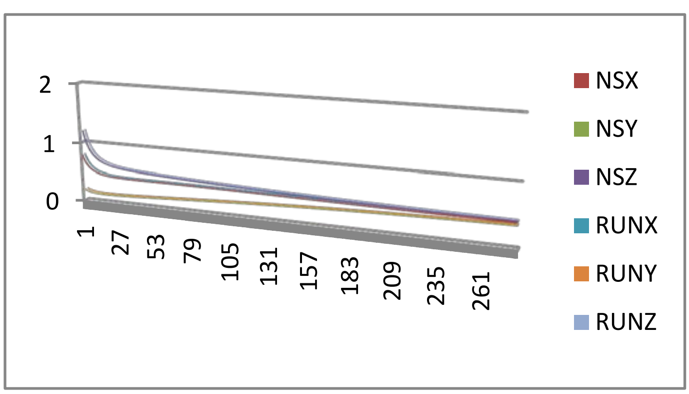

Figure 10 : Orbit of the schemes for $\alpha, \beta \,\epsilon\, (0,1)$ *and* $\alpha\,\beta < 1$

The equilibrium point is (0.4, 0.4, 0.4), $(\alpha = \beta = 0.75)$ with initial value (0.8, 0.2, 1.2)

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which confirms (k, k, k) where $k = \frac{1}{1+\alpha+\beta}$.

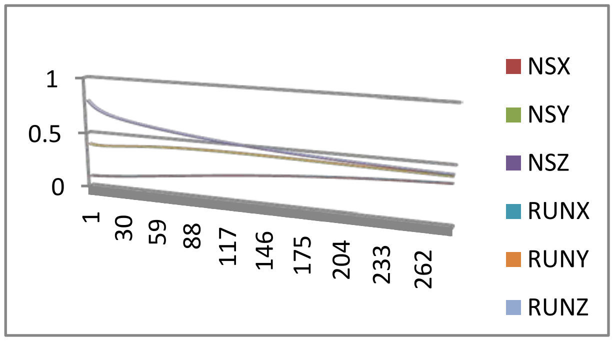

Figure 11 : Orbit of the schemes for $\alpha, \beta \,\epsilon\, (0,1)$ *and* $\alpha\, \beta < 1$

The equilibrium point is (0.4, 0.4, 0.4), $(\alpha = \beta = 0.75)$ with initial value (0.1, 0.4, 0.8) which confirms (k, k, k) where $k = \frac{1}{1+\alpha+\beta}$.

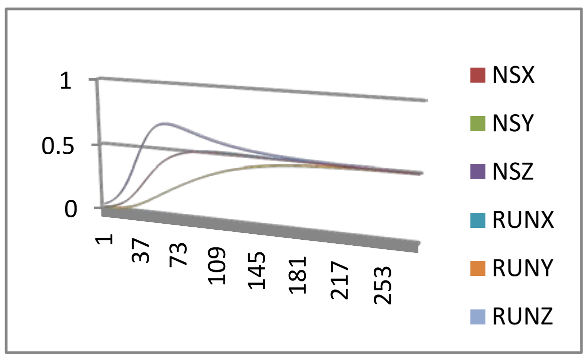

Figure 12 : Orbit of the schemes for $\alpha, \beta \,\epsilon\, (0,1)$ *and* $\alpha\, \beta < 1$

The equilibrium point is (0.5, 0.5, 0.5), $(\alpha = \beta = 0.5)$, initial value is (0.01, 0.003, 0.03) which confirms (k, k, k) where $k = \frac{1}{1+\alpha+\beta}$.

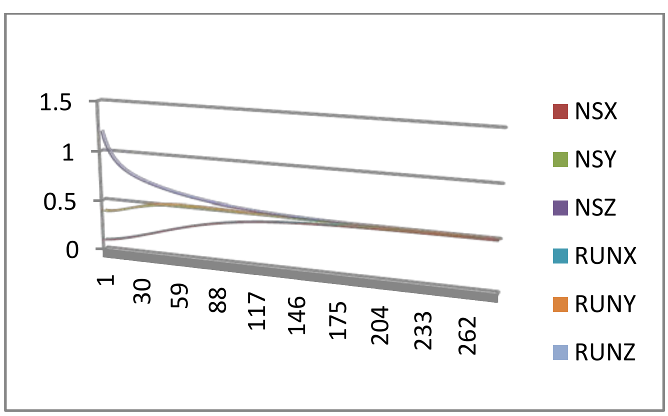

Figure 13 : Orbit of the schemes for $\alpha, \beta \,\epsilon\, (0,1)$ *and* $\alpha\, \beta < 1$

The equilibrium point is (0.5, 0.5, 0.5), $(\alpha = \beta = 0.5)$ with initial value (0.1, 0.4, 1.2) which confirms (k, k, k) where $k = \frac{1}{1+\alpha+\beta}$.

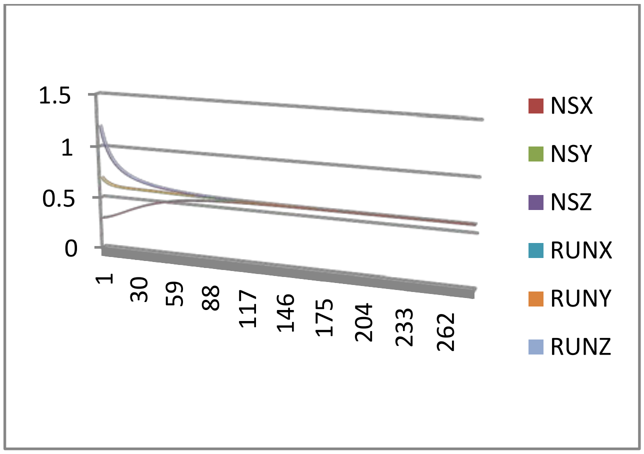

Figure 14 : Orbit of the schemes for $\alpha, \beta \epsilon (0,1)$ *and* $\alpha\, \beta < 1$

The equilibrium point is (0.588, 0.588, 0.588), $(\alpha = \beta = 0.35)$ initial value is (0.3, 0.7, 1.2) which confirms (k, k, k) where k $= \frac{1}{1+\alpha+\beta}$.

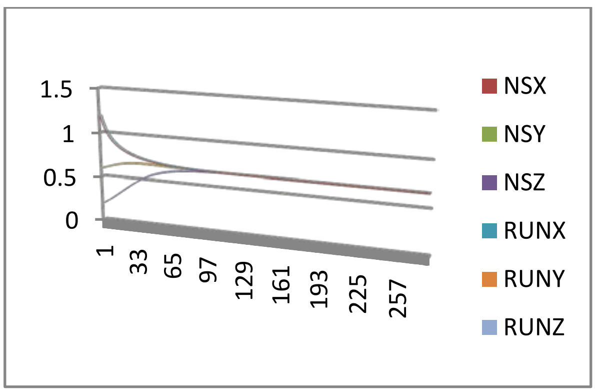

Figure 15 : Orbit of the schemes for $\alpha, \beta \epsilon (0,1)$ *and* $\alpha\, \beta < 1$

The equilibrium point is (0.67, 0.67, 0.67), $(\alpha = \beta = 0.25)$ with initial value (1.2, 0.6, 0.2) which confirms (k, k, k) where k $= \frac{1}{1+\alpha+\beta}$

IV. Discussion of Results

The following were observed from the result of the numerical experiment.

i For $\propto = \beta = 0$, the equilibrium state is $(1,1,1)$. Therefore, for any initial value, it is expected that if the co-existent of the species are either mutually beneficial or of insignificant effect on each other then all species are expected to grow to the full carrying capacity of the ecological environment and the point $(1,1,1)$ is a stable equilibrium.

ii For $\propto = \beta = 1$, the equilibrium state is $(X_1\ , X_2, X_3\ )$ such that $X_1 + X_2 + X_3 = 1$. It may however reduce the habitat to a partially extinct equilibrium state with only the specie with the largest initial value remaining and growing to its full capacity as competition fades away. This equilibrium points depends on the initial value and therefore it is not stable.

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iii For $\propto = 1$, $\beta = 0$ or $\propto = 0$, $\beta = 1$ the equilibrium state is $(0.5, 0.5, 0.5)$ for any initial value. The equilibrium point in this case is stable.

iv For $\alpha, \beta \epsilon (0,1)$ *and* $\alpha \beta < 1$ the equilibrium point is (k, k, k) where $k = \frac{1}{1+\alpha+\beta}$ for any initial value. The equilibrium point here is also stable.

v For $\alpha, \beta > 1$ i.e $\alpha \beta > 1$ the equilibrium point is (k, k, k) where $k = \frac{1}{1+\alpha+\beta}$. If the initial values are the same, the equilibrium points are unstable.

vi For $\alpha, \beta > 1$, $\alpha = \beta$ i.e $\alpha \beta > 1$, the equilibrium point is (k, k, k) where $k = \frac{1}{1+\alpha+\beta}$. If the initial values are the same and the habitat is reduced to one of $(1,0,0)$, $(0,1,0)$ or $(0,0,1)$ depending on the specie with the largest initial value, then the equilibrium point is unstable.

vii For $\alpha, \beta > 1, \alpha \neq \beta$ i.e $\beta > 1$. If the habitat is reduced to one of $(1,0,0)$, $(0,1,0)$ or $(0,0,1)$ depending on the initial values, then the equilibrium points are unstable.

viii The computational results also confirm the trivial equilibrium points.

V. Conclusion

Our results support and confirm the earlier studies on this model and exposed the expected physical situation in the ecological environment. With a suitable control mechanism on the efficiency parameters, ecologists, zoologists etc can apply this model for research and ecological planning. The information obtained here may also be used for pest control in Agrarian science.

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