Global Journal of Science Frontier Research: A
Physics and Space Science
Volume 15 Issue 2 Version 1.0 Year 2015
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

The Two - Variable $(\frac{G'}{G}, \frac{1}{G})$ - Expansion Method for Solving Nonlinear Dynamics of Microtubles - A New Model

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GJSFR-A Classification : *FOR Code: 35A05, 35A20, 65K99, 65Z05, 76R50, 70K70*

THETWOVARIABLEEXPANSIONMETHODFORSOLVINGNONLINEARDYNAMICSOFMICROTUBLESANEWMODEL

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Emad H. M. Zahran [α] & Mostafa M. A. Khater [σ]

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I. Introduction

Many models in mathematics and physics are described by nonlinear differential equations. Nowadays, research in physics devotes much attention to nonlinear partial differential evolution model equations, appearing in various fields of science, especially fluid mechanics, solid-state physics, plasma physics, and nonlinear optics. Large varieties of physical, chemical, and biological phenomena are governed by nonlinear partial differential equations. One of the most exciting advances of nonlinear science and theoretical physics has been the development of methods to look for exact solutions of nonlinear partial differential equations. Exact solutions to nonlinear partial differential equations play an important role in nonlinear science, especially in nonlinear physical science since they can provide much physical information and more insight into the physical aspects of the problem and thus lead to further applications. Nonlinear wave phenomena of dispersion, dissipation, diffusion, reaction and convection are very important in nonlinear wave equations. In recent years, quite a few methods for obtaining explicit traveling and solitary wave solutions of nonlinear evolution equations have been proposed. Such methods are tanh - sech method [1]-[3], extended tanh - method [4]-[6], sine - cosine method [7]-[9], homogeneous balance method [10, 11], F-expansion method [12]-[14], exp-function method [15, 16], trigonometric function series method [17], ($\frac{G'}{G}$) expansion method [18]-[21], Jacobi elliptic function method [22]-[25], The ($\frac{G'}{G},\frac{1}{G}$)-expansion method [26]-[28] and so on.

The objective of this article is to apply. The ($\frac{G'}{G},\frac{1}{G}$)- expansion method for finding the exact traveling wave solution of Nonlinear dynamics of microtubules- a new model which play an important role in biology and mathematical physics.

The rest of this paper is organized as follows: In Section 2, we give the description of The ($\frac{G'}{G},\frac{1}{G}$)-expansion method In Section 3, we use this method to find the exact solutions of the nonlinear evolution equations pointed out above. In Section 4, conclusions are given.

II. Description of Method

Before, we describe the main steps of this method,we need the following remarks [26]-[28]:

Remark 1. If we consider the second order linear ODE:

$$G''(\xi) + \lambda G(\xi) = \mu, \tag{2.1}$$

and set $\phi = \frac{G'}{G}$, $\psi = \frac{1}{G}$, then we get

$$\phi' = -\phi^2 + \mu\psi - \lambda, \quad \psi' = -\phi\psi. \tag{2.2}$$

Remark 2. If $\lambda < 0$, then the general solutions of Eq.(2.1) has the form :

$$G(\xi) = A_1\, sinh(\xi\sqrt{-\lambda}) + A_2\, cosh(\xi\sqrt{-\lambda}) + \frac{\mu}{\lambda}, \tag{2.3}$$

where A_1 and A_2 are arbitrary constants. Consequently, we have

$$\psi^2 = \frac{-\lambda}{\lambda^2\sigma + \mu^2}(\phi^2 - 2\mu\psi + \lambda) \tag{2.4}$$

where $\sigma = A_1^2 - A_2^2$.

Remark 3. If $\lambda > 0$, then the general solutions of Eq.(2.1) has the form:

$$G(\xi) = \frac{\mu}{2}\xi^2 + A_1\xi + A_2, \tag{2.5}$$

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and hence

$$\psi^2 = \frac{1}{A_1^2 - 2\mu A_2}(\phi^2 - 2\mu\psi). \qquad (2.6)$$

where $\sigma = A_1^2 + A_2^2$.

Remark 4. If $\lambda = 0$, then the general solution of Eq.(2.1) has the form:

$$G(\xi) = \frac{\mu}{2}\xi^2 + A_1\xi + A_2, \qquad (2.7)$$

and hence

$$\psi^2 = \frac{1}{A_1^2 - 2\mu A_2}(\phi^2 - 2\mu\psi). \qquad (2.8)$$

Suppose we have the following NLPDEs in the form:

$$F(u, u_t, u_x, u_y, u_z, u_{xx}, ...) = 0, \qquad (2.9)$$

where F is a polynomial in *u(x; y; z; t)* and its partial derivatives. In the following, we give the main steps of the $(\frac{G'}{G}, \frac{1}{G})$-expansion method [26]-[28]:

Step 1. The traveling wave transformation

$$u(x, y, z, t) = u(\xi), \qquad \xi = x + y + z - wt, \quad (2.10)$$

where w is a constant, reduces Eq.(2.9) to an ODE in the form:

$$P(u, u', u'', u''', ...) = 0, \qquad (2.11)$$

where P is a polynomial of $u(\xi)$ and its total derivatives with respect to ξ.

Step 2. Assuming that the solution of Eq.(2.11) can be expressed by a polynomial in the two variables ϕ and ψ as follows:

$$u(\xi) = \sum_{i=0}^{N} a_i\phi^i + \sum_{i=1}^{N} b_i\phi^{i-1}\psi, \qquad (2.12)$$

where $a_i\,(0, 1, 2, ..., N)$ and $b_i\,(i = 1, 2, 3, ..., N)$ are constants to be determined later.

Step 3. Determine the positive integer N in Eq.(2.12) by using the homogeneous balance between the highest-order derivatives and the nonlinear terms in Eq.(2.11).

Step 4. Substitute Eq.(2.12) into Eq.(2.11) along with Eq.(2.2) and Eq.(2.4), the left-hand side Eq.(2.11) can be converted into a polynomial in ϕ and ψ, in which the degree of ψ is not longer than 1. Equating each coefficients of this polynomial to zero, yields a system of algebraic equation which can be solved by using the Maple or Mathematica to get the values of a_i, b_i, w, μ, A_1, A_2 and λ where $\lambda < 0$.

Step 5. Similar to step 4, subsitute Eq.(2.12) into Eq.(2.11) along with Eq.(2.2) and Eq.(2.6) for $\lambda > 0$,(or Eq.(2.2) and Eq.(2.8) for $\lambda = 0$), we obtain the exact solutions of Eq.(2.11) expressed by trigonometric functions(or by rational functions) respectively.

III. Application

Here, we will apply the $(\frac{G'}{G}, \frac{1}{G})$-expansion method described in sec.2 to find the exact traveling wave solutions and then the solitary wave solutions for Nonlinear dynamics of microtubules- a new model [29].

The starting point of the present modelling is the fact that the bonds between dimers within the same PF are significantly stronger than the soft bonds between neighbouring (parallel protofilaments) PFs. This implies that the longitudinal displacements of pertaining dimers in a single PF should cause the longitudinal wave propagating along PF. The averaged impact of soft bonds with collateral PFs is taken to be described by the nonlinear double-well potential.

The present model assumes only one degree of freedom per dimer. This is z_n, a longitudinal displacement of a dimer at a position n.

The Hamiltonian for one PF is represented as

$$H = \sum_n \left[\frac{m}{2}\dot{z}_n^2 + \frac{k}{2}(z_{n+1} - z_2)^2 + V(z_n)\right], \quad (3.1)$$

where dot means the first derivative with respect to time, is a mass of the dimer and is a harmonic constant describing the nearest neighbour interaction between the dimers belonging to the same PF. The first term represents a kinetic energy of the dimer, the second one, which we call harmonic energy, is a potential energy of the chemical interaction between the neighbouring dimers belonging to the same PF and the last term is the combined potential

$$V(z_n) = -Cz_n - \frac{1}{2}Az_n^2 + \frac{1}{4}Bz_n^4, \quad C = qE, \quad (3.2)$$

where E is the magnitude of the intrinsic electric field and *q* represents the excess charge within the dipole. It is assumed that *q* > 0 and *E* > 0. One can recognize an energy of the dimer in the intrinsic electric field *E* at the site n and the well known double-well potential with positive parameters *A* and *B* that should be estimated. The Hamiltonian given by before equations is rather common in physics. The first attempt to use it in nonlinear dynamics of (microtubules) MTs was done almost 20 years ago. To be more precise, the Hamiltonian in [30] would be obtained from before equations if z_n were replaced by u_n Hence, we refer to these two models as u-model and z-model. However, the meanings of u_n in [30] and in the present paper are completely different. The u-model assumes an angular degree of freedom, while the coordinate u_n is a projection of the top of the dimer on the direction of PF. On the other hand, the coordinate z_n is a real displacement of the dimer along x axis. This will be further elaborated later.

Using generalized coordinates z_n and $m\dot{z}_n$ and assuming a continuum approximation $z_n(t)\to z\,(x,t\,)$, we straightforwardly obtain the following nonlinear dynamical equation of motion

$$m\frac{\partial^2 z}{\partial t^2}-kl^2\frac{\partial^2 z}{\partial x^2}-qE-Az+Bz^3+\gamma\frac{\partial z}{\partial t} \quad (3.3)$$

The last term represents a viscosity force with γ being a viscosity coefficient. It is well known that, for a given wave equation, a traveling wave $z\,(\xi)$ is a solution which depends upon x and t only through a unified variable $\xi{=}\kappa x{-}\omega t$, where κ and ω are constants. This allows us to obtain the final dimensionless ordinary differential equation

$$\alpha u''-\rho u'-u+u^3-\eta=0, \quad (3.4)$$

where

$$u'=\frac{du}{d\xi},\ \alpha=\frac{m\omega^2-kl^2\kappa^2}{A},\ z=\sqrt{\frac{A}{B}}u\,,\ \rho=\frac{\gamma\omega}{A}\ and\,\eta=\frac{qE}{A\sqrt{\frac{A}{B}}}.$$

Balancing between u'' and u^3, we get $(n+2=2\,n)\Rightarrow(n=\ 1)$.So that, we assume the solution of Eq.(3.4) by using (2.4), we get:

$$u=a_0+a_1\phi(\xi)+b_1\quad(\xi). \quad (3.5)$$

where a_0, a_1 and b_1 are constants to be determined later.

There are three cases to be discussed as follows:

Case 1. Hyperbolic function solutions ($\lambda<0$).

When ($\lambda<0$), substituting Eq.(3.5) and its derivative into Eq.(3.4) and using Eq.(2.2) and Eq.(2.4), the left-hand side of Eq.(3.4) becomes a polynomial in ϕ and ψ. Setting the coefficients of this polynomial to zero yields a system of algebraic equations in a_0, a_1, b_1, α, η, μ, σ and λ as follows:

$$2\,\alpha\,a_1+{a_1}^3-3\,\frac{a_1{b_1}^2\lambda}{\lambda^2\sigma+\mu^2}=0, \quad (3.6)$$

$$2\,\alpha\,b_1+3\,{a_1}^2b_1-\frac{{b_1}^3\lambda}{\lambda^2\sigma+\mu^2}=0, \quad (3.7)$$

$$\frac{\alpha\,b_1\mu\,\lambda}{\lambda^2\sigma+\mu^2}+\rho\,a_1-3\,\frac{a_0{b_1}^2\lambda}{\lambda^2\sigma+\mu^2}-2\,\frac{{b_1}^3\lambda^2\mu}{(\lambda^2\sigma+\mu^2)^2}+3\,a_0{a_1}^2=0, \quad (3.8)$$

$$-3\,\alpha\,a_1\mu+\rho\,b_1+6\,a_0a_1b_1+6\,\frac{a_1{b_1}^2\lambda\,\mu}{\lambda^2\sigma+\mu^2}=0, \quad (3.9)$$

$$2\,\alpha\,a_1\lambda-a_1+3\,{a_0}^2a_1-3\,\frac{a_1{b_1}^2\lambda^2}{\lambda^2\sigma+\mu^2}=0, \quad (3.10)$$

$$\alpha\,\left(-2\,\frac{b_1\mu^2\lambda}{\lambda^2\sigma+\mu^2}+b_1\lambda\right)-\rho\,a_1\mu-b_1+3\,{a_0}^2b_1+6\,\frac{a_0{b_1}^2\lambda\,\mu}{\lambda^2\sigma+\mu^2}+4\,\frac{{b_1}^3\lambda^2\mu^2}{(\lambda^2\sigma+\mu^2)^2}-\frac{{b_1}^3\lambda^2}{\lambda^2\sigma+\mu^2}=0, \quad (3.11)$$

$$\frac{\alpha\,b_1\mu\,\lambda^2}{\lambda^2\sigma+\mu^2}+\rho\,a_1\lambda-a_0+{a_0}^3-3\,\frac{a_0{b_1}^2\lambda^2}{\lambda^2\sigma+\mu^2}-2\,\frac{{b_1}^3\lambda^3\mu}{(\lambda^2\sigma+\mu^2)^2}-\eta=0. \quad (3.12)$$

Solving above system of algebraic equations by the Maple or Mathematica, we get the following results.

$$\eta=-8\,{a_0}^3+2\,a_0,\ \mu=\frac{\pm\sqrt{-9\,\sigma\,{a_0}^4-3\,{a_0}^2{b_1}^2+6\,\sigma\,{a_0}^2+{b_1}^2-\sigma}}{a_1^2},$$

$$\lambda=\frac{3\,{a_0}^2-1}{{a_1}^2},\ a_0=\frac{-\rho}{6a_1},\ a_1=\pm\sqrt{\frac{-\alpha}{2}},\ b_1=b_1.$$

Substituting these solutions into Eq.(3.5), using Eq.(2.2) and Eq.(2.4), we obtain traveling wave solution of Eq.(3.3) as follows:

$$u\,(\xi)=\frac{-\rho}{6a_1}\pm\sqrt{\frac{\alpha\lambda}{2}}\left[\frac{A_1\cosh\left((\xi)\,\sqrt{-\lambda}\right)+A_2\sinh\left((\xi)\,\sqrt{-\lambda}\right)}{A_1\sinh\left((\xi)\,\sqrt{-\lambda}\right)+A_2\cosh\left((\xi)\,\sqrt{-\lambda}\right)+\frac{\mu}{\lambda}}\right]$$

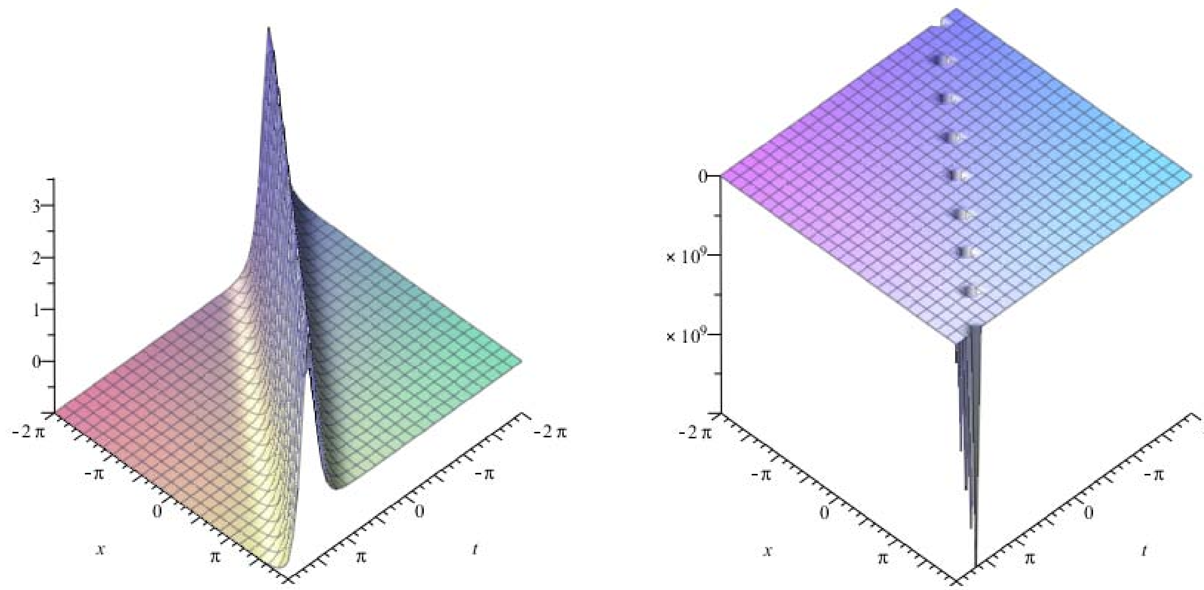
$$+\left[\frac{b_1}{A_1 \sinh\left((\xi)\ \sqrt{-\lambda}\right) + A_2 \cosh\left((\xi)\ \sqrt{-\lambda}\right) + \frac{\mu}{\lambda}}\right]. \tag{3.13}$$

In particular if we set $A_1 = 0$, $A_2 > 0$ and $\mu = 0$ in Eq.(3.13) then we have the solitary solution

$$u(\xi) = \frac{-\rho}{6a_1} \pm \sqrt{\frac{\alpha\lambda}{2}}\, tanh\left(\xi\sqrt{-\lambda}\right) + b_1\, sech\left(\xi\sqrt{-\lambda}\right). \tag{3.14}$$

but, if we set $A_2 = 0$, $A_1 > 0$ and $\mu = 0$ in Eq.(3.13) then we have the solitary solution

$$u(\xi) = \frac{-\rho}{6a_1} \pm \sqrt{\frac{\alpha\lambda}{2}}\, coth\left(\xi\sqrt{-\lambda}\right) + b_1\, csch\left(\xi\sqrt{-\lambda}\right). \tag{3.15}$$



(a)
Eq.(3.14)

(b)
Eq.(3.15)

Figure 1 : Kink Singular Shaped Soliton solution of Eqs.(3.12) and (3.13). When $\alpha = -2,\ a_1 = 1,\ \rho = 3,\ a_0 = \frac{-1}{2}$, $b_1 = 4,\ \lambda = \frac{-1}{4},\ \kappa = 5,\ \omega = 6$

Case 2. Trigonometric function solutions ($\lambda > 0$).

When ($\lambda > 0$), substituting Eq.(3.5) and its derivative into Eq.(3.4) and using Eq.(2.2) andEq.(2.6), the left-hand side of Eq.(3.4) becomes a polynomial in ϕ and ψ. Setting the coefficients of this polynomial to zero yields a system of algebraic equations in $a_0, a_1, b_1, \alpha, \eta, \mu, \sigma$ and λ as follows:

$$2\,\alpha\, a_1 + {a_1}^3 + 3\,\frac{a_1 {b_1}^2 \lambda}{\lambda^2\sigma - \mu^2} = 0, \tag{3.16}$$

$$2\,\alpha\, b_1 + 3\,{a_1}^2 b_1 + \frac{{b_1}^3 \lambda}{\lambda^2\sigma - \mu^2} = 0, \tag{3.17}$$

$$-\frac{\alpha\, b_1 \mu\, \lambda}{\lambda^2\sigma - \mu^2} + \rho\, a_1 + 3\,\frac{a_0 {b_1}^2 \lambda}{\lambda^2\sigma - \mu^2} - 2\,\frac{{b_1}^3 \lambda^2 \mu}{(\lambda^2\sigma - \mu^2)^2} + 3\,a_0 {a_1}^2 = 0, \tag{3.18}$$

$$-3\,\alpha\, a_1 \mu + \rho\, b_1 + 6\,a_0 a_1 b_1 - 6\,\frac{a_1 {b_1}^2 \lambda\, \mu}{\lambda^2\sigma - \mu^2} = 0, \tag{3.19}$$

$$2\,\alpha\, a_1 \lambda - a_1 + 3\,{a_0}^2 a_1 + 3\,\frac{a_1 {b_1}^2 \lambda^2}{\lambda^2\sigma - \mu^2} = 0, \tag{3.20}$$

$$\alpha\left(2\,\frac{b_1\mu^2\lambda}{\lambda^2\sigma-\mu^2}+b_1\lambda\right)-\rho\,a_1\mu-b_1+3\,{a_0}^2b_1-6\,\frac{a_0{b_1}^2\lambda\,\mu}{\lambda^2\sigma-\mu^2}+4\,\frac{{b_1}^3\lambda^2\mu^2}{(\lambda^2\sigma-\mu^2)^2}+\frac{{b_1}^3\lambda^2}{\lambda^2\sigma-\mu^2}=0, \tag{3.21}$$

$$-\frac{\alpha\,b_1\mu\,\lambda^2}{\lambda^2\sigma-\mu^2}+\rho\,a_1\lambda-a_0+{a_0}^3+3\,\frac{a_0{b_1}^2\lambda^2}{\lambda^2\sigma-\mu^2}-2\,\frac{{b_1}^3\lambda^3\mu}{(\lambda^2\sigma-\mu^2)^2}-\eta=0. \tag{3.22}$$

Solving above system of algebraic equations by the Maple or Mathematica, we get the following results.

$$\eta=-8\,{a_0}^3+2\,a_0,\ \mu=\frac{\pm\sqrt{9\,\sigma\,{a_0}^4-3\,{a_0}^2{b_1}^2-6\,\sigma\,{a_0}^2+{b_1}^2+\sigma}}{a_1^2},$$

$$\lambda=\frac{3\,{a_0}^2-1}{{a_1}^2},\ a_0=\frac{-\rho}{6a_1},\ a_1=\pm\sqrt{\frac{-\alpha}{2}},\ b_1=b_1.$$

Substituting these solutions into Eq.(3.5), using Eq.(2.3) and Eq.(2.5), we obtain traveling wave solution of Eq.(3.3) as follows:

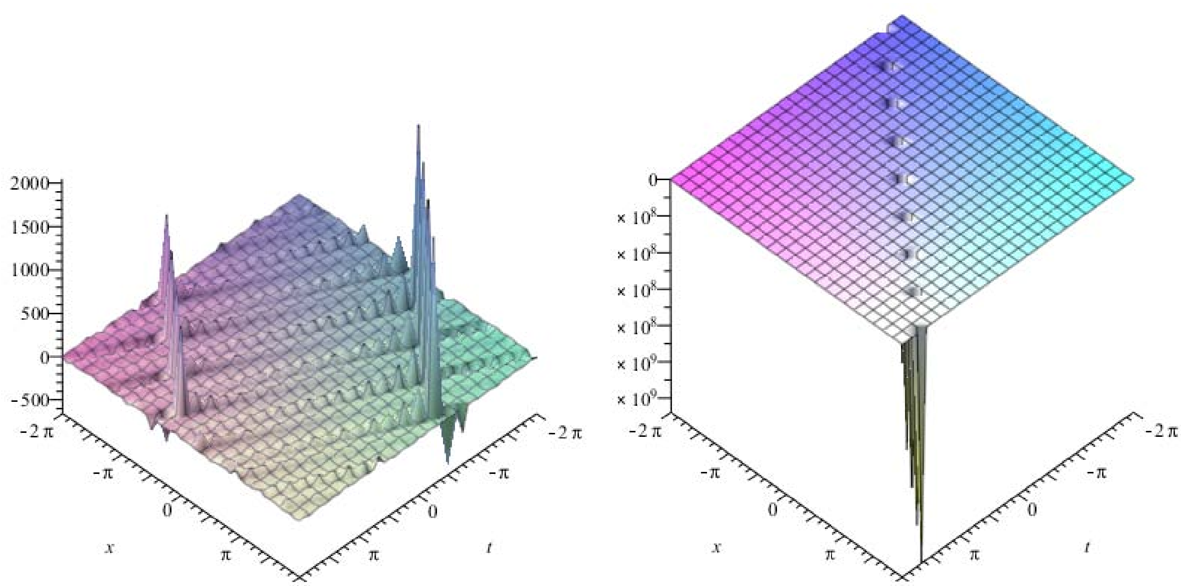
$$u\left(\xi\right)=\tfrac{-\rho}{6a_1}\pm\sqrt{\tfrac{-\alpha\lambda}{2}}\left[\frac{A_1\cos\left((\xi)\sqrt{\lambda}\right)-A_2\sin\left((\xi)\sqrt{\lambda}\right)}{A_1\sin\left((\xi)\sqrt{\lambda}\right)+A_2\cos\left((\xi)\sqrt{\lambda}\right)+\frac{\mu}{\lambda}}\right]$$
$$+\left[\frac{b_1}{A_1\sinh\left((\xi)\ \sqrt{\lambda}\right)+A_2\cos\left((\xi)\ \sqrt{\lambda}\right)+\frac{\mu}{\lambda}}\right]. \tag{3.23}$$

In particular if we set $A_1=0$, $A_2>0$ and $\mu=0$ in Eq.(3.23) then we have the solitary solution

$$u\left(\xi\right)=\tfrac{-\rho}{6a_1}\pm\sqrt{\tfrac{-\alpha\lambda}{2}}\,tanh\left(\xi\sqrt{\lambda}\right)+b_1\,sec\left(\xi\sqrt{\lambda}\right). \tag{3.24}$$

but, if we set $A_2=0$, $A_1>0$ and $\mu=0$ in Eq.(3.23) then we have the solitary solution

$$u\left(\xi\right)=\tfrac{-\rho}{6a_1}\pm\sqrt{\tfrac{-\alpha\lambda}{2}}\,cot\left(\xi\sqrt{\lambda}\right)+b_1\,csc\left(\xi\sqrt{\lambda}\right). \tag{3.25}$$



(a)
Eq.(3.24)

(b)
Eq.(3.25)

Figure 2 : Kink Singular Shaped Soliton solution of Eqs.(3.14) and (3.15). When $\alpha=-2,\ a_1=1,\ \rho=6$, $a_0=-1,\ b_1=4,\ \lambda=2,\ \kappa=5,\ \omega=6$

Case 3. Rational function solutions ($\lambda = 0$).

When ($\lambda = 0$), substituting Eq.(3.5) and its derivative into Eq.(3.4) and using Eq.(2.2) and Eq.(2.8), the left-hand side of Eq.(3.4) becomes a polynomial in ϕ and ψ. Setting the coeficients of this polynomial to zero yields a system of algebraic equations in a_0, a_1, b_1, α, η, μ, σ and λ as follows:

$$2\,\alpha\,a_1 + {a_1}^3 + 3\,\frac{a_1{b_1}^2}{-2\,\mu\,A_2+{A_1}^2} = 0, \tag{3.26}$$

$$2\,\alpha\,b_1 + 3\,{a_1}^2 b_1 + \frac{{b_1}^3}{-2\,\mu\,A_2+{A_1}^2} = 0, \tag{3.27}$$

$$-\frac{\alpha\,b_1\mu}{-2\,\mu\,A_2+{A_1}^2} + \rho\,a_1 + 3\,a_0{a_1}^2 + 3\,\frac{a_0{b_1}^2}{-2\,\mu\,A_2+{A_1}^2} - 2\,\frac{{b_1}^3\mu}{\left(-2\,\mu\,A_2+{A_1}^2\right)^2} = 0, \tag{3.28}$$

$$-3\,\alpha\,a_1\mu + \rho\,b_1 + 6\,a_0a_1b_1 - 6\,\frac{a_1{b_1}^2\mu}{-2\,\mu\,A_2+{A_1}^2} = 0, \tag{3.29}$$

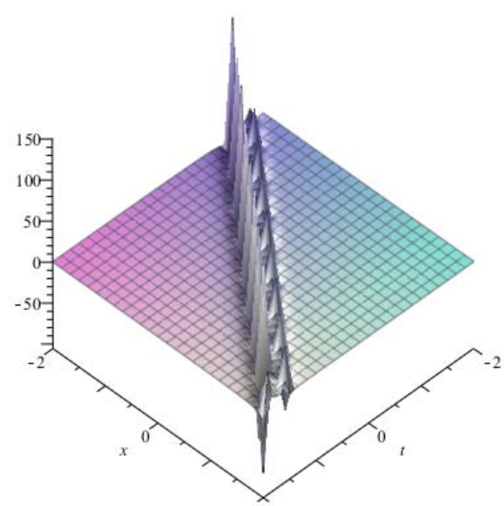
$$3\,{a_0}^2a_1 - a_1 = 0, \tag{3.30}$$

$$2\,\frac{\alpha\,b_1\mu^2}{-2\,\mu\,A_2+{A_1}^2} - \rho\,a_1\mu - b_1 + 3\,{a_0}^2b_1 - 6\,\frac{a_0{b_1}^2\mu}{-2\,\mu\,A_2+{A_1}^2} + 4\,\frac{{b_1}^3\mu^2}{\left(-2\,\mu\,A_2+{A_1}^2\right)^2} = 0, \tag{3.31}$$

$${a_0}^3 - \eta - a_0 = 0. \tag{3.32}$$

Solving above system of algebraic equations by the Maple or Mathematica, we get the following results.

$$\eta = \pm\frac{2\sqrt{3}}{9},\ \alpha = \frac{-\rho^2}{6},\ a_0 = \mp\sqrt{\frac{1}{3}},\ a_1 = \pm\frac{\rho\sqrt{3}}{6},\ b_1 = \pm\frac{\rho}{3}\sqrt{-6\,\mu\,A_2 + 3\,{A_1}^2}.$$



(a)
Eq.(3.33)

Figure 3 : Kink Singular Shaped Soliton solution of Eqs.(3.33). When $a_0 = \sqrt{\frac{1}{3}},\ \rho = 6,\ a_1 = \sqrt{3},\ b_1 = 2\sqrt{15},$ $\mu = -2,\ A_1 = -1,\ A_2 = 1,\ \lambda = \frac{-1}{4},\ \kappa = 5,\ \omega = 6$

Substituting these solutions into Eq.(3.5), using Eq.(2.3) and Eq.(2.6), we obtain traveling wave solution of Eq.(3.3) as follows:

$$u\,(\xi) = \frac{-\rho}{6a_1} \pm \sqrt{\frac{-\alpha}{2}}\left[\frac{\mu\,\xi+A_1}{\frac{\mu}{2}\,\xi^2+A_1\xi+A_2}\right] + \left[\frac{b_1}{\frac{\mu}{2}\,\xi^2 + A_1\xi + A_2}\right]. \tag{3.33}$$

Global Journal of Science Frontier Research (A) Volume XV Issue II Version I Year 2015

- *Remark:* All the obtained results have been checked with Maple 16 by putting them back into the original equation and found correct.

IV. CONCLUSION

The $(\frac{G'}{G},\frac{1}{G})$ expansion method has been applied in this paper to find the exact traveling wave solutions and then the solitary wave solutions of two nonlinear evolution equations, namely, Nonlinear dynamics of microtubules - A new model. Let us compare between our results obtained in the present article with the well-known results obtained by other authors using different methods as follows: Our results of Nonlinear dynamics of microtubules - a new model and The Kundu- Eckhaus equation are new and different from those obtained in [29], [30], [31] and fig. 1, 2 and 3 show the solitary traveling wave solution of Nonlinear dynamics of microtubules - a new model. We can conclude that the $(\frac{G'}{G},\frac{1}{G})$-expansion method is is a very powerful and efficient technique in _nding exact solutions for wide classes of nonlinear problems and can be applied to many other nonlinear evolution equations in mathematical physics. Another possible merit is that the reliability of the method and the reduction in the size of computational domain give this method a wider applicability.

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