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Two Incredible Summation Formula Involving Computational Technique

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Two Incredible Summation Formula Involving Computational Technique

Salahuddin ^α, Upendra Kumar Pandit ^σ & M. P. Chaudhary ^ρ

Abstract- In this paper, we have established two summation formulae with the help of contiguous relation and some derived formulae of Salahuddin et al.

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I. INTRODUCTION AND RESULTS REQUIRED

Special functions and their applications are now incredible in their scope, variety and depth. Not only in their expeditious growth in pure Mathematics and its applications to the traditional fields of Physics, Engineering and Statistics but in new fields of applications like Behavioral Science, Optimization, Biology, Environmental Science and Economics, etc. they are emerging. Summation formulae for hypergeometric function has an important role in applied mathematics.

Prudnikov et al. [2; p.414] derived the following seven summation formulae

$${}_2F_1 \left[\begin{matrix} a, & -a & ; & 1 \\ c & & ; & 2 \end{matrix} \right] = \frac{\sqrt{\pi} \Gamma(c)}{2^c} \left[\frac{1}{\Gamma(\frac{c+a+1}{2}) \Gamma(\frac{c-a}{2})} + \frac{1}{\Gamma(\frac{c+a}{2}) \Gamma(\frac{c-a+1}{2})} \right] \quad (1)$$

$${}_2F_1 \left[\begin{matrix} a, & 1-a & ; & 1 \\ c & & ; & 2 \end{matrix} \right] = \frac{\sqrt{\pi} \Gamma(c)}{2^{c-1}} \left[\frac{1}{\Gamma(\frac{c+a}{2}) \Gamma(\frac{c-a+1}{2})} \right] \quad (2)$$

$${}_2F_1 \left[\begin{matrix} a, & 2-a & ; & 1 \\ c & & ; & 2 \end{matrix} \right] = \frac{\sqrt{\pi} \Gamma(c)}{(a-1) 2^{c-2}} \left[\frac{1}{\Gamma(\frac{c+a-2}{2}) \Gamma(\frac{c-a+1}{2})} - \frac{1}{\Gamma(\frac{c+a-1}{2}) \Gamma(\frac{c-a}{2})} \right] \quad (3)$$

$${}_2F_1 \left[\begin{matrix} a, & 3-a & ; & 1 \\ c & & ; & 2 \end{matrix} \right] = \frac{\sqrt{\pi} \Gamma(c)}{(a-1)(a-2) 2^{c-3}} \left[\frac{(c-2)}{\Gamma(\frac{c+a-2}{2}) \Gamma(\frac{c-a+1}{2})} - \frac{2}{\Gamma(\frac{c+a-3}{2}) \Gamma(\frac{c-a}{2})} \right] \quad (4)$$

$${}_2F_1 \left[\begin{matrix} a, & 4-a & ; & 1 \\ c & & ; & 2 \end{matrix} \right] = \frac{\sqrt{\pi} \Gamma(c)}{(1-a)(2-a)(3-a) 2^{c-4}} \left[\frac{(a-2c+3)}{\Gamma(\frac{c+a-4}{2}) \Gamma(\frac{c-a+1}{2})} + \frac{(a+2c-7)}{\Gamma(\frac{c+a-3}{2}) \Gamma(\frac{c-a}{2})} \right] \quad (5)$$

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$${}_2F_1 \left[\begin{matrix} a, & 5-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix} \right] = \frac{\sqrt{\pi} \Gamma(c)}{2^{c-5} \left\{ \prod_{\gamma=1}^4 (\gamma - a) \right\}} \times \left[\frac{\{2(c-2)(c-4) - (a-1)(a-4)\}}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a-4}{2})} + \frac{(12-4c)}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a-5}{2})} \right] \tag{6}$$

$${}_2F_1 \left[\begin{matrix} a, & 6-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix} \right] = \frac{\sqrt{\pi} \Gamma(c)}{2^{c-6} \left\{ \prod_{\delta=1}^5 (\delta - a) \right\}} \times \left[\frac{(4c^2 + 2ac - a^2 - a - 34c + 62)}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a-5}{2})} - \frac{(4c^2 - 2ac - a^2 + 13a - 22c + 20)}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a-6}{2})} \right] \tag{7}$$

The contiguous relation is defined as Abramowitz et al. [1; p. 558]

$$b {}_2F_1 \left[\begin{matrix} a, & b+1 & ; & z \\ c & & ; & \end{matrix} \right] = (b-c+1) {}_2F_1 \left[\begin{matrix} a, & b & ; & z \\ c & & ; & \end{matrix} \right] + (c-1) {}_2F_1 \left[\begin{matrix} a, & b & ; & z \\ c-1 & & ; & \end{matrix} \right] \tag{8}$$

Salahuddin et al. [3 ; 4] derived the following eleven summation formulae

$${}_2F_1 \left[\begin{matrix} a, & 7-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix} \right] = \frac{\sqrt{\pi} \Gamma(c)}{2^{c-7} \left\{ \prod_{\zeta=1}^6 (\zeta - a) \right\}} \times \left[\frac{1}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a-6}{2})} (-3a^2c + 12a^2 + 21ac - 84a + 4c^3 - 48c^2 + 158c - 120) + \frac{1}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a-7}{2})} (2a^2 - 14a - 8c^2 + 64c - 108) \right] \tag{9}$$

$${}_2F_1 \left[\begin{matrix} a, & 8-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix} \right] = \frac{\sqrt{\pi} \Gamma(c)}{2^{c-8} \left\{ \prod_{\xi=1}^7 (\xi - a) \right\}} \times \left[\frac{1}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a-7}{2})} (-a^3 - 4a^2c + 30a^2 + 4ac^2 - 4ac - 107a + 8c^3 - 124c^2 + 576c - 762) + \frac{1}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a-8}{2})} (-a^3 + 4a^2c - 6a^2 + 4ac^2 - 68ac + 181a - 8c^3 + 92c^2 - 288c + 210) \right] \tag{10}$$

$${}_2F_1 \left[\begin{matrix} a, & 9-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix} \right] = \frac{\sqrt{\pi} \Gamma(c)}{2^{c-9} \left\{ \prod_{\varpi=1}^8 (\varpi - a) \right\}} \times \left[\frac{1}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a-8}{2})} (a^4 - 18a^3 - 8a^2c^2 + 80a^2c - 85a^2 + 72ac^2 - 720ac + 1494a + 8c^4 - 160c^3 + 1056c^2 - 2560c + 1680) + \frac{1}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a-9}{2})} \times (8a^2c - 40a^2 - 72ac + 360a - 16c^3 + 240c^2 - 1072c + 1360) \right] \tag{11}$$

Ref

4. Salahuddin, Khola, R. K.; Certain new Hypergeometric Summation formulae arising from the sum-mation formulae of Salahuddin et al, (communicated).

$$\begin{aligned}
{}_2F_1 \left[\begin{matrix} a, & 10-a & ; & \frac{1}{2} \\ c & & ; & \frac{1}{2} \end{matrix} \right] &= \frac{\sqrt{\pi} \Gamma(c)}{2^{c-10} \left\{ \prod_{v=1}^9 (v-a) \right\}} \times \\
&\times \left[\frac{1}{\Gamma\left(\frac{c-a+1}{2}\right) \Gamma\left(\frac{c+a-10}{2}\right)} (-a^4 - 4a^3c + 42a^3 + 12a^2c^2 - 72a^2c - 107a^2 + 8ac^3 - 252ac^2 + \right. \\
&\quad + 1772ac - 3054a - 16c^4 + 312c^3 - 2000c^2 + 4704c - 3024) + \\
&\quad + \frac{1}{\Gamma\left(\frac{c-a}{2}\right) \Gamma\left(\frac{c+a-9}{2}\right)} (a^4 - 4a^3c + 2a^3 - 12a^2c^2 + 192a^2c - \\
&\quad \left. - 553a^2 + 8ac^3 - 12ac^2 - 868ac + 3406a + 16c^4 - 392c^3 + 3320c^2 - 11224c + 12264) \right] \quad (12)
\end{aligned}$$

$$\begin{aligned}
{}_2F_1 \left[\begin{matrix} a, & 11-a & ; & \frac{1}{2} \\ c & & ; & \frac{1}{2} \end{matrix} \right] &= \frac{\sqrt{\pi} \Gamma(c)}{2^{c-11} \left\{ \prod_{\varphi=1}^{10} (\varphi-a) \right\}} \times \\
&\times \left[\frac{1}{\Gamma\left(\frac{c-a+1}{2}\right) \Gamma\left(\frac{c+a-10}{2}\right)} (5a^4c - 30a^4 - 110a^3c + 660a^3 - 20a^2c^3 + 360a^2c^2 - 1305a^2c - \right. \\
&\quad - 810a^2 + 220ac^3 - 3960ac^2 + 21010ac - 31020a + 16c^5 - 480c^4 + \\
&\quad \left. 5240c^3 - 25200c^2 + 50544c - 30240) + \frac{1}{\Gamma\left(\frac{c-a}{2}\right) \Gamma\left(\frac{c+a-11}{2}\right)} \times \right. \\
&\quad \left. \times (-2a^4 + 44a^3 + 24a^2c^2 - 288a^2c + 530a^2 - 264ac^2 + 3168ac - 8492a - 32c^4 + \right. \\
&\quad \left. 768c^3 - 6352c^2 + 20928c - 22320) \right] \quad (13)
\end{aligned}$$

$$\begin{aligned}
{}_2F_1 \left[\begin{matrix} a, & 12-a & ; & \frac{1}{2} \\ c & & ; & \frac{1}{2} \end{matrix} \right] &= \frac{\sqrt{\pi} \Gamma(c)}{2^{c-12} \left\{ \prod_{\chi=1}^{11} (\chi-a) \right\}} \times \\
&\times \left[\frac{1}{\Gamma\left(\frac{c-a+1}{2}\right) \Gamma\left(\frac{c+a-12}{2}\right)} (a^5 - 6a^4c + 9a^4 - 12a^3c^2 + 300a^3c - 1103a^3 + \right. \\
&\quad + 32a^2c^3 - 408a^2c^2 + 46a^2c + 6351a^2 + 16ac^4 - 800ac^3 + 10364ac^2 - 46852ac + \\
&\quad + 62182a - 32c^5 + 944c^4 - 10112c^3 + 47656c^2 - 93776c + 55440) + \\
&\quad + \frac{1}{\Gamma\left(\frac{c-a}{2}\right) \Gamma\left(\frac{c+a-11}{2}\right)} (a^5 + 6a^4c - 69a^4 - 12a^3c^2 + 12a^3c + 769a^3 - 32a^2c^3 + 840a^2c^2 - \\
&\quad - 5662a^2c + 8301a^2 + 16ac^4 - 32ac^3 - 4612ac^2 + 42380ac - 96002a + \\
&\quad \left. + 32c^5 - 1136c^4 + 15104c^3 - 92536c^2 + 255392c - 245640) \right] \quad (14)
\end{aligned}$$

$$\begin{aligned}
{}_2F_1 \left[\begin{matrix} a, & 13-a & ; & \frac{1}{2} \\ c & & ; & \frac{1}{2} \end{matrix} \right] &= \frac{\sqrt{\pi} \Gamma(c)}{2^{c-13} \left\{ \prod_{\beta=1}^{12} (\beta-a) \right\}} \times \\
&\times \left[\frac{1}{\Gamma\left(\frac{c-a+1}{2}\right) \Gamma\left(\frac{c+a-12}{2}\right)} (-a^6 + 39a^5 + 18a^4c^2 - 252a^4c + 275a^4 - 468a^3c^2 + 6552a^3c \right. \\
&\quad \left. - 18135a^3 - 48a^2c^4 + 1344a^2c^3 - 9834a^2c^2 + 5964a^2c + 74246a^2 + 624ac^4 - 17472ac^3 + \right.
\end{aligned}$$

$$\begin{aligned}
 &+167388ac^2 - 631176ac + 752856a + 32c^6 - 1344c^5 + \\
 &+21824c^4 - 172032c^3 + 674384c^2 - 1187424c + 665280) + \\
 &+ \frac{1}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a-13}{2})} (-12a^4c + 84a^4 + 312a^3c - 2184a^3 + 64a^2c^3 - 1344a^2c^2 + \\
 &+6620a^2c - 2436a^2 - 832ac^3 + 17472ac^2 - 112424ac + 216216a - 64c^5 + \\
 &+2240c^4 - 29312c^3 + 176512c^2 - 478752c + 453600) \Big] \tag{15}
 \end{aligned}$$

$$\begin{aligned}
 {}_2F_1 \left[\begin{matrix} a, & 14 - a & ; & \frac{1}{2} \\ c & & ; & \frac{1}{2} \end{matrix} \right] &= \frac{\sqrt{\pi} \Gamma(c)}{2^{c-14} \left\{ \prod_{\gamma=1}^{13} (\gamma - a) \right\}} \times \\
 &\times \left[\frac{1}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a-14}{2})} (a^6 + 6a^5c - 87a^5 - 24a^4c^2 + 150a^4c + 925a^4 - 32a^3c^3 + 1392a^3c^2 - \right. \\
 &-12706a^3c + 24615a^3 + 80a^2c^4 - 1728a^2c^3 + 5368a^2c^2 + 58986a^2c - 242486a^2 + 32ac^5 - \\
 &-2320ac^4 + 47328ac^3 - 391568ac^2 + 1344076ac - 1496568a - 64c^6 + 2656c^5 - 42560c^4 + \\
 &+330752c^3 - 1278144c^2 + 2222160c - 1235520) + \frac{1}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a-13}{2})} (-a^6 + 6a^5c - \\
 &-3a^5 + 24a^4c^2 - 570a^4c + 2225a^4 - 32a^3c^3 + 48a^3c^2 + 7454a^3c - 39225a^3 - 80a^2c^4 + \\
 &+3072a^2c^3 - 35608a^2c^2 + 133626a^2c - 68104a^2 + \\
 &+32ac^5 - 80ac^4 - 19872ac^3 + 313808ac^2 - 1676564ac + \\
 &+2856228a + 64c^6 - 3104c^5 + 59360c^4 - 566848c^3 + 2810304c^2 - 6724560c + 5897520) \Big] \tag{16}
 \end{aligned}$$

$$\begin{aligned}
 {}_2F_1 \left[\begin{matrix} a, & 15 - a & ; & \frac{1}{2} \\ c & & ; & \frac{1}{2} \end{matrix} \right] &= \frac{\sqrt{\pi} \Gamma(c)}{2^{c-15} \left\{ \prod_{\varepsilon=1}^{14} (\varepsilon - a) \right\}} \times \\
 &\times \left[\frac{1}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a-14}{2})} (-7a^6c + 56a^6 + 315a^5c - 2520a^5 + 56a^4c^3 - 1344a^4c^2 + 5103a^4c + \right. \\
 &+16520a^4 - 1680a^3c^3 + 40320a^3c^2 - 271215a^3c + 449400a^3 - 112a^2c^5 + 4480a^2c^4 - 54040a^2c^3 + \\
 &+150080a^2c^2 + 845824a^2c - 3383296a^2 + 1680ac^5 - 67200ac^4 + 999600ac^3 - 6787200ac^2 + \\
 &+20482140ac - 21070560a + 64c^7 - 3584c^6 + 80864c^5 - \\
 &-940800c^4 + 5987520c^3 - 20296192c^2 + 32464368c - 17297280) + \\
 &+ \frac{1}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a-15}{2})} (2a^6 - 90a^5 - 48a^4c^2 + 768a^4c - \\
 &-1474a^4 + 1440a^3c^2 - 23040a^3c + 77970a^3 + 160a^2c^4 - 5120a^2c^3 + 46640a^2c^2 - \\
 &-90880a^2c - 226192a^2 - 2400ac^4 + 76800ac^3 - 861600ac^2 + 3955200ac - 6138120a - 128c^6 + \\
 &+6144c^5 - 116160c^4 + 1095680c^3 - 5363584c^2 + 12679168c - 11009376) \Big] \tag{17}
 \end{aligned}$$

$$\begin{aligned}
{}_2F_1 \left[\begin{matrix} a, & 16-a & ; & \frac{1}{2} \\ c & & ; & \frac{1}{2} \end{matrix} \right] &= \frac{\sqrt{\pi} \Gamma(c)}{2^{c-16} \left\{ \prod_{\zeta=1}^{15} (\zeta - a) \right\}} \times \\
&\times \left[\frac{1}{\Gamma\left(\frac{c-a+1}{2}\right) \Gamma\left(\frac{c+a-16}{2}\right)} (-a^7 + 8a^6c - 12a^6 + 24a^5c^2 - 792a^5c + 3710a^5 - 80a^4c^3 + 1080a^4c^2 + \right. \\
&+ 6280a^4c - 66600a^4 - 80a^3c^4 + 5280a^3c^3 - 85480a^3c^2 + 435480a^3c - 458929a^3 + 192a^2c^5 - \\
&\quad 6240a^2c^4 + 45200a^2c^3 + 271560a^2c^2 - 3746640a^2c + 8942052a^2 + 64ac^6 - 6336ac^5 + \\
&\quad + 186000ac^4 - 2408160ac^3 + 15005072ac^2 - 42553152ac + 41722740a - 128c^7 + 7104c^6 - \\
&\quad - 158720c^5 + 1827360c^4 - 11505152c^3 + 38596416c^2 - 61194240c + 32432400) + \\
&\quad + \frac{1}{\Gamma\left(\frac{c-a}{2}\right) \Gamma\left(\frac{c+a-15}{2}\right)} (-a^7 - 8a^6c + 124a^6 + 24a^5c^2 - 24a^5c - 2818a^5 + \\
&\quad + 80a^4c^3 - 3000a^4c^2 + 26360a^4c - 40760a^4 - 80a^3c^4 + 160a^3c^3 + 45080a^3c^2 - 534760a^3c + \\
&\quad + 1499471a^3 - 192a^2c^5 + 10080a^2c^4 - 175760a^2c^3 + 1189560a^2c^2 - 2226480a^2c - \\
&\quad - 2760884a^2 + 64ac^6 - 192ac^5 - 75120ac^4 + 1782560ac^3 - \\
&\quad - 16394608ac^2 + 65703616ac - 93008652a + 128c^7 - 8128c^6 + \\
&\quad \left. + 210944c^5 - 2878240c^4 + 22080512c^3 - 94015552c^2 + 202146816c - 165145680) \right] \quad (18)
\end{aligned}$$

$$\begin{aligned}
{}_2F_1 \left[\begin{matrix} a, & 17-a & ; & \frac{1}{2} \\ c & & ; & \frac{1}{2} \end{matrix} \right] &= \frac{\sqrt{\pi} \Gamma(c)}{2^{c-17} \left\{ \prod_{\vartheta=1}^{16} (\vartheta - a) \right\}} \times \\
&\times \left[\frac{1}{\Gamma\left(\frac{c-a+1}{2}\right) \Gamma\left(\frac{c+a-16}{2}\right)} (a^8 - 68a^7 - 32a^6c^2 + 576a^6c - 638a^6 + \right. \\
&+ 1632a^5c^2 - 29376a^5c + 101320a^5 + 160a^4c^4 - 5760a^4c^3 + 44640a^4c^2 + 129600a^4c - 1341071a^4 - \\
&\quad - 5440a^3c^4 + 195840a^3c^3 - 2303840a^3c^2 + 9743040a^3c - 9832052a^3 - 256a^2c^6 + \\
&\quad + 13824a^2c^5 - 246560a^2c^4 + 1411200a^2c^3 + 4297408a^2c^2 - 64103040a^2c + \\
&\quad + 143207628a^2 + 4352ac^6 - 235008ac^5 + 4977600ac^4 - 52289280ac^3 + \\
&\quad + 282566656ac^2 - 727036416ac + 670152240a + 128c^8 - 9216c^7 + 275456c^6 - 4423680c^5 + \\
&\quad + 41249792c^4 - 224907264c^3 + 683065344c^2 - 1014128640c + 518918400 + \\
&\quad + \frac{1}{\Gamma\left(\frac{c-a}{2}\right) \Gamma\left(\frac{c+a-17}{2}\right)} (16a^6c - 144a^6 - 816a^5c + \\
&\quad + 7344a^5 - 160a^4c^3 + 4320a^4c^2 - 22480a^4c - 30960a^4 + 5440a^3c^3 - \\
&\quad - 146880a^3c^2 + 1157360a^3c - 2484720a^3 + 384a^2c^5 - 17280a^2c^4 + 247840a^2c^3 - \\
&\quad - 1092960a^2c^2 - 1901760a^2c + 15669504a^2 - 6528ac^5 + 293760ac^4 - 4999360ac^3 + \\
&\quad + 39804480ac^2 - 146267456ac + 194890176a - 256c^7 + 16128c^6 - 414976c^5 + \\
&\quad \left. + 5610240c^4 - 42628864c^3 + 179788032c^2 - 383195904c + 310867200) \right] \quad (19)
\end{aligned}$$

II. MAIN SUMMATION FORMULAE

$$\begin{aligned}
{}_2F_1 \left[\begin{matrix} a, 18-a ; \\ c \end{matrix} ; \frac{1}{2} \right] &= \frac{\sqrt{\pi} \Gamma(c)}{2^{c-18} \left\{ \prod_{\eta=1}^{17} (\eta-a) \right\}} \times \\
&\times \left[\frac{1}{\Gamma\left(\frac{c-a+1}{2}\right) \Gamma\left(\frac{c+a-18}{2}\right)} (-a^8 - 8a^7c + 148a^7 + 40a^6c^2 - 256a^6c - 3362a^6 + 80a^5c^3 - 4440a^5c^2 + \right. \\
&+ 49664a^5c - 103400a^5 - 240a^4c^4 + 5520a^4c^3 + 18760a^4c^2 - 849520a^4c + 3240271a^4 - 192a^3c^5 + \\
&+ 17760a^3c^4 - 440560a^3c^3 + 4091160a^3c^2 - 12923320a^3c + 3622852a^3 + 448a^2c^6 - 20352a^2c^5 + \\
&+ 253360a^2c^4 + 576240a^2c^3 - 31091248a^2c^2 + 192701168a^2c - 344444908a^2 + 128ac^7 - 16576ac^6 + 660032ac^5 - \\
&- 12228640ac^4 + 118499872ac^3 - 604789504ac^2 + 1488844864ac - 1324543920a - 256c^8 + 18304c^7 - \\
&- 542976c^6 + 8650240c^5 - 79993344c^4 + 432549376c^3 - 1303568384c^2 + 1923025920c - 980179200) + \\
&+ \frac{1}{\Gamma\left(\frac{c-a}{2}\right) \Gamma\left(\frac{c+a-17}{2}\right)} (a^8 - 8a^7c + 4a^7 - 40a^6c^2 + 1264a^6c - 6214a^6 + 80a^5c^3 - 120a^5c^2 - \\
&- 32416a^5c + 213904a^5 + 240a^4c^4 - 12720a^4c^3 + 186440a^4c^2 - 743120a^4c - 456391a^4 - \\
&- 192a^3c^5 + 480a^3c^4 + 216080a^3c^3 - 4278120a^3c^2 + 27569480a^3c - 52277444a^3 - 448a^2c^6 + \\
&+ 30720a^2c^5 - 745840a^2c^4 + 7817520a^2c^3 - 30345632a^2c^2 - 19224224a^2c + 253516684a^2 + \\
&+ 128ac^7 - 448ac^6 - 259264ac^5 + 8556320ac^4 - 118218848ac^3 + 813195488ac^2 - \\
&- 2692403360ac + 3335839536a + 256c^8 - 20608c^7 + 696192c^6 - 12817024c^5 + \\
&+ 139638144c^4 - 913535872c^3 + 3463541888c^2 - 6848013696c + 5284782720) \left. \right] \quad (20)
\end{aligned}$$

$$\begin{aligned}
{}_2F_1 \left[\begin{matrix} a, 19-a ; \\ c \end{matrix} ; \frac{1}{2} \right] &= \frac{\sqrt{\pi} \Gamma(c)}{2^{c-19} \left\{ \prod_{\lambda=1}^{18} (\lambda-a) \right\}} \times \\
&\times \left[\frac{1}{\Gamma\left(\frac{c-a+1}{2}\right) \Gamma\left(\frac{c+a-18}{2}\right)} (9a^8c - 90a^8 - 684a^7c + 6840a^7 - 120a^6c^3 + 3600a^6c^2 - 14046a^6c - 99540a^6 + \right. \\
&+ 6840a^5c^3 - 205200a^5c^2 + 1664856a^5c - 2968560a^5 + 432a^4c^5 - 21600a^4c^4 + 277080a^4c^3 + \\
&+ 327600a^4c^2 - 20793831a^4c + 70898310a^4 - 16416a^3c^5 + 820800a^3c^4 - 14644440a^3c^3 + \\
&+ 111013200a^3c^2 - 315518940a^3c + 131909400a^3 - 576a^2c^7 + 40320a^2c^6 - 992880a^2c^5 + 9324000a^2c^4 + \\
&+ 4429536a^2c^3 - 636886080a^2c^2 + 3695816316a^2c - 6211091160a^2 + 10944ac^7 - \\
&- 766080ac^6 + 21827808ac^5 - 325310400ac^4 + 2707726176ac^3 - 12394025280ac^2 + \\
&+ 28254838896ac - 23908836960a + 256c^9 - 23040c^8 + 880512c^7 - 18627840c^6 + 238347264c^5 - \\
&- 1891123200c^4 + 9158978048c^3 - 25507261440c^2 + 35661692160c - 17643225600) + \\
&+ \frac{1}{\Gamma\left(\frac{c-a}{2}\right) \Gamma\left(\frac{c+a-19}{2}\right)} (-2a^8 + 152a^7 + 80a^6c^2 - 1600a^6c + 3148a^6 -
\end{aligned}$$

$$\begin{aligned}
& -4560a^5c^2 + 91200a^5c - 371488a^5 - 480a^4c^4 + 19200a^4c^3 - 185680a^4c^2 - 126400a^4c + 4559182a^4 + \\
& +18240a^3c^4 - 729600a^3c^3 + 9799440a^3c^2 - 50068800a^3c + 73373288a^3 + 896a^2c^6 - 53760a^2c^5 + \\
& +1107680a^2c^4 - 8467200a^2c^3 - 743936a^2c^2 + 274718720a^2c - 822056088a^2 - \\
& -17024ac^6 + 1021440ac^5 - 24338240ac^4 + 292569600ac^3 - 1853708096ac^2 + \\
& +5798641920ac - 6885423072a - 512c^8 + 40960c^7 - 1374464c^6 + 25123840c^5 - 271685888c^4 + \\
& +1764075520c^3 - 6639757056c^2 + 13042437120c - 10013310720) \quad (21)
\end{aligned}$$

III. DERIVATION OF THE MAIN FORMULAE

Involving the contiguous relation (8) and the formula of Salahuddin et al. (19), one can established the result(20) and on the same way result(21) can be established.

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