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A Class of Regression Estimator with Cum-Dual Product Estimator As Intercept

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A Class of Regression Estimator with Cum-Dual Product Estimator As Intercept

F. B. Adebola [α] & N. A. Adegoke [σ]

Abstract- This paper examines a class of regression estimator with cum-dual product estimator as intercept for estimating the mean of the study variable Y using auxiliary variable X. We obtained the bias and the mean square error (MSE) of the proposed estimator. We also obtained MSE of its asymptotically optimum estimator (AOE). Theoretical and numerical validation of the proposed estimator was done to show it's superiority over the usual simple random sampling estimator and ratio estimator, product estimator, cum-dual ratio and product estimator. It was found that the asymptotic optimal value of the proposed estimator performed better than other competing estimators and performed in exactly the same way as the regression estimator, when compared with the usual simple random estimator for estimating the average sleeping hours of undergraduate students of the department of statistics, Federal University of Technology Akure, Nigeria.

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I. Introduction

In estimating the mean of the study variable Cochran (1940), used the auxiliary information X at the estimation phase to increase the efficiency of the study variable. To estimate the ratio estimator of the population mean or total of the study variable Y, he used additional knowledge on the auxiliary variable X which was positively correlated with Y. When the relationship between the study variable Y and the auxiliary variable X is linear through the origin and Y proportional to X, the ratio estimator will be more efficient than the normal Simple Random Sampling (SRS) Sanjib Choudhury et al (2012). Also, Robson (1957) proposed product estimator and showed that when the relationship between the study variable Y and the auxiliary variable X is linear through the origin and Y is inversely proportional to X, the product estimator will be more efficient than the usual SRS. Murthy 1964 suggested the use of ratio estimator $\bar{y}_p$ when $\frac{\rho C_y}{C_x} > \frac{1}{2}$ and unbiased estimator $\bar{y}$ when $-\frac{1}{2} \leq \rho \frac{C_y}{C_x} \leq \frac{1}{2}$, where C_y, C_x and ρ are coefficients of variation of y, x and correlation between y and x respectively.

Suppose that SRSWOR of n units is drawn from a population of N units to estimate the population mean $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$ of the study variable Y. All the sample units are observed for the variables Y and X. Let (y_i, x_i) where $i = 1,2,3,..,n$ denote the set of the

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observation for the study variable Y and X. Let the sample means $(\bar{x}, \bar{y})$ be unbiased of the population means of the auxiliary variable $\bar{X}$ and study variable $\bar{Y}$ based on the n observations.

The usual product estimator of $\bar{Y}$ is given as $\bar{y}_p = \frac{\bar{y}}{\bar{X}}\bar{x}$ and the usual regression estimator is given as $\bar{y}_{reg} = \bar{y} + \hat{\beta}(\bar{X} - \bar{x})$, where $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, and $\hat{\beta} = \frac{s_{xy}}{s_x^{\ 2}}$ is the estimate slope of regression coefficient of Y and X. Cum-Dual Product estimator given as $\bar{y}_{dP} = \bar{y}\frac{\bar{X}}{\bar{x}^*}$, where $\bar{x}^*$ is the un-sampled auxiliary variable in X given as $\bar{x}^* = \frac{N\bar{X} - n\bar{x}}{N-n}$ was obtained by Bandyopadhyay (1980). The use of auxiliary information in sample surveys was extensively discussed in well-known classical text books such as Cochran(1977), Sukhatme and Sukhatme(1970), Sukhatme, Sukhatme, and Asok (1984), Murthy(1967) and Yates(1960) among others. Recent developments in ratio and product methods of estimation along with their variety of modified forms are lucidly described in detail by Singh (2003).

II. The Proposed Class of Estimator

The proposed a class of regression estimator with dual product as the intercept for estimating population mean $\bar{Y}$, given as;

$$\bar{y}_{pd}^* = \bar{y}\frac{\bar{X}}{\bar{x}^*} + \alpha(\bar{X} - \bar{x}^*) \qquad \text{......................... (1)}$$

Where, α is a suitable scalar.

We obtained the bias and MSE of the proposed estimator $\bar{y}_{pd}^*$ up to the first order approximation, this is obtained by substituting $\bar{x}^* = \frac{N\bar{X} - n\bar{x}}{N-n}$ into equation (1),

$$\bar{y}_{pd}^* = \frac{\bar{y}\bar{X}(N-n)}{N\bar{X} - n\bar{x}} + \alpha(\frac{n\bar{x} - n\bar{X}}{N-n})$$

We write $e_0 = \frac{\bar{y} - \bar{Y}}{\bar{Y}}$ and $e_1 = \frac{\bar{x} - \bar{X}}{\bar{X}}$.

The bias $\bar{y}_{pd}^*$ is given as

$$Bias(\bar{y}_{pd}^*) = \frac{(1-f)}{n}(g^2\bar{Y}^2\frac{S_x^2}{\bar{X}^2} + \bar{Y}g\frac{S_{xy}}{\bar{X}\bar{Y}}) \qquad \text{..................... (1.1)}$$

The MSE of $\bar{y}_{pd}^*$ is given as

$$E(\bar{y}_{pd}^* - \bar{Y})^2 = \left(\frac{1-f}{n}\right)\left(S_y^2 + 2g\bar{Y}\frac{\rho S_x S_y}{\bar{X}\bar{Y}}(\bar{Y} + \alpha\bar{X}) + g^2\frac{S_x^2}{\bar{X}^2}(\bar{Y} + \alpha\bar{X})^2\right)$$

$$MSE\ (\bar{y}_{pd}^*) = \left(\frac{1-f}{n}\right)\left(\bar{Y}^2C_y^2 + 2g\rho\bar{Y}C_xC_y(\bar{Y} + \alpha\bar{X}) + g^2C_x^2(\bar{Y} + \alpha\bar{X})^2\right) \qquad \text{......... (2)}$$

The optimum value of the MSE ($\bar{y}_{pd}^*$) is obtained as follows,

$$\frac{\partial}{\partial\alpha}\text{MSE}\ (\bar{y}_{pd}^*) = \left(\frac{1-f}{n}\right)\left(2g\bar{Y}\frac{S_{xy}}{\bar{X}\bar{Y}}(\bar{X}) + g^2C_x^2(2)(\bar{X})(\bar{Y} + \alpha\bar{X})\right) \qquad \text{...... (3)}$$

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Notes

Set equation (3) to zero; we have

$$2gS_{xy} + 2g^2C_x^2\bar{X}(\bar{Y} + \alpha\bar{X}) = 0$$

$$\alpha = -\left(R + \frac{\beta}{g}\right)$$

The optimal MSE of $\bar{y}_{pd}^{*}$.

$$MSE(\bar{y}_{pd}^{*opt}) = \left(\frac{1-f}{n}\right) S_y^2(1-\rho^2) \quad \text{... ...... (4)}$$

Equation (4) shows that the MSE $(\bar{y}_{pd}^{*opt})$ is the same as the MSE of Regression Estimate of y on X.

Remark

The bias of $\bar{y}_{pd}^{*}$ is the same as the bias of the dual product estimator and when α=0, the MSE $(\bar{y}_{pd}^{*})$ boils down to product cum estimator proposed by Bandyopadhyay (1980). The bias and MSE of $\bar{y}_p^{*}$ are given as

$$\text{Bias}(\bar{y}_p^{*}) = \left(\frac{1-f}{n}\right)(g^2\,\bar{Y}^2C_x^2 + \bar{Y}g\rho C_xC_y)$$

$$\text{MSE}(\bar{y}_p^{*}) = \left(\frac{1-f}{n}\right)(\,\bar{Y}^2C_y^2 + 2g\rho\bar{Y}^2C_xC_y + \bar{Y}g^2C_x^2) \quad \text{................(5)}$$

III. The Efficiency Comparisons

In this section, we compared the MSE of $\bar{y}$ with the MSE of $\bar{y}_{pd}^{*}$. The MSE of $\bar{y}$ under SRS scheme is given as

$$\text{MSE}\;(\bar{y}) = \left(\frac{1-f}{n}\right)\bar{Y}^2C_x^2$$

From equation (2) and (5), $\bar{y}_{pd}^{*}$ is better than $\bar{y}$

If $\text{MSE}\;(\bar{y}_{pd}^{*}) < \text{MSE}\;(\bar{y})$

That is

$$\left(\frac{1-f}{n}\right)(\,\bar{Y}^2C_y^2 + 2g\rho\bar{Y}C_xC_y(\bar{Y}+\alpha\bar{X}) + g^2C_x^2(\bar{Y}+\alpha\bar{X})^2) \le \left(\frac{1-f}{n}\right)\bar{Y}^2C_x^2$$

$$(\bar{Y}+\alpha\bar{X})\left(\,2g\rho\bar{Y}C_xC_y + g^2C_x^2(\bar{Y}+\alpha\bar{X})\right) < 0$$

This holds if

(1) $\bar{Y}+\alpha\bar{X} < 0$ and $2g\rho\bar{Y}C_xC_y + g^2C_x^2(\bar{Y}+\alpha\bar{X}) > 0$

Or

(2) $\bar{Y}+\alpha\bar{X} > 0$ and $2g\rho\bar{Y}C_xC_y + g^2C_x^2(\bar{Y}+\alpha\bar{X}) < 0$

The range of α under which $\bar{y}_{pd}^{*}$ is more efficient than usual SRS $\bar{y}$ is

$$min\left\{-R, -R\left(1+\frac{2\rho C_y}{gC_x}\right)\right\}, max\left\{-R, -R\left(1+\frac{2\rho C_y}{gC_x}\right)\right\}.$$

We also compared $\bar{y}_{pd}^{*}$ with the usual ratio estimator $\bar{y}_R$. The MSE of the $\bar{y}_R$ is given as,

$$MSE(\bar{y}_R) = \left(\frac{1-f}{n}\right)\left(\bar{Y}^2C_y^2 - 2\rho\bar{Y}^2C_xC_y + \bar{Y}^2C_x^2\right)$$

It is found that $\bar{y}_{pd}^*$ will be more efficient than the usual ratio estimator $\bar{y}_R$ if MSE $(\bar{y}_{pd}^*)$ < MSE $(\bar{y}_R)$. That is,

$$\left(\frac{1-f}{n}\right)\left(\bar{Y}^2C_y^2 + 2g\rho\bar{Y}C_xC_y(\bar{Y}+\alpha\bar{X}) + g^2C_x^2(\bar{Y}+\alpha\bar{X})^2\right) \le \left(\frac{1-f}{n}\right)\left(\bar{Y}^2C_y^2 - 2\rho\bar{Y}^2C_xC_y + \bar{Y}^2C_x^2\right)$$

This holds if the following two conditions are satisfied

(1). $(g(\bar{Y}+\alpha\bar{X})+\bar{Y}) < 0$ And $2\rho\bar{Y}C_xC_y + C_x^2(g(\bar{Y}+\alpha\bar{X})-\bar{Y}) > 0.$

or

(2). $(g(\bar{Y}+\alpha\bar{X})+\bar{Y}) > 0$ And $2\rho\bar{Y}C_xC_y + C_x^2(g(\bar{Y}+\alpha\bar{X})-\bar{Y}) < 0.$

This condition holds if $\alpha > -R\left(\frac{N}{n}\right)$ and $\alpha < R\left(\frac{N-2n}{n} - \frac{2\rho C_y}{gC_x}\right)$ or $\alpha < -R\left(\frac{N}{n}\right)$ and $\alpha > R\left(\frac{N-2n}{n} - \frac{2\rho C_y}{gC_x}\right)$

$$\min\left\{-R\left(\frac{N}{n}\right), R\left(\frac{N-2n}{n} - \frac{2\rho C_y}{gC_x}\right)\right\}, \max\left\{-R\left(\frac{N}{n}\right), R\left(\frac{N-2n}{n} - \frac{2\rho C_y}{gC_x}\right)\right\}.$$

We also compared $\bar{y}_{pd}^*$ with the usual product estimator y_P. The MSE of the y_P is given as

$$MSE(\bar{y}_P) = \left(\frac{1-f}{n}\right)\left(\bar{Y}^2C_y^2 + 2\rho\bar{Y}^2C_xC_y + \bar{Y}^2C_x^2\right)$$

It is found that the proposed estimator $\bar{y}_{PD}^*$ will be more efficient than the usual ratio estimator $\bar{y}_P$ if MSE $(\bar{y}_{pd}^*)$ < MSE $(\bar{y}_P)$. That is,

$$\left(\frac{1-f}{n}\right)\left(\bar{Y}^2C_y^2 + 2g\rho\bar{Y}C_xC_y(\bar{Y}+\alpha\bar{X}) + g^2C_x^2(\bar{Y}+\alpha\bar{X})^2\right) \le \left(\frac{1-f}{n}\right)\left(\bar{Y}^2C_y^2 + 2\rho\bar{Y}^2C_xC_y + \bar{Y}^2C_x^2\right)$$

This holds if the following two conditions are satisfied

(1). $(g(\bar{Y}+\alpha\bar{X})-\bar{Y}) < 0$ And $2\rho\bar{Y}C_xC_y + C_x^2(g(\bar{Y}+\alpha\bar{X})+\bar{Y}) > 0.$

Or

(2). $(g(\bar{Y}+\alpha\bar{X})-\bar{Y}) > 0$ And $2\rho\bar{Y}C_xC_y + C_x^2(g(\bar{Y}+\alpha\bar{X})+\bar{Y}) < 0.$

This condition holds if $\alpha > R\left(\frac{N-2n}{n}\right)$ and $\alpha < -R\left(\frac{N}{n} + \frac{2\rho C_y}{gC_x}\right)$ or $\alpha < R\left(\frac{N-2n}{n}\right)$ and $\alpha > -R\left(\frac{N}{n} + \frac{2\rho C_y}{gC_x}\right)$

$$\min\left\{R\left(\frac{N-2n}{n}\right), -R\left(\frac{N}{n} + \frac{2\rho C_y}{gC_x}\right)\right\}, \max\left\{R\left(\frac{N-2n}{n}\right), -R\left(\frac{N}{n} + \frac{2\rho C_y}{gC_x}\right)\right\}.$$

We compared the MSE of the proposed estimator with MSE of dual product estimator from equation (2) and (5) it is found that the proposed estimator $\bar{y}_{pd}^*$ will be more efficient than that of Bandyopadhyay (1980) estimator $\bar{y}_p^*$ if MSE$(\bar{y}_{pd}^*)$ < MSE $(\bar{y}_p^*)$

That is

$$\left(\frac{1-f}{n}\right)\left(\bar{Y}^2C_y^2 + 2g\rho\bar{Y}C_xC_y(\bar{Y}+\alpha\bar{X}) + g^2C_x^2(\bar{Y}+\alpha\bar{X})^2\right) \le \left(\frac{1-f}{n}\right)\left(\bar{Y}^2C_y^2 + 2g\rho\bar{Y}^2C_xC_y + \bar{Y}^2g^2C_x^2\right)$$

Notes

Notes

This holds if

1. $\alpha\bar{X} < 0$ and $2\rho\bar{Y}C_y + gC_x(2\bar{Y} + \alpha\bar{X}) > 0$

Or

2. $\alpha\bar{X} > 0$ and $2\rho\bar{Y}C_y + gC_x(2\bar{Y} + \alpha\bar{X}) < 0$

The range of α under which the proposed estimator $\bar{y}_{pd}^*$ is more efficient than $\bar{y}_p^*$ is

$$min\left\{0, -2R\left(\frac{\rho C_y}{gC_x} + 1\right)\right\}, max\left\{0, -2R\left(\frac{\rho C_y}{gC_x} + 1\right)\right\}$$

Lastly, we compared MSE of $\bar{y}_{pd}^*$ with that of dual to ratio estimator $\bar{y}_R^*$ proposed Srivenkataramana(1980), $\bar{y}_{pd}^*$ will be more efficient than $\bar{y}_R^*$ if

$$\text{MSE }(\bar{y}_{pd}^*) < \text{MSE }(\bar{y}_R^*)$$

$$\left(\frac{1-f}{n}\right)\left(\bar{Y}^2C_y^2 + 2g\rho\bar{Y}C_xC_y(\bar{Y} + \alpha\bar{X}) + g^2C_x^2(\bar{Y} + \alpha\bar{X})^2\right)$$

$$\leq \left(\frac{1-f}{n}\right)\left(\bar{Y}^2C_y^2 - 2g\rho\bar{Y}^2C_xC_y + \bar{Y}^2g^2C_x^2\right)$$

This holds if

1. $2\bar{Y} + \alpha\bar{X} < 0$ and $2\rho\bar{Y}C_y + gC_x(\alpha\bar{X}) > 0$

Or

2. $2\bar{Y} + \alpha\bar{X} > 0$ and $2\rho\bar{Y}C_y + gC_x(\alpha\bar{X}) < 0$

This condition holds if $-2R > \alpha$ and $\frac{-2R\rho C_y}{gC_x} < \alpha$ or $-2R < \alpha$ and $\frac{-2R\rho C_y}{gC_x} > \alpha$

Therefore, the range of α under which the proposed estimator $\bar{y}_{pd}^*$ is more efficient than dual to ratio estimator $\bar{y}_R^*$ is

$$min\left\{-2R, -2R\left(\frac{\rho C_y}{gC_x}\right)\right\}, max\left\{-2R, -2R\left(\frac{\rho C_y}{gC_x}\right)\right\}$$

Comparison of 'AOE' to $\boldsymbol{\bar{y}_{pd}^{*OPT}}$

$\bar{y}_{pd}^{*OPT}$ is more efficient than the other existing estimators $\bar{y}$, the Ratio estimator $\bar{y}_R$, the product estimator $\bar{y}_p$, the dual to ratio estimator $\bar{y}_R$ and the dual to product estimator $\bar{y}_p^*$ since.

$$MSE(\bar{y}) - MSE\left(\bar{y}_{pd}^{*OPT}\right) = \left(\frac{1-f}{n}\right)\left(\bar{Y}^2\rho^2C_y^2\right) > 0$$

$$MSE(\bar{y}_R) - MSE\left(\bar{y}_{pd}^{*OPT}\right) = \left(\frac{1-f}{n}\right)\left(\bar{Y}^2C_x^2\left(1 - \frac{\rho C_y}{C_x}\right)^2\right) > 0$$

$$MSE(\bar{y}_P) - MSE\left(\bar{y}_{pd}^{*OPT}\right) = \left(\frac{1-f}{n}\right)\left(\bar{Y}^2C_x^2\left(1 + \frac{\rho C_y}{C_x}\right)^2\right) > 0$$

$$MSE(\bar{y}_R^*) - MSE\left(\bar{y}_{pd}^{*OPT}\right) = \left(\frac{1-f}{n}\right)\left(\bar{Y}^2C_x^2\left(\frac{\rho C_y}{C_x} - g\right)^2\right) > 0$$

$$MSE(\bar{y}_p^*) - MSE(\bar{y}_{pd}^{*OPT}) = \left(\frac{1-f}{n}\right)\left(\bar{Y}^2 C_x^2 \left(\frac{\rho C_y}{C_x} + g\right)^2\right) > 0$$

Hence, we conclude that the proposed class of estimator $\bar{y}_{pd}^*$ is more efficient than other estimator in case of its optimality.

IV. Numerical Validation

Notes

To illustrate the efficiency of the proposed estimator over the other estimators $\bar{y}$,$\bar{y}_R$,$\bar{y}_p$,$\bar{y}_{R^*}$ *and* $\bar{y}_p^*$. Data on the ages and hours of sleeping by the undergraduate students of the Department of Statistics Federal University of Technology Akure, Ondo State, Nigeria were collected. A sample of 150 out of 461 students of the department was obtained using simple random sampling without replacement. The information on the age of the students was used as auxiliary information to increase the precision of the estimate of the average sleeping hours. The estimate of the average hours of sleeping of the students were obtained and also the 95% confidence intervals of the average hours of sleeping were obtained for the proposed estimator and the other estimators. Table 1.0, gives the estimates of the average sleeping hours and the 95% confidence Interval. As shown in Table 1.0, the proposed estimator performed better than the other estimators, also the width of the confidence interval of the proposed estimator is smallest than the other competing estimators.

Table 1.0. : Average Sleeping Hours and 95% confidence intervals for Different Estimators for the undergraduate Students of Department of Statistics, Federals University of Technology Akure. Nigeria

Estimator	Average Sleeping Hours	LCL	UCL	WIDTH
$\bar{y}$	6.08	5.930386531	6.229613469	0.299226939
$\bar{y}_R$	6.210472103	6.042844235	6.378099971	0.335255737
$\bar{y}_p$	5.952268908	5.778821411	6.125716404	0.346894993
$\bar{y}_R^*$	6.141606636	5.988421023	6.294792249	0.306371226
$\bar{y}_p^*$	6.01901342	5.862732122	6.175290562	0.31255844
$\boldsymbol{\bar{y}_{PD}^*}$	**6.072287**	5.927857	**6.216717**	**0.28886**

The proposed estimator performed the same way as the regression estimator when compare with the usual estimator $\bar{y}$. The average Sleeping Hours and 95% confidence intervals for the proposed estimator and the regression estimator is given below, the two estimators have the same confidence Interval width.

Table 2.0. : Average Sleeping Hours and 95% confidence intervals for the proposed estimators and regression estimators for the undergraduate Students of Department of Statistics, Federals University of Technology Akure. Nigeria

Estimator	Average Sleeping Hours	LCL	UCL	WIDTH
$\boldsymbol{\bar{y}_{PD}^*}$	6.072287	5.927857	6.2167177	0.28886
$\boldsymbol{\bar{y}_{REG}^*}$	6.089652737	5.945222793	6.234082681	0.288859888

Notes

To examine the gain in the efficiency of the proposed estimator $\bar{y}^*_{pd}$ over the estimator $\bar{y}$,$\bar{y}_R$,$\bar{y}_p$, $\bar{y}^*_R$ *and* $\bar{y}^*_{p,}$ we obtained the percentage relative efficiency of different estimator of $\bar{Y}$ with respect to the usual unbiased estimator $\bar{y}$ in Table 2.0. The proposed estimator $\bar{y}^*_{pd}$ performed better than the other estimators $\bar{y}, \bar{y}_R, \bar{y}_p, \bar{y}^*_R$ *and* $\bar{y}^*_p$ and performed exactly the same way as regression estimator.

Table 3.0. : The percentage relative efficiency of different estimator of $\bar{Y}$ with respect to the usual unbiased estimator $\bar{y}$

ESTIMATOR	PERCENATGE RELATIVE FFICIENCY
$\bar{y}$	100
$\bar{y}_R$	79.66158486
$,\bar{y}_p$	74.40554745
$\bar{y}^*_R$	95.39056726
$\bar{y}^*_p$	91.65136111
$\bar{y}^*_{REG}$	107.3067159
$\bar{y}^*_{pd}$	107.3067159

V. Conclusion

We have proposed a class of regression estimator with cum-dual product estimator as intercept for estimating the mean of the study variable Y using auxiliary variable X as in equation (1) and obtained 'AOE' for the proposed estimator. Theoretically, we have demonstrated that proposed estimator is always more efficient than other under the effective ranges of α and its optimum values.

Table 1.0 shows that the proposed estimator performed better than the other estimators as the width of the confidence interval of the proposed estimator is smallest than the other competing estimators. The percentage relative efficiency of different estimator of $\bar{Y}$ with respect to the usual unbiased estimator $\bar{y}$ in Table 2.0 shows that the proposed estimator $\bar{y}^*_{PD}$ performed better than the other estimators $\bar{y}, \bar{y}_R, \bar{y}_p, \bar{y}^*_R$ *and* $\bar{y}^*_p$ and performed exactly the same way as regression estimator when compared to the usual estimator $\bar{y}$. Hence, it is preferred to use the proposed class of estimator in practice.

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