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A Summation Formula of Half Argument in the Light of Hypergeometric Function and Involving Contiguous Relation

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Keywords: *gauss second summation theorem, recurrence relation, prudnikov .*

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A Summation Formula of Half Argument in the Light of Hypergeometric Function and Involving Contiguous Relation

Salahuddin ^α, Upendra Kumar Pandit ^σ & M. P. Chaudhary ^ρ

Abstract- In this paper we have developed a summation formula involving contiguous relation. We have developed this formula using the contiguous relation and involving the formula derived by Salahuddin et. al.[7, p.78-84(8)].

Keywords: gauss second summation theorem, recurrence relation, prudnikov .

I. INTRODUCTION

Generalized Gaussian Hypergeometric function of one variable is defined by

$${}_A F_B \left[\begin{matrix} a_1, a_2, \dots, a_A ; \\ b_1, b_2, \dots, b_B ; \end{matrix} z \right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \dots (a_A)_k z^k}{(b_1)_k (b_2)_k \dots (b_B)_k k!} \quad (1)$$

where the parameters b_1, b_2, \dots, b_B are neither zero nor negative integers and A, B are nonnegative integers and $|z| = 1$

Contiguous Relation is defined by [1,p.558(15.2.14)]

$$(a-b) {}_2F_1 \left[\begin{matrix} a, b ; \\ c ; \end{matrix} z \right] = a {}_2F_1 \left[\begin{matrix} a+1, b ; \\ c ; \end{matrix} z \right] - b {}_2F_1 \left[\begin{matrix} a, b+1 ; \\ c ; \end{matrix} z \right] \quad (2)$$

Gauss second summation theorem is defined by [Prudnikov., 491(7.3.7.5)]

$${}_2F_1 \left[\begin{matrix} a, b ; \\ \frac{a+b+1}{2} ; \end{matrix} \frac{1}{2} \right] = \frac{\Gamma(\frac{a+b+1}{2}) \Gamma(\frac{1}{2})}{\Gamma(\frac{a+1}{2}) \Gamma(\frac{b+1}{2})} \quad (3)$$

$$= \frac{2^{(b-1)} \Gamma(\frac{b}{2}) \Gamma(\frac{a+b+1}{2})}{\Gamma(b) \Gamma(\frac{a+1}{2})} \quad (4)$$

In a monograph of Prudnikov et al., a summation theorem is given in the form [Prudnikov., p.491(7.3.7.8)]

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$${}_2F_1 \left[\begin{matrix} a, b \\ \frac{a+b-1}{2} \end{matrix}; \frac{1}{2} \right] = \sqrt{\pi} \left[\frac{\Gamma(\frac{a+b+1}{2})}{\Gamma(\frac{a+1}{2}) \Gamma(\frac{b+1}{2})} + \frac{2 \Gamma(\frac{a+b-1}{2})}{\Gamma(a) \Gamma(b)} \right] \quad (5)$$

Now using Legendre's duplication formula and Recurrence relation for Gamma function, the above theorem can be written in the form

$${}_2F_1 \left[\begin{matrix} a, b \\ \frac{a+b-1}{2} \end{matrix}; \frac{1}{2} \right] = \frac{2^{(b-1)} \Gamma(\frac{a+b-1}{2})}{\Gamma(b)} \left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} + \frac{2^{(a-b+1)} \Gamma(\frac{a}{2}) \Gamma(\frac{a+1}{2})}{\{\Gamma(a)\}^2} + \frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})} \right] \quad (6)$$

Recurrence relation is defined by

$$\Gamma(\zeta + 1) = \zeta \Gamma(\zeta) \quad (7)$$

II. MAIN SUMMATION FORMULA

$${}_2F_1 \left[\begin{matrix} a, b \\ \frac{a+b+53}{2} \end{matrix}; \frac{1}{2} \right] = \frac{2^b \Gamma(\frac{a+b+53}{2})}{(a-b) \Gamma(b) \left[\prod_{\varrho=1}^{26} \{a-b-(2\varrho-1)\} \right] \left[\prod_{\varsigma=1}^{26} \{a-b+(2\varsigma-1)\} \right]}$$

$$\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+1}{2})} \left\{ \begin{aligned} &99999854876201790200861776103866368000000a \\ &-261083030364832640210848413666477342720000a^2 \\ &+279618009646814556886427718509836369920000a^3 \\ &-171872880997655158069147889848408866816000a^4 \\ &+69892599463741434031897360198231130112000a^5 \\ &-20340875557500786200866985453858605498368a^6 \\ &+4451611783759871693851104858308207443968a^7 \\ &-757666173237536427557360598023720140800a^8 \\ &+102719075765630836883572972193631436800a^9 - 11288918038327853438226939343599042560a^{10} \\ &+1018891067852588033855655097464258560a^{11} - 76251472850207440069588780109004800a^{12} \\ &+4764498207460659434170992086220800a^{13} - 249724853112613454913082574766080a^{14} \\ &+11008905953427255161153210286080a^{15} - 408517765443701267938200780800a^{16} \\ &+12746421102946663778798796800a^{17} - 333365045216510883890462720a^{18} \\ &+7268366473889659454750720a^{19} - 131016677920258968780800a^{20} \\ &+1929092943185759436800a^{21} - 22804003140392714240a^{22} + 211048756139786240a^{23} \\ &-1472113462476800a^{24} + 7273761996800a^{25} - 22682796032a^{26} + 33554432a^{27} \\ &-99999854876201790200861776103866368000000b \\ &+404066213678766534268675482164259717120000a^2b \\ &-549822394273332654567769511461625266176000a^3b \\ &+381071159293812793532752378989514221158400a^4b \\ &-157939037388577550391970404908852148436992a^5b \end{aligned} \right. \right]$$



$$\begin{aligned}
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 &-594408026004197650296995840a^{18}b^2 + 16013256708217828676730880a^{19}b^2 \\
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 &+22159164098878522320542645252760035917824a^6b^3
 \end{aligned}$$

$$\begin{aligned}
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 \end{aligned}$$

$$\begin{aligned}
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$$\begin{aligned}
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 &\quad -3419043095709998387298304a^{12}b^{13} + 2333158805929593929728a^{14}b^{13} \\
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 &+85584944509149990569729140457472a^4b^{14} - 27206295256489447787995173224448a^5b^{14} \\
 &+3021196069940396527037259448320a^6b^{14} - 585531228990167310450642911232a^7b^{14} \\
 &+30484298233062502788797300736a^8b^{14} - 3900633900687211284307902464a^9b^{14} \\
 &+89580217924802207620792320a^{10}b^{14} - 7574803793112568487215104a^{11}b^{14}
 \end{aligned}$$

$$\begin{aligned}
 &+54386736303738120568832a^{12}b^{14} - 2333158805929593929728a^{13}b^{14} \\
 &-11008905953427255161153210286080b^{15} + 24963435732242594101438846074880ab^{15} \\
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 &-4843535994127351779043867361280a^4b^{15} + 753256034497531576746560716800a^5b^{15} \\
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 &-6860758577769942194585600a^{10}b^{15} + 70176433940307252346880a^{11}b^{15} \\
 &-5229493875359434670080a^{12}b^{15} + 408517765443701267938200780800b^{16} \\
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 &-4849933836018830540800a^{11}b^{16} - 12746421102946663778798796800b^{17} \\
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 &+644650471910713451096309760a^5b^{17} - 163671496756811325222420480a^6b^{17} \\
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 &+333365045216510883890462720b^{18} - 817295532601415608429445120ab^{18} \\
 &+594408026004197650296995840a^2b^{18} - 379548558859608116575600640a^3b^{18} \\
 &+74337442189198997313290240a^4b^{18} - 24372114457372901318328320a^5b^{18} \\
 &+1695070926380643343400960a^6b^{18} - 349374120875383838474240a^7b^{18} \\
 &+7618020079469959577600a^8b^{18} - 1084102857463032709120a^9b^{18} \\
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 &-16013256708217828676730880a^2b^{19} + 5199529188585867449466880a^3b^{19} \\
 &-2289867037354645499412480a^4b^{19} + 226283752061401458278400a^5b^{19} \\
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 &-128139585231837756129280a^3b^{20} + 19285830933031786905600a^4b^{20} \\
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 &-6394111847091404800a^6b^{21} + 22804003140392714240b^{22} - 52769444637620305920ab^{22}
 \end{aligned}$$

$$\begin{aligned}
 &+27921301580965478400a^2b^{22} - 17336553823514132480a^3b^{22} + 1546482865343037440a^4b^{22} \\
 &\quad -505580936746762240a^5b^{22} - 211048756139786240b^{23} + 363397445981306880ab^{23} \\
 &\quad -362846751314411520a^2b^{23} + 57224157829529600a^3b^{23} - 24567871606620160a^4b^{23} \\
 &\quad +1472113462476800b^{24} - 3219668043038720ab^{24} + 1124045957365760a^2b^{24} \\
 &\quad -674034551357440a^3b^{24} - 7273761996800b^{25} + 9617505517568ab^{25} - 9039530426368a^2b^{25} \\
 &\quad +22682796032b^{26} - 44459622400ab^{26} - 33554432b^{27} \} - \\
 &\quad -\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})} \left\{ -310314080948366957792258914205439098880000a \right. \\
 &\quad +612497603587902117810878481677389135872000a^2 \\
 &\quad -503822741863665090976307146691686327910400a^3 \\
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 &\quad -89518225778806733026928050625382092636160a^5 \\
 &\quad +24282041395131254495138973325367130980352a^6 \\
 &\quad -4394487149553657057473394585795758653440a^7 \\
 &\quad +726182964465573307177570248167018987520a^8 \\
 &\quad -81435276299387718231014057751777116160a^9 \\
 &\quad +8965635440254224278738824081199595520a^{10} \\
 &\quad -663427431337216157116293844712816640a^{11} + 51102576951919705983915356449669120a^{12} \\
 &\quad -2576655392865578528051933069967360a^{13} + 142609340940634067820065534771200a^{14} \\
 &\quad -4958691543257638644892922019840a^{15} + 199418501102557740088919326720a^{16} \\
 &\quad -4756278235326794989263912960a^{17} + 138660239763515611084226560a^{18} \\
 &\quad -2211990907334704119152640a^{19} + 45919802420941155205120a^{20} - 463529702563837378560a^{21} \\
 &\quad +6576083614239293440a^{22} - 37374896677847040a^{23} + 332006340689920a^{24} - 785173708800a^{25} \\
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 &\quad -394616266661766015597017591327733149859840a^4b \\
 &\quad +156960480469554368144664756915544227053568a^5b \\
 &\quad -37771517456884240262247906277981830512640a^6b \\
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 &\quad -1137938674523730849829002607888548495360a^8b \\
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 &\quad +519057560199763190736018466685440a^{13}b - 203054958013020479395466104012800a^{14}b
 \end{aligned}$$



$$\begin{aligned}
 &+10064021319208990487687777484800a^{15}b - 266335249474739060996843765760a^{16}b \\
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 &-5767518184789770240a^{22}b + 73485140235386880a^{23}b - 191425350205440a^{24}b \\
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 \end{aligned}$$

$$\begin{aligned}
 & -8437876080808569077760a^{20}b^3 + 183067123869936517120a^{21}b^3 - 577083622419333120a^{22}b^3 \\
 & + 8803716589158400a^{23}b^3 - 270616792197416956781799088461973133721600b^4 \\
 & + 394616266661766015597017591327733149859840ab^4 \\
 & - 259911446536700670121718592939741576953856a^2b^4 \\
 & + 63905408280242551217291468498363071070208a^3b^4 \\
 & - 616418473296098301228874072929002848256a^5b^4 \\
 & + 2935706885164912256789605705429045739520a^6b^4 \\
 & - 583695913641103911133456646518693953536a^7b^4 \\
 & + 121015507818773637252775337243977973760a^8b^4 \\
 & - 12449927514514579375420569608779726848a^9b^4 \\
 & + 1625170598344439075291044800194674688a^{10}b^4 - 100885427133907938046424140671352832a^{11}b^4 \\
 & + 9135758528333274263544976470179840a^{12}b^4 - 357950824573904251331631137161216a^{13}b^4 \\
 & + 23504488858569839583712361054208a^{14}b^4 - 582141530495391051207388692480a^{15}b^4 \\
 & + 28326609126153579412051722240a^{16}b^4 - 427374230669356228222648320a^{17}b^4 \\
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 & - 29971450697709184177947295085529906806784a^2b^6 \\
 & + 10473496743174773054928924706338213199872a^3b^6
 \end{aligned}$$

$$\begin{aligned}
 & -2935706885164912256789605705429045739520a^4b^6 \\
 & +318174961732473100252631415052834963456a^5b^6 \\
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 & +3004198395013711238494181351615365120a^8b^6 \\
 & -331234878834507336859643640809521152a^9b^6 + 49851909584221051030583163540733952a^{10}b^6 \\
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 & -2620792820961326191498947905051846246400a^3b^7 \\
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 & +44526781962200234385810271233900544a^9b^7 - 3604063591273581486417705326084096a^{10}b^7 \\
 & +471909233158210376518257407426560a^{11}b^7 - 19547180151009272739563541889024a^{12}b^7 \\
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 \end{aligned}$$

$$\begin{aligned}
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 &\quad -10937139947410589681120345849856a^{10}b^9 + 2224863935189757001820308242432a^{11}b^9 \\
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 &\quad -471909233158210376518257407426560a^7b^{11} + 23329420395466521314816390332416a^8b^{11} \\
 &\quad -2224863935189757001820308242432a^9b^{11} + 39870747960828959451605106688a^{10}b^{11} \\
 &\quad -76288801336508014026293248a^{12}b^{11} + 12159956545687393587429376a^{13}b^{11} \\
 &\quad -131175949596112787079168a^{14}b^{11} + 10796374452354961899520a^{15}b^{11} \\
 &\quad -51102576951919705983915356449669120b^{12} + 76253621358412274045975527340113920ab^{12} \\
 &-69410567545684129682563660792201216a^2b^{12} + 23699017900042243132880893287333888a^3b^{12} \\
 &\quad -9135758528333274263544976470179840a^4b^{12} + 1301606807662544916091610788265984a^5b^{12} \\
 &\quad -285563832649812888015331466936320a^6b^{12} + 19547180151009272739563541889024a^7b^{12}
 \end{aligned}$$

$$\begin{aligned}
 & -2686379768320812013946604093440a^8b^{12} + 86738819729241228172981174272a^9b^{12} \\
 & -7146708382512041317841764352a^{10}b^{12} + 76288801336508014026293248a^{11}b^{12} \\
 & -58570331404025668304896a^{13}b^{12} + 8367190200575095472128a^{14}b^{12} \\
 & +2576655392865578528051933069967360b^{13} - 5190575601997631907360184666685440ab^{13} \\
 & +3102140127936413336812977030955008a^2b^{13} - 1749208935885534358913960479031296a^3b^{13} \\
 & +357950824573904251331631137161216a^4b^{13} - 104220584730912563853529437962240a^5b^{13} \\
 & +9613224596120555843077825101824a^6b^{13} - 1722742697371432755553525301248a^7b^{13} \\
 & +75683242373738193318102171648a^8b^{13} - 8863958380064078156529664000a^9b^{13} \\
 & +168640988401952358114787328a^{10}b^{13} - 12159956545687393587429376a^{11}b^{13} \\
 & +58570331404025668304896a^{12}b^{13} - 142609340940634067820065534771200b^{14} \\
 & +203054958013020479395466104012800ab^{14} - 187795264949971795332713334964224a^2b^{14} \\
 & +59205538114853411380075343904768a^3b^{14} - 23504488858569839583712361054208a^4b^{14} \\
 & +2975311002755349756228937973760a^5b^{14} - 685795695238085936055972790272a^6b^{14} \\
 & +39674370552251605811655081984a^7b^{14} - 5953860942612692115205914624a^8b^{14} \\
 & +152042642318901839690465280a^9b^{14} - 15413983805627179103944704a^{10}b^{14} \\
 & +131175949596112787079168a^{11}b^{14} - 8367190200575095472128a^{12}b^{14} \\
 & +4958691543257638644892922019840b^{15} - 10064021319208990487687777484800ab^{15} \\
 & +5582987274544695336269135216640a^2b^{15} - 3205477994625171638413971947520a^3b^{15} \\
 & +582141530495391051207388692480a^4b^{15} - 174900292183819738556368158720a^5b^{15} \\
 & +13455764508981224472513085440a^6b^{15} - 2538148804503169898863656960a^7b^{15} \\
 & +83883671431664376710430720a^8b^{15} - 10747790767319364570972160a^9b^{15} \\
 & +121459212588993321369600a^{10}b^{15} - 10796374452354961899520a^{11}b^{15} \\
 & -199418501102557740088919326720b^{16} + 266335249474739060996843765760ab^{16} \\
 & -248679648338502677328769843200a^2b^{16} + 69940933668835563196687319040a^3b^{16} \\
 & -28326609126153579412051722240a^4b^{16} + 2987801946086821450481664000a^5b^{16} \\
 & -711931763489024161118945280a^6b^{16} + 30583404478512969673605120a^7b^{16} \\
 & -4839657219458185235005440a^8b^{16} + 69839047238671159787520a^9b^{16} \\
 & -7759894137630128865280a^{10}b^{16} + 4756278235326794989263912960b^{17} \\
 & -9675847843307482882926182400ab^{17} + 4837559454713544055644487680a^2b^{17} \\
 & -2809096463303815084501893120a^3b^{17} + 427374230669356228222648320a^4b^{17} \\
 & -131219075021515078710067200a^5b^{17} + 7490861667562275345530880a^6b^{17} \\
 & -1463229249471584088883200a^7b^{17} + 26980744799677411491840a^8b^{17} \\
 & -3651714888296531230720a^9b^{17} - 138660239763515611084226560b^{18} \\
 & +169571332713142044862709760ab^{18} - 158942424126743343309783040a^2b^{18} \\
 & +37837109081978063667855360a^3b^{18} - 15523186878326101308866560a^4b^{18}
 \end{aligned}$$

$$\begin{aligned}
 &+1221886412463089469358080a^5b^{18} - 298016857553661130178560a^6b^{18} \\
 &\quad +7140203435432198799360a^7b^{18} - 1172003089149224550400a^8b^{18} \\
 &\quad +2211990907334704119152640b^{19} - 4485965640391204053975040ab^{19} \\
 &\quad +1928548363149141186969600a^2b^{19} - 1125679730658013997957120a^3b^{19} \\
 &\quad +129185857941808978329600a^4b^{19} - 40236592507791228272640a^5b^{19} \\
 &+1289052948373627207680a^6b^{19} - 258125376719505326080a^7b^{19} - 45919802420941155205120b^{20} \\
 &\quad +49436825533319668039680ab^{20} - 46230448188340222033920a^2b^{20} \\
 &+8437876080808569077760a^3b^{20} - 3483594491371066490880a^4b^{20} + 155654873014704537600a^5b^{20} \\
 &\quad - 38561412985535856640a^6b^{20} + 463529702563837378560b^{21} - 930620040245286010880ab^{21} \\
 &+314180812148582645760a^2b^{21} - 183067123869936517120a^3b^{21} + 12133942481922293760a^4b^{21} \\
 &\quad - 3806727053152092160a^5b^{21} - 6576083614239293440b^{22} + 5767518184789770240ab^{22} \\
 &\quad - 5336938763727667200a^2b^{22} + 577083622419333120a^3b^{22} - 237920440822005760a^4b^{22} \\
 &\quad + 37374896677847040b^{23} - 73485140235386880ab^{23} + 15280736651182080a^2b^{23} \\
 &\quad - 8803716589158400a^3b^{23} - 332006340689920b^{24} + 191425350205440ab^{24} \\
 &\quad - 172930147287040a^2b^{24} + 785173708800b^{25} - 1479616233472ab^{25} - 3489660928b^{26} \Big\} \quad (8)
 \end{aligned}$$

III. DERIVATION OF THE SUMMATION FORMULA

Putting $c = \frac{a+b+53}{2}$ and $z = 12$ in equation (2), we get

$$(a - b) {}_2F_1 \left[\begin{matrix} a, b \\ \frac{a+b+53}{2} \end{matrix}; \frac{1}{2} \right] = a {}_2F_1 \left[\begin{matrix} a + 1, b \\ \frac{a+b+53}{2} \end{matrix}; \frac{1}{2} \right] - b {}_2F_1 \left[\begin{matrix} a, b + 1 \\ \frac{a+b+53}{2} \end{matrix}; \frac{1}{2} \right]$$

Now involving the formula derived by Salahuddin et. al.[7, p.78-84(8)], the summation formula is obtained.

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