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On Non-Invariant Hypersurfaces of $\delta-$lorentzian Trans-Sasakian Manifolds

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Abstract- The object of the present paper is to study non-invariant Hypersurfaces of $\delta-$Lorentzian trans-Sasakian Manifolds equipped with $(f, g, u, v, \lambda)-$ structure and some properties obeyed by this structure are obtained also. The necessary and sufficient conditions have been otained for totally umbilical non-invariant hypersurfaces with (f, g, u, v, λ) –structure of δ–Lorentzian trans-Sasakian Manifold to be totally geodesic.

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ONNONINVARIANTHYPERSURFACESOFLORENTZIANTRANSSASAKIANMANIFOLDS

On Non-Invariant Hypersurfaces of $\delta-$lorentzian Trans-Sasakian Manifolds

Shyam Kishor [α] & Puneet Kumar Gupt [σ]

Abstract- The object of the present paper is to study non-invariant Hypersurfaces of $\boldsymbol{\delta}-$Lorentzian trans-Sasakian Manifolds equipped with $(f,g,u,v,\lambda)-$ structure and some properties obeyed by this structure are obtained also. The necessary and sufficient conditions have been obtained for totally umbilical non-invariant hypersurfaces with (f,g,u,v,λ) –structure of $\boldsymbol{\delta}-$Lorentzian trans-Sasakian Manifold to be totally geodesic.

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I. Introduction

Recently, many authors have studied Lorentzian $\alpha-$Sasakian manifold [1] and Lorentzian $\beta-$Kenmotsu manifolds [7], [3]. S.S.Pujar and V.J.Khairnar [12] have initiated the study of Lorentzian Trans-Sasakian manifolds and studied the basic results with some of its properties. Earlier to this, S.S.Pujar [13] has initiated the study of $\delta-$Lorentzian, $\alpha-$Sasakian manifold [3] and $\delta-$Lorentzian $\beta-$Kenmotsu manifolds [4].

In 2010, S.S.Shukla and D.D. Singh [14] have introduced the notion of $\varepsilon-$trans-Sasakian manifolds and studied its basic results and using these results some of its properties were studied. Earlier to this in 1969 Takahashi [16] had introduced the notaion of almost contact metric manifold equipped with pseudo Riemannian metric. In particular he studied the Sasakian manifolds equipped with Riemannian metric g. These indefinite almost contact metric manifolds and indefinite Sasakian manifolds are also known as $\varepsilon-$almost contact metric manifolds and $\varepsilon-$Sasakian manifolds respectively.

Recently, it has been observed that there does not exists a light like surface in the $\varepsilon-$Sasakian manifolds ([8], [16]). On the other hand in almost para contact manifold defined by Motsumoto [6] , the semi Riemannian manifold has the index 1 and the structure vector field ξ is always a time like. This motivated Tripathi et. al [8] to introduce $\varepsilon-$almost para contact structure where the vector field ξ is space like or time like according as $\varepsilon = 1$ or $\varepsilon = -1$.

In 1970, S.I.Goldberg et. al [10] introduced the notion of a non-invariant hypersurfaces of an almost contact manifold in which the transform of a tangent vector of the hypersurface by the $(1,1)$ structure tensor field ϕ defining the almost contact structure is never tangent to the hypersurface.

The notion of (f,g,u,v,λ) -structure was given by K.Yano [4]. It is well known that a hypersurface of an almost contact metric manifold always admits a (f,g,u,v,λ) -structure ([5] [2]). Goldberg et. al [10] proved that there always exists a (f,g,u,v,λ) -structure on a non-invariant hypersurface of an almost contact metric manifold.

Ref
3. G. T. Srinivas, Venkatesh and C.S. Bagewadi, On Lorentzian $\beta-$ Kenmotsu manifolds, General Mathematics, 18(4)(2010), 61 – 69:

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They also proved that there does not exist invariant hypersurface of a contact manifold. R.Prasad [9] studied the non-invariant hypersurfaces of a trans-Sasakian manifolds. Non-invariant hypersurfaces of nearly trans-Sasakian manifold have been studied by S.Kishor et. al [11]. In the present paper, we study the non-invariant hypersurfaces of $\delta-$Lorentzian trans-Sasakian manifolds.

II. Preliminaries

A $(2n+1)$ dimensional manifold $\widetilde{M}$, is said to be the $\delta-$almost contact metric manifold if it admits a $(1,1)$ tensor field ϕ, a structure tensor field ξ, a $1-$form η, and an indefinite metric g such that

$$\phi^2 = I + \eta \otimes \xi, \eta(\xi) = -1, \phi \circ \xi = 0, \eta \circ \phi = 0 \tag{2.1}$$

$$g(\xi, \xi) = -\delta, \eta(X) = \delta g(X, \xi) \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X, Y) + \delta \eta(X) \eta(Y) \tag{2.3}$$

$$g(X, \phi Y) = g(\phi X, Y) \tag{2.4}$$

for all $X, Y \in T\widetilde{M}$, where δ is such that $\delta^2 = 1$.

The above structure $(\phi, \xi, \eta, g, \delta)$ on $\widetilde{M}$ is called the $\delta-$Lorentzian structure on $\widetilde{M}$.

A $\delta-$Lorentzian manifold with structure $(\phi, \xi, \eta, g, \delta)$ is said to be $\delta-$Lorentzian trans-Sasakian manifold $\widetilde{M}$ of type (α, β) if it satisfies the condition

$$\left(\widetilde{\nabla}_X \phi\right) Y = \alpha \{g(X, Y)\xi - \delta\eta(Y) X\} + \beta \{g(\phi X, Y)\xi - \delta\eta(Y)\phi X\} \tag{2.5}$$

for any vector fields X and Y on $\widetilde{M}$. where $\widetilde{\nabla}$ is the operator of covariant differentiation with respect to g. From above, we have

$$\widetilde{\nabla}_X \xi = \delta(-\alpha\phi X - \beta(X + \eta(X)\xi)) \tag{2.6}$$

and

$$\left(\widetilde{\nabla}_X \eta\right) Y = \alpha g(\phi X, Y) + \beta\{g(X, Y) + \delta\eta(X)\eta(Y)\} \tag{2.7}$$

A hypersurface of an almost contact metric manifold $\widetilde{M}$ is called a non-invariant hypersurface, if the transform of a tangent vector of the hypersurface under the action of $(1,1)$ tensor field ϕ defining the contact structure is never tangent to the hypersurface. Let X be a tangent vector on non-invariant hypersurface of an almost contact metric manifold $\widetilde{M}$, then ϕX is never tangent to the hypersurface.

Let M be a non-invariant hypersurface of an almost contact metric manifold. Now, if we define the following

$$\phi X = fX + u(X)\hat{N}, \tag{2.8}$$

$$\phi\hat{N} = -U, \tag{2.9}$$

$$\xi = V + \lambda\hat{N}, \lambda = \eta\left(\hat{N}\right), \tag{2.10}$$

$$\eta(X) = v(X), \tag{2.11}$$

Ref

9. R. Prasad, On Non-invariant Hypersurfaces of Trans-Sasakian Manifold, J. Inter. Acad. Phys. Sci.

Notes

where, f is a $(1,1)$ tensor field, u, v are $1-$form, $\hat{N}$ is a unit normal to the hypersurface, $X \in TM$ and $u(X) \neq 0$, then we get an induced $(f, g, u, v, \lambda)-$structure on M satisfying the conditions

$$f^2 = -I + u \otimes U + v \otimes V, \tag{2.12}$$

$$fU = -\lambda V, fV = \lambda U, \tag{2.13}$$

$$u \circ f = \lambda v, v \circ f = -\lambda u, \tag{2.14}$$

$$u(U) = 1 - \lambda^2, u(V) = v(U) = 0, v(V) = 1 - \lambda^2, \tag{2.15}$$

$$g(fX, fY) = g(X, Y - u(X)u(Y) - v(X)v(Y)), \tag{2.16}$$

$$g(X, fY) = -g(fX, Y), g(X, U) = u(X), \tag{2.17}$$

$$g(X, V) = v(X), \tag{2.18}$$

for all $X, Y \in TM$, where $\lambda = \eta\left(\hat{N}\right)$.

The Gauss and Weingarten formulae are given by

$$\widetilde{\nabla}_X Y = \nabla_X Y + \sigma(X, Y)\hat{N}, \tag{2.19}$$

$$\widetilde{\nabla}_X \hat{N} = -A_{\hat{N}} X, \tag{2.20}$$

for all $X, Y \in TM$, where $\widetilde{\nabla}$ and ∇ are the Riemannian and induced Riemannian connections on $\widetilde{M}$ and M respectively and $\hat{N}$ is the unit normal vector in the normal bundle $T^{\perp}M$. In this formula σ is the second fundamental form on M related to $A_{\hat{N}}$ by

$$\sigma(X, Y) = g\left(A_{\hat{N}} X, Y\right), \text{ for all } X, Y \in TM.$$

III. Some Properties of Non-Invariant Hypersurfaces

Lemma 1. :*Let M be a non-invariant hypersurface with $(f, g, u, v, \lambda)-$structure of $\delta-$Lorentzian trans-Sasakian manifold $\widetilde{M}$. Then*

$$\left(\widetilde{\nabla}_X \phi\right) Y = (\nabla_X f) Y - u(Y)\left(A_{\hat{N}} X\right) + \sigma(X, Y) U + ((\nabla_X u) Y + \sigma(X, fY)) \hat{N} \tag{3.1}$$

$$\left(\widetilde{\nabla}_X \eta\right) Y = (\nabla_X v) Y - \lambda\sigma(X, Y) \tag{3.2}$$

$$\widetilde{\nabla}_X \xi = \nabla_X V - \lambda A_{\hat{N}} X + (\sigma(X, V) + X\lambda)\hat{N} \tag{3.3}$$

Proof. Consider

$$\begin{aligned}
\left(\widetilde{\nabla}_X \phi\right) Y &= \left(\widetilde{\nabla}_X \phi Y\right) - \phi\left(\widetilde{\nabla}_X Y\right) \\
&= \widetilde{\nabla}_X\left(fY + u(Y)\hat{N}\right) - \phi\left(\nabla_X Y + \sigma(X, Y)\hat{N}\right) \\
&= \widetilde{\nabla}_X (fY) + \widetilde{\nabla}_X\left(u(Y)\hat{N}\right) - \phi(\nabla_X Y) - \sigma(X, Y)\phi\left(\hat{N}\right)
\end{aligned}$$

$$\begin{aligned} &= \nabla_X (fY) + \sigma(X, fY)\hat{N} + u(Y)\left(\widetilde{\nabla}_X \hat{N}\right) + \left(\widetilde{\nabla}_X u(Y)\right)\hat{N} - f(\nabla_X Y) \\ &\quad -u(\nabla_X Y)\hat{N} + \sigma(X, Y)U \end{aligned}$$

which gives

$$\left(\widetilde{\nabla}_X \phi\right) Y = (\nabla_X f) Y - u(Y)\left(A_{\hat{N}} X\right) + \sigma(X, Y)U + ((\nabla_X u)Y + \sigma(X, fY))\hat{N}$$

Also we have,

$$\begin{aligned} \left(\widetilde{\nabla}_X \eta\right) Y &= \widetilde{\nabla}_X \eta(Y) - \eta\left(\widetilde{\nabla}_X Y\right) \\ &= \widetilde{\nabla}_X (v(Y)) - \eta\left(\nabla_X Y + \sigma(X, Y)\hat{N}\right) \\ &= \nabla_X (v(Y)) + \sigma(X, v(Y))\hat{N} - \eta(\nabla_X Y) - \sigma(X, Y)\eta\left(\hat{N}\right) \\ &= \nabla_X (v(Y)) - v(\nabla_X Y) - \lambda\sigma(X, Y) \end{aligned}$$

$$\left(\widetilde{\nabla}_X \eta\right) Y = (\nabla_X v) Y - \lambda\sigma(X, Y)$$

Further, consider

$$\begin{aligned} \widetilde{\nabla}_X \xi &= \nabla_X \xi + \sigma(X, \xi)\hat{N} \\ &= \nabla_X V + \nabla_X \lambda\hat{N} + \sigma(X, V)\hat{N} \\ &= \nabla_X V + \lambda\nabla_X \hat{N} + (X\lambda)\hat{N} + \sigma(X, V)\hat{N} \end{aligned}$$

which gives

$$\widetilde{\nabla}_X \xi = \nabla_X V - \lambda A_{\hat{N}} X + (\sigma(X, V) + X\lambda)\hat{N}$$

□

Theorem 1.: *Let M be a non-invariant hypersurface with $(f, g, u, v, \lambda)-$structure of $\delta-$Lorentzian trans-Sasakian manifold $\widetilde{M}$. Then*

$$\sigma(X, \xi)U = \alpha\delta f^2 X - \alpha\delta u(X)U + \delta\beta f(X) + f(\nabla_X \xi) \tag{3.4}$$

$$u(\nabla_X \xi) = -\alpha\delta u(fX) - \beta\delta u(X) \tag{3.5}$$

Proof. :Consider

$$\begin{aligned} \left(\widetilde{\nabla}_X \phi\right)\xi &= \widetilde{\nabla}_X \phi\xi - \phi\left(\widetilde{\nabla}_X \xi\right) \\ &= -\phi(\delta(-\alpha\phi X - \beta(X + \eta(X)\xi))) \end{aligned}$$

or

$$\left(\widetilde{\nabla}_X \phi\right)\xi = \alpha\delta f^2 X + \alpha\delta u(fX)\hat{N} - \alpha\delta u(X)U + \delta\beta f(X) + \beta\delta u(X)\hat{N}$$

and we know that the relation

$$\begin{aligned} \left(\widetilde{\nabla}_X \phi\right)\xi &= \widetilde{\nabla}_X \phi\xi - \phi\left(\widetilde{\nabla}_X \xi\right) \\ &= -\phi\left(\nabla_X \xi + \sigma(X, \xi)\hat{N}\right) \\ &= -\phi(\nabla_X \xi) + \sigma(X, \xi)U \\ &= -f(\nabla_X \xi) - u(\nabla_X \xi)\hat{N} + \sigma(X, \xi)U \end{aligned}$$

from above two equation,we have

$$\begin{aligned} -f(\nabla_X \xi) - u(\nabla_X \xi)\hat{N} + \sigma(X, \xi)U &= \alpha\delta f^2 X + \alpha\delta u(fX)\hat{N} - \alpha\delta u(X)U \\ &\quad +\delta\beta f(X) + \beta\delta u(X)\hat{N} \end{aligned}$$

Notes

Euating tangential and normal parts on both side, we get

$$\sigma(X,\xi)U = \alpha\delta f^2 X - \alpha\delta u(X)U + \delta\beta f(X) + f(\nabla_X \xi)$$

and

$$u(\nabla_X \xi) = -\alpha\delta u(fX) - \beta\delta u(X)$$

□

Theorem 2.: *Let M be a non-invariant hypersurface with $(f,g,u,v,\lambda)-$structure of $\delta-$Lorentzian trans-Sasakian manifold $\widetilde{M}$. Then*

$$\begin{aligned}(\nabla_X f)Y &= u(Y)(A_{\hat{N}}X) - \sigma(X,Y)U + \alpha(g(X,Y)V - \delta v(Y)X) \\ &\quad +\beta(g(fX,Y)V - \delta v(Y)fX)\end{aligned} \quad (3.6)$$

$$(\nabla_X u)Y = \alpha\lambda g(X,Y) + \beta(\lambda g(fX,Y) - \delta u(X)v(Y)) - \sigma(X,fY) \quad (3.7)$$

$$\nabla_X V = \lambda A_{\hat{N}}X - \delta\alpha fX - \delta\beta(X + v(X)V) \quad (3.8)$$

$$\sigma(X,V) = -\delta\alpha u(X) - \delta\lambda\beta v(X) - X\lambda \quad (3.9)$$

$$(\nabla_X v)Y = \lambda\sigma(X,Y) + \alpha g(fX,Y) + \beta\{g(X,Y) - \delta v(X)v(Y)\} \quad (3.10)$$

Proof. : Using $(2.8), (2.10)$ in (2.5) and (3.1) we obtain

$$\begin{aligned}&(\nabla_X f)Y - u(Y)(A_{\hat{N}}X) + \sigma(X,Y)U + ((\nabla_X u)Y + \sigma(X,fY))\hat{N} \\ =\ &\alpha g(X,Y)V + \alpha\lambda g(X,Y)\hat{N} - \alpha\delta v(Y)X + \beta g(fX,Y)V \\ &+\beta\lambda g(fX,Y)\hat{N} - \beta\delta v(Y)fX - \beta\delta v(Y)u(X)\hat{N}\end{aligned}$$

Equating tangential and normal parts in the above equation, we get (3.6) and (3.7) respectively.

Using equation $(2.6), (2.8)$ and (2.11) we get,

$$\widetilde{\nabla}_X \xi = -\delta\alpha fX - \delta\alpha u(X)\hat{N} - \delta\beta X - \delta\beta v(X)V - \lambda\delta\beta v(X)\hat{N}$$

and also we have,

$$\widetilde{\nabla}_X \xi = \nabla_X V - \lambda A_{\hat{N}}X + (\sigma(X,V) + X\lambda)\hat{N}$$

Equating the tangential and normal part of the above two equation, we get (3.8) and (3.9).

In last using $(2.7), (2.8)$ and (3.2) we get (3.10)

□

Theorem 3. :*Let M be a non-invariant hypersurface with $(f,g,u,v,\lambda)-$structure of $\delta-$Lorentzian trans-Sasakian manifold $\widetilde{M}$. Then*

$$\begin{aligned}(\widetilde{\nabla}_X \phi)Y &= \alpha(g(X,Y)V - \delta v(Y)X) + \beta(g(fX,Y)V - \delta v(Y)fX) \\ &\quad + \{\alpha(\lambda g(X,Y)) + \beta(\lambda g(fX,Y) - \delta u(X)v(Y))\}\hat{N}\end{aligned} \quad (3.11)$$

Proof. :Consider

$$\begin{aligned}(\widetilde{\nabla}_X \phi)Y &= (\widetilde{\nabla}_X \phi Y) - \phi(\widetilde{\nabla}_X Y) \\ &= \widetilde{\nabla}_X\left(fY + u(Y)\hat{N}\right) - \phi\left(\nabla_X Y + \sigma(X,Y)\hat{N}\right) \\ &= \widetilde{\nabla}_X(fY) + \widetilde{\nabla}_X\left(u(Y)\hat{N}\right) - \phi(\nabla_X Y) - \sigma(X,Y)\phi\left(\hat{N}\right) \\ &= \nabla_X(fY) + \sigma(X,fY)\hat{N} + u(Y)\left(\widetilde{\nabla}_X\hat{N}\right) + (\widetilde{\nabla}_X u(Y))\hat{N} - f(\nabla_X Y) \\ &\quad -u(\nabla_X Y)\hat{N} + \sigma(X,Y)U\end{aligned}$$

Notes

which gives

$$\left(\widetilde{\nabla}_X \phi\right) Y = \left(\nabla_X f\right) Y - u\left(Y\right)\left(A_{\hat{N}} X\right) + \sigma\left(X, Y\right) U + \left(\left(\nabla_X u\right) Y + \sigma\left(X, fY\right)\right) \hat{N} \tag{3.12}$$

and we have also from (3.6) and (3.7)

$$\begin{aligned} \left(\nabla_X f\right) Y &= u\left(Y\right)\left(A_{\hat{N}} X\right) - \sigma\left(X, Y\right) U + \alpha\left(g\left(X, Y\right) V - \delta v\left(Y\right) X\right) \\ &\quad + \beta\left(g\left(fX, Y\right) V - \delta v\left(Y\right) fX\right) \end{aligned} \tag{3.13}$$

and

$$\left(\nabla_X u\right) Y = \alpha\lambda g\left(X, Y\right) + \beta\left(\lambda g\left(fX, Y\right) - \delta u\left(X\right) v\left(Y\right)\right) - \sigma\left(X, fY\right) \tag{3.14}$$

now equation (3.12), (3.13), and (3.14) enables us to deduce (3.11) □

Theorem 4. :*Let M be a totally umbilical noninvariant hypersurface with (f, g, u, v, λ) −structure of δ−Lorentzian trans-Sasakian manifold. Then it is totally geodesic if and only if*

$$\delta\alpha u\left(X\right) + \delta\lambda\beta v\left(X\right) + X\lambda = 0 \tag{3.15}$$

Proof. :From equation (2.6) we have,

$$\widetilde{\nabla}_X \xi = \delta\left(-\alpha\phi X - \beta\left(X + \eta\left(X\right)\xi\right)\right)$$

Using (2.8) and (2.11) in above equation we get

$$\begin{aligned} \widetilde{\nabla}_X \xi &= -\delta\alpha f X - \delta\alpha u\left(X\right)\hat{N} - \delta\beta X - \delta\beta v\left(X\right) V \\ &\quad -\delta\lambda\beta v\left(X\right)\hat{N} \end{aligned}$$

Equating the normal parts of above equation and equation (3.3) we obtain

$$\sigma\left(X, V\right) = -\delta\alpha u\left(X\right) - \delta\lambda\beta v\left(X\right) - X\lambda \tag{3.16}$$

If M is totally umbilical, then $A_{\hat{N}} = \zeta I$
where ζ is Kahlerian metric

$$\sigma\left(X, Y\right) = g\left(A_{\hat{N}} X, Y\right) = g\left(\zeta X, Y\right) = \zeta g\left(X, Y\right)$$

$$\sigma\left(X, V\right) = \zeta g\left(X, V\right) = \zeta v\left(X\right) \tag{3.17}$$

Then, from (3.13) and (3.14) we get

$$\delta\alpha u\left(X\right) + \delta\lambda\beta v\left(X\right) + X\lambda + \zeta v\left(X\right) = 0$$

If M is totally geodesic, i.e. $\zeta = 0$
then,

$$\delta\alpha u\left(X\right) + \delta\lambda\beta v\left(X\right) + X\lambda = 0$$

□

Theorem 5. :*Let M be a non-invariant hypersurface with (f, g, u, v, λ) −structure of δ−Lorentzian trans-Sasakian manifold $\widetilde{M}$. If U is parallel, then we have*

$$f\left(A_{\hat{N}} X\right) + \delta\lambda\alpha X + \delta\lambda\beta f X = 0 \tag{3.18}$$

Proof. :Consider

$$\left(\widetilde{\nabla}_X \phi\right)\hat{N} = \widetilde{\nabla}_X\left(\phi\hat{N}\right) - \phi\left(\widetilde{\nabla}_X \hat{N}\right)$$

Using (2.9), (2.19) and (2.20) we get

$$\left(\widetilde{\nabla}_X \phi\right)\hat{N} = -\nabla_X U + f\left(A_{\hat{N}} X\right) \tag{3.19}$$

and from (2.5) we write

$$\left(\widetilde{\nabla}_X \phi\right)\hat{N} = \alpha\left\{g\left(X, \hat{N}\right)\xi - \delta\eta\left(\hat{N}\right) X\right\} + \beta\left\{g\left(\phi X, \hat{N}\right)\xi - \delta\eta\left(\hat{N}\right)\phi X\right\}$$

Notes

Using (2.10) in above equation we get,

$$\left(\widetilde{\nabla}_X\phi\right)\hat{N} = -\delta\alpha\lambda X - \beta\delta\lambda\phi X \tag{3.20}$$

Using (3.19) and (3.20), we get

$$\begin{aligned}\nabla_X U &= \delta\alpha\lambda X + \beta\delta\lambda\phi X + f\left(A_{\hat{N}}X\right)\\ &= \delta\alpha\lambda X + f\left(A_{\hat{N}}X\right) + \beta\delta\lambda f X + \beta\delta\lambda u\left(X\right)\hat{N}\end{aligned}$$

If U is parallel,then $\nabla_X U = 0$

$$\delta\alpha\lambda X + f\left(A_{\hat{N}}X\right) + \beta\delta\lambda f X + \beta\delta\lambda u\left(X\right)\hat{N} = 0$$

Now, equating the tangential part, we have the result. □

Notes

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