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# The Question of Space-Time Singularities in General Relativity and Einstein's Errors

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### THE QUE STION OF SPACETIMES INGULARITIES INGENERAL RELATIVITY AND EINSTEINSER RORS

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## The Question of Space-Time Singularities in General Relativity and Einstein's Errors

#### C. Y. Lo

Abstract- In general relativity, the existence of space-time singularities plays a central role on the notion of black holes and the expanding universe. However, these two speculations have not been firmly verified in spite of the efforts of generations of physicists. The existence of space-time singularities is due to the spacetime singularity theorems of Hawking and Penrose, whose implicit physical assumption is the general validity of  $E = mc^2$  that leads to the unique sign for all the couplings. However, recently it is found that such an assumption is not supported by various experiments. In particular, the electromagnetic energy is not equivalent to mass. In fact, the Einstein equation with massive sources has no dynamic solution unless the gravitational energy-stress tensor with an anti-gravity coupling is added to the source. Moreover, for the electromagnetic wave to have a physically gravitational effect, the related Einstein equation must additionally have a photonic energy-stress tensor with an antigravity coupling. Thus, Einstein's understanding of general relativity is inadequate. Since the energy conditions in the space-time singularity theorems actually cannot be satisfied in physics, these mathematical theorems are actually irrelevant to physics. Their claims have been proven as nonsense in physics. Further more, recognizing the nonexistence of dynamic solution for the Einstein equation is the first step to the unification of gravitation and electromagnetism. Many overlooked the crucial charge-mass interaction. Due to inadequate understanding of the principle of causality and non-linear mathematics, Einstein failed to show his unification. He has made three major errors: 1) He is mistaken that the Einstein equation has dynamic solutions. 2) He speculated that  $E = mc^2$  was generally valid. 3) He invalidly rejected repulsive gravitation, which is supported by experiments. Also, the Physical Review, the Proceeding of the Royal Society A, the Annals of Physics and the Chinese Physics. accepted the space-time singularity theorems as valid because theorists make errors in the non-linear mathematics and physics.

*Keywords:* anti-gravity coupling; dynamic solution; gravitational radiation; repulsive gravitation; principle of causality.

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"Unthinking respect for authority is the greatest enemy of truth." – A. Einstein

#### I. INTRODUCTION

n physics, the existence of singularities suggests problematic assumptions. Nevertheless, in current theory of general relativity, the existence of spacetime singularities plays a central role on the notion of black holes and the expanding universe (see Appendix A). However, these two speculations have not been firmly verified in spite of the efforts of generations of physicists.<sup>1)</sup> Thus, one may question the general validity of the theory of general relativity in spite of its earlier success.

The existence of space-time singularities is due to the spacetime singularity theorems of Hawking and Penrose [1]. The mathematical validity of these theorems is highly reliable because Penrose have won his arguments in mathematics against the theoretical physicist, E. M. Lifshitz [2] in a long dispute. Moreover, the static Einstein equation in general relativity has passed various tests with surprises. Since the physical assumptions on the energy conditions of these theorems seem to be very natural, there would be little doubt on the validity of these assumptions.

An implicit assumption on these singularity theorems is that all the coupling constants have the same sign.<sup>2)</sup> Such an assumption would be necessarily valid if the formula  $E = mc^2$  is unconditional. However, recently it has been found that the general validity of this formula is questionable. In contrast to Einstein's prediction [3], a piece of heated-up metal actually has reduced weight [4]. Moreover, a charged capacitor also has reduced weight, <sup>3)</sup> which is proportional to the square of the difference in the electric potential of the capacitor [5, 6]. Theoretically, it has been found that the equivalence between mass and the electromagnetic energy is in conflict with the Einstein equation because the electromagnetic energy-stress tensor is traceless [7]. Thus, the assumption of unique coupling sign is questionable.

In this paper, it will be shown that the assumption of unique coupling sign is, indeed, not valid for the dynamic case. Therefore, the space-time singularity theorems of Hawking and Penrose are actually irrelevant to physics. A root of the problem is, however, that many relativists have lost their touch with experiments, in addition to inadequacy in non-linear mathematics.

#### II. The Space-Time Singularity Theorems and the Assumption of Unique Sign of Couplings

Let us examine the energy conditions in the singularity theorems. These theorems [1] are listed as the following:

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Theorem 1. Let (M,  $g_{ab}$ ) be a globally hyperbolic spacetime with  $R_{ab}\xi^a\xi^b \ge 0$  for all timelike  $\xi^a$ , which will be the case if Einstein equation is satisfied with the strong energy condition holding for matter. Suppose there exists a smooth (or at least C<sup>2</sup>) spacelike Cauchy surface  $\Sigma$  for which the trace of the extrinsic curvature (for the past directed normal geodesic congruence) satisfies  $0 > C \ge K$  everywhere C is a constant. Then no past directed timelike curve from  $\Sigma$  can have length greater than 3/|C|. In particular, all past directed timelike geodesic are incomplete.

Theorem 2. Let (M,  $g_{ab}$ ) be a strongly causal spacetime with  $R_{ab}\xi^{a}\xi^{b} \ge 0$  for all timelike  $\xi^{a}$ , as will be the case if Einstein's equation is satisfied with the strong energy condition holding for matter. Suppose there exists a compact, edgeless, achronal smooth spacelike hypersurface S such that for the past directed normal geodesic congruence form S we have 0 > K everywhere on S. Let C denote the maximum value for K, so 0 > C $\ge K$  everywhere on S. Then at least one inextendible past directed timelike geodesic from S has length no greater that 3/|C|.

Theorem 3. Let (M,  $g_{ab}$ ) be a connected, globally hyperbolic spacetime with a noncompact Cauchy surface  $\Sigma$ . Suppose  $R_{ab}k^ak^b \geq 0$  for all null  $k^a$ , as will be the case if (M,  $g_{ab}$ ) is a solution of Einstein's equation with matter satisfying the weak or strong energy condition. Suppose, further, that M contains a trapped

surface T. Let  $0 > \theta_0$  denote the maximum value of  $\theta$  for both sets of orthogonal geodesic on T. Then at least one inextendible future directed orthogonal null geodesic from T has affine length no greater than  $2/|\theta_0|$ .

Theorem 4. Suppose a spacetime (M,  $g_{ab}$ ) satisfies the following four conditions. (1)  $R_{ab}v^av^b \ge 0$  for all timelike and null  $v^a$ , as will be the case if Einstein's equation is satisfied with the strong energy condition holding for matter. (2) The timelike and null generic conditions are satisfied. (3) No closed timelike curve exists. (4) At least one of the three properties holds: (a) (M,  $g_{ab}$ ) posses a compact achronal set without edge [i.e., (M,  $g_{ab}$ ) is a closed universe], (b) (M,  $g_{ab}$ ) possesses a trapped surface, or (c) there exists a point p  $\epsilon$  M such that the expansion of the future (or past) directed null geodesics emanating from p becomes negative along each geodesic in this congruence. Then (M,  $g_{ab}$ ) must contain at least one incomplete timelike or null geodesic.

Originally, the energy condition is related to the energy-momentum tensor  $T_{ab}$ . According to the Einstein equation [1]

$$G_{ab} \equiv R_{ab} - (1/2) g_{ab} R = 4\pi T_{ab},$$
 (1)

one would have

$$R_{ab} = 8\pi [T_{ab} - (1/2)g_{ab} T]$$
 where  $T = g^{ab}T_{ab}$  (2)

Then,

$$R_{ab}\xi^{a}\xi^{b} = 8\pi [T_{ab} - (1/2)g_{ab} T]\xi^{a}\xi^{b} = 8\pi [T_{ab}\xi^{a}\xi^{b} + (1/2)T], \text{ for a unit timelike } \xi^{a}$$
(3)

It is believed that for all physically reasonable classical matter the energy condition is non-negative, i.e.,

$$\mathsf{T}_{ab}\,\xi^a\xi^b \ge 0 \tag{4}$$

for all timelike  $\xi^a$ . This assumption is known as the weak energy condition. However, it also seems physically reasonable that the stress of matter will not become so large and negative as to make the right-hand side of eq. (3) negative. This assumption,

$$T_{ab}\,\xi^a\xi^b \ge -(1/2)T\tag{5}$$

for all unit timelike unit vector  $\xi^a$ , is known as the strong energy condition. An implicit assumption of these energy-conditions (3)-(5) is that all the coupling constants have the same sign. However, as will be shown such an assumption leads to the invalidity of the Einstein equation because of the non-existence of dynamic solutions.

To illustrate this, we shall first show examples that for the case of gravitational waves; there is no bounded dynamic solution. Now, consider a well-known metric obtained by Bondi, Pirani, & Robinson [8] as follows:

$$ds^{2} = e^{2\varphi} \left( d\tau^{2} - d\xi^{2} \right) - u^{2} \begin{bmatrix} \cosh 2\beta \left( d\eta^{2} + d\zeta^{2} \right) \\ + \sinh 2\beta \cos 2\theta \left( d\eta^{2} - d\zeta^{2} \right) \\ -2\sinh 2\beta \sin 2\theta d\eta d\zeta \end{bmatrix}$$
(6a)

where  $\varphi$ ,  $\beta$  and  $\theta$  are functions of u ( $\tau$  -  $\xi$ ). It satisfies the differential equation (i.e., their Eq. [2.8]),

$$2\phi' = u\left(\beta'^2 + \theta'^2 \sinh^2 2\beta\right) \tag{6b}$$

which is a special cases of  $G_{\mu\nu} = 0$ . They claimed this is a wave from a distant source and weak gravity invalid. The metric is irreducibly unbounded because of the factor  $u^2$ . And linearization of (6b) does not make sense since u is not bounded.

Moreover, when gravity is absent, it is necessary to have  $\phi = \sinh 2\beta = \sin 2\theta = 0$ . These would reduce (6a) to

$$ds^{2} = (d\tau^{2} - d\xi^{2}) - u^{2}(d\eta^{2} + d\zeta^{2})$$
 (6c)

However, this metric is not equivalent to the flat metric. Thus, metric (6c) violates the principle of causality (see Appendix B).

This challenges the view that both Einstein's notion of weak gravity and his covariance principle are

valid. These conflicting views are supported respectively by the editors of the "Royal Society Proceedings A" and the "Physical Review D"; thus there is no general consensus. Note that Einstein's covariance principle has been proven invalid with counter examples [9].

#### The Non-Existence of a Bounded III. Dynamic Solution for a Two-Body PROBLEM IN GENERAL RELATIVITY

According to the principle of causality, weak sources would produce a weak field, i.e.,

$$g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}$$
, where  $1 >> |\gamma_{\mu\nu}|$  (7)

and  $\eta_{uv}$  is the flat metric. However, eq. (7) is valid, only if the Einstein equation is valid. Since the strength of a source can always be reduced, to show the nonexistence of a dynamic solution, it is sufficient to show the case of weak gravity.

Unfortunately, many believe that condition (7) for weak gravity is always valid for the Einstein equation. They believed that an approximate weak solution can be derived through the approach of the field equation being linearized. The linearized Einstein equation with the linearized harmonic gauge  $\partial^{\mu} \gamma_{\mu\nu} = 0$  is

$$\frac{1}{2}\partial^{\alpha}\partial_{\alpha}\overline{\gamma}_{\mu\nu} = \kappa T_{\mu\nu} \quad \text{where} \quad \overline{\gamma}_{\mu\nu} = \gamma_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}(\eta^{\text{cd}}\gamma_{\text{cd}}), \tag{8}$$

Note that we have

$$G_{\mu\nu} = G_{\mu\nu}^{(1)} + G_{\mu\nu}^{(2)} \quad \text{and} \qquad G^{(1)}{}_{\mu\nu} = \frac{1}{2} \partial^{\alpha} \partial_{\alpha} \overline{\gamma}_{\mu\nu} + H^{(1)}{}_{\mu\nu} , \qquad (9)$$

where

$$H^{(1)}_{\mu\nu} = -\frac{1}{2}\partial^{\alpha} [\partial_{\mu}\overline{\gamma}_{\nu\alpha} + \partial_{\nu}\overline{\gamma}_{\mu\alpha}] + \frac{1}{2}\eta_{\mu\nu}\partial^{\alpha}\partial^{\beta}\overline{\gamma}_{\alpha\beta}$$

The linearized vacuum Einstein equation means  $G^{(1)}_{\mu\nu}[\gamma^{(1)}_{\alpha\beta}] = 0$ . Thus, to have a solution of the second order we must correct  $\gamma^{(1)}_{\mu\nu}$  by adding to it the term  $\gamma^{(2)}_{\mu\nu}$  that satisfies

$$G_{\mu\nu}^{(1)}[\gamma^{(2)}{}_{\alpha\beta}] + G_{\mu\nu}^{(2)}[\gamma_{\alpha\beta}] = 0, \quad \text{where} \quad \gamma_{\mu\nu} = \gamma^{(1)}{}_{\mu\nu} + \gamma^{(2)}{}_{\mu\nu} \tag{10}$$

which is the correct form of eq. (4.4.52) in Wald's book [1] (Wald did not distinguish  $\gamma_{\mu\nu}$  from  $\gamma^{(1)}_{\mu\nu}$ ).<sup>4)</sup> However, detailed calculation shows that this equation does not have a solution for the dynamic case [10-13].<sup>5)</sup> In fact, as shown by the example in the last section, for a

1

dynamic case, the linealized equation and the Einstein equation are independent equations [14].

It was believed that the linear Maxwell-Newton Approximation [11] (or the linearized Einstein equation [15, 16])

appears to be justified and the faith on the dynamic

solutions maintained. It was not recognized until 1995

[11] that such a symptom of divergence shows the

gravity. Consider,  $G^{(2)}_{\mu\nu}$  ( $G_{\mu\nu} \equiv G_{\mu\nu}^{(1)} + G_{\mu\nu}^{(2)}$ ) is at least of

second order in terms of the metric elements. For an

absence of bounded physical dynamic solutions.

$$\frac{1}{2}\partial^{\circ}\partial_{c}\bar{\gamma}_{\mu\nu} = K T(m)_{\mu\nu}, \text{ where } \bar{\gamma}_{\mu\nu} = \gamma_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}(\eta^{cd}\gamma_{cd})$$
(11a)

and

$$\overline{\gamma}_{\mu\nu}(\mathbf{x}^{i}, \mathbf{t}) = \frac{K}{2\pi} \int \frac{1}{R} \mathsf{T}_{\mu\nu}[\mathbf{y}^{i}, (\mathbf{t} - \mathbf{R})] \mathrm{d}^{3}\mathbf{y}, \quad \text{where} \quad \mathbf{R}^{2} = \sum_{i=1}^{3} (x^{i} - y^{i})^{2}.$$
(11b)

provides the first-order approximation for the Einstein equation (1). However, this belief was verified for the static case only.

The Cauchy data of eq.(1) must satisfy four constraint equations,  $G_{\mu t} = -KT(m)_{\mu t}$  ( $\mu = x, y, z, t$ ) since G<sub>ut</sub> contains only first-order time derivatives [17]. This shows that (11a) would be dynamically incompatible with Einstein equation (1).

In 1957, Fock [18] pointed out that, in harmonic coordinates, there are divergent logarithmic deviations from expected linearized behavior of the radiation. This was misinterpreted to mean merely that the contribution of the complicated nonlinear terms in the Einstein equation cannot be dealt with satisfactorily following this method and that another approach is needed.

D) Subsequently, vacuum solutions that do not involve logarithmic deviation were founded by Bondi, Pirani & Robinson [8] in 1959. Thus, the incorrect interpretation

Equation (11) shows that a gravitational wave is bounded and is related to the dynamic of the source. These are useful to prove that eq. (11), as the first-order approximation for a dynamic problem, is incompatible with the Einstein equation (1). According to the principle of causality, it is sufficient to (consider the case of weak isolated system located near the origin of the space coordinate system,  $G^{(2)}_{\mu t}$  at large r (= [x<sup>2</sup> + y<sup>2</sup> + z<sup>2</sup>]<sup>1/2</sup>) is of O(K<sup>2</sup>/r<sup>2</sup>) [1, 17, 19].

One may obtain some general characteristics of a dynamic solution for an isolated system as follows:

 The characteristics of some physical quantities of an isolated system:

For an isolated system consisting of particles with typical mass  $\overline{M}$ , separation  $\overline{r}$ , and velocities  $\overline{v}$ , Weinberg [17] estimated, <sup>6)</sup> the power radiated at a frequency  $\omega$  of order  $\overline{v}/\overline{r}$  will be of order

$$\mathsf{P} \approx \kappa (\overline{v} / \overline{r})^6 \overline{M}^2 \overline{r}^4 \text{ or } \mathsf{P} \approx \overline{M} \overline{v}^8 / \overline{r}, \qquad (12)$$

since  $\kappa \overline{M} / \overline{r}$  is of order  $\overline{v}^2$ . The typical deceleration  $\overline{a}_{rad}$  of particles in the system owing this energy loss is given by the power P divided by the momentum  $\overline{M} \ \overline{v}$ , or  $\overline{a}_{rad} \approx \overline{v}^7 / \overline{r}$ . This may be compared with the accelerations computed in Newtonian mechanics, which are of order  $\overline{v}^2 / \overline{r}$ , and with the post-Newtonian correction of  $\overline{v}^4 / \overline{r}$ . Since radiation reaction is smaller

than the post-Newtonian effects by a factor  $\overline{v}^{3}$ , if  $\overline{v} << c$ , the velocity of light, the neglect of radiation reaction is perfectly justified. This allows us to consider the motion of a particle in an isolated system as almost periodic.

Consider two particles of equal mass with an almost circular orbit in the x-y plane whose origin is the center of the circle (i.e., the orbits are a circle if radiation is neglected). Thus, the principle of causality implies that the metric  $g_{\mu\nu}$  is weak and very close to the flat metric at distance far from the source and that  $g_{\mu\nu}$  (x, y, z, t') is an almost periodic function of t' (= t - r/c).

2) The expansion of a bounded dynamic solution g  $_{\mu\nu}$  for an isolated weak gravitational source:

According to eq. (11), a first-order approximation of metric  $g_{\mu\nu}$  (x, y, z, t') is bounded and almost periodic since  $T_{\mu\nu}$  is. Physically, the principle of causality requires  $g_{\mu\nu}$  to be almost periodic in time since the motion of a source particle is. Such a metric  $g_{\mu\nu}$  is asymptotically flat for a large distance r, and the expansion of a bounded dynamic solution is:

$$g_{\mu\nu}(n^{x}, n^{y}, n^{z}, r, t') = \eta_{\mu\nu} + \sum_{k=1}^{\infty} f_{\mu\nu}{}^{(k)}(n^{x}, n^{y}, n^{z}, t')/r^{k}, \text{ where } n^{\nu} = x^{\nu}/r.$$
(13a)

3) The non-existence of dynamic solutions:

It follows expansion (13a) that the non-zero time average of  $G^{(1)}_{\ \mu t}$  would be of O(1/r<sup>3</sup>) due to

$$\partial_{\mu}n^{\nu} = (\delta^{\nu}{}_{\mu} + n^{\nu}n_{\mu})/r, \qquad (13b)$$

since the term of O(1/r<sup>2</sup>), being a sum of derivatives with respect to t', can have a zero time-average. If  $G^{(2)}_{\mu t}$  is of O(K<sup>2</sup>/r<sup>2</sup>) and has a nonzero time-average, consistency

can be achieved only if another term of time-average  $O(K^2/r^2)$  at vacuum be added to the source of the Einstein equation (1). Note that there is no plane-wave solution for  $G_{\mu\nu} = 0$  [20].

It will be shown that there is no dynamic solution for the Einstein equation with a massive source. Let us define

$$\gamma_{\mu\nu} = \gamma^{(1)}{}_{\mu\nu} + \gamma^{(2)}{}_{\mu\nu} \; ; \qquad \quad \overline{\gamma}^{(i)}{}_{\mu\nu} = \gamma^{(i)}{}_{\mu\nu} - \frac{1}{2} \, \eta_{\mu\nu} \, (\gamma^{(i)}{}_{cd} \, \eta^{cd}), \qquad \text{where } i = 1, 2 \; ;$$

and

$$\frac{1}{2}\partial^{\alpha}\partial_{\alpha}\bar{\gamma}^{(1)}{}_{\mu\nu} = K T(m)_{\mu\nu}.$$
(14)

Then  $\bar{\gamma}^{(1)}{}_{\mu\nu}$  is of a first-order; and  $\gamma^{(2)}{}_{\mu\nu}$  is finite. On the other hand, from the Einstein equation (1), one has

$$\frac{1}{2} \partial^{\alpha} \partial_{\alpha} \bar{\gamma}^{(2)}{}_{\mu\nu} + \mathsf{H}^{(1)}{}_{\mu\nu} + \mathsf{G}^{(2)}{}_{\mu\nu} = 0$$
(15)

Note that, for a dynamic case, equation (15) may not be satisfied. If (14) is a first-order approximation,  $G^{(2)}_{\mu\nu}$  has a nonzero time-average of  $O(K^2/r^2)$  [1] (but  $[\partial^{\alpha}\partial_{\alpha}\,\bar{\gamma}^{\ (2)}_{\mu\nu}/2 + H^{(1)}_{\mu\nu}]$  would have zero time-average); and thus  $\bar{\gamma}^{\ (2)}_{\mu\nu}$  cannot have a solution.

However, if  $\bar{\gamma}^{(2)}_{\mu\nu}$  is also of the first-order of K, one cannot estimate  $G^{(2)}_{\mu\nu}$  by assuming that  $\bar{\gamma}^{(1)}_{\mu\nu}$  provides a first-order approximation. For example, equation (11) does not provide the first approximation

for the static Schwarzschild solution, although it can be transformed to a form such that (11) provides a first-order approximation [15]. According to eq.(9),  $\bar{\gamma}^{(2)}_{\mu\nu}$  will be a second order term if the sum H<sup>(1)</sup><sub>µv</sub> is of second order. From (9), this would require  $\partial^{\mu}\bar{\gamma}_{\mu\nu}$  being of second order. For weak gravity, it is known that a coordinate transformation would turn  $\partial^{\mu}\bar{\gamma}_{\mu\nu}$  to a second order term [17, 18, 21]. (Eq. [15] implies that  $\partial^{c}\partial_{c}\bar{\gamma}^{(2)}_{\mu\nu} - \partial^{c}[\partial_{\nu}\bar{\gamma}_{\mu c} + \partial_{\mu}\bar{\gamma}_{\nu c}] + \eta_{\mu\nu}\partial^{\alpha}\partial^{\beta}\bar{\gamma}_{\alpha\beta}$  would be of second order.) Thus, it is possible to turn (14) to become an equation for a first-order approximation for weak gravity.

Since it has been proven that (11) necessarily gives a first-order approximation [15], a failure of such a coordinate transformation means only that such a solution is not valid in physics. Moreover, for the dynamic of massive matter, experiment [22] supports the fact that Maxwell-Newton Approximation (11) is related to a dynamic solution of weak gravity [16]. Thus, theoretical considerations as well as experiments eliminate other unverified speculations thought to be possible since 1957.

As shown, the difficulty comes from the assumption of boundedness, which allows the existence of a bounded first-order approximation, which in turn implies that a time-average of the radiative part of  $G^{(2)}_{\mu\nu}$  is non-zero. The present method has an advantage over Fock's approach to obtaining logarithmic divergence [18] for being simple and clear.

In short, according to Einstein's radiation formula, a time average of  $G^{(2)}_{\mu t}$  is non-zero and of  $O(K^2/r^2)$  [17]. Although (11) implies  $G^{(1)}_{\mu t}$  is of order  $K^2$ , its terms of  $O(1/r^2)$  can have a zero time average because  $G^{(1)}_{\mu t}$  is linear on the metric elements. Thus, the Einstein equation (1) in vacuum cannot be satisfied. Nevertheless, a static metric can satisfy (1), since both  $G^{(1)}_{\mu\nu}$  and  $G^{(2)}_{\mu\nu}$  are of  $O(K^2/r^4)$  in vacuum. Note that  $G_{\mu t} = KT(m)_{\mu t}$  are constraints on the initial data.

In conclusion, assuming the existence of dynamic solutions of weak gravity for Einstein equation (1) [8, 18, 21, 23-28] is invalid. This means that general relativity has not yet totally superseded Newtonian gravity [12]. This illustrates also that theorists should not carelessly follow the erroneous and groundless claims of Christodoulou and Klainerman (see Section 6).

Nevertheless, because of inadequacy in mathematics, many theorists following Einstein's error, claimed or believed that there are dynamic solutions. A good example is that Misner, Thorne and Wheeler [19] claimed they have an explicit bounded dynamic solution of the following form,

$$-ds^{2} = c^{2}dt^{2} - dx^{2} - L^{2}\left(e^{2\beta}dy^{2} + e^{-2\beta}dz^{2}\right)$$
(16)

where L = L(u),  $\beta = \beta(u)$ , u = ct - x, and c is the light speed. Then, the Einstein equation  $G_{\mu\nu} = 0$  becomes

$$\frac{d^2L}{du^2} + L \left(\frac{d\beta}{du}\right)^2 = 0 \tag{17}$$

Misner et al. [19] claimed that Eq. (17) has a bounded approximate solution, compatible with a linearization of metric (16). However, it has been shown with undergraduate mathematics [29] that Misner et al. are incorrect and Eq. (17) does not have a physical solution that satisfies Einstein's requirement on weak gravity. In fact, L(u) is unbounded even for a very small  $\beta(u)$ .

On the other hand, from the Maxwell-Newton approximation in vacuum, Einstein [30] obtained a solution as follows:

$$-ds^{2} = c^{2}dt^{2} - dx^{2} - (1 + 2\phi)dy^{2} - (1 - 2\phi)dz^{2}$$
(18)

where  $\phi$  is a bounded function of  $u \ (= ct - x)$ . Note that metric (18) is the linearization of metric (16) if  $\phi = \beta(u)$ .

Thus, the waves illustrate that the linearization is not valid for the dynamic case when gravitational waves are involved.

Moreover, Misner et al. [19] also make other serious errors in physics as shown in their eq. (40.14) for the proper time measured by an earth-based clock, but other theorists such as Wald [1] and Weinberg [17] did not make the same mistake.

#### IV. The Anti-Gravity Coupling and Invalidity of the Space-Time Singularity Theorems to Physics

From the above analysis, there is a conflict between the Einstein equation, which has no dynamic solution and its linearized equation, which has a dynamic solution. The conflict is due to that the second order terms  $G^{(2)}_{\mu\nu}$  cannot be eliminated in the Non-linear Einstein equation. Thus, a simple solution is the 1995 update of the Einstein equation [10] as follows:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = K [T(m)_{\mu\nu} - t(g)_{\mu\nu}], \qquad (19)$$

where  $t(g)_{\mu\nu}$  is the energy-stress tensors for gravity. Then, from (19), the equation in vacuum is

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -K t(g)_{\mu\nu}$$
(19')

Note that  $t(g)_{\mu\nu}$  is equivalent to  $G^{(2)}{}_{\mu\nu}$  (and Einstein's gravitational pseudotensor) in terms of his radiation formula.

When gravitational wave is present, the gravitational energy-stress tensor  $t(g)_{\mu\nu}$  is non-zero. Thus, a radiation does carry energy-momentum as physics requires. This explains also that the absence of an anti-gravity coupling which is determined by Einstein's radiation formula, is the physical reason that the 1915 Einstein equation (1) is incompatible with radiation.

Note that the radiation of the binary pulsar can be calculated without detailed knowledge of  $t(g)_{\mu\nu}$ . From (19'), the approximate value of  $t(g)_{\mu\nu}$  at vacuum can be calculated through  $G_{\mu\nu}/K$  as before since the first-order approximation of  $g_{\mu\nu}$  can be calculated through (11). In view of the facts that  $Kt(g)_{\mu\nu}$  is of the fifth order in a post-Newtonian approximation, that the deceleration due to radiation is of the three and a half order in a post-Newtonian approximation [17] and that the perihelion of Mercury was successfully calculated with the secondorder approximation from (1), the orbits of the binary pulsar can be calculated with the second-order post-Newtonian approximation of (19) by using (1). Thus, the calculation approaches of Damour and Taylor [31, 32] would be essentially valid except that they did not realize the crucial fact that (11) is actually an approximation of the updated equation (19) [33].

In light of the above, the Hulse-Taylor experiments support the anti-gravity coupling being crucial to the existence of the gravitational wave [34], and (11) being an approximation of weak waves generated by massive matter. Thus, it has been experimentally verified that Einstein equation (1) is not compatible with radiation, but the updated Einstein equation is.

The 1995 updated Einstein equation actually was first proposed by Lorentz [35] and Levi-Civita [36] as follows:

$$\kappa t(g)_{ab} = G_{ab} + \kappa T_{ab}$$
(20)

where  $T_{ab}$  is the sum of other massive energy-stress tensors. Then, the gravitational energy-stress tensor takes a covariant form, However, Einstein [37] objected to this form on the grounds that his field equation implies  $t(g)_{ab} = 0$ . Now, Einstein is wrong since his equation is proven invalid for the dynamic case. An independent evidence for unboundedness is that the calculated radiation depends on the approached chosen [38]. Thus, eq. (19) should be called the Lorentz-Levi-Einstein equation. While eq. (19) is consistent with the linearized equation for the massive case and can do an approximate calculation for the gravitational radiation, it is still not clear that it is the exact equation. For this, our position is that this is the best we can get so far. Further verification can be done only after the exact form of the gravitational energy-stress tensor  $t(g)_{ab}$  is known. Moreover, if the unique sign for couplings could be attributed to a general  $E = mc^2$ ,<sup>7)</sup> the non-unique signs of coupling would suggest that  $E = mc^2$  is only conditionally valid. It has been shown explicitly that the electromagnetic energy is not equivalent to mass.

It should be noted that the anti-gravity coupling is a general feature that would appear in where the gravitational wave is present. For instance, it is necessary to appear in the Einstein equation for the gravitational waves generated by an electromagnetic wave [39, 40]. For the validity of the calculation on light bending, it is necessary that an electromagnetic wave would generate a negligible gravitational wave because this was implicitly assumed in such a calculation. For this case the related equation is the following:

$$G_{ab} = K[T(E)_{ab} - T(p)_{ab}], \text{ and } T_{ab} = -T(g)_{ab} = T(E)_{ab} - T(P)_{ab},$$
(21)

where T(E) <sub>ab</sub> and T(P) <sub>ab</sub> are the energy-stress tensors for the electromagnetic wave and the related photons.<sup>8)</sup> The photonic energy-stress is necessary; otherwise there is no bounded gravitational wave solution for equation (21) [39, 40]. Thus, the anti-gravity coupling must be present for any dynamic case and Einstein's understanding in general relativity was still incomplete.

In Einstein's initial assumption, the photons consist of only electromagnetic energy. If the photons consist of only electromagnetic energy, there is a conflict since the photonic energy can be equivalent to mass and the electromagnetic energy-stress tensor is traceless. Now, this conflict is resolved since the photonic energy is the sum of electro-magnetic energy and gravitational energy. Both quantum theory and relativity are based on the phenomena of light. It is gravity that makes the notion of photons compatible with electromagnetic waves. Einstein probably would smile heartily for this.

It should be noted that the existence of an antigravity coupling means the energy conditions in the singularity theorems [1] are not valid for a dynamic situation. Thus, the existence of singularity is not certain, and the claim of inevitably breaking of general relativity is baseless since these singularity theorems have been proven to be unrealistic in physics. In other words, the contributions of Hawking and Penrose to physics in terms of those theorems are essentially zero if not negative. Apparently, both Hawking and Penrose also did not understand the principle of causality adequately, and therefore they accepted unbounded solutions as valid in physics. Moreover, Penrose even accepted solutions that include unphysical parameters. [34]

#### V. E = MC<sup>2</sup>, the Reissner-Nordstrom Metric, and the Question of Black Holes

The existence of the anti-gravity coupling raised a question whether the formula  $E = mc^2$  is unconditionally valid. It is found that this is only a speculation that Einstein failed to prove (1905-1909) [41]. Moreover, since the electromagnetic energy-stress tensor is traceless, an electromagnetic energy-stress tensor would generate gravitation which does not change the Ricci curvature R in the Einstein equation. However, nobody seriously studied gravitation generated by the electromagnetic energy, although the Riessner-Nordstrom metric [19] for a charged particle would answer the above issues.<sup>9</sup>

Now, let us examine the Reissner-Nordstrom metric [19] (with c = 1) as follows:

$$-ds^{2} = \left(1 - \frac{2M}{r} + \frac{q^{2}}{r^{2}}\right)dt^{2} - \left(1 - \frac{2M}{r} + \frac{q^{2}}{r^{2}}\right)^{-1}dr^{2} - r^{2}d\Omega^{2}$$
(22)

where q and M are the charge and mass of a particle, and r is the radial distance (in terms of the Euclideanlike structure [42]) from the particle center. In this metric (22), the gravitational components generated by electricity have not only a very different radial coordinate dependence but also a different sign that makes it a new repulsive gravity in general relativity [43]. Thus, general relativity must be extended to include the unification of electromagnetism and gravitation [43].

Nevertheless, some argued that the effective mass could be considered as

$$M - q^2/2r$$
, (23)

because the total electric energy outside a sphere of radius r is  $q^2/2r$ , and thus (23) could be interpreted as supporting  $m = E/c^2$ . If the electric energy has a mass equivalence, an increase of such energy should lead to an increment of gravitational strength. However, from metric (22), the strength of a gravitational force decreases everywhere after an increase of the electric energy.

Moreover, the gravitational forces would be different from the force created by the "effective mass"  $M - q^2/2r$  because

$$-\frac{1}{2}\frac{\partial}{\partial r}\left(1-\frac{2M}{r}+\frac{q^{2}}{r^{2}}\right) = -(\frac{M}{r^{2}}-\frac{q^{2}}{r^{3}}) > -\frac{1}{r^{2}}\left(M-\frac{q^{2}}{2r}\right)_{(33)}$$

Thus Will was defeated because he could not defend his interpretation of  $m = E/c^2$  [7].

The validity of  $E = mc^2$  was questioned because for the binary pulsars experiment the coupling constants necessarily have different signs [11]. Nevertheless, with supports from editorials of Nature, the Physical Review D, and Science, Will continued to misinterpret the formula. Also, some theorists [44, 45] argued that M in (22) includes the external electric energy.

For instance, Herrera, Santos, & Skea [45], also argued that M in (31) involves the electric energy. They follow the error of Whittaker [46] and Tolman [47] who believed the equivalence of mass and electric energy. Then they obtained a metric that would imply a charged ball would increase its weight as the charge Q increased, in disagreement with experiments [48].

The above approach is essentially the same as that of Pekeris [44], who gets a similar metric in 1982. The difference is due to that Pekeris requires that  $|g_{\mu\nu}| = g = -1$ . Thus, the approach of Herrera et al. [45] is essentially what Pekeris had done. Apparently, theorists have run out of ways that can be used against the repulsive force. Nevertheless, Nobel Laureate 't Hooft even claimed incorrectly that the electric energy of an electron contributed to the inertial mass of an electron [49].

On the other hand, if the mass  ${\sf M}$  is just the inertial mass of the particle, the weight of a charged

metal ball can be reduced [50]. Thus, as Lo expected [7], experiments of Tsipenyuk and Andreev on two metal balls [48] rejects the claims of Herrera et al. [45] since the charged ball has reduced weight. This is an experimental direct proof. We recommend that the detailed investigation of such experiments [50] should be continued such that this static case of general relativity is fully verified.

Note that the appearing of the repulsive gravitation is important because it would solve a puzzle as to why we have never seen a black hole. If gravity is always attractive to mass, Wheeler simulation convinces him that a black hole must be formed [2]. <sup>10)</sup> Another piece of information for the existence of black holes is the existence of space-time singularities, proven by Hawking and Penrose [1]. Now, because the necessity of the existence of the anti-gravity coupling the energy conditions of their theorems cannot be satisfied. Thus, their space-time singularity theorems are actually irrelevant to physics.

More important, this repulsive force is crucial for establishing the unification of gravitation and electromagnetism [43].

#### VI. DISCUSSIONS AND CONCLUSIONS

In current theory of general relativity, essentially only the case of massive sources is studied, due to Einstein's speculation on mass and energy,  $E = mc^2$ being unconditionally true. Moreover, because of such speculation, physicists accepted the assumption that all the coupling constants have the same sign. For instance, such assumption was implicitly included in the energy-conditions used in the space-time singularity theorems of Hawking and Penrose [1]. In turn, such theorems would imply that general relativity is inapplicable for microscopic phenomena. The problem actually is due to a lack of understanding the non-linear mathematics. However, many incorrectly blame the problem as due to the classical nature of general relativity. Furthermore, this leads to their speculation that there was a conflict between general relativity and quantum mechanics.

In spite of the efforts of generations of physicists, there is no experimental evidence to support their claims (see Appendix A). Nevertheless, the Spacetime singularity theorems provide a convenient excuse for the Big Bang Theory of the expanding universe and the notion of black holes. However, although Penrose has won his arguments against the theorist, E. M. Lifshitz [2], the problem is, however, not in mathematics. A time tested practice in physics is that if the conclusion is unusual, one should go back to examine their assumptions. However, the singularity theorems have been treated as exceptions.

Consequently, many believed that there was a conflict between general relativity and quantum

mechanics. The fact is, however, that not only this is not true but also general relativity actually necessitates the existence of photons [39, 40].

Nevertheless, Einstein has commanded such a faith and thus nobody ever questions his speculation  $E = mc^2$  as unconditional although Einstein has failed to prove it for years (1905-1909) [41]. Moreover, people over-looked that  $E = mc^2$  being unconditionally true is in direct conflict with general relativity because of the fact that the electromagnetic energy-momentum tensor is traceless [43]. Some theorists even supply their own errors that lead to further confusion [34].

According to their theorems, Hawking and Penrose claimed that general relativity is unsuitable for microscopic phenomena [1]. However, they were not aware that the assumption of unique coupling sign also implies that general relativity is also not suitable for macroscopic phenomena [39, 40]. This is so because in the light bending, it is implicitly assumed that the gravity due to the light ray is negligible [15, 16]. Since the energy-stress tensor of an electromagnetic wave can be a source term, gravitational waves would be generated [51]. However, if such gravity is not negligible or having no solution due to the assumption of unique coupling sign, then general relativity would not be valid even for macroscopic phenomena. Since Einstein was not aware of this, his understanding in general relativity also needs to be improved.

A major problem is that Einstein and his followers do not understand non-linear mathematics. As Gullstrand [52] pointed out, Einstein's calculation on the perihelion of Mercury is problematic. Nevertheless, the coincidence between his calculation and observation was the source of Einstein's confidence in his theory. Understandably, almost the whole physics community was against Gullstrand, Ironically, Gullstrand is right. In fact, many failed to understand that the Einstein equation does not have a bounded solution for a twobody problem [13] as in Newtonian theory.<sup>11)</sup> Moreover, for the dynamic case, the Einstein equation and the linearzied equation are actually not compatible, but independent equations [14].<sup>12)</sup> Another problem was Einstein's partially verified formula  $E = mc^2$  which turns out to be invalid for the electromagnetic energy.

Einstein's false confidence leads to serious problems. One of them is the acceptance of space-time singularity theorems of Hawking and Penrose.<sup>13)</sup> They serve as the justification for the Big Bang Theory that has been known to have many problems [53, 54], and black holes that have not been observed. Hawking and Penrose invalidly blame their claim of inapplicability to microscopic problem as due to being a classical theory. Moreover, the Wheeler School, in particular the errors of Christodoulou [55],<sup>14)</sup> have been proven as creating further confusions [34] that even misled the 1993 Nobel Prize Committee for physics [56] to claim erroneously

that the Einstein equation has bounded dynamic solutions [10-13].<sup>5)</sup>

The formula  $E = mc^2$  was proved for the case of light rays [15]. However, it remains to reconcile that the electromagnetic energy-stress tensor is traceless. Now, the notion of a photonic energy-stress tensor of massless particles solves this puzzle since the photonic energy can be equivalent to mass because photons are massless particles, but the electromagnetic energy alone cannot. Note that both quantum theory and relativity are based on the phenomena of light. Now, it is gravity in general relativity that makes photons necessary for electromagnetic waves, and thus Einstein's photonic proposal is inadequate.

The important results from this analysis are: 1) The invalid covariance principle confuses mathematics and physics. 2) The electromagnetic energy is not equivalent to mass. 3) The photons include energy from its gravitational components. <sup>9)</sup> 4) Einstein's general relativity is invalid for the dynamic case, for which it remains to be rectified and completed in at least two aspects: a) The exact form of the gravitational energystress tensor is not known; and b) The radiation reaction force is also not known. Since the photons include gravitational energy, general relativity is clearly compatible with quantum theory.

Thus, the space-time singularity theorems of Hawking and Penrose are actually irrelevant to physics. Using the same invalid unique sign assumption, the positive energy theorem of Schoen and Yau actually does not include the case of the dynamic solutions [57, 58].<sup>15)</sup> Thus, such a theorem is misleading because it is based on an invalid assumption.<sup>16)</sup>. In fact, recognizing E = mc<sup>2</sup> as only conditionally valid [6], is crucial to identify the charge-mass interaction. Moreover, such a force is coupled to the charge square, and thus such a force exists naturally in a five-dimensional theory of Lo et al. [59].

Einstein failed to show such unification because of his three shortcomings: 1) He failed to see, as Maxwell showed, that unification is necessary to have new interactions. 2) He has mistaken that  $E = mc^2$  was unconditional. 3) Einstein invalidly rejected the repulsive gravitation [7]. Einstein's invalid covariance principle also added confusions [9].<sup>17)</sup> However, this new repulsive force can also be detected from a charged capacitor. Thus, unification of electromagnetism and gravitation beyond Einstein is confirmed [6]. Hence, Einstein turns out to be the biggest winner from the rectification of his errors.<sup>18)</sup>

The earliest error of Einstein started from his inadequate notion of photons which include only electromagnetic energy. This led him to believe erroneously that  $E = mc^2$  could be generally valid [15]. Consequently, he rejected repulsive gravitation [7] and believed all the coupling constants had the same sign.

Einstein incorrectly believed that his test particle approach could be derivable as a limit of a dynamic solution [81] because he has never derived one with nonlinear mathematics. <sup>19)</sup> A serious problem in general relativity is that many rush to obtain new conclusions without careful deliberations [10].

Clearly, the errors of Einstein and his followers are the obstacle of progress in physics. However, if one has any doubt on my claims, it would be beneficial for him to find examples to support his objections.<sup>20</sup> Note also that the outstanding work on repulsive gravitation of Musha [5] was not recognized for a long time because of the theoretical errors. Nevertheless, the work of Liu [4] makes it clear that the weight reduction of charged capacitor is due to the repulsive gravitation [6].

Journals such as the Physical Review, the Proceedings of the Royal Society A, the Annals of Physics, and the Chinese Physics accepted the singularity theorems because theorists make mistakes in non-linear mathematics and related physics. Experimental supports and explicit calculations of the present derivation make clear that they are wrong.

#### VII. Acknowledgments

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Appendix A: On Interpretations of Hubble's Law and Einstein's Theory of Measurement

Hubble's law is often considered as the observational evidence of an expanding universe. Then, the further a galaxies is from the Milky Way, the faster it appears to recede. However, Hubble himself concluded in 1936 that the Galaxies are stationary [60]. In fact, such a receding velocity is incompatible with the local light speeds used in deriving the light bending. Thus, such a questionable assumption that has been pointed out by Whitehead [61] and also proven as theoretically invalid.

Note that the Doppler redshifts of the light from receding Galaxies is based on an implicit assumption

$$\mathsf{R} = \int_{1}^{2} a(\tau) \, \sqrt{dx^{2} + dy^{2} + dz^{2}} = a(\tau) L,$$

Then

$$v = \frac{dR}{d\tau} = \frac{da}{d\tau}L + \frac{dL}{d\tau}a = \frac{da}{d\tau}\frac{R}{a} = HR , \qquad (A7)$$

that there is no expansion for the space coordinates. Moreover, the receding velocity is incompatible with the light speeds used in deriving the light bending. In short, the notion of expanding universe is a production due to an inadequate understanding of a physical space. Thus, it is questionable that such a universe is related to the reality.

#### A.1. Hubble's Law

Hubble observed from light emitted by galaxies that the redshifts S are linearly proportion to the distance L from the Milky Way as,

$$S = H L$$
 (A1)

where H is the Hubble constant although the redshifts of distant galaxies will deviate from this linear law slightly.

#### A.2. The Redshifts

In terms of a theory from general relativity, it is well known that this law can be derived with the following metric [1],

$$ds^{2} = -d\tau^{2} + a^{2}(\tau) \{ dx^{2} + dy^{2} + dz^{2} \},$$
 (A2)

since

$$S = \frac{\lambda_2 - \lambda_1}{\lambda_1} = \frac{\omega_1}{\omega_2} - 1 = \frac{a(\tau_2)}{a(\tau_1)} - 1,$$
 (A3)

where  $\omega_1$  is the frequency of a photon emitted at event  $P_1$  at time  $\tau_1$ , and  $\omega_2$  is the frequency of the photon observed at  $P_2$  at time  $\tau_2$ . Furthermore, for nearby galaxies, one has

$$a(\tau_2) \approx a(\tau_1) + (\tau_2 - \tau_1)\dot{a}$$
 and  $\tau_2 - \tau_1 \approx R$  (A4)

Thus,

$$S = \frac{\dot{a}}{a}L = H L, \text{ and } H = \frac{\dot{a}}{a}$$
(A5)

Note that Hubble's Law need not be related to the Doppler redshifts. In fact, Hubble rejected such an interpretation [61].

#### A.3. Hubble's Law and the Doppler Redshifts

If one chooses to define the distance between two points as

where 
$$L = \int_{1}^{2} \sqrt{dx^2 + dy^2 + dz^2}$$
 (A6)

Thus,

This means that the redshifts could be superficially considered as a Doppler effect.

#### A.4. Remarks

However, if we define the distance as L, there is actually no receding velocity since L is fixed (i.e.,  $dL/d\tau = 0$ ).

Thus, whether Hubble's Law represents the effects of an expanding universe is a matter of the interpretation of the local distance. From the above analysis, the crucial point is what a valid physical velocity in a physical space is.

It should be noted that  $dL/d\tau = 0$  means that the space coordinates are independent of physics. In other words, the physical space has a Euclidean-like structure, which is independent of time. However, since L between any two space-points is fixed, the notion of an expanding universe, if it means anything, is just an illusion. Moreover, the validity of (A6) as the physical distance has no known experimental supports since it is not even clearly measurable. Also, a problem is that the notion of velocity in (A7) would be incompatible with the light speeds in the calculation of light bending experiment.

### A.5. The Coordinates of an Einstein Physical Space, and Definition of Velocity

If the Riemannian space is embedded in a higher dimensional flat space [62], then the coordinates  $dx^{\mu}$  are determined by

$$ds^{2} = g_{\mu\nu} dx^{\mu} dx^{\nu}, \qquad \text{ or } -g_{tt} dt^{2} + g_{ij} dx^{i} dx^{j} \qquad (A9)$$

such as the surface of a sphere in a three dimensional Euclidean space. For a physical space since the metric is a variable function, it is impossible to determine the coordinates with the metric. Moreover, it has been proven [42] that a frame of reference with the Euclidean-like structure must exist for a physical space.

For a spherical mass distribution with the center at the origin, the metric with the isotropic gauge is,

$$-ds^{2} = -[(1 - M\kappa/2r)^{2}/(1 + M\kappa/2r)^{2}]c^{2}dt^{2} + (1 + M\kappa/2r)^{4}(dx^{2} + dy^{2} + dz^{2})$$
(A10)

where  $\kappa = G/c^2$  ( $G = 6.67 \times 10^{-8} \text{ erg cm/gm}^2$ ), M is the total mass, and  $r = \sqrt{x^2 + y^2 + z^2}$ . Then, if the equivalence principle is satisfied, the light speeds are determined by ds<sup>2</sup> = 0 [12, 13], i.e.,

$$\frac{\sqrt{dx^2 + dy^2 + dz^2}}{dt} = c \frac{1 - M\kappa/2r}{(1 + M\kappa/2r)^3}$$
(A11)

However, such a definition of light speeds is incompatible with the definition of velocity (A7). Since this light speed is supported by observations, definition (A7) is invalid in physics.

Nevertheless, Liu [63] has defined light speeds, which is more compatible with (A7), as

$$\frac{\sqrt{g_{ij}dx^i dx^j}}{dt} = c\frac{1 - M\kappa/2r}{1 + M\kappa/2r}$$
(A12)

for metric (A10). However, (A12) implies only half of the deflection implied by (A11) [15].

The above analysis also explains why many current theorists insist on that the light speeds are not defined even though Einstein defined them clearly in his 1916 paper [15] as well as in his book [16]. The light speeds are well defined although diffeomorphic metrics give different sets of light speeds for the same frame of reference. However, Einstein defines light speeds after the assumption that his equivalence principle is satisfied. Thus, at most only one of such metrics is valid in physics.

Moreover, it has been proven that the Maxwell-Newton Approximation gives the valid first order approximation of the physical metric [11-13]. Since metric (A10) is compatible with the Maxwell-Newton approximation, the first order of light speed (A11) is valid in physics. Thus, the speculation that local light speeds are not well defined is proven incorrect. In essence, the velocity definition (A7), which leads to the notion of the Doppler redshifts, has been rejected by experiments.

#### a) Discussions

One may ask what causes such redshifts that are roughly proportional to the distances from the observer. One possibility is that the scatterings of a light ray along its path to the observer. In physics, it is known that different scatterings are common causes for losing energy of a particle, and for the case of photons it means redshifts. Unfortunately, to test such a conjecture is not possible because no current theory of gravity is capable of handling the inelastic scatterings of lights.

Nevertheless, the assumption that observed redshifts could be due to inelastic scatterings may help to explain some puzzles of observed facts [64]. For instance, younger objects such as star forming galaxies have higher intrinsic redshifts, and objects with the same path length to the observer have much different redshifts while all parts of the object have about the same amount of redshifts. For those interested in alternative cosmology theories, there are the plasma universe model [54] and others [65].

### Appendix B: The Principle of Causality and the Physics of Plane-Waves

There are two aspects in causality: its relevance and its time ordering. In time ordering, a cause event must happen before its effects. This is further restricted by relativistic causality that no cause event can propagate faster than the light speed in vacuum. The time-tested assumption that phenomena can be explained in terms of identifiable causes will be called the principle of causality. This is the basis of relevance for all scientific investigations.

Thus, the principle of causality implies that any parameter in a solution for physics must be related to some physical causes. Moreover, Einstein's notion of weak gravity is also based on the principle of causality that implies a weak source would produce a weak gravity. Here this principle will be elucidated first in connection with symmetries of a field, the boundedness of a field solution, and consequently in the validity of a field equation in physics.

In practice, when the considered field is absent, physical properties are ascribed to the space-time as in a "normal" state. For example, the electromagnetic field is zero in a normal state. Then, any deviation from the normal state must have physically identifiable causes. Thus, the principle of causality implies that the symmetry must be preserved if no cause breaks it. The implication of causality to symmetry has been used in deriving the inverse square law from Gauss's law.

The normal state of a space-time metric is the flat metric in special relativity. Thus, if a metric does not possess a symmetry, then there must be physical cause(s) which has broken such a symmetry. For a spherically symmetric mass, causality requires that the metric is spherically symmetric and asymptotically flat. Also, since gravity must have a cause, a weak cause can lead to only weak gravity. Therefore, Einstein's notion of weak gravity is also a consequence of the principle of causality.

However, the physical cause(s) should not be confused with the mathematical source term in the field equation. In general relativity, the cause of gravity is the physical matter itself, but not its energy tensors in the source term of Einstein's field equation. The energystress tensors (for example the perfect fluid model) may explicitly depend on the metric. Since nothing should be a cause of itself, such a source tensor does not represent the cause of a metric. For the accompanying gravitational wave of an electromagnetic wave, the physical cause is the electromagnetic wave. Thus, one should not infer the symmetries of the metric based on the source term (instead of its causes) although their symmetries are not unrelated.

Moreover, inferences based on the source term can be misleading. The source term may have higher symmetries than those of the cause and the metric. For instance, a transverse electromagnetic plane-wave (1) is not rotationally invariant with respect to the z-direction of propagation. But the related electromagnetic energystress tensor component  $T(E)_{tt}$  for a circularly polarized wave is rotationally invariant. This assumption violates causality and results in theoretical difficulties.

Classical electrodynamics implies that the flat metric is an accurate approximation, caused by the presence of weak electromagnetic waves. This physical requirement is supported by the principle of causality which implies such a metric to be a bounded periodic function. However, this required boundedness is not satisfied by solutions in the literature [66-68]. These solutions also violate causality directly since they involve parameters without any physical cause [67]. They also do not satisfy the equivalence principle [69, 70] although they are Lorentz manifolds.

A necessary and sufficient condition for satisfying the equivalence principle <sup>21)</sup> is that a time-like geodesic represents a physical free falling; but the mathematical existence of local Minkowski spaces is only necessary. A major problem in general relativity is that many theorists and journals do not understand related physics, in particular, the principle of causality adequately.

#### Endnotes

- Although a Nobel Prize in Physics has been awarded to S. Perlmutter, B. PSchmidt, and A. G. Riess for the accelerating expansion of the universe in 2011, the expanding universe actually has not been verified (See Appendix A).
- 2. The existence of anti-gravity coupling has been considered by Lorentz [35], Levi-Civita [36] and Pauli [71].
- 3. Thus, the observation of Galileo that all neutral matter falls in the same rate under gravitation is actually incomplete [72].
- 4. Wald has never provided a dynamic solution [73].
- 5. The proof on the non-existence of dynamic solution [11] was published in 1995 Astrophys. J. when S. Chandrasekhar, a Nobel Laureate, was the editor-inchief. This was two years after the 1993 Nobel Prize was awarded to Hulse and Taylor. Thus, Chandrasekhar also agreed that there are problems in the 1993 Nobel prize in physics.
- Prof. S. Weinberg taught us that general relativity 6. must be understood in terms of physics, and thus summarized the viewpoints and tradition of M.I.T on general relativity. This tradition has a long history, starting from N. Rosen and A. Einstein's paper of 1937, followed by H. Yilmaz, advocated by V. F. Weisskopf and P. Morrison, and so on. It is a pleasure to be able to contribute to such an outstanding tradition. However, to repair such a tradition is urgently needed since it has been broken by the Wheeler School [34] after Prof. Morrison passed away. This would not be an easy task for MIT since Einstein also has mistakes. And Harvard University as well as Princeton Advanced Studies were also sources of errors [57].
- For a thorough discussion on the relation between the mass and the total energy of a particle, one can read the paper of L. B. Okun [74]. However, since Okun believed that E= mc<sup>2</sup> is generally true [75], Okun did not understand that the electromagnetic energy is not equivalent to mass [75]. Nevertheless,

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Okun has some interesting thought [76] about the energy of photon related to gravity, and would question to interpret the Hubble's law as due to receding of the stars.

- 8. Now, we understand that Einstein's proposal of the photons is inadequate, and moreover it is a necessary consequence of general relativity. It is interesting that this is a major triumph of general relativity, but shows also a major error of Einstein in general relativity [77]. The Chinese Physics also mistaken that a bounded solution can be obtained by perturbation.
- 9. I have reported these to MIT President Hockfield and the subsequent President Reif. They have promised to up-grade the related education in gravitation.
- 10. The unconditional validity of  $E = mc^2$  is responsible for the invalid notion that energy always produces attractive gravity [2; p.488]. Thus, the notion of black holes could be considered as due to the errors of Einstein.
- 11. Because of inadequacy in non-linear mathematics, theorists such as Hod [78], like many other "theorists", carelessly failed in recognizing the problem is that there is no bounded dynamic solution [10-13] since linearization for the static case is valid. Thus, they claimed that general relativity has superseded Newtonian theory in every aspects. Unfortunately, this is simply not true although Einstein also wrongly believed this as Gullstrand [52] pointed out.
- Bertschinger [79] also does not know that, for the dynamic case, the linearized equation and the nonlinear Einstein equation have only unrelated solutions [14]. Because he does not understand non-linear mathematics, Bertschinger erroneously [14, 34] believed that the linearization of Einstein equation was in general mathematically valid [79].
- 13. The space-time singularity theorems made sense only if the Einstein equation has bounded dynamic solutions. This is why Hawking and Penrose and their followers had to believe the existence of bounded dynamic solutions.
- 14. The Ph. D. degree advisor of D. Christodoulou is J. A. Wheeler, whose mathematics has been shown also in his book Gravitation [19] having errors in crucial arguments and unreliable at the undergraduate level. In fact, mathematician Perlick, [80, 81] has pointed out the book of Christodoulou and Klainerman is incomprehensible. Accordingly, the honors awarded to Christodoulou, in fact, reflected, the blind faith toward Einstein and accumulated errors in general relativity [82]. These expose that many theorists just do not understand non-linear mathematics. In fact, Christodoulou has

never completed the construction of dynamics solutions [83]. In short, the contributions of Christodoulou to general relativity are just errors.

- 15. Michael Francis Atiyah has been president of the Royal Society (1990-1995), and President of the Royal Society of Edinburgh (2005-2008). Since 1997, he has been an honorary professor at the University of Edinburgh (Wikipedia). However, like many mathematicians, clearly he does not understand general relativity just as Hilbert [84] did not and thus, Yau and Witten were awarded the Fields Medal for their errors [58].
- 16. Due to making the same erroneous assumptions in physics (claiming the existence of dynamic solution for the Einstein equation), S. T. Yau had organized seminars for Hawking in Hong Kong and other cities of China. Now, he claimed, however, that he is no longer interested in general relativity. Thus, he ignores that his positive mass theorem [85] is misleading in physics. [57, 58]. Moreover, later the same error was made by Witten [86]. This error may also explain the fact that there is little progress in the string theory. Thus, it seems, many mathematicians at the top as well as physicists "at the top" have made errors in general relativity.
- 17. Due to the influence of L. Z. Fang and C. N. Yang, currently in China few understand general relativity after P. Y. Zhou [87]. Yang is against Zhou, who correctly pointed out that Einstein's covariance principle is invalid [34]. However, as Weinberg [88] pointed out, Yang's understanding on the gauge theories is incorrect [34]. Moreover, Yang is also proven wrong by explicit examples [89].
- Apparently, Einstein did not know that his unification was that close to confirmation. If he had known this, he may not be that willing to go by rejecting the modern medicine to prolong his life [90].
- 19. It has never occurred to Einstein that his field equation is invalid for the dynamic case, and this is a main reason that he failed to see the necessary unification of gravitation and electromagnetism.
- I claim a statement in physics is wrong only under two conditions: 1) it is logically not self-consistent; and 2) it is against by experiments or observations. Otherwise, I claim them as disagreements only.
- 21. Eric J. Weinberg, editor of the Physical Review D, invalidly insists that there is no difference in physics between Einstein's equivalence principle and Pauli's version [71] although Einstein pointed out that Pauli's is a misinterpretation [91]. Eric does not understand the functional analysis related to Einstein's equivalent principle [34], and the nonlinear mathematics with related physics. His incompetence is a main reason that APS is lack behind in the physics of gravitation [92].

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