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On Special Pairs of Pythagorean Triangles

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18. A. A. K. Majumdar, On some pseudo smarandache function related triangles, pythagorean triangles hold a special fascination, scientia majna, 4(3), 95-105, 2008

On Special Pairs of Pythagorean Triangles

M. A. Gopalan [α], S. Vidhyalakshmi [σ], N. Thiruniraiselvi [ρ] & R. Presenna [ω]

Abstract- We search for pairs of Pythagorean triangles such that, in each pair, twice the difference between their perimeters is expressed in terms of special polygonal numbers.

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Notations used:

$$t_{m,n} = n\left(1 + \frac{(n-1)(m-2)}{2}\right)$$

$$s_n = 6n(n-1)+1$$

$$cs_n = n^2 + (n-1)^2$$

I. Introduction

Number is the essence of mathematical calculation. Varieties of numbers have variety of range and richness, many of which can be explained very easily but extremely difficult to prove. One of the varieties of numbers that have fascinated mathematicians and the lovers of maths is the pythagorean number as they provide limitless supply of exciting and interesting properties. In other word, the method of obtaining three non-zero integers x,y and z under certain relations satisfying the equation $x^2 + y^2 = z^2$ has been a matter of interest to various mathematicians. For an elaborate review of various properties are may refer [1-17]. In [18], the author proves existence of an infinite family of pairs of dissimilar Pythagorean triangles that are pseudo smarandache related. For problems on pairs of Pythagorean triangles one may refer [1,5].

In this communication concerns with the problem of determining pairs of Pythagorean triangles wherein each pair 2-times the difference between the perimeters is expressed interms of special polygonal numbers.

II. Method of Analysis

Let $T_1(\alpha_1, \beta_1, \gamma_1)$ and $T_2(\alpha_2, \beta_2, \gamma_2)$ be two distinct Pythagorean triangles where $\alpha_1 = 2mq, \beta_1 = m^2 - q^2, \gamma_1 = m^2 + q^2$ and $\alpha_2 = 2pq, \beta_2 = p^2 - q^2, \gamma_2 = p^2 + q^2$, m>q>0;p>q>0

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Let $P_1 = 2m(m+q), P_2 = 2p(p+q)$ be their perimeters respectively. We illustrated below the process of obtaining pair of Pythagorean triangles such that 2 times the difference between their perimeters is expressed in terms of a special polygonal number.

Illustration 1

The assumption $2(P_1 - P_2) = (3\alpha - 1)^2$ (1)

gives $(2m+q)^2 = (2p+q)^2 + (3\alpha-1)^2$ (2)

which is in the form of well know Pythagorean equation and it is satisfied by

$$3\alpha - 1 = 2RS, 2p+q = R^2 - S^2, 2m+q = R^2 + S^2, R > S > 0 \quad (3)$$

Solving the above system of equations (3), we have

$$\alpha = \left(\frac{2RS+1}{3}\right), m = p + S^2, q = R^2 - S^2 - 2p$$

As our main thrust is on integers, note that α is an integer when

$$R = 3(k_1 + k_2) - 2, S = 3k_2 - 2$$

and thus

$$\begin{aligned} \alpha &= (k_1+k_2)(6k_2-4) - 4k_2 + 3 \\ m &= p + (3k_2-2)^2 \\ q &= 9k_1^2 + 18k_1k_2 - 12k_1 - 2p \end{aligned} \quad (4)$$

The conditions m>q>0, p>q>0 lead to

$$2p < 9k_1^2 + 18k_1k_2 - 12k_1 < 3p \quad (5)$$

Choose k_1, k_2, p such that (5) is satisfied. Substituting the corresponding values of k_1, k_2, p in (4) and in the value of m, q, α are obtained. Thus, it is observed that

$$2(P_1 - P_2) = \begin{cases} 3t_{8,\alpha} + 1 \\ s_\alpha + 3t_{4,\alpha} \end{cases}$$

A few examples are given below in Table -1

k_1	k_2	p	m	q	α	$2(P_1-P_2)$	$3t_{8,\alpha}+1$	$s_\alpha + 3t_{4,\alpha}$
1	1	7	8	1	3	64	$3(21)+1$	$37+3(3)^2$
1	2	16	32	1	19	3136	$3(1045)+1$	$2053+3(19)^2$
1	2	15	31	3	19	3136	$3(1045)+1$	$2053+3(19)^2$
1	2	14	30	5	19	3136	$3(1045)+1$	$2053+3(19)^2$
1	2	13	29	7	19	3136	$3(1045)+1$	$2053+3(19)^2$
1	2	12	28	9	19	3136	$3(1045)+1$	$2053+3(19)^2$
1	3	25	74	1	47	19600	$3(6533)+1$	$12973+3(47)^2$
1	3	24	73	3	47	19600	$3(6533)+1$	$12973+3(47)^2$

Notes

Notes

1	3	23	72	5	47	19600	$3(6533)+1$	$12973+3(47)^2$
1	3	22	71	7	47	19600	$3(6533)+1$	$12973+3(47)^2$
1	3	21	70	9	47	19600	$3(6533)+1$	$12973+3(47)^2$
1	3	20	69	11	47	19600	$3(6533)+1$	$12973+3(47)^2$
1	3	19	68	13	47	19600	$3(6533)+1$	$12973+3(47)^2$
1	3	18	67	15	47	19600	$3(6533)+1$	$12973+3(47)^2$

It is worth to note that there are only finitely many pairs of Pythagorean triangles such that in each pair, twice the difference between their perimeters is expressed in terms of a special polygonal number.

Illustration- 2

Assume $2(P_1 - P_2) = (5\alpha - 2)^2$

Proceeding as in **illustration 1**, it is seen that there are 3 values for α satisfying the equation $\alpha = \left(\frac{2(RS+1)}{5}\right)$ and thus, there are 3 sets of triples (m, q, α) which are represented as follows

SET 1:

$$\alpha = 2(k + k_1)(5k_1 - 4) - 12k_1 + 10$$
$$m = p + (5k_1 - 4)^2$$
$$q = (10k_1 + 5k - 10)(5k - 2) - 2p$$

The condition to be satisfied by k_1, k_2 and p is $2p < (10k_1 + 5k - 10)(5k - 2) < 3p$

SET 2:

$$\alpha = 2(k + k_1)(5k_1 - 3) - 6k_1 + 4$$
$$m = p + (5k_1 - 3)^2$$
$$q = (5k + 10k_1 - 6)(5k) - 2p$$

The condition to be satisfied by k_1, k_2 and p is $2p < (10k_1 + 5k - 6)(5k) < 3p$

SET 3:

$$\alpha = 2(k + k_1)(5k_1 - 2) - 4k_1 + 2$$
$$m = p + (5k_1 - 2)^2$$
$$q = (5k + 10k_1 - 4)(5k) - 2p$$

The condition to be satisfied by k_1, k_2 and p is $2p < (10k_1 + 5k - 4)(5k) < 3p$

From each of the above 3 sets it is observed that $2(P_1 - P_2) = 5t_{12,\alpha} + 4$

Illustration 3

Assume $2(P_1 - P_2) = (\alpha - 1)^2$

For this choice, the values for m,q, α satisfying the equation

$(2m + q)^2 = (2p + q)^2 + (\alpha - 1)^2$ are given by

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$$\alpha = 2rs + 1, m = s^2 + p, q = r^2 - s^2 - 2p$$

The conditions m>q>0,p>q>0 lead to

$$2p < r^2 - s^2 < 3p \qquad (6)$$

Choose r,s,p satisfying (6), it is seen that

$$2(P_1 - P_2) = (cs_{\alpha} - t_{4,\alpha})$$

Notes

III. Conclusion

In this paper, we have obtained finitely many pairs of Pythagorean triangles where each pair connects 2-times the difference between the perimeters with special polygonal numbers. To conclude, one may search for the connections between special numbers and the other characterizations of Pythagorean triangle.

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