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Thermomechanical Response of Transversely Isotropic Thermoelastic Solids with Two Temperature and without Energy Dissipation Due to Time Harmonic Sources

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Thermomechanical Response of Transversely Isotropic Thermoelastic Solids with Two Temperature and without Energy Dissipation Due to Time Harmonic Sources

Nidhi Sharma [α], Rajneesh Kumar [σ] & Parveen Lata [ρ]

Abstract- The paper is concerned with two dimensional deformation in a homogeneous, transversely isotropic thermoelastic solids without energy dissipation and with two temperatures due to various sources. Assuming the disturbances to be harmonically time-dependent, the transformed solution is obtained in the frequency domain. The application of a time harmonic concentrated and distributed sources have been considered to show the utility of the solution obtained. The transformed components of displacements, stresses and conductive temperature distribution so obtained are inverted numerically using a numerical inversion technique. Effect of anisotropy and two temperature on the resulting expressions are depicted graphically.

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I. Introduction

Thermoelasticity with two temperatures is one of the non classical theories of thermomechanics of elastic solids. The main difference of this theory with respect to the classical one is a thermal dependence. During the last few decades, an intense amount of attention has been paid to the theories of generalized thermoelasticity as they attempt to overcome the shortcomings of the classical coupled theory of thermoelasticity, i.e., infinite speed of propagation of thermoelasticity disturbances, unsatisfactory response of a solid body to short laser action, and poor description of thermoelastic behaviour at low temperature.

Green and Naghdi [5] and [6] proposed three new thermoelastic theories based on an entropy equality rather than usual entropy inequality and proposed three models which are subsequently referred to as GN-I, II, and III models. The linearised version of model-I corresponds to classical Thermoelastic model. In model -II, the internal rate of production entropy is taken to be identically zero implying no dissipation of thermal energy . This model admits un-damped thermoelastic waves in a thermoelastic material and is best known as theory of thermoelasticity without energy dissipation. The principal feature of this theory is in contrast to classical thermoelasticity associated with Fourier's law of heat conduction, the heat flow does not involve energy dissipation. This theory permits the transmission of heat as thermal waves at finite speed. Model-III includes the previous two models as special cases and admits

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dissipation of energy in general. In context of Green and Naghdi model many applications have been found. Chandrasekharaiah and Srinath [1] discussed the thermoelastic waves without energy dissipation in an unbounded body with a spherical cavity.

Youssef [18] constructed a new theory of generalized thermoelasticity by taking into account two-temperature generalized thermoelasticity theory for a homogeneous and isotropic body without energy dissipation. Youssef [22] also obtained variational principle of two temperature thermoelasticity without energy dissipation. Chen and Gurtin [2], Chen et al. [3] and [4] have formulated a theory of heat conduction in deformable bodies which depends upon two distinct temperatures, the conductive temperature φ and the thermo dynamical temperature T. For time independent situations, the difference between these two temperatures is proportional to the heat supply, and in absence of heat supply, the two temperatures are identical. For time dependent problems, the two temperatures are different, regardless of the presence of heat supply. The two temperatures T, φ and the strain are found to have representations in the form of a travelling wave plus a response, which occurs instantaneously throughout the body.

Warren and Chen [17] investigated the wave propagation in the two temperature theory of thermoelasticity. Quintanilla [16] proved some theorems in thermoelasticity with two temperatures. Youssef AI-Lehaibi [19] and Youssef AI -Harby [20] investigated various problems on the basis of two temperature thermoelasticity. Kumar and Deswal [8] studied the surface wave propagation in a micropolar thermoelastic medium without energy dissipation. Kaushal, Kumar and Miglani [9] discussed response of frequency domain in generalized thermoelasticity with two temperatures. Sharma and Kumar [12] discussed elastodynamic response and interactions of generalised thermoelastic diffusion due to inclined load. Sharma, Kumar and Ram[13] discussed dynamical behaviour of generalized thermoelastic diffusion with two relaxation times in frequency domain. Kumar and Kansal [10] discussed propagation of cylindrical Rayleigh waves in a transversely isotropic thermoelastic diffusive solid half-space. Kumar, Sharma and Garg [11] analyzed effect of two temperature on reflection coefficient in micropolar thermoelastic media with and without energy dissipation. No attempt has been made so far to examine the thermomechanical response in transversely isotropic thermoelastic solid with two temperature and without energy dissipation in frequency domain.

The deformation at any point of the medium is useful to analyze the deformation field around mining tremors and drilling into the crust of earth. It can also contribute to the theoretical consideration of the seismic and volcanic sources since it can account for the deformation field in the entire volume surrounding the source region. The purpose of the present paper is to determine the expression for components of displacement, normal stress, tangential stress and conductive temperature, when the time -harmonic mechanical or thermal source is applied, by applying Integral transform techniques. The present model is useful for understanding the nature of interaction between mechanical and thermal fields since most of the structural elements of heavy industries are often subjected to mechanical and thermal stresses at an elevated temperature.

II. Basic Equations

Following Youssef [21] the constitutive relations and field equations in absence of body forces and heat sources are

$$t_{ij} = C_{ijkl}\, e_{kl} - \beta_{ij} T \tag{1}$$

$$C_{ijkl}\, e_{kl,j} - \beta_{ij} T_{,j} = \rho \ddot{u}_i \tag{2}$$

$$K_{ij}\, \varphi_{,ij} = \beta_{ij} T_0 \ddot{e_{ij}} + \rho C_E \ddot{T} \tag{3}$$

Where

$$T = \varphi - a_{ij}\, \varphi_{,ij} \tag{4}$$

Ref

1. Chandrasekharaiah, D. S., and Srinath, K. S; Thermoelastic waves without energy dissipation in an unbounded body with a spherical cavity, The International Journal of Mathematics and Mathematical Sciences,(2000), 23 (8), 555-562.

$$\beta_{ij} = C_{ijkl}\,\alpha_{ij} \tag{5}$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \qquad i,j = 1,2,3 \tag{6}$$

Here

$C_{ijkl}\,(C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk})$ are elastic parameters, β_{ij} is the thermal tensor, T is the temperature, T_0 is the reference temperature, t_{ij} are the components of stress tensor, e_{kl} are the components of strain tensor, u_i are the displacement components, ρ is the density, C_E is the specific heat, K_{ij} is the materialistic constant, a_{ij} are the two temperature parameters, α_{ij} is the coefficient of linear thermal expansion.

Ref

14. Slaughter, W. S; The linearised theory of elasticity,Birkhausar (2002).

III. Formulation and Solution of the Problem

We consider a homogeneous, transversely isotropic thermoelastic body initially at uniform temperature T_0. We take a rectangular Cartesian co-ordinate system (x_1, x_2, x_3) with x_3 axis pointing normally into the half space, which is thus represented by $x_3 \geq 0$. We consider the plane such that all particles on a line parallel to x_2 −axis are equally displaced, so that the field component $u_2 = 0$ and u_1, u_3 and φ are independent of x_2 .we have used appropriate transformations following Slaughter [14] on the set of equations (1)-(3) to derive the equations for transversely isotropic thermoelastic solid with two temperature and without energy dissipation and we restrict our analysis to the two dimensional problem with

$$\vec{u} = (u_1, 0, u_3) \tag{7}$$

$$c_{11}\frac{\partial^2 u_1}{\partial x_1^2} + c_{44}\frac{\partial^2 u_1}{\partial x_3^2} + (c_{13} + c_{44})\frac{\partial^2 u_3}{\partial x_1 \partial x_3} - \beta_1 \frac{\partial}{\partial x_1}\left\{\varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x_1^2} + a_3 \frac{\partial^2 \varphi}{\partial x_3^2}\right)\right\} = \rho \frac{\partial^2 u_1}{\partial t^2} \tag{8}$$

$$(c_{13} + c_{44})\frac{\partial^2 u_1}{\partial x_1 \partial x_3} + c_{44}\frac{\partial^2 u_3}{\partial x_1^2} + c_{33}\frac{\partial^2 u_3}{\partial x_3^2} - \beta_3 \frac{\partial}{\partial x_3}\left\{\varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x_1^2} + a_3 \frac{\partial^2 \varphi}{\partial x_3^2}\right)\right\} = \rho \frac{\partial^2 u_3}{\partial t^2} \tag{9}$$

$$k_1 \frac{\partial^2 \varphi}{\partial x_1^2} + k_3 \frac{\partial^2 \varphi}{\partial x_3^2} = T_0 \frac{\partial^2}{\partial t^2}\left(\beta_1 \frac{\partial u_1}{\partial x_1} + \beta_3 \frac{\partial u_3}{\partial x_3}\right) + \rho C_E \frac{\partial^2}{\partial t^2}\left\{\varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x_1^2} + a_3 \frac{\partial^2 \varphi}{\partial x_3^2}\right)\right\} \tag{10}$$

$$t_{11} = c_{11}e_{11} + c_{13}e_{33} - \beta_1 T$$

$$t_{33} = c_{13}e_{11} + c_{33}e_{33} - \beta_3 T$$

$$t_{13} = 2c_{44}e_{13} \tag{11}$$

where

$$e_{11} = \frac{\partial u_1}{\partial x_1}, \qquad e_{33} = \frac{\partial u_3}{\partial x_3}, \qquad e_{13} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right),\,, \quad T = \varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x_1^2} + a_3 \frac{\partial^2 \varphi}{\partial x_3^2}\right)$$

$$\beta_{ij} = \beta_i \delta_{ij}\ , \qquad K_{ij} = K_i \delta_{ij}$$

$$\beta_1 = (c_{11} + c_{12})\alpha_1 + c_{13}\alpha_3\ , \qquad \beta_3 = 2c_{13}\alpha_1 + c_{33}\alpha_3$$

In the above equations we use the contracting subscript notations $(1 \to 11, 2 \to 22, 3 \to 33, 4 \to 23, 5 \to 13, 6 \to 12)$ to relate

$$c_{ijkl}\ to\ c_{mn} \tag{12}$$

The initial and regularity conditions are given by

$u_1(x_1, x_3, 0) = 0 = \dot{u_1}(x_1, x_3, 0)$

$u_3(x_1, x_3, 0) = 0 = \dot{u_3}(x_1, x_3, 0)$

$$\varphi(x_1, x_3, 0) = 0 = \dot{\varphi}(x_1, x_3, 0) \qquad For \quad x_3 \geq 0, \quad -\infty < x_1 < \infty \tag{13}$$

$$u_1(x_1, x_3, t) = u_3(x_1, x_3, t) = \varphi(x_1, x_3, t) = 0\ for\ t > 0\ when\ x_3 \to \infty \tag{14}$$

Assuming the harmonic behaviour as

$$(u_1, u_3, \varphi)(x_1, x_3, t) = (u_1, u_3, \varphi)(x_1, x_3)e^{i\omega t} \tag{15}$$

where ω is the angular frequency.
To facilitate the solution, following dimensionless quantities are introduced:

$$x_1' = \frac{x_1}{L}, \quad x_3' = \frac{x_3}{L}, \quad u_1' = \frac{\rho c_1^2}{L\beta_1 T_0}u_1, \quad u_3' = \frac{\rho c_1^2}{L\beta_1 T_0}u_3, \quad T' = \frac{T}{T_0}, \quad t' = \frac{c_1}{L}t, \quad t_{33}' = \frac{t_{33}}{\beta_1 T_0},$$

$$t_{31}' = \frac{t_{31}}{\beta_1 T_0}, \quad \varphi' = \frac{\varphi}{T_0}, \quad a_1' = \frac{a_1}{L}, \quad a_3' = \frac{a_3}{L}, \quad F_1' = \frac{F_1}{\beta_1 T_0}, \quad F_2' = \frac{F_2}{T_0}, \quad \omega' = \frac{\omega}{L} \tag{16}$$

where $c_1^2 = \frac{c_{11}}{\rho}$ and L is a constant of dimension of length.

The equations (8)-(10) with the aid of (12) ,(15) and (16) recast into the following form (after suppressing the primes)

$$\frac{\partial^2 u_1}{\partial x_1^2} + \delta_1 \frac{\partial^2 u_1}{\partial x_3^2} + \delta_2 \frac{\partial^2 u_3}{\partial x_1 \partial x_3} - \left[1 - \left(a_1 \frac{\partial^2}{\partial x_1^2} + a_3 \frac{\partial^2}{\partial x_3^2}\right)\right]\frac{\partial \varphi}{\partial x_1} = -\omega^2 u_1 \tag{17}$$

$$\delta_4 \frac{\partial^2 u_3}{\partial x_3^2} + \delta_1 \frac{\partial^2 u_3}{\partial x_1^2} + \delta_2 \frac{\partial^2 u_1}{\partial x_1 \partial x_3} - p_5\left[1 - \left(a_1 \frac{\partial^2}{\partial x_1^2} + a_3 \frac{\partial^2}{\partial x_3^2}\right)\right]\frac{\partial \varphi}{\partial x_3} == -\omega^2 u_3 \tag{18}$$

$$\frac{\partial^2 \varphi}{\partial x_1^2} + p_3 \frac{\partial^2 \varphi}{\partial x_3^2} + \zeta_1 \omega^2 \frac{\partial u_1}{\partial x_1} + \zeta_2 \omega^2 \frac{\partial u_3}{\partial x_3} = -\zeta_3 \left[1 - \left(a_1 \frac{\partial^2}{\partial x_1^2} + a_3 \frac{\partial^2}{\partial x_3^2}\right)\right]\omega^2 \varphi \tag{19}$$

where

$$\delta_1 = \frac{c_{44}}{c_{11}}, \quad \delta_2 = \frac{c_{13}+c_{44}}{c_{11}}, \quad \delta_4 = \frac{c_{33}}{c_{11}}, \quad p_5 = \frac{\beta_3}{\beta_1}, \quad p_3 = \frac{k_3}{k_1}, \quad \zeta_1 = \frac{T_0\beta_1^2}{k_1\rho}, \quad \zeta_2 = \frac{T_0\beta_3\beta_1}{k_1\rho}, \quad \zeta_3 = \frac{C_E c_{11}}{k_1}$$

Applying Fourier transform defined by

$$\hat{f}(\xi, x_3, \omega) = \int_{-\infty}^{\infty} \bar{f}(x_1, x_3, \omega)e^{i\xi x_1}dx_1 \tag{20}$$

on equations (17)-(19), we obtain a system of 3 homogeneous equations in terms of $\hat{u}_1$, $\hat{u}_3$ and $\hat{\varphi}$ which yield a non trivial solution if determinant of the coefficient ($\hat{u}_1$, $\hat{u}_3$,$\hat{\varphi}$) vanishes i.e. we obtain the following characteristic equation

$$\left(\frac{d^6}{dx_3^6} + Q\frac{d^4}{dx_3^4} + R\frac{d^2}{dx_3^2} + S\right)(\hat{u}_1, \hat{u}_3, \hat{\varphi}) = 0 \tag{21}$$

$Q = \frac{1}{P}(\xi^2 E + F)$

$R = \frac{1}{P}(G\xi^4 + H\xi^2 + I)$

$S = \frac{1}{P}(J\xi^6 + L\xi^4 + M\xi^2 + N)$

Where $P = \delta_1(-\delta_4\zeta_3 a_3\omega^2 - \delta_4 p_3 + \zeta_2 p_5 a_3\omega^2)$

$E = \omega^2\{\zeta_3 a_3(\delta_4 + \delta_1 b_1 - \delta_2^2) + \zeta_3 a_1 - \zeta_2(p_5 a_3 + \delta_1 p_5 a_1) + \delta_4 p_5 a_3\zeta_1 + \delta_4 a_3(\zeta_2 - \zeta_1)\} + p_3(\delta_4 + \delta_1 b_1 - \delta_2^2) + \delta_1\delta_4$

$F = \omega^4\{\zeta_3 a_3(\delta_4 - \delta_1) + \delta_1\delta_4\zeta_2 p_5 a_3\} + \omega^2\{p_3(\delta_4 - \delta_1) + \delta_1\delta_4(\zeta_3 - \delta_1\zeta_2 p_5)\}$

$G = \omega^2\{a_1(\zeta_2 p_5 - p_5\zeta_1\delta_2 - \delta_2\zeta_2 + \zeta_1\delta_4 - \delta_1^2\zeta_3 - a_3\delta_4 + \zeta_3\delta_2^2) + a_3(-\delta_1\zeta_3 + \delta_1\zeta_1) - \delta_1 p_3 - \delta_1^2 - \delta_4 + \delta_2^2\}$

$H = \omega^4\{a_3(\delta_1\zeta_3 + \zeta_3 - \zeta_1) + a_1(\zeta_3\delta_4 + \delta_1\zeta_3)\} + \omega^2\{p_5(\zeta_2 - \zeta_1\delta_2 - a_1\zeta_2\zeta_2) - \zeta_2\delta_2 + \zeta_1\delta_4 + p_3(\zeta_1 + 1) - \delta_1^2\zeta_3 + \delta_1 + \delta_4 - \zeta_3\delta_4 + \zeta_3\delta_2^2\}$

$I = -\omega^6\zeta_3 a_3 - \omega^4(\zeta_2 p_5 + p_3 - \delta_1\zeta_3 + \zeta_3\delta_4)$

$J = a_1\delta_1\omega^2(\zeta_3 - \zeta_1) + \delta_1$

$L = \omega^4(a_1\zeta_1 - a_1\zeta_3 - \delta_1\zeta_3 a_1) + \omega^2(\delta_1\zeta_3 - \delta_1 - \delta_1\zeta_1 - 1)$

Notes

Notes

$M = \omega^6\zeta_3 a_1 + \omega^4(-\zeta_3\delta_1 - \zeta_3 + 1 + \zeta_1)$

$N = \omega^6\zeta_3$

The roots of the equation (21) are $\pm\lambda_i$ ($i = 1,2,3$) satisfying the radiation condition that $\hat{u}_1, \hat{u}_3, \hat{\varphi} \to 0$ as $x_3 \to \infty$, the solution of the equation (21) can be written as

$$\hat{u}_1 = A_1 e^{-\lambda_1 x_3} + A_2 e^{-\lambda_2 x_3} + A_3 e^{-\lambda_3 x_3} \tag{22}$$

$$\hat{u}_3 = d_1 A_1 e^{-\lambda_1 x_3} + d_2 A_2 e^{-\lambda_2 x_3} + d_3 A_3 e^{-\lambda_3 x_3} \tag{23}$$

$$\hat{\varphi} = l_1 A_1 e^{-\lambda_1 x_3} + l_2 A_2 e^{-\lambda_2 x_3} + l_3 A_3 e^{-\lambda_3 x_3} \tag{24}$$

where

$$d_i = \frac{-\lambda_i^3 P^* - \lambda_i Q^*}{\lambda_1^4 R^* + \lambda_1^2 S^* + T^*} \quad i = 1,2,3 \tag{25}$$

$$l_i = \frac{\lambda_i^2 P^{**} + Q^{**}}{\lambda_1^4 R^* + \lambda_1^2 S^* + T^*} \quad i = 1,2,3 \tag{26}$$

Where $P^* = i\xi\{(\zeta_1 p_5 a_3\omega^2 - \delta_2(\zeta_3 a_3\omega^2 + p_3)\}$

$Q^* = \delta_2(\xi^2 + \zeta_3\omega^2 + \zeta_3\omega^2 a_1\xi^2) - p_5\zeta_1(1 + a_1\xi^2)\omega^2$

$R^* = \zeta_2 p_5 a_3\omega^2 - \delta_4(\zeta_3 a_3\omega^2 + p_3)$

$S^* = (\xi^2 + \zeta_3\omega^2 + \zeta_3\omega^2 a_1\xi^2)\delta_4 + (\delta_1\xi^2 - \omega^2)(a_3\zeta_3\omega^2 + p_3) - \zeta_2 p_5\omega^2(1 + a_1\xi^2)$

$T^* = -(\delta_1\xi^2 - \omega^2)(\xi^2 + \zeta_3\omega^2 + \zeta_3\omega^2 a_1\xi^2)$

$P^{**} = -(\zeta_2\delta_2 - \zeta_1\delta_4)\omega^2 i\xi$

$Q^{**} = -\zeta_1\omega^2(\delta_1\xi^2 - \omega^2)$

IV. Applications

On the half-space surface($x_3 = 0$) normal point force and thermal point source, which are assumed to be time harmonic, are applied. We consider two types of boundary conditions, as follows

Case 1. The normal force on the surface of half-space

The boundary conditions in this case are

(1) $t_{33}(x_1, x_3, t) = -F_1\psi_1(x)e^{i\omega t}$

(2) $t_{31}(x_1, x_3, t) = 0$

(3) $\frac{\partial\, \varphi(x_1, x_3, t)}{\partial x_3} = 0$ at $x_3 = 0$ (27)

where F_1 is the magnitude of the force applied, $\psi_1(x)$ specify the source distribution function along x_1 axis.

Case 2. The thermal source on the surface of half-space

When the plane boundary is stress free and subjected to thermal point source, the boundary conditions in this case are

(1) $t_{33}(x_1, x_3, t) = 0$

(2) $t_{31}(x_1, x_3, t) = 0$

(3) $\frac{\partial\, \varphi(x_1, x_3, t)}{\partial x_3} = F_2\psi_1(x)e^{i\omega t}$ at $x_3 = 0$ (28)

where F_2 is the constant temperature applied on the boundary, $\psi_1(x)$ specify the source distribution function along x_1 axis.

a) Green's function

To synthesize the Green's function, i.e. the solution due to concentrated normal force and thermal source on the half-space is obtained by setting

$$\psi_1(x) = \delta(x) \tag{29}$$

In equations (27) and (28). Applying the Fourier transform defined by (20) on the equation (29) gives

$$\hat{\psi}_1(\xi) = 1 \tag{30}$$

Subcase 1(a). Mechanical force

Substitute the values of $\hat{u}_1, \widehat{u}_3$ *and* $\hat{\varphi}$ from (22)-(24) in the boundary conditions (27) and with the aid of (1), (4)-(7), (12), (15) ,(16) and (20),we obtain the components of displacement, normal stress, tangential stress and conductive temperature as

$$\hat{u}_1 = \frac{F_1\hat{\psi}_1(\xi)}{\Delta}\left(-M_{11}e^{-\lambda_1 x_3} + M_{12}e^{-\lambda_2 x_3} - M_{13}e^{-\lambda_3 x_3}\right)e^{i\omega t} \tag{31}$$

$$\hat{u}_3 = \frac{F_1\hat{\psi}_1(\xi)}{\Delta}\left(-d_1M_{11}e^{-\lambda_1 x_3} + d_2M_{12}e^{-\lambda_2 x_3} - d_3M_{13}e^{-\lambda_3 x_3}\right)e^{i\omega t} \tag{32}$$

$$\hat{\varphi} = \frac{F_1\hat{\psi}_1(\xi)}{\Delta}\left(-l_1M_{11}e^{-\lambda_1 x_3} + l_2M_{12}e^{-\lambda_2 x_3} - l_3M_{13}e^{-\lambda_3 x_3}\right)e^{i\omega t} \tag{33}$$

$$\widehat{t_{33}} = \frac{F_1\hat{\psi}_1(\xi)}{\Delta}\left(-\Delta_{11}M_{11}e^{-\lambda_1 x_3} + \Delta_{12}M_{12}e^{-\lambda_2 x_3} - \Delta_{13}M_{13}e^{-\lambda_3 x_3}\right)e^{i\omega t} \tag{34}$$

$$\widehat{t_{31}} = \frac{F_1\hat{\psi}_1(\xi)}{\Delta}\left(-\Delta_{21}M_{11}e^{-\lambda_1 x_3} + \Delta_{22}M_{12}e^{-\lambda_2 x_3} - \Delta_{23}M_{13}e^{-\lambda_3 x_3}\right)e^{i\omega t} \tag{35}$$

where

$$M_{11} = \Delta_{22}\Delta_{33} - \Delta_{32}\Delta_{23}\ , \quad M_{12} = \Delta_{21}\Delta_{33} - \Delta_{23}\Delta_{31}, \quad M_{13} = \Delta_{21}\Delta_{32} - \Delta_{22}\Delta_{31}$$

$$M_{21} = \Delta_{12}\Delta_{33} - \Delta_{13}\Delta_{22}, \quad M_{22} = \Delta_{11}\Delta_{33} - \Delta_{13}\Delta_{31}, \quad M_{23} = \Delta_{11}\Delta_{32} - \Delta_{12}\Delta_{31}$$

$$\Delta_{1j} = \frac{c_{31}}{\rho c_1^2}i\xi - \frac{c_{33}}{\rho c_1^2}d_j\lambda_j - \frac{\beta_3}{\beta_1}l_j + \frac{\beta_3}{\beta_1 T_0}a_3 l_j {\lambda_j}^2 - \frac{\beta_3}{\beta_1}l_j a_1\xi^2, \ \ j = 1,2,3$$

$$\Delta_{2j} = -\frac{c_{44}}{\rho c_1^2}\lambda_j + \frac{c_{44}}{\rho c_1^2}i\xi d_j \quad j = 1,2,3$$

$$\Delta_{3j} = l_j\ \lambda_j \ \ j = 1,2,3$$

$$\Delta = \Delta_{11}M_{11} - \Delta_{12}M_{12} + \Delta_{13}M_{13}$$

Subcase 2(a). Thermal source on the surface of half-space

Making use of (1), (4)-(7), (12),(15) and (16)in B.C. (28) , and applying Fourier Transform defined by (20) and substituting the values of $\hat{u}_1, \widehat{u}_3$ *and* $\hat{\varphi}$ from (22)-(24) in the resulting equations, we obtain the components of displacement, normal stress, tangential stress and conductive temperature are as given by equations (31)-(35) with M_{11}, M_{12} *and* M_{13} replaced by M_{31}, M_{32} and M_{33} respectively and F_1 replaced by F_2.

Where $M_{31} = \Delta_{12}\Delta_{23} - \Delta_{13}\Delta_{22}$, $M_{32} = \Delta_{11}\Delta_{23} - \Delta_{13}\Delta_{21}$, $M_{33} = \Delta_{11}\Delta_{22} - \Delta_{12}$ (36)

b). Influence function

The method to obtain the half-space influence function, i.e. the solution due to distributed load applied on the half space is obtained by setting

$$\psi_1(x) = \begin{cases} 1 \ if \ |x| \le m \\ 0 \ if \ |x| > m \end{cases} \tag{37}$$

In equations (27) and (28). The Fourier transforms of $\psi_1(x)$ with respect to the pair (x, ξ) for the case of a uniform strip load of non dimensional width 2m applied at origin of co-ordinate system $x_1 = x_3 = 0$ in the dimensionless form after suppressing the primes becomes

$$\hat{\psi}_1(\xi) = \left[2\sin(\xi m) / \xi \right], \xi \neq 0 \tag{38}$$

The expressions for displacement, stresses and conductive temperature can be obtained for uniformly distributed normal force and thermal source by replacing $\hat{\psi}_1(\xi)$ from (38) respectively in equations (31)-(35) along with (36)

V. Particular Cases

(i): If $a_1 = a_3 = 0$, from equations (31)-(35) ,we obtain the corresponding expressions for displacement, stresses and conductive temperature in thermoelastic solid without energy dissipation.

(ii) If we take $a_1 = a_3 = a$, $c_{11} = \lambda + 2\mu = c_{33}$, $c_{12} = c_{13} = \lambda$, $c_{44} = \mu$, $\beta_1 = \beta_3 = \beta$, $\alpha_1 = \alpha_3 = \alpha$, $K_1 = K_3 = K$ in equations $(31) - (35)$ we obtain the corresponding expressions for displacements, stresses and conductive temperature for isotropic thermoelastic solid without energy dissipation.

VI. Inversion of the Transformation

To obtain the solution of the problem in physical domain, we must invert the transforms in equations (31)-(35) . Here the displacement components, normal and tangential stresses and conductive temperature are functions of x_3 and the parameters of Fourier transforms ξ and hence are of the form $f(\xi, x_3)$. To obtain the function $f(x_1, x_3)$ in the physical domain, we first invert the Fourier transform as used by Sharma, Kumar and Ram [13] using

$$f(x_1, x_3) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\xi x_1} \hat{f}(\xi, x_3) d\xi = \frac{1}{2\pi} \int_{-\infty}^{\infty} |\cos(\xi x_1) f_e - i sin(\xi x_1) f_o| \, d\xi \tag{39}$$

Where f_e and f_o are respectively the even and odd parts of $\hat{f}$ (ξ, x_3). The method for evaluating this integral is described in Press et al. [15] . It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

VII. Numerical Results and Discussion

Copper material is chosen for the purpose of numerical calculation which is transversely isotropic. Physical data for a single crystal of copper is given by

$c_{11} = 18.78 \times 10^{10}\ Kgm^{-1}s^{-2}$, $c_{12} = 8.76 \times 10^{10}\ Kgm^{-1}s^{-2}$, $c_{13} = 8.0 \times 10^{10}\ Kgm^{-1}s^{-2}$
$c_{33} = 17.2 \times 10^{10}\ Kgm^{-1}s^{-2}$, $c_{44} = 5.06 \times 10^{10}\ Kgm^{-1}s^{-2}$ $C_E = 0.6331 \times 10^3 JKg^{-1}K^{-1}$
$\alpha_1 = 2.98 \times 10^{-5} K^{-1}$, $\alpha_3 = 2.4 \times 10^{-5} K^{-1}$, $\rho = 8.954 \times 10^3 Kgm^{-3}$,
$k_1 = 0.02 \times 10^2 Nsec^{-2}deg^{-1}$, $k_3 = 0.04 \times 10^2 Nsec^{-2}deg^{-1}$

Following Dhaliwal and Singh [5], magnesium crystal is chosen for the purpose of numerical calculation(isotropic solid). In case of magnesium crystal like material for numerical calculations, the physical constants used are

$$\lambda = 2.17 \times 10^{10} Nm^2, \quad \mu = 3.278 \times 10^{10} Nm^2 \ , \quad K = 0.02 \times 10^2 Nsec^{-2}deg^{-1}$$

Ref

15. Press,W.H., Teukolshy,S.A., Vellerling,W.T.,and Flannery, B.P; Numerical recipes in Fortran, Cambridge University Press, Cambridge,(1986).

$\omega_1 = 3.58 \times 10^{11} S^{-1}$ $\beta = 2.68 \times 10^{6}\ Nm^{-2}deg^{-1}$, $\rho = 1.74 \times 10^{3} Kgm^{-3}$

$T_0 = 298\mathrm{K}$, $C_E = 1.04 \times 10^{3} Jkg^{-1}deg^{-1}$

The values of normal displacement u_3, normal force stress t_{33} , tangential stress t_{31} and conductive temperature φ for a transversely isotropic thermoelastic solid (TIT) and for isotropic thermoelastic solid (IT) are presented graphically for the non-dimensional frequencies ω=.25, ω=.5 and ω=.75. Two temperature parameter for (TIT) are taken as $a_1 = 0.03$ and $a_3 = 0.05$ whereas for (IT) ,the two temperature parameter are taken as $a_1 = a_3 = 0.04$

1). The solid line, small dashed line and long dashed line , respectively corresponds to isotropic solid with frequencies ω=.25, ω=.5 and ω=.75 respectively and $a_1 = a_3 = 0.04$

2). The solid line with centre symbol circle , the small dashed line with centre symbol diamond and the long dashed line with centre symbol cross respectively correspond to transversely isotropic solid with frequencies ω=.25, ω=.5 and ω=.75 respectively and $a_1 = 0.03$ and $a_3 = 0.05$

a) Normal force on the surface of half-space

i. *Concentrated force*

Fig.1 shows the variations of the normal displacement u_3. The values of u_3 (TIT), follow oscillatory pattern for ω=.75 and for ω=.5, whereas for ω=.25,variations are very small owing to scale of graph. For u_3(IT), corresponding to the three frequencies, behaviour is oscillatory with difference in the magnitude. Fig.2 depicts the values of normal stress t_{33}. Near the loading surface, the values of t_{33} (TIT) increase sharply corresponding to the three frequencies but away from the loading surface, these oscillate for ω=.5 and ω=.75, however for ω=.25, it is descending oscillatory. For t_{33} (IT), small variations are observed corresponding to three frequencies. Fig.3 describes the variations of tangential stress t_{31}. For both the mediums (i.e. IT and TIT), variations in t_{31} are oscillatory for ω=.5 and ω=.75 where as for ω=.25, it increases near the loading surface and then decreases i.e. somehow oscillates. Fig.4 interprets the variations of conductive temperature φ .The values of φ (IT), for ω=.25 and ω=.5 increase sharply near the loading surface and then decrease i.e. are oscillatory with difference in magnitude whereas for ω=.75 it also oscillates with small magnitude. φ (TIT) shows small oscillations for ω=.5 and ω=.75 whereas variations for ω=.25 are very small in the whole range.

ii. *Uniformly Distributed force*

Fig. 5-8 show the characteristics for uniformly distributed force. It is depicted from Fig.5-Fig.8 that the distribution curves for u_3, normal stress t_{33}, tangential stress t_{31} and conductive temperature φ for uniformly distributed force, follow same trends as in case of concentrated force for both the mediums with difference in magnitudes in their respective patterns.

b) Thermal source on the surface of half-space

i. *Concentrated Thermal Source*

Fig.9 shows the variations of normal displacement u_3 when concentrated thermal source is applied. It is depicted that the variations in u_3 for both the mediums follow oscillatory pattern corresponding to the three frequencies with difference in their magnitude, except for ω=.25 (TIT). In case ω=.25 (TIT), small variations are observed. Fig.10. explains variations of normal stress t_{33}, near the loading surface, values of t_{33} (IT) increase, whereas a decrease is seen in t_{33}(TIT) ,but away from the loading surface, behaviour is oscillatory in the whole range with difference in their magnitudes corresponding to the three frequencies. Fig.11 displays the picture about the behaviour of tangential stress t_{31}, here for t_{31} (IT) ,there is a sharp increase in the range $0 \leq x \leq 2$ for ω=.5 and ω=.75 and afterwards pattern is oscillatory, whereas for

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t_{31}(TIT), oscillatory pattern is observed in the whole range corresponding to these frequencies. For ω=.25, different type of variations are observed as compared with ω=.5 and ω=.75 .Fig. 12 shows the movements of conductive temperature φ , here for both the mediums oscillatory variations are depicted for ω=.5 and ω=.75 whereas there are small variations for ω=.25(TIT) and movements are oscillatory for ω=.25 (IT).

ii. *Uniformly Distributed thermal source*

Fig.13 –Fig.16 show that variations in normal displacement u_3, normal stress t_{33}, tangential stress t_{31} and the conductive temperature φ for both the mediums are of similar pattern as in case of concentrated thermal source with change in magnitude. In some figures, these appear as the mirror image of the figures of concentrated thermal source.

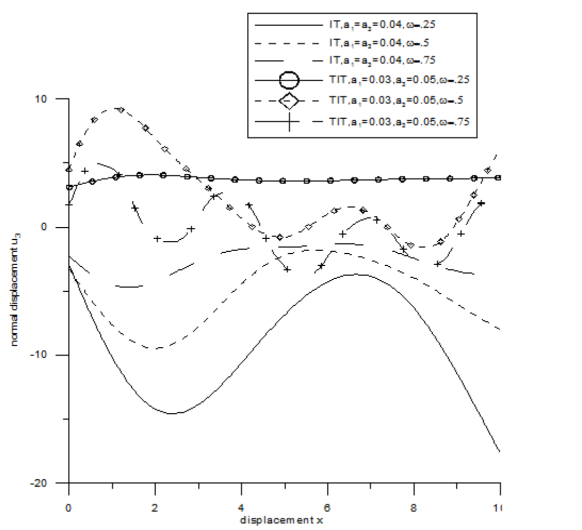

Figure 1 : Variation of Normal Displacement U_3 with Distance X(Concentrated Force)

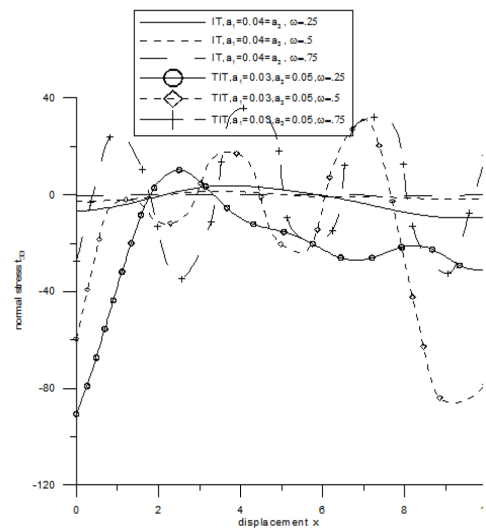

Figure 2 : Variation of Normal Stress t_{33} with Distance X(Concentrated Force)

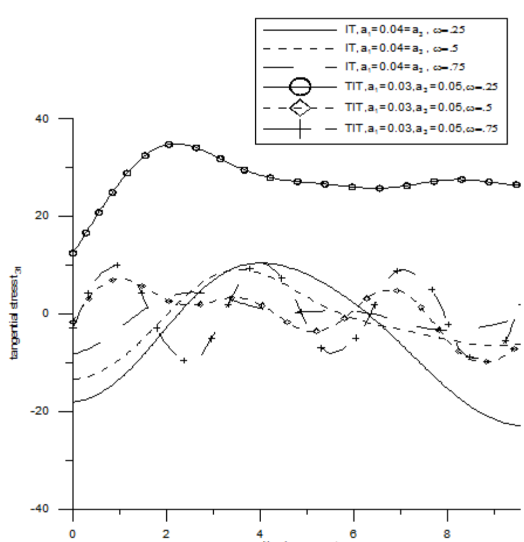

Figure 3 : Variation of Tangential Stress t_{31} with Distance X(Concentrated Force)

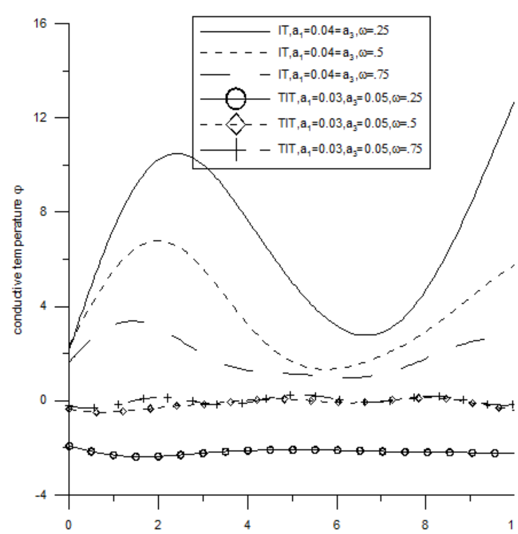

Figure 4 : Variation of Conductive Temperature φ with Distance X(Concentrated Force)

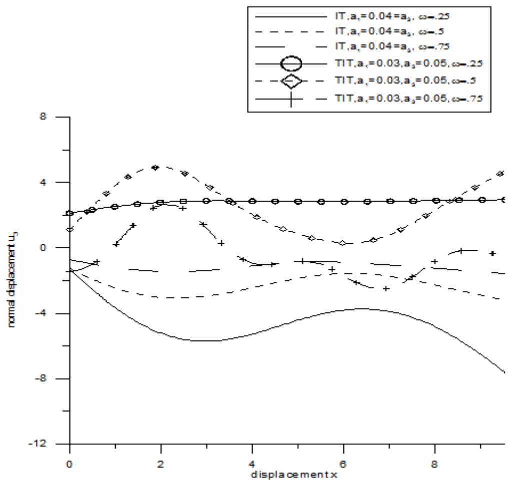

Figure 5 : *Variation of Normal Displacement U_3 with Distance X(Uniformly Distributed Force)*

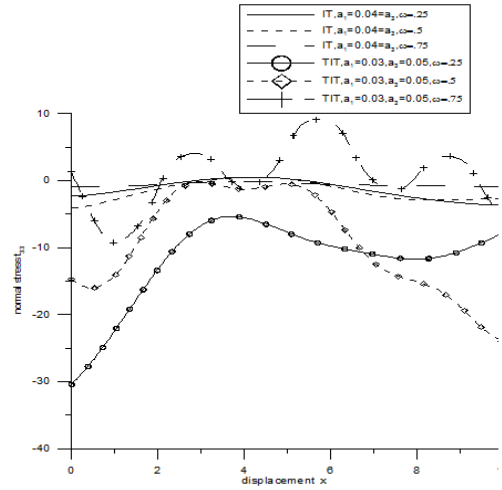

Figure 6 : *Variation of Normal Stress t_{33} with Distance X(Uniformly Distributed Force)*

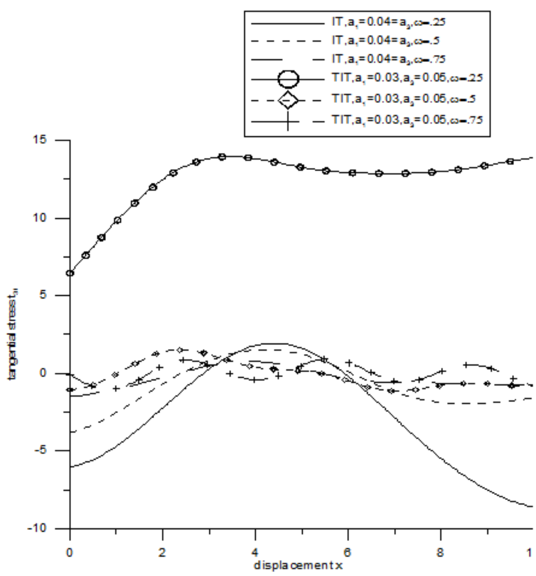

Figure 7 : *Variation of Tangential Stress t_{31} with Distance X(Uniformly Distributed Force)*

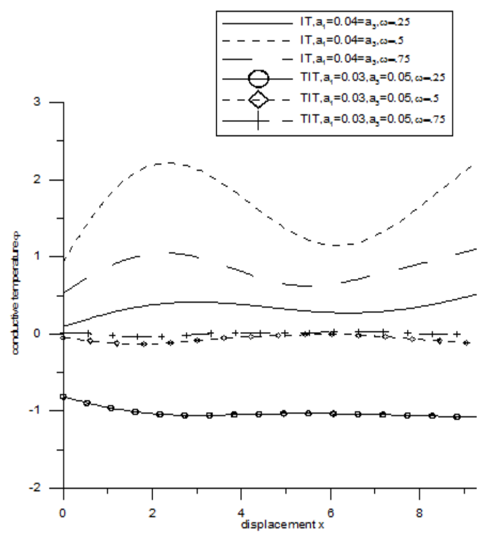

Figure 8 : *Variation of Conductive Temperature φ with Distance X(Uniformly Distributed Force)*

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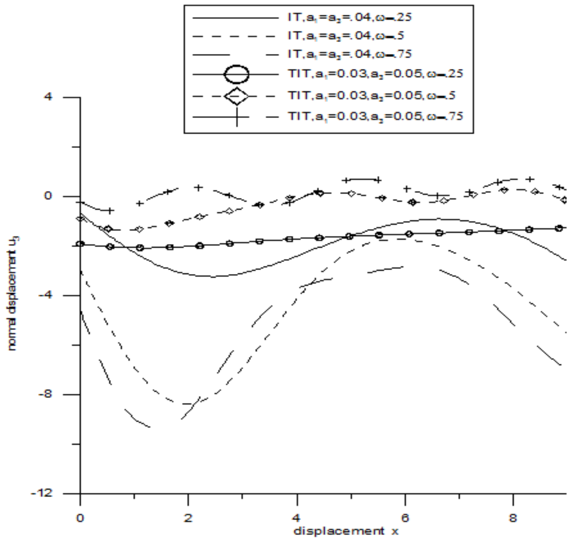

Figure 9 : *Variation of Normal Displacement* U_3 *with Distance X(Concentrated Thermal Source)*

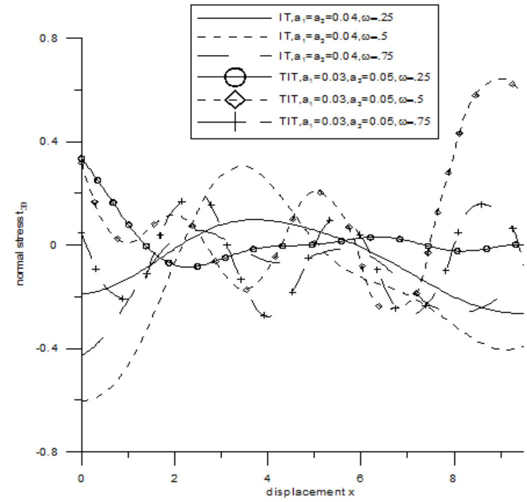

Figure 10 : *Variation of Normal Stress* t_{33} *with Distance X(Concentrated Thermal Source)*

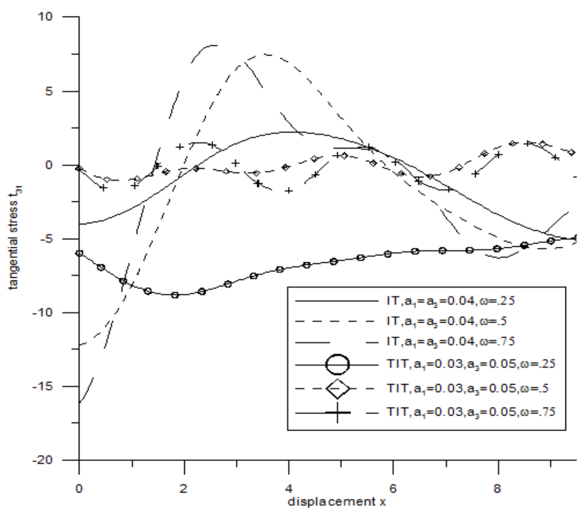

Figure 11 : *Variation of Tangential Stress* t_{31} *with Distance X(Concentrated Thermal Source)*

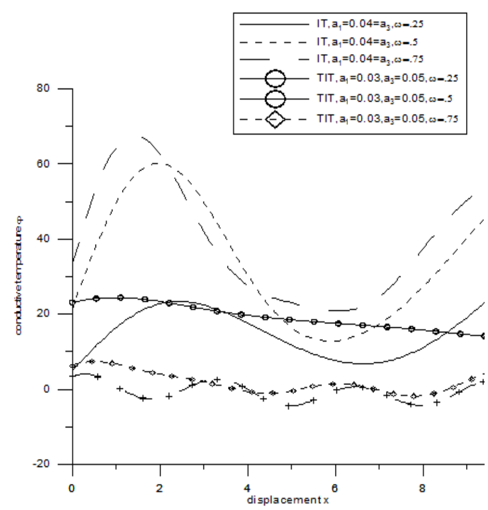

Figure 12 : *Variation of Conductive Temperature* φ *with Distance X(Concentrated Thermal Source)*

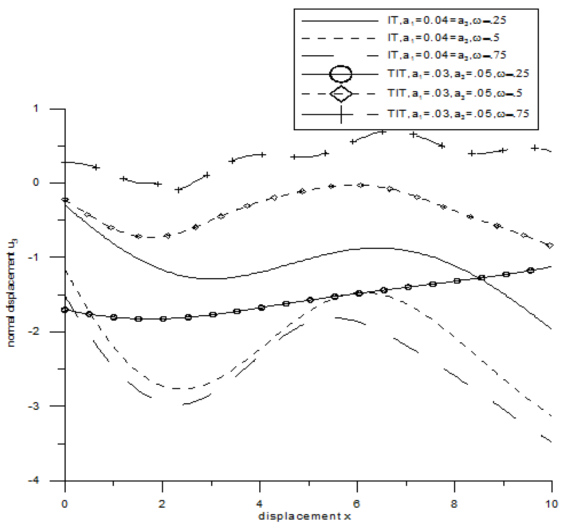

Figure 13 : Variation of Normal Displacement U_3 with Distance X(Uniformly Distributed Thermal Source)

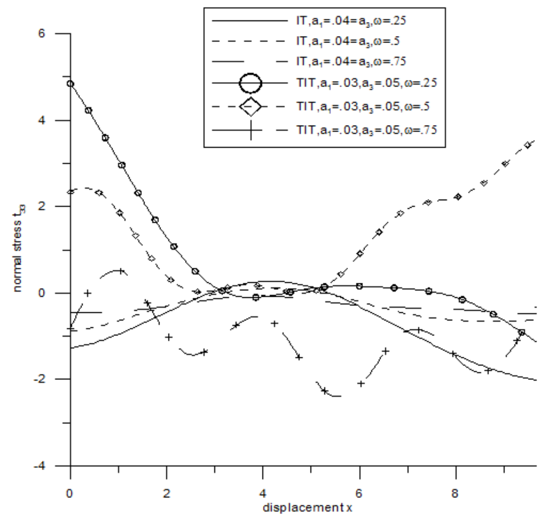

Figure 14 : Variation of Normal Stress t_{33} with Distance X(Uniformly Distributed Thermal Source)

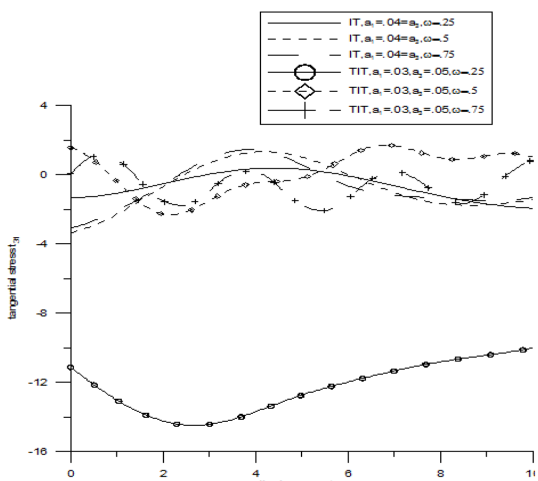

Figure 15 : Variation of Tangential Stress t_{31} with Distance X(Uniformly Distributed Thermal Source)

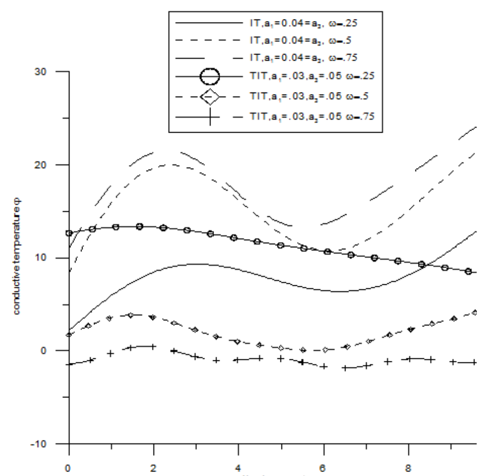

Figure 16 : Variation of Conductive Temperature φ with Distance X(Uniformly Distributed Thermal Source)

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VII. Conclusion

From the graphs, it is observed that effect of anisotropy plays important role in the deformation of the body. As disturbance travels through the constituents of the medium, it suffers sudden changes resulting in an inconsistent/non uniform pattern of graphs. Anisotropy has significant impact on components of normal displacement, normal stress, tangential stress and conductive temperature. It is observed from the figures(1-8) that the trends in the variations of the characteristics mentioned are similar with difference in their magnitude when the mechanical forces (i.e. concentrated or distributed forces)are applied , where as the trends are also similar when thermal sources (i.e. concentrated or distributed forces) are applied as is observed in figures (8-16). The trend of curves exhibits the properties of the medium and satisfies requisite condition of the problem. It can also contribute to the theoretical considerations of the seismic and volcanic sources since it can account for deformation fields in the entire volume surrounding the source region.

Notes

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