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By Roman Bihun & Gregory Tsehelyk
Ivan Franko National University of Lviv, Ukraine

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Numerical Method for Finding All Points of Extremum of Random as Smooth and Non-Smooth Functions of One Variable

Roman Bihun [α] & Gregory Tsehelyk [σ]

Abstract- A device of non-classic Newton's minorant and their graphs of functions of two real table-like variables have been introduced and a new numerical method for finding extremum of random as smooth and non-smooth functions of one real variable has been constructed.

Keywords: minorant and majorant of function, numerical analysis, optimization method.

I. Introduction

In [2,3] a device of non-classical Newton's majorants and diagrams of functions given in tabular form is constructed and its usage for: the approximation of functions; construction, calculation of the definite integrals and numerical methods for solving the Cauchy problem for ordinary differential equations and their systems, accurate to a certain class of functions is discussed (leaving aside the rounding transaction); optimization both smooth and non-smooth logarithmically concave functions of one and several real variables.

In [1,4] for the first time a device of non-classical Newton's minorants of functions given in tabular form is constructed, which is used for the approximation of functions and development of numerical optimization methods as smooth and non-smooth logarithmically convex functions of one and two real variables.

II. Device of Non-Classical Newton's Majorants and Minorants of Functions, Given in Tabular Form, and Their Diagrams

Let consider the function of a real variable $y = f(x)$, which defined its values at some points x_i, $i = 0,1, \dots, n$:

$$f(x_i) = y_i, \quad i = 0,\ 1,\ \dots,n. \tag{1}$$

Let

$$|y_i| = a_i \le M, \quad i = 0,\ 1,\ \dots,n, \quad a_1 \cdot a_n \ne 0, \tag{2}$$

where M – certain constant.

Definition 1. Point $P_i\ (x_i, -\ln a_i)$ coordinates $x = x_i$, $y = -\ln a_i$ in space xy called *bitmaps value function* $y = f(x)$ in the point $x = x_i$.

Assume that the points of the image P_i of the function $y = f(x)$ at points x_i, $i = 0,\ 1,\ \dots,\ n$, in plane xy are built. From every point P_i we draw a half-line in

Author α σ: Faculty of applied mathematics and informatics, Ivan Franko National University of Lviv. e-mails: bigunroman@ukr.net, kafmmsep@lnu.edu.ua

positive direction of the axis Oy, perpendicular to the axis Ox. The set of these half-lines is denoted by S, and its convex hull – by $C(S)$. For each point $x \in [x_0,\ x_n]$ we define the point $B_x(x,\ \chi_x)$, where

$$\chi_x = \inf_{(x,y)\ \in\ C(S)} y.$$

The set of points $B_x(x,\ \chi_x)$, $x \in [x_0,\ x_n]$, forms a line δ_f, which limits $C(S)$ below. This line is continuous, convex, broken and its equation is

$$y = \chi(x), \quad x \in [x_0,\ x_n],$$

where $\chi(x) = \chi_x$.

Definition 2. Broken line δ_f, defined on the interval $[x_0,\ x_n]$, called *non-classical Newton's diagram* of function $y = f(x)$ on this interval.

Newton's diagram δ_f of function $y = f(x)$ has the following properties:

– each vertex δ_f is placed in one of the *bitmaps* P_i of value of the function $y = f(x)$ at the point x_i, $i = 0,\ 1,\ \ldots,\ n$;

– each bitmap P_i, $i = 0,\ 1,\ \ldots,\ n$, is located on δ_f or above it.

Let

$$M_f(x) = \exp\ (-\chi(x)), \quad x \in [x_0,\ x_n].$$

Then for each point x_i, $i = 0,\ 1,\ \ldots,\ n$, the inequality is performed

$$|f(x_i)| \ = a_i \leq M_f(x_i).$$

In fact, with the construction of δ_f follows that

$$-\ln\ |f(x_i)| \ \geq \chi(x_i),$$

or

$$|f(x_i)| \leq \exp\ (-\chi(x_i)) = M_f(x_i).$$

Besides,

$$M_f(x_0) = \ |f(x_0)|, \quad M_f(x_n) = \ |f(x_n)|.$$

Definition 3. Function $y = M_f(x)$, defined on the interval $[x_0,\ x_n]$, called *non-classical Newton's majorant* of function $y = f(x)$ on this interval.

Let

$$M_f(x_i) = T_i, \qquad i = 0,\ 1,\ \ldots,\ n.$$

Definition 4. Values

$$R_i = \left(\frac{T_{i-1}}{T_i}\right)^{\frac{1}{x_i - x_{i-1}}} \quad (i = 1,\ 2,\ \ldots,\ n;\ R_0 = 0)$$

and

$$D_i = \frac{R_{i+1}}{R_i} \quad (i = 1,\ 3,\ \ldots,\ n-1;\ D_0 = D_n = \infty)$$

called, respectively, *i-th numerical inclination* and *i-th deviation* of Newton's diagram δ_f.

Notes

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Definition 5. If the bitmap P_i, $i = 0, 1, \ldots, n$, is located at the top of δ_f, then index i is called *vertex index*, if it is placed on δ_f, - then it is called *diagram index* of δ_f. Indexes $i = 0$ та $i = n$ belong to vertex indexes.

The set of vertex indices we denote by I, and the set of diagram indexes – by G. Obviously, $I \subset G$ and $T_i = a_i$ for all $i \in G$.

Newton's diagram was constructed for function given in nine points in fig.1.

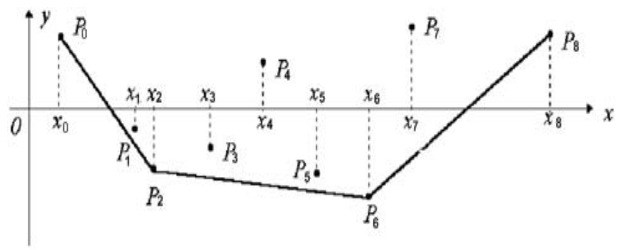

Figure 1 : Newton's diagram for function given in nine points

Now let the points of the image P_i of the function $y = f(x)$ at points x_i, $i = 0, 1, \ldots, n$, in plane xy are built. From every point P_i we draw a half-line in negative direction of the axis Oy, perpendicular to the axis oci Ox. The set of these half-lines is denoted by S, and its convex hull – by $C(S)$. For each point $x \in [x_0, x_n]$ we define the point $D_x(x, \chi_x)$, where

$$\chi_x = \sup_{(x,y) \in C(S)} y.$$

The set of points $D_x(x, \chi_x)$, $x \in [x_0, x_n]$ forms a line δ_f, which limits $C(S)$ top. This line is continuous, concave, broken and its equation is

$$y = \chi(x), x \in [x_0, x_n],$$

where $\chi(x) = \chi_x$.

Let $m_f(x) = \exp(-\chi(x)), x \in [x_0, x_n]$.

Then for each point x_i, $i = 0, 1, \ldots, n$, the inequality is performed

$$m_f(x_i) \leq |f(x_i)| = a_i.$$

In fact, with the construction of δ_f follows that

$$-\ln|f(x_i)| \leq \chi(x_i),$$

or

$$|f(x_i)| \geq \exp(-\chi(x_i)) = m_f(x_i).$$

Besides,

$$m_f(x_0) = |f(x_0)|, m_f(x_n) = |f(x_n)|.$$

Definition 6. The function $y = m_f(x)$, defined on the interval $[x_0, x_n]$, called *non-classical Newton's minorant* of function $y = f(x)$ on this interval, and broken line δ_f – its diagram.

Newton's minorant diagram was constructed for function given in nine points in fig.2.

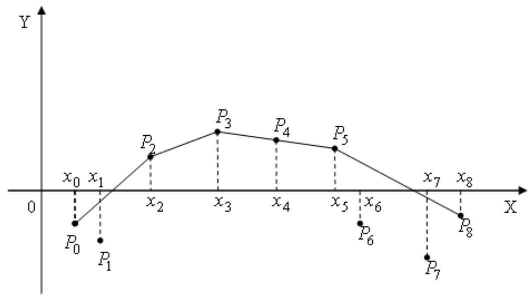

Figure 2 : Newton's minorant diagram for function given in nine points

Newton's minorant diagram δ_f of function $y = f(x)$ has the following properties:
– each vertex δ_f is placed in one of the *bitmaps* P_i of value of the function $y = f(x)$ at the point x_i, $i = 0, 1, \ldots, n$;
– each bitmap P_i, $i = 0, 1, \ldots, n$, is located on δ_f or below it.
Let

$$m_f(x_i) = t_i, \quad i = 0,1, \ldots, n.$$

Definition 7. Values

$$r_i = \left(\frac{t_{i-1}}{t_i}\right)^{\frac{1}{x_i - x_{i-1}}} \quad (i = 1,2, \ldots, n; \quad r_0 = \infty)$$

and

$$d_i = \frac{r_{i+1}}{r_i} \quad (i = 1,2, \ldots, n-1; \quad d_0 = d_n = 0)$$

called, respectively, *i-th numerical inclination* and *i-th deviation* of Newton's minorant diagram δ_f.

Let $f(x)$ is logarithmically concave function on the interval $[a, b]$. Let us choose on the interval $[a, b]$ points system $x_0, x_1, \ldots, x_n$, where $x_k = x_0 + kh$ $(k = 0, 1, \ldots, n)$, $x_0 = a$, $h = \frac{b-a}{n}$, and find the value of the function $y = f(x)$ at these points. Let

$$f(x_i) = c_i, \quad i = 0, 1, \ldots, n.$$

Since the $f(x)$ – logarithmically concave function on the interval $[a, b]$, then numerical inclinations of Newton's majorants, which were built on the values of the function at the points $x_1, x_2, \ldots, x_n$, are determined by the formula

$$R_k = \left(\frac{c_{k-1}}{c_k}\right)^{\frac{1}{h}} \quad (k = 1, 2, \ldots, n; R_0 = 0).$$

In this case

$$R_1 < R_2 < \ldots < R_n.$$

Deviations D_k of Newton's majorants will satisfy the condition

$$D_k > 1 \quad (k = 1, 2, \ldots, n-1; D_0 = D_n = \infty).$$

If for some index k $(0 < k < n)$ the conditions $R_k \leq 1$, $R_{k+1} > 1$ accomplish, then the point x_k with accuracy $\varepsilon < h$ is a maximum point of function $f(x)$.

Notes

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Now let $f(x)$ is logarithmically convex function on the interval $[a, b]$. Similarly choose on the interval $[a,\ b]$ points system $x_0,\ x_1, \ldots,\ x_n$, where $x_k = x_0 + kh$ $(k = 0,\ 1, \ldots,\ n)$, $x_0 = a$, $h = \frac{b-a}{n}$, and find the value of the function $y = f(x)$ at these points. Let

$$f(x_i) = c_i, \quad i = 0,\ 1,\ \ldots,\ n.$$

Since the $f(x)$ – logarithmically convex function on the interval $[a,\ b]$, then numerical inclinations of Newton's minorants, which were built on the values of the function at the points $x_1,\ x_2,\ \ldots,\ x_n$, are determined by the formula

$$r_k = \left(\frac{c_{k-1}}{c_k}\right)^{\frac{1}{h}} \quad (k = 1,\ 2,\ \ldots,\ n;\ r_0 = \infty).$$

In this case

$$r_1 > r_2 > \ . \ . \ . > r_n.$$

Deviations d_k of Newton's minorants will satisfy the condition

$$0 < d_k < 1 \quad (k = 1,\ 2,\ \ldots,\ n-1;\ d_0 = d_n = 0).$$

If for some index k $(0 < k < n)$ the conditions $r_k \geq 1$, $r_{k+1} < 1$ accomplish, then the point x_k with accuracy $\varepsilon < h$ is a minimum point of function $f(x)$.

III. Numerical Method for Finding All Points of Extremum of Random as Smooth and Non-Smooth Functions at Preset Interval

Let we have to find all points of extremum of function $y = f(x)$ at preset interval $[a, b]$. We assume that $f(x) > 0$ for all $x \in [a, b]$.

Choose on the interval $[a, b]$ points system $x_0, x_1, \ldots, x_n$, where $x_k = x_0 + kh$ $(k = 0,1, \ldots, n)$, $x_0 = a$, $h = \frac{b-a}{n}$, and find the value of function $y = f(x)$ at these points. Let

$$f(x_i) = c_i\,, \quad i = 0,1, \ldots, n.$$

Put

$$\tilde{r}_k = \left(\frac{c_{k-1}}{c_k}\right)^{\frac{1}{h}}, \qquad k = 1,2, \ldots, n.$$

Then on the intervals $[\alpha, \beta] \in [a, b]$, where the function $f(x)$ is convex,

$$\tilde{r}_i \geq \tilde{r}_{i+1},$$

and the intervals where the function $f(x)$ concave,

$$\tilde{r}_i \leq \tilde{r}_{i+1},$$

a) Algorithm of the method

The algorithm of the method consists of series of steps. In the first step we choose the point x_0 and x_1 and find $\tilde{r}_1$. Then the following two possible cases:

$$1) \qquad \tilde{r}_1 \leq 1, \qquad\qquad 2)\ \tilde{r}_1 > 1$$

In the first case we calculate $\tilde{r}_2, \tilde{r}_3, \ldots$ until for some i $(i \geq 1)$ condition $\tilde{r}_{i+1} > 1$ does not perform. Then point x_i with accuracy $\varepsilon < h$ is taken as a point of local maximum of function $f(x)$.

In the second case we calculate $\tilde{r}_2, \tilde{r}_3, \ldots$ until for some i $(i \geq 1)$ condition $\tilde{r}_{i+1} < 1$ does not perform. Then point x_i with accuracy $\varepsilon < h$ s taken as a point of local minimum of function $f(x)$. In the second step the point as a starting point x_i, found in the first step. Then, if $\tilde{r}_{i+1} \leq 1$, we search $\tilde{r}_{i+2}, \tilde{r}_{i+3}, \ldots$ until for some k $(k > 1)$ condition $\tilde{r}_{i+k} > 1$ does not perform. The point x_{i+k-1} is taken as a point of local maximum with accuracy $\varepsilon < h$ of the function $f(x)$. If $\tilde{r}_{i+1} > 1$, we search $\tilde{r}_{i+2}, \tilde{r}_{i+3}, \ldots$ until for some k $(k > 1)$ condition $\tilde{r}_{i+k} \leq 1$ does not performed. The point x_{i+k-1} is taken as a point of local maximum with accuracy $\varepsilon < h$ of the function $f(x)$.

Notes

The process ends when we found the point x_l, which is a point of local extremum, and the sequence $\tilde{r}_{l+1}, \tilde{r}_{l+2}, \ldots, \tilde{r}_n$ is either decreasing or increasing.

b) Example

We will consider the problem of function optimization

$$f(x) = 8x^6 - 3x^5 - 4x^4 + x^3 - 5x^2 + 4x + 10; \qquad (3)$$

on the interval $[-1; 1]$ with step $h = 0{,}1$ $(n = 20)$.

The graph of this function is shown in fig. 3.

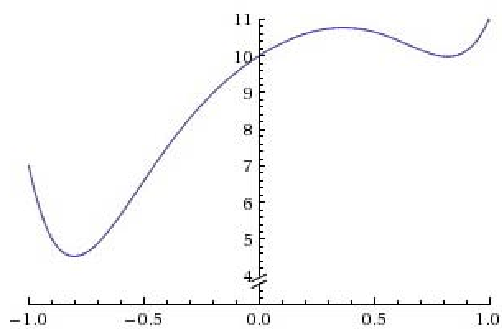

Figure 3 : The graph of function (3)

Values x_i and $\tilde{r}_i$ $(i = 0{,}1, \ldots, 20)$ are given in table 1.

Table 1 : Values for function (3)

i	x_i	$\tilde{r}_i$
0	-1	
1	-0,9	27,81566
2	-0,8	2,79197
3	-0,7	0,46336
4	-0,6	0,22772
5	-0,5	0,22189
6	-0,4	0,27695
7	-0,3	0,35992
8	-0,2	0,45347
9	-0,1	0,54546
10	0	0,63011
11	0,1	0,70852
12	0,2	0,78726
13	0,3	0,87542
14	0,4	0,98111
15	0,5	1,10548

16	0,6	1,23041
17	0,7	1,29707
18	0,8	1,19776
19	0,9	0,86411
20	1	0,44072

Notes

Let describe one iteration of the algorithm in detail. First, we choose a points x_0 and x_1 by initial. Then we find $\tilde{r}_1 = 27{,}81566 > 1$. Therefore compute $\tilde{r}_2, \tilde{r}_3, \ldots$ until for some i $(i \geq 1)$ condition $\tilde{r}_{i+1} < 1$ performs. We obtain $\tilde{r}_3 = 0{,}46336$ and take x_2 for local minimum point.

After completing the required number of iterations, we will find 3 extremum points: x_2, x_{14}, x_{18}. Function (3) reaches a local minimum at points x_2, x_{18}, and local maximum at point x_{14}.

IV. Conclusion

In this paper, using device of non-classical Newton's majorants and minorants of functions of one real variable given in tabular form, numerical method for finding all points of extremum of random as smooth and nonsmooth functions of one real variable at the selected interval is constructed, also example of this method is shown.

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