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Highlights

Existence of Coriolis Force

High Energy K X-Ray Satellites

Discovering Thoughts, Inventing Future

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Contents of the Issue

i. Copyright Notice
ii. Editorial Board Members
iii. Chief Author and Dean
iv. Contents of the Issue

1. High Energy K X-Ray Satellites. *1-8*
2. 3-Point Distribution Functions in the Statistical Theory in MHD Turbulent flow for Velocity, Magnetic Temperature and Concentration under going a First Order Reaction. *9-41*
3. The Real Universe. *43-50*
4. Statistical Theory for Three-Point Distribution Functions of Certain Variables in MHD Turbulent Flow in Existence of Coriolis Force in a First Order Reaction. *51-86*
5. The Two - Variable $(\frac{G'}{G},\frac{1}{G})$ - Expansion Method for Solving Nonlinear Dynamics of Microtubles - A New Model. *87-94*

v. Fellows and Auxiliary Memberships
vi. Process of Submission of Research Paper
vii. Preferred Author Guidelines
viii. Index

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High Energy K X-Ray Satellites

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Abstract- This study focus on High Energy K X-ray satellites which are a particular category of characteristic X-ray. They are emitted when an atom that has undergone multiple ionization de excites. Simultaneous single ionization in the K-shell and multiple ionization in the L-shell gives rise to K_α X-ray satellites. These can be studied only by high energy resolution instruments like crystal spectrometry. Experimental investigations were carried out in this field by several workers using photon, electron, and ion excitation modes. The theoretical models to predict their energies and intensities were developed. And also this work can show a clear discrepancy between theoretical and experimental results in the case of satellites formation from different shells. In case of experimental instrumentation, WDXRF is the most accurate for determining the energy and intensity of X-ray satellites. The basic source of data was literature done by different scholars.

Keywords: x-ray satellites; spectrometers; energy ratio; intensity ratio.

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HIGHENERGYKXRAYSATELLITES

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Zewdu Alamineh Fetene

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Keywords: x-ray satellites, spectrometers, energy ratio, intensity ratio.

I. Introduction

X-ray spectroscopy has developed over the years in to a powerful analytical tool with applications in a wide range of fields like elemental analysis, study of chemical structure, X-ray astronomy, plasma physics etc. All aspects of X-ray spectra should be understood in the finest detail possible to achieve maximum accuracy in such analytical studies. Investigation of X-ray satellites helps to further our understanding of inner shell ionization. Basically this is happen when an atom ionized simultaneously in different shells de-excites, X-ray satellites are produced. The K_α X-ray satellites or non-diagram lines were first observed by electron bombardment by Siegbahn and Stenstrom[1]. Wentzel[2] was the first to interpret these lines as arising from multi-ionized atoms having one or more L-shell vacancies in addition to one K-shell vacancy. The absence of L-shell electrons reduces the screening of the nuclear potential felt by the remaining electrons and increases their binding energies. K X-rays emitted due to transitions of electrons from such states will be at higher energies than the normal diagram line KL^0 and such lines are called $K\alpha$ satellite lines. Depending on the number of L-shell vacancies in addition to a single K-shell vacancy, these satellites are designated as $K_\alpha L^1 (K_\alpha L)$, $K_\alpha L^2$ $(K_\alpha LL)$, $K_\alpha L^3$ $(K_\alpha LLL)$, ….. , $K_\alpha L^n$ $(K_\alpha LLL\ldots\ldots nL)$ (where n is the number of L vacancies). When n = 0, it represents the normal diagram line $K\alpha L^0$ (i.e. $K_{\alpha1\alpha2}$). Similarly K_β satellites can be designated as $K_\beta L^1$, $K_\beta L^2 \ldots K_\beta L^n$.

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It was widely believed that K_α X-ray satellites or non-diagram lines found in electron excitation do not appear in fluorescent excitation[3]. However, later experimental work showed that this was not true[4, 5]. Further it was also concluded from measurements of charge states of ions formed following photo-ionization, that two or more electrons could be ejected with considerable probability, when one photon is absorbed[6]. In the early days for the measurement of the K_α X-ray satellites, photographic technique was used but subsequently crystal spectrometers in combination with continuous flow proportional counters were being used.

Several theoretical formalisms[7] are available for computing the energy shift of $K\alpha L^1$ from the diagram line $K\alpha L^0$: (a) Non-relativistic Hartee-Fock calculations, (b) simple analytical model of Burch *et al* [8] (c) modified version of Burch, including relativistic effects, (d) self-consistent field calculations of Bhattacharya *et al* [9]. In addition to these there is a semi-empirical formula proposed by Torok[10] for the prediction of these energy shifts, later discussed under theoretical models in detail.

Generally, these paper states the production of K X-ray satellites, the correlation between experimental and theoretical results of energy shifts for different Z-values, and the dependence on mode of excitation, chemical effects and Zsystematics of relative intensity and energy shifts where the experimental results from different literature have been considered.

II. Experimental Details

Based on the fact that, when a beam of X-rays is directed on to a sample, secondary fluorescent X-rays are emitted at a series of wavelengths characteristic of the element under investigation. The individual components of the fluorescent X-radiation are separated by means of an analyzing crystal, which diffracts them at different angles according to their wavelengths. Each wavelength refers to a different X-ray, the energy and intensity of which can be measured by means of a suitable detector placed in the appropriate position. The basic facilities needed for such investigation are, X-ray wavelength dispersive crystal spectrometer. The crystal is so oriented as to reflect only one wavelength at a given angle. By slowly rotating the crystal at one half the angular speed of the detector, the various wavelengths from the parallel beam of target X-rays are reflected, one by one, as the crystal makes the appropriate angle for each wavelength. The intensity at each wavelength can then be measured with a suitable detector.

a) Description of Crystal Spectrometer

This system consists of a plane crystal, a continuous flow type proportional counter with a thin (1 micron) polypropylene window and an X-ray generator with a suitable X-ray tube (Rh X-ray tube in the present case). The system has a built-in provision to change the crystal by operating an external switch without disturbing the optimum conditions. Basically X-rays from the tube, incident on the target, produce characteristic X-rays of the target, which are made to fall on the plane crystal. The diffracted rays from the crystal are detected by the continuous flow type proportional counter maintained at a constant pressure (1atmos) [11].

The radiation detector and the analyzing crystal are mounted on a high precision goniometer which is an instrument used to rotate both X-ray tube and the detector from 0° to 148° (2θ) under a microprocessor control. The goniometer is driven by a stepping motor and it is possible to scan its full angular range, in either direction. The goniometer is normally made to scan from a lower angle to a higher angle. The fast bi-directional facility available enables the goniometer to be used for specific measurement at predetermined angles. In the present experiment, in order to have correct peak identification by way of angular accuracy, the goniometer is set to proceed always in the same direction to approach the selected angle. The goniometer has the facility of scanning the spectrum in 2θ steps of 0.01° to 1° with a required predetermined time setting at each position[12].

The crystal attenuator unit carries nine such terminals each related to an attenuator for a particular type of analyzing crystal, a mains connector and an earth connection. X-rays are attenuated by air, and for this reason, spectrometry is normally performed in a low vacuum. The vacuum system is designed to maintain a vacuum in the sample and crystal chambers while measurements are being made. The system consists of a vacuum pump, the air lock, the sample chamber, the crystal chamber, the air filter and three valves. The air-lock provides the means of transferring the sample to be analyzed from the loading position in the air lock to the sample turret without disturbing the vacuum. It is also provided with several interlocking and safety features to prevent faulty operation. The sample turret switch is a four position rotary with one position corresponding to each of the four samples. When the switch is set to a selected position, the sample turret will be rotated by its motor until the sample of interest is in the analyzing position.

During experimental investigations the following should be considered: *Self-Absorption in the Sample: -* To correct for such an absorption[13], the following expression is used:

$$I = Io(\mu_\rho t)^{-1}(1 - e^{-\mu_\rho t})$$

Where I_0 = corrected intensity, I = observed intensity, μ_ρ = Mass Absorption Coefficient of the target materials for the X-rays under study and t = thickness of the sample (gm/cm^2)

$$(\mu_\rho)_{compound} = \sum_i W_i(\mu_\rho)_i$$

where, W_i is the fractional weight age and $(\mu_\rho)_i$ is the Mass Absorption Coefficient of the constituent elements;

b) Crystal Reflectivity

The reflectivity of the crystal depends upon the wavelength of the X-rays under analysis. This correction is carried out assuming that the reflectivity is proportional to the cube of the wavelength[14]. The correction for reflection is small, so the net correction is still smaller; *Window Absorption: -* This correction is carried out employing the basic relation for absorption

$$Io = Ie^{-\mu_\rho t}$$

I is the observed intensity, μ_ρ is the Mass Absorption Coefficient of the window material, t is the thickness of the window (gm/cm^2); *Efficiency of the Detector*:- Because of the differences in the energies of X-ray lines there may be differences in the quantum counting efficiency of the counter for these energies. This efficiency is usually determined by estimating the absorption in the gas of the counter.

III. Discussion: Theoretical Computations based on Some Models

a) Satellite Energies

Deustch[14] has computed the energy shift of $K_\alpha L^1$ satellite relative to $K_\alpha L^0$ diagram line using four theoretical models for the elements in the Z range 10-32. The salient features of these four models and Semi-empirical Formula of Torok *et al*[15] are presented briefly as; *HF- the Non-Relativistic Hartee–Fock Calculations*:- In this model intermediate coupling scheme is adopted and only single configurations are employed. As the contribution of 2S spectator hole to the satellite spectrum was experimentally determined 8, 16 to be negligible for the z range under consideration only 2p spectator hole configurations were considered. The energies for initial and final configurations were computed using $MCHF_{78}$ programme developed by Froese [17]. The satellite energy shift was calculated from the following equation:

$$\Delta E_1 = [E(1S^{-1}2P^{-1}) - E(2P^{-1})] - [E(1S^{-1}) - E(2P^{-1})],$$

where $E(nl)$ and $E(nl\,n'l')$ were taken as the total energy of the nl and $nl\,n'l'$ Configurations respectively as obtained from the programmed; *B-S – the Simple Analytical Model of Burch* :- In this model the effective

Global Journal of Science Frontier Research (A) Volume XV Issue II Version I Year 2015

charge of the long lived 2p spectator vacancy is uniformly smeared over the 2p shell. This charged shell is the source of a perturbing electrostatic potential which shifts the inner levels of the atom none uniformly causing the energy shifts of the satellite line relative to the spectator less case of the diagram line. According to Burch *et al* [8] a vacancy the 2p shell produces a perturbing potential,

$$V_{2p}(r) = r^{-1} - 4\pi r^{-1} \int \rho_{2p}(r')(R'^2 - rr')dr',$$

(where an integral tendes from r→∞) where using hydrogen like wave functions,

$$\rho_{2p}(r) = \psi^2{}_{2p}(r) \propto r^2 \exp(-Zer)$$

After integration and normalization (by checking using $\frac{\partial^2 V}{\partial r^2} = -4\pi\rho$ above $V_{2p}(r)$ equation will have)

$$V_{2p}(x) = \left(\frac{Ze}{x}\right)\left[1 - \frac{1}{24}e^{-x}(x^3 + 6x^2 + 18x + 24)\right],$$

Where $x = Zer/\alpha_0$ The binding energy shifts are given by

$$\Delta K = \langle 1s|V_{2p}|1s\rangle$$

$$\Delta L = \langle 2p|V_{2p}|2p\rangle$$

$$\Delta M = \langle 3p|V_{2p}|3p\rangle$$

The removal of the 2p electron increases the binding energy of each of the levels. The un shifted and shifted K X-ray energies are indicated, making no distinction between $K_{\alpha1}$ and $K_{\alpha2}$. We use the sudden approximation in which the transition energies are defined as difference in binding energies. From these the $K_\alpha L^1$ energy shift can be calculated as follows:

$\Delta K_\alpha L^1 = \Delta K - \Delta L$, so this model yields for the shift,

$$\Delta K_\alpha L^1 = 1.66Ze$$

Where Z_e is the effective charge[17] given by $Z_e = Z - \sigma$, where σ is the Slater [18] screening constant which is taken as 4.15 for all Z; *B-HF- Model* :- This is a modified version of the previous model of Burch, developed by Deustch[14] where relativistic effects were included and Z dependent screening constant obtained from HF calculations was used in place of the constant value of 4.15; **SCF- Self Consistent Field model:-** This model developed by Bhattaracharya *et al*[9] further improved version of Burch's model where analytic self-consistent field wave functions were employed in place of the screened hydrogenic functions used in all the previous models. Relativistic effects were included in this model; *S-E- Semi – Empirical Formula of Torok et al:-* This semi-empirical formula based on graphical systematic developed with an experimental data base obtained by different authors by different modes of excitation. It gives the shift as

$$\Delta E(K_\alpha L^1) = 1.530(Z - 6.828,$$

Where 6.828 is the screening constant taken in place of 4.15, which Torok *et al*[15] claim resulted in better agreement with experimental values.

Experimental energy shifts of KL satellites and the calculated values after $\Delta(K_\alpha L) = 1.66Z_L = 1.66 (Z - 4.15)$[15] have a significant difference. Here $Z_L = (Z - 4.15)$ is the effective charge at the L shell determined from the Slater screening rules with i spectator holes one has to multiply this Δ value by i, shows the equation superimposed on the experimental values. At seven vacancies the agreement with the experiments is quite good. For better agreement other Z dependent effective charges were used by several authors obtaining much better agreement for single vacancy shift. These investigators did not report the application of their results for more vacancies.

From a systematic of the $K_\alpha L^i$ satellites and from a single spectator hole, we obtained another similar equation, again for equally spaced satellites,

$$\Delta(K_\alpha L) = 1.530(Z - 6.828)$$

The following simple empirical calculation (with an imposed effective Z) of the energy shift for the case of i spectator L vacancies was obtained:-

$$\Delta E(K_\alpha L^i) = i \times 1.530[Z + 0.5(i - 1) - 6.828]$$

Where i, = 1, 2, 3,…7

In the other cases, employing the simple analytical model of Burch *et al*[9] described above, $K_\beta L^1$ satellite energy shifts can be computed from,

$$\Delta K_\beta L^1 = \Delta K - \Delta M.$$

From that model, for hydrogen like wave functions, this turns out to be

$$\Delta K_\beta L^1 = 4.38Ze,\ \ (\Delta \text{ Shifts in the binding energies})$$

Using this relation these satellite energy shifts are computed for Z range 19-25 taking the Slater's screening constant $\sigma = 4.15$ in the relation $Ze = Z - \sigma$ and also taking $\sigma = 6.828$ as suggested by Torok *et al*[15].

Table 1 : Theoretical $K_\alpha L^1$ energy shifts[19, 20]

Z	H.F.	B.HF.	B-S	Bh	S.E.
10	5.2	8.6	9.7	5.6	4.8
11	6.7	10.4	11.4	7.3	6.4
12	8.2	12.1	13.0	9.0	7.9
13	9.5	13.8	14.7	10.7	9.4
14	10.8	15.5	16.4	12.5	11.0
15	12.1	17.2	18.0	14.2	12.5
16	13.4	18.8	19.7	15.9	14.0
17	14.6	20.5	21.3	17.6	15.6
18	15.8	22.1	23.0	19.4	17.1
19	17.3	23.8	24.7	21.1	18.6
20	18.3	25.4	26.3	22.8	20.2
21	20.4	27.0	28.0	24.5	21.7
22	22.0	28.7	29.8	26.3	23.2
23	23.5	30.3	31.3	28.0	24.7
24	25.1	31.9	33.0	29.7	26.3
25	26.4	33.5	34.6	31.4	27.8

Table 2 : Theoretical $K_\beta L^1$ energy shifts[9, 19]

Atomic number	$Ze = Z - 4.15$	$\Delta K_\beta L^1$	$Ze = Z - 6.828$	$\Delta K_\beta L^1$
19	14.85	65.0	12.172	53.3
20	15.85	69.4	13.172	57.7
21	16.85	73.8	14.172	62.1
22	17.85	78.2	15.172	66.4
23	18.85	82.6	16.172	70.8
24	19.85	86.9	17.172	75.2
25	20.85	91.3	18.172	79.6

b) Satellite Relative Intensities

According to the Wentzel-Druyvesteyn theory[21], there exists a possibility for not only one, but two or more photoelectrons being emitted from the atom in X-ray excitation. The resulting anomalous multihole states produce satellites while the normal single inner-hole states produce the diagram line. The Richtmeyer[22] theory suggests that in the anomalous states, which produce satellites, the outer electrons of the atom are excited but are still bound to the atom. No explanation is provided in these theories as to how the anomalous states arise. Nevertheless, it is common to both the theories that some sort of an anomalous state, which corresponds to an anomalous configuration of mainly the outer electrons, is responsible for a satellite line. These anomalous states can be termed as "Valence-Electron-Configuration" (VEC) states[5]. The assumption is that the formation of an inner hole occurs so quickly that the rapid change in the Coulomb field experienced by electrons, other than the photoelectron, gives rise to the anomalous configuration. The change in the Hamiltonian, attributable to the production of an inner vacancy or hole, is regarded as a Sudden approximation.

In the Sudden approximation, $W(nl)$, the relative probability for anomalous or VEC states, which are the

Global Journal of Science Frontier Research (A) Volume XV Issue II Version I Year 2015

initial states of the satellites, corresponding to an anomalous configuration of electrons originally in the nl shell, is given by

$$W(nl) = \frac{[1 - P(nl)]}{P(nl)}$$

$P(nl)$ is the probability that the electrons in shell nl stay in their orbits (keep their quantum numbers n, l, m_l, m_s) during the excitation:

$$P(nl) = \left| \int_0^\infty R_{nl}(r) R_{nl}^1(r) r^2 dr \right|^{2p}$$

Where, P is the number of electrons in the shell. R_{nl} (r) and $R^l{}_{nl}$ (r) are the radial functions of the ground and the excitation state spin orbitals, respectively. The intensity ratio of the $K\alpha L^n$ satellite group and the $K_{\alpha 1,2}$ line is given by

$$\frac{IK_\alpha L^n}{IK_{\alpha 1,2}} = \frac{f_s}{f_0}[W(2s) + W(2s)W(2p)] + \frac{5f_p}{6f_o}W(2p)$$

in which f_0 is the oscillator strength of transitions between the normal states and f_s is the mean oscillator strength of transition between anomalous states which correspond to excitation or ionization of L_I-shell electrons and L_I, L_{II} and L_{III}- shell electrons, simultaneously. The oscillator strength f_0 is assumed to change slightly when the outer electrons become excited or ionized. In the case of excited states the excitation probability can be calculated in accordance with the Sudden approximation, if

$$2\pi[W_{nl\,n'l'} - W_{nl}]\frac{\tau_{n'l'}}{h} \square\ 1$$

Where W_{nl} is the energy of the normal single hole state, $W_{nl\,n'l'}$ is the energy of the anomalous state with an extra n′l′ and $\tau_{n'1'}$ is the time of transit of the ejected photoelectron past the n′l′ shell.

According to Carlson and Krause[23], the probability of the simultaneous ionization of L and K shells should be constant in photo ionization process and comparable with sudden approximation probability, if

$$(E_{KL} - E_K)t < 0.4h/2\pi$$

Here t is the time of transit of the K photoelectron past L-shell. E_{KL} and E_K are the excitation energies of the KL and K X-ray states, respectively. It can be shown that the time t in the validity criterion corresponds to an excitation energy of the incoming X-rays which is more than two times the threshold energy E_k. Based on the sudden approximation principle and assuming the Neon core and neglecting exchange effects Aberg[20] has calculated $\frac{K_\alpha L^1}{K_\alpha L^0}$ intensity ratio for low Z elements up to Z = 20.

c) *Dependence on Mode of Excitation*

No appreciable changes were observed in the energy shifts of the satellites with variation in the mode of excitation. But it was shown that relative intensity of the x-ray satellites does depend on the mode of excitation. Compared to photon excitation slightly higher relative intensity was reported by electron excitation in the elements Ti, V, Cr, and Mn (Table 3). Even in the ion mode of excitations a dependence on charge state and modes of the projectile was observed by Watson *et al*[24].

d) *Projectile Dependence of Energy Shifts*

The satellite energy shifts of Ti, V, Cr, and Mn between the projectiles photon and electron excitation related with other five theoretical models are listed in Table 3.

Table 3 : The property of energy shift for two modes of excitation[4, 25]

Elements	Z	Energy shift by photon excitation	Energy shift by electron excitation	HF	BS	B-HF	SCF	SE
Ti	22	25.3±1.0	28.7±0.2	22.0	29.6	28.7	26.3	23.2
V	23	26.2±1.0	28.3±0.2	23.5	31.3	30.3	28.0	24.7
Cr	24	27.9±1.0	29.6±0.2	25.1	33.0	31.9	29.7	26.3
Mn	25	29.6±1.0	31.4±0.2	26.4	34.6	33.5	31.4	27.8

Here the energy shifts $K_\alpha L^1$ excitation by electron and photon is in agreement which means the energy shift of the satellite is independent of mode of excitation or projectiles. Among the theoretical estimation BS and B-HF values are called overestimated the shifts while HF is underestimated. And also SE is slightly underestimated. The excellent agreement with the analytic SCF calculations of Bhattacharya *et al*[9].

e) *Projectile Dependence of Intensity Ratio*

Kawatsura[26] investigated projectile dependence and also chemical effects on X-ray satellites using Be and BeO targets. In the case of inner shell ionization for the Be atom is described by the direct coulomb excitation for light ion bombardment. For the BeO target, the spectra became more complicated. Both K_α diagram and K_α^2 hyper satellite lines became broader and shift to lower energy and then new satellite lines appear in higher energy side of each main peak. The ratio of intensities for low energy components of the satellite groups to those of high energy components increase with increasing projectile Z 27. In the case of the KL^2 and KL^3 satellite groups, the peaks under consideration are composed of lines from a number of different terms and one would expect preferential population to have a much smaller net effect. Difference in relative intensity ratio of KL^1 obtained by photon and electron excitation modes was reported.

Table 4 : The property of intensity ratio related with two modes of excitation[4, 28]

Element	Intensity ratio by photon excitation	Intensity ratio by electron excitation
K	2.61±0.13	3.90±0.2
Ca	2.31±0.12	3.36±0.17
Sc	1.80±0.09	2.78±0.13
Ti	1.66±0.08	2.20±0.11
V	1.54±0.08	1.80±0.09
Cr	1.36±0.07	1.37±0.07
Mn	1.07±0.05	1.10±0.05

From the Table 4, intensity observed by photon excitation is less than electron excitation. But we see that the difference goes on decrease as Z increase. These shows relative intensity of K X-ray satellite is found to be dependent on the mode of excitation being higher for electron than photon excitation. Watson *et al*[29] studied how $K_\alpha L^n$ relative intensity varies with different projectiles in Al, Cl and K.

Table 5 : Relative K_α X-ray satellite intensity for Al, Cl, and K using 1.7MeV/amu[29]

Target	Projectile	No. of L shell vacancy							
		0	1	2	3	4	5	6	7
Al	H	0.858	0.142	_	_	_	_	_	_
	He	0.664	0.285	0.051	_	_	_	_	_
	C	0.068	0.215	0.366	0.239	0.090	0.023	_	_
	O	0.045	0.128	0.312	0.293	0.168	0.053	_	_
Cl	He	0.680	0.320	_	_	_	_	_	_
	C	0.053	0.229	0.363	0.256	0.094	0.005	_	_
	O	0.028	0.133	0.304	0.316	0.170	0.049	_	_
K	He	0.765	0.235	_	_	_	_	_	_
	C	0.074	0.307	0.353	0.208	0.058	_	_	_
	O	0.045	0.185	0.316	0.271	0.142	0.042	_	_

They have shown that, X-ray satellite intensity (Table 5) depends on targets, projectiles, and number of L shell vacancies. Intensity under He projectile decrease as a number of vacancy increase; while directly correlated with atomic number. In the case of C and O projectile the value of intensity is not consistent when Z increases.

f) *Chemical Effects*

The influence of chemical bond[26] on the energy shifts and the relative intensities of X-ray satellites is an interesting aspect of the study of these satellites. The relative intensities are particularly more susceptible to the chemical environment than the energy shifts. In some previous studies[30] it was shown that the satellite relative intensities are higher in compound than in the elements.

A noticeable chemical effect was observed in the case fluorine K. Ram Narayana *et al*[11]. When $K_\alpha L^1$ relative intensity was measured in five fluorine compounds, on anomalous redaction of this relative intensity was observed in the case of potassium fluoride and strontium fluoride. This is attracted to phenomena called resonance electron transfer.

It can be seen that $K_\alpha L^1$ line relative intensity with respect to that of K_α diagram line is very much lower in the case of KF and SrF_2 compared to other compounds. This anomalously low value of the relative intensity was observed in these compounds in the study of Watson *et al*[31] by ion excitation. The anomalous decrease in intensity of this satellite line in these compounds is explained as being due to the 'resonance electron transfer phenomena' proposed by Watson.

In these phenomena electrons will be transferred from the outer np level of neighboring metal ion to the 2p level of fluorine ions which have been excited to $1s^{-1}$ $2p^{-1}$ states by the impact of photons. In the ground state, 2p level of the fluorine atom is confined to a narrow valance band. Impact of photons causes simultaneous ionization of 1s and 2p orbital's and these results in an increase in the 2p binding energy. Thus the 2p level of the fluorine atom is brought from the valance band to a level in close proximity with the outer np levels of the metal ion. When the energy difference between the outer levels of the metal ions and the excited level of 2p vacancy of the fluorine ion in a fluorine compound becomes small enough, electron transfer from the metal ion to the ligand fluorine atom can take place thus filling up the 2p vacancy in the fluorine atom. In such a situation the spectator vacancy, necessary for the emission of a satellite line, disappear and consequently a diagram line is emitted in its place. This results in reduction in the intensity of the satellite line and enhancement of the intensity of the diagram line, thus giving a lower $K_\alpha L^1$ relative intensity ratio.

g) Z Systematics ↔ Z Dependence of Relative intensities

Sattar *et al*[12]measured energies and relative intensities of $K_{\alpha3}$ and $K_{\alpha4}$ X-ray satellites of sulphur and some sulphides by photon excitation. He compiled the experimental values of other elements in the Z range 11-17 and plotted $K_{\alpha4}/K_{\alpha3}$ intensity ratio as a function of Z. It can be seen that as Z increases from 11, $K_{\alpha4}/K_{\alpha3}$ intensity ratio first decreases up to Z=14 and then increases sharply as it approaches Z=18. Therefore it was hypothesized that this ratio increases as the inert gas i.e., stable configuration is reached. To confirm this there was a need to measure this intensity ratio in Ne (Z=10) and Ar (Z=18).

h) Z Dependence of Energy Shift

Raju *et al*[19] supplemented their data with those reported by other authors on K_α hyper satellite energy shift relative to the K_α diagram line in the Z range 12-30 and studied the variation of this with respect to Z. They found the relationship to be linear. They obtained from the plot of this Δ(E) versus Z, the following empirical relationship

$$\Delta E(K_\alpha^h) = -3.0 + 10.048Z$$

They also analyzed variation energy shift of $K_\beta L^1$ satellite relative to K_β diagram line as a function of Z and found the relationship to be linear. They compared these values with the theoretical values calculated using the formula of Burch *et al* [8],

$$\Delta E\left(K_\beta L^1\right) = 18.541 + 2.304(Z - 6.828)$$

Where, σ is Slater's screening constant for L shell originally taken to be 4.15; the shift thus calculated was much lower than the experimental values. Using[15] σ =6.828, in the computation of this energy shift resulted in much improvement in agreement with theoretical values, but still the energy shift is overestimated.

IV. Conclusions

WDXRF spectrometry is a crucial instrument for determining the energy shift and intensity of x-ray satellites because the instrument have eight different analyzing crystals and also other basic components like collimators. Due to this instrument the production of a satellites have been registered for a single-vacancy states existing in the shells is called X-ray satellites by critical consideration of self- absorption in the sample, crystal reflectivity, window absorption and efficiency of the detector.

From the basic theoretical investigations, the energy shift calculated by SCF approach was in better agreement with energy shift by electron excitation. From different mode of excitation; the energy shift by photon was less than by electron excitation for the same element. And also the relative intensities were particularly more susceptible to the chemical environment than the energy shift.

Generally, a review of the literature on X-ray satellites shows that, so far the research on these aspects is carried out on elements up to the atomic number 32. Beyond this as the wavelength of the satellites become smaller and smaller, crystals of matching 2d are to be used; this poses a challenge. Also, very few studies are found in literature on L X-ray satellites and hyper satellites. These studies can be extended to cover them. Because a number of synchrotron facilities are being developed throughout the world; using these tunable hard X-ray sources, energy dependence of these processes can be studied more efficiently. Finally, I believe that all these studies help to improve further knowledge on inner shell ionization processes.

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3-Point Distribution Functions in the Statistical Theory in MHD Turbulent flow for Velocity, Magnetic Temperature and Concentration under going a First Order Reaction

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Abstract- In this paper, an attempt is made to study the three-point distribution function for simultaneous velocity, magnetic temperature and concentration fields in MHD turbulence under going a first order reaction. The various properties of constructed distribution functions have been discussed. Through out the study, the transport equation for three-point distribution functions in MHD turbulent flow under going a first order reaction has been obtained. The obtained equation is compared with the first equation of BBGKY hierarchy of equations and the closure difficulty is to be removed as in the case of ordinary turbulence.

Keywords: *magnetic temperature, concentration, three-point distribution functions, MHD turbulent flow, first order reactant.*

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Abstract- In this paper, an attempt is made to study the three-point distribution function for simultaneous velocity, magnetic temperature and concentration fields in MHD turbulence under going a first order reaction. The various properties of constructed distribution functions have been discussed. Through out the study, the transport equation for three-point distribution functions in MHD turbulent flow under going a first order reaction has been obtained. The obtained equation is compared with the first equation of BBGKY hierarchy of equations and the closure difficulty is to be removed as in the case of ordinary turbulence.

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I. Introduction

In molecular kinetic theory in physics, a particle's distribution function is a function of several variables. Particle distribution functions are used in plasma physics to describe wave-particle interactions and velocity-space instabilities. Distribution functions are also used in fluid mechanics, statistical mechanics and nuclear physics. In the past Hopf (1952), Kraichanan (1959), Edward (1964) and Herring (1965) have been discussed various analytical theories in the statistical theory of turbulence. Lundgren (1967, 1969) derived a hierarchy of coupled equations for multi-point turbulence velocity distribution functions, which resemble with BBGKY hierarchy of equations of Ta-You (1966) in the kinetic theory of gasses.

Kishore (1978) studied the Distributions functions in the statistical theory of MHD turbulence of an incompressible fluid. Pope (1979) studied the statistical theory of turbulence flames. Pope (1981) derived the transport equation for the joint probability density function of velocity and scalars in turbulent flow.

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Kollman and Janicka (1982) derived the transport equation for the probability density function of a scalar in turbulent shear flow and considered a closure model based on gradient – flux model. Kishore and Singh (1984) derived the transport equation for the bivariate joint distribution function of velocity and temperature in turbulent flow. Also Kishore and Singh (1985) have been derived the transport equation for the joint distribution function of velocity, temperature and concentration in convective turbulent flow.

In the next, some researchers included coriolis force and first order reaction rate in their works. Dixit and Upadhyay (1989) considered the distribution functions in the statistical theory of MHD turbulence of an incompressible fluid in the presence of the coriolis force. Sarker and Kishore (1991) discussed the distribution functions in the statistical theory of convective MHD turbulence of an incompressible fluid. Also Sarker and Kishore (1999) studied the distribution functions in the statistical theory of convective MHD turbulence of mixture of a miscible incompressible fluid.

In the continuation, the following some researchers included first order reaction in their works. In many cases, they also considered Coriolis force and dust particles in their works. Azad and Sarker (2004a) discussed statistical theory of certain distribution functions in MHD turbulence in a rotating system in presence of dust particles. Sarker and Azad (2004b) studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time in a rotating system. Sarker and Azad(2006), Islam and Sarker (2007) studied distribution functions in the statistical theory of MHD turbulence for velocity and concentration undergoing a first order reaction. Azad *et al* (2009b, 2009c) studied the first order reactant in Magneto-hydrodynamic turbulence before the final Period of decay with dust particles and rotating System. Aziz et al (2009d, 2010c) discussed the first order reactant in Magneto- hydrodynamic turbulence before the final period of decay for the case of multi-point and multi-time taking rotating system and dust particles. Aziz et al (2010a, 2010b) studied the statistical theory of certain Distribution Functions in MHD turbulent flow

Global Journal of Science Frontier Research (A) Volume XV Issue II Version I Year 2015

undergoing a first order reaction in presence of dust particles and rotating system separately. Azad et al (2011) studied the statistical theory of certain distribution Functions in MHD turbulent flow for velocity and concentration undergoing a first order reaction in a rotating system. Azad et al (2012) derived the transport equatoin for the joint distribution function of velocity, temperature and concentration in convective tubulent flow in presence of dust particles. Bkar Pk. et al (2012) studed the First-order reactant in homogeneou dusty fluid turbulence prior to the ultimate phase of decay for four-point correlation in a rotating system. Molla et al (2013) derived the Transport equation for the joint distribution functions of velocity, temperature and concentration in convective turbulent flow in presence of coriolis force. Bkar Pk. et al (2013a,2013b) discussed the first-order reactant in homogeneous turbulence prior to the ultimate phase of decay for four-point correlation with dust particle and rotating system. Very recent Azad et al (2014a) derived the transport equations of three point distribution functions in MHD turbulent flow for velocity, magnetic temperature and concentration, Azad and Nazmul (2014b) considered the transport equations of three point distribution functions in MHD turbulent flow for velocity, magnetic temperature and concentration in a rotating system, Nazmul and Azad (2014) studied the transport equations of three-point distribution functions in MHD turbulent flow for velocity, magnetic temperature and concentration in presence of dust particles. Azad and Mumtahinah (2014) further has been studied the transport equatoin for the joint distribution functions in convective tubulent flow in presence of dust particles undergoing a first order reaction.Very recently, Bkar Pk. et al (2015) considering the effects of first-order reactant on MHD turbulence at four-point correlation. Azad et al (2015) derived a transport equation for the joint distribution functions of certain variables in convective dusty fluid turbulent flow in a rotating system under going a first order reaction. Bkar et al (2015a) studied 4-point correlations of dusty fluid MHD turbulent flow in a 1st order reaction. Most of the above researchers have done their research for two point distribution functions in the statistical theory in MHD turbulence.

But in this paper, we have tried to do this research for three-point distribution functions in the statistical theory in MHD turbulence in a first order reaction. In this paper, the main purpose is to study the statistical theory of three-point distribution function for simultaneous velocity, magnetic, temperature, concentration fields in MHD turbulence under going a first order reaction. Through out the study, the transport equations for evolution of distribution functions have been derived. Various properties of the distribution function have been discussed for solving the problem. The obtained three-point transport equation is compared with the first equation of BBGKY hierarchy of equations and the closure difficulty is to be removed as in the case of ordinary turbulence.

II. Formulation of the Problem

The equations of motion and continuity for viscous incompressible MHD turbulent flow constant reaction rate, the diffusion equations for the temperature and concentration are given by

$$\frac{\partial u_\alpha}{\partial t}+\frac{\partial}{\partial x_\beta}\left(u_\alpha u_\beta-h_\alpha h_\beta\right)=-\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha}\int\left[\frac{\partial u'_\alpha}{\partial x'_\beta}\frac{\partial u'_\beta}{\partial x'_\alpha}-\frac{\partial h'_\alpha}{\partial x'_\beta}\frac{\partial h'_\beta}{\partial x'_\alpha}\right]\frac{d\bar{x}'}{\left|\bar{x}'-\bar{x}\right|}+\nu\nabla^2 u_\alpha \tag{1}$$

$$\frac{\partial h_\alpha}{\partial t}+\frac{\partial}{\partial x_\beta}\left(h_\alpha u_\beta-u_\alpha h_\beta\right)=\lambda\nabla^2 h_\alpha\,, \tag{2}$$

$$\frac{\partial\theta}{\partial t}+u_\beta\frac{\partial\theta}{\partial x_\beta}=\gamma\nabla^2\theta\,, \tag{3}$$

$$\frac{\partial c}{\partial t}+u_\beta\frac{\partial c}{\partial x_\beta}=D\nabla^2 c-Rc \tag{4}$$

$$\text{with}\ \frac{\partial u_\alpha}{\partial x_\alpha}=\frac{\partial v_\alpha}{\partial x_\alpha}=\frac{\partial h_\alpha}{\partial x_\alpha}=0\,, \tag{5}$$

where

$u_\alpha(x,t)=\alpha$ – component of turbulent velocity, $h_\alpha(x,t)=\alpha$ – component of magnetic field $\theta(x,t)=$ temperature fluctuation, c=concentration of contaminants, $P(\hat{x},t)=$ hydrodynamic pressure, $\rho=$fluid density, ν= Kinematic viscosity, $\lambda=(4\pi\mu\sigma)^{-1}$, magnetic diffusivity, $\gamma=\frac{k_T}{\rho c_p}$, thermal diffusivity; $c_p=$ specific heat at constant pressure, k_T =thermal conductivity, σ =electrical conductivity, μ=magnetic permeability, D=diffusive co-efficient for

Global Journal of Science Frontier Research (A) Volume XV Issue II Version I Year 2015

contaminants. R=constant reaction rate'.The repeated suffices are assumed over the values 1, 2 and 3 and unrepeated suffices may take any of these values. In the whole process *u, h* and x are the vector quantities.

Here, we have considered that the turbulence and the concentration fields are homogeneous, the chemical reaction and the local mass transfer have no effect on the velocity field. The reaction rate and the diffusivity are constant. It is also considered a large ensemble of identical fluids in which each member is an infinite incompressible reacting and heat conducting fluid in turbulent state. The fluid velocity *u*, Alfven velocity *h*, temperature θ and concentration c are randomly distributed functions of position and time and satisfy their field. Different members of ensemble are subjected to different initial conditions and the aim is to find out a way by which we can determine the ensemble averages at the initial time.

Certain microscopic properties of conducting fluids such as total energy, total pressure, stress tensor which are nothing but ensemble averages at a particular time can be determined with the help of the distribution functions (defined as the averaged distribution functions with the help of Dirac delta-functions). The present aim is to construct distribution function for its evolution of three-point distribution functions in MHD turbulent flow in a first order reaction, study its properties and derive a transport equation for the 3-point distribution functions of velocity, magnetic temperature and concentration in MHD turbulent flow in a first order reaction.

III. Distribution Function in MHD Turbulence and their Properties

In MHD turbulence, it is considered that the fluid velocity *u*, Alfven velocity *h*, temperature θ and concentration c at each point of the flow field. Corresponding to each point of the flow field, there are four measurable characteristics represent by the four variables by v, g, ϕ and ψ denote the pairs of these variables at the points $\bar{x}^{(1)}, \bar{x}^{(2)}, ------, \bar{x}^{(n)}$ as $\left(\bar{v}^{(1)}, \bar{g}^{(1)}, \phi^{(1)}, \psi^{(1)}\right), \left(\bar{v}^{(2)}, \bar{g}^{(2)}, \phi^{(2)}, \psi^{(2)}\right)$, --- $\left(\bar{v}^{(n)}, \bar{g}^{(n)}, \phi^{(n)}, \psi^{(n)}\right)$ at a fixed instant of time.

It is possible that the same pair may be occurred more than once; therefore, it simplifies the problem by an assumption that the distribution is discrete (in the sense that no pairs occur more than once). Symbolically we can express the bivariate distribution as

$$\left\{ \left(\bar{v}^{(1)}, \bar{g}^{(1)}, \phi^{(1)}, \psi^{(1)}\right), \left(\bar{v}^{(2)}, \bar{g}^{(2)}, \phi^{(2)}, \psi^{(2)}\right) ------ \left(\bar{v}^{(n)}, \bar{g}^{(n)}, \phi^{(n)}, \psi^{(n)}\right) \right\}$$

Instead of considering discrete points in the flow field, if it is considered the continuous distribution of the variables $\bar{v}, \bar{g}, \phi$ and ψ over the entire flow field, statistically behavior of the fluid may be described by the distribution function $F\left(\bar{v}, \bar{g}, \phi, \psi\right)$ which is normalized so that

$$\int F\left(\bar{v}, \bar{g}, \phi, \psi\right) d\bar{v}\, d\bar{g}\, d\phi\, d\psi = 1$$

where the integration ranges over all the possible values of v, g, ϕ and ψ. We shall make use of the same normalization condition for the discrete distributions also.

The distribution functions of the above quantities can be defined in terms of Dirac delta function.

The one-point distribution function $F_1^{(1)}\left(v^{(1)}, g^{(1)}, \phi^{(1)}, \psi^{(1)}\right)$, defined so that $F_1^{(1)}\left(v^{(1)}, g^{(1)}, \phi^{(1)}, \psi^{(1)}\right) dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)}$ is the probability that the fluid velocity, Alfven velocity, temperature and concentration at a time t are in the element $dv^{(1)}$ about $v^{(1)}$, $dg^{(1)}$ about $g^{(1)}$, $d\phi^{(1)}$ about $\phi^{(1)}$ and $d\psi^{(1)}$ about $\psi^{(1)}$ respectively and is given by

$$F_1^{(1)}\left(v^{(1)}, g^{(1)}, \phi^{(1)}, \psi^{(1)}\right) = \left\langle \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(h^{(1)} - g^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(c^{(1)} - \psi^{(1)}\right) \right\rangle \tag{6}$$

where δ is the Dirac delta-function defined as $\int \delta\left(\bar{u} - \bar{v}\right) d\bar{v} = \begin{cases} 1 & \text{at the point } \bar{u} = \bar{v} \\ 0 & \text{elsewhere} \end{cases}$

Two-point distribution function is given by

$$F_2^{(1,2)} = \left\langle \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(h^{(1)} - g^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(c^{(1)} - \psi^{(1)}\right) \delta\left(u^{(2)} - v^{(2)}\right) \delta\left(h^{(2)} - g^{(2)}\right) \delta\left(\theta^{(2)} - \phi^{(2)}\right) \delta\left(c^{(2)} - \psi^{(2)}\right) \right\rangle \tag{7}$$

and three point distribution function is given by

$$F_3^{(1,2,3)} = \left\langle \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(h^{(1)} - g^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(c^{(1)} - \psi^{(1)}\right) \delta\left(u^{(2)} - v^{(2)}\right) \delta\left(h^{(2)} - g^{(2)}\right) \right.$$

$$\left. \times \delta\left(\theta^{(2)} - \phi^{(2)}\right) \delta\left(c^{(2)} - \psi^{(2)}\right) \delta\left(u^{(3)} - v^{(3)}\right) \delta\left(h^{(3)} - g^{(3)}\right) \delta\left(\theta^{(3)} - \phi^{(3)}\right) \delta\left(c^{(3)} - \psi^{(3)}\right) \right\rangle \tag{8}$$

Similarly, we can define an infinite numbers of multi-point distribution functions $F_4^{(1,2,3,4)}$, $F_5^{(1,2,3,4,5)}$ and so on. The following properties of the constructed distribution functions can be deduced from the above definitions:

a) Reduction Properties

Integration with respect to pair of variables at one-point lowers the order of distribution function by one. For example,

$$\int F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} = 1,$$

$$\int F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} = F_1^{(1)},$$

$$\int F_3^{(1,2,3)} dv^{(3)} dg^{(3)} d\phi^{(3)} d\psi^{(3)} = F_2^{(1,2)}$$

And so on. Also the integration with respect to any one of the variables, reduces the number of Delta-functions from the distribution function by one as

$$\int F_1^{(1)} dv^{(1)} = \left\langle \delta\left(h^{(1)} - g^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(c^{(1)} - \psi^{(1)}\right) \right\rangle,$$

$$\int F_1^{(1)} dg^{(1)} = \left\langle \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(c^{(1)} - \psi^{(1)}\right) \right\rangle,$$

$$\int F_1^{(1)} d\phi^{(1)} = \left\langle \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(h^{(1)} - g^{(1)}\right) \delta\left(c^{(1)} - \psi^{(1)}\right) \right\rangle, \text{ and}$$

$$\int F_2^{(1,2)} dv^{(2)} = \left\langle \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(h^{(1)} - g^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(c^{(1)} - \psi^{(1)}\right) \delta\left(h^{(2)} - g^{(2)}\right) \delta\left(\theta^{(2)} - \phi^{(2)}\right) \delta\left(c^{(2)} - \psi^{(2)}\right) \right\rangle$$

b) Separation Properties

If two points are far apart from each other in the flow field, the pairs of variables at these points are statistically independent of each other i.e.,

$$\lim_{\left|\bar{x}^{(2)} \to \bar{x}^{(1)}\right| \to \infty} F_2^{(1,2)} = F_1^{(1)} F_1^{(2)} \text{ and similarly,}$$

$$\lim_{\left|\bar{x}^{(3)} \to \bar{x}^{(2)}\right| \to \infty} F_3^{(1,2,3)} = F_2^{(1,2)} F_1^{(3)} \text{ etc.}$$

When two points coincide in the flow field, the components at these points should be obviously the same that is $F_2^{(1,2)}$ must be zero.

Thus $\bar{v}^{(2)} = \bar{v}^{(1)}$, $g^{(2)} = g^{(1)}$, $\phi^{(2)} = \phi^{(1)}$ and $\psi^{(2)} = \psi^{(1)}$, but $F_2^{(1,2)}$ must also have the property.

$$\int F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} = F_1^{(1)}$$

and hence it follows that

c) Co-incidence Properties

$$\lim_{\left|\bar{x}^{(2)} \to \bar{x}^{(1)}\right| \to \infty} \int F_2^{(1,2)} = F_1^{(1)} \delta\left(v^{(2)} - v^{(1)}\right) \delta\left(g^{(2)} - g^{(1)}\right) \delta\left(\phi^{(2)} - \phi^{(1)}\right) \delta\left(\psi^{(2)} - \psi^{(1)}\right)$$

Similarly,

$$\lim_{\left|\bar{x}^{(3)} \to \bar{x}^{(2)}\right| \to \infty} \int F_3^{(1,2,3)} = F_2^{(1,2)} \delta\left(v^{(3)} - v^{(1)}\right) \delta\left(g^{(3)} - g^{(1)}\right) \delta\left(\phi^{(3)} - \phi^{(1)}\right) \delta\left(\psi^{(3)} - \psi^{(1)}\right) \text{ etc.}$$

d) Symmetric Conditions

$$F_n^{(1,2,r,------s,-----n)} = F_n^{(1,2,-----s,---r,---n)}.$$

e) Incompressibility Conditions

$$\text{(i)} \int \frac{\partial F_n^{(1,2,---n)}}{\partial x_\alpha^{(r)}} v_\alpha^{(r)} d\bar{v}^{(r)} d\bar{h}^{(r)} = 0 \qquad \text{(ii)} \int \frac{\partial F_n^{(1,2,---n)}}{\partial x_\alpha^{(r)}} h_\alpha^{(r)} d\bar{v}^{(r)} d\bar{h}^{(r)} = 0$$

IV. Continuity Equation in Terms of Distribution Functions

The continuity equations can be easily expressed in terms of distribution functions. An infinite number of continuity equations can be derived for the convective MHD turbulent flow and are obtained directly by using $\operatorname{div} u = 0$

Taking ensemble average of equation (5), we get

$$0 = \left\langle \frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}} \right\rangle = \left\langle \frac{\partial}{\partial x_\alpha^{(1)}} u_\alpha^{(1)} \int F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \right\rangle$$

$$=\frac{\partial}{\partial x_\alpha^{(1)}}\langle\ u_\alpha^{(1)}\int F_1^{(1)}dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)}\ \rangle=\frac{\partial}{\partial x_\alpha^{(1)}}\int\ \langle\ u_\alpha^{(1)}\ \rangle\langle\ F_1^{(1)}\ \rangle dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)}$$

$$=\frac{\partial}{\partial x_\alpha^{(1)}}\int\ v_\alpha^{(1)}F_1^{(1)}dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)}=\int\ \frac{\partial F_1^{(1)}}{\partial x_\alpha^{(1)}}v_\alpha^{(1)}dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)} \tag{9}$$

and similarly,

$$0=\int\ \frac{\partial F_1^{(1)}}{\partial x_\alpha^{(1)}}g_\alpha^{(1)}dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)} \tag{10}$$

Equation (15) and (16) are the first order continuity equations in which only one point distribution function is involved. For second-order continuity equations, if we multiply the continuity equation by

$$\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

and if we take the ensemble average, we obtain

$$o=\langle\ \delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}}\ \rangle$$

$$=\frac{\partial}{\partial x_\alpha^{(1)}}\langle\ \delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)u_\alpha^{(1)}\ \rangle$$

$$=\frac{\partial}{\partial x_\alpha^{(1)}}[\ \int\langle\ u_\alpha^{(1)}\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)$$

$$\times\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\ \rangle dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)}\ ]$$

$$=\frac{\partial}{\partial x_\alpha^{(1)}}\int v_\alpha^{(1)}F_2^{(1,2)}dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)} \tag{11}$$

and similarly,

$$o=\frac{\partial}{\partial x_\alpha^{(1)}}\int g_\alpha^{(1)}F_2^{(1,2)}dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)} \tag{12}$$

The Nth – order continuity equations are

$$o=\frac{\partial}{\partial x_\alpha^{(1)}}\int v_\alpha^{(1)}F_N^{(1,2,---,N)}dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)} \tag{13}$$

and

$$o=\frac{\partial}{\partial x_\alpha^{(1)}}\int g_\alpha^{(1)}F_N^{(1,2,.........,N)}dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)} \tag{14}$$

The continuity equations are symmetric in their arguments i.e.

$$\frac{\partial}{\partial x_\alpha^{(r)}}\left(v_\alpha^{(r)}F_N^{(1,2,..........;r.N)}dv^{(r)}dg^{(r)}d\phi^{(r)}d\psi^{(r)}\right)=\frac{\partial}{\partial x_\alpha^{(s)}}\int v_\alpha^{(s)}F_N^{(1,2,.....r,s,.....N)}dv^{(s)}dg^{(s)}d\phi^{(s)}d\psi^{(s)} \tag{15}$$

Since the divergence property is an important property and it is easily verified by the use of the property of distribution function as

$$\frac{\partial}{\partial x_\alpha^{(1)}}\int v_\alpha^{(1)}F_1^{(1)}dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)}\ \frac{\partial}{\partial x_\alpha^{(1)}}\langle\ u_\alpha^{(1)}\ \rangle=\langle\ \frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}}\ \rangle=o \tag{16}$$

and all the properties of the distribution function obtained in section (4) can also be verified.

V. Equations for Evolution of One – Point Distribution Function $F_1^{(1)}$:

We shall make use of equations (1) - (4) to convert these into a set of equations for the variation of the distribution function with time. This, in fact, is done by making use of the definitions of the constructed distribution functions, differentiating them partially with respect the right-hand side of the equation so obtained and lastly replacing the time derivative of u, h, θ and c from the equations (1) - (4).

Differentiating equation (6) with respect to time we get

$$\frac{\partial F_1^{(1)}}{\partial t} = \frac{\partial}{\partial t}\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\right\rangle$$

$$=\left\langle \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\frac{\partial}{\partial t}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle$$

$$+\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\frac{\partial}{\partial t}\delta\left(h^{(1)}-g^{(1)}\right)\right\rangle$$

$$+\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\frac{\partial}{\partial t}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\right\rangle$$

$$+\left\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\frac{\partial}{\partial t}\delta\left(c^{(1)}-\psi^{(1)}\right)\right\rangle$$

$$=\left\langle -\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\frac{\partial u^{(1)}}{\partial t}\frac{\partial}{\partial v^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\right\rangle$$

$$+\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\frac{\partial h^{(1)}}{\partial t}\frac{\partial}{\partial g^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\right\rangle$$

$$+\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\frac{\partial \theta^{(1)}}{\partial t}\frac{\partial}{\partial \phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\right\rangle$$

$$+\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\frac{\partial c^{(1)}}{\partial t}\frac{\partial}{\partial \psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\right\rangle \tag{17}$$

Using equations (1) to (4) in the equation (17), we get

$$\frac{\partial F_1^{(1)}}{\partial t} = \left\langle -\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\right\{ -\frac{\partial}{\partial x_\beta^{(1)}}\left(u_\alpha^{(1)}u_\beta^{(1)}-h_\alpha^{(1)}h_\beta^{(1)}\right)$$

$$-\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(1)}}\int \left[\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}}-\frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}}\right]\frac{d\overline{x}'}{\left|\overline{x}'-\overline{x}\right|}+\nu\nabla^2 u_\alpha^{(1)}\ \Big\}\times\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)$$

$$+\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\right\{ -\frac{\partial}{\partial x_\beta^{(1)}}\left(h_\alpha^{(1)}u_\beta^{(1)}-u_\alpha^{(1)}h_\beta^{(1)}\right)+\lambda\nabla^2 h_\alpha^{(1)}\ \Big\}$$

$$\times\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\right\rangle+\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\right\{ -u_\beta^{(1)}\frac{\partial \theta^{(1)}}{\partial x_\beta^{(1)}}+\gamma\nabla^2\theta^{(1)}\ \Big\}$$

$$\times\frac{\partial}{\partial \phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\right\rangle+\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\right\{ -u_\beta^{(1)}\frac{\partial c^{(1)}}{\partial x_\beta^{(1)}}+D\nabla^2 c-Rc^1\Big\}$$

$$\times\frac{\partial}{\partial \psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\right\rangle$$

Global Journal of Science Frontier Research (A) Volume XV Issue II Version I Year 2015

$$=\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\frac{\partial u_\alpha^{(1)}u_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$+\langle-\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\frac{\partial h_\alpha^{(1)}h_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$+\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(1)}}\int\ [\ \frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}}-\frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}}\ ]$$

$$\times\frac{d\overline{x}'}{\left|\overline{x}'-\overline{x}\right|}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$+\langle-\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\times\nu\nabla^2u_\alpha^{(1)}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\times\frac{\partial h_\alpha^{(1)}u_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\ \rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\times\frac{\partial u_\alpha^{(1)}h_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\ \rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\times\lambda\nabla^2h_\alpha^{(1)}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\ \rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\times u_\beta^{(1)}\frac{\partial\theta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\ \rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\times\gamma\nabla^2\theta^{(1)}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\ \rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\times u_\beta^{(1)}\frac{\partial c^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\ \rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\times D\nabla^2c^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\ \rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\times Rc^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\ \rangle \tag{18}$$

We may simplify various terms in the above equation as that they can be expressed in terms of one point and two point distribution functions with the help of the properties of the distribution function and the continuity equation in terms distribution function. Equation (18) becomes

$$\frac{\partial F_1^{(1)}}{\partial t}+v_\beta^{(1)}\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}+g_\beta^{(1)}(\ \frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}\ )\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}-\frac{\partial}{\partial v_\alpha^{(1)}}[\ \frac{1}{4\pi}\int\ \frac{\partial}{\partial x_\alpha^{(1)}}(\ \frac{1}{\left|\overline{x}^{(2)}-\overline{x}^{(1)}\right|}\ )$$

$$\times(\ \frac{\partial v_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial v_\beta^{(2)}}{\partial x_\alpha^{(2)}}-\frac{\partial g_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial g_\beta^{(2)}}{\partial x_\alpha^{(2)}}\ )F_2^{(1,2)}dx^{(2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+\nu\frac{\partial}{\partial v_\alpha^{(1)}}\lim_{\bar{x}(2)\to\bar{x}(1)}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\int v_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+\lambda\frac{\partial}{\partial g_\alpha^{(1)}}\lim_{\bar{x}(2)\to\bar{x}(1)}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\int g_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+\gamma\frac{\partial}{\partial \phi^{(1)}}\lim_{\bar{x}(2)\to\bar{x}(1)}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\int \phi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+D\frac{\partial}{\partial \psi^{(1)}}\lim_{\bar{x}(2)\to\bar{x}(1)}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\int \psi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}-R\psi^{(1)}\frac{\partial}{\partial \psi^{(1)}}F_1^{(1)}=0 \tag{19}$$

This is the transport equation for evolution of one –point distribution function $F_1^{(1)}$ under going a first order chemical reaction in MHD turbulent flow.

VI. Equation for Two-Point Distribution Function $F_2^{(1,2)}$

Differentiating equation (7) with respect to time, we get,

$$\frac{\partial F_2^{(1,2)}}{\partial t}=\frac{\partial}{\partial t}\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)$$
$$\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle$$
$$=\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\frac{\partial}{\partial t}\delta\left(u^{(1)}-v^{(1)}\right)\rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)$$
$$\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\frac{\partial}{\partial t}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$
$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\frac{\partial}{\partial t}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)$$
$$\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\frac{\partial}{\partial t}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle$$
$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\frac{\partial}{\partial t}\delta\left(u^{(2)}-v^{(2)}\right)\rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)$$
$$\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\frac{\partial}{\partial t}\delta\left(h^{(2)}-g^{(2)}\right)\rangle$$
$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\frac{\partial}{\partial t}\delta\left(\theta^{(2)}-\phi^{(2)}\right)\rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)$$
$$\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\frac{\partial}{\partial t}\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle$$

$$=\langle -\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\frac{\partial u^{(1)}}{\partial t}\frac{\partial}{\partial v^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\frac{\partial h^{(1)}}{\partial t}\frac{\partial}{\partial g^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\frac{\partial \theta^{(1)}}{\partial t}\frac{\partial}{\partial \phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\frac{\partial c^{(1)}}{\partial t}\frac{\partial}{\partial \psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\frac{\partial u^{(2)}}{\partial t}\frac{\partial}{\partial v^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\frac{\partial h^{(2)}}{\partial t}\frac{\partial}{\partial g^{(2)}}\delta\left(h^{(2)}-g^{(2)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\frac{\partial \theta^{(2)}}{\partial t}\frac{\partial}{\partial \phi^{(2)}}\delta\left(\theta^{(2)}-\phi^{(2)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\frac{\partial c^{(2)}}{\partial t}\frac{\partial}{\partial \psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle$$

Using equations (1) to (4) we get,

$$\frac{\partial F_2^{(1,2)}}{\partial t}=\langle -\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\Big\{-\frac{\partial}{\partial x_\beta^{(1)}}\left(u_\alpha^{(1)}u_\beta^{(1)}-h_\alpha^{(1)}h_\beta^{(1)}\right)-\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(1)}}\int\left[\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}}-\frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}}\right]$$

$$\times\frac{d\bar{x}''}{|\bar{x}''-\bar{x}|}+\nu\nabla^2 u_\alpha^{(1)}\Big\}\times\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\Big\{-\frac{\partial}{\partial x_\beta^{(1)}}\left(h_\alpha^{(1)}u_\beta^{(1)}-u_\alpha^{(1)}h_\beta^{(1)}\right)+\lambda\nabla^2 h_\alpha^{(1)}\Big\}\times\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\{-u_\beta^{(1)}\frac{\partial\theta^{(1)}}{\partial x_\beta^{(1)}}+\gamma\nabla^2\theta^{(1)}\;\}\times\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\rangle+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)$$

$$\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\{-u_\beta^{(1)}\frac{\partial c^{(1)}}{\partial x_\beta^{(1)}}+D\nabla^2c^{(1)}-Rc^{(1)}\;\}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\{\;-\frac{\partial}{\partial x_\beta^{(2)}}\left(u_\alpha^{(2)}u_\beta^{(2)}-h_\alpha^{(2)}h_\beta^{(2)}\right)-\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(2)}}\int\left[\;\frac{\partial u_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial u_\beta^{(2)}}{\partial x_\alpha^{(2)}}-\frac{\partial h_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial h_\beta^{(2)}}{\partial x_\alpha^{(2)}}\;\right]\frac{d\bar{x}''}{\left|\bar{x}''-\bar{x}'\right|}$$

$$+\nu\nabla^2u_\alpha^{(2)}\;\}\times\frac{\partial}{\partial v_\alpha^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\{\;-\frac{\partial}{\partial x_\beta^{(2)}}\left(h_\alpha^{(2)}u_\beta^{(2)}-u_\alpha^{(2)}h_\beta^{(2)}\right)+\lambda\nabla^2h_\alpha^{(2)}\;\}\times\frac{\partial}{\partial g_\alpha^{(2)}}\delta\left(h^{(2)}-g^{(2)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\{-u_\beta^{(2)}\frac{\partial\theta^{(2)}}{\partial x_\beta^{(2)}}+\gamma\nabla^2\theta^{(2)}\;\}\times\frac{\partial}{\partial\phi^{(2)}}\delta\left(\theta^{(2)}-\phi^{(2)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\{-u_\beta^{(2)}\frac{\partial c^{(2)}}{\partial x_\beta^{(2)}}+D\nabla^2c^{(2)}-Rc^{(2)}\;\}\times\frac{\partial}{\partial\psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle$$

$$=\langle\;\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\frac{\partial u_\alpha^{(1)}u_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle-\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\frac{\partial h_\alpha^{(1)}h_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle\;\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(1)}}\int\left[\;\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}}-\frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}}\;\right]\frac{d\bar{x}''}{\left|\bar{x}''-\bar{x}\right|}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle-\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\times\nu\nabla^2u_\alpha^{(1)}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\frac{\partial h_\alpha^{(1)}u_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\ \rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\frac{\partial u_\alpha^{(1)}h_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\ \rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\lambda\nabla^2 h_\alpha^{(1)}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\ \rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times u_\beta^{(1)}\frac{\partial\theta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\ \rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\gamma\nabla^2\theta^{(1)}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\ \rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times u_\beta^{(1)}\frac{\partial c^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\ \rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times D\nabla^2 c^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\ \rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times Rc^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\ \rangle$$

$$+\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\frac{\partial u_\alpha^{(2)}u_\beta^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial v_\alpha^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\ \rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\frac{\partial h_\alpha^{(2)}h_\beta^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial v_\alpha^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\ \rangle$$

$$+\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(2)}}\int\left[\ \frac{\partial u_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial u_\beta^{(2)}}{\partial x_\alpha^{(2)}}-\frac{\partial h_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial h_\beta^{(2)}}{\partial x_\alpha^{(2)}}\ \right]\frac{d\bar{x}''}{\left|\bar{x}''-\bar{x}'\right|}\frac{\partial}{\partial v_\alpha^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\ \rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times \nu\nabla^2 u_\alpha^{(2)}\frac{\partial}{\partial v_\alpha^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$
$$+\langle \ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\frac{\partial h_\alpha^{(2)}u_\beta^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial g_\alpha^{(2)}}\delta\left(h^{(2)}-g^{(2)}\right)\rangle$$
$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\frac{\partial u_\alpha^{(2)}h_\beta^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial g_\alpha^{(2)}}\delta\left(h^{(2)}-g^{(2)}\right)\rangle$$
$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\lambda\nabla^2 h_\alpha^{(2)}\frac{\partial}{\partial g_\alpha^{(2)}}\delta\left(h^{(2)}-g^{(2)}\right)\rangle$$
$$+\langle \ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times u_\beta^{(2)}\frac{\partial\theta^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial\phi^{(2)}}\delta\left(\theta^{(2)}-\phi^{(2)}\right)\rangle$$
$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\gamma\nabla^2\theta^{(2)}\frac{\partial}{\partial\phi^{(2)}}\delta\left(\theta^{(2)}-\phi^{(2)}\right)\rangle$$
$$+\langle \ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$
$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\times u_\beta^{(2)}\frac{\partial c^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial\psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle$$
$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$
$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\times D\nabla^2 c^{(2)}\frac{\partial}{\partial\psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle$$
$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$
$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\times Rc^{(2)}\frac{\partial}{\partial\psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle \qquad (20)$$

After simplify the various terms of equation (20) with the help of the properties of the distribution function and the continuity equation in terms distribution function as that they can be expressed in terms of one point and two point distribution functions, we get the transport equation for two point distribution function $F_2^{(1,2)}(v,g,\phi,\psi)$ in MHD turbulent flow in a 1st order chemical reaction as

$$\frac{\partial F_2^{(1,2)}}{\partial t}+\left(\ v_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}+v_\beta^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}\ \right)F_2^{(1,2)}+g_\beta^{(1)}\left(\ \frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}\ \right)\frac{\partial}{\partial x_\beta^{(1)}}F_2^{(1,2)}$$

$$+g_{\beta}^{(2)}\left(\frac{\partial g_{\alpha}^{(2)}}{\partial v_{\alpha}^{(2)}}+\frac{\partial v_{\alpha}^{(2)}}{\partial g_{\alpha}^{(2)}}\right)\frac{\partial}{\partial x_{\beta}^{(2)}}F_2^{(1,2)}-\frac{\partial}{\partial v_{\alpha}^{(1)}}\Big[\frac{1}{4\pi}\int\frac{\partial}{\partial x_{\alpha}^{(1)}}\Big(\frac{1}{\left|\bar{x}^{(3)}-\bar{x}^{(1)}\right|}\Big)$$

$$\times\left(\frac{\partial v_{\alpha}^{(3)}}{\partial x_{\beta}^{(3)}}\frac{\partial v_{\beta}^{(3)}}{\partial x_{\alpha}^{(3)}}-\frac{\partial g_{\alpha}^{(3)}}{\partial x_{\beta}^{(3)}}\frac{\partial g_{\beta}^{(3)}}{\partial x_{\alpha}^{(3)}}\right)F_3^{(1,2,3)}dx^{(3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{3)}\Big]$$

$$-\frac{\partial}{\partial v_{\alpha}^{(2)}}\Big[\frac{1}{4\pi}\int\frac{\partial}{\partial x_{\alpha}^{(2)}}\Big(\frac{1}{\left|\bar{x}^{(3)}-\bar{x}^{(2)}\right|}\Big)\left(\frac{\partial v_{\alpha}^{(3)}}{\partial x_{\beta}^{(3)}}\frac{\partial v_{\beta}^{(3)}}{\partial x_{\alpha}^{(3)}}-\frac{\partial g_{\alpha}^{(3)}}{\partial x_{\beta}^{(3)}}\frac{\partial g_{\beta}^{(3)}}{\partial x_{\alpha}^{(3)}}\right)$$

$$\times F_3^{(1,2,3)}dx^{(3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{3)}\Big]$$

$$+\nu\Big(\frac{\partial}{\partial v_{\alpha}^{(1)}}\lim_{\bar{x}^{(3)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial v_{\alpha}^{(2)}}\lim_{\bar{x}^{(3)}\to\bar{x}^{(2)}}\Big)\frac{\partial^2}{\partial x_{\beta}^{(3)}\partial x_{\beta}^{(3)}}\int v_{\alpha}^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)}$$

$$+\lambda\Big(\frac{\partial}{\partial g_{\alpha}^{(1)}}\lim_{\bar{x}^{(3)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial g_{\alpha}^{(2)}}\lim_{\bar{x}^{(3)}\to\bar{x}^{(2)}}\Big)\frac{\partial^2}{\partial x_{\beta}^{(3)}\partial x_{\beta}^{(3)}}\int g_{\alpha}^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)}$$

$$+\gamma\Big(\frac{\partial}{\partial \phi^{(1)}}\lim_{\bar{x}^{(3)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial \phi^{(2)}}\lim_{\bar{x}^{(3)}\to\bar{x}^{(2)}}\Big)\frac{\partial^2}{\partial x_{\beta}^{(3)}\partial x_{\beta}^{(3)}}\int \phi^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)}$$

$$+D\Big(\frac{\partial}{\partial \psi^{(1)}}\lim_{\bar{x}^{(3)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial \psi^{(2)}}\lim_{\bar{x}^{(3)}\to\bar{x}^{(2)}}\Big)\frac{\partial^2}{\partial x_{\beta}^{(3)}\partial x_{\beta}^{(3)}}\int \psi^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)}$$

$$-R\psi^{(1)}\frac{\partial}{\partial\psi^{(1)}}F_2^{(1,2)}-R\psi^{(2)}\frac{\partial}{\partial\psi^{(2)}}F_2^{(1,2)}=0 \tag{21}$$

VII. Equations for Three-Point Distribution Function $F_3^{(1,2,3)}$

Due to get the transport equation for three- point distribution function $F_3^{(1,2,3)}(v,g,\phi,\psi)$ in MHD turbulent flow in a 1st order chemical reaction, again we differentiating equation (8) with respect to time, we get

$$\frac{\partial F_3^{(1,2,3)}}{\partial t}=\frac{\partial}{\partial t}\Big\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\Big\rangle$$

$$=\Big\langle\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial}{\partial t}\delta\left(u^{(1)}-v^{(1)}\right)\Big\rangle$$

$$+\Big\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial}{\partial t}\delta\left(h^{(1)}-g^{(1)}\right)\Big\rangle$$

$$+\Big\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial}{\partial t}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\Big\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial}{\partial t}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle$$
$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial}{\partial t}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$
$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial}{\partial t}\delta\left(h^{(2)}-g^{(2)}\right)\rangle$$
$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial}{\partial t}\delta\left(\theta^{(2)}-\phi^{(2)}\right)\rangle$$
$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$
$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial}{\partial t}\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle \quad *$$
$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$
$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial}{\partial t}\delta\left(u^{(3)}-v^{(3)}\right)\rangle$$
$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$
$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial}{\partial t}\delta\left(h^{(3)}-g^{(3)}\right)\rangle$$
$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$
$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial}{\partial t}\delta\left(\theta^{(3)}-\phi^{(3)}\right)\rangle$$
$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$
$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\frac{\partial}{\partial t}\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle$$
$$=\langle-\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial u^{(1)}}{\partial t}\frac{\partial}{\partial v^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$
$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$
$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial h^{(1)}}{\partial t}\frac{\partial}{\partial g^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$
$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$
$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial\theta^{(1)}}{\partial t}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$
$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial c^{(1)}}{\partial t}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$
$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial u^{(2)}}{\partial t}\frac{\partial}{\partial v^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$
$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial\theta^{(2)}}{\partial t}\frac{\partial}{\partial\phi^{(2)}}\delta\left(\theta^{(2)}-\phi^{(2)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial c^{(2)}}{\partial t}\frac{\partial}{\partial\psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial u^{(3)}}{\partial t}\frac{\partial}{\partial v^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial h^{(3)}}{\partial t}\frac{\partial}{\partial g^{(3)}}\delta\left(h^{(3)}-g^{(3)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial\theta^{(3)}}{\partial t}\frac{\partial}{\partial\phi^{(3)}}\delta\left(\theta^{(3)}-\phi^{(3)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\frac{\partial c^{(3)}}{\partial t}\frac{\partial}{\partial\psi^{(3)}}\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle$$

Using equations (1) to (4), we get from the above equation

$$\frac{\partial F_3^{(1,2,3)}}{\partial t}=\langle-\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$
$$\left\{-\frac{\partial}{\partial x_\beta^{(1)}}\left(u_\alpha^{(1)}u_\beta^{(1)}-h_\alpha^{(1)}h_\beta^{(1)}\right)-\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(1)}}\int\left[\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}}-\frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}}\right]\frac{d\overline{x}'''}{\left|\overline{x}'''-\overline{x}^{n}\right|}+\nu\nabla^2u_\alpha^{(1)}\right\}\times\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$\{ -\frac{\partial}{\partial x_\beta^{(1)}}\left(h_\alpha^{(1)}u_\beta^{(1)}-u_\alpha^{(1)}h_\beta^{(1)}\right)+\lambda\nabla^2 h_\alpha^{(1)} \}\times\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$\times\{-u_\beta^{(1)}\frac{\partial\theta^{(1)}}{\partial x_\beta^{(1)}}+\gamma\nabla^2\theta^{(1)} \}\times\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$\{-u_\beta^{(1)}\frac{\partial c^{(1)}}{\partial x_\beta^{(1)}}+D\nabla^2 c^{(1)}-Rc^{(1)}\}\times\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle$$

$$\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$\{ -\frac{\partial}{\partial x_\beta^{(2)}}\left(u_\alpha^{(2)}u_\beta^{(2)}-h_\alpha^{(2)}h_\beta^{(2)}\right)-\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(2)}}\int\left[\frac{\partial u_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial u_\beta^{(2)}}{\partial x_\alpha^{(2)}}-\frac{\partial h_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial h_\beta^{(2)}}{\partial x_\alpha^{(2)}}\right]\frac{d\bar{x}'''}{\left|\bar{x}'''-\bar{x}'\right|}$$

$$+v\nabla^2 u_\alpha^{(2)} \}\times\frac{\partial}{\partial v_\alpha^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$\{-u_\beta^{(2)}\frac{\partial c^{(2)}}{\partial x_\beta^{(2)}}+D\nabla^2 c^{(2)}-Rc^{(2)} \}\frac{\partial}{\partial\psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle$$

$$\{ -\frac{\partial}{\partial x_\beta^{(2)}}\left(h_\alpha^{(2)}u_\beta^{(2)}-u_\alpha^{(2)}h_\beta^{(2)}\right)+\lambda\nabla^2 h_\alpha^{(2)} \}\frac{\partial}{\partial g_\alpha^{(2)}}\delta\left(h^{(2)}-g^{(2)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$\times\{-u_\beta^{(2)}\frac{\partial\theta^{(2)}}{\partial x_\beta^{(2)}}+\gamma\nabla^2\theta^{(2)} \}\times\frac{\partial}{\partial\phi^{(2)}}\delta\left(\theta^{(2)}-\phi^{(2)}\right)\rangle$$

$$\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$\{ -\frac{\partial}{\partial x_\beta^{(3)}}\left(u_\alpha^{(3)}u_\beta^{(3)}-h_\alpha^{(3)}h_\beta^{(3)}\right)-\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(3)}}\int\left[\frac{\partial u_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial u_\beta^{(3)}}{\partial x_\alpha^{(3)}}-\frac{\partial h_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial h_\beta^{(3)}}{\partial x_\alpha^{(3)}}\right]\frac{d\bar{x}'''}{\left|\bar{x}'''-\bar{x}''\right|}$$

$$+\nu\nabla^2 u_\alpha^{(3)} \ \} \times \frac{\partial}{\partial v_\alpha^{(3)}} \delta\left(u^{(3)} - v^{(3)}\right) \rangle$$

$$+\langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)$$
$$\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(c^{(2)} - \psi^{(2)}\right)\delta\left(u^{(3)} - v^{(3)}\right)\delta\left(\theta^{(3)} - \phi^{(3)}\right)\delta\left(c^{(3)} - \psi^{(3)}\right)$$
$$\{ \ -\frac{\partial}{\partial x_\beta^{(3)}}\left(h_\alpha^{(3)} u_\beta^{(3)} - u_\alpha^{(3)} h_\beta^{(3)}\right) + \lambda\nabla^2 h_\alpha^{(3)} \ \} \frac{\partial}{\partial g_\alpha^{(3)}} \delta\left(h^{(3)} - g^{(3)}\right) \rangle$$

$$+\langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)$$
$$\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(c^{(2)} - \psi^{(2)}\right)\delta\left(u^{(3)} - v^{(3)}\right)\delta\left(h^{(3)} - g^{(3)}\right)\delta\left(c^{(3)} - \psi^{(3)}\right)$$
$$\times \left\{ -u_\beta^{(3)} \frac{\partial \theta^{(3)}}{\partial x_\beta^{(3)}} + \gamma\nabla^2 \theta^{(3)} \ \right\} \times \frac{\partial}{\partial \phi^{(3)}} \delta\left(\theta^{(3)} - \phi^{(3)}\right) \rangle$$

$$+\langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)$$
$$\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(c^{(2)} - \psi^{(2)}\right)\delta\left(u^{(3)} - v^{(3)}\right)\delta\left(h^{(3)} - g^{(3)}\right)\delta\left(\theta^{(3)} - \phi^{(3)}\right)$$
$$\left\{ -u_\beta^{(3)} \frac{\partial c^{(3)}}{\partial x_\beta^{(3)}} + D\nabla^2 c^{(3)} - Rc^{(3)} \ \right\} \frac{\partial}{\partial \psi^{(3)}} \delta\left(c^{(3)} - \psi^{(3)}\right) \rangle$$

$$= \langle \ \delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(c^{(2)} - \psi^{(2)}\right)$$
$$\delta\left(u^{(3)} - v^{(3)}\right)\delta\left(h^{(3)} - g^{(3)}\right)\delta\left(\theta^{(3)} - \phi^{(3)}\right)\delta\left(c^{(3)} - \psi^{(3)}\right) \times \frac{\partial u_\alpha^{(1)} u_\beta^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial v_\alpha^{(1)}} \delta\left(u^{(1)} - v^{(1)}\right) \rangle$$

$$+\langle -\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(c^{(2)} - \psi^{(2)}\right)$$
$$\delta\left(u^{(3)} - v^{(3)}\right)\delta\left(h^{(3)} - g^{(3)}\right)\delta\left(\theta^{(3)} - \phi^{(3)}\right)\delta\left(c^{(3)} - \psi^{(3)}\right) \times \frac{\partial h_\alpha^{(1)} h_\beta^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial v_\alpha^{(1)}} \delta\left(u^{(1)} - v^{(1)}\right) \rangle$$

$$+\langle \ \delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(c^{(2)} - \psi^{(2)}\right)$$
$$\delta\left(u^{(3)} - v^{(3)}\right)\delta\left(h^{(3)} - g^{(3)}\right)\delta\left(\theta^{(3)} - \phi^{(3)}\right)\delta\left(c^{(3)} - \psi^{(3)}\right) \times \frac{1}{4\pi} \frac{\partial}{\partial x_\alpha^{(1)}} \int \left[\ \frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}} - \frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}} \ \right]$$
$$\times \frac{d\bar{x}'''}{\left|\bar{x}''' - \bar{x}\right|} \frac{\partial}{\partial v_\alpha^{(1)}} \delta\left(u^{(1)} - v^{(1)}\right) \rangle + \langle -\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)$$
$$\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(c^{(2)} - \psi^{(2)}\right)\delta\left(u^{(3)} - v^{(3)}\right)\delta\left(h^{(3)} - g^{(3)}\right)\delta\left(\theta^{(3)} - \phi^{(3)}\right)\delta\left(c^{(3)} - \psi^{(3)}\right) \times \nu\nabla^2 u_\alpha^{(1)} \frac{\partial}{\partial v_\alpha^{(1)}} \delta\left(u^{(1)} - v^{(1)}\right) \rangle$$

$$+\langle \ \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(c^{(2)} - \psi^{(2)}\right)$$
$$\delta\left(u^{(3)} - v^{(3)}\right)\delta\left(h^{(3)} - g^{(3)}\right)\delta\left(\theta^{(3)} - \phi^{(3)}\right)\delta\left(c^{(3)} - \psi^{(3)}\right) \times \frac{\partial h_\alpha^{(1)} u_\beta^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial g_\alpha^{(1)}} \delta\left(h^{(1)} - g^{(1)}\right) \rangle$$

$$+\langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(c^{(2)} - \psi^{(2)}\right)$$
$$\delta\left(u^{(3)} - v^{(3)}\right)\delta\left(h^{(3)} - g^{(3)}\right)\delta\left(\theta^{(3)} - \phi^{(3)}\right)\delta\left(c^{(3)} - \psi^{(3)}\right) \times \frac{\partial u_\alpha^{(1)} h_\beta^{(1)}}{\partial x_\beta^{(1)}} \frac{\partial}{\partial g_\alpha^{(1)}} \delta\left(h^{(1)} - g^{(1)}\right) \rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times \lambda\nabla^2 h_\alpha^{(1)}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(1)}\frac{\partial\theta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times \gamma\nabla^2\theta^{(1)}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(1)}\frac{\partial c^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times D\nabla^2 c^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle$$

$$+\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times Rc^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle$$

$$+\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial u_\alpha^{(2)}u_\beta^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial v_\alpha^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial h_\alpha^{(2)}h_\beta^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial v_\alpha^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$

$$+\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(2)}}\int\left[\frac{\partial u_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial u_\beta^{(2)}}{\partial x_\alpha^{(2)}}-\frac{\partial h_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial h_\beta^{(2)}}{\partial x_\alpha^{(2)}}\right]$$

$$\times\frac{d\overline{x}'''}{\left|\overline{x}'''-\overline{x}'\right|}\frac{\partial}{\partial v_\alpha^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times \nu\nabla^2 u_\alpha^{(2)}\frac{\partial}{\partial v_\alpha^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial h_\alpha^{(2)}u_\beta^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial g_\alpha^{(2)}}\delta\left(h^{(2)}-g^{(2)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial u_\alpha^{(2)}h_\beta^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial g_\alpha^{(2)}}\delta\left(h^{(2)}-g^{(2)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\lambda\nabla^2h_\alpha^{(2)}\frac{\partial}{\partial g_\alpha^{(2)}}\delta\left(h^{(2)}-g^{(2)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(2)}\frac{\partial\theta^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial\phi^{(2)}}\delta\left(\theta^{(2)}-\phi^{(2)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\gamma\nabla^2\theta^{(2)}\frac{\partial}{\partial\phi^{(2)}}\delta\left(\theta^{(2)}-\phi^{(2)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(2)}\frac{\partial c^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial\psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times D\nabla^2c^{(2)}\frac{\partial}{\partial\psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle$$

$$+\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times Rc^{(2)}\frac{\partial}{\partial\psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle$$

$$+\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial u_\alpha^{(3)}u_\beta^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial}{\partial v_\alpha^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial h_\alpha^{(3)}h_\beta^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial}{\partial v_\alpha^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\rangle$$

$$+\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(3)}}\int\left[\frac{\partial u_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial u_\beta^{(3)}}{\partial x_\alpha^{(3)}}-\frac{\partial h_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial h_\beta^{(3)}}{\partial x_\alpha^{(3)}}\right]$$

$$\times\frac{d\overline{x}'''}{\left|\overline{x}'''-\overline{x}''\right|}\frac{\partial}{\partial v_\alpha^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\nu\nabla^2u_\alpha^{(3)}\frac{\partial}{\partial v_\alpha^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial h_\alpha^{(3)}u_\beta^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial}{\partial g_\alpha^{(3)}}\delta\left(h^{(3)}-g^{(3)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial u_\alpha^{(3)}h_\beta^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial}{\partial g_\alpha^{(3)}}\delta\left(h^{(3)}-g^{(3)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\lambda\nabla^2h_\alpha^{(3)}\frac{\partial}{\partial g_\alpha^{(3)}}\delta\left(h^{(3)}-g^{(3)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(3)}\frac{\partial\theta^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial}{\partial\phi^{(3)}}\delta\left(\theta^{(3)}-\phi^{(3)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\gamma\nabla^2\theta^{(3)}\frac{\partial}{\partial\phi^{(3)}}\delta\left(\theta^{(3)}-\phi^{(3)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\times u_\beta^{(3)}\frac{\partial c^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial}{\partial\psi^{(3)}}\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\times D\nabla^2c^{(3)}\frac{\partial}{\partial\psi^{(3)}}\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle$$

$$+\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\times Rc^{(3)}\frac{\partial}{\partial\psi^{(3)}}\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle \tag{22}$$

Various terms in the above equation can be simplified as that they may be expressed in terms of one-, two-, three and four - point distribution functions.

The 1st term in the above equation is simplified as follows

$$\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial u_\alpha^{(1)}u_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$=\left\langle u_\beta^{(1)}\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\right.$$

$$\left.\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \right\rangle$$

$$=\left\langle -u_\beta^{(1)}\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\right.$$

$$\left.\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial u_\alpha^{(1)}}{\partial v_\alpha^{(1)}}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \right\rangle;\ (\text{since}\ \frac{\partial u_\alpha^{(1)}}{\partial v_\alpha^{(1)}}=1)$$

$$=\langle -\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle \tag{23}$$

Similarly, 5th, 8th and 10th terms of right hand-side of equation (22) can be simplified as follows;

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial h_\alpha^{(1)}u_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\ \rangle \tag{24}$$

$$=\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\ \rangle$$

8th term,

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(1)}\frac{\partial\theta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\ \rangle$$

$$=\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\ \rangle \tag{25}$$

and 10th term

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(1)}\frac{\partial c^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\ \rangle$$

$$=\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle \tag{26}$$

Adding these equations from (23) to (26), we get

$$\langle-\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle$$

$$=-\frac{\partial}{\partial x_\beta^{(1)}}\langle u_\beta^{(1)}\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle$$

$$=-\frac{\partial}{\partial x_\beta^{(1)}}v_\beta^{(1)}F_3^{(1,2,3)} \quad \text{[Applying the properties of distribution functions]} \tag{27}$$

Similarly, 13th, 17th, 20th and 22nd terms of right hand-side of equation (22) can be simplified as follows;

$$\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial u_\alpha^{(2)}u_\beta^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial v_\alpha^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$

$$=\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle \tag{28}$$

17th term,

$$\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial h_\alpha^{(2)}u_\beta^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial g_\alpha^{(2)}}\delta\left(h^{(2)}-g^{(2)}\right)\rangle$$

$$=\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}\delta\left(h^{(2)}-g^{(2)}\right)\rangle \tag{29}$$

20th term,

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(2)}\frac{\partial\theta^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial\phi^{(2)}}\delta\left(\theta^{(2)}-\phi^{(2)}\right)\ \rangle$$

$$=\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}\delta\left(\theta^{(2)}-\phi^{(2)}\right)\ \rangle \tag{30}$$

And 22nd term,

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(2)}\frac{\partial c^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial\psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\ \rangle$$

$$=\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\ \rangle \tag{31}$$

Adding equations (28) to (31), we get

$$-\frac{\partial}{\partial x_\beta^{(2)}}\langle u_\beta^{(2)}\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\ \rangle$$

$$=-v_\beta^{(2)}\frac{\partial F_3^{(1,2,3)}}{\partial x_\beta^{(2)}} \tag{32}$$

Similarly, 25th, 29th, 32nd and 34th terms of right hand-side of equation (67) can be simplified as follows;

$$\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial u_\alpha^{(3)}u_\beta^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial}{\partial v_\alpha^{(2)}}\delta\left(u^{(3)}-v^{(3)}\right)\ \rangle$$

$$=\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(3)}\frac{\partial}{\partial x_\beta^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\ \rangle \tag{33}$$

29th term,

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial h_\alpha^{(3)}u_\beta^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial}{\partial g_\alpha^{(3)}}\delta\left(h^{(3)}-g^{(3)}\right)\ \rangle$$

$$= \langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(3)}\frac{\partial}{\partial x_\beta^{(3)}}\delta\left(h^{(3)}-g^{(3)}\right)\rangle \tag{34}$$

32nd term,

$$= \langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(3)}\frac{\partial}{\partial x_\beta^{(3)}}\delta\left(\theta^{(3)}-\phi^{(3)}\right)\rangle$$

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(3)}\frac{\partial\theta^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial}{\partial\phi^{(3)}}\delta\left(\theta^{(3)}-\phi^{(3)}\right)\rangle \tag{35}$$

and 34th term,

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\times u_\beta^{(3)}\frac{\partial c^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial}{\partial\psi^{(3)}}\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle$$

$$= \langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\times u_\beta^{(3)}\frac{\partial}{\partial x_\beta^{(3)}}\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle \tag{36}$$

Adding equations (33) to (36), we get

$$-\frac{\partial}{\partial x_\beta^{(3)}}\left\langle u_\beta^{(3)}\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\right.$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle$$

$$= -\ v_\beta^{(3)}\ \frac{\partial F_3^{(1,2,3)}}{\partial x_\beta^{(3)}} \tag{37}$$

Similarly, 2nd, 6th, 14th, 18th, 26th and 30th terms of right hand-side of equation (67) can be simplified as follows;

$$\langle -\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial h_\alpha^{(1)}h_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$= -\ g_\beta^{(1)}\ \frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}\ \frac{\partial F_3^{(1,2,3)}}{\partial x_\beta^{(1)}} \tag{38}$$

6th term,

$$\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial u_\alpha^{(1)}h_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial g_\alpha}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$

$$= - g_{\beta}^{(1)} \frac{\partial v_{\alpha}^{(1)}}{\partial g_{\alpha}^{(1)}} \frac{\partial F_3^{(1,2,3)}}{\partial x_{\beta}^{(1)}} \tag{39}$$

14th term,

$$\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial h_{\alpha}^{(2)}h_{\beta}^{(2)}}{\partial x_{\beta}^{(2)}}\frac{\partial}{\partial v_{\alpha}^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$

$$= - g_{\beta}^{(2)} \frac{\partial g_{\alpha}^{(2)}}{\partial v_{\alpha}^{(2)}} \frac{\partial F_3^{(1,2,3)}}{\partial x_{\beta}^{(2)}} \tag{40}$$

18th term,

$$\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial u_{\alpha}^{(2)}h_{\beta}^{(2)}}{\partial x_{\beta}^{(2)}}\frac{\partial}{\partial g_{\alpha}^{(2)}}\delta\left(h^{(2)}-g^{(2)}\right)\rangle$$

$$= - g_{\beta}^{(2)} \frac{\partial v_{\alpha}^{(2)}}{\partial g_{\alpha}^{(2)}} \frac{\partial F_3^{(1,2,3)}}{\partial x_{\beta}^{(2)}} \tag{41}$$

26th term,

$$\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial h_{\alpha}^{(3)}h_{\beta}^{(3)}}{\partial x_{\beta}^{(3)}}\frac{\partial}{\partial v_{\alpha}^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\rangle$$

$$= - g_{\beta}^{(3)} \frac{\partial g_{\alpha}^{(3)}}{\partial v_{\alpha}^{(3)}} \frac{\partial F_3^{(1,2,3)}}{\partial x_{\beta}^{(3)}} \tag{42}$$

and 30th term,

$$\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial u_{\alpha}^{(3)}h_{\beta}^{(3)}}{\partial x_{\beta}^{(3)}}\frac{\partial}{\partial g_{\alpha}^{(3)}}\delta\left(h^{(3)}-g^{(3)}\right)\rangle$$

$$= - g_{\beta}^{(3)} \frac{\partial v_{\alpha}^{(3)}}{\partial g_{\alpha}^{(3)}} \frac{\partial F_3^{(1,2,3)}}{\partial x_{\beta}^{(3)}} \tag{43}$$

Fourth term can be reduced as

$$\langle -\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\nu\nabla^2 u_{\alpha}^{(1)}\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)$$

$$= -\nu \frac{\partial}{\partial v_\alpha^{(1)}} \langle \nabla^2 u_\alpha^{(1)} [\ \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)$$

$$\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(c^{(2)} - \psi^{(2)}\right)\delta\left(u^{(3)} - v^{(3)}\right)\delta\left(h^{(3)} - g^{(3)}\right)\delta\left(\theta^{(3)} - \phi^{(3)}\right)\delta\left(c^{(3)} - \psi^{(3)}\right)\ ]\ \rangle$$

$$= -\nu \frac{\partial}{\partial v_\alpha^{(1)}} \frac{\partial^2}{\partial x_\beta^{(1)} \partial x_\beta^{(1)}} \langle u_\alpha^{(1)} [\ \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\delta\left(u^{(2)} - v^{(2)}\right)$$

$$\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(c^{(2)} - \psi^{(2)}\right)\delta\left(u^{(3)} - v^{(3)}\right)\delta\left(h^{(3)} - g^{(3)}\right)\delta\left(\theta^{(3)} - \phi^{(3)}\right)\delta\left(c^{(3)} - \psi^{(3)}\right)\ ]\ \rangle$$

$$= -\nu \frac{\partial}{\partial v_\alpha^{(1)}} \lim_{\underline{x}^{(4)} \to \underline{x}^{(1)}} \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \langle u_\alpha^{(4)} [\ \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)$$

$$\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(c^{(2)} - \psi^{(2)}\right)\delta\left(u^{(3)} - v^{(3)}\right)\delta\left(h^{(3)} - g^{(3)}\right)\delta\left(\theta^{(3)} - \phi^{(3)}\right)\delta\left(c^{(3)} - \psi^{(3)}\right)\ ]\ \rangle$$

$$= -\nu \frac{\partial}{\partial v_\alpha^{(1)}} \lim_{\underline{x}^{(4)} \to \underline{x}^{(1)}} \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \langle \int u_\alpha^{(4)} \delta\left(u^{(4)} - v^{(4)}\right)\delta\left(h^{(4)} - g^{(4)}\right)\delta\left(\theta^{(4)} - \phi^{(4)}\right)\delta\left(c^{(4)} - \psi^{(4)}\right)$$

$$\delta\left(u^{(3)} - v^{(3)}\right)\delta\left(h^{(3)} - g^{(3)}\right)\delta\left(\theta^{(3)} - \phi^{(3)}\right)\delta\left(c^{(3)} - \psi^{(3)}\right)\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)$$

$$\delta\left(c^{(2)} - \psi^{(2)}\right)\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right) dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)}\ \rangle$$

$$= -\nu \frac{\partial}{\partial v_\alpha^{(1)}} \lim_{\underline{x}^{(4)} \to \underline{x}^{(1)}} \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int v_\alpha^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \tag{44}$$

Similarly, 7th ,9th ,11th ,16th ,19rd,21th ,23rd ,28th ,31st ,33rd and 35th terms of right hand-side of equation (67) can be simplified as follows;

$$\langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(c^{(2)} - \psi^{(2)}\right)$$

$$\delta\left(u^{(3)} - v^{(3)}\right)\delta\left(h^{(3)} - g^{(3)}\right)\delta\left(\theta^{(3)} - \phi^{(3)}\right)\delta\left(c^{(3)} - \psi^{(3)}\right) \times \lambda \nabla^2 h_\alpha^{(1)} \frac{\partial}{\partial g_\alpha^{(1)}} \delta\left(h^{(1)} - g^{(1)}\right)\ \rangle$$

$$= -\lambda \frac{\partial}{\partial g_\alpha^{(1)}} \lim_{\underline{x}^{(4)} \to \underline{x}^{(1)}} \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int g_\alpha^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \tag{45}$$

9th term,

$$\langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(c^{(2)} - \psi^{(2)}\right)$$

$$\delta\left(u^{(3)} - v^{(3)}\right)\delta\left(h^{(3)} - g^{(3)}\right)\delta\left(\theta^{(3)} - \phi^{(3)}\right)\delta\left(c^{(3)} - \psi^{(3)}\right) \times \gamma \nabla^2 \theta^{(1)} \frac{\partial}{\partial \phi^{(1)}} \delta\left(\theta^{(1)} - \phi^{(1)}\right)\ \rangle$$

$$= -\gamma \frac{\partial}{\partial \phi^{(1)}} \lim_{\underline{x}^{(4)} \to \underline{x}^{(1)}} \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int \phi^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \tag{46}$$

11th term,

$$+\langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(c^{(2)} - \psi^{(2)}\right)$$

$$\delta\left(u^{(3)} - v^{(3)}\right)\delta\left(h^{(3)} - g^{(3)}\right)\delta\left(\theta^{(3)} - \phi^{(3)}\right)\delta\left(c^{(3)} - \psi^{(3)}\right) \times D \nabla^2 c^{(1)} \frac{\partial}{\partial \psi^{(1)}} \delta\left(c^{(1)} - \psi^{(1)}\right)\ \rangle$$

$$= -D\frac{\partial}{\partial \psi^{(1)}} \lim_{\overline{x}^{(4)} \to \overline{x}^{(1)}} \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(3)}} \int \psi^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \tag{47}$$

16th term,

$$\langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(c^{(2)} - \psi^{(2)}\right)$$

$$\delta\left(u^{(3)} - v^{(3)}\right)\delta\left(h^{(3)} - g^{(3)}\right)\delta\left(\theta^{(3)} - \phi^{(3)}\right)\delta\left(c^{(3)} - \psi^{(3)}\right) \times \nu \nabla^2 u_\alpha^{(2)} \frac{\partial}{\partial v_\alpha^{(2)}} \delta\left(u^{(2)} - v^{(2)}\right) \rangle$$

$$= -\nu \frac{\partial}{\partial v_\alpha^{(2)}} \lim_{\overline{x}^{(4)} \to \overline{x}^{(2)}} \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int v_\alpha^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \tag{48}$$

19th term,

$$\langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(c^{(2)} - \psi^{(2)}\right)$$

$$\delta\left(u^{(3)} - v^{(3)}\right)\delta\left(h^{(3)} - g^{(3)}\right)\delta\left(\theta^{(3)} - \phi^{(3)}\right)\delta\left(c^{(3)} - \psi^{(3)}\right) \times \lambda \nabla^2 h_\alpha^{(2)} \frac{\partial}{\partial g_\alpha^{(2)}} \delta\left(h^{(2)} - g^{(2)}\right) \rangle$$

$$= -\lambda \frac{\partial}{\partial g_\alpha^{(2)}} \lim_{\overline{x}^{(4)} \to \overline{x}^{(2)}} \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int g_\alpha^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \tag{49}$$

21st term,

$$\langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(c^{(2)} - \psi^{(2)}\right)$$

$$\delta\left(u^{(3)} - v^{(3)}\right)\delta\left(h^{(3)} - g^{(3)}\right)\delta\left(\theta^{(3)} - \phi^{(3)}\right)\delta\left(c^{(3)} - \psi^{(3)}\right) \times \gamma \nabla^2 \theta^{(2)} \frac{\partial}{\partial \phi^{(2)}} \delta\left(\theta^{(2)} - \phi^{(2)}\right) \rangle$$

$$= -\gamma \frac{\partial}{\partial \phi^{(2)}} \lim_{\overline{x}^{(4)} \to \overline{x}^{(2)}} \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int \phi^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \tag{50}$$

23rd term,

$$\langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)$$

$$\delta\left(u^{(3)} - v^{(3)}\right)\delta\left(h^{(3)} - g^{(3)}\right)\delta\left(\theta^{(3)} - \phi^{(3)}\right)\delta\left(c^{(3)} - \psi^{(3)}\right) \times D \nabla^2 c^{(2)} \frac{\partial}{\partial \psi^{(2)}} \delta\left(c^{(2)} - \psi^{(2)}\right) \rangle$$

$$= -D \frac{\partial}{\partial \psi^{(2)}} \lim_{\overline{x}^{(4)} \to \overline{x}^{(2)}} \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int \psi^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \tag{51}$$

28th term,

$$\langle -\delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\delta\left(u^{(2)} - v^{(2)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\delta\left(\theta^{(2)} - \phi^{(2)}\right)$$

$$\delta\left(c^{(2)} - \psi^{(2)}\right)\delta\left(h^{(3)} - g^{(3)}\right)\delta\left(\theta^{(3)} - \phi^{(3)}\right)\delta\left(c^{(3)} - \psi^{(3)}\right) \times \nu \nabla^2 u_\alpha^{(3)} \frac{\partial}{\partial v_\alpha^{(3)}} \delta\left(u^{(3)} - v^{(3)}\right) \rangle$$

$$=-\nu\frac{\partial}{\partial v_{\alpha}^{(3)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(3)}}\frac{\partial^2}{\partial x_{\beta}^{(4)}\partial x_{\beta}^{(4)}}\int v_{\alpha}^{(4)}F_4^{(1,2,3,4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)} \tag{52}$$

31st term,

$$\langle -\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})$$

$$\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\times\lambda\nabla^2 h_{\alpha}^{(3)}\frac{\partial}{\partial g_{\alpha}^{(3)}}\delta(h^{(3)}-g^{(3)})\rangle$$

$$=-\lambda\frac{\partial}{\partial g_{\alpha}^{(3)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(3)}}\frac{\partial^2}{\partial x_{\beta}^{(4)}\partial x_{\beta}^{(4)}}\int g_{\alpha}^{(4)}F_4^{(1,2,3,4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)} \tag{53}$$

33rd term,

$$\langle -\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})$$

$$\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(c^{(3)}-\psi^{(3)})\times\gamma\nabla^2\theta^{(3)}\frac{\partial}{\partial\phi^{(3)}}\delta(\theta^{(3)}-\phi^{(3)})\rangle$$

$$=-\gamma\frac{\partial}{\partial\phi^{(3)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(3)}}\frac{\partial^2}{\partial x_{\beta}^{(4)}\partial x_{\beta}^{(4)}}\int\phi^{(4)}F_4^{(1,2,3,4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)} \tag{54}$$

35th term,

$$\langle -\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})$$

$$\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\times D\nabla^2 c^{(3)}\frac{\partial}{\partial\psi^{(3)}}\delta(c^{(3)}-\psi^{(3)})\rangle$$

$$=-D\frac{\partial}{\partial\psi^{(3)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(3)}}\frac{\partial^2}{\partial x_{\beta}^{(4)}\partial x_{\beta}^{(4)}}\int\psi^{(4)}F_4^{(1,2,3,4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)} \tag{55}$$

We reduce the third term of right hand side of equation (22),

$$\langle\ \delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})$$

$$\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\times\frac{1}{4\pi}\frac{\partial}{\partial x_{\alpha}^{(1)}}\int\left[\frac{\partial u_{\alpha}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial u_{\beta}^{(1)}}{\partial x_{\alpha}^{(1)}}-\frac{\partial h_{\alpha}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial h_{\beta}^{(1)}}{\partial x_{\alpha}^{(1)}}\right]$$

$$\times\frac{d\bar{x}'''}{|\bar{x}'''-\bar{x}|}\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta(u^{(1)}-v^{(1)})\ \rangle$$

$$=\frac{\partial}{\partial v_{\alpha}^{(1)}}\left[\frac{1}{4\pi}\int\frac{\partial}{\partial x_{\alpha}^{(1)}}\left(\frac{1}{|\bar{x}^{(4)}-\bar{x}^{(1)}|}\right)\left(\frac{\partial v_{\alpha}^{(4)}}{\partial x_{\beta}^{(4)}}\frac{\partial v_{\beta}^{(4)}}{\partial x_{\alpha}^{(4)}}-\frac{\partial g_{\alpha}^{(4)}}{\partial x_{\beta}^{(4)}}\frac{\partial g_{\beta}^{(4)}}{\partial x_{\alpha}^{(4)}}\right)F_4^{(1,2,3,4)}dx^{(4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)}\right] \tag{56}$$

15th term,

$$\langle\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$\times\frac{1}{4\pi}\frac{\partial}{\partial x_{\alpha}^{(2)}}\int\left[\frac{\partial u_{\alpha}^{(2)}}{\partial x_{\beta}^{(2)}}\frac{\partial u_{\beta}^{(2)}}{\partial x_{\alpha}^{(2)}}-\frac{\partial h_{\alpha}^{(2)}}{\partial x_{\beta}^{(2)}}\frac{\partial h_{\beta}^{(2)}}{\partial x_{\alpha}^{(2)}}\right]\times\frac{d\overline{x}'''}{\left|\overline{x}'''-\overline{x}'\right|}\frac{\partial}{\partial v_{\alpha}^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$

$$=\frac{\partial}{\partial v_{\alpha}^{(2)}}\left[\frac{1}{4\pi}\int\frac{\partial}{\partial x_{\alpha}^{(2)}}\left(\frac{1}{\left|\overline{x}^{(4)}-\overline{x}^{(2)}\right|}\right)\left(\frac{\partial v_{\alpha}^{(4)}}{\partial x_{\beta}^{(4)}}\frac{\partial v_{\beta}^{(4)}}{\partial x_{\alpha}^{(4)}}-\frac{\partial g_{\alpha}^{(4)}}{\partial x_{\beta}^{(4)}}\frac{\partial g_{\beta}^{(4)}}{\partial x_{\alpha}^{(4)}}\right)F_4^{(1,2,3,4)}dx^{(4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)}\right] \tag{57}$$

Similarly, 27th term,

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$\times\frac{1}{4\pi}\frac{\partial}{\partial x_{\alpha}^{(3)}}\int\left[\frac{\partial u_{\alpha}^{(3)}}{\partial x_{\beta}^{(3)}}\frac{\partial u_{\beta}^{(3)}}{\partial x_{\alpha}^{(3)}}-\frac{\partial h_{\alpha}^{(3)}}{\partial x_{\beta}^{(3)}}\frac{\partial h_{\beta}^{(3)}}{\partial x_{\alpha}^{(3)}}\right]\frac{d\overline{x}'''}{\left|\overline{x}'''-\overline{x}''\right|}\frac{\partial}{\partial v_{\alpha}^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\rangle$$

$$=\frac{\partial}{\partial v_{\alpha}^{(3)}}\left[\frac{1}{4\pi}\int\frac{\partial}{\partial x_{\alpha}^{(3)}}\left(\frac{1}{\left|\overline{x}^{(4)}-\overline{x}^{(3)}\right|}\right)\left(\frac{\partial v_{\alpha}^{(4)}}{\partial x_{\beta}^{(4)}}\frac{\partial v_{\beta}^{(4)}}{\partial x_{\alpha}^{(4)}}-\frac{\partial g_{\alpha}^{(4)}}{\partial x_{\beta}^{(4)}}\frac{\partial g_{\beta}^{(4)}}{\partial x_{\alpha}^{(4)}}\right)F_4^{(1,2,3,4)}dx^{(4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)}\right] \tag{58}$$

12th term of Equation (22)

$$\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times Rc^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle=R\psi^{(1)}\frac{\partial}{\partial\psi^{(1)}}F_3^{(1,2,3)} \tag{59}$$

24th term of Equation (22)

$$\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times Rc^{(2)}\frac{\partial}{\partial\psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle=R\psi^{(2)}\frac{\partial}{\partial\psi^{(2)}}F_3^{(1,2,3)} \tag{60}$$

36th term of Equation (22)

$$\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\times Rc^{(3)}\frac{\partial}{\partial\psi^{(3)}}\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle=R\psi^{(3)}\frac{\partial}{\partial\psi^{(3)}}F_3^{(1,2,3)} \tag{61}$$

VIII. Results and Discussions

Substituting the results (23) – (61) in equation (22) we get the transport equation for three- point distribution function $F_3^{(1,2,3)}(v,g,\phi,\psi)$ in MHD turbulent flow in a first order reaction as

$$\frac{\partial F_3^{(1,2,3)}}{\partial t}+\left(v_{\beta}^{(1)}\frac{\partial}{\partial x_{\beta}^{(1)}}+v_{\beta}^{(2)}\frac{\partial}{\partial x_{\beta}^{(2)}}v_{\beta}^{(3)}\frac{\partial}{\partial x_{\beta}^{(3)}}\right)F_3^{(1,2,3)}+\left[g_{\beta}^{(1)}\left(\frac{\partial g_{\alpha}^{(1)}}{\partial v_{\alpha}^{(1)}}+\frac{\partial v_{\alpha}^{(1)}}{\partial g_{\alpha}^{(1)}}\right)\frac{\partial}{\partial x_{\beta}^{(1)}}\right.$$

$$\left.+g_{\beta}^{(2)}\left(\frac{\partial g_{\alpha}^{(2)}}{\partial v_{\alpha}^{(2)}}+\frac{\partial v_{\alpha}^{(2)}}{\partial g_{\alpha}^{(2)}}\right)\frac{\partial}{\partial x_{\beta}^{(2)}}+g_{\beta}^{(3)}\left(\frac{\partial g_{\alpha}^{(3)}}{\partial v_{\alpha}^{(3)}}+\frac{\partial v_{\alpha}^{(3)}}{\partial g_{\alpha}^{(3)}}\right)\frac{\partial}{\partial x_{\beta}^{(3)}}\right]F_3^{(1,2,3)}$$

$$+\nu\left(\frac{\partial}{\partial v_\alpha^{(1)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial v_\alpha^{(2)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(2)}}+\frac{\partial}{\partial v_\alpha^{(3)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(3)}}\right)$$

$$\times\frac{\partial^2}{\partial x_\beta^{(4)}\partial x_\beta^{(4)}}\int v_\alpha^{(4)}F_4^{(1,2,3,4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)}$$

$$+\lambda\left(\frac{\partial}{\partial g_\alpha^{(1)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial g_\alpha^{(2)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(2)}}+\frac{\partial}{\partial g_\alpha^{(3)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(3)}}\right)$$

$$\times\frac{\partial^2}{\partial x_\beta^{(4)}\partial x_\beta^{(4)}}\int g_\alpha^{(4)}F_4^{(1,2,3,4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)}$$

$$+\gamma\left(\frac{\partial}{\partial \phi^{(1)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial \phi^{(2)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(2)}}+\frac{\partial}{\partial \phi^{(3)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(3)}}\right)$$

$$\times\frac{\partial^2}{\partial x_\beta^{(4)}\partial x_\beta^{(4)}}\int \phi^{(4)}F_4^{(1,2,3,4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)}$$

$$+D\left(\frac{\partial}{\partial \psi^{(1)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial \psi^{(2)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(2)}}+\frac{\partial}{\partial \psi^{(3)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(3)}}\right)$$

$$\times\frac{\partial^2}{\partial x_\beta^{(4)}\partial x_\beta^{(4)}}\int \psi^{(4)}F_4^{(1,2,3,4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)}$$

$$-\left[\frac{\partial}{\partial v_\alpha^{(1)}}\left\{\frac{1}{4\pi}\int\frac{\partial}{\partial x_\alpha^{(1)}}\left(\frac{1}{\left|\bar{x}^{(4)}-\bar{x}^{(1)}\right|}\right)\right\}+\frac{\partial}{\partial v_\alpha^{(2)}}\left\{\frac{1}{4\pi}\int\frac{\partial}{\partial x_\alpha^{(2)}}\left(\frac{1}{\left|\bar{x}^{(4)}-\bar{x}^{(2)}\right|}\right)\right\}\right.$$

$$+\frac{\partial}{\partial v_\alpha^{(3)}}\left\{\frac{1}{4\pi}\int\frac{\partial}{\partial x_\alpha^{(3)}}\left(\frac{1}{\left|\bar{x}^{(4)}-\bar{x}^{(3)}\right|}\right)\right\}\times\left(\frac{\partial v_\alpha^{(4)}}{\partial x_\beta^{(4)}}\frac{\partial v_\beta^{(4)}}{\partial x_\alpha^{(4)}}-\frac{\partial g_\alpha^{(4)}}{\partial x_\beta^{(4)}}\frac{\partial g_\beta^{(4)}}{\partial x_\alpha^{(4)}}\right)F_4^{(1,2,3,4)}$$

$$\left.\times dx^{(4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)}\right]-R\left(\psi^{(1)}\frac{\partial}{\partial\psi^{(1)}}+\psi^{(2)}\frac{\partial}{\partial\psi^{(2)}}+\psi^{(3)}\frac{\partial}{\partial\psi^{(3)}}\right)F_3^{(1,2,3)}=0 \tag{62}$$

Continuing this way, we can derive the equations for evolution of $F_4^{(1,2,3,4)}$, $F_5^{(1,2,3,4,5)}$ and so on. Logically it is possible to have an equation for every F_n (n is an integer) but the system of equations so obtained is not closed. Certain approximations will be required thus obtained.

If R=0,i.e the reaction rate is absent, the transport equation for three- point distribution function in MHD turbulent flow in a first order reaction (62) becomes to Azad et al (2014a).

If we drop the viscous, magnetic and thermal diffusive and concentration terms from the three point evolution equation (62), we have

$$\frac{\partial F_3^{(1,2,3)}}{\partial t}+\left(v_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}+v_\beta^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}v_\beta^{(3)}\frac{\partial}{\partial x_\beta^{(3)}}\right)F_3^{(1,2,3)}+\left[g_\beta^{(1)}\left(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}\right)\frac{\partial}{\partial x_\beta^{(1)}}\right.$$

$$
\begin{aligned}
&+g_{\beta}^{(2)}\left(\frac{\partial g_{\alpha}^{(2)}}{\partial v_{\alpha}^{(2)}}+\frac{\partial v_{\alpha}^{(2)}}{\partial g_{\alpha}^{(2)}}\right)\frac{\partial}{\partial x_{\beta}^{(2)}}+g_{\beta}^{(3)}\left(\frac{\partial g_{\alpha}^{(3)}}{\partial v_{\alpha}^{(3)}}+\frac{\partial v_{\alpha}^{(3)}}{\partial g_{\alpha}^{(3)}}\right)\frac{\partial}{\partial x_{\beta}^{(3)}}\Big]F_3^{(1,2,3)}\\
&-\Big[\frac{\partial}{\partial v_{\alpha}^{(1)}}\Big\{\frac{1}{4\pi}\int\frac{\partial}{\partial x_{\alpha}^{(1)}}\Big(\frac{1}{\left|\overline{x}^{(4)}-\overline{x}^{(1)}\right|}\Big)\Big\}+\frac{\partial}{\partial v_{\alpha}^{(2)}}\Big\{\frac{1}{4\pi}\int\frac{\partial}{\partial x_{\alpha}^{(2)}}\Big(\frac{1}{\left|\overline{x}^{(4)}-\overline{x}^{(2)}\right|}\Big)\Big\}\\
&+\frac{\partial}{\partial v_{\alpha}^{(3)}}\Big\{\frac{1}{4\pi}\int\frac{\partial}{\partial x_{\alpha}^{(3)}}\Big(\frac{1}{\left|\overline{x}^{(4)}-\overline{x}^{(3)}\right|}\Big)\Big\}\times\Big(\frac{\partial v_{\alpha}^{(4)}}{\partial x_{\beta}^{(4)}}\frac{\partial v_{\beta}^{(4)}}{\partial x_{\alpha}^{(4)}}-\frac{\partial g_{\alpha}^{(4)}}{\partial x_{\beta}^{(4)}}\frac{\partial g_{\beta}^{(4)}}{\partial x_{\alpha}^{(4)}}\Big)F_4^{(1,2,3,4)}\\
&\times dx^{(4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)}\Big]=0
\end{aligned}
\tag{63}
$$

The existence of the term

$$\left(\frac{\partial g_{\alpha}^{(1)}}{\partial v_{\alpha}^{(1)}}+\frac{\partial v_{\alpha}^{(1)}}{\partial g_{\alpha}^{(1)}}\right),\ \left(\frac{\partial g_{\alpha}^{(2)}}{\partial v_{\alpha}^{(2)}}+\frac{\partial v_{\alpha}^{(2)}}{\partial g_{\alpha}^{(2)}}\right)\text{ and }\left(\frac{\partial g_{\alpha}^{(3)}}{\partial v_{\alpha}^{(3)}}+\frac{\partial v_{\alpha}^{(3)}}{\partial g_{\alpha}^{(3)}}\right)$$

can be explained on the basis that two characteristics of the flow field are related to each other and describe the interaction between the two modes (velocity and magnetic) at point $x^{(1)}$, $x^{(2)}$ and $x^{(3)}$.

We can exhibit an analogy of this equation with the 1st equation in BBGKY hierarchy in the kinetic theory of gases. The first equation of BBGKY hierarchy is given Lundgren (1969) as

$$\frac{\partial F_1^{(1)}}{\partial t}+\frac{1}{m}v_{\beta}^{(1)}\frac{\partial}{\partial x_{\beta}^{(1)}}F_1^{(1)}=n\iint\frac{\partial\psi_{1,2}}{\partial x_{\alpha}^{(1)}}\frac{\partial F_2^{(1,2)}}{\partial v_{\alpha}^{(1)}}d\overline{x}^{(2)}d\overline{v}^{(2)} \tag{64}$$

where $\psi_{1,2}=\psi\left|v_{\alpha}^{(2)}-v_{\alpha}^{(1)}\right|$ is the inter molecular potential.

Some approximations are required, if we want to close the system of equations for the distribution functions. In the case of collection of ionized particles, i.e. in plasma turbulence, it can be provided closure form easily by decomposing $F_2^{(1,2)}$ as $F_1^{(1)} F_1^{(2)}$. But it will be possible if there is no interaction or correlation between two particles. If we decompose $F_2^{(1,2)}$ as

$F_2^{(1,2)} = (1+\in) F_1^{(1)} F_1^{(2)}$

and

$F_3^{(1,2,3)} = (1+\in)^2 F_1^{(1)} F_1^{(2)} F_1^{(3)}$

Also

$F_4^{(1,2,3,4)} = (1+\in)^3 F_1^{(1)} F_1^{(2)} F_1^{(3)} F_1^{(4)}$

where $\in$ is the correlation coefficient between the particles. If there is no correlation between the particles, $\in$ will be zero and distribution function can be decomposed in usual way. Here we are considering such type of approximation only to provide closed from of the equation.

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The Real Universe

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Abstract- The findings of strong gravitational lensing by Sgr A* indicate gravitation power/energy exist in the center of our milky way, though it is invisible. The presence of energy equal presence of mass ($M= E/C^2$, C^2 is constant). However, current perception of particles can be massless e.g. gauge bosons, the photon and the gluon (1) which create a centerpiece of confusion when we come to study dark matter and origin of universe. In this paper, the author will explain why all particles possess mass which is supported partially by 2013 Nobel Prize in Physics- the Higgs mechanism, which gives mass to fundamental particles. The big bang is the greatest event happened in the universe; it is unthinkable that such event left no trace in space; we must be blindfolded by something. In this paper, the author will point out what to look for so that we won't miss out on the Universe greatest party.

Keywords: particles, dark matters, black hole, gamma ray, big bang, origin of universe, time, space, odds, numerous bangs, endless cycles.

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THEREALUNIVERSE

Strictly as per the compliance and regulations of :

The Real Universe

Edwin Zong

Abstract- The findings of strong gravitational lensing by Sgr A* indicate gravitation power/energy exist in the center of our milky way, though it is invisible. The presence of energy equal presence of mass (M= E/C², C² is constant). However, current perception of particles can be massless e.g. gauge bosons, the photon and the gluon (1) which create a centerpiece of confusion when we come to study dark matter and origin of universe. In this paper, the author will explain why all particles possess mass which is supported partially by 2013 Nobel Prize in Physics- the Higgs mechanism, which gives mass to fundamental particles. The big bang is the greatest event happened in the universe; it is unthinkable that such event left no trace in space; we must be blindfolded by something. In this paper, the author will point out what to look for so that we won't miss out on the Universe greatest party. Few will argue our matters (whether visible or invisible) are all made from particles. Many doubt that electromagnetic radiation or particles follow same laws of basic physics. Those doubts directly lead to a set of principles that land us in a fantasy world of uncertainty. However, the correct understanding of fundamental particles is the key for understanding big bang (2). In this paper, the author will pave a clear road map for how to study our fundamental particles, particular Gamma Ray. Furthermore, the author will explain the reasons that dark matter/black hole is made from gamma ray nucleo synthesis! The author also will explain the reasons that our current background cosmic ray/gamma ray originated directly from big bang (for most part)! The master of infrared/fire power set human apart from animals and land us on top of the world. The master of the gamma-ray will undoubtedly land us on top of the universe. The Physics will unite all science branches. The particle physics will lead artificial intelligence to its terminal end in the universe civilization of all kind.

Keywords: particles, dark matters, black hole, gamma ray, big bang, origin of universe, time, space, odds, numerous bangs, endless cycles.

I. Introduction

All matters (include fundamental particles) intrinsically possess positional energy/power and kinetic energy/power. For better understanding, the author here describes all positional energy as orbiting energy. The usual term of gravity means earth's gravity affects objects that are positioned on earth, but it really means that positioned objects are orbiting around earth. The single orbiting energy/power unifies matter and particles, because particles possess orbiting power when they are coupled. Those orbiting energy of particles are same as gravity, which should be scaled in gravity term. Furthermore, the gravity affects particles in same way as does to their heavier mass brothers, even though the effects on heavier matter are negligible. It's that reason, any study of particle must be in a lab environment of 0G0K (Zero Gravity Zero Kelvin). The study of particles behavior in a non-0G0K environment is invalid(3). The exception is to generate new particle in a contaminated lab environment.0G0K is beyond human reach, but it will not stop us from understanding those little creatures. The author can assure you, when they are coupled, all particles orbit in a circular way, as it appears for any orbiting celestial body. When particles are liberated from their orbits, they will fly out in a straight way in a void space. The flying path of liberated particle can be bent. The particle will be affected by gravity if foreign particle or mass exist, exemplified as gravitational lens phenomenon, that refers to a distribution of matter (such as a cluster of galaxies) between a distant source and an observer, that is capable of bending the light from the source, as it travels towards the observer. That is one of the direct evidence that light particle possesses mass. The gravity only exists among masses whether big or negligible small.

The light particles can be absorbed as well, it can be absorbed by inorganic and organic material. The liberated particles advance in a wave way, a straight direction in a void space. We often describe them as electromagnetic radiation or ray. We often know maters better, because many of them are visible to our naked eyes, e.g. soccer. When matter breaks down to an invisible level, many panic. Some create principle of uncertainty for those invisible devils, but the author assures you, invisible particles follow same laws of physics and mathematics odds of random. Many bizarre behaviors of observed particles are resulted from contaminated labs (non-zero gravity, non-absolute zero temperature).

The conservative of energy dictates that liberated particles possess greater kinetic energy when they leave their orbit. The orbiting energy they once possess now fuels their kinetic energy. The destiny of liberated particles is to reunite other particles. The reunion will eventually lead to recapture its positional energy/power. The fate of particle closely mirrors our universe visible matters. The destiny of cosmos objects is destruction/big bang, every destruction/big bang breeds new born of cosmos objects/construction. The universe is in an endless cycle of matters/energy, where universe law applies both visible and invisible matters. There is no mystery in universe. Other than gravity

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profiling, we also need to know the intrinsic temperature of particle. Particle physics are the cutting edge of all science, but its focus should be on profiling particles in a correct way-quantifying its G and K.

II. Big Bang, γ-Ray, Dark Matter, Black Hole

The universe is a complete and enclosed system, because there is nothing existing outside universe which may interferes universe matters/energy. The conservation of mass and energy perfectly apply universe as a unit. We can safely say that universe energy and mass are constant. The universe is never a boring place, there are many events happening in the universe, but nothing is more significant than a big bang. First, few will argue the big bang gives out the greatest kinetic energy. The question is where the big blast of kinetic energy goes today? The kinetic energy cannot disappear without trace. Second, the big bang spits out matters which carry tremendous energy, where are those high energy matters today? Again, the matters cannot disappear without a trace.

Admittedly, much of a big bang's kinetic energy transforms into positional energy (conservation of energy) after the big bang. This is directly evidenced by numerous stars, planets etc. All those positional energy possessed by today's celestial mass originated from the big bang's kinetic energy; however, it is not a complete story here. There are numerous circulating celestial mass which carry tremendous kinetic energy besides their positional energy. Furthermore, there come cosmic rays! Those cosmic rays scatter with no pairing; they are free standing loners. The cosmic rays carry little positional energy but tremendous kinetic energy. Among them, the Gamma Ray is most significant kinetic energy form existing in the universe. The big bang Energy can be safely construed as pure kinetic energy (those kinetic energy is transformed from positional energy possessed in pre-big bang Mass). Now we can put all the pieces of puzzle together, it solves a very simple question in a simple equation.

$$E_{bigbang} = E_k + E_p$$

The E_k is the sum of kinetic energy presents in the current universe. E_p is the sum of positional energy presents in the current universe.

The matter that carries kinetic energy can be considered "visible", because we have a way to detect them directly, e.g. its body temperature. The temperature is a good example of kinetic energy. The other good example of kinetic energy is orbiting planets. The speed prevents them from falling into their orbiting mass. Many objects possess kinetic energy also possess significant positional energy e.g. sun. The sun carries kinetic energy and it has gravity (a form of positional energy). Many objects possess kinetic energy, but own little or none positional energy. Luckily, they possess kinetic energy, so we can detect them. Those creatures perfectly meet the needs of the big bang explosion. Among all those creatures, we can clearly see nothing is more qualified than Liberated Gamma Ray/Particles to fit into this role. The liberated Gamma Ray/Particles stand for free from orbiting (e.g. Radioactive decay).

What confuses many scientists today are objects that possess positional energy but little kinetic energy. Some of those objects that possess positional energy also possess kinetic energy e.g. sun. Those are considered visible and detectable. What about objects possessing positional energy but have little or no kinetic energy? Furthermore, when their positional energy powerful enough, they will absorb kinetic energy of any kind, pure or partial. They will have no lights reflex at all. If those objects present right in front of us, we can touch them but not able to see them. While we touch those objects, we will most likely be absorbed into them and dissembled to particles. If they present far away, beyond our physical reach, the objects could be a nightmare for scientists. The author agrees that dark matter is a perfect name for them. They are matters but invisible.

The big explosion/big bang is the most extreme event happening here and there in the universe. It experiences moment of silent/quiet followed by a gigantic explosion (positional energy transforms into kinetic energy in energy term for a blast). Nothing is more qualified than dark matter to be a pre-bang mass which meets the requirement for feeding a big bang.

The black hole is made of dark matters. The so-called dark/black simply means invisible to human eyes. Invisible doesn't equal to non-existing. Dark matters emit little lights, their temperature are very low as they are evidenced by their presence in the center of Milky Way. A black hole of 4.5 × 1022 kg (about the mass of the Moon) would be in equilibrium at 2.7 kelvin, absorbing as much radiation as it emits (4). For comparison, at the core of our sun, the temperature can reach more than 27 million degrees F (15 million degrees C). The sun only holds our solar system together, but the black hole holds our milky way together. Our milky way is said to have 100 billion stars. It is undoubtedly to say, our black hole possesses enormous amount of gravitation power, which means enormous amounts of mass. The black hole is no mystery. The gravitation power is part of positional power. However, the recent observations of the star S14 (S0-16) circulates the center of our milky way indicate that our black hole radius is no more than 6.25 light-hours. The only widely hypothesized type of object that can contain 4.1 million solar masses in a volume of that small size is a black hole. (5)

The black hole of Milky Way is incredible cold comparing to our solar center (4). So, given such low kinetic energy/low temperature in the black hole of Milky

Global Journal of Science Frontier Research (A) Volume XV Issue II Version I Year 2015

Way, which form does its energy exist? According to conservation of energy, it must be in its positional energy. The author can therefore assure you that dark matters/black hole possesses enormous amounts of orbiting energy/positional energy, which is directly evidenced by Milky Way staying together. Without black hole, any galaxy will scatter away.

The life span of galaxy is lot longer than our solar system, which dictates our milky way's black hole cannot emit too much energy/mass out too fast or outpace its absorbing new mass/energy, while it needs to be continued to be fed by outside masses/energy. The gigantic level of black hole gravity ensures its longer life in two ways. One, few lights escape its gravity grip. Two, tremendous amount of celestial mass are continuously pulled in. Our sun, on other hand, it sheds tremendous amounts of energy/mass continuously, but it is not fed by outside masses at any significant level. The sun has much shorter life span vs. black hole. The mass's make up of sun is different from black hole as well.

Dark matters and black holes are no magic vs. their visible cousins. The denser element means heavier element. What is heavier? It means greater gravitation power. Few will argue dark matters are compactly packed creatures. The real question is where they get such striking energy/gravitation power from? According to conservation of energy, energy cannot be created or destroyed. Their gravitation energy must come from their kinetic energy they once owned.

Who owns highest kinetic energy in the universe? The liberated particles own highest kinetic energy directly from a big bang! Now we can put final piece of puzzle in its place. Without seeing its happening, the author can assure you two things. 1. The dark matter/black hole is compacted (non-liberated or orbit-binding) gamma ray/particles nucleo synthesis! 2. The dark matter/black hole/non-liberated Gamma Ray are those original Gamma Ray/Particles directly come from big bang! The compacted or non-liberated gamma ray particles exist in dark matter of black hole is understandable, because tremendous numbers of γ-ray/particles are gathering in various dense areas (space is not vacuumed during pre or post big bang. The free standing γ-ray will be coupled or orbited among each other where they gather. The mass of orbit-binding gamma ray acquired tremendous positional power from such gathering. Those density induced γ-ray gathering/concentration triggers massive gamma ray/particles nucleo synthesis! Which is termed as dark matters/black hole. The free standing or non-orbiting γ-ray will continue to fly in a straight line (almost pure kinetic energy with little positional energy) until it meets its mate or potential coupling partner. As a matter of fact, the free standing loner γ-ray still exists in a tremendous amount in today's space- it is called cosmic ray!

Furthermore, our current cosmos Gamma ray (not all of them) is indeed from a big bang! It is partially supported by fact that the energies of gamma rays from astronomical sources range to over 10 TeV. That energy is far too large to result from radioactive decay. (6) some cosmos gamma ray are from pulsar and black hole though. However, gamma-ray pulsars and rare occurrence of gamma ray burst from black hole are very rare events in the universe. It is evidenced by recent observation: there have been only about one hundred gamma-ray pulsars identified out of about 1800 known pulsars (7)(8). Base on observation, the sources of most GRBs are billions of light years away from Earth, implying that the explosions are both extremely energetic (a typical burst releases as much energy in a few seconds as the Sun will in its entire 10-billion-year lifetime) and extremely rare (a few per galaxy per million years).(9) Given the facts that the universe space is infinite big and it is full of diffuse gamma radiation, it can be reasoned that the pulsars and black holes are not sufficient source for such extensive universe background gamma radiation. Secondly, the radioactive decay is not a right candidate power house for cosmos γ-ray background either. The only option left here is a big bang. The big bang provides most of those cosmos γ-ray today. The remaining cosmos γ-ray is provided by big bang indirectly. The issue remaining here is that some of those cosmos γ-ray might not come from the big bang which created our galaxy, some of those cosmos γ-ray may come from foreign big bangs if it can stay single/free long enough and it can escape being captured by cosmos mass/energy along its way.

The minute part of cosmos γ-ray may even come from alien artificial intelligence. The author here can assure you black matter/black hole are made from gamma ray nucleosynthesis that are originated directly from big bang. The gigantic gravitation power which black hole possesses today directly comes from a big bang! There will be no other appropriate candidate source powerful enough to fuel black hole such energy at that astonishing level. It is evidenced by galaxy's black hole. In the universe space, there is no single isolated structure that is bigger than a galaxy. All galaxies are hold together by its black hole. All galaxies descend from a big bang directly, so does its black hole. As a matter of fact, all galaxies are made from gamma ray of some kind as well.

Secondly, gamma ray nucleo synthesis existing in black matter/black hole remain domicile which presents little kinetic power, however, the continue feeding of new celestial mass may trigger some of its gamma ray particles to be liberated from their orbiting position and fly out with tremendous kinetic power, which is evidenced by gamma ray burst from a black hole. It is supported by NASA SWIFT project's findings of gamma ray burst (GRB) that is directly related to a black hole.

a) Dark Matter and Big Bang

The dark Matter is Matter that possesses little kinetic energy/power, so that it is hard to be detected by conventional detectors, e.g. telescope. The dark energy is positional energy, not kinetic energy. The kinetic energy is readily detected by our conventional detectors. However, there is a consensus that our universe dark matter makes up about 27% and 68% of the Universe is dark energy. What 68% tells us? It tells us that more than half of energy is still in Gamma Ray. 27% mass is in Gamma Ray nucleo synthesis of some kind! The dark energy is carried by Dark matters. The dark is no mystery. The author here will crack the nut for you. 68% of the universe energy is dark energy; it is so significant sources of energy that cannot be supplied by any other sources but a big bang itself! The greatest kinetic energy deprived from a big blast turned into positional energy shortly after explosion. The author's point is supported by Big Bang nucleo synthesis- The first nuclei were formed about three minutes after the Big Bang! Nucleo synthesis is the process that creates new atomic nuclei from pre-existing nucleons, primarily protons and neutrons. The newly formed new atomic nuclei are perfect example of pure kinetic energy captured into positional energy/orbiting energy! The Big Bang Nucleo synthesis not only provides food for baby galaxies' visible matters but also breeds dark matters that hold galaxies visible matter together!

The positional energy possessed by dark matters apparently is necessary for galaxies to form and stay together; otherwise, all matters/energy will scatter all over places without having a chance to grow into anything significant in its size. Therefore, the "visible matters" which have chances to form must "follow" dark matters formation which occurs first, not vice versa. The author's view is supported by the dark ages of the universe — an era of darkness that existed before the first stars and galaxies ever appear. Many consider this dark age of the universe is mystery, the author think it is the most rational process in nature's evolution development.

The dark Matter forms first immediately after a big bang. The greatest kinetic energy starts to turn into positional energy during this period of time/ nucleosynthesis. Once positional energy establishes, matters starts to gather and circulate around dark matter. The dark matter merges as well, the greater dark matter, the greater mass can be collected around the dark matter. This is the model of baby galaxy's birth process.

As the author described earlier, the big bang does not occur in a vacuum space. The existing cosmos mass/cosmos ray will interact with a big bang's ray/Gamma ray, the interaction facilitate "post bang" kinetic energy transform into positional energy. The "so called" dark matter possesses so little kinetic energy for us to detect, the dark matter mustbe something unusual to our earthlings. Most materials which we are grown up with are matters that possess both kinetic energy and positional energy, though pure kinetic energy is very common to us, e.g. solar lights.

The pure positional energy, on the other hand, is not common on earth. However, it is not mystery and it is not dangerous if it is on a small scale. The author applaud European Particle accelerator, and assure the rest of us that black hole created in a human lab will not swallow earth! It is the game of quantity, a drop of water is not going to drown a giant elephant! The black hole is nothing but a compacted Gamma Ray nucleo synthesis. The author will address it over and over in this paper, because Gamma Ray Particles are the only suitable candidate for possessing greatest power in a very small package. As human, we are well aware of high power Gamma Ray during Gamma Ray Burst, however, the author can reasonable assure you if we pair liberated Gamma Ray/Particle together, its mighty kinetic energy will transform into mighty positional energy(γ ray nucleosynthesis), a perfect candidate for dark matter/black hole.

The universe energy is constant and it exists in two forms. Since we human have tools readily detect kinetic energy in distance, we know gamma ray is the matter possessing the most powerful kinetic energy in a very small package. We also know that a great bang is the most powerful explosion in universe. So gamma ray matches big blast! Now, since the positional energy is not readily to be detected. However, we know that the energy is constant. The greatest positional energy must come from the greatest kinetic energy! What will be the matter possessing most positional energy? The only rationale answer for this is Gamma Ray nucleo synthesis!

According to our recent cosmos findings, we know that there are black hole and dark matters which hold our visible mass together. The black hole/dark matters are regions/masses which possess most powerful positional energy in the universe. The author here safely say that black hole/dark matter is made from "non-liberated" gamma ray nucleosynthesis for most part. Since gamma ray particles are so small, it is very understandable that black hole doesn't take much space! The author's description is supported by recent discovery of Milky Way Black hole whose radius is no more than 6.25 light-hours! 6.25 light hours are plenty room for its dark matter holding something together that is sized 100,000–120,000 light-years in diameter which contains 100–400 billion stars, aka milky way, a home for human sapiens.

b) Gamma Ray and Big Bang

It is the big bang that kicks off Gamma Ray first. The diffuse cosmic back ground Gamma Ray is not originated during the period of Big Bang's nucleo synthesis (also known as primordial nucleo synthesis). The nucleo synthesis does happen, the process requires kinetic energy to be transformed into positional energy after particles start to mingle and gather around the dense regions. The mingle means when particles start to pair/orbit from each other that result in the orbiting power/energy. According to conservation of energy, the new orbiting power must come from somewhere other than from nothing. The only source is its kinetic energy. The evolution of a big bang simply means that some of its greatest kinetic energy starts to be captured into its form of orbiting energy/gravity power during its cooling off stage, when galaxies start to be created. The destiny of big bang is another big bang. The gravity power will eventually collect enough mass and creates another new pre-big bang black hole. The pre-big bang black hole is so enormous that it will dwarf our Milky Way's black hole as a drop of water vs. ocean. The enormous orbiting power that a black hole/dark matter possesses will eventually be released and transformed into kinetic energy, which presents itself in a spectacular gamma bomb show/big bang explosion, which will dwarf gamma ray burst (GRB) as a 3 watts light bulb vs. the center of sun.

We can reasonably believe that the pre-Big Bang Mass must contain mostly positional energy rather than kinetic energy, because soon it will kick off as an explosion. Accordingly, the explosion must be in a form of mostly kinetic energy, rather than in a form of positional energy. Before and during explosion, there is no logical reason to think that the universe space of any moment must be complete vacant. Here are the reasons. 1. The collection/preparation stage of pre-Big Bang Mass only requires a quantifiable amount of mass/energy to trigger a big blast. There is no reason and need to stripe the space to a total void. ⑵2. The big bang is not the only event happening in the universe at any moment. There is mass that is constant exchanging from region to region regardless of status of a particular region. There are numerous big bangs. The various regions of universe are at different stage of mass/energy evolution process. Therefore, it further reduces possibilities of striping a space to a complete void even for a region that is ripened for a big blast.

My description of universe big blast is indirectly supported by general belief that primordial stage of clustering and merging exists. If the space were total void, the blast will send homogeneous Gamma Ray/Particles in a perfect sphere shape; there would be no area of density difference which is the key inducer for clustering and merging. The author reasons that non-void space which provides the density difference. Those density differences continue to exist before and during a big blast. Those density differences are necessity to induce clustering and merging which lead to dark matters, black holes and galaxies. Furthermore, those density differences spread out in space follow the mathematic rule of random. If the space were total void, it would have gradual changing appearance of cosmic ray spread in a sphere shape. Without those random existing of cosmos mass before and during big blast, we would not be here, neither is our Milky Way.

c) Gamma Ray and US

Gamma Ray is a good stuff. Gamma rays, electromagnetic radiation of an extremely high frequency / high energy photons, are produced by a number of astronomical processes in which very high-energy electrons are produced, that in turn cause secondary gamma rays via bremsstrahlung, inverse Compton scattering and synchrotron radiation. Gamma rays typically have frequencies above 10 exahertz (or >1019 Hz), and therefore have energies above 100 keV and wavelengths less than 10 picometers, which is less than the diameter of an atom. They can also be produced by the decay of atomic nuclei known as gamma decay. Gamma Ray also can be produced in our nuclear reaction etc.

There are 4 stages of utilizing energy in our human history. The First Stage is horse power stage. The humans mostly use animal power to supplement man's power. The second stage is Molecular Power stage. The humans mostly utilize fire power by burning fuels. The third stage is Nuclear Power stage. The humans mostly harvest energy from sub-atom nucleus. The fourth stage will be sub-nucleus Particle Power stage. The author applaud European Organization for Nuclear Research (CERN)'s effort to crackdown the sub-nucleus particles. The Large Hadron Collider (LHC), the world's largest and most powerful particle collider is the first step for humans entering the Era of Particle Power. The danger comes when particle power fall into a wrong hand. To protect our way of living and beloved human civilization, the international task to prevent particle power proliferation soon must be in its place!

Comparing to pure kinetic energy, the pure positional energy is always hard to be detected in cosmos. The kinetic energy comes to us, while positional energy sits there. When this happens, the cosmos Gamma Ray serves perfect tool for us to notice the existence of positional energy/gravitation. The author's point is supported by phenomenon as gravitation lens. Furthermore, we shall be able to quantify the positional energy/gravitation by measuring gravitation lens effects carefully. The calculation will provide us not only the gravitation power but also the actual mass that invisible object possesses.

The earthlings are only one step from Gamma Ray communication. The Gamma Ray is much more powerful source that we can use to communicate cross

Global Journal of Science Frontier Research (A) Volume XV Issue II Version I Year 2015

vast cosmos distance. Humans are restricted by our physical forms which stop us from traveling in space. However, the electronic form of human in Gamma Ray will easily overcome the barriers that physical form of human face today.

Gamma Ray weapon will be the ultimate weapon dwarfing our current nuclear weapon systems as a catfish vs. Tyrannosaurus rex. Imagine that the GPS locate a target; Gamma Ray will evaporate it like mini big bang that hit a home run! The characteristic of gamma ray weapon is to leave no messy blood with a potential of recycling enemy's physical body and their weaponry. The nuclear weapon is more readily a source for γ ray nucleo synthesis. The era of γ particle warfare will become energy recycle enterprise. Not only the superior civilization send out cyber generals and solders (electronic form of human fighters) but also evaporate enemy and their arsenals into dark matter, take them home and sell them to a local utility company. The recycle of enemy energy is not a new concept. The very first war of human civilization involved capturing enemy soldiers and sold them to slave market to recycle their man power. In the era of particle power, enemy's sub-atom particle energy will be captured and recycled. The fight will eventually become bloodless and painless.

If there is any intelligent life exists outside earth, the Gamma Ray in space will be the main target to study. Gamma Ray is the most powerful energy carrier which is capable to travel the vastest distance. When the cosmos Gamma Rays reach us, some of them may carry information, or some Gamma Rays are alien life in their electronic form. Our human race is only one step below the ultimate level of terminal end of artificial intelligence. If there is superior alien life that exists somewhere in the universe, the only hope that they have to travel among galaxies or escape a big bang is to store their information in the gamma ray. The gamma ray is the most powerful ray available in the universe. Comparing to other types of ray, the Gamma ray might last longer and reach further. However, the artificial Gamma Ray with information may never escape a big bang. It is, though, worth our effort to collect cosmos Gamma Ray and look for clues that may mean something other than existing as a form of pure energy unit.

In a speed of light, all mass will break down into particles. The dream of human carrier reaching speed of light is just fantasy(2). The human becomes unique and distinguish because of his/her personal memory and emotion that is associated with such memory. Those distinguished memories are nothing but an electronic form that is developed in our brain cells at first place. Once we are able to upload and pack such electronic memory/emotion into a light form, we will be able to travel in a speed of light.

The only intelligent alien life that is superior to our human are the ones who master the Gamma Ray. The alien gamma rays wander in space, with hope to reach the receptors. They would not be able to complete their travel unless they find particular receiving receptors which allow them to down load. The colonization of cosmos will start in a form of Gamma Ray first, not a "May Flower" wooden boat. This sounds like a fiction, but it is science at its ultimate level.

Other than for the potential of information storage, The Gamma Ray engine will be an ultimate power horse for inter stars/inter galaxies travel due to γ ray/fuels that are readily available in space.

The Gamma Ray will be an excellent detector for invisible mass. It is high energy, which travels fast and far, it will detect any positional energy nearby on its route due to the gravity lensing effects, or the bend of light. The Gamma Ray radar detector will be the ultimate detector available in the universe. There will be no stealth plane, or dark matters that can escape from its detection.

d) What We are Made From

It has been said that we are made from star dust. This is not a quite correct answer. It is the inorganic matters that are made from star dust.

In about three minutes, The Big Bang nucleosynthesis produced most of the helium, lithium, and deuterium in the Universe, and perhaps some of the beryllium and boron.(10)(11)(12)The heavier elements are produced in stars through the process of nuclear fusion (13)see stellar nucleo synthesis for details. Isotopes such as lithium-6, as well as some beryllium and boron are produced in space through cosmic ray spallation.(14)This occurs when a high-energy proton strikes an atomic nucleus, causing large numbers of nucleons to be ejected. Elements heavier than iron are produced in supernovae through the r-process and in AGB stars through the s-process, both of which involve the capture of neutrons by atomic nuclei.(15) Elements such as lead formed largely through the radioactive decay of heavier elements.(16) Those elements exemplify what is called atom evolution.

The Organic Matters branch off from inorganic matters. The process requires light energy. The light energy could come from solar or earth itself. The most popular photosynthesis requires green chlorophyll pigments. In plants, these proteins are held inside organelles called chloroplasts, which are most abundant in leaf cells, while in bacteria they are embedded in the plasma membrane. The first photosynthetic organisms probably evolved early in the evolutionary history of life and most likely used reducing agents such as hydrogen or hydrogen sulfide as sources of electrons, rather than water.(17) Cyanobacteria appeared later, and the excess oxygen they produced contributed to the oxygen catastrophe,(18) which rendered the evolution of complex life possible.

The Organic Matters are mass possessing both kinetic energy and positional energy. To become alive and grow, the living beings require additional matter that possesses almost pure form of kinetic energy -light energy. However, the light energy is no alien to us. Neither is dark matter! Dark matter is just opposite from light energy, which possess positional energy mostly with very minimum kinetic energy. It is evidenced by universe background temperature. If there is temperature, there is kinetic energy.

Light has mass; it is evidenced by gaining slightly weight after photosynthesis. The current model of photosynthesis is perfect example that nature set up for us. When we enter the era of γ particle power, we will be able to utilize γ ray for photosynthesis.

e) Time and Space

Time is just a record of history that describes an evolution in process or movement in matter; if you were to stop time, you would see a snapshot or momentary freeze picture of matter physics -- a halted progress in matter/energy. The speed of matter/mass or their evolution can never stop; therefore, time will never stop. There is no disappearance of matter/mass/energy; there is only transformation of the engaging or dissembling with external or internal mass/energy.(2) The Universe Mass and Energy are in an endless cycles, accordingly, time has no beginning and no ending.

Space is just a void where matters float. The Space is not a mass. The space and time can never be bent.

f) Mathematic Odds and Déjà Vu

Since the universe matters are all made of same particles, whether they are gods particles or ghost particles, there is no mystery or magic force in our universe. It is all about odds in the universe where math comes to play. From the most primitive particles to the most sophisticated chemical structures, matter will never vanish. It just exists in different forms by pure odds.(2)Given the infinite size of the universe, the probabilities are most likely infinite as well: “anything is possible.” Therefore, at any moment, if you take a snapshot, you can always find materials/particles of same physical character at different places in other parts of universe simultaneously, which gives people a sensation of déjà vu. Similarly, when you look into a large crowd, I am sure some déjà vu is going to play right in front of your eyes -- two faces will appear to be identical.(2)Two Identical person does not mean they are same person. Similarly, if you look deep in the universe, you might find another “you”, the childhood memory, however, could be different.

g) Unify All Science Branches and Philosophy

The study of universe will eventually lead to great reunion of all science branches and philosophy as organic intelligence heads to its terminal stage.

III. Results Analysis

The universe is a complete and enclosed system, because there is nothing existing outside universe which may interferes universe matters/energy. The conservation of mass and energy perfectly apply universe as a unit. We can safely say that universe energy and mass are constant.

$$E_{bigbang} = E_k + E_p$$

The Ek is the sum of kinetic energy presents in the current universe. Ep is the sum of positional energy presents in the current universe.

The big bang is a greatest event happened in the universe; it is an almost pure kinetic energy show. Soon after a big blast, some of its kinetic energy starts to transfer itself to the positional energy (γ ray nucleosynthesis). Some γ ray nucleosynthesis trapped at subatomic stage and evolves into dark matter (significant positional energy, little kinetic energy). Some γ ray nucleosynthesis goes on to form galaxies and us. Some γ ray remains free standing status and becomes cosmos background ray.

The cosmos background ray may have diversified sources, due to supernova. It may also have organic intelligence originating, though the artificial gamma ray would be very little to none at its scale vs. a great bang or a supernova.

IV. Discussion

In this theoretical study, the author paved a road to disclose our universe mysteries by mapping its energy evolution. Under dictations of the law of classic physics, the author reasons what dark matter/black hole is made of- γ ray nucleosynthesis. Furthermore, the author describes the mode of mass evolution in our universe by analyzing its energy composition. In consideration of infinite size of the universe, the author postulates numerous big bangs exist in our universe with endless mass/energy cycles. (2)

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Statistical Theory for Three-Point Distribution Functions of Certain Variables in MHD Turbulent Flow in Existence of Coriolis Force in a First Order Reaction

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Abstract- In this paper, the three-point distribution functions for simultaneous velocity, magnetic, temperature and concentration fields in MHD turbulent flow in presence of coriolis force under going a first order reaction have been studied. The various properties of constructed distribution functions have been discussed. From beginning to end of the study, the transport equation for three-point distribution function under going a first order reaction has been obtained. The resulting equation is compared with the first equation of BBGKY hierarchy of equations and the closure difficulty is to be removed as in the case of ordinary turbulence.

Keywords: *coriolis force, magnetic temperature, concentration, three-point distribution functions, MHD turbulent flow, first order reactant.*

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Abstract- In this paper, the three-point distribution functions for simultaneous velocity, magnetic, temperature and concentration fields in MHD turbulent flow in presence of coriolis force under going a first order reaction have been studied. The various properties of constructed distribution functions have been discussed. From beginning to end of the study, the transport equation for three-point distribution function under going a first order reaction has been obtained. The resulting equation is compared with the first equation of BBGKY hierarchy of equations and the closure difficulty is to be removed as in the case of ordinary turbulence.

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I. Introduction

Particle's distribution function is a function of several variables. It has to make use of in plasma physics to describe wave-particle interactions and velocity-space instabilities. It is also used in fluid mechanics, statistical mechanics and nuclear physics. A distribution function may be specialized with respect to a particular set of dimensions. It may attribute non-isotropic temperatures, in which each term in the exponent is divided by a different temperature. In the past, numerous researchers like as Hopf (1952) Kraichanan (1959), Edward (1964) and Herring (1965), Lundgren (1967, 1969) had done their work on the statistical theory of turbulence.

Kishore (1978) studied the Distributions functions in the statistical theory of MHD turbulence of an incompressible fluid. Pope (1979) studied the statistical theory of turbulence flames. Pope (1981) derived the transport equation for the joint probability density function of velocity and scalars in turbulent flow. Kollman and Janicka (1982) derived the transport equation for the probability density function of a scalar in turbulent shear flow and considered a closure model based on gradient – flux model. Kishore and Singh (1984) derived the transport equation for the bivariate joint distribution function of velocity and temperature in turbulent flow. Also Kishore and Singh (1985) have been derived the transport equation for the joint distribution function of velocity, temperature and concentration in convective turbulent flow. The Coriolis force acts to change the direction of a moving body to the right in the Northern Hemisphere and to the left in the Southern Hemisphere. This deflection is not only instrumental in the large-scale atmospheric circulation, the development of storms, and the sea-breeze circulation. Afterward, the following some researchers had included coriolis force and first order reaction rate in their works.

Dixit and Upadhyay (1989) considered the distribution functions in the statistical theory of MHD turbulence of an incompressible fluid in the presence of the coriolis force. Sarker and Kishore (1991) discussed the distribution functions in the statistical theory of convective MHD turbulence of an incompressible fluid. Also Sarker and Kishore (1999) studied the distribution functions in the statistical theory of convective MHD turbulence of mixture of a miscible incompressible fluid. Sarker and Islam (2002) studied the Distribution functions in the statistical theory of convective MHD turbulence of an incompressible fluid in a rotating system. In the continuation of the above researcher, Azad and Sarker (2004a) discussed statistical theory of certain distribution functions in MHD turbulence in a rotating system in presence of dust particles. Sarker and Azad (2004b) studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time in a rotating system. Azad and Sarker(2008) studied the decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi- point and multi- time in a rotating system and dust particles. Azad and Sarker(2009a) had measured the decay of temperature fluctuations in MHD turbulence before the final period in a rotating system. Chemical reaction is usually occurred in a fluid. It is stated that first-order reaction is defined as a reaction that proceeds at a rate that depends linearly only on one reactant concentration Very recent, the following some

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researchers had included first order reaction rate in their own works Sarker and Azad(2006), Islam and Sarker (2007) studied distribution functions in the statistical theory of MHD turbulence for velocity and concentration undergoing a first order reaction. Azad et al (2009b, 2009c) studied the first order reactant in Magneto-hydrodynamic turbulence before the final Period of decay with dust particles and rotating System. Aziz et al (2009d, 2010c) discussed the first order reactant in Magneto- hydrodynamic turbulence before the final period of decay for the case of multi-point and multi-time taking rotating system and dust particles. Aziz et al (2010a, 2010b) studied the statistical theory of certain Distribution Functions in MHD turbulent flow undergoing a first order reaction in presence of dust particles and rotating system separately. Azad et al (2011) studied the statistical theory of certain distribution Functions in MHD turbulent flow for velocity and concentration undergoing a first order reaction in a rotating system. Azad et al (2012) derived the transport equatoin for the joint distribution function of velocity, temperature and concentration in convective tubulent flow in presence of dust particles. Molla et al (2012) studied the decay of temperature fluctuations in homogeneous turbulenc before the final period in a rotating system. Bkar Pk. et al (2012) studed the First-order reactant in homogeneou dusty fluid turbulence prior to the ultimate phase of decay for four-point correlation in a rotating system. Azad and Mumtahinah(2013) studied the decay of temperature fluctuations in dusty fluid homogeneous turbulence prior to final period. Bkar Pk. et al (2013a,2013b) discussed the first-order reactant in homogeneous turbulence prior to the ultimate phase of decay for four-point correlation with dust particle and rotating system. Bkar Pk.et al (2013,2013c, 2013d) studied the decay of MHD turbulence before the final period for four- point correlation in a rotating system and dust particles. Very recent Azad et al (2014a) derived the transport equations of three point distribution functions in MHD turbulent flow for velocity, magnetic temperature and concentration, Azad and Nazmul (2014b) considered the transport equations of three point distribution functions in MHD turbulent flow for velocity, magnetic temperature and concentration in a rotating system, Nazmul and Azad (2014) studied the transport equations of three-point distribution functions in MHD turbulent flow for velocity, magnetic temperature and concentration in presence of dust particles. Azad and Mumtahinah (2014) further has been studied the transport equatoin for the joint distribution functions in convective tubulent flow in presence of dust particles undergoing a first order reaction.Very recently, Bkar Pk. et al (2015) considering the effects of first-order reactant on MHD turbulence at four-point correlation. Azad et al (2015) derived a transport equation for the joint distribution functions of certain variables in convective dusty fluid turbulent flow in a rotating system under going a first order reaction. Bkar Pk, et al (2015a) studied the 4-point correlations of dusty fluid MHD turbulent flow in a 1st order reaction. Most of the above researchers have done their research for two point distribution functions in the statistical theory in MHD turbulence.

But in this paper, we have tried to do this research for three-point distribution functions in the statistical theory in MHD turbulence in a first order reaction in presence of coriolis force

In present paper, the main purpose is to study the statistical theory of three-point distribution function for simultaneous velocity, magnetic, temperature, concentration fields in MHD turbulence in a rotating system under going a first order reaction. Through out the study, the transport equations for evolution of distribution functions have been derived and various properties of the distribution function have been discussed.

II. Formulation of the Problem

The equations of motion and continuity for viscous incompressible MHD turbulent flow in a rotating system with constant reaction rate, the diffusion equations for the temperature and concentration are given by

$$\frac{\partial u_\alpha}{\partial t}+\frac{\partial}{\partial x_\beta}\left(u_\alpha u_\beta - h_\alpha h_\beta\right)=-\frac{\partial w}{\partial x_\alpha}+\nu\nabla^2 u_\alpha - 2\in_{m\alpha\beta}\Omega_m u_\alpha \tag{1}$$

$$\frac{\partial h_\alpha}{\partial t}+\frac{\partial}{\partial x_\beta}\left(h_\alpha u_\beta - u_\alpha h_\beta\right)=\lambda\nabla^2 h_\alpha\ , \tag{2}$$

$$\frac{\partial \theta}{\partial t}+u_\beta\frac{\partial \theta}{\partial x_\beta}=\gamma\nabla^2\theta\ , \tag{3}$$

$$\frac{\partial c}{\partial t}+u_\beta\frac{\partial c}{\partial x_\beta}=D\nabla^2 c - Rc \tag{4}$$

with $\frac{\partial u_\alpha}{\partial x_\alpha} = \frac{\partial v_\alpha}{\partial x_\alpha} = \frac{\partial h_\alpha}{\partial x_\alpha} = 0$, (5)

where

$u_\alpha(x,t)$, α – component of turbulent velocity; $h_\alpha(x,t)$, α – component of magnetic field; $\theta(x,t)$, temperature fluctuation; c, concentration of contaminants; $\in_{m\alpha\beta}$, alternating tensor;

$w(\hat{x},t) = P/\rho + \frac{1}{2}|\bar{h}|^2 + \frac{1}{2}|\hat{\Omega}\times\hat{x}|^2$, total pressure; $P(\hat{x},t)$, hydrodynamic pressure; ρ, fluid density; Ω, angular velocity of a uniform rotation; ν, Kinematic viscosity; $\lambda = (4\pi\mu\sigma)^{-1}$, magnetic diffusivity; $\gamma = \frac{k_T}{\rho c_p}$, thermal diffusivity; c_p, specific heat at constant pressure; k_T, thermal conductivity; σ, electrical conductivity; μ, magnetic permeability; D,diffusive co-efficient for contaminants; R, constant reaction rate.

The repeated suffices are assumed over the values 1, 2 and 3 and unrepeated suffices may take any of these values. In the whole process u, h and x are the vector quantities.

The total pressure w which, occurs in equation (1) may be eliminated with the help of the equation obtained by taking the divergence of equation (1)

$$\nabla^2 w = -\frac{\partial^2}{\partial x_\alpha \partial x_\beta}(u_\alpha u_\beta - h_\alpha h_\beta) = -\left[\frac{\partial u_\alpha}{\partial x_\beta}\frac{\partial u_\beta}{\partial x_\alpha} - \frac{\partial h_\alpha}{\partial x_\beta}\frac{\partial h_\beta}{\partial x_\alpha}\right] \tag{6}$$

In a conducting infinite fluid only the particular solution of the Equation (6) is related, so that

$$w = \frac{1}{4\pi}\int \left[\frac{\partial u'_\alpha}{\partial x'_\beta}\frac{\partial u'_\beta}{\partial x'_\alpha} - \frac{\partial h'_\alpha}{\partial x'_\beta}\frac{\partial h'_\beta}{\partial x'_\alpha}\right]\frac{\partial \bar{x}'}{|\bar{x}' - \bar{x}|} \tag{7}$$

Hence equation (1) – (4) becomes

$$\frac{\partial u_\alpha}{\partial t} + \frac{\partial}{\partial x_\beta}(u_\alpha u_\beta - h_\alpha h_\beta) = -\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha}\int \left[\frac{\partial u'_\alpha}{\partial x'_\beta}\frac{\partial u'_\beta}{\partial x'_\alpha} - \frac{\partial h'_\alpha}{\partial x'_\beta}\frac{\partial h'_\beta}{\partial x'_\alpha}\right]\frac{d\bar{x}'}{|\bar{x}' - \bar{x}|} + \nu\nabla^2 u_\alpha - 2\in_{m\alpha\beta}\Omega_m u_\alpha \tag{8}$$

$$\frac{\partial h_\alpha}{\partial t} + \frac{\partial}{\partial x_\beta}(h_\alpha u_\beta - u_\alpha h_\beta) = \lambda\nabla^2 h_\alpha, \tag{9}$$

$$\frac{\partial \theta}{\partial t} + u_\beta\frac{\partial \theta}{\partial x_\beta} = \gamma\nabla^2\theta, \tag{10}$$

$$\frac{\partial c}{\partial t} + u_\beta\frac{\partial c}{\partial x_\beta} = D\nabla^2 c - Rc, \tag{11}$$

Certain microscopic properties of conducting fluids such as total energy, total pressure, stress tensor which are nothing but ensemble averages at a particular time can be determined with the help of the distribution functions (defined as the averaged distribution functions with the help of Dirac delta-functions). The present aim is to construct a 3-point distribution functions in MHD turbulent flow in a rotating system under going a first order reaction, study its properties and derive a transport equation for the joint distribution function of velocity, temperature and concentration in MHD turbulent flow in a rotating system under going a first order reaction.

III. Distribution Function in MHD Turbulence and their Properties

In MHD turbulence, it is considered that the fluid velocity u, Alfven velocity h, temperature θ and concentration c at each point of the flow field. Corresponding to each point of the flow field, there are four measurable characteristics represent by the four variables by v, g, ϕ and ψ denote the pairs of these variables at the points $\bar{x}^{(1)}, \bar{x}^{(2)}, -----, \bar{x}^{(n)}$ as $(\bar{v}^{(1)}, \bar{g}^{(1)}, \phi^{(1)}, \psi^{(1)})$, $(\bar{v}^{(2)}, \bar{g}^{(2)}, \phi^{(2)}, \psi^{(2)})$, --- $(\bar{v}^{(n)}, \bar{g}^{(n)}, \phi^{(n)}, \psi^{(n)})$ at a fixed instant of time.

It is possible that the same pair may be occurred more than once; therefore, it simplifies the problem by an assumption that the distribution is discrete (in the sense that no pairs occur more than once). Symbolically we can express the bivariate distribution as

$$\left\{ \left(\bar{v}^{(1)}, \bar{g}^{(1)}, \phi^{(1)}, \psi^{(1)}\right), \left(\bar{v}^{(2)}, \bar{g}^{(2)}, \phi^{(2)}, \psi^{(2)}\right), ------ \left(\bar{v}^{(n)}, \bar{g}^{(n)}, \phi^{(n)}, \psi^{(n)}\right) \right\}$$

Instead of considering discrete points in the flow field, if it is considered the continuous distribution of the variables $\bar{v}, \bar{g}, \phi$ and ψ over the entire flow field, statistically behavior of the fluid may be described by the distribution function $F(\bar{v}, \bar{g}, \phi, \psi)$ which is normalized so that

$$\int F(\bar{v}, \bar{g}, \phi, \psi) d\bar{v} d\bar{g} d\phi d\psi = 1$$

where the integration ranges over all the possible values of v, g, ϕ and ψ. We shall make use of the same normalization condition for the discrete distributions also.

The distribution functions of the above quantities can be defined in terms of Dirac delta function.

The one-point distribution function $F_1^{(1)}\left(v^{(1)}, g^{(1)}, \phi^{(1)}, \psi^{(1)}\right)$, defined so that $F_1^{(1)}\left(v^{(1)}, g^{(1)}, \phi^{(1)}, \psi^{(1)}\right) dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)}$ is the probability that the fluid velocity, Alfven velocity, temperature and concentration at a time t are in the element $dv^{(1)}$ about $v^{(1)}$, $dg^{(1)}$ about $g^{(1)}$, $d\phi^{(1)}$ about $\phi^{(1)}$ and $d\psi^{(1)}$ about $\psi^{(1)}$ respectively and is given by

$$F_1^{(1)}\left(v^{(1)}, g^{(1)}, \phi^{(1)}, \psi^{(1)}\right) = \left\langle \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(h^{(1)} - g^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(c^{(1)} - \psi^{(1)}\right) \right\rangle \tag{12}$$

where δ is the Dirac delta-function defined as

$$\int \delta(\bar{u} - \bar{v}) d\bar{v} = \begin{cases} 1 & \text{at the point } \bar{u} = \bar{v} \\ 0 & \text{elsewhere} \end{cases}$$

Two-point distribution function is given by

$$F_2^{(1,2)} = \left\langle \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(h^{(1)} - g^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(c^{(1)} - \psi^{(1)}\right) \delta\left(u^{(2)} - v^{(2)}\right) \delta\left(h^{(2)} - g^{(2)}\right) \delta\left(\theta^{(2)} - \phi^{(2)}\right) \delta\left(c^{(2)} - \psi^{(2)}\right) \right\rangle \tag{13}$$

and three point distribution function is given by

$$F_3^{(1,2,3)} = \left\langle \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(h^{(1)} - g^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(c^{(1)} - \psi^{(1)}\right) \delta\left(u^{(2)} - v^{(2)}\right) \delta\left(h^{(2)} - g^{(2)}\right) \right.$$
$$\left. \times \delta\left(\theta^{(2)} - \phi^{(2)}\right) \delta\left(c^{(2)} - \psi^{(2)}\right) \delta\left(u^{(3)} - v^{(3)}\right) \delta\left(h^{(3)} - g^{(3)}\right) \delta\left(\theta^{(3)} - \phi^{(3)}\right) \delta\left(c^{(3)} - \psi^{(3)}\right) \right\rangle \tag{14}$$

Similarly, we can define an infinite numbers of multi-point distribution functions $F_4^{(1,2,3,4)}$, $F_5^{(1,2,3,4,5)}$ and so on. The following properties of the constructed distribution functions can be deduced from the above definitions:

a) Reduction Properties

Integration with respect to pair of variables at one-point lowers the order of distribution function by one. For example,

$$\int F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} = 1,$$

$$\int F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} = F_1^{(1)},$$

$$\int F_3^{(1,2,3)} dv^{(3)} dg^{(3)} d\phi^{(3)} d\psi^{(3)} = F_2^{(1,2)}$$

And so on. Also the integration with respect to any one of the variables, reduces the number of Delta-functions from the distribution function by one as

$$\int F_1^{(1)} dv^{(1)} = \left\langle \delta\left(h^{(1)} - g^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(c^{(1)} - \psi^{(1)}\right) \right\rangle,$$

$$\int F_1^{(1)} dg^{(1)} = \left\langle \delta\left(u^{(1)} - v^{(1)}\right) \delta\left(\theta^{(1)} - \phi^{(1)}\right) \delta\left(c^{(1)} - \psi^{(1)}\right) \right\rangle,$$

$$\int F_1^{(1)} d\phi^{(1)} = \left\langle \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right) \right\rangle ,$$

and

$$\int F_2^{(1,2)} dv^{(2)} = \left\langle \delta\left(u^{(1)} - v^{(1)}\right)\delta\left(h^{(1)} - g^{(1)}\right)\delta\left(\theta^{(1)} - \phi^{(1)}\right)\delta\left(c^{(1)} - \psi^{(1)}\right)\delta\left(h^{(2)} - g^{(2)}\right)\right.$$
$$\left.\delta\left(\theta^{(2)} - \phi^{(2)}\right)\delta\left(c^{(2)} - \psi^{(2)}\right) \right\rangle$$

b) Separation Properties

If two points are far apart from each other in the flow field, the pairs of variables at these points are statistically independent of each other i.e.,

$$\lim_{\left|\vec{x}^{(2)} \to \vec{x}^{(1)}\right| \to \infty} F_2^{(1,2)} = F_1^{(1)} F_1^{(2)}$$

and similarly,

$$\lim_{\left|\vec{x}^{(3)} \to \vec{x}^{(2)}\right| \to \infty} F_3^{(1,2,3)} = F_2^{(1,2)} F_1^{(3)} \quad \text{etc.}$$

c) Co-incidence Properties

When two points coincide in the flow field, the components at these points should be obviously the same that is $F_2^{(1,2)}$ must be zero.

Thus $\bar{v}^{(2)} = \bar{v}^{(1)}$, $g^{(2)} = g^{(1)}$, $\phi^{(2)} = \phi^{(1)}$ and $\psi^{(2)} = \psi^{(1)}$, but $F_2^{(1,2)}$ must also have the property.

$$\int F_2^{(1,2)} dv^{(2)} dg^{(2)} d\phi^{(2)} d\psi^{(2)} = F_1^{(1)}$$

and hence it follows that

$$\lim_{\left|\bar{x}^{(2)} \to \bar{x}^{(1)}\right| \to \infty} \int F_2^{(1,2)} = F_1^{(1)} \delta\left(v^{(2)} - v^{(1)}\right)\delta\left(g^{(2)} - g^{(1)}\right)\delta\left(\phi^{(2)} - \phi^{(1)}\right)\delta\left(\psi^{(2)} - \psi^{(1)}\right)$$

Similarly,

$$\lim_{\left|\bar{x}^{(3)} \to \bar{x}^{(2)}\right| \to \infty} \int F_3^{(1,2,3)} = F_2^{(1,2)} \delta\left(v^{(3)} - v^{(1)}\right)\delta\left(g^{(3)} - g^{(1)}\right)\delta\left(\phi^{(3)} - \phi^{(1)}\right)\delta\left(\psi^{(3)} - \psi^{(1)}\right) \quad \text{etc.}$$

d) Symmetric Conditions

$$F_n^{(1,2,r,------s,-----n)} = F_n^{(1,2,-----s,---r,---n)} .$$

e) Incompressibility Conditions

(i) $\int \dfrac{\partial F_n^{(1,2,---n)}}{\partial x_\alpha^{(r)}} v_\alpha^{(r)} d\bar{v}^{(r)} d\bar{h}^{(r)} = 0$ (ii) $\int \dfrac{\partial F_n^{(1,2,---n)}}{\partial x_\alpha^{(r)}} h_\alpha^{(r)} d\bar{v}^{(r)} d\bar{h}^{(r)} = 0$

Continuity Equation in Terms of Distribution Functions

The continuity equations can be easily expressed in terms of distribution functions. An infinite number of continuity equations can be derived for the convective MHD turbulent flow directly by using div $u = 0$

Taking ensemble average of equation (5), we get

$$0 = \left\langle \frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}} \right\rangle = \left\langle \frac{\partial}{\partial x_\alpha^{(1)}} u_\alpha^{(1)} \int F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \right\rangle = \frac{\partial}{\partial x_\alpha^{(1)}} \left\langle u_\alpha^{(1)} \int F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \right\rangle$$

$$= \frac{\partial}{\partial x_\alpha^{(1)}} \int \left\langle u_\alpha^{(1)} \right\rangle \left\langle F_1^{(1)} \right\rangle dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} = \frac{\partial}{\partial x_\alpha^{(1)}} \int v_\alpha^{(1)} F_1^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)}$$

$$= \int \frac{\partial F_1^{(1)}}{\partial x_\alpha^{(1)}} v_\alpha^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \tag{15}$$

and similarly,

$$0 = \int \frac{\partial F_1^{(1)}}{\partial x_\alpha^{(1)}} g_\alpha^{(1)} dv^{(1)} dg^{(1)} d\phi^{(1)} d\psi^{(1)} \tag{16}$$

Equation (15) and (16) are the first order continuity equations in which only one point distribution function is involved.

For second-order continuity equations, if we multiply the continuity equation by

$$\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

and if we take the ensemble average, we obtain

$$o=\left\langle\ \delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}}\ \right\rangle$$

$$=\frac{\partial}{\partial x_\alpha^{(1)}}\left\langle\ \delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)u_\alpha^{(1)}\ \right\rangle$$

$$=\frac{\partial}{\partial x_\alpha^{(1)}}\Big[\ \int\Big\langle\ u_\alpha^{(1)}\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)$$

$$\times\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\Big\rangle dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)}\ \Big]$$

$$=\frac{\partial}{\partial x_\alpha^{(1)}}\int v_\alpha^{(1)}F_2^{(1,2)}dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)} \tag{17}$$

and similarly,

$$o=\frac{\partial}{\partial x_\alpha^{(1)}}\int g_\alpha^{(1)}F_2^{(1,2)}dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)} \tag{18}$$

The Nth – order continuity equations are

$$o=\frac{\partial}{\partial x_\alpha^{(1)}}\int v_\alpha^{(1)}F_N^{(1,2,---,N)}dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)} \tag{19}$$

and

$$o=\frac{\partial}{\partial x_\alpha^{(1)}}\int g_\alpha^{(1)}F_N^{(1,2,.........,N)}dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)} \tag{20}$$

The continuity equations are symmetric in their arguments i.e.

$$\frac{\partial}{\partial x_\alpha^{(r)}}\left(v_\alpha^{(r)}F_N^{(1,2,..........;r.N)}dv^{(r)}dg^{(r)}d\phi^{(r)}d\psi^{(r)}\right)=\frac{\partial}{\partial x_\alpha^{(s)}}\int v_\alpha^{(s)}F_N^{(1,2,.....r,s,.....N)}dv^{(s)}dg^{(s)}d\phi^{(s)}d\psi^{(s)} \tag{21}$$

Since the divergence property is an important property and it is easily verified by the use of the property of distribution function as

$$\frac{\partial}{\partial x_\alpha^{(1)}}\int v_\alpha^{(1)}F_1^{(1)}dv^{(1)}dg^{(1)}d\phi^{(1)}d\psi^{(1)}\ \frac{\partial}{\partial x_\alpha^{(1)}}\left\langle\ u_\alpha^{(1)}\ \right\rangle=\left\langle\ \frac{\partial u_\alpha^{(1)}}{\partial x_\alpha^{(1)}}\ \right\rangle=o \tag{22}$$

and all the properties of the distribution function obtained in section (4) can also be verified.

IV. Equations for Evolution of One – Point Distribution Function $F_1^{(1)}$

It shall make use of equations (8) - (11) to convert these into a set of equations for the variation of the distribution function with time. This, in fact, is done by making use of the definitions of the constructed distribution functions, differentiating equation (12) partially with respect to time, making some suitable operations on the right-hand side of the equation so obtained and lastly replacing the time derivative of u,h,θ and c from the equations (8) - (11),

we get,

$$\frac{\partial F_1^{(1)}}{\partial t}=\frac{\partial}{\partial t}\left\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\ \right\rangle$$

$$=\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\frac{\partial}{\partial t}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\frac{\partial}{\partial t}\delta\left(h^{(1)}-g^{(1)}\right)\ \rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\frac{\partial}{\partial t}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\ \rangle+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\frac{\partial}{\partial t}\delta\left(c^{(1)}-\psi^{(1)}\right)\ \rangle$$

$$=\langle-\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\frac{\partial u^{(1)}}{\partial t}\frac{\partial}{\partial v^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\frac{\partial h^{(1)}}{\partial t}\frac{\partial}{\partial g^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\ \rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\frac{\partial \theta^{(1)}}{\partial t}\frac{\partial}{\partial \phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\ \rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\frac{\partial c^{(1)}}{\partial t}\frac{\partial}{\partial \psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\ \rangle$$

$$\frac{\partial F_1^{(1)}}{\partial t}=\langle-\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\{\ -\frac{\partial}{\partial x_\beta^{(1)}}\left(u_\alpha^{(1)}u_\beta^{(1)}-h_\alpha^{(1)}h_\beta^{(1)}\right)$$

$$-\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(1)}}\int\Big[\ \frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}}-\frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}}\ \Big]\frac{d\bar{x}'}{|\bar{x}'-\bar{x}|}+\nu\nabla^2 u_\alpha^{(1)}-2\in_{m\alpha\beta}\Omega_m u_\alpha^{(1)}\ \}\times\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\{\ -\frac{\partial}{\partial x_\beta^{(1)}}\left(h_\alpha^{(1)}u_\beta^{(1)}-u_\alpha^{(1)}h_\beta^{(1)}\right)+\lambda\nabla^2 h_\alpha^{(1)}\ \}$$

$$\times\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\ \rangle+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\{-u_\beta^{(1)}\frac{\partial\theta^{(1)}}{\partial x_\beta^{(1)}}+\gamma\nabla^2\theta^{(1)}\ \}$$

$$\times\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\ \rangle+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\{-u_\beta^{(1)}\frac{\partial c^{(1)}}{\partial x_\beta^{(1)}}+D\nabla^2 c-Rc^1\}\times\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\ \rangle$$

$$=\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\frac{\partial u_\alpha^{(1)}u_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$+\langle-\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\frac{\partial h_\alpha^{(1)}h_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$+\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(1)}}\int\Big[\ \frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}}-\frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}}\ \Big]\times\frac{d\bar{x}'}{|\bar{x}'-\bar{x}|}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$+\langle-\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\times\nu\nabla^2 u_\alpha^{(1)}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$+\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\times 2\in_{m\alpha\beta}\Omega_m u_\alpha^{(1)}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\times\frac{\partial h_\alpha^{(1)}u_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\times\frac{\partial u_\alpha^{(1)}h_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\times\lambda\nabla^2 h_\alpha^{(1)}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\times u_\beta^{(1)}\frac{\partial\theta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\times\gamma\nabla^2\theta^{(1)}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\times u_\beta^{(1)}\frac{\partial c^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\times D\nabla^2 c^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\times Rc^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle \tag{23}$$

Various terms in the above equation can be simplified with the help of the properties of distribution functions and continuity equation as that they may be expressed in terms of one point and two point distribution functions. So after simplifying the equation (24), we get the transport equation for one point distribution function in MHD turbulent flow in a rotating system under going a first order reaction as

$$\frac{\partial F_1^{(1)}}{\partial t}+v_\beta^{(1)}\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}+g_\beta^{(1)}\left(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}\right)\frac{\partial F_1^{(1)}}{\partial x_\beta^{(1)}}-\frac{\partial}{\partial v_\alpha^{(1)}}\left[\frac{1}{4\pi}\int\frac{\partial}{\partial x_\alpha^{(1)}}\left(\frac{1}{\left|\overline{x}^{(2)}-\overline{x}^{(1)}\right|}\right)\right.$$

$$\times\left(\frac{\partial v_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial v_\beta^{(2)}}{\partial x_\alpha^{(2)}}-\frac{\partial g_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial g_\beta^{(2)}}{\partial x_\alpha^{(2)}}\right)F_2^{(1,2)}dx^{(2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+\nu\frac{\partial}{\partial v_\alpha^{(1)}}\lim_{\overline{x}^{(2)}\to\overline{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\int v_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+\lambda\frac{\partial}{\partial g_\alpha^{(1)}}\lim_{\overline{x}^{(2)}\to\overline{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\int g_\alpha^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+\gamma\frac{\partial}{\partial\phi^{(1)}}\lim_{\overline{x}^{(2)}\to\overline{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\int\phi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+D\frac{\partial}{\partial\psi^{(1)}}\lim_{\overline{x}^{(2)}\to\overline{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(2)}\partial x_\beta^{(2)}}\int\psi^{(2)}F_2^{(1,2)}dv^{(2)}dg^{(2)}d\phi^{(2)}d\psi^{(2)}$$

$$+2\in_{m\alpha\beta}\Omega_m F_1^{(1)}-R\psi^{(1)}\frac{\partial}{\partial\psi^{(1)}}F_1^{(1)}=0 \tag{24}$$

V. Equations for Two-Point Distribution Function $F_2^{(1,2)}$

Due to derive the transport equation of two-point distribution function $F_2^{(1,2)}$ differentiating equation (13) partially with respect to time, making use of the definitions of the constructed distribution functions, making some suitable operations on the right-hand side of the equation so obtained and lastly replacing the time derivative of u,h,θ and c from the equations (8) - (11), we get,

$$\frac{\partial F_2^{(1,2)}}{\partial t}==\langle-\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})$$

$$\delta(c^{(2)}-\psi^{(2)})\{-\frac{\partial}{\partial x_\beta^{(1)}}(u_\alpha^{(1)}u_\beta^{(1)}-h_\alpha^{(1)}h_\beta^{(1)})-\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(1)}}\int[\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}}-\frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}}]$$

$$\times\frac{d\bar{x}''}{|\bar{x}''-\bar{x}|}+\nu\nabla^2u_\alpha^{(1)}-2\in_{m\alpha\beta}\Omega_m u_\alpha^{(1)}\}\times\frac{\partial}{\partial v_\alpha^{(1)}}\delta(u^{(1)}-v^{(1)})\rangle$$

$$+\langle-\delta(u^{(1)}-v^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})$$

$$\delta(c^{(2)}-\psi^{(2)})\{-\frac{\partial}{\partial x_\beta^{(1)}}(h_\alpha^{(1)}u_\beta^{(1)}-u_\alpha^{(1)}h_\beta^{(1)})+\lambda\nabla^2h_\alpha^{(1)}\}\times\frac{\partial}{\partial g_\alpha^{(1)}}\delta(h^{(1)}-g^{(1)})\rangle$$

$$+\langle-\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})$$

$$\{-u_\beta^{(1)}\frac{\partial\theta^{(1)}}{\partial x_\beta^{(1)}}+\gamma\nabla^2\theta^{(1)}\}\times\frac{\partial}{\partial\phi^{(1)}}\delta(\theta^{(1)}-\phi^{(1)})\rangle+\langle-\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(u^{(2)}-v^{(2)})$$

$$\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})\{-u_\beta^{(1)}\frac{\partial c^{(1)}}{\partial x_\beta^{(1)}}+D\nabla^2c^{(1)}-Rc^{(1)}\}\frac{\partial}{\partial\psi^{(1)}}\delta(c^{(1)}-\psi^{(1)})\rangle$$

$$+\langle-\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})$$

$$\{-\frac{\partial}{\partial x_\beta^{(2)}}(u_\alpha^{(2)}u_\beta^{(2)}-h_\alpha^{(2)}h_\beta^{(2)})-\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(2)}}\int[\frac{\partial u_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial u_\beta^{(2)}}{\partial x_\alpha^{(2)}}-\frac{\partial h_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial h_\beta^{(2)}}{\partial x_\alpha^{(2)}}]\frac{d\bar{x}''}{|\bar{x}''-\bar{x}'|}$$

$$+\nu\nabla^2u_\alpha^{(2)}-2\in_{m\alpha\beta}\Omega_m u_\alpha^{(2)}\}\times\frac{\partial}{\partial v_\alpha^{(2)}}\delta(u^{(2)}-v^{(2)})\rangle$$

$$+\langle-\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(\theta^{(2)}-\phi^{(2)})$$

$$\delta(c^{(2)}-\psi^{(2)})\{-\frac{\partial}{\partial x_\beta^{(2)}}(h_\alpha^{(2)}u_\beta^{(2)}-u_\alpha^{(2)}h_\beta^{(2)})+\lambda\nabla^2h_\alpha^{(2)}\}\times\frac{\partial}{\partial g_\alpha^{(2)}}\delta(h^{(2)}-g^{(2)})\rangle$$

$$+\langle-\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})$$

$$\delta(c^{(2)}-\psi^{(2)})\{-u_\beta^{(2)}\frac{\partial\theta^{(2)}}{\partial x_\beta^{(2)}}+\gamma\nabla^2\theta^{(2)}\}\times\frac{\partial}{\partial\phi^{(2)}}\delta(\theta^{(2)}-\phi^{(2)})\rangle$$

$$+\langle-\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\left\{-u_{\beta}^{(2)}\frac{\partial c^{(2)}}{\partial x_{\beta}^{(2)}}+D\nabla^{2}c^{(2)}-Rc^{(2)}\right\}\times\frac{\partial}{\partial\psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle$$

$$=\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\frac{\partial u_{\alpha}^{(1)}u_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle-\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\frac{\partial h_{\alpha}^{(1)}h_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\frac{1}{4\pi}\frac{\partial}{\partial x_{\alpha}^{(1)}}\int\left[\frac{\partial u_{\alpha}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial u_{\beta}^{(1)}}{\partial x_{\alpha}^{(1)}}-\frac{\partial h_{\alpha}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial h_{\beta}^{(1)}}{\partial x_{\alpha}^{(1)}}\right]\frac{d\bar{x}''}{\left|\bar{x}''-\bar{x}\right|}\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle-\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\times\nu\nabla^{2}u_{\alpha}^{(1)}\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times 2\in_{m\alpha\beta}\Omega_{m}u_{\alpha}^{(1)}\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\frac{\partial h_{\alpha}^{(1)}u_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial g_{\alpha}^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\frac{\partial u_{\alpha}^{(1)}h_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial g_{\alpha}^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\lambda\nabla^{2}h_{\alpha}^{(1)}\frac{\partial}{\partial g_{\alpha}^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times u_{\beta}^{(1)}\frac{\partial\theta^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\rangle$$

$$+\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\gamma\nabla^{2}\theta^{(1)}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times u_{\beta}^{(1)}\frac{\partial c^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times D\nabla^2 c^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times Rc^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle$$

$$+\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\times\frac{\partial u_\alpha^{(2)}u_\beta^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial v_\alpha^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\frac{\partial h_\alpha^{(2)}h_\beta^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial v_\alpha^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$

$$+\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(2)}}\int\left[\frac{\partial u_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial u_\beta^{(2)}}{\partial x_\alpha^{(2)}}-\frac{\partial h_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial h_\beta^{(2)}}{\partial x_\alpha^{(2)}}\right]\frac{d\bar{x}''}{\left|\bar{x}''-\bar{x}'\right|}\frac{\partial}{\partial v_\alpha^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\nu\nabla^2 u_\alpha^{(2)}\frac{\partial}{\partial v_\alpha^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$

$$+\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times 2\in_{m\alpha\beta}\Omega_m u_\alpha^{(2)}\frac{\partial}{\partial v_\alpha^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\frac{\partial h_\alpha^{(2)}u_\beta^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial g_\alpha^{(2)}}\delta\left(h^{(2)}-g^{(2)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\frac{\partial u_\alpha^{(2)}h_\beta^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial g_\alpha^{(2)}}\delta\left(h^{(2)}-g^{(2)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times\lambda\nabla^2 h_\alpha^{(2)}\frac{\partial}{\partial g_\alpha^{(2)}}\delta\left(h^{(2)}-g^{(2)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\times u_\beta^{(2)}\frac{\partial\theta^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial\phi^{(2)}}\delta\left(\theta^{(2)}-\phi^{(2)}\right)\rangle$$

$$+\langle -\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})$$
$$\delta(c^{(2)}-\psi^{(2)})\times\gamma\nabla^2\theta^{(2)}\frac{\partial}{\partial\phi^{(2)}}\delta(\theta^{(2)}-\phi^{(2)})\rangle$$
$$+\langle \delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})$$
$$\delta(\theta^{(2)}-\phi^{(2)})\times u_\beta^{(2)}\frac{\partial c^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial\psi^{(2)}}\delta(c^{(2)}-\psi^{(2)})\rangle$$
$$+\langle -\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})$$
$$\delta(\theta^{(2)}-\phi^{(2)})\times D\nabla^2c^{(2)}\frac{\partial}{\partial\psi^{(2)}}\delta(c^{(2)}-\psi^{(2)})\rangle$$
$$+\langle -\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})$$
$$\delta(\theta^{(2)}-\phi^{(2)})\times Rc^{(2)}\frac{\partial}{\partial\psi^{(2)}}\delta(c^{(2)}-\psi^{(2)})\rangle \tag{25}$$

After simplifying the various terms in equation (26), we get the transport equation for two -point distribution function $F_2^{(1,2)}(v,g,\phi,\psi)$ in MHD turbulent flow in a rotating system in a first order chemical reaction

$$\frac{\partial F_2^{(1,2)}}{\partial t}+(v_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}+v_\beta^{(2)}\frac{\partial}{\partial x_\beta^{(2)}})F_2^{(1,2)}+g_\beta^{(1)}(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}})\frac{\partial}{\partial x_\beta^{(1)}}F_2^{(1,2)}$$
$$+g_\beta^{(2)}(\frac{\partial g_\alpha^{(2)}}{\partial v_\alpha^{(2)}}+\frac{\partial v_\alpha^{(2)}}{\partial g_\alpha^{(2)}})\frac{\partial}{\partial x_\beta^{(2)}}F_2^{(1,2)}-\frac{\partial}{\partial v_\alpha^{(1)}}[\frac{1}{4\pi}\int\frac{\partial}{\partial x_\alpha^{(1)}}(\frac{1}{|\bar{x}^{(3)}-\bar{x}^{(1)}|})\times(\frac{\partial v_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial v_\beta^{(3)}}{\partial x_\alpha^{(3)}}-\frac{\partial g_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial g_\beta^{(3)}}{\partial x_\alpha^{(3)}})F_3^{(1,2,3)}dx^{(3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{3)}]$$
$$-\frac{\partial}{\partial v_\alpha^{(2)}}[\frac{1}{4\pi}\int\frac{\partial}{\partial x_\alpha^{(2)}}(\frac{1}{|\bar{x}^{(3)}-\bar{x}^{(2)}|})(\frac{\partial v_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial v_\beta^{(3)}}{\partial x_\alpha^{(3)}}-\frac{\partial g_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial g_\beta^{(3)}}{\partial x_\alpha^{(3)}})$$
$$\times F_3^{(1,2,3)}dx^{(3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{3)}]$$
$$+\nu(\frac{\partial}{\partial v_\alpha^{(1)}}\lim_{\bar{x}^{(3)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial v_\alpha^{(2)}}\lim_{\bar{x}^{(3)}\to\bar{x}^{(2)}})\frac{\partial^2}{\partial x_\beta^{(3)}\partial x_\beta^{(3)}}\int v_\alpha^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)}$$
$$+\lambda(\frac{\partial}{\partial g_\alpha^{(1)}}\lim_{\bar{x}^{(3)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial g_\alpha^{(2)}}\lim_{\bar{x}^{(3)}\to\bar{x}^{(2)}})\frac{\partial^2}{\partial x_\beta^{(3)}\partial x_\beta^{(3)}}\int g_\alpha^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)}$$
$$+\gamma(\frac{\partial}{\partial\phi^{(1)}}\lim_{\bar{x}^{(3)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial\phi^{(2)}}\lim_{\bar{x}^{(3)}\to\bar{x}^{(2)}})\frac{\partial^2}{\partial x_\beta^{(3)}\partial x_\beta^{(3)}}\int\phi^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)}$$
$$+D(\frac{\partial}{\partial\psi^{(1)}}\lim_{\bar{x}^{(3)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial\psi^{(2)}}\lim_{\bar{x}^{(3)}\to\bar{x}^{(2)}})\frac{\partial^2}{\partial x_\beta^{(3)}\partial x_\beta^{(3)}}\int\psi^{(3)}F_3^{(1,2,3)}dv^{(3)}dg^{(3)}d\phi^{(3)}d\psi^{(3)} \tag{26}$$

$$+4\in_{m\alpha\beta}\Omega_m F_2^{(1,2)} - R\psi^{(1)}\frac{\partial}{\partial\psi^{(1)}}F_2^{(1,2)} - R\psi^{(2)}\frac{\partial}{\partial\psi^{(2)}}F_2^{(1,2)} = 0$$

VI. Equations for Three-Point Distribution Function $F_3^{(1,2,3)}$

It shall make use of equations (8) - (11) to convert these into a set of equations for the variation of the distribution function with time. This, in fact, is done by making use of the definitions of the constructed distribution functions, differentiating equation (14) partially with respect to time, making some suitable operations on the right-hand side of the equation so obtained and lastly replacing the time derivative of u, h, θ and c from the equations (8) - (11), we get

$$\frac{\partial F_3^{(1,2,3)}}{\partial t} = \frac{\partial}{\partial t}\langle\ \delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})$$
$$\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\rangle$$

$$=\langle\ \delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})$$
$$\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\frac{\partial}{\partial t}\delta(u^{(1)}-v^{(1)})\ \rangle$$

$$+\langle\ \delta(u^{(1)}-v^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})$$
$$\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\frac{\partial}{\partial t}\delta(h^{(1)}-g^{(1)})\ \rangle$$

$$+\langle\ \delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})$$
$$\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\frac{\partial}{\partial t}\delta(\theta^{(1)}-\phi^{(1)})\ \rangle$$

$$+\langle\ \delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})$$
$$\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\frac{\partial}{\partial t}\delta(c^{(1)}-\psi^{(1)})\ \rangle$$

$$+\langle\ \delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})$$
$$\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\frac{\partial}{\partial t}\delta(u^{(2)}-v^{(2)})\ \rangle$$

$$+\langle\ \delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(\theta^{(2)}-\phi^{(2)})$$
$$\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\frac{\partial}{\partial t}\delta(h^{(2)}-g^{(2)})\ \rangle$$

$$+\langle\ \delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})$$
$$\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\frac{\partial}{\partial t}\delta(\theta^{(2)}-\phi^{(2)})\ \rangle$$

$$+\langle\ \delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})$$
$$\delta(\theta^{(2)}-\phi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\frac{\partial}{\partial t}\delta(c^{(2)}-\psi^{(2)})\rangle$$

$$+\langle\ \delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})$$
$$\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\frac{\partial}{\partial t}\delta(u^{(3)}-v^{(3)})\ \rangle$$
$$+\langle\ \delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})$$
$$\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\frac{\partial}{\partial t}\delta(h^{(3)}-g^{(3)})\ \rangle$$
$$+\langle\ \delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})$$
$$\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(c^{(3)}-\psi^{(3)})\frac{\partial}{\partial t}\delta(\theta^{(3)}-\phi^{(3)})\ \rangle$$
$$+\langle\ \delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})$$
$$\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\frac{\partial}{\partial t}\delta(c^{(3)}-\psi^{(3)})\rangle$$
$$=\langle-\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})$$
$$\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\frac{\partial u^{(1)}}{\partial t}\frac{\partial}{\partial v^{(1)}}\delta(u^{(1)}-v^{(1)})\ \rangle$$
$$+\langle-\delta(u^{(1)}-v^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})$$
$$\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\frac{\partial h^{(1)}}{\partial t}\frac{\partial}{\partial g^{(1)}}\delta(h^{(1)}-g^{(1)})\ \rangle$$
$$+\langle-\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})$$
$$\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\frac{\partial \theta^{(1)}}{\partial t}\frac{\partial}{\partial \phi^{(1)}}\delta(\theta^{(1)}-\phi^{(1)})\ \rangle$$
$$+\langle-\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})$$
$$\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\frac{\partial c^{(1)}}{\partial t}\frac{\partial}{\partial \psi^{(1)}}\delta(c^{(1)}-\psi^{(1)})\ \rangle$$
$$+\langle-\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})$$
$$\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\frac{\partial u^{(2)}}{\partial t}\frac{\partial}{\partial v^{(2)}}\delta(u^{(2)}-v^{(2)})\ \rangle$$
$$+\langle-\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})$$
$$\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\frac{\partial h^{(2)}}{\partial t}\frac{\partial}{\partial g^{(2)}}\delta(h^{(2)}-g^{(2)})\ \rangle$$
$$+\langle-\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(c^{(2)}-\psi^{(2)})$$
$$\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\frac{\partial \theta^{(2)}}{\partial t}\frac{\partial}{\partial \phi^{(2)}}\delta(\theta^{(2)}-\phi^{(2)})\ \rangle$$
$$+\langle-\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})$$
$$\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\frac{\partial c^{(2)}}{\partial t}\frac{\partial}{\partial \psi^{(2)}}\delta(c^{(2)}-\psi^{(2)})\ \rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial u^{(3)}}{\partial t}\frac{\partial}{\partial v^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial h^{(3)}}{\partial t}\frac{\partial}{\partial g^{(3)}}\delta\left(h^{(3)}-g^{(3)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\frac{\partial \theta^{(3)}}{\partial t}\frac{\partial}{\partial \phi^{(3)}}\delta\left(\theta^{(3)}-\phi^{(3)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\frac{\partial c^{(3)}}{\partial t}\frac{\partial}{\partial \psi^{(3)}}\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle$$

Using equations (8) to (11), we get from the above equation

$$\frac{\partial F_3^{(1,2,3)}}{\partial t}=\langle -\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$\{ -\frac{\partial}{\partial x_\beta^{(1)}}\left(u_\alpha^{(1)}u_\beta^{(1)}-h_\alpha^{(1)}h_\beta^{(1)}\right)-\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(1)}}\int \left[\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}}-\frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}}\right]\frac{d\overline{x}'''}{\left|\overline{x}'''-\overline{x}^n\right|}$$

$$+\nu\nabla^2 u_\alpha^{(1)}-2\in_{m\alpha\beta}\Omega_m u_\alpha^{(1)}\ \}\times\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$\{ -\frac{\partial}{\partial x_\beta^{(1)}}\left(h_\alpha^{(1)}u_\beta^{(1)}-u_\alpha^{(1)}h_\beta^{(1)}\right)+\lambda\nabla^2 h_\alpha^{(1)}\ \}\times\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$\times\left\{-u_\beta^{(1)}\frac{\partial \theta^{(1)}}{\partial x_\beta^{(1)}}+\gamma\nabla^2\theta^{(1)}\ \right\}\times\frac{\partial}{\partial \phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$\left\{-u_\beta^{(1)}\frac{\partial c^{(1)}}{\partial x_\beta^{(1)}}+D\nabla^2 c^{(1)}-Rc^{(1)}\right\}\times\frac{\partial}{\partial \psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle$$

$$\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$\left\{ -\frac{\partial}{\partial x_\beta^{(2)}}\left(u_\alpha^{(2)}u_\beta^{(2)}-h_\alpha^{(2)}h_\beta^{(2)}\right)-\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(2)}}\int\left[\frac{\partial u_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial u_\beta^{(2)}}{\partial x_\alpha^{(2)}}-\frac{\partial h_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial h_\beta^{(2)}}{\partial x_\alpha^{(2)}}\right]\frac{d\overline{x}'''}{\left|\overline{x}'''-\overline{x}'\right|}\right.$$

$$\left.+\nu\nabla^2 u_\alpha^{(2)}-2\in_{m\alpha\beta}\Omega_m u_\alpha^{(2)}\right\}\times\frac{\partial}{\partial v_\alpha^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\Big\rangle$$

$$+\Big\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$+\Big\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$\left\{-u_\beta^{(2)}\frac{\partial c^{(2)}}{\partial x_\beta^{(2)}}+D\nabla^2 c^{(2)}-Rc^{(2)}\right\}\frac{\partial}{\partial\psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\Big\rangle$$

$$\left\{-\frac{\partial}{\partial x_\beta^{(2)}}\left(h_\alpha^{(2)}u_\beta^{(2)}-u_\alpha^{(2)}h_\beta^{(2)}\right)+\lambda\nabla^2 h_\alpha^{(2)}\right\}\frac{\partial}{\partial g_\alpha^{(2)}}\delta\left(h^{(2)}-g^{(2)}\right)\Big\rangle$$

$$+\Big\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$\times\left\{-u_\beta^{(2)}\frac{\partial\theta^{(2)}}{\partial x_\beta^{(2)}}+\gamma\nabla^2\theta^{(2)}\right\}\times\frac{\partial}{\partial\phi^{(2)}}\delta\left(\theta^{(2)}-\phi^{(2)}\right)\Big\rangle$$

$$\Big\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$\left\{ -\frac{\partial}{\partial x_\beta^{(3)}}\left(u_\alpha^{(3)}u_\beta^{(3)}-h_\alpha^{(3)}h_\beta^{(3)}\right)-\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(3)}}\int\left[\frac{\partial u_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial u_\beta^{(3)}}{\partial x_\alpha^{(3)}}-\frac{\partial h_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial h_\beta^{(3)}}{\partial x_\alpha^{(3)}}\right]\frac{d\overline{x}'''}{\left|\overline{x}'''-\overline{x}''\right|}\right.$$

$$\left.+\nu\nabla^2 u_\alpha^{(3)}-2\in_{m\alpha\beta}\Omega_m u_\alpha^{(3)}\right\}\times\frac{\partial}{\partial v_\alpha^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\Big\rangle$$

$$+\Big\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$\left\{-\frac{\partial}{\partial x_\beta^{(3)}}\left(h_\alpha^{(3)}u_\beta^{(3)}-u_\alpha^{(3)}h_\beta^{(3)}\right)+\lambda\nabla^2 h_\alpha^{(3)}\right\}\frac{\partial}{\partial g_\alpha^{(3)}}\delta\left(h^{(3)}-g^{(3)}\right)\Big\rangle$$

$$+\Big\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$\times\left\{-u_\beta^{(3)}\frac{\partial\theta^{(3)}}{\partial x_\beta^{(3)}}+\gamma\nabla^2\theta^{(3)}\right\}\times\frac{\partial}{\partial\phi^{(3)}}\delta\left(\theta^{(3)}-\phi^{(3)}\right)\Big\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)$$

$$\left\{-u_{\beta}^{(3)}\frac{\partial c^{(3)}}{\partial x_{\beta}^{(3)}}+D\nabla^{2}c^{(3)}-Rc^{(3)}\right\}\frac{\partial}{\partial\psi^{(3)}}\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle$$

$$=\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial u_{\alpha}^{(1)}u_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle -\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial h_{\alpha}^{(1)}h_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{1}{4\pi}\frac{\partial}{\partial x_{\alpha}^{(1)}}\int\left[\ \frac{\partial u_{\alpha}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial u_{\beta}^{(1)}}{\partial x_{\alpha}^{(1)}}-\frac{\partial h_{\alpha}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial h_{\beta}^{(1)}}{\partial x_{\alpha}^{(1)}}\ \right]$$

$$\times\frac{d\overline{x}'''}{\left|\overline{x}'''-\overline{x}\right|}\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle+\langle -\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\nu\nabla^{2}u_{\alpha}^{(1)}\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times 2\in_{m\alpha\beta}\Omega_{m}u_{\alpha}^{(1)}\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial h_{\alpha}^{(1)}u_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial g_{\alpha}^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial u_{\alpha}^{(1)}h_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial g_{\alpha}^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\lambda\nabla^{2}h_{\alpha}^{(1)}\frac{\partial}{\partial g_{\alpha}^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_{\beta}^{(1)}\frac{\partial\theta^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\gamma\nabla^{2}\theta^{(1)}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_{\beta}^{(1)}\frac{\partial c^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial \psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times D\nabla^{2}c^{(1)}\frac{\partial}{\partial \psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle$$

$$+\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times Rc^{(1)}\frac{\partial}{\partial \psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle$$

$$+\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial u_{\alpha}^{(2)}u_{\beta}^{(2)}}{\partial x_{\beta}^{(2)}}\frac{\partial}{\partial v_{\alpha}^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial h_{\alpha}^{(2)}h_{\beta}^{(2)}}{\partial x_{\beta}^{(2)}}\frac{\partial}{\partial v_{\alpha}^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$

$$+\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{1}{4\pi}\frac{\partial}{\partial x_{\alpha}^{(2)}}\int\left[\ \frac{\partial u_{\alpha}^{(2)}}{\partial x_{\beta}^{(2)}}\frac{\partial u_{\beta}^{(2)}}{\partial x_{\alpha}^{(2)}}-\frac{\partial h_{\alpha}^{(2)}}{\partial x_{\beta}^{(2)}}\frac{\partial h_{\beta}^{(2)}}{\partial x_{\alpha}^{(2)}}\ \right]$$

$$\times\frac{d\overline{x}'''}{\left|\overline{x}'''-\overline{x}'\right|}\frac{\partial}{\partial v_{\alpha}^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\nu\nabla^{2}u_{\alpha}^{(2)}\frac{\partial}{\partial v_{\alpha}^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$

$$+\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times 2\in_{m\alpha\beta}\Omega_{m}u_{\alpha}^{(2)}\frac{\partial}{\partial v_{\alpha}^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$

$$+\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial h_{\alpha}^{(2)}u_{\beta}^{(2)}}{\partial x_{\beta}^{(2)}}\frac{\partial}{\partial g_{\alpha}^{(2)}}\delta\left(h^{(2)}-g^{(2)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial u_{\alpha}^{(2)}h_{\beta}^{(2)}}{\partial x_{\beta}^{(2)}}\frac{\partial}{\partial g_{\alpha}^{(2)}}\delta\left(h^{(2)}-g^{(2)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times \lambda\nabla^2 h_\alpha^{(2)}\frac{\partial}{\partial g_\alpha^{(2)}}\delta\left(h^{(2)}-g^{(2)}\right)\rangle$$

$$+\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(2)}\frac{\partial \theta^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial \phi^{(2)}}\delta\left(\theta^{(2)}-\phi^{(2)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times \gamma\nabla^2 \theta^{(2)}\frac{\partial}{\partial \phi^{(2)}}\delta\left(\theta^{(2)}-\phi^{(2)}\right)\rangle$$

$$+\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(2)}\frac{\partial c^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial \psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times D\nabla^2 c^{(2)}\frac{\partial}{\partial \psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle$$

$$+\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times Rc^{(2)}\frac{\partial}{\partial \psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle$$

$$+\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial u_\alpha^{(3)}u_\beta^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial}{\partial v_\alpha^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial h_\alpha^{(3)}h_\beta^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial}{\partial v_\alpha^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\rangle$$

$$+\langle \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(3)}}\int\left[\frac{\partial u_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial u_\beta^{(3)}}{\partial x_\alpha^{(3)}}-\frac{\partial h_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial h_\beta^{(3)}}{\partial x_\alpha^{(3)}}\right]^{***}$$

$$\times\frac{d\overline{x}'''}{\left|\overline{x}'''-\overline{x}''\right|}\frac{\partial}{\partial v_\alpha^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\rangle$$

$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times \nu\nabla^2 u_\alpha^{(3)}\frac{\partial}{\partial v_\alpha^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\rangle$$

$$+\langle \delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})$$

$$\delta(c^{(2)}-\psi^{(2)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\times 2\in_{m\alpha\beta}\Omega_m u_\alpha^{(3)}\frac{\partial}{\partial v_\alpha^{(3)}}\delta(u^{(3)}-v^{(3)})\rangle$$

$$+\langle \delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})$$

$$\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\times\frac{\partial h_\alpha^{(3)}u_\beta^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial}{\partial g_\alpha^{(3)}}\delta(h^{(3)}-g^{(3)})\rangle$$

$$+\langle -\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})$$

$$\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\times\frac{\partial u_\alpha^{(3)}h_\beta^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial}{\partial g_\alpha^{(3)}}\delta(h^{(3)}-g^{(3)})\rangle$$

$$+\langle -\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})$$

$$\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\times\lambda\nabla^2 h_\alpha^{(3)}\frac{\partial}{\partial g_\alpha^{(3)}}\delta(h^{(3)}-g^{(3)})\rangle$$

$$+\langle \delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})$$

$$\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(c^{(3)}-\psi^{(3)})\times u_\beta^{(3)}\frac{\partial\theta^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial}{\partial\phi^{(3)}}\delta(\theta^{(3)}-\phi^{(3)})\rangle$$

$$+\langle -\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})$$

$$\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(c^{(3)}-\psi^{(3)})\times\gamma\nabla^2\theta^{(3)}\frac{\partial}{\partial\phi^{(3)}}\delta(\theta^{(3)}-\phi^{(3)})\rangle$$

$$+\langle \delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})$$

$$\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\times u_\beta^{(3)}\frac{\partial c^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial}{\partial\psi^{(3)}}\delta(c^{(3)}-\psi^{(3)})\rangle$$

$$+\langle -\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})$$

$$\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\times D\nabla^2 c^{(3)}\frac{\partial}{\partial\psi^{(3)}}\delta(c^{(3)}-\psi^{(3)})\rangle$$

$$+\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})$$

$$\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\times Rc^{(3)}\frac{\partial}{\partial\psi^{(3)}}\delta(c^{(3)}-\psi^{(3)})\rangle \tag{27}$$

Various terms in the above equation can be simplified as that they may be expressed in terms of one-, two-, three- and four - point distribution functions.

The 1st term in the above equation is simplified as follows

$$\langle \ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial u_\alpha^{(1)}u_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$=\left\langle u_\beta^{(1)}\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\right.$$

$$\left.\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \right\rangle$$

$$=\left\langle -u_\beta^{(1)}\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\right.$$

$$\left.\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial u_\alpha^{(1)}}{\partial v_\alpha^{(1)}}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \right\rangle;(\text{since }\frac{\partial u_\alpha^{(1)}}{\partial v_\alpha^{(1)}}=1)$$

$$=\left\langle -\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\right.$$

$$\left.\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \right\rangle \tag{28}$$

Similarly, 6th, 9th and 11th terms of right hand-side of equation (27) can be simplified as follows;

$$\langle \ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial h_\alpha^{(1)}u_\beta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial g_\alpha^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\ \rangle$$

$$=\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\right.$$

$$\left.\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\ \right\rangle \tag{29}$$

9th term,

$$\langle \ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(1)}\frac{\partial\theta^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial\phi^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\ \rangle$$

$$=\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\right.$$

$$\left.\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\ \right\rangle \tag{30}$$

And 11th term

$$\langle \ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(1)}\frac{\partial c^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\ \rangle$$

$$=\left\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\right.$$

$$\left.\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\ \right\rangle \tag{31}$$

Adding these equations from (28) to (31), we get

$$\langle -\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$
$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$
$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$
$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$
$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$
$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(\theta^{(1)}-\phi^{(1)}\right)\rangle$$
$$+\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$
$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle$$
$$=-\frac{\partial}{\partial x_\beta^{(1)}}\langle u_\beta^{(1)}\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$
$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle$$

$=-\dfrac{\partial}{\partial x_\beta^{(1)}}v_\beta^{(1)}F_3^{(1,2,3)}$ [Applying the properties of distribution functions] (32)

Similarly, 14th, 19th, 22nd and 24th terms of right hand-side of equation (27) can be simplified as follows;

$$\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$
$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial u_\alpha^{(2)}u_\beta^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial v_\alpha^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$
$$=\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$
$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle \tag{33}$$

19th term,

$$\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$
$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial h_\alpha^{(2)}u_\beta^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial g_\alpha^{(2)}}\delta\left(h^{(2)}-g^{(2)}\right)\rangle \tag{34}$$

22nd term,

$$\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$
$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(2)}\frac{\partial\theta^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial\phi^{(2)}}\delta\left(\theta^{(2)}-\phi^{(2)}\right)\rangle$$

$$=\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}\delta\left(\theta^{(2)}-\phi^{(2)}\right)\rangle \tag{35}$$

And 24th term,

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(2)}\frac{\partial c^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial \psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle$$

$$=\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle \tag{36}$$

Adding equations (33) to (36), we get

$$-\frac{\partial}{\partial x_\beta^{(2)}}\langle u_\beta^{(2)}\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle$$

$$=-v_\beta^{(2)}\frac{\partial F_3^{(1,2,3)}}{\partial x_\beta^{(2)}} \tag{37}$$

Similarly, 27th, 32nd, 35th and 37th terms of right hand-side of equation (27) can be simplified as follows;

$$\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial u_\alpha^{(3)}u_\beta^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial}{\partial v_\alpha^{(2)}}\delta\left(u^{(3)}-v^{(3)}\right)\rangle$$

$$=\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(3)}\frac{\partial}{\partial x_\beta^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\rangle \tag{38}$$

32nd term,

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial h_\alpha^{(3)}u_\beta^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial}{\partial g_\alpha^{(3)}}\delta\left(h^{(3)}-g^{(3)}\right)\rangle$$

$$=\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_\beta^{(3)}\frac{\partial}{\partial x_\beta^{(3)}}\delta\left(h^{(3)}-g^{(3)}\right)\rangle \tag{39}$$

35th term,

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_{\beta}^{(3)}\frac{\partial\theta^{(3)}}{\partial x_{\beta}^{(3)}}\frac{\partial}{\partial\phi^{(3)}}\delta\left(\theta^{(3)}-\phi^{(3)}\right)\ \rangle$$

$$=\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times u_{\beta}^{(3)}\frac{\partial}{\partial x_{\beta}^{(3)}}\delta\left(\theta^{(3)}-\phi^{(3)}\right)\ \rangle \tag{40}$$

and 37th term,

$$\langle\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\times u_{\beta}^{(3)}\frac{\partial c^{(3)}}{\partial x_{\beta}^{(3)}}\frac{\partial}{\partial\psi^{(3)}}\delta\left(c^{(3)}-\psi^{(3)}\right)\ \rangle$$

$$=\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\times u_{\beta}^{(3)}\frac{\partial}{\partial x_{\beta}^{(3)}}\delta\left(c^{(3)}-\psi^{(3)}\right)\ \rangle \tag{41}$$

Adding equations (38) to (41), we get

$$-\frac{\partial}{\partial x_{\beta}^{(3)}}\langle u_{\beta}^{(3)}\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\ \rangle$$

$$= -\ v_{\beta}^{(3)}\ \frac{\partial F_3^{(1,2,3)}}{\partial x_{\beta}^{(3)}} \tag{42}$$

Similarly, 2nd, 7th, 15th, 20th, 28th and 33rd terms of right hand-side of equation (27) can be simplified as follows;

$$\langle-\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial h_{\alpha}^{(1)}h_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial v_{\alpha}^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\ \rangle$$

$$= -\ g_{\beta}^{(1)}\ \frac{\partial g_{\alpha}^{(1)}}{\partial v_{\alpha}^{(1)}}\ \frac{\partial F_3^{(1,2,3)}}{\partial x_{\beta}^{(1)}} \tag{43}$$

7th term,

$$\langle-\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial u_{\alpha}^{(1)}h_{\beta}^{(1)}}{\partial x_{\beta}^{(1)}}\frac{\partial}{\partial g_{\alpha}}\delta\left(h^{(1)}-g^{(1)}\right)\rangle$$

$$= -\ g_{\beta}^{(1)}\ \frac{\partial v_{\alpha}^{(1)}}{\partial g_{\alpha}^{(1)}}\ \frac{\partial F_3^{(1,2,3)}}{\partial x_{\beta}^{(1)}} \tag{44}$$

15th term,

$$\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial h_\alpha^{(2)}h_\beta^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial v_\alpha^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$

$$= - g_\beta^{(2)}\frac{\partial g_\alpha^{(2)}}{\partial v_\alpha^{(2)}}\frac{\partial F_3^{(1,2,3)}}{\partial x_\beta^{(2)}} \tag{45}$$

20th term,

$$\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial u_\alpha^{(2)}h_\beta^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial}{\partial g_\alpha^{(2)}}\delta\left(h^{(2)}-g^{(2)}\right)\rangle$$

$$= - g_\beta^{(2)}\frac{\partial v_\alpha^{(2)}}{\partial g_\alpha^{(2)}}\frac{\partial F_3^{(1,2,3)}}{\partial x_\beta^{(2)}} \tag{46}$$

28th term,

$$\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial h_\alpha^{(3)}h_\beta^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial}{\partial v_\alpha^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\rangle$$

$$= - g_\beta^{(3)}\frac{\partial g_\alpha^{(3)}}{\partial v_\alpha^{(3)}}\frac{\partial F_3^{(1,2,3)}}{\partial x_\beta^{(3)}} \tag{47}$$

and 33rd term,

$$\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{\partial u_\alpha^{(3)}h_\beta^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial}{\partial g_\alpha^{(3)}}\delta\left(h^{(3)}-g^{(3)}\right)\rangle$$

$$= - g_\beta^{(3)}\frac{\partial v_\alpha^{(3)}}{\partial g_\alpha^{(3)}}\frac{\partial F_3^{(1,2,3)}}{\partial x_\beta^{(3)}} \tag{48}$$

Fourth term can be reduced as

$$\langle -\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\nu\nabla^2 u_\alpha^{(1)}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)$$

$$=-\nu\frac{\partial}{\partial v_\alpha^{(1)}}\langle\nabla^2 u_\alpha^{(1)}[\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)$$

$$\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\ ]\ \rangle$$

$$=-\nu\frac{\partial}{\partial v_\alpha^{(1)}}\frac{\partial^2}{\partial x_\beta^{(1)}\partial x_\beta^{(1)}}\langle u_\alpha^{(1)}[\ \delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)$$

$$\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\ ]\ \rangle$$

$$=-\nu\frac{\partial}{\partial v_\alpha^{(1)}}\lim_{\overline{x}^{(4)}\to\overline{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(4)}\partial x_\beta^{(4)}}\langle u_\alpha^{(4)}[\ \delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})$$

$$\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\ ]\rangle$$

$$=-\nu\frac{\partial}{\partial v_\alpha^{(1)}}\lim_{\overline{x}^{(4)}\to\overline{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(4)}\partial x_\beta^{(4)}}\langle\int u_\alpha^{(4)}\delta(u^{(4)}-v^{(4)})\delta(h^{(4)}-g^{(4)})\delta(\theta^{(4)}-\phi^{(4)})\delta(c^{(4)}-\psi^{(4)})$$

$$\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})$$

$$\delta(c^{(2)}-\psi^{(2)})\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)}\ \rangle$$

$$=-\nu\frac{\partial}{\partial v_\alpha^{(1)}}\lim_{\overline{x}^{(4)}\to\overline{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(4)}\partial x_\beta^{(4)}}\int v_\alpha^{(4)}F_4^{(1,2,3,4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)} \tag{49}$$

Similarly, 8th ,10th ,12th ,17th ,21st ,23rd ,25th ,30th ,34th ,36th and 38th terms of right hand-side of equation (27) can be simplified as follows;

$$\langle-\delta(u^{(1)}-v^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})$$

$$\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\times\lambda\nabla^2h_\alpha^{(1)}\frac{\partial}{\partial g_\alpha^{(1)}}\delta(h^{(1)}-g^{(1)})\ \rangle$$

$$=-\lambda\frac{\partial}{\partial g_\alpha^{(1)}}\lim_{\overline{x}^{(4)}\to\overline{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(4)}\partial x_\beta^{(4)}}\int g_\alpha^{(4)}F_4^{(1,2,3,4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)} \tag{50}$$

10th term,

$$\langle-\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(c^{(1)}-\psi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})$$

$$\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\times\gamma\nabla^2\theta^{(1)}\frac{\partial}{\partial\phi^{(1)}}\delta(\theta^{(1)}-\phi^{(1)})\ \rangle$$

$$=-\gamma\frac{\partial}{\partial\phi^{(1)}}\lim_{\overline{x}^{(4)}\to\overline{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(4)}\partial x_\beta^{(4)}}\int\phi^{(4)}F_4^{(1,2,3,4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)} \tag{51}$$

12th term,

$$+\langle-\delta(u^{(1)}-v^{(1)})\delta(h^{(1)}-g^{(1)})\delta(\theta^{(1)}-\phi^{(1)})\delta(u^{(2)}-v^{(2)})\delta(h^{(2)}-g^{(2)})\delta(\theta^{(2)}-\phi^{(2)})\delta(c^{(2)}-\psi^{(2)})$$

$$\delta(u^{(3)}-v^{(3)})\delta(h^{(3)}-g^{(3)})\delta(\theta^{(3)}-\phi^{(3)})\delta(c^{(3)}-\psi^{(3)})\times D\nabla^2c^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta(c^{(1)}-\psi^{(1)})\ \rangle$$

$$=-D\frac{\partial}{\partial\psi^{(1)}}\lim_{\overline{x}^{(4)}\to\overline{x}^{(1)}}\frac{\partial^2}{\partial x_\beta^{(4)}\partial x_\beta^{(3)}}\int\psi^{(4)}F_4^{(1,2,3,4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)} \tag{52}$$

17th term,

$$\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times \nu\nabla^2 u_\alpha^{(2)}\frac{\partial}{\partial v_\alpha^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$

$$=-\nu\frac{\partial}{\partial v_\alpha^{(2)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(2)}}\frac{\partial^2}{\partial x_\beta^{(4)}\partial x_\beta^{(4)}}\int v_\alpha^{(4)}F_4^{(1,2,3,4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)} \tag{53}$$

21st term,

$$\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times \lambda\nabla^2 h_\alpha^{(2)}\frac{\partial}{\partial g_\alpha^{(2)}}\delta\left(h^{(2)}-g^{(2)}\right)\rangle$$

$$=-\lambda\frac{\partial}{\partial g_\alpha^{(2)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(2)}}\frac{\partial^2}{\partial x_\beta^{(4)}\partial x_\beta^{(4)}}\int g_\alpha^{(4)}F_4^{(1,2,3,4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)} \tag{54}$$

23rd term,

$$\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times \gamma\nabla^2 \theta^{(2)}\frac{\partial}{\partial \phi^{(2)}}\delta\left(\theta^{(2)}-\phi^{(2)}\right)\rangle$$

$$=-\gamma\frac{\partial}{\partial \phi^{(2)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(2)}}\frac{\partial^2}{\partial x_\beta^{(4)}\partial x_\beta^{(4)}}\int \phi^{(4)}F_4^{(1,2,3,4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)} \tag{55}$$

25th term,

$$\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times D\nabla^2 c^{(2)}\frac{\partial}{\partial \psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle$$

$$=-D\frac{\partial}{\partial \psi^{(2)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(2)}}\frac{\partial^2}{\partial x_\beta^{(4)}\partial x_\beta^{(4)}}\int \psi^{(4)}F_4^{(1,2,3,4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)} \tag{56}$$

31st term,

$$\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times \nu\nabla^2 u_\alpha^{(3)}\frac{\partial}{\partial v_\alpha^{(3)}}\delta\left(u^{(3)}-v^{(3)}\right)\rangle$$

$$=-\nu\frac{\partial}{\partial v_\alpha^{(3)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(3)}}\frac{\partial^2}{\partial x_\beta^{(4)}\partial x_\beta^{(4)}}\int v_\alpha^{(4)}F_4^{(1,2,3,4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)} \tag{57}$$

34th term,

Global Journal of Science Frontier Research (A) Volume XV Issue II Version I Year 2015

$$\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\lambda\nabla^2 h_\alpha^{(3)}\frac{\partial}{\partial g_\alpha^{(3)}}\delta\left(h^{(3)}-g^{(3)}\right)\rangle$$

$$=-\lambda\frac{\partial}{\partial g_\alpha^{(3)}}\lim_{\overline{x}^{(4)}\to\overline{x}^{(3)}}\frac{\partial^2}{\partial x_\beta^{(4)}\partial x_\beta^{(4)}}\int g_\alpha^{(4)}F_4^{(1,2,3,4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)} \tag{58}$$

36th term,

$$\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\gamma\nabla^2\theta^{(3)}\frac{\partial}{\partial\phi^{(3)}}\delta\left(\theta^{(3)}-\phi^{(3)}\right)\rangle$$

$$=-\gamma\frac{\partial}{\partial\phi^{(3)}}\lim_{\overline{x}^{(4)}\to\overline{x}^{(3)}}\frac{\partial^2}{\partial x_\beta^{(4)}\partial x_\beta^{(4)}}\int\phi^{(4)}F_4^{(1,2,3,4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)} \tag{59}$$

38th term,

$$\langle -\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(c^{(2)}-\psi^{(2)}\right)\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\times D\nabla^2 c^{(3)}\frac{\partial}{\partial\psi^{(3)}}\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle$$

$$=-D\frac{\partial}{\partial\psi^{(3)}}\lim_{\overline{x}^{(4)}\to\overline{x}^{(3)}}\frac{\partial^2}{\partial x_\beta^{(4)}\partial x_\beta^{(4)}}\int\psi^{(4)}F_4^{(1,2,3,4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)} \tag{60}$$

We reduce the third term of right hand side of equation (27),

$$\langle\ \delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(1)}}\int\left[\frac{\partial u_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial u_\beta^{(1)}}{\partial x_\alpha^{(1)}}-\frac{\partial h_\alpha^{(1)}}{\partial x_\beta^{(1)}}\frac{\partial h_\beta^{(1)}}{\partial x_\alpha^{(1)}}\right]$$

$$\times\frac{d\overline{x}'''}{\left|\overline{x}'''-\overline{x}\right|}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\left(u^{(1)}-v^{(1)}\right)\rangle$$

$$=\frac{\partial}{\partial v_\alpha^{(1)}}\left[\frac{1}{4\pi}\int\frac{\partial}{\partial x_\alpha^{(1)}}\left(\frac{1}{\left|\overline{x}^{(4)}-\overline{x}^{(1)}\right|}\right)\left(\frac{\partial v_\alpha^{(4)}}{\partial x_\beta^{(4)}}\frac{\partial v_\beta^{(4)}}{\partial x_\alpha^{(4)}}-\frac{\partial g_\alpha^{(4)}}{\partial x_\beta^{(4)}}\frac{\partial g_\beta^{(4)}}{\partial x_\alpha^{(4)}}\right)F_4^{(1,2,3,4)}dx^{(4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)}\right] \tag{61}$$

16th term,

$$\langle\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)$$

$$\times\frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(2)}}\int\left[\frac{\partial u_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial u_\beta^{(2)}}{\partial x_\alpha^{(2)}}-\frac{\partial h_\alpha^{(2)}}{\partial x_\beta^{(2)}}\frac{\partial h_\beta^{(2)}}{\partial x_\alpha^{(2)}}\right]\times\frac{d\overline{x}'''}{\left|\overline{x}'''-\overline{x}'\right|}\frac{\partial}{\partial v_\alpha^{(2)}}\delta\left(u^{(2)}-v^{(2)}\right)\rangle$$

$$= \frac{\partial}{\partial v_\alpha^{(2)}}\Big[\frac{1}{4\pi}\int \frac{\partial}{\partial x_\alpha^{(2)}}\Big(\frac{1}{\left|\overline{x}^{(4)}-\overline{x}^{(2)}\right|}\Big)\Big(\frac{\partial v_\alpha^{(4)}}{\partial x_\beta^{(4)}}\frac{\partial v_\beta^{(4)}}{\partial x_\alpha^{(4)}} - \frac{\partial g_\alpha^{(4)}}{\partial x_\beta^{(4)}}\frac{\partial g_\beta^{(4)}}{\partial x_\alpha^{(4)}}\Big)F_4^{(1,2,3,4)}dx^{(4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)}\Big] \tag{62}$$

Similarly, 29th term,

$$\Big\langle \delta\big(u^{(1)}-v^{(1)}\big)\delta\big(h^{(1)}-g^{(1)}\big)\delta\big(\theta^{(1)}-\phi^{(1)}\big)\delta\big(c^{(1)}-\psi^{(1)}\big)\delta\big(u^{(2)}-v^{(2)}\big)\delta\big(h^{(2)}-g^{(2)}\big)$$

$$\delta\big(\theta^{(2)}-\phi^{(2)}\big)\delta\big(c^{(2)}-\psi^{(2)}\big)\delta\big(h^{(3)}-g^{(3)}\big)\delta\big(\theta^{(3)}-\phi^{(3)}\big)\delta\big(c^{(3)}-\psi^{(3)}\big)$$

$$\times \frac{1}{4\pi}\frac{\partial}{\partial x_\alpha^{(3)}}\int \Big[\frac{\partial u_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial u_\beta^{(3)}}{\partial x_\alpha^{(3)}} - \frac{\partial h_\alpha^{(3)}}{\partial x_\beta^{(3)}}\frac{\partial h_\beta^{(3)}}{\partial x_\alpha^{(3)}}\Big]\frac{d\overline{x}'''}{\left|\overline{x}'''-\overline{x}''\right|}\frac{\partial}{\partial v_\alpha^{(3)}}\delta\big(u^{(3)}-v^{(3)}\big)\Big\rangle$$

$$= \frac{\partial}{\partial v_\alpha^{(3)}}\Big[\frac{1}{4\pi}\int \frac{\partial}{\partial x_\alpha^{(3)}}\Big(\frac{1}{\left|\overline{x}^{(4)}-\overline{x}^{(3)}\right|}\Big)\Big(\frac{\partial v_\alpha^{(4)}}{\partial x_\beta^{(4)}}\frac{\partial v_\beta^{(4)}}{\partial x_\alpha^{(4)}} - \frac{\partial g_\alpha^{(4)}}{\partial x_\beta^{(4)}}\frac{\partial g_\beta^{(4)}}{\partial x_\alpha^{(4)}}\Big)F_4^{(1,2,3,4)}dx^{(4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)}\Big] \tag{63}$$

Fifth term of right hand side of equation (27),

$$\Big\langle \delta\big(h^{(1)}-g^{(1)}\big)\delta\big(\theta^{(1)}-\phi^{(1)}\big)\delta\big(c^{(1)}-\psi^{(1)}\big)\delta\big(u^{(2)}-v^{(2)}\big)\delta\big(h^{(2)}-g^{(2)}\big)\delta\big(\theta^{(2)}-\phi^{(2)}\big)\delta\big(c^{(2)}-\psi^{(2)}\big)$$

$$\delta\big(u^{(3)}-v^{(3)}\big)\delta\big(h^{(3)}-g^{(3)}\big)\delta\big(\theta^{(3)}-\phi^{(3)}\big)\delta\big(c^{(3)}-\psi^{(3)}\big)\times 2\in_{m\alpha\beta}\Omega_m u_\alpha^{(1)}\frac{\partial}{\partial v_\alpha^{(1)}}\delta\big(u^{(1)}-v^{(1)}\big)\Big\rangle$$

$$=\Big\langle 2\in_{m\alpha\beta}\Omega_m u_\alpha^{(1)}\frac{\partial}{\partial v_\alpha^{(1)}}\Big[\delta\big(u^{(1)}-v^{(1)}\big)\delta\big(h^{(1)}-g^{(1)}\big)\delta\big(\theta^{(1)}-\phi^{(1)}\big)\delta\big(c^{(1)}-\psi^{(1)}\big)\delta\big(u^{(2)}-v^{(2)}\big)$$

$$\delta\big(h^{(2)}-g^{(2)}\big)\delta\big(\theta^{(2)}-\phi^{(2)}\big)\delta\big(c^{(2)}-\psi^{(2)}\big)\delta\big(u^{(3)}-v^{(3)}\big)\delta\big(h^{(3)}-g^{(3)}\big)\delta\big(\theta^{(3)}-\phi^{(3)}\big)\delta\big(c^{(3)}-\psi^{(3)}\big)\Big]\Big\rangle$$

$$=2\in_{m\alpha\beta}\Omega_m\frac{\partial}{\partial v_\alpha^{(1)}}\Big\langle u_\alpha^{(1)}\delta\big(u^{(1)}-v^{(1)}\big)\delta\big(h^{(1)}-g^{(1)}\big)\delta\big(\theta^{(1)}-\phi^{(1)}\big)\delta\big(c^{(1)}-\psi^{(1)}\big)\delta\big(u^{(2)}-v^{(2)}\big)$$

$$\delta\big(h^{(2)}-g^{(2)}\big)\delta\big(\theta^{(2)}-\phi^{(2)}\big)\delta\big(c^{(2)}-\psi^{(2)}\big)\delta\big(u^{(3)}-v^{(3)}\big)\delta\big(h^{(3)}-g^{(3)}\big)\delta\big(\theta^{(3)}-\phi^{(3)}\big)\delta\big(c^{(3)}-\psi^{(3)}\big)\Big\rangle$$

$$=2\in_{m\alpha\beta}\Omega_m\frac{\partial u_\alpha^{(1)}}{\partial v_\alpha^{(1)}}\Big\langle \delta\big(u^{(1)}-v^{(1)}\big)\delta\big(h^{(1)}-g^{(1)}\big)\delta\big(\theta^{(1)}-\phi^{(1)}\big)\delta\big(c^{(1)}-\psi^{(1)}\big)\delta\big(u^{(2)}-v^{(2)}\big)\delta\big(h^{(2)}-g^{(2)}\big)$$

$$\delta\big(\theta^{(2)}-\phi^{(2)}\big)\delta\big(c^{(2)}-\psi^{(2)}\big)\delta\big(u^{(3)}-v^{(3)}\big)\delta\big(h^{(3)}-g^{(3)}\big)\delta\big(\theta^{(3)}-\phi^{(3)}\big)\delta\big(c^{(3)}-\psi^{(3)}\big)\Big\rangle$$

$$=2\in_{m\alpha\beta}\Omega_m F_3^{(1,2,3)} \tag{64}$$

Similarly, 18th and 31rd terms of right hand side of equation (27),

$$\Big\langle\delta\big(u^{(1)}-v^{(1)}\big)\delta\big(h^{(1)}-g^{(1)}\big)\delta\big(\theta^{(1)}-\phi^{(1)}\big)\delta\big(c^{(1)}-\psi^{(1)}\big)\delta\big(h^{(2)}-g^{(2)}\big)\delta\big(\theta^{(2)}-\phi^{(2)}\big)\delta\big(c^{(2)}-\psi^{(2)}\big)$$

$$\delta\big(u^{(3)}-v^{(3)}\big)\delta\big(h^{(3)}-g^{(3)}\big)\delta\big(\theta^{(3)}-\phi^{(3)}\big)\delta\big(c^{(3)}-\psi^{(3)}\big)\times 2\in_{m\alpha\beta}\Omega_m u_\alpha^{(2)}\frac{\partial}{\partial v_\alpha^{(2)}}\delta\big(u^{(2)}-v^{(2)}\big)\Big\rangle$$

$$=2\in_{m\alpha\beta}\Omega_m F_3^{(1,2,3)} \tag{65}$$

31rd term,

$$\Big\langle\delta\big(u^{(1)}-v^{(1)}\big)\delta\big(h^{(1)}-g^{(1)}\big)\delta\big(\theta^{(1)}-\phi^{(1)}\big)\delta\big(c^{(1)}-\psi^{(1)}\big)\delta\big(u^{(2)}-v^{(2)}\big)\delta\big(h^{(2)}-g^{(2)}\big)\delta\big(\theta^{(2)}-\phi^{(2)}\big)$$

$$\delta\big(c^{(2)}-\psi^{(2)}\big)\delta\big(h^{(3)}-g^{(3)}\big)\delta\big(\theta^{(3)}-\phi^{(3)}\big)\delta\big(c^{(3)}-\psi^{(3)}\big)\times 2\in_{m\alpha\beta}\Omega_m u_\alpha^{(3)}\frac{\partial}{\partial v_\alpha^{(3)}}\delta\big(u^{(3)}-v^{(3)}\big)\Big\rangle$$

$$=2\in_{m\alpha\beta}\Omega_m F_3^{(1,2,3)} \tag{66}$$

13[th] term of Equation (27)

$$\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)\delta\left(c^{(2)}-\psi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times Rc^{(1)}\frac{\partial}{\partial\psi^{(1)}}\delta\left(c^{(1)}-\psi^{(1)}\right)\rangle \quad (67)$$

$$=R\psi^{(1)}\frac{\partial}{\partial\psi^{(1)}}F_3^{(1,2,3)}$$

26[th] term of Equation (27)

$$\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\delta\left(c^{(3)}-\psi^{(3)}\right)\times Rc^{(2)}\frac{\partial}{\partial\psi^{(2)}}\delta\left(c^{(2)}-\psi^{(2)}\right)\rangle \quad (68)$$

$$=R\psi^{(2)}\frac{\partial}{\partial\psi^{(2)}}F_3^{(1,2,3)}$$

39[th] term of Equation (27)

$$\delta\left(u^{(1)}-v^{(1)}\right)\delta\left(h^{(1)}-g^{(1)}\right)\delta\left(\theta^{(1)}-\phi^{(1)}\right)\delta\left(c^{(1)}-\psi^{(1)}\right)\delta\left(u^{(2)}-v^{(2)}\right)\delta\left(h^{(2)}-g^{(2)}\right)\delta\left(\theta^{(2)}-\phi^{(2)}\right)$$

$$\delta\left(u^{(3)}-v^{(3)}\right)\delta\left(h^{(3)}-g^{(3)}\right)\delta\left(\theta^{(3)}-\phi^{(3)}\right)\times Rc^{(3)}\frac{\partial}{\partial\psi^{(3)}}\delta\left(c^{(3)}-\psi^{(3)}\right)\rangle \quad (69)$$

$$=R\psi^{(3)}\frac{\partial}{\partial\psi^{(3)}}F_3^{(1,2,3)}$$

VII. Results

Substituting the results (28) – (69) in equation (27) we get the transport equation for three- point distribution function $F_3^{(1,2,3)}(v,g,\phi,\psi)$ in MHD turbulent flow in a rotating system under going a first order reaction as

$$\frac{\partial F_3^{(1,2,3)}}{\partial t}+\left(v_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}+v_\beta^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}v_\beta^{(3)}\frac{\partial}{\partial x_\beta^{(3)}}\right)F_3^{(1,2,3)}+\Big[\,g_\beta^{(1)}\left(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}\right)\frac{\partial}{\partial x_\beta^{(1)}}$$

$$+g_\beta^{(2)}\left(\frac{\partial g_\alpha^{(2)}}{\partial v_\alpha^{(2)}}+\frac{\partial v_\alpha^{(2)}}{\partial g_\alpha^{(2)}}\right)\frac{\partial}{\partial x_\beta^{(2)}}+g_\beta^{(3)}\left(\frac{\partial g_\alpha^{(3)}}{\partial v_\alpha^{(3)}}+\frac{\partial v_\alpha^{(3)}}{\partial g_\alpha^{(3)}}\right)\frac{\partial}{\partial x_\beta^{(3)}}\Big]F_3^{(1,2,3)}$$

$$+v\left(\frac{\partial}{\partial v_\alpha^{(1)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial v_\alpha^{(2)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(2)}}+\frac{\partial}{\partial v_\alpha^{(3)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(3)}}\right)$$

$$\times\frac{\partial^2}{\partial x_\beta^{(4)}\partial x_\beta^{(4)}}\int v_\alpha^{(4)}F_4^{(1,2,3,4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)}$$

$$+\lambda\left(\frac{\partial}{\partial g_\alpha^{(1)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(1)}}+\frac{\partial}{\partial g_\alpha^{(2)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(2)}}+\frac{\partial}{\partial g_\alpha^{(3)}}\lim_{\bar{x}^{(4)}\to\bar{x}^{(3)}}\right)$$

$$\times \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int g_\alpha^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)}$$

$$+\gamma\Big(\frac{\partial}{\partial \phi^{(1)}} \lim_{\bar{x}^{(4)} \to \bar{x}^{(1)}} + \frac{\partial}{\partial \phi^{(2)}} \lim_{\bar{x}^{(4)} \to \bar{x}^{(2)}} + \frac{\partial}{\partial \phi^{(3)}} \lim_{\bar{x}^{(4)} \to \bar{x}^{(3)}} \Big)$$

$$\times \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int \phi^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)}$$

$$+D\Big(\frac{\partial}{\partial \psi^{(1)}} \lim_{\bar{x}^{(4)} \to \bar{x}^{(1)}} + \frac{\partial}{\partial \psi^{(2)}} \lim_{\bar{x}^{(4)} \to \bar{x}^{(2)}} + \frac{\partial}{\partial \psi^{(3)}} \lim_{\bar{x}^{(4)} \to \bar{x}^{(3)}} \Big)$$

$$\times \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int \psi^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)}$$

$$-\Big[\frac{\partial}{\partial v_\alpha^{(1)}} \Big\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(1)}} \Big(\frac{1}{\left|\bar{x}^{(4)} - \bar{x}^{(1)}\right|} \Big) \Big\} + \frac{\partial}{\partial v_\alpha^{(2)}} \Big\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(2)}} \Big(\frac{1}{\left|\bar{x}^{(4)} - \bar{x}^{(2)}\right|} \Big) \Big\}$$

$$+ \frac{\partial}{\partial v_\alpha^{(3)}} \Big\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(3)}} \Big(\frac{1}{\left|\bar{x}^{(4)} - \bar{x}^{(3)}\right|} \Big) \Big\} \times \Big(\frac{\partial v_\alpha^{(4)}}{\partial x_\beta^{(4)}} \frac{\partial v_\beta^{(4)}}{\partial x_\alpha^{(4)}} - \frac{\partial g_\alpha^{(4)}}{\partial x_\beta^{(4)}} \frac{\partial g_\beta^{(4)}}{\partial x_\alpha^{(4)}} \Big) F_4^{(1,2,3,4)}$$

$$\times dx^{(4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \Big] + 6 \in_{m\alpha\beta} \Omega_m F_3^{(1,2,3)}$$

$$- R\Big(\psi^{(1)} \frac{\partial}{\partial \psi^{(1)}} + \psi^{(2)} \frac{\partial}{\partial \psi^{(2)}} + \psi^{(3)} \frac{\partial}{\partial \psi^{(3)}}\Big) F_3^{(1,2,3)} = 0 \tag{70}$$

Continuing this way, we can derive the equations for evolution of $F_4^{(1,2,3,4)}$, $F_5^{(1,2,3,4,5)}$ and so on. Logically it is possible to have an equation for every F_n (n is an integer) but the system of equations so obtained is not closed. Certain approximations will be required thus obtained.

VIII. Discussions

If R=0,i.e the reaction rate is absent, the transport equation for three- point distribution function in MHD turbulent flow (113) becomes

$$\frac{\partial F_3^{(1,2,3)}}{\partial t} + \Big(v_\beta^{(1)} \frac{\partial}{\partial x_\beta^{(1)}} + v_\beta^{(2)} \frac{\partial}{\partial x_\beta^{(2)}} v_\beta^{(3)} \frac{\partial}{\partial x_\beta^{(3)}} \Big) F_3^{(1,2,3)} + \Big[g_\beta^{(1)} \Big(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}} + \frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}} \Big) \frac{\partial}{\partial x_\beta^{(1)}}$$

$$+ g_\beta^{(2)} \Big(\frac{\partial g_\alpha^{(2)}}{\partial v_\alpha^{(2)}} + \frac{\partial v_\alpha^{(2)}}{\partial g_\alpha^{(2)}} \Big) \frac{\partial}{\partial x_\beta^{(2)}} + g_\beta^{(3)} \Big(\frac{\partial g_\alpha^{(3)}}{\partial v_\alpha^{(3)}} + \frac{\partial v_\alpha^{(3)}}{\partial g_\alpha^{(3)}} \Big) \frac{\partial}{\partial x_\beta^{(3)}} \Big] F_3^{(1,2,3)}$$

$$+\nu\Big(\frac{\partial}{\partial v_\alpha^{(1)}} \lim_{\bar{x}^{(4)} \to \bar{x}^{(1)}} + \frac{\partial}{\partial v_\alpha^{(2)}} \lim_{\bar{x}^{(4)} \to \bar{x}^{(2)}} + \frac{\partial}{\partial v_\alpha^{(3)}} \lim_{\bar{x}^{(4)} \to \bar{x}^{(3)}} \Big)$$

$$\times \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int v_\alpha^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)}$$

$$+\lambda\left(\frac{\partial}{\partial g_\alpha^{(1)}} \lim_{\overline{x}^{(4)} \to \overline{x}^{(1)}} + \frac{\partial}{\partial g_\alpha^{(2)}} \lim_{\overline{x}^{(4)} \to \overline{x}^{(2)}} + \frac{\partial}{\partial g_\alpha^{(3)}} \lim_{\overline{x}^{(4)} \to \overline{x}^{(3)}} \right)$$

$$\times \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int g_\alpha^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)}$$

$$+\gamma\left(\frac{\partial}{\partial \phi^{(1)}} \lim_{\overline{x}^{(4)} \to \overline{x}^{(1)}} + \frac{\partial}{\partial \phi^{(2)}} \lim_{\overline{x}^{(4)} \to \overline{x}^{(2)}} + \frac{\partial}{\partial \phi^{(3)}} \lim_{\overline{x}^{(4)} \to \overline{x}^{(3)}} \right)$$

$$\times \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int \phi^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)}$$

$$+D\left(\frac{\partial}{\partial \psi^{(1)}} \lim_{\overline{x}^{(4)} \to \overline{x}^{(1)}} + \frac{\partial}{\partial \psi^{(2)}} \lim_{\overline{x}^{(4)} \to \overline{x}^{(2)}} + \frac{\partial}{\partial \psi^{(3)}} \lim_{\overline{x}^{(4)} \to \overline{x}^{(3)}} \right)$$

$$\times \frac{\partial^2}{\partial x_\beta^{(4)} \partial x_\beta^{(4)}} \int \psi^{(4)} F_4^{(1,2,3,4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)}$$

$$-\left[\frac{\partial}{\partial v_\alpha^{(1)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(1)}} \left(\frac{1}{\left|\overline{x}^{(4)} - \overline{x}^{(1)}\right|} \right) \right\} + \frac{\partial}{\partial v_\alpha^{(2)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(2)}} \left(\frac{1}{\left|\overline{x}^{(4)} - \overline{x}^{(2)}\right|} \right) \right\} \right.$$

$$+ \frac{\partial}{\partial v_\alpha^{(3)}} \left\{ \frac{1}{4\pi} \int \frac{\partial}{\partial x_\alpha^{(3)}} \left(\frac{1}{\left|\overline{x}^{(4)} - \overline{x}^{(3)}\right|} \right) \right\} \times \left(\frac{\partial v_\alpha^{(4)}}{\partial x_\beta^{(4)}} \frac{\partial v_\beta^{(4)}}{\partial x_\alpha^{(4)}} - \frac{\partial g_\alpha^{(4)}}{\partial x_\beta^{(4)}} \frac{\partial g_\beta^{(4)}}{\partial x_\alpha^{(4)}} \right) F_4^{(1,2,3,4)}$$

$$\left. \times dx^{(4)} dv^{(4)} dg^{(4)} d\phi^{(4)} d\psi^{(4)} \right] + 6 \in_{m\alpha\beta} \Omega_m F_3^{(1,2,3)} \qquad (71)$$

This was obtained earlier by Azad et al (2014b)

In the absence of coriolis force, $\Omega_m = 0$, the transport equation for three- point distribution function in MHD turbulent flow (114) becomes

$$\frac{\partial F_3^{(1,2,3)}}{\partial t} + \left(v_\beta^{(1)} \frac{\partial}{\partial x_\beta^{(1)}} + v_\beta^{(2)} \frac{\partial}{\partial x_\beta^{(2)}} v_\beta^{(3)} \frac{\partial}{\partial x_\beta^{(3)}} \right) F_3^{(1,2,3)} + \left[g_\beta^{(1)} \left(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}} + \frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}} \right) \frac{\partial}{\partial x_\beta^{(1)}} \right.$$

$$\left. + g_\beta^{(2)} \left(\frac{\partial g_\alpha^{(2)}}{\partial v_\alpha^{(2)}} + \frac{\partial v_\alpha^{(2)}}{\partial g_\alpha^{(2)}} \right) \frac{\partial}{\partial x_\beta^{(2)}} + g_\beta^{(3)} \left(\frac{\partial g_\alpha^{(3)}}{\partial v_\alpha^{(3)}} + \frac{\partial v_\alpha^{(3)}}{\partial g_\alpha^{(3)}} \right) \frac{\partial}{\partial x_\beta^{(3)}} \right] F_3^{(1,2,3)}$$

$$+\nu\Big(\frac{\partial}{\partial v_\alpha^{(1)}}\lim_{\overline{x}^{(4)}\to\overline{x}^{(1)}}+\frac{\partial}{\partial v_\alpha^{(2)}}\lim_{\overline{x}^{(4)}\to\overline{x}^{(2)}}+\frac{\partial}{\partial v_\alpha^{(3)}}\lim_{\overline{x}^{(4)}\to\overline{x}^{(3)}}\Big)$$

$$\times\frac{\partial^2}{\partial x_\beta^{(4)}\partial x_\beta^{(4)}}\int v_\alpha^{(4)}F_4^{(1,2,3,4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)}$$

$$+\lambda\Big(\frac{\partial}{\partial g_\alpha^{(1)}}\lim_{\overline{x}^{(4)}\to\overline{x}^{(1)}}+\frac{\partial}{\partial g_\alpha^{(2)}}\lim_{\overline{x}^{(4)}\to\overline{x}^{(2)}}+\frac{\partial}{\partial g_\alpha^{(3)}}\lim_{\overline{x}^{(4)}\to\overline{x}^{(3)}}\Big)$$

$$\times\frac{\partial^2}{\partial x_\beta^{(4)}\partial x_\beta^{(4)}}\int g_\alpha^{(4)}F_4^{(1,2,3,4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)}$$

$$+\gamma\Big(\frac{\partial}{\partial \phi^{(1)}}\lim_{\overline{x}^{(4)}\to\overline{x}^{(1)}}+\frac{\partial}{\partial \phi^{(2)}}\lim_{\overline{x}^{(4)}\to\overline{x}^{(2)}}+\frac{\partial}{\partial \phi^{(3)}}\lim_{\overline{x}^{(4)}\to\overline{x}^{(3)}}\Big)$$

$$\times\frac{\partial^2}{\partial x_\beta^{(4)}\partial x_\beta^{(4)}}\int \phi^{(4)}F_4^{(1,2,3,4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)}$$

$$+D\Big(\frac{\partial}{\partial \psi^{(1)}}\lim_{\overline{x}^{(4)}\to\overline{x}^{(1)}}+\frac{\partial}{\partial \psi^{(2)}}\lim_{\overline{x}^{(4)}\to\overline{x}^{(2)}}+\frac{\partial}{\partial \psi^{(3)}}\lim_{\overline{x}^{(4)}\to\overline{x}^{(3)}}\Big)$$

$$\times\frac{\partial^2}{\partial x_\beta^{(4)}\partial x_\beta^{(4)}}\int \psi^{(4)}F_4^{(1,2,3,4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)}$$

$$-\Big[\frac{\partial}{\partial v_\alpha^{(1)}}\Big\{\frac{1}{4\pi}\int\frac{\partial}{\partial x_\alpha^{(1)}}\Big(\frac{1}{\left|\overline{x}^{(4)}-\overline{x}^{(1)}\right|}\Big)\Big\}+\frac{\partial}{\partial v_\alpha^{(2)}}\Big\{\frac{1}{4\pi}\int\frac{\partial}{\partial x_\alpha^{(2)}}\Big(\frac{1}{\left|\overline{x}^{(4)}-\overline{x}^{(2)}\right|}\Big)\Big\}$$

$$+\frac{\partial}{\partial v_\alpha^{(3)}}\Big\{\frac{1}{4\pi}\int\frac{\partial}{\partial x_\alpha^{(3)}}\Big(\frac{1}{\left|\overline{x}^{(4)}-\overline{x}^{(3)}\right|}\Big)\Big\}\times\Big(\frac{\partial v_\alpha^{(4)}}{\partial x_\beta^{(4)}}\frac{\partial v_\beta^{(4)}}{\partial x_\alpha^{(4)}}-\frac{\partial g_\alpha^{(4)}}{\partial x_\beta^{(4)}}\frac{\partial g_\beta^{(4)}}{\partial x_\alpha^{(4)}}\Big)F_4^{(1,2,3,4)}$$

$$\times dx^{(4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)}\ \Big] \tag{72}$$

It was obtained earlier by Azad et al (2014a).

Equations (113)-(115) are hierarchies of coupled equations for multi-point MHD turbulence velocity distribution functions, which derived by Lundgren (1967, 1969) and resemble with BBGKY hierarchy of equations of Ta-You (1966) in the kinetic theory of gasses.

If we drop the viscous, magnetic and thermal diffusive and concentration terms from the three point evolution equation (113), we have

$$\frac{\partial F_3^{(1,2,3)}}{\partial t}+\Big(v_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}+v_\beta^{(2)}\frac{\partial}{\partial x_\beta^{(2)}}v_\beta^{(3)}\frac{\partial}{\partial x_\beta^{(3)}}\Big)F_3^{(1,2,3)}+\Big[g_\beta^{(1)}\Big(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}\Big)\frac{\partial}{\partial x_\beta^{(1)}}$$

$$+g_\beta^{(2)}\Big(\frac{\partial g_\alpha^{(2)}}{\partial v_\alpha^{(2)}}+\frac{\partial v_\alpha^{(2)}}{\partial g_\alpha^{(2)}}\Big)\frac{\partial}{\partial x_\beta^{(2)}}+g_\beta^{(3)}\Big(\frac{\partial g_\alpha^{(3)}}{\partial v_\alpha^{(3)}}+\frac{\partial v_\alpha^{(3)}}{\partial g_\alpha^{(3)}}\Big)\frac{\partial}{\partial x_\beta^{(3)}}\Big]F_3^{(1,2,3)}$$

$$-\Big[\frac{\partial}{\partial v_\alpha^{(1)}}\Big\{\frac{1}{4\pi}\int\frac{\partial}{\partial x_\alpha^{(1)}}\Big(\frac{1}{\left|\bar{x}^{(4)}-\bar{x}^{(1)}\right|}\Big)\Big\}+\frac{\partial}{\partial v_\alpha^{(2)}}\Big\{\frac{1}{4\pi}\int\frac{\partial}{\partial x_\alpha^{(2)}}\Big(\frac{1}{\left|\bar{x}^{(4)}-\bar{x}^{(2)}\right|}\Big)\Big\}$$

$$+\frac{\partial}{\partial v_\alpha^{(3)}}\Big\{\frac{1}{4\pi}\int\frac{\partial}{\partial x_\alpha^{(3)}}\Big(\frac{1}{\left|\bar{x}^{(4)}-\bar{x}^{(3)}\right|}\Big)\Big\}\times\Big(\frac{\partial v_\alpha^{(4)}}{\partial x_\beta^{(4)}}\frac{\partial v_\beta^{(4)}}{\partial x_\alpha^{(4)}}-\frac{\partial g_\alpha^{(4)}}{\partial x_\beta^{(4)}}\frac{\partial g_\beta^{(4)}}{\partial x_\alpha^{(4)}}\Big)F_4^{(1,2,3,4)} \tag{73}$$

$$\times dx^{(4)}dv^{(4)}dg^{(4)}d\phi^{(4)}d\psi^{(4)}\Big]=0$$

The existence of the term

$$\Big(\frac{\partial g_\alpha^{(1)}}{\partial v_\alpha^{(1)}}+\frac{\partial v_\alpha^{(1)}}{\partial g_\alpha^{(1)}}\Big),\ \Big(\frac{\partial g_\alpha^{(2)}}{\partial v_\alpha^{(2)}}+\frac{\partial v_\alpha^{(2)}}{\partial g_\alpha^{(2)}}\Big)\text{ and }\Big(\frac{\partial g_\alpha^{(3)}}{\partial v_\alpha^{(3)}}+\frac{\partial v_\alpha^{(3)}}{\partial g_\alpha^{(3)}}\Big)$$

can be explained on the basis that two characteristics of the flow field are related to each other and describe the interaction between the two modes (velocity and magnetic) at point $x^{(1)}$, $x^{(2)}$ and $x^{(3)}$.

We can exhibit an analogy of this equation with the 1st equation in BBGKY hierarchy in the kinetic theory of gases. The first equation of BBGKY hierarchy is given Lundgren (1969) as

$$\frac{\partial F_1^{(1)}}{\partial t}+\frac{1}{m}v_\beta^{(1)}\frac{\partial}{\partial x_\beta^{(1)}}F_1^{(1)}=n\iint\frac{\partial\psi_{1,2}}{\partial x_\alpha^{(1)}}\frac{\partial F_2^{(1,2)}}{\partial v_\alpha^{(1)}}d\bar{x}^{(2)}d\bar{v}^{(2)} \tag{74}$$

where $\psi_{1,2}=\psi\left|v_\alpha^{(2)}-v_\alpha^{(1)}\right|$ is the inter molecular potential.

Some approximations are required, if we want to close the system of equations for the distribution functions. In the case of collection of ionized particles, i.e. in plasma turbulence, it can be provided closure form easily by decomposing $F_2^{(1,2)}$ as $F_1^{(1)}F_1^{(2)}$. But it will be possible if there is no interaction or correlation between two particles. If we decompose $F_2^{(1,2)}$ as

$F_2^{(1,2)}=(1+\in)F_1^{(1)}F_1^{(2)}$

and

$F_3^{(1,2,3)}=(1+\in)^2F_1^{(1)}F_1^{(2)}F_1^{(3)}$

Also

$F_4^{(1,2,3,4)}=(1+\in)^3F_1^{(1)}F_1^{(2)}F_1^{(3)}F_1^{(4)}$

where $\in$ is the correlation coefficient between the particles. If there is no correlation between the particles, $\in$ will be zero and distribution function can be decomposed in usual way. Here we are considering such type of approximation only to provide closed from of the equation.

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The Two - Variable $(\frac{G'}{G},\frac{1}{G})$ - Expansion Method for Solving Nonlinear Dynamics of Microtubles - A New Model

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Abstract- In this paper, we employ the ($\frac{G'}{G},\frac{1}{G}$)-expansion method to find the exact traveling wave solutions involving parameters of nonlinear dynamics of microtubulesa New Model . When these parameters are taken to be special values, the solitary wave solutions are derived from the exact traveling wave solutions. It is shown that the proposed method provides a more powerful mathematical tool for constructing exact traveling wave solutions for many other nonlinear evolution equations.

Keywords: The $(\frac{G'}{G},\frac{1}{G})$*-expansion method; Nonlinear dynamics of microtubules; Traveling wave solutions; Solitary wave solutions; Kink-anti kink shaped.*

GJSFR-A Classification : *FOR Code: 35A05, 35A20, 65K99, 65Z05, 76R50, 70K70*

THETWOVARIABLEEXPANSIONMETHODFORSOLVINGNONLINEARDYNAMICSOFMICROTUBLESANEWMODEL

Strictly as per the compliance and regulations of :

The Two - Variable $(\frac{G'}{G}, \frac{1}{G})$ - Expansion Method for Solving Nonlinear Dynamics of Microtubles - A New Model

Emad H. M. Zahran [α] & Mostafa M. A. Khater [σ]

Abstract- In this paper, we employ the ($\frac{G'}{G},\frac{1}{G}$)-expansion method to find the exact traveling wave solutions involving parameters of nonlinear dynamics of microtubulesa New Model. When these parameters are taken to be special values, the solitary wave solutions are derived from the exact traveling wave solutions. It is shown that the proposed method provides a more powerful mathematical tool for constructing exact traveling wave solutions for many other nonlinear evolution equations.

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I. Introduction

Many models in mathematics and physics are described by nonlinear differential equations. Nowadays, research in physics devotes much attention to nonlinear partial differential evolution model equations, appearing in various fields of science, especially fluid mechanics, solid-state physics, plasma physics, and nonlinear optics. Large varieties of physical, chemical, and biological phenomena are governed by nonlinear partial differential equations. One of the most exciting advances of nonlinear science and theoretical physics has been the development of methods to look for exact solutions of nonlinear partial differential equations. Exact solutions to nonlinear partial differential equations play an important role in nonlinear science, especially in nonlinear physical science since they can provide much physical information and more insight into the physical aspects of the problem and thus lead to further applications. Nonlinear wave phenomena of dispersion, dissipation, diffusion, reaction and convection are very important in nonlinear wave equations. In recent years, quite a few methods for obtaining explicit traveling and solitary wave solutions of nonlinear evolution equations have been proposed. Such methods are tanh - sech method [1]-[3], extended tanh - method [4]-[6], sine - cosine method [7]-[9], homogeneous balance method [10, 11], F-expansion method [12]-[14], exp-function method [15, 16], trigonometric function series method [17], ($\frac{G'}{G}$) expansion method [18]-[21], Jacobi elliptic function method [22]-[25], The ($\frac{G'}{G},\frac{1}{G}$)-expansion method [26]-[28] and so on.

The objective of this article is to apply. The ($\frac{G'}{G},\frac{1}{G}$)- expansion method for finding the exact traveling wave solution of Nonlinear dynamics of microtubules- a new model which play an important role in biology and mathematical physics.

The rest of this paper is organized as follows: In Section 2, we give the description of The ($\frac{G'}{G},\frac{1}{G}$)-expansion method In Section 3, we use this method to find the exact solutions of the nonlinear evolution equations pointed out above. In Section 4, conclusions are given.

II. Description of Method

Before, we describe the main steps of this method,we need the following remarks [26]-[28]:

Remark 1. If we consider the second order linear ODE:

$$G''(\xi) + \lambda G(\xi) = \mu, \tag{2.1}$$

and set $\phi = \frac{G'}{G}$, $\psi = \frac{1}{G}$, then we get

$$\phi' = -\phi^2 + \mu\psi - \lambda, \quad \psi' = -\phi\psi. \tag{2.2}$$

Remark 2. If $\lambda < 0$, then the general solutions of Eq.(2.1) has the form :

$$G(\xi) = A_1 \, sinh(\xi\sqrt{-\lambda}) + A_2 \, cosh(\xi\sqrt{-\lambda}) + \frac{\mu}{\lambda}, \tag{2.3}$$

where A_1 and A_2 are arbitrary constants. Consequently, we have

$$\psi^2 = \frac{-\lambda}{\lambda^2\sigma + \mu^2}(\phi^2 - 2\mu\psi + \lambda) \tag{2.4}$$

where $\sigma = A_1^2 - A_2^2$.

Remark 3. If $\lambda > 0$, then the general solutions of Eq.(2.1) has the form:

$$G(\xi) = \frac{\mu}{2}\xi^2 + A_1\xi + A_2, \tag{2.5}$$

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and hence

$$\psi^2 = \frac{1}{A_1^2 - 2\mu A_2}(\phi^2 - 2\mu\psi). \qquad (2.6)$$

where $\sigma = A_1^2 + A_2^2$.

Remark 4. If $\lambda = 0$, then the general solution of Eq.(2.1) has the form:

$$G(\xi) = \frac{\mu}{2}\xi^2 + A_1\xi + A_2, \qquad (2.7)$$

and hence

$$\psi^2 = \frac{1}{A_1^2 - 2\mu A_2}(\phi^2 - 2\mu\psi). \qquad (2.8)$$

Suppose we have the following NLPDEs in the form:

$$F(u, u_t, u_x, u_y, u_z, u_{xx}, ...) = 0, \qquad (2.9)$$

where F is a polynomial in *u(x; y; z; t)* and its partial derivatives. In the following, we give the main steps of the $(\frac{G'}{G},\frac{1}{G})$-expansion method [26]-[28]:

Step 1. The traveling wave transformation

$$u(x,y,z,t) = u(\xi), \qquad \xi = x + y + z - wt, \quad (2.10)$$

where w is a constant, reduces Eq.(2.9) to an ODE in the form:

$$P(u, u', u'', u''', ...) = 0, \qquad (2.11)$$

where P is a polynomial of $u(\xi)$ and its total derivatives with respect to ξ.

Step 2. Assuming that the solution of Eq.(2.11) can be expressed by a polynomial in the two variables ϕ and ψ as follows:

$$u(\xi) = \sum_{i=0}^{N} a_i\phi^i + \sum_{i=1}^{N} b_i\phi^{i-1}\psi, \qquad (2.12)$$

where $a_i\,(0,1,2,....,N)$ and $b_i\ (i = 1,2,3,...,N)$ are constants to be determined later.

Step 3. Determine the positive integer N in Eq.(2.12) by using the homogeneous balance between the highest-order derivatives and the nonlinear terms in Eq.(2.11).

Step 4. Substitute Eq.(2.12) into Eq.(2.11) along with Eq.(2.2) and Eq.(2.4), the left-hand side Eq.(2.11) can be converted into a polynomial in ϕ and ψ, in which the degree of ψ is not longer than 1. Equating each coefficients of this polynomial to zero, yields a system of algebraic equation which can be solved by using the Maple or Mathematica to get the values of a_i, b_i, w, μ, A_1, A_2 and λ where $\lambda < 0$.

Step 5. Similar to step 4, subsitute Eq.(2.12) into Eq.(2.11) along with Eq.(2.2) and Eq.(2.6) for $\lambda > 0$,(or Eq.(2.2) and Eq.(2.8) for $\lambda = 0$), we obtain the exact solutions of Eq.(2.11) expressed by trigonometric functions(or by rational functions) respectively.

III. Application

Here, we will apply the $(\frac{G'}{G},\frac{1}{G})$-expansion method described in sec.2 to find the exact traveling wave solutions and then the solitary wave solutions for Nonlinear dynamics of microtubules- a new model [29].

The starting point of the present modelling is the fact that the bonds between dimers within the same PF are significantly stronger than the soft bonds between neighbouring (parallel protofilaments) PFs. This implies that the longitudinal displacements of pertaining dimers in a single PF should cause the longitudinal wave propagating along PF. The averaged impact of soft bonds with collateral PFs is taken to be described by the nonlinear double-well potential.

The present model assumes only one degree of freedom per dimer. This is z_n, a longitudinal displacement of a dimer at a position n.

The Hamiltonian for one PF is represented as

$$H = \sum_n \left[\frac{m}{2}\dot{z}_n^2 + \frac{k}{2}\left(z_{n+1} - z_2\right)^2 + V\left(z_n\right)\right], \quad (3.1)$$

where dot means the first derivative with respect to time, is a mass of the dimer and is a harmonic constant describing the nearest neighbour interaction between the dimers belonging to the same PF. The first term represents a kinetic energy of the dimer, the second one, which we call harmonic energy, is a potential energy of the chemical interaction between the neighbouring dimers belonging to the same PF and the last term is the combined potential

$$V\left(z_n\right) = -Cz_n - \frac{1}{2}Az_n^2 + \frac{1}{4}Bz_n^4, \;\; C = qE, \quad (3.2)$$

where E is the magnitude of the intrinsic electric field and *q* represents the excess charge within the dipole. It is assumed that *q* > 0 and *E* > 0. One can recognize an energy of the dimer in the intrinsic electric field *E* at the site n and the well known double-well potential with positive parameters *A* and *B* that should be estimated. The Hamiltonian given by before equations is rather common in physics. The first attempt to use it in nonlinear dynamics of (microtubules) MTs was done almost 20 years ago. To be more precise, the Hamiltonian in [30] would be obtained from before equations if z_n were replaced by u_n Hence, we refer to these two models as u-model and z-model. However, the meanings of u_n in [30] and in the present paper are completely different. The u-model assumes an angular degree of freedom, while the coordinate u_n is a projection of the top of the dimer on the direction of PF. On the other hand, the coordinate z_n is a real displacement of the dimer along x axis. This will be further elaborated later.

Using generalized coordinates z_n and $m\dot{z}_n$ and assuming a continuum approximation $z_n(t) \to z(x,t)$, we straightforwardly obtain the following nonlinear dynamical equation of motion

$$m\frac{\partial^2 z}{\partial t^2} - kl^2\frac{\partial^2 z}{\partial x^2} - qE - Az + Bz^3 + \gamma\frac{\partial z}{\partial t} \quad (3.3)$$

The last term represents a viscosity force with γ being a viscosity coefficient. It is well known that, for a given wave equation, a traveling wave $z(\xi)$ is a solution which depends upon x and t only through a unified variable $\xi = \kappa x - \omega t$, where κ and ω are constants. This allows us to obtain the final dimensionless ordinary differential equation

$$\alpha u'' - \rho u' - u + u^3 - \eta = 0, \quad (3.4)$$

where

$$u' = \frac{du}{d\xi},\ \alpha = \frac{m\omega^2 - kl^2\kappa^2}{A},\ z = \sqrt{\frac{A}{B}}u,\ \rho = \frac{\gamma\omega}{A}\ and\, \eta = \frac{qE}{A\sqrt{\frac{A}{B}}}.$$

Balancing between u'' and u^3, we get $(n+2 = 2n) \Rightarrow (n = 1)$.So that, we assume the solution of Eq.(3.4) by using (2.4), we get:

$$u = a_0 + a_1\phi(\xi) + b_1 \quad (\xi). \quad (3.5)$$

where a_0, a_1 and b_1 are constants to be determined later.

There are three cases to be discussed as follows:

Case 1. Hyperbolic function solutions ($\lambda < 0$).

When ($\lambda < 0$), substituting Eq.(3.5) and its derivative into Eq.(3.4) and using Eq.(2.2) and Eq.(2.4), the left-hand side of Eq.(3.4) becomes a polynomial in ϕ and ψ. Setting the coefficients of this polynomial to zero yields a system of algebraic equations in a_0, a_1, b_1, α, η, μ, σ and λ as follows:

$$2\,\alpha\,a_1 + a_1{}^3 - 3\,\frac{a_1 b_1{}^2\lambda}{\lambda^2\sigma+\mu^2} = 0, \quad (3.6)$$

$$2\,\alpha\,b_1 + 3\,a_1{}^2 b_1 - \frac{b_1{}^3\lambda}{\lambda^2\sigma+\mu^2} = 0, \quad (3.7)$$

$$\frac{\alpha\,b_1\mu\,\lambda}{\lambda^2\sigma+\mu^2} + \rho\,a_1 - 3\,\frac{a_0 b_1{}^2\lambda}{\lambda^2\sigma+\mu^2} - 2\,\frac{b_1{}^3\lambda^2\mu}{(\lambda^2\sigma+\mu^2)^2} + 3\,a_0 a_1{}^2 = 0, \quad (3.8)$$

$$-3\,\alpha\,a_1\mu + \rho\,b_1 + 6\,a_0 a_1 b_1 + 6\,\frac{a_1 b_1{}^2\lambda\,\mu}{\lambda^2\sigma+\mu^2} = 0, \quad (3.9)$$

$$2\,\alpha\,a_1\lambda - a_1 + 3\,a_0{}^2 a_1 - 3\,\frac{a_1 b_1{}^2\lambda^2}{\lambda^2\sigma+\mu^2} = 0, \quad (3.10)$$

$$\alpha\left(-2\,\frac{b_1\mu^2\lambda}{\lambda^2\sigma+\mu^2} + b_1\lambda\right) - \rho\,a_1\mu - b_1 + 3\,a_0{}^2 b_1 + 6\,\frac{a_0 b_1{}^2\lambda\,\mu}{\lambda^2\sigma+\mu^2} + 4\,\frac{b_1{}^3\lambda^2\mu^2}{(\lambda^2\sigma+\mu^2)^2} - \frac{b_1{}^3\lambda^2}{\lambda^2\sigma+\mu^2} = 0, \quad (3.11)$$

$$\frac{\alpha\,b_1\mu\,\lambda^2}{\lambda^2\sigma+\mu^2} + \rho\,a_1\lambda - a_0 + a_0{}^3 - 3\,\frac{a_0 b_1{}^2\lambda^2}{\lambda^2\sigma+\mu^2} - 2\,\frac{b_1{}^3\lambda^3\mu}{(\lambda^2\sigma+\mu^2)^2} - \eta = 0. \quad (3.12)$$

Solving above system of algebraic equations by the Maple or Mathematica, we get the following results.

$$\eta = -8\,a_0{}^3 + 2\,a_0,\ \mu = \frac{\pm\sqrt{-9\,\sigma\,a_0{}^4 - 3\,a_0{}^2 b_1{}^2 + 6\,\sigma\,a_0{}^2 + b_1{}^2 - \sigma}}{a_1^2},$$

$$\lambda = \frac{3\,a_0{}^2 - 1}{a_1{}^2},\ a_0 = \frac{-\rho}{6a_1},\ a_1 = \pm\sqrt{\frac{-\alpha}{2}},\ b_1 = b_1.$$

Substituting these solutions into Eq.(3.5), using Eq.(2.2) and Eq.(2.4), we obtain traveling wave solution of Eq.(3.3) as follows:

$$u(\xi) = \frac{-\rho}{6a_1} \pm \sqrt{\frac{\alpha\lambda}{2}}\left[\frac{A_1\cosh\left((\xi)\sqrt{-\lambda}\right) + A_2\sinh\left((\xi)\sqrt{-\lambda}\right)}{A_1\sinh\left((\xi)\sqrt{-\lambda}\right) + A_2\cosh\left((\xi)\sqrt{-\lambda}\right) + \frac{\mu}{\lambda}}\right]$$

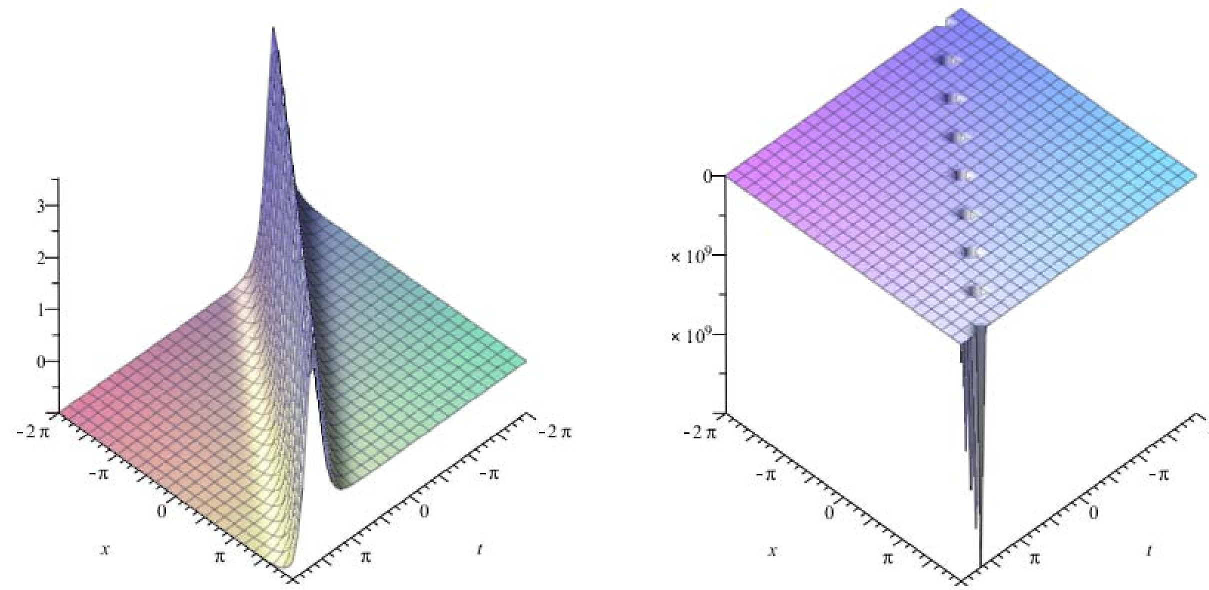
$$+\left[\frac{b_1}{A_1\sinh\left((\xi)\ \sqrt{-\lambda}\right)+A_2\cosh\left((\xi)\ \sqrt{-\lambda}\right)+\frac{\mu}{\lambda}}\right]. \tag{3.13}$$

In particular if we set $A_1=0$, $A_2>0$ and $\mu=0$ in Eq.(3.13) then we have the solitary solution

$$u(\xi)=\frac{-\rho}{6a_1}\pm\sqrt{\frac{\alpha\lambda}{2}}\,tanh\left(\xi\sqrt{-\lambda}\right)+b_1\,sech\left(\xi\sqrt{-\lambda}\right). \tag{3.14}$$

but, if we set $A_2=0$, $A_1>0$ and $\mu=0$ in Eq.(3.13) then we have the solitary solution

$$u(\xi)=\frac{-\rho}{6a_1}\pm\sqrt{\frac{\alpha\lambda}{2}}\,coth\left(\xi\sqrt{-\lambda}\right)+b_1\,csch\left(\xi\sqrt{-\lambda}\right). \tag{3.15}$$



(a)
Eq.(3.14)

(b)
Eq.(3.15)

Figure 1 : Kink Singular Shaped Soliton solution of Eqs.(3.12) and (3.13). When $\alpha=-2,\ a_1=1,\ \rho=3,\ a_0=\frac{-1}{2},$ $b_1=4,\ \lambda=\frac{-1}{4},\ \kappa=5,\ \omega=6$

Case 2. Trigonometric function solutions ($\lambda>0$).

When ($\lambda>0$), substituting Eq.(3.5) and its derivative into Eq.(3.4) and using Eq.(2.2) andEq.(2.6), the left-hand side of Eq.(3.4) becomes a polynomial in ϕ and ψ. Setting the coefficients of this polynomial to zero yields a system of algebraic equations in $a_0,\ a_1,\ b_1,$ $\alpha,\ \eta,\ \mu,\ \sigma$ and λ as follows:

$$2\,\alpha\,a_1+{a_1}^3+3\,\frac{a_1{b_1}^2\lambda}{\lambda^2\sigma-\mu^2}=0, \tag{3.16}$$

$$2\,\alpha\,b_1+3\,{a_1}^2b_1+\frac{{b_1}^3\lambda}{\lambda^2\sigma-\mu^2}=0, \tag{3.17}$$

$$-\frac{\alpha\,b_1\mu\,\lambda}{\lambda^2\sigma-\mu^2}+\rho\,a_1+3\,\frac{a_0{b_1}^2\lambda}{\lambda^2\sigma-\mu^2}-2\,\frac{{b_1}^3\lambda^2\mu}{(\lambda^2\sigma-\mu^2)^2}+3\,a_0{a_1}^2=0, \tag{3.18}$$

$$-3\,\alpha\,a_1\mu+\rho\,b_1+6\,a_0a_1b_1-6\,\frac{a_1{b_1}^2\lambda\,\mu}{\lambda^2\sigma-\mu^2}=0, \tag{3.19}$$

$$2\,\alpha\,a_1\lambda-a_1+3\,{a_0}^2a_1+3\,\frac{a_1{b_1}^2\lambda^2}{\lambda^2\sigma-\mu^2}=0, \tag{3.20}$$

$$\alpha \left(2\,\frac{b_1\mu^2\lambda}{\lambda^2\sigma-\mu^2}+b_1\lambda\right)-\rho\,a_1\mu-b_1+3\,{a_0}^2b_1-6\,\frac{a_0{b_1}^2\lambda\,\mu}{\lambda^2\sigma-\mu^2}+4\,\frac{{b_1}^3\lambda^2\mu^2}{(\lambda^2\sigma-\mu^2)^2}+\frac{{b_1}^3\lambda^2}{\lambda^2\sigma-\mu^2}=0, \tag{3.21}$$

$$-\frac{\alpha\,b_1\mu\,\lambda^2}{\lambda^2\sigma-\mu^2}+\rho\,a_1\lambda-a_0+{a_0}^3+3\,\frac{a_0{b_1}^2\lambda^2}{\lambda^2\sigma-\mu^2}-2\,\frac{{b_1}^3\lambda^3\mu}{(\lambda^2\sigma-\mu^2)^2}-\eta=0. \tag{3.22}$$

Solving above system of algebraic equations by the Maple or Mathematica, we get the following results.

$$\eta=-8\,{a_0}^3+2\,a_0,\ \mu=\frac{\pm\sqrt{9\,\sigma\,{a_0}^4-3\,{a_0}^2{b_1}^2-6\,\sigma\,{a_0}^2+{b_1}^2+\sigma}}{a_1^2},$$

$$\lambda=\frac{3\,{a_0}^2-1}{{a_1}^2},\ a_0=\frac{-\rho}{6a_1},\ a_1=\pm\sqrt{\frac{-\alpha}{2}},\ b_1=b_1.$$

Substituting these solutions into Eq.(3.5), using Eq.(2.3) and Eq.(2.5), we obtain traveling wave solution of Eq.(3.3) as follows:

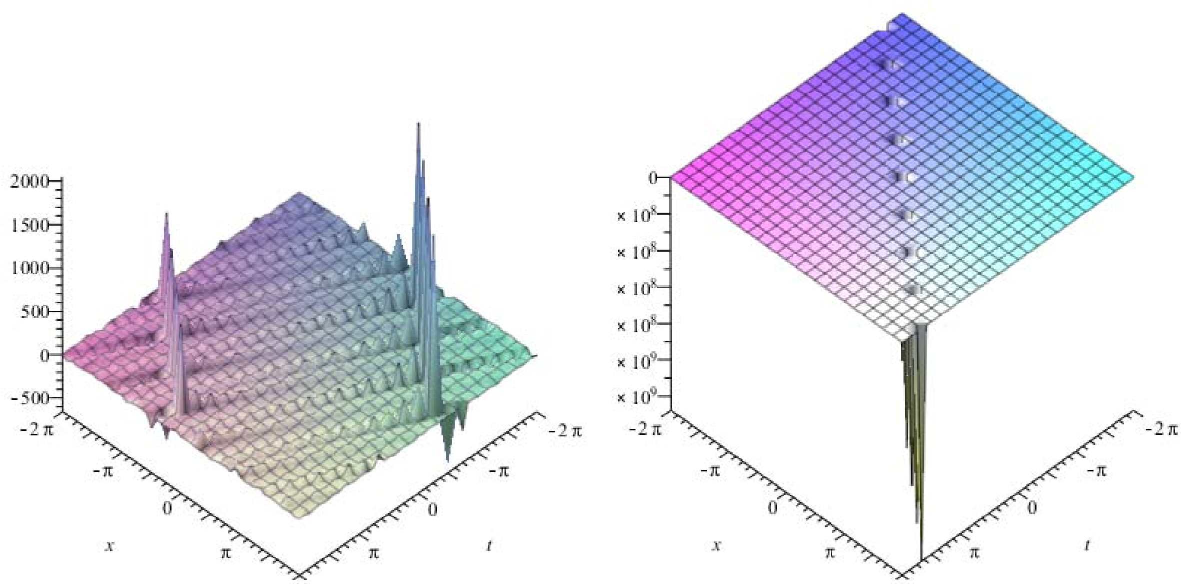
$$\begin{aligned} u\,(\xi)=\tfrac{-\rho}{6a_1}\pm\sqrt{\tfrac{-\alpha\lambda}{2}}&\left[\frac{A_1\cos\left((\xi)\,\sqrt{\lambda}\right)-A_2\sin\left((\xi)\,\sqrt{\lambda}\right)}{A_1\sin\left((\xi)\,\sqrt{\lambda}\right)+A_2\cos\left((\xi)\,\sqrt{\lambda}\right)+\frac{\mu}{\lambda}}\right]\\ +&\left[\frac{b_1}{A_1\sinh\left((\xi)\,\sqrt{\lambda}\right)+A_2\cos\left((\xi)\,\sqrt{\lambda}\right)+\frac{\mu}{\lambda}}\right]. \end{aligned} \tag{3.23}$$

In particular if we set $A_1 = 0$, $A_2 > 0$ and $\mu = 0$ in Eq.(3.23) then we have the solitary solution

$$u\,(\xi)=\tfrac{-\rho}{6a_1}\pm\sqrt{\tfrac{-\alpha\lambda}{2}}\,tanh\left(\xi\sqrt{\lambda}\right)+b_1\,sec\left(\xi\sqrt{\lambda}\right). \tag{3.24}$$

but, if we set $A_2 = 0$, $A_1 > 0$ and $\mu = 0$ in Eq.(3.23) then we have the solitary solution

$$u\,(\xi)=\tfrac{-\rho}{6a_1}\pm\sqrt{\tfrac{-\alpha\lambda}{2}}\,cot\left(\xi\sqrt{\lambda}\right)+b_1\,csc\left(\xi\sqrt{\lambda}\right). \tag{3.25}$$



(a) Eq.(3.24)

(b) Eq.(3.25)

Figure 2 : Kink Singular Shaped Soliton solution of Eqs.(3.14) and (3.15). When $\alpha=-2,\ a_1=1,\ \rho=6$, $a_0=-1,\ b_1=4,\ \lambda=2,\ \kappa=5,\ \omega=6$

Case 3. Rational function solutions ($\lambda = 0$).

When ($\lambda = 0$), substituting Eq.(3.5) and its derivative into Eq.(3.4) and using Eq.(2.2) and Eq.(2.8), the left-hand side of Eq.(3.4) becomes a polynomial in ϕ and ψ. Setting the coeficients of this polynomial to zero yields a system of algebraic equations in a_0, a_1, b_1, α, η, μ, σ and λ as follows:

$$2\,\alpha\,a_1 + {a_1}^3 + 3\,\frac{a_1{b_1}^2}{-2\,\mu\,A_2 + {A_1}^2} = 0, \tag{3.26}$$

$$2\,\alpha\,b_1 + 3\,{a_1}^2 b_1 + \frac{{b_1}^3}{-2\,\mu\,A_2 + {A_1}^2} = 0, \tag{3.27}$$

$$-\frac{\alpha\,b_1\mu}{-2\,\mu\,A_2 + {A_1}^2} + \rho\,a_1 + 3\,a_0{a_1}^2 + 3\,\frac{a_0{b_1}^2}{-2\,\mu\,A_2 + {A_1}^2} - 2\,\frac{{b_1}^3\mu}{\left(-2\,\mu\,A_2 + {A_1}^2\right)^2} = 0, \tag{3.28}$$

$$-3\,\alpha\,a_1\mu + \rho\,b_1 + 6\,a_0a_1b_1 - 6\,\frac{a_1{b_1}^2\mu}{-2\,\mu\,A_2 + {A_1}^2} = 0, \tag{3.29}$$

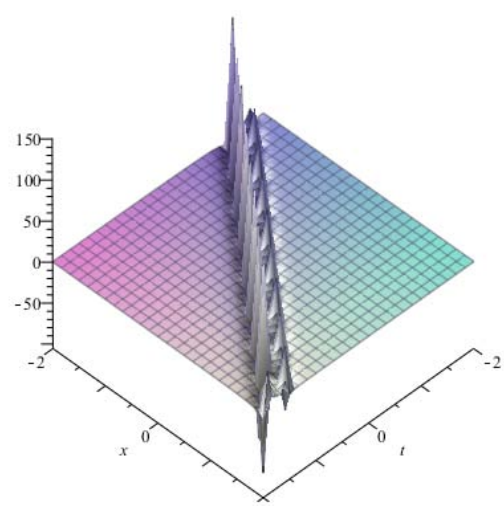
$$3\,{a_0}^2a_1 - a_1 = 0, \tag{3.30}$$

$$2\,\frac{\alpha\,b_1\mu^2}{-2\,\mu\,A_2 + {A_1}^2} - \rho\,a_1\mu - b_1 + 3\,{a_0}^2b_1 - 6\,\frac{a_0{b_1}^2\mu}{-2\,\mu\,A_2 + {A_1}^2} + 4\,\frac{{b_1}^3\mu^2}{\left(-2\,\mu\,A_2 + {A_1}^2\right)^2} = 0, \tag{3.31}$$

$${a_0}^3 - \eta - a_0 = 0. \tag{3.32}$$

Solving above system of algebraic equations by the Maple or Mathematica, we get the following results.

$$\eta = \pm\frac{2\sqrt{3}}{9},\ \alpha = \frac{-\rho^2}{6},\ a_0 = \mp\sqrt{\frac{1}{3}},\ a_1 = \pm\frac{\rho\sqrt{3}}{6},\ b_1 = \pm\frac{\rho}{3}\sqrt{-6\,\mu\,A_2 + 3\,{A_1}^2}.$$



(a)
Eq.(3.33)

Figure 3 : Kink Singular Shaped Soliton solution of Eqs.(3.33). When $a_0 = \sqrt{\frac{1}{3}}$, $\rho = 6$, $a_1 = \sqrt{3}$, $b_1 = 2\sqrt{15}$, $\mu = -2$, $A_1 = -1$, $A_2 = 1$, $\lambda = \frac{-1}{4}$, $\kappa = 5$, $\omega = 6$

Substituting these solutions into Eq.(3.5), using Eq.(2.3) and Eq.(2.6), we obtain traveling wave solution of Eq.(3.3) as follows:

$$u\left(\xi\right) = \frac{-\rho}{6a_1} \pm \sqrt{\frac{-\alpha}{2}}\left[\frac{\mu\,\xi + A_1}{\frac{\mu}{2}\,\xi^2 + A_1\xi + A_2}\right] + \left[\frac{b_1}{\frac{\mu}{2}\,\xi^2 + A_1\xi + A_2}\right]. \tag{3.33}$$

- *Remark:* All the obtained results have been checked with Maple 16 by putting them back into the original equation and found correct.

IV. Conclusion

The $(\frac{G'}{G},\frac{1}{G})$ expansion method has been applied in this paper to find the exact traveling wave solutions and then the solitary wave solutions of two nonlinear evolution equations, namely, Nonlinear dynamics of microtubules - A new model. Let us compare between our results obtained in the present article with the well-known results obtained by other authors using different methods as follows: Our results of Nonlinear dynamics of microtubules - a new model and The Kundu- Eckhaus equation are new and different from those obtained in [29], [30], [31] and fig. 1, 2 and 3 show the solitary traveling wave solution of Nonlinear dynamics of microtubules - a new model. We can conclude that the $(\frac{G'}{G},\frac{1}{G})$- expansion method is is a very powerful and efficient technique in _nding exact solutions for wide classes of nonlinear problems and can be applied to many other nonlinear evolution equations in mathematical physics. Another possible merit is that the reliability of the method and the reduction in the size of computational domain give this method a wider applicability.

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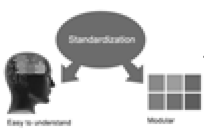

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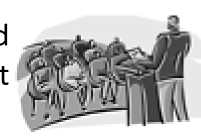
The IBOARS can initially review research papers of their institute and recommend them to publish with respective journal of Global Journals. It can also review the papers of other institutions after obtaining our consent. The second review will be done by peer reviewer of Global Journals Incorporation (USA) The Board is at liberty to appoint a peer reviewer with the approval of chairperson after consulting us.
The author fees of such paper may be waived off up to 40%.



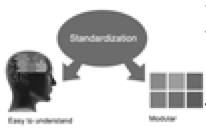
The Global Journals Incorporation (USA) at its discretion can also refer double blind peer reviewed paper at their end to the board for the verification and to get recommendation for final stage of acceptance of publication.

The IBOARS can organize symposium/seminar/conference in their country on behalf of Global Journals Incorporation (USA)-OARS (USA). The terms and conditions can be discussed separately.

The Board can also play vital role by exploring and giving valuable suggestions regarding the Standards of “Open Association of Research Society, U.S.A (OARS)” so that proper amendment can take place for the benefit of entire research community. We shall provide details of particular standard only on receipt of request from the Board.

The board members can also join us as Individual Fellow with 40% discount on total fees applicable to Individual Fellow. They will be entitled to avail all the benefits as declared. Please visit Individual Fellow-sub menu of GlobalJournals.org to have more relevant details.

We shall provide you intimation regarding launching of e-version of journal of your stream time to time. This may be utilized in your library for the enrichment of knowledge of your students as well as it can also be helpful for the concerned faculty members.



After nomination of your institution as "Institutional Fellow" and constantly functioning successfully for one year, we can consider giving recognition to your institute to function as Regional/Zonal office on our behalf.

The board can also take up the additional allied activities for betterment after our consultation.

The following entitlements are applicable to individual Fellows:

Open Association of Research Society, U.S.A (OARS) By-laws states that an individual Fellow may use the designations as applicable, or the corresponding initials. The Credentials of individual Fellow and Associate designations signify that the individual has gained knowledge of the fundamental concepts. One is magnanimous and proficient in an expertise course covering the professional code of conduct, and follows recognized standards of practice.

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We shall provide print version of 12 issues of any three journals [as per your requirement] out of our 38 journals worth $ 2376 USD.

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- The professional accredited with Fellow honor, is entitled to various benefits viz. name, fame, honor, regular flow of income, secured bright future, social status etc.

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- The Fellow can organize symposium/seminar/conference on behalf of Global Journals Incorporation (USA) and he/she can also attend the same organized by other institutes on behalf of Global Journals.
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- • This individual has learned the basic methods of applying those concepts and techniques to common challenging situations. This individual has further demonstrated an in–depth understanding of the application of suitable techniques to a particular area of research practice.

Note :

"

- In future, if the board feels the necessity to change any board member, the same can be done with the consent of the chairperson along with anyone board member without our approval.
- In case, the chairperson needs to be replaced then consent of 2/3rd board members are required and they are also required to jointly pass the resolution copy of which should be sent to us. In such case, it will be compulsory to obtain our approval before replacement.
- In case of "Difference of Opinion [if any]" among the Board members, our decision will be final and binding to everyone.

"

Process of submission of Research Paper

The Area or field of specialization may or may not be of any category as mentioned in 'Scope of Journal' menu of the GlobalJournals.org website. There are 37 Research Journal categorized with Six parental Journals GJCST, GJMR, GJRE, GJMBR, GJSFR, GJHSS. For Authors should prefer the mentioned categories. There are three widely used systems UDC, DDC and LCC. The details are available as 'Knowledge Abstract' at Home page. The major advantage of this coding is that, the research work will be exposed to and shared with all over the world as we are being abstracted and indexed worldwide.

The paper should be in proper format. The format can be downloaded from first page of 'Author Guideline' Menu. The Author is expected to follow the general rules as mentioned in this menu. The paper should be written in MS-Word Format (*.DOC,*.DOCX).

The Author can submit the paper either online or offline. The authors should prefer online submission.Online Submission: There are three ways to submit your paper:

(A) (I) First, register yourself using top right corner of Home page then Login. If you are already registered, then login using your username and password.

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1. General,

2. Ethical Guidelines,

3. Submission of Manuscripts,

4. Manuscript's Category,

5. Structure and Format of Manuscript,

6. After Acceptance.

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The recommended size of original research paper is less than seven thousand words, review papers fewer than seven thousands words also.Preparation of research paper or how to write research paper, are major hurdle, while writing manuscript. The research articles and research letters should be fewer than three thousand words, the structure original research paper; sometime review paper should be as follows:

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(g) Discussion should cover the implications and consequences, not just recapitulating the results; conclusions should be summarizing.

(h) Brief Acknowledgements.

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It is vital, that authors take care in submitting a manuscript that is written in simple language and adheres to published guidelines.

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24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

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27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

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30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

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32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page
- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.

- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

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	A-B	**C-D**	**E-F**
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Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

Index

B

Bent · 62, 68
Bizarre · 62

C

Catastrophe · 67

D

Descartes. · 5

F

Foucault. · 1

I

Intrinsic · 63, 123
Invariant · 1, 4

P

Possesses · 62, 63, 64, 65, 66, 68

S

Stepping · 7

T

Tremendous · 63, 64
Turbulent · 16, 59, 60, 70, 120, 121

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