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New Solitary Wave Solution

Highlights

Space-Time Singularities

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Discovering Thoughts, Inventing Future

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Bi-Directional EPR Correlation in Cosmology and Planckeon Origin of Dark Energy

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Abstract- A quantum paradox of nonlocal Einstein-Podolsky-Rosen correlation between counter-propagating pair of polarization-entangled photons emitted from a common source S and detected at points P and Q is solved outside the EPR's reality criterion of local causality but within the framework of time-symmetric quantum electrodynamics allowing the bi-directional signal transmission P↔S↔Q on the double-light cone where the future and the past cones. share common light paths connecting the photon source S and the detection points P and Q. A cosmlogical implication of the bi-directional signal transmission P↔Q without common source S in cosmology and possible Planckeon orgin of dark energy in the upper hemisphere of semiclosed Friedman uiverse, joined on to an asymptotically flat outer space, are also discussed.

Keywords: *quantum theory, relativity, causality, locality, correlation, cosmology, dark energy.*

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Bi-Directional EPR Correlation in Cosmology and Planckeon Origin of Dark Energy

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Abstract- A quantum paradox of nonlocal Einstein-Podolsky-Rosen correlation between counter-propagating pair of polarization-entangled photons emitted from a common source S and detected at points P and Q is solved outside the EPR's reality criterion of local causality but within the framework of time-symmetric quantum electrodynamics allowing the bi-directional signal transmission P↔S↔Q on the double-light cone where the future and the past cones. share common light paths connecting the photon source S and the detection points P and Q. A cosmlogical implication of the bi-directional signal transmission P↔Q without common source S in cosmology and possible Planckeon orgin of dark energy in the upper hemisphere of semiclosed Friedman uiverse, joined on to an asymptotically flat outer space, are also discussed.

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I. Introduction

Since the advent of quantum mechanics in the mid-1920s there have benn continued interpretational controversies surrounding its counter-intuitive nature such as the wave-particle duality and the instantaneous collapse of the particle wave function at the detection point. But the paradox of nonlocal EPR[1] correlation between distant events without nonlocal interactions has been more problematic in recent times by Bell's experimental non locality test[2,3] proposed in 1964, though the paradox was first noticed by Schrödinger[4] and discussed in the dialogue between Einstein and Bohr[5] at 1935 Solvay Council. In emphasis of the signal transmission in EPR correlation Cavaicanti and Wiseman[6] asked: "What Bohr could have told Einstein at Solvay had he known about Bell experiments ?" In his recollection in 1990 Bell[7] wrote: "Suppose quantum mechanics were found to *resist* precise formulation. Suppose that when formulation beyond FAPP (For All Practical Purposes)[8] is attempted, we find an unmovable finger obstinately pointing outside the subject....to the Mind of the Observer..., or only Gravitation ?" We here show that the solution of quantum paradoxes can be found outside the EPR's reality criterion of local cauality[9] but within the framework of time-symmetric quantum electrodynamics for finite spacetme.[10] A cosmlogical implication of the bi-directional signal transmission P↔Q without common source S in the inflationary cosmology and a possible orgin of dark energy are also discussed.

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II. EPR Correlation in EPR Loophole

At the Solvay council EPR asked: "Are there spooky actions at a distance in quantum dmechanics ?" Recently, Yin et al.[11] led by Q. Zhang measured a superluminal speed of spooky acions between counter-propagating pair of photons emitted from an opticaly pumped atom in spin 0 state. During the measurement the locality and the freedom-of-choice loopholes of previous expriments were maximally closed by observing a 12-hour continuous violation of Bell's numerical expression (inequality) to EPR's reality criterion of local causality and separability of distant events. Let the spacetime positions of the photon source and the detection points be $S(x_S, t_S)$, $P(x_P, t_P)$ and $Q(x_Q, t_Q)$. Then the lower bound of the speed c_S of the spooky actions

$$c_S = |x_Q - x_P|/|t_Q - t_P| \qquad (1)$$

can be superluminal as $|t_Q - t_P| \to 0$. Here we can see a local and causal link P←S→Q ($t_S < t_P \approx t_Q$) and the nonlocal and a causal (spooky) link P→Q ($t_P < t_Q$). Let ε_P and ε_Q be the unit polarization vectors of photons measured at P and Q. The experiments verified the quantum expectations of the correlation function $C_{QM(}(\boldsymbol{\varepsilon}_P,\boldsymbol{\varepsilon}_Q) = \boldsymbol{\varepsilon}_P\boldsymbol{\varepsilon}_Q = \cos\theta$, where θ is the Hilbert space angle between $\boldsymbol{\varepsilon}_P$ and $\boldsymbol{\varepsilon}_Q$, and showed a clear rejection of classical theories obeying Bell's inequlities. The experiments also confirmed the insenstivty of C_{QM} to observer's delayed decision as to which direction to measure each photon's polarization at P and Q after the photon left the source at S—too late for a message to reach the opposite photon,[8] making the causal link P←S→Q improbable and the bi-directional link P↔S↔Q probable in the loophole of EPR's reality criterion of local causlity between P and Q,

III. EPR Correlation on Double-Light Cone

Dirac[12] defined the two-point corrlation function or propagator $\Delta(x,t)$ between S(0,0) and P(x,t), and visualized the signal transmission S↔P on the light cone with the origin S as vertex:

$$(\partial^2/c^2\partial^2t^2 - \partial^2/^2\partial^2x^2)\Delta(x,t) = 0, \qquad (2)$$

$$\Delta(x,t) = \alpha(t)\delta(c^2t^2 - x^2)$$

$$= [\delta(ct - x) - \delta(ct - x)]$$

$$= \Delta_{future} - \Delta_{past} = \Delta_{ret} - \Delta_{advt}, \quad (3)$$

where $\alpha(t) = t/|t| = 1$ $(t > 0)$; $= -1$ $(t < 0)$. There an electron at S(0, 0) moves under the retarded (causal) action Δ_{ret} of a charged partcle at P on the past light cone $\delta(ct + x)$ of S as well as the advanced (retrocausal) action Δ_{adv} of a charged particle at Q on the future light cone $\delta(ct - x)$ of S, giving a divergence-free radiation damping of the electron at S. The bi-directional EPR link, $P(x_P, t_P) \leftrightarrow S(x_S, t_S) \leftrightarrow Q(x_Q, t_Q)$, can be visualized on the future light cone of the optically pumped metastable atom at S(0, 0) by replacing the step function $\alpha(t)$ by the square (step-up and down) function $\beta(t) = 0$ $(t < t_S)$; $= 1$ $(t_S < t < t_{P/Q})$; $= 1$ $(t > t_{P/Q})$

$$\Delta(|x_{P/Q} - x_S|, |t_{P/Q} - t_S|)$$
$$= [\delta(c|t_{P/Q} - t_S| - |x_{P/Q} - x_S|)$$
$$-\delta[(c|t_{P/Q} - t_S| - |x_{P/Q} - x_S|]/|x_{P/Q} - x_S| . \quad (4)$$

The double-light cone[13] $[\delta_{ret} - \delta_{adv}]$ in Eq.(5) tells that the detection point P/Q on the left/right arms of the future light cone of S is reached by retarded wave $\exp i(\omega t - kt)$ from S while advanced wave $\exp i(\omega t + kx)$ from P/Q reaches S on the right/left arms of the past light cone of P/Q, forming a bi-directional sinusoidal wave, $\exp i\omega t \sin kx$, standing in phase between S and P/Q with nodes fixed in space at $x = n\pi/k$ (n = integer).

IV. Bi-Directional Microscope

"Is the star (moon) there when nobody looks" asked Tetrode (Mermin).[14] At the 1947 Solvay Council Heisenberg proposed a thought experiment measuring the electron position on microscope's object plane. There the photon wave collapsing at S in the retinaof the oberver entails the retrocollapse (appearance) of an elecon at P scattering the photon to be observed at S. That is, the electron is not at P when nobody looks. at S. This point was emphasized by Weizäcker[15] in his delayed-choice thought experiment measuring the transverse photon momentum on the focal plane of Heisenberg's microscope. If the microscope is very long, the observer at S can make choice as to which property of the electron, position or momentum, to measure after the scattering process has taken place at P. To see the bi-directional signal transmission S↔P in microscope we write Eq.(4) in momentum space[14]

$$\Delta_{\omega,k}(|x_P - x_S|, |t_P - t_S|)$$
$$= [\exp ikc|t_P - t_S| \, |\sin k|x_P - x_S|]/|k|, \quad (5)$$

getting an uncertainty relation between photon momentum $p = \hbar k$ and the microscope length

$$p|x_P - x_S| = nh/2, \quad (6)$$

where n is the number of nodes of the sinusoidal wave standing between S and P.

V. Cosmological EPR Correlation

In an attempt to resolve EPR problem Dirac[17,18] revived early ideas of aether transmitting light signal between distant points separated by spacelike distance, but was rejected by Einstein as it could not be fitted in his 4-dimensional formulation of relativity. Dirac[18] further poposed a bi-directional EPR connection between points P and Q located on the 4-dimensional hyperboloid $(ct)^2 - r^2 = l_{pl}^2$ crossing the light cone at $ct = l_{pl}$ at $r = 0$ with spacelike velocity:

$$dr/dt = ct/r = c(1 + l_{pl}^2/r^2)^{1/2}, \quad (7)$$

defining 3-dimensional Lorenz sphere $r^2 = l_{pl}^2$. We can likewise embed the Lorenz sphere into the radial line element ds of de Sitter universe in Reissner-Nordstroem form:[20]

$$ds^2 = c^2 g_{tt} dt^2 - g_{rr} dr^{22},$$
$$g_{tt} = g_{rr}^{-1} = (1 - \Lambda c^2/r^2 + l_{pl}^2/r^2)^{1/2}, \quad (8)$$

where Λ is the cosmological constant. Putting $ds^2 = 0$, we get light velocity:

$$dr/dt = cg_{tt}/g_{rr} = c(1 - \Lambda r^2/c^2 + l_{pl}^2/r^2). \quad (9)$$

We find that dr/dt is space like at $r \sim l_{pl}$ but decreases towards $dr/dt = c$ at $r = (cl_{pl})^{1/2}\Lambda^{1/4} \sim 10^{-12}\Lambda^{1/4}$ cm. From there dr/dt continues to decrease towards $dr/dt = c$, but rises again to space like velocity at the comological horizon $r = c\Lambda^{1/2}$ after a brief interlude of subluminal period.

Hawking[19] proposed a cyclic Lorenz-de Sitter model where an expanding and contracting universe starts and ends on the 4-dimensional Lorenz sphere $\tau^2 + r^2 = l_{pl}^2$ with imaginary time $\tau = it$. There the Hubble expansion $\sim \exp Ht$ is replaced by a cosmological wave function of radius $\sim \exp iH\tau \sim \exp i\Lambda\tau$, determining the temperature $T \sim H$, entropy $S \sim \Lambda^{-2}$ and the energy of radiation $E \sim \hbar c/R$ created by the de Sitter black hole capturing negative energy components of Zittering electrons[20] at temerture T.

In high dimensional string theory,[21] the parallel orbifold branes collide periodically in cycle, expanding and contracting with dark energy Λ.

VI. Mass Defect of Semiclosed Friedman Universe

The idea of semicosed Friedman universe was proposed by Zel'dovich and others[21] in 1970s as a possible model of quasistellar radio sources evolving from and joined-on to preexisting asymptotically flat space. The expansion history of the semicosed universe is dictated by the Hubble constant $H = 8\pi G/\rho_\Lambda$ and the dimensionless density parameter $\Omega_\Lambda = \rho_\Lambda/\rho_{c\Lambda}$ where ρ_Λ is the energy density and $\rho_{c\Lambda}$ the critical density.

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For $0 < \Omega_\Lambda < 0.5$ the expanding univese in lower hemisphere is joined onto asymptotically flat outer space through Schwarzschild throat; for $0.5 < \Omega_\Lambda < 1$ the contracting upper hemisphere is joined onto outer space through double-valued Schwarzschild bottleneck; for $\Omega_\Lambda \sim 1$ the almost closed universe is joined onto asymptotically flat space extending to infinity through Planck scale throat. For an observer comoving with cosmological expansion and contraction history of semicosed Friedman universe can be described by the radial line element ds of the universe in Reissner-Nordstroem form:

$$ds^2 = c^2 g_{tt} dt^2 - g_{rr} dr^2,$$

$$g_{tt} = g_{rr}^{-1} = (1 - r^2/r_g^2 + l_{pl}^2/r^2)^{1/2}, \qquad (10)$$

where $r_g = 3c^2/8\pi G\rho_\Lambda$. The light velocity $dr/dt = c(1 - \Lambda r^2/c^2 + l_{pl}^2/r^2)$ is space like at $r \sim l_{pl}$. As r increases from $r = l_{pl}$, dr/dt decreases towads c at $r = (r_g l_{pl})^{1/2} \sim 10^2$cm for $r_g \sim 10^{28}$cm, to be compared with the radius of causally related small region $\sim$ 10cm in inflationary model. With further increase of r towards horizon $r = r_g$, dr/dt reaches spacelike velocity $r = r_g$ after a long interlude of subluminal period: $l_{pl} << r << r_g$. In this period a detailed description of the semiclosed Friedman universe can be given by using the integral $\int dx (1 - r^2/r_g^2)^{1/2} = \sin^{-1}x$ to calculate the proper mass M_p and volume V_p of the universe:

$$M_p = \rho_\Lambda V_p = 2\pi\rho_\Lambda \int^R r^2 g_{rr} dr = (3/2)(R/r_g)^3[\sin^{-1}(R/r_g)$$
$$- (R/r_g)(1 - (R^2/r_g^2)^{1/2})]M, \qquad (11)$$

where $M = (4\pi R^3/3)\rho_\Lambda = \rho_\Lambda V$ is the Newtonian mass and volume V. Eq.(11) tells that the proper radius $R_p = \int^R g_{rr} r dr$ and volume $V_p = (4\pi R_p^3/3)$ increases with the increase of the world radius from $r \sim 0$, where $\sin_p^{-1}(R/r_g) \sim 0$, until V_p fills the half of the closed universe (lower hemisphere) of the closed Friedman universe, where $\sin^{-1}(l_{pl}/r_g) = \pi/2$. With further increase of r, R_p decreases towards $R_p \sim l_{pl}$, where $\sin^{-1}(l_{pl}/r_g) \sim \pi$, forming a gravitational semiclosure with $V_p \sim M_p \sim 0$ (with upper hemisphere) having Planck surface l_{pl}^2 and mass m_{pl}, creating Planck scale black holes liberating dark energy $E \sim \hbar c/l_{pl}$ or recreating a black Lorenz sphere outside the gravitational radius $R = r_g$, liberating dark energy $E \sim \hbar c/R$ with information content $(R/l_{pl})^2$ in asymptoticlly flat outer space.[22]

VII. Source of Dark Energy

We note that the negative equation of state $\rho_\Lambda + p_\Lambda c^2 < 0$ required by the dark energy is satisfied in the upper hemisphere of the semiclosed Friedman universe. There the density of gravitationally bound pairs of quantized metric fluctuations, or graviational Bohr atoms, dominate by creating negative attractive potential $Gm_{pl}^2/l_{pl} = \hbar c/l_{pl} = m_{pl}c^2$ capturing positive rest mass energy $m_{pl}c^p$ of single metric fluctuation, or Planckeons. In the lower hemisphere, where the positive equation of state $\rho_\Lambda + p\, c^2 > 0$ is satisfied, the free Planckeons prevails. The evolutionarily earlier upper hemishere is chracterized by the density parameter $0.5 < \Omega_\Lambda = (R/r_g)^2 = 1$ and the less earlier lower hemisphere by $0 < \Omega_m < 0.5$. The recently updated density parameters[23] fall into these ranges: $\Omega_\Lambda \sim 0.685$ and $\Omega_m \sim 0.266$. Adding the evolutionarily recent atomicmatter $\Omega_{atom} \sim 0.049$, we have $\Omega_{tot} = \Omega_\Lambda + \Omega_m + \Omega_{atom} = 0.965 \sim 1$ indicating the asymptotic flatness of the extragalactic space required for the asymptotic solutions of Einstein equations to be found useful.

VIII. Conclusion

We have shown that the quantum paradox of EPR correlation between distant points P and Q sharing a comon source S arises outside EPR's reality criterion of local causality, and can be solved within the framework of time-symmetric and relativistic quantum electrodynamics for finite spacetime[10] with singular boundary conditions allowing the bi-directional signal transmission P↔S↔Q on the double-light cone where the future and the past cones share a common light path connecting S and P/Q. A cosmlogical implication of the bi-directional and superluminal signal transmission P↔Q without a common source S in explaining the observed homogeneity of the universe and the possible Planckeon orgin of dark energy in the upper hemisphere of the semiclosed Friedman uiverse, joined onto an asymptotically flat space, are also discussed to explain the asymptotic flatness of the extragalactic space. In conclusion Planckeon origin of dark energħ in the upper hemisphere of the semiclosed Friedman universe is proposed.

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High Energy K X-Ray Hypersatellites

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Abstract- This study focus on the weak lines that appear on high energy side of the diagram lines which is called High Energy K X-ray hypersatellites i.e., a particular category of characteristic X-ray due to splitting of fine structure levels. It's resulted from the interaction of spin of an electron with the spin of the nucleus. They are emitted when an atom that has undergone a spontaneous transitions from the higher hyperfine level to the lower one to a radiation of $\approx 1.42 \times 10^9 Hz$ frequency and $\approx 21\ cm$ wavelength. Simultaneous double ionization in the K-shell and multiple ionization in the L-shell gives rise to K_α X-ray hypersatellites. These can be studied only by high energy resolution instruments like WDXRF spectrometry. Fundamental experimental procedures were outlined in this field by several workers due to different excitation modes. The theoretical models to predict their energies and intensities were developed. And also this review can show a clear discrepancy between theoretical and experimental results in the case of hyper satellites formation from different shells. In case of experimental instrumentation, WDXRF is the most accurate for determining the energy and intensity of X-ray hypersatellites. All sources of data was literature done by different scholars.

Keywords: *X-ray hyper satellites, energy ratio, intensity ratio.*

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Keywords: X-ray hyper satellites, energy ratio, intensity ratio.

I. Introduction

X-ray satellites emitted in the radioactive decay of double-vacancy states when the two initial vacancies are located in the same shell say K shell are of particular interest. Such satellites were named by Briand *et al*[1] as the hyper satellites. The denotation of the spectra as $K^h{}_{\alpha 1,2}$ and $K^h{}_{\beta 1,3}$ originate in the single electron K-spectator-hole transitions $1s^{-2} \rightarrow 1s^{-1}\,2p^{-1}$ and $1s^{-2} \rightarrow 1s^{-1}3p^{-1}$, respectively. They were first investigated by ion excitation and later by photon excitation using X-ray tubes. High-resolution measurements of heavy-ion-induced K hyper satellites were found to represent a sensitive tool for studying the relativistic and QED effects in atoms[2].

More recently, it has become possible to investigate the double-K-shell photo ionization process by measuring the K hyper satellite X-ray emission of light and mid-heavy elements irradiated with intense synchrotron radiation beams[3]. In double photo ionization despite of the electron shake-off process, the electron knock-out process appears, in which the first ionized electron can kick out a second bound electron leading to formation of a double-K-vacancy state. As a consequence, double K-shell photo ionization can only result from electron-electron correlation effects[4, 5].

An atom with one of its shells empty is called a hollow atom. These satellites can give information on the energy level structure of the electron shell of the hollow atom. They can be produced using target bombardment by high energy heavy ions. But when electrons are ejected, the resulting hollow atom is very difficult to control so that using photo excitation by monochromatic synchrotron radiation is preferable. But photo excitation method produces small probability for the creation of a double K-vacancy and is a disadvantage.

Well resolved photo excitation studies of the hyper satellite lines are very rare because the creation probability of hollow atom via photo excitation is very small. For example, for a Z≈30 atoms, the cross-section for the production of hyper satellite lines is $\approx 10^{-4}$ of that of the $K_{\alpha1,2}$ and $K_{\beta1,3}$ diagram fluorescence lines. However, study of hyper satellites using photo ionization is also reported by Raju *et al*[6] for elements in the Z range 19-25. Studies of the formation mechanism and the electronic structure of hollow atoms give important new insight in to fundamental issues like inter and intra shell electron correlations, the effects of relativity, and the Breit interaction. In addition to their importance for basic atomic physics, hollow atoms have important applications for studies of e.g. the electronic structure of surface, or systems far from equilibrium[7]. Hollow atoms have even been proposed as a way of achieving population inversion and lasing in the hard X-ray range of wavelengths[8, 9].

Relativistic effects also play a major role in the above mentioned transitions, due to the involvement of the highly relativistic K shell. While the effect of Breit interaction on almost all atomic transitions is less than 1%, for the hyper satellite transitions this effect can be as large as 20% in heavy atoms[10]. Therefore the hyper satellite lines offer a very rare opportunity to study in detail the Breit interaction, one of the least studied of all atomic interactions. The experiment done for a formation of hollow atom by bombarding high energy heavy ion, the number of electrons ejected and the state of the resulting hollow atoms are very difficult to control[11,12,13]. In this experiment photon excitation by monochromatic synchrotron radiation, this allows the measurement of a clean and high resolution hyper satellite spectrum and gives the possibility to select the exciting photon energy accurately. The disadvantage of the photo ionization method is the small probability for the creation of a double K vacancy, rendering high intensity synchrotron

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radiation sources vital for these experiments. A hollow atom can be producing either a single[14]or a multiphoton[15] process. Two photons are used to excite two K electrons nearly simultaneously. In a single photon excitation experiment, which was the method used in this work, both electrons are excited as a result of a single photon absorption.

The hyper satellites in these studies were very poor both in intensity and resolution. Richard *et al*[16] were the first to study hyper satellites of Calcium by heavy ion bombardment using the crystal spectrometer. This was followed by the work of Yohkoawaya[17] who measured K^2L^n/KL^n intensity ratio of Nickel, Iron, Chromium and Titanium by ion bombardment. Briand *et al*[18] observed hypersatellites in Copper, Nickel and Iron in a study by electron bombardment. Keski-Rahkonen *et al*[19] studied hyper satellite spectra of Magnesium, Vanadium, Chromium, Manganese and Iron by photon excitation. Pure Copper K_α hyper satellite spectrum generated by photo excitation using Synchrotron radiation is reported by Diamant *et al*[3].

In general, experimental procedures, theoretical computations based on some models, and some investigations carried out in this field at various laboratories in the world are examined with special references to chemical effects and Z systematics.

II. Experimental Procedures

The procedure, for utilizing WDXRF, adopted in recording the spectra is to introduce the sample in the sample holder and to record the intensities in 2θ steps of suitable angular intervals ranging from 0.01 to 0.05^0. In general, good statistics are maintained in counting. In each case, the experiment is repeated at least four times, using fresh sample each time.

The collected data are first smoothed using sliding least square fitting and then the spectra are corrected for background using an appropriate computer programmer. The deconvolution of the peaks and estimation of areas under different peaks is carried out by one of the codes like 'PEAK-FIT'[20] software version 4.11 of Systat Software Inc. Energy calibration of the crystal is done by taking spectra of some element standards and assuming the established values of the diagram lines and Calibration parameters are determined. Using these energies of the X-ray is determined.

The estimated intensities, however, do not represent the absolute intensities, as they have to be corrected for various effects. Since the present interest is on the relative intensities and the energy regions covered for each element for the satellites are very small, the final total correction on relative basis is small. Hence normally no corrections need be applied[21]. However, corrections like Self-Absorption in the Sample, Crystal Reflectivity, Window Absorption, and Efficiency of the Detector are applied, even though small, in the present investigations.

III. Theoretical Computations based on some Models

Different theoretical models are developed for the computation of energy shifts of hyper satellites relative to their respective diagram lines. The theoretical explanation of hyper satellite energy shifts can be defined by using calculations like;

D-H-S calculations: -Chen *et al*[2] estimated the energy shifts of the Kα hyper satellites $K_{\alpha1}{}^h$ and $K_{\alpha2}{}^h$ from the K_α diagram line. They have completed the energy shift of K_α hyper satellites relative to the diagram line in the intermediate coupling scheme using Dirac–Hartee-Slater wave functions and incorporating the full Breit interaction and the final state splitting produced by the Coulomb and Breit interactions in addition to the electrostatic interaction. The Breit interaction operator used was

$$H_{Br} = \frac{-1}{r_{12}}[\,\overline{\alpha_1}.\overline{\alpha_2} cos\omega r_{12} + (1 - cos\omega r_{12})]$$

$$\overline{\alpha_1}.\overline{\alpha_2} = Diracmatrices$$

Where r_{12} = distance between the two interacting electrons

For atoms with double inner shell vacancies, the multiplet splitting can be found by evaluating the corresponding coupled two hole matrix elements of the electrostatic and Briet interaction operators. As closed shells do not contribute to the multiplet splitting[22] the splitting of the double hole states is determined by the coupled – two- hole states alone. The sum of the electrostatic and Breit operators is,

$$H_{CBr} = \frac{1 - \overline{\alpha_1}\,.\overline{\alpha_2}}{r_{12}} cos\omega r_{12}$$

Hence, the energy matrix of the electrostatic and Breit operators between the anti-symmetrized j-j coupled two hole states, which can be separated in to direct and exchange matrix elements is,

$$\langle j_1 j_2 JM | r_{12}^{-1}(1 - \overline{\alpha_1}.\overline{\alpha_2}\,) cos\omega r_{12} | j_1 j_2 JM \rangle = D - E$$

Chen[2] *et al* have calculated these shifts with some gaps in the Z range of present interest. Values are available for Calcium (Z=20) and Manganese (Z=25). The values for the other elements are interpolated from the values given in the *Table 1*.The theoretical hyper satellite energy shift varies smoothly with Z in this region.

Table 1: Theoretical Energy shifts (in eV) of the $K_\alpha^h L^0$ hyper satellites with respect to the $K_\alpha L^0$ diagram line[23]

Atomic number	$K_{\alpha1}^h — K_{\alpha1}$	$K_{\alpha2}^h — K_{\alpha2}$
18	184.4	175.5
20	206.3	197.4
25	260.7	254.9
30	317.3	314.0
36	388.0	387.6
40	438.9	439.1
45	506.6	506.4

M.C.D.F.Caliculations: -Multi Configuration Dirac Fock calculations made in the intermediate coupling scheme takes in to consideration the relativistic effects. But Breit interaction is omitted from the final splitting calculations. The Breit operator is expressed as,

$$\overline{H_{Br}} = -\frac{1}{2_{r_{12}}}[\,\overline{\alpha_1}.\overline{\alpha_2} + \frac{1}{r_{12}^2}(\overline{\alpha_1}.\overline{r_{12}})(\overline{\alpha_2}.\overline{r_{12}})]$$

Where α_i Dirac matrices and r_{12} is the distance between the two interacting electrons. In the MCDF calculations only the average contribution of this Breit operator which is valid only in the long wave length limit[24] is taken into account. According to Chen[2] these calculations tend to overestimate the hyper satellite energy shifts.

IV. Dependence on Mode of Excitation, Chemical Effects and Z Systematics

A review of the literature shows that experimental investigations were carried out on hyper satellites on low and medium Z elements (mostly up to Z=32). In these studies attention was paid to the aspects of dependence on mode of excitation[25, 26], chemical effect[27] and Z systematics. Related to this for any element and chemical compounds regarding to their atomic number and projectiles has different energy shift and intensity ratio. When we see the oxidation number of any pure element is zero while the oxidation number of compounds are higher relative to one the other and also even if the oxidation number increase or decrease there is no variation in energy shift while the value of the intensity ratio increase or decrease.

Dependence on Mode of Excitation: 1) Projectile Dependence of Energy Shifts:- The K_α satellite energies are listed in *Tables 2.* In the case of the Al satellites, no systematic variation of the peak energies with projectile atomic number could be discerned.

Table 2 : K X-ray energies for Al, Cl and K[28]

X-ray peak	Initial state vacancy configuration	Al	Cl	K
		X-ray energy		
1	$1s^{-1}$	1486.6	2621.9	3312.9
2	$1s^{-1}2s^{-1}$ $1s^{-1}2p^{-1}$	1496.9(±0.1)	2640.2(±1.4)	3336.0(±3.2)
3	$1s^{-1}2s^{-2}$ $1s^{-1}2s^{-1}2p^{-1}$ $1s^{-1}2p^{-2}$	1507.9(±0.2)	2658.8(±1.8)	3357.1(±4.0)
4	$1s^{-1}2s^{-2}2p^{-1}$ $1s^{-1}2s^{-1}2p^{-2}$ $1s^{-1}2p^{-3}$	1521.3(±0.3)	2678.5(±2.2)	3379.1(±4.7)
5	$1s^{-1}2s^{-2}2p^{-2}$ $1s^{-1}2s^{-1}2p^{-3}$ $1s^{-1}2p^{-4}$	1535.1(±0.3)	2699.1(±2.0)	3402.3(±5.7)
6	$1s^{-1}2s^{-2}2p^{-3}$ $1s\text{-}12s^{-1}2p^{-4}$ $1s^{-1}2p^{-5}$	1549.9(±0.5)	2720.3(±2.8)	3427.5(±5.3)
7	$1s^{-1}2s^{-2}2p^{-4}$ $1s^{-1}2s^{-1}2p^{-5}$	_	2743.5(±2.7)	3452(±5.6)
8	$1s^{-1}2s^{-2}2p^{-5}$	_	2769.8(±3.6)	3481.2(±5.6)

The listed energies are the averages obtained for all runs with all projectiles and the indicated errors are root mean square deviations. The absolute error in the satellite energies for Cl and K are essentially the same as those given for Al. *2) Projectile Dependence of Intensity Ratio:* - Watson *et al*[28] studied how $K_\alpha L^n$ relative intensity varies with different projectiles in Al, Cl and K.

Table 3 : Relative K_α X-ray satellite intensity for Al, Cl, and K using 1.7MeV/amu[28]

Target	Projectile	No. of L shell vacancy							
		0	1	2	3	4	5	6	7
Al	H	0.858	0.142	_	_	_	_	_	_
	He	0.664	0.285	0.051	_	_	_	_	_
	C	0.068	0.215	0.366	0.239	0.090	0.023	_	_
	O	0.045	0.128	0.312	0.293	0.168	0.053	_	_
Cl	He	0.680	0.320	_	_	_	_	_	_
	C	0.053	0.229	0.363	0.256	0.094	0.005	_	_
	O	0.028	0.133	0.304	0.316	0.170	0.049	_	_
K	He	0.765	0.235	_	_	_	_	_	_
	C	0.074	0.307	0.353	0.208	0.058	_	_	_
	O	0.045	0.185	0.316	0.271	0.142	0.042	_	_

From the *Table 3* in all Al, Cl and K targets the intensity under He projectile decrease as a number of vacancy increase; while directly correlated with atomic number. As we see under the *Table 3* in the case of C and O projectile the value of intensity is not consistent when Z increases. *3) Chemical Effects:* - A noticeable chemical effect was observed in the case fluorine K. Ram Narayana *et al*[29]. *4)Z Systematics (Z Dependence of Relative intensities):* - Raju *et al*[6] measured energies and intensities of K_α hyper satellites and K_β satellites of the elements in the Z range 19-25 by photon excitation. The relative intensity of the K_α hyper satellite with respect to that of the K_α diagram line is related to the ratio of double K-shell to single K-shell ionization. This intensity ratio was plotted as a function of Z. It decreases smoothly and exponentially with Z. The K_β satellite relative intensity with respect to that of K_β diagram line was plotted as a function of Z. This also was observed to vary exponentially and smoothly with Z. *5)Z Dependence of Energy Shift:* -Raju *et al*[6]supplemented their data with those reported by other authors on K_α hyper satellite energy shift relative to the K_α diagram line in the Z range 12-30 and studied the variation of this with respect to Z. They found the relationship to be linear. They obtained from the plot of this Δ(E) versus Z, the following empirical relationship

$$\Delta E(K_\alpha^h) = -3.0 + 10.048Z$$

V. Conclusions

WDXRF has an auxiliary collimator mounted in front of the detector helps in improving the resolution, analyzing crystal spectrometer is effected and data acquired by a personal computer making use of Philips super QMS windows based software, a suitable voltage and current to operate easily, sample changers for fast data collection etc. used to determining the energy shift and intensity of x-ray satellites. Due to this instrument the production of a satellites have been registered for a double-vacancy states existing in the K-shells is called X-ray hyper satellites.

When the atom is doubly ionized in K-shell and if one of these holes is filled by transitions from outer shell K X-ray hyper satellites (weak lines) are emitted. These lines appear on high energy side of the diagram lines. Study of K X-ray hyper satellites provides information on the; intra atomic relations, excitation dynamics, relaxation and other factors influencing X-ray emissions.

Energies and relative intensities of hyper satellites of high Z-elements were determined by various researchers using crystal spectrometer by applying photon, electron, and ion excitation modes. Investigations were carried out to examine chemical shifts, it was found that relative intensities are susceptible to the chemical environment but energies are not much affected by it.

Generally, from different literature, studies were not cover an energy from small Z-elements which have smaller wavelength of the satellites. So that the existing few studies about hyper satellites can be extended to cover all elements. Because a number of synchrotron facilities are being developed throughout the world; using these tunable hard X-ray sources, energy dependence of these processes can be studied more efficiently.

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On the New Solitary Wave Solution of the Generalized Hirota-Satsuma Couple KdV System

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Abstract- In this article, we employ $\left(\frac{G'}{G}\right)$ - expansion method for the generalized Hirota - Satsuma couple KdV system to find the exact traveling wave solutions involving parameters with the aid of Maple 16. When these parameters are taken special values, the solitary wave solutions are derived from the exact traveling wave solutions. It is shown that the $\left(\frac{G'}{G}\right)$- expansion method provides an effective and a more powerful mathematical tool for solving nonlinear evolution equations in mathematical physics. Comparison between our results and the well-known results will be presented.

Keywords: *the generalized hirota-satsuma couple KdV system, the $\left(\frac{G'}{G}\right)$ - expansion method, traveling wave solutions, solitary wave solutions, bell singular soliton solutions.*

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I. Introduction

No one can deny the important role which played by the nonlinear partial differential equations in the description of many and a wide variety of phenomena not only in physical phenomena, but also in plasma, fluid mechanics, optical fibers, solid state physics, chemical kinetics and geochemistry phenomena. So that, during the past five decades, a lot of method was discovered by a diverse group of scientists to solve the nonlinear partial differential equations. For examples tanh - sech method [12],[16] and [18], extended tanh - method [13], [6] and [20], sine – cosine method [19], [17] and [22], homogeneous balance method [4], the $\exp(-\varphi(\xi))$ expansion Method [11], Jacobi elliptic function method [3], [5], [14] and [24], F-expansion method [2], [21] and [9], exp-function method [8] and [7], trigonometric function series method [32], $(\frac{G'}{G})$ expansion method [10], [15], [29] and [26], the modi_ed simple equation method [1], [27], [30], [28], [31] and [25] and so on.

The objective of this article is to apply the $\left(\frac{G'}{G}\right)$ expansion method for finding the exact traveling wave solution of the generalized Hirota-Satsuma couple KdV system [23], which play an important role in mathematical physics.

The rest of this paper is organized as follows: In section 2, we give the description of the modified simple equation method. In section 3, we use this method to find the exact solutions of the nonlinear evolution equations pointed out above. In section 5, conclusions are given.

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II. Description of Method

Consider the following nonlinear evolution equation

$$F(u, u_t, u_x, u_{tt}, u_{xx}, ...) = 0, \qquad (2.1)$$

where F is a polynomial in u(x; t) and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method :

Step 1. We use the wave transformation

$$u(x,t) = u(\xi), \qquad \xi = x - ct, \qquad (2.2)$$

where c is a constant, to reduce Eq.(2.1) to the following ODE:

$$P(u, u', u'', u''', ...) = 0, \qquad (2.3)$$

where P is a polynomial in $u(\xi)$and its total derivatives, while $\left\{u' = \frac{du}{d\xi}\right\}$.

Step 2. Suppose that the solution of Eq.(2.3) has the form:

$$U(\xi) = a_0 + \sum_{j=1}^{m} a_j \left(\frac{G'}{G}\right)^j, \quad a_m \neq 0, \quad (2.4)$$

where a_0 and a_j, for $(j = 1, 2, 3, ..., m)$, are constants to be determined later, $G(\xi)$ satisfies a second order linear ordinary differential equation (LODE):

$$G'' + \lambda G' + \mu G = 0, \qquad (2.5)$$

where λ and μ are arbitrary constants. The positive integer M can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in Eq.(2.3). Moreover precisely, we define the degree of $u(\xi)$ as $D(u(\xi)) = m$, which gives rise to degree of other expression as follows:

$$D\left(\frac{d^q u}{d\xi^q}\right) = m + q,\ D\left(u^p\left(\frac{d^q u}{d\xi^q}\right)^s\right) = mp + s\,(m + q)$$

Therefore, we can find the value of m in Eq.(2.4)

Step 3. Substitute Eq.(2.4) along Eq.(2.5) into Eq.(2.3) and collecting all the terms of the same power $\left(\frac{G'}{G}\right)^j$ $j = 0, 1, 2, 3, ...$ and equating them to zero, we obtain a system of algebraic equations, which can be solved by Maple or Mathematica to get the values of a_j, since $a_m \neq 0$. The solution of Eq.(2.5) depending on whether $\lambda^2 - 4\mu > 0,\ \lambda^2 - 4\mu < 0,\ \lambda^2 - 4\mu = 0$ are given as

Case 1. When $\lambda^2 - 4\mu > 0$

$$\left(\frac{G'}{G}\right) = \frac{\sqrt{\lambda^2 - 4\mu}}{2}\left(\frac{A_1 sinh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi\ + A_2 cosh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi}{A_1 cosh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi\ + A_2 sinh(\frac{1}{2}\sqrt{\lambda^2 - 4\mu})\xi}\right) - \frac{\lambda}{2}. \tag{2.6}$$

Case 2. When $\lambda^2 - 4\mu < 0$

$$\left(\frac{G'}{G}\right) = \frac{\sqrt{4\mu - \lambda^2}}{2}\left(\frac{-A_1 sin(\frac{1}{2}\sqrt{4\mu - \lambda^2})\xi\ + A_2 cos(\frac{1}{2}\sqrt{4\mu - \lambda^2})\xi}{A_1 cos(\frac{1}{2}\sqrt{4\mu - \lambda^2})\xi\ + A_2 sin(\frac{1}{2}\sqrt{4\mu - \lambda^2})\xi}\right) - \frac{\lambda}{2}. \tag{2.7}$$

Case 3. When $\lambda^2 - 4\mu = 0$

$$\left(\frac{G'}{G}\right) = \frac{A_2}{A_1 + A_2\xi} - \frac{\lambda}{2}. \tag{2.8}$$

The above results can be written in simplified forms as
Case 1. When $\lambda^2 - 4\mu > 0$

$$\left(\frac{G'}{G}\right) = \frac{\sqrt{\lambda^2 - 4\mu}}{2} tanh(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi + \xi_0) - \frac{\lambda}{2}, \quad tanh\,\xi_0 = \frac{A_1}{A_2}, \quad \left|\frac{A_1}{A_2}\right| > 1. \tag{2.9}$$

Case 2. When $\lambda^2 - 4\mu < 0$

$$\left(\frac{G'}{G}\right) = \frac{\sqrt{\lambda^2 - 4\mu}}{2} coth(\frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi + \xi_0) - \frac{\lambda}{2}, \quad coth\,\xi_0 = \frac{A_1}{A_2}, \quad \left|\frac{A_1}{A_2}\right| < 1. \tag{2.10}$$

Case 3. When $\lambda^2 - 4\mu = 0$

$$\left(\frac{G'}{G}\right) = \frac{\sqrt{\lambda^2 - 4\mu}}{2} cot\left(\frac{\sqrt{4\mu - \lambda^2}}{2}\xi + \xi_0\right) - \frac{\lambda}{2}, \quad cot\,\xi_0 = \frac{A_2}{A_1}. \tag{2.11}$$

Step 4. substituting these values and the solutions of Eq.(2.5) into Eq.(2.3) we obtain the exact solutions of Eq.(2.1).

III. Application

Here, we will apply the $\left(\frac{G'}{G}\right)$-expansion method described in Sec.2 to find the exact traveling wave solutions and the solitary wave solutions of the generalized Hirota-Satsuma couple KdV system[23]. We consider the generalized Hirota-Satsuma couple KdV system

$$\begin{cases} u_t = \frac{1}{4}u_{xxx} + 3uu_x + 3\left(-v^2 + w\right)_x, \\ v_t = -\frac{1}{2}v_{xxx} - 3uv_x, \\ w_t = -\frac{1}{2}w_{xxx} - 3uw_x. \end{cases} \tag{3.1}$$

When $w = 0$, Eq.(3.1) reduce to be the well known Hirota-Satsuma couple KdV equation. Using the

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wave transformation $u(x,t)=u(\xi), v(x,t)=v(\xi), w(x,t) = w(\xi)$, $\xi = k(x-\lambda_1 t)$ carries the partial differential equation (3.1) into the ordinary differential equation

$$\begin{cases} -\lambda_1 k\, u' = \frac{1}{4}k^3 u''' + 3\,k\,u\,u' + 3\,k\left(-v^2+w\right)', \\ -\lambda_1 k\, v' = -\frac{1}{2}k^3 v''' - 3\,k\,u\,v', \\ -\lambda_1 k\, w' = -\frac{1}{2}k^3 w''' - 3\,k\,u\,w'. \end{cases} \tag{3.2}$$

Suppose we have the relations between (u and v) and (w and v) $\Rightarrow \left(u = \alpha v^2 + \beta v + \gamma\right)$ and $(w = Av + B)$ where α, β, γ, A and B are arbitrary constants. Substituting this relations into second and third equations of Eq.(3.2) and integrating them , we get the same equation and integrate it once again we obtain

$$k^2 v'^2 = -2\alpha v^4 - 2\beta v^3 + 2\left(\lambda_1 - 3\gamma\right) v^2 + 2c_1 v + c_2, \tag{3.3}$$

where c_1 and c_2 is the arbitrary constants of integration, and hence, we obtain

$$\begin{aligned} k^2 u'' &= 2\alpha k^2 v'^2 + k^2\left(2\alpha v + \beta\right) v'' \\ &= 2\alpha\left[-\alpha v^4 - 2\beta v^3 + 2\left(\lambda_1 - 3\gamma\right) v^2 + 2c_1 v + c_2\right] \\ &+ \left(2\alpha v + \beta\right)\left[-2\alpha v^3 - 3\beta v^2 + 2\left(\lambda_1 - 3\gamma\right) v + c_1\right]. \end{aligned} \tag{3.4}$$

So that, we have

$$P'' + lP - mP^3 = 0. \tag{3.5}$$

Where

$$c_1 = \frac{1}{2\alpha^2\left(\beta^2 + 2\lambda_1\alpha\beta - 6\alpha\beta\gamma\right)}, \quad v(\xi) = aP(\xi) - \frac{\beta}{2\alpha}, \quad \alpha = \frac{\beta^2 - 4}{4\left(\gamma - \lambda_1\right)}, \quad A = \frac{4\beta\left(\lambda_1 - \gamma\right)}{\beta^2 - 4},$$

$$\begin{aligned} B = \frac{1}{6\left(-\gamma + \lambda_1\right)\left(\beta^2 - 4\right)^2} &(16c_3\lambda_1\beta^2 - 2c_3\lambda_1\beta^4 - 16c_3\gamma\beta^2 + 3c_3\gamma\beta^4 + 56\lambda_1^2\gamma\beta^2 \\ &- 48\gamma^2\lambda_1\beta^2 - 16c_2 + c_2\beta^6 - 12c_2\beta^4 + 12c_2\beta^2 - 16\gamma^2\lambda_1 - 32\lambda_1^2\gamma - 8\lambda_1^3\beta^2 + \beta^4\gamma^3 \\ &- 2\beta^4\lambda_1^3 + 32c_3\gamma - 32c_3\lambda_1 + 48\gamma^3 + \beta^4\gamma^2\lambda_1), \end{aligned}$$

$$l = \frac{-a}{k^2}\left(\frac{3\beta^2}{2\alpha} + 2\lambda_1 - 6\gamma\right), \quad m = \frac{-2\alpha a^3}{k^2}.$$

Balancing between the highest order derivatives and nonlinear terms appearing in P'' and $P^3 \Rightarrow (N+2 = 3N) \Rightarrow (N=1)$ So that, by using Eq.(2.4) we get the formal solution of Eq.(3.5)

$$P(\xi) = a_0 + a_1\left(\frac{G'}{G}\right). \tag{3.6}$$

Substituting Eq.(3.6) and its derivative into Eq.(3.5) and collecting all term with the same power of $\left(\frac{G'}{G}\right)^3, \left(\frac{G'}{G}\right)^2, \left(\frac{G'}{G}\right)^1, \left(\frac{G'}{G}\right)^0$ we obtained:

$$\left(\frac{G'}{G}\right)^3 : 2\,a_1 + m{a_1}^3 = 0, \tag{3.7}$$

$$\left(\frac{G'}{G}\right)^2 : 3\,a_1\lambda + 3\,m a_0 {a_1}^2 = 0, \tag{3.8}$$

$$\left(\frac{G'}{G}\right)^1 : a_1\lambda^2 + 2\,a_1\mu + l a_1 + 3\,m{a_0}^2 a_1 = 0, \tag{3.9}$$

$$\left(\frac{G'}{G}\right)^0 : a_1\lambda\,\mu + l a_0 + m{a_0}^3 = 0 \tag{3.10}$$

Solving above system of algebraic equations by using Maple program, we obtain

$$l = \frac{1}{2}\lambda^2 - 2\,\mu,\; a_1 = \pm\sqrt{\frac{-2}{m}},\; a_0 = \pm\lambda\sqrt{\frac{-1}{2m}}\; since\; (m < 0).$$

So that, the exact traveling wave solution

$$P(\xi) = \pm\lambda\sqrt{\frac{-1}{2m}} \pm \sqrt{\frac{-2}{m}}\left(\frac{G'}{G}\right). \tag{3.11}$$

here, we discuss the three cases:

Case 1. When $\lambda^2 - 4\mu > 0$

$$P(\xi) = \pm\lambda\sqrt{\frac{-1}{2m}} \pm \sqrt{\frac{-2}{m}}\left[\frac{\sqrt{\lambda^2-4\mu}}{2}\quad \frac{A_1 sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu})\xi\, + A_2 cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu})\xi}{A_1 cosh(\frac{1}{2}\sqrt{\lambda^2-4\mu})\xi\, + A_2 sinh(\frac{1}{2}\sqrt{\lambda^2-4\mu})\xi}\right) - \frac{\lambda}{2}\right] \tag{3.12}$$

Case 2. When $\lambda^2 - 4\mu < 0$

$$P(\xi) = \pm\lambda\sqrt{\frac{-1}{2m}} \pm \sqrt{\frac{-2}{m}}\left[\frac{\sqrt{4\mu-\lambda^2}}{2}\quad \frac{-A_1 sin(\frac{1}{2}\sqrt{4\mu-\lambda^2})\xi\, + A_2 cos(\frac{1}{2}\sqrt{4\mu-\lambda^2})\xi}{A_1 cos(\frac{1}{2}\sqrt{4\mu-\lambda^2})\xi\, + A_2 sin(\frac{1}{2}\sqrt{4\mu-\lambda^2})\xi}\right) - \frac{\lambda}{2}\right]. \tag{3.13}$$

Case 3. When $\lambda^2 - 4\mu = 0$

$$P(\xi) = \pm\lambda\sqrt{\frac{-1}{2m}} \pm \sqrt{\frac{-2}{m}}\frac{A_2}{A_1 + A_2\xi} - \frac{\lambda}{2}. \tag{3.14}$$

The above results can be written in simplified forms as

Case 1. When $\lambda^2 - 4\mu > 0$

$$P(\xi) = \pm\lambda\sqrt{\frac{-1}{2m}} \pm \sqrt{\frac{-2}{m}}\frac{\sqrt{\lambda^2-4\mu}}{2}\left[tanh(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi + \xi_0) - \frac{\lambda}{2}\right]. \tag{3.15}$$

Case 2. When

$$P(\xi) = \pm\lambda\sqrt{\frac{-1}{2m}} \pm \sqrt{\frac{-2}{m}}\frac{\sqrt{\lambda^2-4\mu}}{2}\left[coth(\frac{\sqrt{\lambda^2-4\mu}}{2}\xi + \xi_0) - \frac{\lambda}{2}\right]. \tag{3.16}$$

Case 3. When $\lambda^2 - 4\mu = 0$

$$P(\xi) = \pm\lambda\sqrt{\frac{-1}{2m}} \pm \sqrt{\frac{-2}{m}}\frac{\sqrt{\lambda^2-4\mu}}{2}\left[cot\quad \frac{\sqrt{4\mu-\lambda^2}}{2}\xi + \xi_0\right) - \frac{\lambda}{2}\right]. \tag{3.17}$$

Note that:

All the obtained results have been checked with Maple 16 by putting them back into the original equation and found correct.

IV. Physical Interpretations of the Solutions

In this section, we depict the graph and signify the obtained solutions to the generalized Hirota-Satsuma couple KdV system. Now, we will discuss all possible physical significances for parameter.

case 1.
when $m = \frac{-1}{2}$, $\lambda = 3$, $\mu = 2$, $A_1 = 4$, $A_2 = 5$, $k = 1$, $\lambda_{-2} \Rightarrow$ we obtain the hyperbolic function solution and represent singular kink type solitary wave solution.

case 2.
when $m = \frac{-1}{2}$, $\lambda = 2$, $\mu = 3$, $A_1 = 4$, $A_2 = 5$, $k = 1$, $\lambda_{-2} \Rightarrow$ we obtain the trigonometric function solution and represent bell solitary wave solution.

case 1.
when $m = \frac{-1}{2}$, $\lambda = 2$, $\mu = 1$, $A_1 = 4$, $A_2 = 5$, $k = 1$, $\lambda_{-2} \Rightarrow$ we obtain the rational function solution and represent singular kink type solitary wave solution.

V. Conclusion

The $\left(\frac{G'}{G}\right)$ - expansion method has been successfully used to find the exact traveling wave solutions of some nonlinear evolution equations. As an application, the traveling wave solutions for the generalized Hirota-Satsuma couple KdV system which have been constructed using the $\left(\frac{G'}{G}\right)$ -expansion method. Let us compare between our results obtained in the present article with the well-known results obtained by other authors using different methods as follows: Our results of the generalized Hirota-Satsuma couple KdV system are new and different from those obtained in [23] It can be concluded that this method is reliable and propose a variety of exact solutions NPDEs. The performance of this method is effective and can be applied to many other nonlinear evolution equations. The solutions represent the solitary traveling wave solution for the generalized Hirota-Satsuma couple KdV system.

Competing Interests

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors. The author did not have any competing interests in this research.

Author's Contributions

All parts contained in the research carried out by the researcher through hard work and a review of the various references and contributions in the field of mathematics and the physical Applied.

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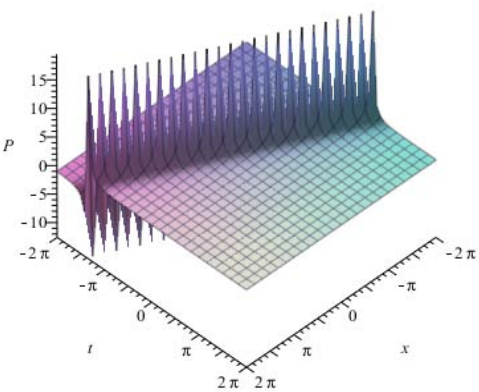
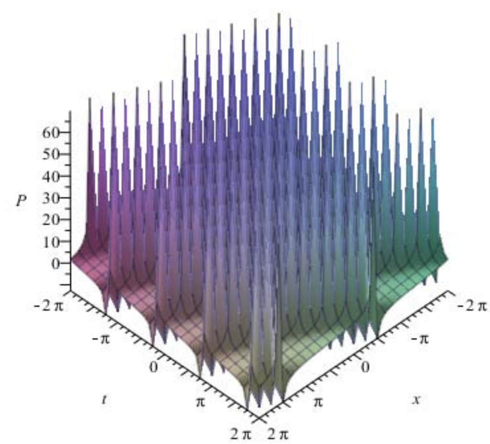
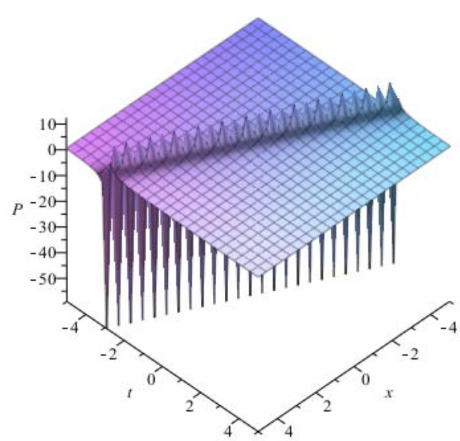


(a)
Eq.(3.12)



(b)
Eq.(3.13)



(c)
Eq.(3.14)

Figure 1 : The Solitary wave solution of Eqs.(3.12),(3.13) and (3.14)

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Quintessential Nature of the Fine-Structure Constant

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Abstract- An introduction is given to the geometry and harmonics of the Golden Apex in the Great Pyramid, with the metaphysical and mathematical determination of the fine-structure constant of electromagnetic interactions. Newton's gravitational constant is also presented in harmonic form and other fundamental physical constants are then found related to the quintessential geometry of the Golden Apex in the Great Pyramid.

Keywords: *quintessence, fine-structure constant, great pyramid, physical constants, electromagnetism.*

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QUINTESSENTIALNATUREOFTHEFINESTRUCTURECONSTANT

Strictly as per the compliance and regulations of :

Quintessential Nature of the Fine-Structure Constant

Michael A. Sherbon

Abstract- An introduction is given to the geometry and harmonics of the Golden Apex in the Great Pyramid, with the metaphysical and mathematical determination of the fine-structure constant of electromagnetic interactions. Newton's gravitational constant is also presented in harmonic form and other fundamental physical constants are then found related to the quintessential geometry of the Golden Apex in the Great Pyramid.

Keywords: quintessence, fine-structure constant, great pyramid, physical constants, electromagnetism.

I. Introduction

The geometry of the Great Pyramid of Giza, as stated by Eckhart Schmitz in *The Great Pyramid of Giza*, "... demonstrates extraordinary precision in relaying highly accurate geodetic knowledge..." and "it is evident, with a very high degree of probability, that the design parameters were expressly intent on conveying this advanced knowledge." [1]. In *The Essence of the Cabalah*, William Eisen describes the fundamental geometry of what was known as the Golden Apex of the Great Pyramid and the ancient *pre*-pharaonic science of the Agashan Masters [2].

II. Golden Geometry of the Great Pyramid

The Golden Apex of the Great Pyramid is the side of a square in the upper part of the capstone resulting from four exponential curves at the base of the Great Pyramid [2]. Golden Apex of the Great Pyramid A:

$$A = e^{\pi} + 7\pi - 1 \cong 0.1495 \tag{1}$$

The inverse of the Golden Apex A^{-1} is a harmonic of Newton's gravitational constant G. $A^{-1} \cong \phi\sqrt{2\pi e} \cong \phi/\ln(4/\pi)$ and $4/\pi \cong \sqrt{11A}$. $A+1 \cong R \cong 1.152$, the approximate radius of the regular heptagon with the side equal to one. $A \cong \pi/\sqrt{440} \cong \sqrt{\pi/140}$, the base of the Great Pyramid is approximately 440 cubits, and the height is $2 \times 140 = 280$ cubits. $140/360 \cong e/7 \cong \sqrt{A}$ and $4 \times 440 \simeq 2\pi \times 280$. The height of the Golden Apex, $4A/2\pi \cong S^{-2}$, the silver constant of the heptagon, see Eq. (4).

$$RA \cong \sqrt{\phi}/e^2 \cong \ln(\pi/\sqrt{7}) \tag{2}$$

The golden ratio approximates the squaring of the circle and is also closely related to the heptagon geometry. $2R = \csc(\pi/7)$ and $R \cong \phi/\sqrt{2} \cong 13/7\,\phi \cong \ln\pi \cong 2/\sqrt{3} \cong 7/\sqrt{37}$. The $\tan S^{-1} \cong \pi^{-1}$. In the *Timaeus*, Plato "considered the golden section to be the most binding of all mathematical relationships and the key to the physics of the cosmos," quoted by Robert Schoch and Robert McNally in *Pyramid Quest* [3]. Quintessence is associated with the dynamic aether and also embraces the four known forces of nature.

$$Q = C/\sqrt{2} \cong \sqrt{e}/\phi \cong 1 + \alpha\phi^2, \tag{3}$$

where $\alpha \cong 1/137$ is the fine-structure constant and quintessence $Q \cong 377/370 \cong \sqrt{11}/S \cong 1.019$. C^2 is the approximate inner diameter of the regular heptagon with side equal to one. $C^2 \cong \cot(\pi/7) \cong 1/\ln\phi$. Also, $CQ \cong \sqrt{\phi}R \cong \csc\alpha^{-1}$. With the golden ratio $\phi \cong A + CQ$ and $\phi^2 \cong A + C + Q$. In elementary form, $22 = 1 + (3 \times 7) \cong 7\pi$. Also, $C = \sqrt{2}Q \cong 1.441$.

Aristotle coined the term quinta essentia for Plato's fifth element, often represented by the dodecahedron. The quintessence is described in Malcolm Macleod's geometry of angular momentum model as the harmonic Q and also has units from the square root of Planck angular momentum [4]. The golden ratio is an approximate harmonic of the Planck length in meters and harmonics of fundamental units have a geometric basis in ancient metrology [5]. R is a harmonic of half the Great Pyramid base length in meters, and CQ is a harmonic of the height in meters. The $\ln A^{-1} \cong \sec Q \cong 6/\pi \cong \pi/\sqrt{e}$, the cube-sphere ratio. With golden ratio overlay software, a golden spiral centered on the Eye of Ra in Robert Temple's plan of the Giza Plateau, is shown to pass through both the Golden Apex of the Great Pyramid and the Great Sphinx of Giza.

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Ratio of the length of the sides of Temple's Perfect Square of Giza/Shadow Square of Giza is approximately equal to Q [6]. $Q \cong \sqrt{7A} \cong \ln\sqrt{R/A} \cong A + R^{-1}$ and $\ln(S/Q) \cong \pi/e$. The tan $Q \cong \phi$.

$$A \cong \sqrt{11}/7\pi \cong \sqrt{e}/11 \cong 2\,\pi\,\alpha\,S \quad (4)$$

The silver constant S from the regular heptagon, $S = 4\cos^2(\pi/7) \cong 2\sqrt{2}R \cong 2\tan Q \cong \tan\sqrt{\phi} \cong 3.247$. Also, $CQ \cong \sqrt{7/S}$. $1+3+7=11$. $2\pi\alpha$ is the ratio between the Compton wavelength of the electron in hydrogen and the Bohr radius. From the hyperdimensional aether, elementary charge and spin angular momentum (determining the fine structure constant) is manifested through the proportions of the golden ratio. The Golden Apex $A \cong \sqrt{S}/12$, $AS^2 \cong 11/7 \cong \pi/2$ and $\alpha^{-1} \cong A^{-2}\tan 72°$, with the regular pentagon angle. $S \cong \sqrt{7\pi}/C$ and $C^2 \cong R\sqrt{S} \cong \cot(\pi/7)$.

Jean-Paul and Robert Bauval describe in the *Secret Chamber Revisited* how prime numbers 7 and 11 are significant keys to the Great Pyramid [7]. The number seven was "especially dedicated to Sirius," [1, 8, 9]. Half the face apex angle of the Great Pyramid is approximately 32° [10]. $11 \times 11 = 121$ and $121/137 \cong \sqrt{2}\tan 32°$. $G_a \cong 1/\sqrt{RA} \cong 2\sqrt{C}$. $\ln 11 \cong 2.4$, where G_a is the golden angle in radians. From the pyramid face apex angle, $\tan 64° \cong \sqrt{11}/\phi \cong 137/67$.

The special number 528 translates as the *Key* to the *Pyramid of Light* [11, 12]. The harmonic $5.28 \cong 37/7 \cong 2\sqrt{7} \cong e\pi/\phi$. Also, $2\pi \cong \ln(528)$ and $5.28 \cong 2\pi/\sqrt{C} \cong \pi/4A$. Found in several places, $2.64'$ is the width and height of the Ark of the Covenant, the proportioned Golden Apex height and section lengths above the $5.28'$ rise in the Grand Gallery. $2.64 \cong Q/\sqrt{A} \cong \sqrt{7}$.

III. Mathematics of the Fine-Structure Constant

The fine-structure constant was introduced by Arnold Sommerfeld with the addition of elliptic orbits to Bohr's atomic model [13]. The fine-structure constant determines the strength of the electromagnetic interaction and being related to quintessence it is involved in all aetheric phenomena. The usual definition for alpha, the fine-structure constant is $\alpha = e^2/\hbar c$ in *cgs* units. Mark Rohrbaugh's review of Nassim Haramein's work motivated the question of recombining the classical electron radius $r_e = e^2/m_e c^2$ with the formulation by Haramein for the proton charge radius, from the modified Eq. (31) $r_p = 4\,\hbar/m_p\,c$ [14].

$$\alpha = 4/(m_p/m_e)(r_p/r_e) \quad (5)$$

When substituting the reference value for alpha and using the latest reference values for the proton and electron mass with the classical radius of the electron, gives a value for the proton radius $r_p \cong 0.8412$ fm. The proton/electron radius ratio: $r_p/r_e \cong \tanh S^{-1} \cong 2A$, with the Golden Apex of the Great Pyramid. The inverse ratio: $r_e/r_p \cong 5\pi/\sqrt{7\pi}$. The inradius of the regular pentagon, $\sqrt{25+10\sqrt{5}}/10$, is approximately equal to $-e^{\pi} + 7\pi + \ln 2\pi$. Also, the heptagon diameter divided by the pentagon inradius is approximately equal to r_e/r_p. The approximate value for the inverse fine-structure constant from Eqs. (9-11):

$$\alpha^{-1} \cong 137.035\,999\,168 \quad (6)$$

Latest value by Aoyama et al, $\alpha^{-1} \cong 137.035\,999\,157$ (41), determined from quantum electrodynamic theory and experiment [15]. M. Temple Richmond says that in the esoteric tradition 137 is a representation of the Law of One, the Three Cosmic Laws and the Seven Rays [8]. Given G_w is the Wilbraham-Gibbs constant and the sinc function $\operatorname{sinc} x = \sin x/x$, then:

$$G_W = \int_0^{\pi} \operatorname{sinc} x\,dx \cong \phi \ln\pi \cong e\sin\alpha^{-1} \quad (7)$$

The Wilbraham-Gibbs constant $G_w \cong \sec(1) \cong \phi^2/\sqrt{2} \cong 1.852$. The Wilbraham-Gibbs constant is related to the overshoot of Fourier sums in Gibbs phenomena [16]. $\ln G_w \cong 1/\phi$ and $137 \cong (37+37)G_w$. $G_a/G_w \cong KA \cong 3/2R \cong \sqrt{e/\phi}$, where K is the polygon circumscribing constant. Again with K, the Wilbraham-Gibbs constant $G_w \cong \pi/(K-7)$. The polygon circumscribing constant K [13]:

$$K = \frac{\pi}{2}\prod_{n=1}^{\infty}\operatorname{sinc}\left(\frac{2\pi}{2n+1}\right) = \prod_{n=3}^{\infty}\sec\left(\frac{\pi}{n}\right) \quad (8)$$

The polygon circumscribing constant $K \cong 2 + 3\sqrt{5} \cong \sqrt{11}\,\phi^2 \cong 8.7$. $KA \cong R + A \cong \sqrt{e/\phi} \cong \phi^2/2$. Also, $1+\phi^{-2} \cong K/2\pi \cong \sqrt{6/\pi} \cong \sinh^2(1)$. $KA + RA \cong CQ$ and $K \cong 10/R$. A curious relationship between the *Key* harmonic 528 and the polygon circumscribing constant K, $528/140 \cong K/D$, where $D = 2R$ is the heptagon diameter. The internal angle of the nonagon is 140° and is also the central angle in the ancient Egyptian hieroglyph for gold [10]. $D \cong 85/37 \cong \sqrt{\phi^2+\phi^2}$. $85/11 \cong R/A$ and $528/85 \cong 2\pi$. $440/85 \cong \pi\sqrt{e}$, $137/85 \cong \phi$ and $12 \times 44 = 528$. $528/440 \cong \pi/\phi^2$, see [13]. $504/280 \cong \sqrt{S}$ and $504/440 \cong \ln \pi$. $528/504 \cong 7A$. With the angular harmonic, $\sin \alpha^{-1} \cong 504/85K = 7!/(713+137)K$.

$$\sin \alpha^{-1} \cong 7!/(713+137)K, \tag{9}$$

with the same approximate value for the inverse fine-structure constant as Eq. (6). Plato's favorite symbolic number $5040 = 7!$. $504/396 \cong 108/85 \cong \sqrt{\phi}$. 7920 represents the canonical harmonic of the Earth's diameter. $7920/5040 = 11/7$ and $25920/7920 = (14+22)/11$. 25920 represents the precession of the equinoxes. $2592/1370 \cong 6/\pi$. $792/528 = 3/2$, $792/396 = 2$ and $792/264 = 3$. The harmonic of Newton's gravitational constant, $A^{-1} \cong \ln 792$. $14 \times 20 = 280$ and $22 \times 20 = 440$ $DK \cong 20$. $504 = 7 \times 8 \times 9$ and $\sqrt{14} \cong SR$. Also, $\sin \alpha^{-1} \cong 2 \sin(\pi/9)$. CQ is the approximate circumradius of the nonagon. Other relationships include $\csc \alpha^{-1} - \sin \alpha^{-1} \cong \pi/4$ and $\tan^{-1}(4/\pi) \cong 51.85^{\circ} \cong 2 \times 25.92^{\circ}$, the Great Pyramid base angle. $4/\pi \cong \sqrt{S/2} \cong \sqrt{\phi}$ and $RA \cong 3/2K$. $2.592 \cong 1/\sqrt{A} \cong \sqrt{7}/Q$. $D\sqrt{A} \cong 8/9$ the proportion for "squaring the circle." $D = 2R \cong \csc(\pi/7)$ and $R \cong \cot^2(\alpha^{-1})$. In degrees, the modern golden angle, $G_a = 360^{\circ}/\phi^2$ and the related $26.57^{\circ} \cong \tan^{-1}(1/2)$ is the ancient Golden Angle of Resurrection according to Robert Temple [6]. Along with the Golden Angle of Resurrection, Robert Temple states the Pythagorean Comma P_c was one of the greatest secrets of the ancient Egyptians [6], $P_c \cong \sqrt{2\phi/\pi} \cong \sqrt{7}/\phi^2 \cong 370/365$. $Q^5 \cong P_c{}^7$ and $P_c = (3/2)^{12}/2^7 \cong 1.0136$. Also, $2^{7/12} \cong 3/2 \cong ARK$. Another form of calculation involves the prime constant [13], described as a binary expansion corresponding to an indicator function for the set of prime numbers. The inverse fine-structure constant:

$$\alpha^{-1} \cong 157 - 337P/7, \tag{10}$$

with the same approximate value for the inverse fine-structure constant as determined in Eq. (6), having three prime numbers and the prime constant. The prime constant $P \cong \sqrt{RA} \cong \phi^2/2\pi \cong 220/528 \cong 0.4147$. $PQ \cong 2\sqrt{2}A$ and $PK \cong 2\sqrt{S}$. $KA \cong P\pi \cong e \ln \phi$. $85^2 + 132^2 = 157^2$. Grand Gallery length in the Great Pyramid is 157'. $137 - 90 = 47$ and $47 + 85 = 132$. $132/90 = 528/360 \cong CQ$. $157/288 \cong 288/528$. $180/157 \cong R$ and $337/180 \cong \phi R$. With the *Key*, $528/337 \cong \pi/2$ and $157/528 \cong 2A$, approximate proton/electron radius ratio. With pyramid base, $440/337 \cong \phi^2/2$ and $440/157 \cong \sqrt{3}\phi$. The $\sin 140^{\circ} = \sin 40^{\circ} \cong \sqrt{P}$. $528/140 \cong K/D \cong 7/G_w$ and $140/360 \cong e/7$. The $\sec 40^{\circ} \cong P\sqrt{3S} \cong KA$.

$$\alpha^{-\phi} \cong 2867.2867 + 28672^{-\eta}, \tag{11}$$

which also gives the same approximate value for the inverse fine-structure constant as Eq. (6), where $\eta \cong 365/365.24$ and $\eta^{\phi} \cong -\sin(14/3) \cong 286.7/287 \cong (\sqrt{2}-1)/P$, inverse of the proposed "pyramid inch" by Taylor and Smyth [17]. The Solar Year harmonic is 365.24. The proton/neutron mass ratio is approximately equal to $m_p/m_n \cong \eta^2$. From the pyramid capstone height, $8 \times 286.7 \cong 2\pi \times 365$ [18]. $2.867/2.592 \cong \sqrt{11}/3$ and $25920/2867 \cong 2\pi C$. The ascending and descending passages of the Great Pyramid are displaced 287" to the east [3]. $365/287 \cong \sqrt{\phi}$. $365/286.7 \cong 4/\pi \cong \sqrt{S/2}$ and $528/287 \cong S/\sqrt{\pi} \cong \sqrt{11/S}$. $(3 \times 37)/287 \cong \sqrt{A}$. $P_c{}^{\phi} \cong Q$ and $P_c Q \cong (64/63)^2 \cong \sec(1/4)$. The inverse of the Eye of Horus fractions is $64/63$ [6]. With the polygon circumscribing constant, $\sqrt{7\pi - K} \cong \sqrt{2} + \sqrt{5} \cong 3.65$, again from the harmonic 365 of the capstone height [18]. $504/365 \cong \sqrt{5}/\phi$ and $(286.7 \times 8)/2\pi \cong 365$. $2.867 \cong 3/7A \cong 9/\pi \cong \phi\sqrt{\pi}$. Another expression with alpha, $\alpha^{-\phi} \cong 3\pi 22^3/35$, having prime factors of 3, 5, 7 and 11 with a value approximately the same as Eq. (6). Also, $\alpha^{-1} \cong (\pi/A)^{\phi}$ and again, $2.867 \cong 2\sqrt{2}P_c \cong G_w\sqrt{G_a} \cong (4+\sqrt{3})/2$. The Golden Apex $A \cong (1+\sqrt{5})^{-\phi} \cong 432/2867$. With the Wilbraham-Gibbs constant, $G_w \cong 528/286.7$ and $2867/528 \cong S/4A \cong 2e$. The harmonic of the Solar Year again, $365.24 \cong 2\pi K/A$, $365.24/140 \cong \phi^2$ and $365.24/37 \cong \pi^2$.

IV. Conclusion

The height of the gold pyramidion thought to be on the original Great Pyramid, as described by John Michell (with the metrology of Algernon Berriman) [19], is $0.152 \cong AQ \cong 11.7/(7 \times 11)$, $\sqrt{137} \cong 11.7$ and the tenth part of the Greek cubit of $1.52 \cong \pi - \phi$. The modern golden angle in radians, $G_a \cong 365/152$. This small pyramidion was supposed to be the final top and possibly made of something like transmuted gold, similar to the legendary Golden Sun Disc of Mu. Reports about stargates, resurrection and the replication of golden pyramids in the hyperdimensional aether suggest the ancient Egyptian initiates were well-informed by Thoth, "Architect of the Great Pyramid." They knew about the mathematical constants, fundamental physical constants and advanced mathematical functions inherent in the analysis and applications of the golden ratio vortices of the quintessential dynamic aether [20]-[23]. According to Manly P. Hall, the capstone of the Great Pyramid was associated with the Eye of Horus. "The exact science of human regeneration is ... when the Spirit Fire is lifted up through the ... spinal column ... passes into the pituitary body (Isis), where it invokes Ra (the pineal gland) and ... the Eye of Horus is opened." [24].

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Cv Bipartite Entanglement of Non-Degenerate Three-Level Laser with Squeezed Modes Pumped by Coherent Light

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Keywords: *continuous variables, bipartite, entanglement, quadrature squeezing, correlations, two-mode subharmonic generator.*

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CVBIPARTITEENTANGLEMENTOFNONDEGENERATETHREELEVELLASERWITHSQUEEZEDMODESPUMPEDBYCOHERENTLIGHT

Strictly as per the compliance and regulations of :

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Getachew A. Gebru [α] & Solomon Getahun [σ]

Abstract- We analyze the CV bipartite entanglement of the light generated by a coherently pumped non degenerate three-level laser with a two-mode subharmonic generator coupled to a two-mode vacuum reservoir via a single-port mirror, whose open cavity contains N non-degenerate three- level cascade atoms. We carry out our analysis by putting the noise operators associated with a vacuum reservoir in normal order. It is found that the photon-state of the system is strongly entangled at steady state where as the atom state of the system is not entangled. We have also shown that as the stimulated decay constant increases, the degree of entanglement increases. In addition, we have established that the photons in the laser cavity are highly correlated and the photon correlation and entanglement increases as the amplitude of the coherent light driving the pump mode increases. Moreover, we have realized that the presence of the subharmonic generator leads to an increase in the degree of entanglement and correlations.

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I. Introduction

One of the most fundamentally interesting and intriguing phenomena associated with the composite quantum system is entanglement. In recent years, the topic of continuous-variable entanglement has received a significant amount of attention as it plays an important role in all branches of quantum information processing [1]. The efficiency of quantum information schemes highly depends on the degree of entanglement. A two-mode subharmonic generator at and above threshold has been theoretically predicted to be a source of light in an entangled state [2,3]. Recently, the experimental realization of the entanglement in twomode subharmonic generator has been demonstared by Zhang et al [4]. On the other hand, Xiong et al [5] have recently proposed a scheme for an entanglement based on a non-degenerate three level laser when the three-level atoms are injected at the lower level and the top and bottom levels are coupled by a strong coherent light. They have found that a non-degenerate three-level laser can generate light in an entangled state employing the entanglement criteria for Bipartite continuous-variable state [5].

Moreover, Tan et al [6] extended the work of xiong et al. and examined and the generation and evolution of the entangled ligth in the Wigner representation using the sufficient and necessary inseparability criteria for a two-mode Gaussian state proposed by Duan et al [5] and simon [7]. Tesfa [8] have considered a similar system when the atomic coherence is induced by superposition of atomic states and analyzed the entanglement at steady state. Furthermore, Ooi [9] has studied the steady-state entanglement in a two-mode Λ laser.

More recently, Eyob [10] has studied continuous-variable entanglement in non-degenerate three-level laser with a parametric amplifier. In this model the injected atomic coherence introduced by initially preparing the atoms in a coherent superposition of the top and bottom levels. in addition, to exhibiting a two-mode squeezed light, this combined system produces light in an entangled state. In one model of such a laser, three-level atoms initially in the upper level are injected at a constant rate into the cavity and removed after they have decayed due to spontaneous emission. It appears to be quite difficult to prepare the atoms in a coherent superpositions of the top and bottom levels before they are injected into the laser cavity. Beside, it should certainly be hard to find out that the atoms have decayed spontaneously before they are removed from the cavity.

In order to avoid the aforementioned problems, Fesseha [11] have considered that N two-level atoms available in a closed cavity are pumped to the top level by means of electron bombardment. He has shown that the light generated by this laser operating well above threshold is coherent and the light generated by the same laser operating below threshold is chaotic. In addition, Fesseha [12,13] has studied the squeezing and the statistical properties of the light produced by a degenerate three- level laser with the atoms in a closed cavity and pumped by electron bombardment. He has shown that the maximum quadrature squeezing of the light generated by the laser, operating far below threshold, is 50% below the coherent-state level. Alternatively, the three-level atoms available in a closed cavity and pumped by coherent light also generated

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squeezed light under certain conditions, with the maximum quadrature squeezing being 43% below the coherent-state level. In view of these results, better squeezing is found from the laser, in which the atoms are pumped by electron bombardment than by coherent light.

In this paper, we seek to study CV bipartite entanglement for the light generated by a coherently pumped non-degenerate three-level laser with a two-mode subharmonic generator coupled to a two-mode vacuum reservoir via a single-port mirror whose open cavity contains N non-degenerate three-level cascade atoms. In order to carry out our analysis, we put the noise operators associated with the vacuum reservoir in the normal order and by considering the interaction of the three-level atoms with a two-mode vacuum reservoir outside the cavity. We then first drive the quantum Langevin equations for the cavity mode operators. We next determine the equations of evolution of the expectation values of atomic operators employing the pertinent master equation. Applying the steady-state solutions of the equations of evolution of atomic and cavity mode operators, we analyze the CV atomic and photon state entanglement as well as atom and photon state correlations.

II. Model and Dynamics of Atomic and Cavity Mode Operators

We consider a coherently pumped non-degenerate three-level laser with two-mode subharmonic generator coupled to a two-mode vacuum reservoir whose cavity contains N non-degenerate three-level atoms in cascade configuration as depicted in Fig 1. For the sake of convenience, we denote the top, middle, and bottom levels of these atoms by $|2\rangle_j$, $|1\rangle_j$ and $|0\rangle_j$ respectively. We seek to represent the light emitted from the top level by $\hat{a}_2$ and the light emitted from the middle by $\hat{a}_1$. In addition, in order to expedite the cascading process, it is assumed that the parity of energy levels $|2\rangle_j$ and $|0\rangle_j$ is the same, where as that of $|1\rangle_j$, is different. This entails that direct transition between energy level $|2\rangle_j$ and $|0\rangle_j$ are electric dipole forbidden but due to parity difference, the transition between $|2\rangle_j \rightarrow |1\rangle_j$, and $|1\rangle_j \rightarrow |0\rangle_j$ are allowed.

The interaction of one of the three-level atoms with light modes a_1 and a_2 can be described at resonance by the Hamiltonian

$$\hat{H}_1(t) = ig\Big[\hat{\sigma}_1^{\dagger j}(t)\hat{a}_1(t) - \hat{a}_1^\dagger(t)\hat{\sigma}_1^j(t) + \hat{\sigma}_2^{\dagger j}(t)\hat{a}_2(t) - \hat{a}_2^\dagger(t)\hat{\sigma}_2^j(t)\Big], \tag{1}$$

where

$$\hat{\sigma}_1^j = |0\rangle_{jj}\langle 1| \tag{2}$$

and

$$\hat{\sigma}_2^j = |1\rangle_{jj}\langle 2| \tag{3}$$

are lowering atomic operators, $\hat{a}_1(t)$ and $\hat{a}_2(t)$ is the annihilation operators for light modes a_1 and a_2, and g is the coupling constant between the atom and the cavity modes.

On the other hand, a pump mode photon of frequency (ω_3) directly interacts with the nonlinear crystal (NLC) to produce the signal-idler photon pairs having different frequencies as the two cavity modes. Furthermore, we consider the case for which the pump mode emerging from the NLC (or two-mode subharmonic) does not couple the top and bottom levels. This could be realized by putting on the right-hand side of the NLC a screen which absorbs the pump mode. The top and bottom levels of the three-level atoms are coupled by a strong driving coherent light with the frequency (ω_4). The coupling of the top and bottom levels of a three-level atom by coherent light can be described at resonance by the Hamiltonian

$$\hat{H}_2(t) = \frac{i\Omega}{2}\left(\hat{\sigma}_0^{\dagger j}(t) - \hat{\sigma}_0^j(t)\right), \tag{4}$$

in which

$$\hat{\sigma}_0^j = |0\rangle_{jj}\langle 2| \tag{5}$$

and

$$\Omega = 2\mu_0\lambda_0. \tag{6}$$

Here, μ_0 is the amplitude of the driving coherent light and λ_0 is the coupling constant between the driving coherent light and the three-level atom. Moreover, in a two-mode subharmonic generator, a pump photon of frequency ω_3 is down converted into highly correlated signal and idler photons with frequencies ω_1 and ω_2 such as $\omega_3 = \omega_1 + \omega_2$ [13]. This quantum optical process leads to the generation of squeezed light. With the pump mode treated classically represented by a real and constant c-number μ, the process of two-mode subharmonic generation can be described by the Hamiltonian

$$\hat{H}_3(t) = i\varepsilon\left(\hat{a}_1^\dagger\hat{a}_2^\dagger - \hat{a}_1\hat{a}_2\right), \tag{7}$$

in which $\varepsilon = 2\eta_0\mu$, with η_0 is the coupling constant between the pump mode and nonlinear crystal and μ is proportional to the amplitude of the coherent light driving the pump mode. Thus upon combining Eqs. (1), (4), and (7), the interaction of the three-level atoms with the cavity modes and the driving coherent light, and the parametric down-conversion can be described by the Hamiltonian

$$\hat{H}_s(t) = ig\Big[\hat{\sigma}_1^{\dagger j}(t)\hat{a}_1(t) - \hat{a}_1^{\dagger}(t)\hat{\sigma}_1^{j}(t) + \hat{\sigma}_2^{\dagger j}(t)\hat{a}_2(t) - \hat{a}_2^{\dagger}(t)\hat{\sigma}_2^{j}(t)\Big] + i\varepsilon\Big(\hat{a}_1^{\dagger}\hat{a}_2^{\dagger} - \hat{a}_1\hat{a}_2\Big) + \frac{i\Omega}{2}\Big(\hat{\sigma}_0^{\dagger j}(t) - \hat{\sigma}_0^{j}(t)\Big). \quad (8)$$

The master equation for a pair of cavity modes coupled to a two-mode vacuum reservoir has the form [14]

$$\frac{d}{dt}\hat{\rho}(t) = -i\Big[\hat{H}_s(t), \hat{\rho}(t)\Big] + \frac{\beta}{2}\Big[2\hat{\sigma}_1^{j}\hat{\rho}\hat{\sigma}_1^{\dagger j} - \hat{\sigma}_1^{\dagger j}\hat{\sigma}_1^{j}\hat{\rho} - \hat{\rho}\hat{\sigma}_1^{\dagger j}\hat{\sigma}_1^{j}\Big] + \frac{\beta}{2}\Big[2\hat{\sigma}_0^{j}\hat{\rho}\hat{\sigma}_0^{\dagger j} - \hat{\sigma}_0^{\dagger j}\hat{\sigma}_0^{j}\hat{\rho} - \hat{\rho}\hat{\sigma}_0^{\dagger j}\hat{\sigma}_0^{j}\Big], \quad (9)$$

where γ is the spontaneous emission decay constant. Now with the aid of Eq. (8), one can put Eq. (9) in the form

$$\begin{aligned}\frac{d}{dt}\hat{\rho}(t) = {} & g\Big[\hat{\sigma}_1^{\dagger j}\hat{a}_1\hat{\rho} - \hat{a}_1^{\dagger}\hat{\sigma}_1^{j}\hat{\rho} + \hat{\sigma}_2^{\dagger j}\hat{a}_2\hat{\rho} - \hat{a}_2^{\dagger}\hat{\sigma}_2^{j}\hat{\rho}\Big] \\ & -g\Big[\hat{\rho}\hat{\sigma}_1^{\dagger j}\hat{a}_1 - \hat{\rho}\hat{a}_1^{\dagger}\hat{\sigma}_1^{j} + \hat{\rho}\hat{\sigma}_2^{\dagger j}\hat{a}_2 - \hat{\rho}\hat{a}_2^{\dagger}\hat{\sigma}_2^{j}\Big] \\ & +\varepsilon\Big[\hat{a}_1^{\dagger}\hat{a}_2^{\dagger}\hat{\rho} - \hat{a}_1\hat{a}_2\hat{\rho} - \hat{\rho}\hat{a}_1^{\dagger}\hat{a}_2^{\dagger} + \hat{\rho}\hat{a}_1\hat{a}_2\Big] \\ & +\frac{\Omega}{2}\Big[\hat{\sigma}_0^{\dagger j}\hat{\rho} - \hat{\sigma}_0^{j}\hat{\rho} + \hat{\rho}\hat{\sigma}_0^{j} - \hat{\rho}\hat{\sigma}_0^{\dagger j}\Big] \\ & +\frac{\beta}{2}\Big[2\hat{\sigma}_1^{j}\hat{\rho}\hat{\sigma}_1^{\dagger j} - \hat{\sigma}_1^{\dagger j}\hat{\sigma}_1^{j}\hat{\rho} - \hat{\rho}\hat{\sigma}_1^{\dagger j}\hat{\sigma}_1^{j}\Big] \\ & +\frac{\beta}{2}\Big[2\hat{\sigma}_0^{j}\hat{\rho}\hat{\sigma}_0^{\dagger j} - \hat{\sigma}_0^{\dagger j}\hat{\sigma}_0^{j}\hat{\rho} - \hat{\rho}\hat{\sigma}_0^{\dagger j}\hat{\sigma}_0^{j}\Big] \quad (10)\end{aligned}$$

We recall that the laser cavity is coupled to a two-mode vacuum reservoir via a single-port mirror. In addition, we carry out our analysis by putting the noise operators associated with the vacuum reservoir in normal order. Thus the noise operators will not have any effect on the dynamics of the cavity mode operators [13]. In view of this, we can drop the noise operators and write the quantum Langevin equation for the operators $\hat{a}_1$ and $\hat{a}_2$ as

$$\frac{d}{dt}\hat{a}_1(t) = -\frac{k}{2}\hat{a}_1(t) - i\Big[\hat{a}_1(t), \hat{H}_s(t)\Big] \quad (11)$$

and

$$\frac{d}{dt}\hat{a}_2(t) = -\frac{k}{2}\hat{a}_2(t) - i\Big[\hat{a}_2(t), \hat{H}_s(t)\Big], \quad (12)$$

where k is the cavity damping constant for the light modes a_1 and a_2. Then with the aid of Eqs. (8), (11), and (12), we easily

$$\frac{d}{dt}\hat{a}_2(t) = -\frac{k}{2}\hat{a}_1(t) + \varepsilon\hat{a}_2^{\dagger}(t) - g\hat{\sigma}_1^{j}, \quad (13)$$

$$\frac{d}{dt}\hat{a}_2(t) = -\frac{k}{2}\hat{a}_2(t) + \varepsilon\hat{a}_1^{\dagger}(t) - g\hat{\sigma}_2^{j}. \quad (14)$$

Making use of the pertinent master equation and the fact that $\frac{d}{dt}\langle\hat{A}\rangle = Tr\left(\frac{d\hat{\rho}(t)}{dt}\hat{A}\right)$ where ($\hat{A}$ is an operator), it is not difficult to verify that

$$\frac{d}{dt}\left\langle\hat{\sigma}_1^{j}\right\rangle = g\Big[\left\langle\hat{n}_0^{j}\hat{a}_1\right\rangle - \left\langle\hat{n}_1^{j}\hat{a}_1\right\rangle - \langle\hat{a}_2^{\dagger}\hat{\sigma}_0^{j}\rangle\Big] - \frac{\Omega}{2}\langle\hat{\sigma}_2^{\dagger j}\rangle - \frac{\beta}{2}\langle\hat{\sigma}_1^{j}\rangle, \quad (15)$$

$$\frac{d}{dt}\left\langle\hat{\sigma}_2^{j}\right\rangle = g\Big[\left\langle\hat{n}_1^{j}\hat{a}_2\right\rangle - \left\langle\hat{n}_2^{j}\hat{a}_2\right\rangle + \langle\hat{a}_1^{\dagger}\hat{\sigma}_0^{j}\rangle\Big] + \frac{\Omega}{2}\langle\hat{\sigma}_1^{\dagger j}\rangle - \frac{\beta}{2}\langle\hat{\sigma}_2^{j}\rangle, \quad (16)$$

$$\frac{d}{dt}\langle\hat{\sigma}_0^j\rangle = g\Big[\langle\hat{\sigma}_1^j\hat{a}_2\rangle - \langle\hat{\sigma}_2^j\hat{a}_1\rangle\Big] + \frac{\Omega}{2}\Big[\langle\hat{n}_0^j\rangle - \langle\hat{n}_2^j\rangle\Big] - \frac{\beta}{2}\langle\hat{\sigma}_0^j\rangle, \quad (17)$$

$$\frac{d}{dt}\langle\hat{n}_1^j\rangle = g\Big[\langle\hat{\sigma}_1^{\dagger j}\hat{a}_1\rangle + \langle\hat{a}_1^\dagger\hat{\sigma}_1^j\rangle - \langle\hat{\sigma}_2^{\dagger j}\hat{a}_2\rangle - \langle\hat{a}_2^\dagger\hat{\sigma}_2^j\rangle\Big] + \beta\Big[\langle\hat{n}_2^j - \langle\hat{n}_1^j\rangle\Big], \quad (18)$$

$$\frac{d}{dt}\langle\hat{n}_2^j\rangle = g\Big[\langle\hat{\sigma}_2^{\dagger j}\hat{a}_2\rangle + \langle\hat{a}_2^\dagger\hat{\sigma}_2^j\rangle\Big] + \frac{\Omega}{2}\Big[\langle\hat{\sigma}_0^{\dagger j}\rangle + \langle\hat{\sigma}_0^j\rangle\Big] - \beta\langle\hat{n}_2^j\rangle, \quad (19)$$

$$\frac{d}{dt}\langle\hat{n}_0^j\rangle = -g\Big[\langle\hat{\sigma}_1^{\dagger j}\hat{a}_1\rangle + \langle\hat{a}_1^\dagger\hat{\sigma}_1^j\rangle\Big] - \frac{\Omega}{2}\Big[\langle\hat{\sigma}_0^{\dagger j}\rangle + \langle\hat{\sigma}_0^j\rangle\Big] + \beta\Big[\langle\hat{n}_1^j\rangle + \langle\hat{n}_2^j\rangle\Big], \quad (20)$$

where

$$\hat{n}_0^j = |0\rangle_{jj}\langle 0|, \quad (21)$$

$$\hat{n}_1^j = |1\rangle_{jj}\langle 1|, \quad (22)$$

$$\hat{n}_2^j = |2\rangle_{jj}\langle 2|. \quad (23)$$

We see that Eqs. (15)-(20) are nonlinear and coupled differential equations. Therefore, it is not possible to obtain the exact time-dependent solutions. We intend to overcome this problem by applying the large-time approximation [13]. Then using this approximation scheme, we get from Eqs. (13) and (14) the approximately valid relations

$$\hat{a}_1 = \frac{2\varepsilon}{k}\hat{a}_2^\dagger - \frac{2g}{k}\hat{\sigma}_1^j \quad (24)$$

and

$$\hat{a}_2 = \frac{2\varepsilon}{k}\hat{a}_1^\dagger - \frac{2g}{k}\hat{\sigma}_2^j. \quad (25)$$

Evidently, these turn out to be exact relations at steady state. Solving these equation simultaneously, one easily verify that

$$\hat{a}_1 = -\frac{4\varepsilon g}{(k^2-4\varepsilon^2)}\hat{\sigma}_2^{\dagger j} - \frac{2gk}{(k^2-4\varepsilon^2)}\hat{\sigma}_1^j \quad (26)$$

and

$$\hat{a}_2 = -\frac{4\varepsilon g}{(k^2-4\varepsilon^2)}\hat{\sigma}_1^{\dagger j} - \frac{2gk}{(k^2-4\varepsilon^2)}\hat{\sigma}_2^j. \quad (27)$$

Now introducing (26) and (27), into Eqs. (15)-(20), we get

$$\frac{d}{dt}\langle\hat{\sigma}_1^j\rangle = -\frac{1}{2}\Big[\beta + \frac{\gamma_c k^2}{k^2-4\varepsilon^2}\Big]\langle\hat{\sigma}_1^j\rangle - \frac{\Omega}{2}\langle\hat{\sigma}_2^{\dagger j}\rangle, \quad (28)$$

$$\frac{d}{dt}\langle\hat{\sigma}_2^j\rangle = -\frac{1}{2}\Big[\beta + \frac{\gamma_c k^2}{k^2-4\varepsilon^2}\Big]\langle\hat{\sigma}_2^j\rangle - \frac{1}{2}\Big[\frac{2\gamma_c\varepsilon k}{k^2-4\varepsilon^2} - \Omega\Big]\langle\hat{\sigma}_1^{\dagger j}\rangle, \quad (29)$$

$$\frac{d}{dt}\langle\hat{\sigma}_0^j\rangle = -\frac{1}{2}\Big[\beta + \frac{\gamma_c k^2}{k^2-4\varepsilon^2}\Big]\langle\hat{\sigma}_0^j\rangle + \frac{\gamma_c\varepsilon k}{k^2-4\varepsilon^2}\Big[\langle\hat{n}_1^j\rangle - \langle\hat{n}_0^j\rangle\Big] + \frac{\Omega}{2}\Big[\langle\hat{n}_0^j\rangle - \langle\hat{n}_2^j\rangle\Big], \quad (30)$$

$$\frac{d}{dt}\langle\hat{n}_1^j\rangle = -\Big[\beta + \frac{\gamma_c k^2}{k^2-4\varepsilon^2}\Big] \times \Big[\langle\hat{n}_1^j\rangle - \langle\hat{n}_2^j\rangle\Big] + \frac{\gamma_c\varepsilon k}{k^2-4\varepsilon^2}\Big[\langle\hat{\sigma}_0^{\dagger j}\rangle + \langle\hat{\sigma}_0^j\rangle\Big], \quad (31)$$

$$\frac{d}{dt}\langle\hat{n}_2^j\rangle = -\Big[\beta + \frac{\gamma_c k^2}{k^2-4\varepsilon^2}\Big]\langle\hat{n}_2^j\rangle - \frac{1}{2}\Big[\frac{2\gamma_c\varepsilon k}{k^2-4\varepsilon^2} - \Omega\Big] \times \Big[\langle\hat{\sigma}_0^{\dagger j}\rangle + \langle\hat{\sigma}_0^j\rangle\Big], \quad (32)$$

$$\frac{d}{dt}\langle\hat{n}_0^j\rangle = \left[\beta + \frac{\gamma_c k^2}{k^2-4\varepsilon^2}\right]\langle\hat{n}_1^j\rangle - \frac{\Omega}{2}\left[\langle\hat{\sigma}_0^{\dagger j}\rangle + \langle\hat{\sigma}_0^j\rangle\right] + \beta\langle\hat{n}_2^j\rangle, \tag{33}$$

where

$$\gamma_c = \frac{4g^2}{k} \tag{34}$$

is the stimulated emission decay constant.

We next sum Eqs. (29)-(33) over the N three-level atoms, so that

$$\frac{d}{dt}\langle\hat{\Sigma}_1\rangle = -\frac{1}{2}\left[\beta + \frac{\gamma_c k^2}{k^2-4\varepsilon^2}\right]\langle\hat{\Sigma}_1\rangle - \frac{\Omega}{2}\langle\hat{\Sigma}_2^\dagger\rangle, \tag{35}$$

$$\frac{d}{dt}\langle\hat{\Sigma}_2\rangle = -\frac{1}{2}\left[\beta + \frac{\gamma_c k^2}{k^2-4\varepsilon^2}\right]\langle\hat{\Sigma}_2\rangle - \frac{1}{2}\left[\frac{2\gamma_c\varepsilon k}{k^2-4\varepsilon^2} - \Omega\right]\langle\hat{\Sigma}_1^\dagger\rangle, \tag{36}$$

$$\frac{d}{dt}\langle\hat{\Sigma}_0\rangle = -\frac{1}{2}\left[\beta + \frac{\gamma_c k^2}{k^2-4\varepsilon^2}\right]\langle\hat{\Sigma}_0\rangle + \frac{\gamma_c\varepsilon k}{k^2-4\varepsilon^2}\left[\langle\hat{N}_1\rangle - \langle\hat{N}_0\rangle\right] + \frac{\Omega}{2}\left[\langle\hat{N}_0\rangle - \langle\hat{N}_2\rangle\right], \tag{37}$$

$$\frac{d}{dt}\langle\hat{N}_1\rangle = -\left[\beta + \frac{\gamma_c k^2}{k^2-4\varepsilon^2}\right] \times \left[\langle\hat{N}_1\rangle - \langle\hat{N}_2\rangle\right] + \frac{\gamma_c\varepsilon k}{k^2-4\varepsilon^2}\left[\langle\hat{\Sigma}_0^\dagger\rangle + \langle\hat{\Sigma}_0\rangle\right], \tag{38}$$

$$\frac{d}{dt}\langle\hat{N}_2\rangle = -\left[\beta + \frac{\gamma_c k^2}{k^2-4\varepsilon^2}\right]\langle\hat{n}_2^j\rangle - \frac{1}{2}\left[\frac{2\gamma_c\varepsilon k}{k^2-4\varepsilon^2} - \Omega\right] \times \left[\langle\hat{\Sigma}_0^\dagger\rangle + \langle\hat{\Sigma}_0\rangle\right], \tag{39}$$

$$\frac{d}{dt}\langle\hat{N}_0\rangle = \left[\beta + \frac{\gamma_c k^2}{k^2-4\varepsilon^2}\right]\langle\hat{N}_1\rangle - \frac{\Omega}{2}\left[\langle\hat{\Sigma}_0^\dagger\rangle + \langle\hat{\Sigma}_0\rangle\right] + \beta\langle\hat{N}_2\rangle, \tag{40}$$

in which

$$\hat{\Sigma}_1 = \sum_{j=1}^{N}\hat{\sigma}_1^j, \tag{41}$$

$$\hat{\Sigma}_2 = \sum_{j=1}^{N}\hat{\sigma}_2^j, \tag{42}$$

$$\hat{\Sigma}_0 = \sum_{j=1}^{N}\hat{\sigma}_0^j, \tag{43}$$

$$\hat{N}_0 = \sum_{j=1}^{N}\hat{n}_0^j, \tag{44}$$

$$\hat{N}_1 = \sum_{j=1}^{N}\hat{n}_1^j, \tag{45}$$

$$\hat{N}_2 = \sum_{j=1}^{N}\hat{n}_2^j, \tag{46}$$

with the operators $\hat{N}_2$, $\hat{N}_1$, and $\hat{N}_0$ representing the number of atoms in the top, middle, and bottom levels. In addition, employing the completeness relation

$$\hat{n}_0^j + \hat{n}_1^j + \hat{n}_2^j = \hat{I}, \tag{47}$$

we easily arrive at

$$\langle\hat{N}_0\rangle + \langle\hat{N}_1\rangle + \langle\hat{N}_2\rangle = N. \tag{48}$$

Furthermore, applying the definition given by Eq. (2) and setting for any j

$$\hat{\sigma}_1^j = |0\rangle\langle 1|, \tag{49}$$

we have

$$\hat{\Sigma}_1 = N|0\rangle\langle 1|. \tag{50}$$

Following the same procedure, one can easily find

$$\hat{\Sigma}_2 = N|1\rangle\langle 2|, \tag{51}$$

$$\hat{\Sigma}_0 = N|0\rangle\langle 2|, \tag{52}$$

$$\hat{N}_0 = N|0\rangle\langle 0|, \tag{53}$$

$$\hat{N}_1 = N|1\rangle\langle 1|, \tag{54}$$

$$\hat{N}_2 = N|2\rangle\langle 2|. \quad (55)$$

Moreover, using the definition

$$\hat{\Sigma} = \hat{\Sigma}_1 + \hat{\Sigma}_2 \quad (56)$$

and taking into account Eqs. (50)-(55), it can be readily established that

$$\hat{\Sigma}^\dagger\hat{\Sigma} = N(\hat{N}_1 + \hat{N}_2), \quad (57)$$

$$\hat{\Sigma}\hat{\Sigma}^\dagger = N(\hat{N}_0 + \hat{N}_1), \quad (58)$$

$$\hat{\Sigma}^2 = N\hat{\Sigma}_0. \quad (59)$$

We next seek to calculate the expectation value of the atomic operators $\hat{\Sigma}_0$, $\hat{\Sigma}_1$, and $\hat{\Sigma}_2$, To this end, applying the large time approximation scheme to Eqs. (36) and (37), we easily get

$$\langle\hat{\Sigma}_0\rangle = \frac{1}{(\gamma_c k^2 + \beta k^2 - 4\beta\varepsilon^2)}\Big\{\Omega\left[k^2 - 4\varepsilon^2\right] \times \left[N - \langle\hat{N}_1\rangle - 2\langle\hat{N}_2\rangle\right] - 2\gamma_c\varepsilon k\left[N - \langle\hat{N}_2\rangle - 2\langle\hat{N}_1\rangle\right]\Big\}, \quad (60)$$

$$\langle\hat{\Sigma}_2\rangle = \frac{1}{2}\left[\frac{\Omega k^2 - 4\Omega\varepsilon^2 - 2\gamma_c\varepsilon k}{\gamma_c k^2 + \beta k^2 - 4\beta\varepsilon^2}\right] \times\langle\hat{\Sigma}_1^\dagger\rangle \quad (61)$$

and in view of the adjoint of (61), Eq. (35) takes the form

$$\frac{d}{dt}\langle\hat{\Sigma}_1(t)\rangle = -\frac{1}{2}\eta\langle\hat{\Sigma}_1(t)\rangle, \quad (62)$$

where

$$\eta = \beta + \frac{\gamma_c k^2}{k^2 - 4\varepsilon^2} + \frac{\Omega}{2}\left[\frac{\Omega k^2 - 4\Omega\varepsilon^2 - 2\gamma_c\varepsilon k}{\gamma_c k^2 + \beta k^2 - 4\beta\varepsilon^2}\right]. \quad (63)$$

We notice that the steady-state solution of Eq. (62) for η different from zero is

$$\langle\hat{\Sigma}_1(t)\rangle = 0. \quad (64)$$

Now on account of (64), one can write (61) in the form

$$\langle\hat{\Sigma}_2(t)\rangle = 0. \quad (65)$$

III. CORRELATIONS

In this section we seek to analyze the degree of photon number and atomic-number correlation. In addition, we wish to study the entanglement of photon-state and atomic states in the laser cavity. In the cascading transition from energy level $|2\rangle_j$ to $|0\rangle_j$ via $|1\rangle_j$, a correlation between the two emitted photons a_1 and a_2 can readily be established. Hence the photon number correlation for the cavity modes can be defined as

$$g\left(\hat{n}_1, \hat{n}_2\right)_p = \frac{\langle\hat{a}_1^\dagger(t)\hat{a}_1(t)\hat{a}_2^\dagger(t)\hat{a}_2(t)\rangle}{\langle\hat{a}_1^\dagger(t)\hat{a}_1(t)\rangle\langle\hat{a}_2^\dagger(t)\hat{a}_2(t)\rangle}. \quad (66)$$

On the other hand, using Eqs. (13) and (14) together with (26) and (27), the equation of evolution of cavity mode operators $\hat{a}_1$ and $\hat{a}_2$ can be rewritten as

$$\frac{d}{dt}\hat{a}_1(t) = -\frac{1}{2}\left[\frac{k^2 - 4\varepsilon^2}{k}\right]\hat{a}_1(t) - g\hat{\sigma}_1^j - \frac{2g\varepsilon}{k}\hat{\sigma}_2^{\dagger j}, \quad (67)$$

$$\frac{d}{dt}\hat{a}_2(t) = -\frac{1}{2}\left[\frac{k^2 - 4\varepsilon^2}{k}\right]\hat{a}_2(t) - g\hat{\sigma}_2^j - \frac{2g\varepsilon}{k}\hat{\sigma}_1^{\dagger j}. \quad (68)$$

Applying the steady state solution of Eqs. (67) and (68), one readily established the commutation relation of thecavity mode operator $\hat{a}_1$ and $\hat{a}_1^\dagger$ as well as $\hat{a}_2$ and $\hat{a}_2^\dagger$.

Hence, we notice that

$$\left[\hat{a}_1, \hat{a}_1^\dagger\right]_j = \frac{\gamma_c k}{(k^2 - 4\varepsilon^2)^2}\Big[k^2\left(\hat{n}_0^j - \hat{n}_1^j\right) + 4\varepsilon^2\left(\hat{n}_2^j - \hat{n}_1^j\right) + 2\varepsilon k\left(\hat{\sigma}_0^{\dagger j} + \hat{\sigma}_0^{\dagger j}\right)\Big], \quad (69)$$

$$\left[\hat{a}_2, \hat{a}_2^\dagger\right]_j = \frac{\gamma_c k}{(k^2 - 4\varepsilon^2)^2}\Big[k^2\left(\hat{n}_1^j - \hat{n}_2^j\right) + 4\varepsilon^2\left(\hat{n}_1^j - \hat{n}_0^j\right) - 2\varepsilon k\left(\hat{\sigma}_0^{\dagger j} + \hat{\sigma}_0^{\dagger j}\right)\Big], \quad (70)$$

and summing over all atoms, we obtain

$$\left[\hat{a}_1,\hat{a}_1^\dagger\right] = \frac{\gamma_c k}{(k^2-4\varepsilon^2)^2}\Big[k^2\left(\hat{N}_0-\hat{N}_1\right) +4\varepsilon^2\left(\hat{N}_2-\hat{N}_1\right) +2\varepsilon k\left(\hat{\Sigma}_0^\dagger+\hat{\Sigma}_0\right)\Big], \quad (71)$$

$$\left[\hat{a}_2,\hat{a}_2^\dagger\right] = \frac{\gamma_c k}{(k^2-4\varepsilon^2)^2}\Big[k^2\left(\hat{N}_1-\hat{N}_2\right) +4\varepsilon^2\left(\hat{N}_1-\hat{N}_0\right) -2\varepsilon k\left(\hat{\Sigma}_0^\dagger+\hat{\Sigma}_0\right)\Big], \quad (72)$$

where

$$\left[\hat{a}_i,\hat{a}_k^\dagger\right] = \delta_{ik}\sum_{j=1}^{N}\left[\hat{a}_i,\hat{a}_k^\dagger\right]_j \quad (73)$$

stands for the commutators of $\left(\hat{a}_1,\hat{a}_1^\dagger\right)$ and $\left(\hat{a}_2,\hat{a}_2^\dagger\right)$ when the cavity light modes α_1 and α_2 is interacting with all the N three-level atoms.

In the presence of N three-level atoms, we rewrite Eqs. (67) and (68) as

$$\frac{d}{dt}\hat{a}_1(t) = -\frac{1}{2}\left[\frac{k^2-4\varepsilon^2}{k}\right]\hat{a}_1(t) +\lambda_1'\hat{\Sigma}_1+\lambda_1''\hat{\Sigma}_2^\dagger, \quad (74)$$

$$\frac{d}{dt}\hat{a}_2(t) = -\frac{1}{2}\left[\frac{k^2-4\varepsilon^2}{k}\right]\hat{a}_2(t) +\lambda_2'\hat{\Sigma}_2+\lambda_2''\hat{\Sigma}_1^\dagger, \quad (75)$$

in which λ_1', λ_1'', λ_2' and λ_2'' are a constant whose values remains to be fixed. The steady-state solution of Eqs. (74) and (75) is

$$\hat{a}_1 = \frac{2\lambda_1' k}{(k^2-4\varepsilon^2)}\hat{\Sigma}_1+\frac{2\lambda_1'' k}{(k^2-4\varepsilon^2)}\hat{\Sigma}_2^\dagger, \quad (76)$$

$$\hat{a}_2 = \frac{2\lambda_2' k}{(k^2-4\varepsilon^2)}\hat{\Sigma}_2+\frac{2\lambda_2'' k}{(k^2-4\varepsilon^2)}\hat{\Sigma}_1^\dagger. \quad (77)$$

On account of Eq. (76) and (77), the commutation relation for the cavity mode operators is

$$\left[\hat{a}_1,\hat{a}_1^\dagger\right] = \frac{4Nk^2}{(k^2-4\varepsilon^2)^2}\Big[\lambda_1'^2\left(\hat{N}_0-\hat{N}_1\right) +\lambda_1''^2\left(\hat{N}_2-\hat{N}_1\right) +\lambda_1'\lambda_1''\left(\hat{\Sigma}_0^\dagger+\hat{\Sigma}_0\right)\Big], \quad (78)$$

$$\left[\hat{a}_2,\hat{a}_2^\dagger\right] = \frac{4Nk^2}{(k^2-4\varepsilon^2)^2}\Big[\lambda_2'^2\left(\hat{N}_1-\hat{N}_2\right) +\lambda_2''^2\left(\hat{N}_1-\hat{N}_0\right) -\lambda_1'\lambda_1''\left(\hat{\Sigma}_0^\dagger+\hat{\Sigma}_0\right)\Big]. \quad (79)$$

On comparing Eqs. (71) and (78) together with (72) and (79), shows that

$$\lambda_1' = \lambda_2' = \frac{g}{\sqrt{N}} \quad (80)$$

and

$$\lambda_1'' = \lambda_2'' = \frac{2g\varepsilon}{k\sqrt{N}}. \quad (81)$$

Then Eqs. (76) and (77) can be written as

$$\hat{a}_1 = \frac{2kg}{\sqrt{N}\,(k^2-4\varepsilon^2)}\hat{\Sigma}_1 +\frac{4g\varepsilon}{\sqrt{N}\,(k^2-4\varepsilon^2)}\hat{\Sigma}_2^\dagger \quad (82)$$

and

$$\hat{a}_2 = \frac{2kg}{\sqrt{N}\,(k^2-4\varepsilon^2)}\hat{\Sigma}_2 +\frac{4g\varepsilon}{\sqrt{N}\,(k^2-4\varepsilon^2)}\hat{\Sigma}_1^\dagger. \quad (83)$$

Furthermore, the expectation value of the solution of Eqs. (74) and (75) together with (80) and (81) is expressible as

$$\langle\hat{a}_1(t)\rangle = \langle\hat{a}_1(0)\rangle e^{-\frac{1}{2}\eta_0 t}+\frac{g}{\sqrt{N}}e^{-\frac{1}{2}\eta_0 t} \times\int_o^t e^{\frac{1}{2}\eta_0 t'}\langle\hat{\Sigma}_1(t')\rangle +\frac{2g\varepsilon}{k\sqrt{N}}e^{-\frac{1}{2}\eta_0 t}\int_o^t e^{\frac{1}{2}\eta_0 t'}\langle\hat{\Sigma}_2^\dagger(t')\rangle \quad (84)$$

and

$$\langle \hat{a}_2(t)\rangle = \langle \hat{a}_2(0)\rangle e^{-\frac{1}{2}\eta_0 t} + \frac{g}{\sqrt{N}} e^{-\frac{1}{2}\eta_0 t} \int_o^t e^{\frac{1}{2}\eta_0 t'} \langle \hat{\Sigma}_2(t')\rangle + \frac{2g\varepsilon}{k\sqrt{N}} e^{-\frac{1}{2}\eta_0 t} \int_o^t e^{\frac{1}{2}\eta_0 t'} \langle \hat{\Sigma}_1^\dagger(t')\rangle, \quad (85)$$

where

$$\eta_o = \frac{k^2 - 4\varepsilon^2}{k}. \quad (86)$$

Now in view of Eqs. (64) and (65) with the assumption that the cavity light is initially in a vacuum state, Eqs. (84) and (85) goes over into

$$\langle \hat{a}_1(t)\rangle = \langle \hat{a}_2(t)\rangle = 0. \quad (87)$$

On account of this result as well as Eqs. (74) and (75) that $\hat{a}_1(t)$ and $\hat{a}_2(t)$ are Gaussian variables with zero mean. Then Eq. (66) can be rewritten as

$$g\left(\hat{n}_1, \hat{n}_2\right)_p = 1 + \frac{\langle \hat{a}_1^\dagger(t)\hat{a}_2^\dagger(t)\rangle\langle \hat{a}_1(t)\hat{a}_2(t)\rangle}{\langle \hat{a}_1^\dagger(t)\hat{a}_1(t)\rangle\langle \hat{a}_2^\dagger(t)\hat{a}_2(t)\rangle} + \frac{\langle \hat{a}_1^\dagger(t)\hat{a}_2(t)\rangle\langle \hat{a}_2^\dagger(t)\hat{a}_1(t)\rangle}{\langle \hat{a}_1^\dagger(t)\hat{a}_1(t)\rangle\langle \hat{a}_2^\dagger(t)\hat{a}_2(t)\rangle}. \quad (88)$$

Thus employing Eqs. (82) and (83) together with (50) and (51) along with (88), the photon-number correlation turns out to be

$$g\left(\hat{n}_1, \hat{n}_2\right)_p = 1 + \frac{W_1}{W_2}, \quad (89)$$

where

$$W_1 = 4\varepsilon k\Big[\left(k^2 + 4\varepsilon^2\right)\langle \hat{\Sigma}_0\rangle + 2\varepsilon k\left(\langle \hat{N}_2\rangle + \langle \hat{N}_0\rangle\right)\Big], \quad (90)$$

$$W_2 = \Big[k^2 + 4\varepsilon^2\Big]\Big[k^2\langle \hat{N}_2\rangle + 4\varepsilon^2\langle \hat{N}_0\rangle + 4\varepsilon k\langle \hat{\Sigma}_0\rangle\Big], \quad (91)$$

in which $\langle \hat{\Sigma}_0\rangle$ is given by Eq. (60). Moreover, the atom-number correlation is defined by

$$g\left(\hat{n}_1, \hat{n}_2\right)_a = \frac{\langle \hat{\Sigma}_1^\dagger \hat{\Sigma}_1 \hat{\Sigma}_2^\dagger \hat{\Sigma}_2\rangle}{\langle \hat{\Sigma}_1^\dagger \hat{\Sigma}_1\rangle\langle \hat{\Sigma}_2^\dagger \hat{\Sigma}_2\rangle}. \quad (92)$$

We recall that the atomic operators $\hat{\Sigma}_1$ and $\hat{\Sigma}_2$ are Gaussian variables with zero mean. Hence Eq. (92) can be rewritten as

$$g\left(\hat{n}_1, \hat{n}_2\right)_a = 1 + \frac{\langle \hat{\Sigma}_1^\dagger \hat{\Sigma}_2^\dagger\rangle\langle \hat{\Sigma}_1 \hat{\Sigma}_2\rangle}{\langle \hat{\Sigma}_1^\dagger \hat{\Sigma}_1\rangle\langle \hat{\Sigma}_2^\dagger \hat{\Sigma}_2\rangle} + \frac{\langle \hat{\Sigma}_1^\dagger \hat{\Sigma}_2\rangle\langle \hat{\Sigma}_2^\dagger \hat{\Sigma}_1\rangle}{\langle \hat{\Sigma}_1^\dagger \hat{\Sigma}_1\rangle\langle \hat{\Sigma}_2^\dagger \hat{\Sigma}_2\rangle}. \quad (93)$$

Thus in view of Eqs. (50) and (51), we obtain

$$g\left(\hat{n}_1, \hat{n}_2\right)_a = 1. \quad (94)$$

We immediately see that the maximum degree of photon number correlation observed when more atoms in the lower energy level than on the upper level. This occurs when the three-level laser is operating below threshold. On the other hand, we note that Eq. (94) that unlike the photon-number correlation, the atoms in the laser cavity are not correlated. Moreover, we point out that in the absence of subharmonic generator, one can never realize correlated photons in the laser cavity.

IV. Entanglement Quantification

Here, we seek to analyze the entanglement of photon-states and atomic states in the laser cavity. Quantum entanglement is a physical phenomenon that occurs when pairs or groups of particles cannot be described independently instead, a quantum state may be given for the system as a whole. Measurements of physical properties such as position, momentum, spin polariza- tion, etc performed on entangled particles are found to be appropriately correlated.

A pair of particles is taken to be entangled in quantum theory, if its states cannot be expressed as a product of the states of its individual constituents. The preparation and manipulation of these entangled states that have non-classical and non-local properties lead to better understanding of the basic quantum principles. It is in this spirit that this section is devoted to the analysis of the entanglement of the two modes (photon-states). In other words, it is a well-known fact that a quantum system is said to be entangled, if it is not separable. That is, if the density operator for the combined state cannot be described as a combination of the product density operators of the constituents,

$$\hat{\rho} \neq \sum_k P_k \hat{\rho}_k^{(1)} \otimes \hat{\rho}_k^{(2)}, \quad (95)$$

in which $P_k \gg 0$ and $\sum_k P_k = 1$ to verify the normalization of the combined density states. On the other hand, on entangled continuous variable (CV) state can be expressed as a common eigenstate of a pair of EPR-type operators [14] such as $\hat{x}_2 - \hat{x}_1$ and $\hat{p}_2 + \hat{p}_1$. The total variance of these two operators reduces to zero for maximally entangled CV states. According to the inseparable criteria given by Duan et al [5], cavity photon-states of a system are entangled, if the sum of the variance of a pair of EPR-like operators,

$$\hat{s} = \hat{x}_2 - \hat{x}_1 \tag{96}$$

and

$$\hat{t} = \hat{p}_2 + \hat{p}_1, \tag{97}$$

where

$$\hat{x}_1 = \frac{1}{\sqrt{2}}\left(\hat{a}_1 + \hat{a}_1^\dagger\right), \tag{98}$$

$$\hat{x}_2 = \frac{1}{\sqrt{2}}\left(\hat{a}_2 + \hat{a}_2^\dagger\right), \tag{99}$$

$$\hat{p}_1 = \frac{i}{\sqrt{2}}\left(\hat{a}_1^\dagger - \hat{a}_1\right), \tag{100}$$

$$\hat{p}_2 = \frac{i}{\sqrt{2}}\left(\hat{a}_2^\dagger - \hat{a}_2\right), \tag{101}$$

are quadrature operators for modes α_1 and α_2, satisfy

$$(\Delta s)^2 + (\Delta t)^2 < 2N \tag{102}$$

and recalling the cavity mode operators $\hat{a}_1$ and $\hat{a}_2$ are Gaussian variables with zero mean, we readily get

$$(\Delta s)^2 + (\Delta t)^2 = \Big[\langle \hat{a}_1^\dagger \hat{a}_1\rangle + \langle \hat{a}_1 \hat{a}_1^\dagger\rangle + \langle \hat{a}_2^\dagger \hat{a}_2\rangle + \langle \hat{a}_2 \hat{a}_2^\dagger\rangle\Big] - \Big[\langle \hat{a}_1 \hat{a}_2\rangle + \langle \hat{a}_1^\dagger \hat{a}_2^\dagger\rangle + \langle \hat{a}_2 \hat{a}_1\rangle + \langle \hat{a}_2^\dagger \hat{a}_1^\dagger\rangle\Big]. \tag{103}$$

Thus with the aid of Eqs. (82) and (83) along with (50) and (51), we arrive at

$$(\Delta s)^2 + (\Delta t)^2 = \frac{\gamma_c k}{(k^2 - 4\varepsilon^2)^2} \times \Big[k^2 + 4\varepsilon^2 - 4\varepsilon k\Big] \times \Big[2N - \langle \hat{N}_0\rangle - \langle \hat{N}_2\rangle - 2\langle \hat{\Sigma}_0\rangle\Big]. \tag{104}$$

Upon setting $\varepsilon = 0$, we see that

$$(\Delta s)^2 + (\Delta t)^2 = \frac{\gamma_c}{k}\Big[2N - \langle \hat{N}_0\rangle - \langle \hat{N}_2\rangle - 2\langle \hat{\Sigma}_0\rangle_0\Big], \tag{105}$$

where

$$\langle \hat{\Sigma}_0\rangle_0 = \frac{\Omega}{(\gamma_c + \beta)}\Big[N - \langle \hat{N}_1\rangle - 2\langle \hat{N}_2\rangle\Big] \tag{106}$$

On the other hand, cavity atomic-states of a system are entangled, if the sum of the variance of a pair of EPR-like operators,

$$\hat{u} = \hat{x}_2' - \hat{x}_1' \tag{107}$$

and

$$\hat{v} = \hat{p}_2' + \hat{p}_1', \tag{108}$$

where

$$\hat{x}_1' = \frac{1}{\sqrt{2}}\left(\hat{\Sigma}_1 + \hat{\Sigma}_1^\dagger\right), \tag{109}$$

$$\hat{x}_2' = \frac{1}{\sqrt{2}}\left(\hat{\Sigma}_2 + \hat{\Sigma}_2^\dagger\right), \tag{110}$$

$$\hat{p}_1' = \frac{i}{\sqrt{2}}\left(\hat{\Sigma}_1^\dagger - \hat{\Sigma}_1\right), \tag{111}$$

$$\hat{p}_2' = \frac{i}{\sqrt{2}}\left(\hat{\Sigma}_2^\dagger - \hat{\Sigma}_2\right), \tag{112}$$

are quadrature operators for the cavity atoms, satisfy

$$(\Delta u)^2 + (\Delta v)^2 < 2N^2. \tag{113}$$

Since $\hat{\Sigma}_1$ and $\hat{\Sigma}_2$ are Gaussian variables with zero mean, so one can easily verify that

$$(\Delta u)^2 + (\Delta v)^2 = \Big[\langle \hat{\Sigma}_1^\dagger \hat{\Sigma}_1\rangle + \langle \hat{\Sigma}_1 \hat{\Sigma}_1^\dagger\rangle + \langle \hat{\Sigma}_2^\dagger \hat{\Sigma}_2\rangle + \langle \hat{\Sigma}_2 \hat{\Sigma}_2^\dagger\rangle\Big] - \Big[\langle \hat{\Sigma}_2^\dagger \hat{\Sigma}_1^\dagger\rangle + \langle \hat{\Sigma}_1 \hat{\Sigma}_2\rangle\Big]. \tag{114}$$

Now with the aid of (50) and (51), Eq. (114) takes the form

$$(\Delta u)^2 + (\Delta v)^2 = N\Big[2N - \langle \hat{N}_0\rangle - \langle \hat{N}_2\rangle - 2\langle \hat{\Sigma}_0\rangle\Big]. \tag{115}$$

V. Conclusion

In this paper we have studied a coherently driven non-degenerate three-level laser with, two-mode subharmonic generator, coupled to a two-mode vacuum reservoir via a single-port mirror whose open cavity contains N non-degenerate three-level atoms. We carried out our analysis by putting the noise operators associated with the vacuum reservoir in normal order and by considering the interaction of the three-level atoms with the vacuum reservoir outside the cavity. Results show that the presence of parametric ampli_er is to increase the squeezing and the mean photon number of the two-mode cavity light significantly. It is found that the photon-states of the system is strongly entangled at steady state where as the atomic state of the system is not entangled. We have also shown that as the stimulated decay constant increases, the degree of entanglement decreases. In addition, we have established that the photons in the laser cavity are highly correlated and the degree of photon number correlation and entanglement increases as the amplitude of the coherent light driving the pump mode increases. Moreover, we have shown that the presence of the subharmonic generator leads to an increase in the degree of entanglement and correlation. Moreover, we point out that in the absence of subharmonic generator, one can never realize correlated photons.

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The Question of Space-Time Singularities in General Relativity and Einstein's Errors

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Abstract- In general relativity, the existence of space-time singularities plays a central role on the notion of black holes and the expanding universe. However, these two speculations have not been firmly verified in spite of the efforts of generations of physicists. The existence of space-time singularities is due to the spacetime singularity theorems of Hawking and Penrose, whose implicit physical assumption is the general validity of $E = mc^2$ that leads to the unique sign for all the couplings. However, recently it is found that such an assumption is not supported by various experiments. In particular, the electromagnetic energy is not equivalent to mass. In fact, the Einstein equation with massive sources has no dynamic solution unless the gravitational energy-stress tensor with an anti-gravity coupling is added to the source.

Keywords: *anti-gravity coupling; dynamic solution; gravitational radiation; repulsive gravitation; principle of causality.*

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THEQUESTIONOFSPACETIMESINGULARITIESINGENERALRELATIVITYANDEINSTEINSERRORS

Strictly as per the compliance and regulations of :

The Question of Space-Time Singularities in General Relativity and Einstein's Errors

C. Y. Lo

Abstract- In general relativity, the existence of space-time singularities plays a central role on the notion of black holes and the expanding universe. However, these two speculations have not been firmly verified in spite of the efforts of generations of physicists. The existence of space-time singularities is due to the spacetime singularity theorems of Hawking and Penrose, whose implicit physical assumption is the general validity of $E = mc^2$ that leads to the unique sign for all the couplings. However, recently it is found that such an assumption is not supported by various experiments. In particular, the electromagnetic energy is not equivalent to mass. In fact, the Einstein equation with massive sources has no dynamic solution unless the gravitational energy-stress tensor with an anti-gravity coupling is added to the source. Moreover, for the electromagnetic wave to have a physically gravitational effect, the related Einstein equation must additionally have a photonic energy-stress tensor with an anti-gravity coupling. Thus, Einstein's understanding of general relativity is inadequate. Since the energy conditions in the space-time singularity theorems actually cannot be satisfied in physics, these mathematical theorems are actually irrelevant to physics. Their claims have been proven as nonsense in physics. Further more, recognizing the nonexistence of dynamic solution for the Einstein equation is the first step to the unification of gravitation and electromagnetism. Many overlooked the crucial charge-mass interaction. Due to inadequate understanding of the principle of causality and non-linear mathematics, Einstein failed to show his unification. He has made three major errors: 1) He is mistaken that the Einstein equation has dynamic solutions. 2) He speculated that $E = mc^2$ was generally valid. 3) He invalidly rejected repulsive gravitation, which is supported by experiments. Also, the Physical Review, the Proceeding of the Royal Society A, the Annals of Physics and the Chinese Physics. accepted the space-time singularity theorems as valid because theorists make errors in the non-linear mathematics and physics.

Keywords: anti-gravity coupling; dynamic solution; gravitational radiation; repulsive gravitation; principle of causality.

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"Unthinking respect for authority is the greatest enemy of truth." – A. Einstein

I. Introduction

In physics, the existence of singularities suggests problematic assumptions. Nevertheless, in current theory of general relativity, the existence of space-time singularities plays a central role on the notion of black holes and the expanding universe (see Appendix A). However, these two speculations have not been firmly verified in spite of the efforts of generations of physicists.[1)] Thus, one may question the general validity of the theory of general relativity in spite of its earlier success.

The existence of space-time singularities is due to the spacetime singularity theorems of Hawking and Penrose [1]. The mathematical validity of these theorems is highly reliable because Penrose have won his arguments in mathematics against the theoretical physicist, E. M. Lifshitz [2] in a long dispute. Moreover, the static Einstein equation in general relativity has passed various tests with surprises. Since the physical assumptions on the energy conditions of these theorems seem to be very natural, there would be little doubt on the validity of these assumptions.

An implicit assumption on these singularity theorems is that all the coupling constants have the same sign.[2)] Such an assumption would be necessarily valid if the formula $E = mc^2$ is unconditional. However, recently it has been found that the general validity of this formula is questionable. In contrast to Einstein's prediction [3], a piece of heated-up metal actually has reduced weight [4]. Moreover, a charged capacitor also has reduced weight, [3)] which is proportional to the square of the difference in the electric potential of the capacitor [5, 6]. Theoretically, it has been found that the equivalence between mass and the electromagnetic energy is in conflict with the Einstein equation because the electromagnetic energy-stress tensor is traceless [7]. Thus, the assumption of unique coupling sign is questionable.

In this paper, it will be shown that the assumption of unique coupling sign is, indeed, not valid for the dynamic case. Therefore, the space-time singularity theorems of Hawking and Penrose are actually irrelevant to physics. A root of the problem is, however, that many relativists have lost their touch with experiments, in addition to inadequacy in non-linear mathematics.

II. The Space-Time Singularity Theorems and the Assumption of Unique Sign of Couplings

Let us examine the energy conditions in the singularity theorems. These theorems [1] are listed as the following:

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Theorem 1. Let (M, g_{ab}) be a globally hyperbolic spacetime with $R_{ab}\xi^a\xi^b \geq 0$ for all timelike ξ^a, which will be the case if Einstein equation is satisfied with the strong energy condition holding for matter. Suppose there exists a smooth (or at least C^2) spacelike Cauchy surface Σ for which the trace of the extrinsic curvature (for the past directed normal geodesic congruence) satisfies $0 > C \geq K$ everywhere C is a constant. Then no past directed timelike curve from Σ can have length greater than $3/|C|$. In particular, all past directed timelike geodesic are incomplete.

Theorem 2. Let (M, g_{ab}) be a strongly causal spacetime with $R_{ab}\xi^a\xi^b \geq 0$ for all timelike ξ^a, as will be the case if Einstein's equation is satisfied with the strong energy condition holding for matter. Suppose there exists a compact, edgeless, achronal smooth spacelike hypersurface S such that for the past directed normal geodesic congruence form S we have $0 > K$ everywhere on S. Let C denote the maximum value for K, so $0 > C \geq K$ everywhere on S. Then at least one inextendible past directed timelike geodesic from S has length no greater that $3/|C|$.

Theorem 3. Let (M, g_{ab}) be a connected, globally hyperbolic spacetime with a noncompact Cauchy surface Σ. Suppose $R_{ab}k^ak^b \geq 0$ for all null k^a, as will be the case if (M, g_{ab}) is a solution of Einstein's equation with matter satisfying the weak or strong energy condition. Suppose, further, that M contains a trapped surface T. Let $0 > \theta_0$ denote the maximum value of θ for both sets of orthogonal geodesic on T. Then at least one inextendible future directed orthogonal null geodesic from T has affine length no greater than $2/|\theta_0|$.

Theorem 4. Suppose a spacetime (M, g_{ab}) satisfies the following four conditions. (1) $R_{ab}v^av^b \geq 0$ for all timelike and null v^a, as will be the case if Einstein's equation is satisfied with the strong energy condition holding for matter. (2) The timelike and null generic conditions are satisfied. (3) No closed timelike curve exists. (4) At least one of the three properties holds: (a) (M, g_{ab}) posses a compact achronal set without edge [i.e., (M, g_{ab}) is a closed universe], (b) (M, g_{ab}) possesses a trapped surface, or (c) there exists a point $p \in M$ such that the expansion of the future (or past) directed null geodesics emanating from p becomes negative along each geodesic in this congruence. Then (M, g_{ab}) must contain at least one incomplete timelike or null geodesic.

Originally, the energy condition is related to the energy-momentum tensor T_{ab}. According to the Einstein equation [1]

$$G_{ab} \equiv R_{ab} - (1/2)\, g_{ab} R = 4\pi T_{ab}, \tag{1}$$

one would have

$$R_{ab} = 8\pi\, [T_{ab} - (1/2) g_{ab} T] \quad \text{where} \quad T = g^{ab}T_{ab} \tag{2}$$

Then,

$$R_{ab}\xi^a\xi^b = 8\pi\, [T_{ab} - (1/2)g_{ab} T]\, \xi^a\xi^b = 8\pi\, [T_{ab}\xi^a\xi^b + (1/2)T], \quad \text{for a unit timelike } \xi^a \tag{3}$$

It is believed that for all physically reasonable classical matter the energy condition is non-negative, i.e.,

$$T_{ab}\, \xi^a\xi^b \geq 0 \tag{4}$$

for all timelike ξ^a. This assumption is known as the weak energy condition. However, it also seems physically reasonable that the stress of matter will not become so large and negative as to make the right-hand side of eq. (3) negative. This assumption,

$$T_{ab}\, \xi^a\xi^b \geq -(1/2)T \tag{5}$$

for all unit timelike unit vector ξ^a, is known as the strong energy condition. An implicit assumption of these energy-conditions (3)-(5) is that all the coupling constants have the same sign. However, as will be shown such an assumption leads to the invalidity of the Einstein equation because of the non-existence of dynamic solutions.

To illustrate this, we shall first show examples that for the case of gravitational waves; there is no bounded dynamic solution. Now, consider a well-known metric obtained by Bondi, Pirani, & Robinson [8] as follows:

$$ds^2 = e^{2\varphi}\left(d\tau^2 - d\xi^2\right) - u^2 \begin{bmatrix} \cosh 2\beta\left(d\eta^2 + d\varsigma^2\right) \\ +\sinh 2\beta \cos 2\theta\left(d\eta^2 - d\varsigma^2\right) \\ -2\sinh 2\beta \sin 2\theta\, d\eta\, d\varsigma \end{bmatrix} \tag{6a}$$

where φ, β and θ are functions of u $(\tau - \xi)$. It satisfies the differential equation (i.e., their Eq. [2.8]),

$$2\phi' = u\left(\beta'^2 + \theta'^2 \sinh^2 2\beta\right) \tag{6b}$$

which is a special cases of $G_{\mu\nu} = 0$. They claimed this is a wave from a distant source and weak gravity invalid. The metric is irreducibly unbounded because of the factor u^2. And linearization of (6b) does not make sense since u is not bounded.

Moreover, when gravity is absent, it is necessary to have $\phi = \sinh 2\beta = \sin 2\theta = 0$. These would reduce (6a) to

$$ds^2 = (d\tau^2 - d\xi^2) - u^2(d\eta^2 + d\zeta^2) \tag{6c}$$

However, this metric is not equivalent to the flat metric. Thus, metric (6c) violates the principle of causality (see Appendix B).

This challenges the view that both Einstein's notion of weak gravity and his covariance principle are valid. These conflicting views are supported respectively by the editors of the "Royal Society Proceedings A" and the "Physical Review D"; thus there is no general consensus. Note that Einstein's covariance principle has been proven invalid with counter examples [9].

III. The Non-Existence of a Bounded Dynamic Solution for a Two-Body Problem in General Relativity

According to the principle of causality, weak sources would produce a weak field, i.e.,

$$g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}, \text{ where } 1 >> |\gamma_{\mu\nu}| \tag{7}$$

and $\eta_{\mu\nu}$ is the flat metric. However, eq. (7) is valid, only if the Einstein equation is valid. Since the strength of a source can always be reduced, to show the non-existence of a dynamic solution, it is sufficient to show the case of weak gravity.

Unfortunately, many believe that condition (7) for weak gravity is always valid for the Einstein equation. They believed that an approximate weak solution can be derived through the approach of the field equation being linearized. The linearized Einstein equation with the linearized harmonic gauge $\partial^{\mu}\bar{\gamma}_{\mu\nu} = 0$ is

$$\frac{1}{2}\partial^{\alpha}\partial_{\alpha}\bar{\gamma}_{\mu\nu} = \kappa T_{\mu\nu} \quad \text{where} \quad \bar{\gamma}_{\mu\nu} = \gamma_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}(\eta^{cd}\gamma_{cd}), \tag{8}$$

Note that we have

$$G_{\mu\nu} = G_{\mu\nu}^{(1)} + G_{\mu\nu}^{(2)} \quad \text{and} \quad G^{(1)}{}_{\mu\nu} = \frac{1}{2}\partial^{\alpha}\partial_{\alpha}\bar{\gamma}_{\mu\nu} + H^{(1)}{}_{\mu\nu}, \tag{9}$$

where

$$H^{(1)}_{\mu\nu} = -\frac{1}{2}\partial^{\alpha}[\partial_{\mu}\bar{\gamma}_{\nu\alpha} + \partial_{\nu}\bar{\gamma}_{\mu\alpha}] + \frac{1}{2}\eta_{\mu\nu}\partial^{\alpha}\partial^{\beta}\bar{\gamma}_{\alpha\beta}$$

The linearized vacuum Einstein equation means $G^{(1)}{}_{\mu\nu}[\gamma^{(1)}{}_{\alpha\beta}] = 0$. Thus, to have a solution of the second order we must correct $\gamma^{(1)}{}_{\mu\nu}$ by adding to it the term $\gamma^{(2)}{}_{\mu\nu}$ that satisfies

$$G^{(1)}_{\mu\nu}[\gamma^{(2)}{}_{\alpha\beta}] + G^{(2)}_{\mu\nu}[\gamma_{\alpha\beta}] = 0, \quad \text{where} \quad \gamma_{\mu\nu} = \gamma^{(1)}{}_{\mu\nu} + \gamma^{(2)}{}_{\mu\nu} \tag{10}$$

which is the correct form of eq. (4.4.52) in Wald's book [1] (Wald did not distinguish $\gamma_{\mu\nu}$ from $\gamma^{(1)}{}_{\mu\nu}$).[4)] However, detailed calculation shows that this equation does not have a solution for the dynamic case [10-13].[5)] In fact, as shown by the example in the last section, for a dynamic case, the linealized equation and the Einstein equation are independent equations [14].

It was believed that the linear Maxwell-Newton Approximation [11] (or the linearized Einstein equation [15, 16])

$$\frac{1}{2}\partial^{c}\partial_{c}\bar{\gamma}_{\mu\nu} = K\,T(m)_{\mu\nu}, \quad \text{where} \quad \bar{\gamma}_{\mu\nu} = \gamma_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}(\eta^{cd}\gamma_{cd}) \tag{11a}$$

and

$$\bar{\gamma}_{\mu\nu}(x^i, t) = \frac{K}{2\pi}\int \frac{1}{R} T_{\mu\nu}[y^i, (t - R)]d^3y, \quad \text{where} \quad R^2 = \sum_{i=1}^{3}(x^i - y^i)^2. \tag{11b}$$

provides the first-order approximation for the Einstein equation (1). However, this belief was verified for the static case only.

The Cauchy data of eq.(1) must satisfy four constraint equations, $G_{\mu t} = -KT(m)_{\mu t}$ ($\mu = x, y, z, t$) since $G_{\mu t}$ contains only first-order time derivatives [17]. This shows that (11a) would be dynamically incompatible with Einstein equation (1).

In 1957, Fock [18] pointed out that, in harmonic coordinates, there are divergent logarithmic deviations from expected linearized behavior of the radiation. This was misinterpreted to mean merely that the contribution of the complicated nonlinear terms in the Einstein equation cannot be dealt with satisfactorily following this method and that another approach is needed. Subsequently, vacuum solutions that do not involve logarithmic deviation were founded by Bondi, Pirani & Robinson [8] in 1959. Thus, the incorrect interpretation appears to be justified and the faith on the dynamic solutions maintained. It was not recognized until 1995 [11] that such a symptom of divergence shows the absence of bounded physical dynamic solutions.

Equation (11) shows that a gravitational wave is bounded and is related to the dynamic of the source. These are useful to prove that eq. (11), as the first-order approximation for a dynamic problem, is incompatible with the Einstein equation (1). According to the principle of causality, it is sufficient to (consider the case of weak gravity. Consider, $G^{(2)}{}_{\mu\nu}$ ($G_{\mu\nu} \equiv G_{\mu\nu}^{(1)} + G_{\mu\nu}^{(2)}$) is at least of second order in terms of the metric elements. For an isolated system located near the origin of the space coordinate system, $G^{(2)}_{\mu t}$ at large r (= $[x^2 + y^2 + z^2]^{1/2}$) is of $O(K^2/r^2)$ [1, 17, 19].

One may obtain some general characteristics of a dynamic solution for an isolated system as follows:

1) The characteristics of some physical quantities of an isolated system:

For an isolated system consisting of particles with typical mass $\overline{M}$, separation $\overline{r}$, and velocities $\overline{v}$, Weinberg [17] estimated, [6)] the power radiated at a frequency ω of order $\overline{v}/\overline{r}$ will be of order

$$P \approx \kappa(\overline{v}/\overline{r})^6 \overline{M}^2 \overline{r}^4 \text{ or } P \approx \overline{M}\ \overline{v}^8/\overline{r}, \qquad (12)$$

since $\kappa \overline{M}/\overline{r}$ is of order $\overline{v}^2$. The typical deceleration $\overline{a}_{rad}$ of particles in the system owing this energy loss is given by the power P divided by the momentum $\overline{M}\ \overline{v}$, or $\overline{a}_{rad} \approx \overline{v}^7/\overline{r}$. This may be compared with the accelerations computed in Newtonian mechanics, which are of order $\overline{v}^2/\overline{r}$, and with the post-Newtonian correction of $\overline{v}^4/\overline{r}$. Since radiation reaction is smaller than the post-Newtonian effects by a factor $\overline{v}^3$, if $\overline{v}$ << c, the velocity of light, the neglect of radiation reaction is perfectly justified. This allows us to consider the motion of a particle in an isolated system as almost periodic.

Consider two particles of equal mass with an almost circular orbit in the x-y plane whose origin is the center of the circle (i.e., the orbits are a circle if radiation is neglected). Thus, the principle of causality implies that the metric $g_{\mu\nu}$ is weak and very close to the flat metric at distance far from the source and that $g_{\mu\nu}$ (x, y, z, t') is an almost periodic function of t' (= t - r/c).

2) The expansion of a bounded dynamic solution $g_{\mu\nu}$ for an isolated weak gravitational source:

According to eq. (11), a first-order approximation of metric $g_{\mu\nu}$ (x, y, z, t') is bounded and almost periodic since $T_{\mu\nu}$ is. Physically, the principle of causality requires $g_{\mu\nu}$ to be almost periodic in time since the motion of a source particle is. Such a metric $g_{\mu\nu}$ is asymptotically flat for a large distance r, and the expansion of a bounded dynamic solution is:

$$g_{\mu\nu}(n^x, n^y, n^z, r, t') = \eta_{\mu\nu} + \sum_{k=1}^{\infty} f_{\mu\nu}{}^{(k)}(n^x, n^y, n^z, t')/r^k, \quad \text{where} \quad n^\nu = x^\nu/r. \qquad (13a)$$

3) The non-existence of dynamic solutions:

It follows expansion (13a) that the non-zero time average of $G^{(1)}_{\mu t}$ would be of $O(1/r^3)$ due to

$$\partial_\mu n^\nu = (\delta^\nu{}_\mu + n^\nu n_\mu)/r, \qquad (13b)$$

since the term of $O(1/r^2)$, being a sum of derivatives with respect to t', can have a zero time-average. If $G^{(2)}_{\mu t}$ is of $O(K^2/r^2)$ and has a nonzero time-average, consistency can be achieved only if another term of time-average $O(K^2/r^2)$ at vacuum be added to the source of the Einstein equation (1). Note that there is no plane-wave solution for $G_{\mu\nu} = 0$ [20].

It will be shown that there is no dynamic solution for the Einstein equation with a massive source. Let us define

$$\gamma_{\mu\nu} = \gamma^{(1)}{}_{\mu\nu} + \gamma^{(2)}{}_{\mu\nu}\,; \qquad \bar{\gamma}^{(i)}{}_{\mu\nu} = \gamma^{(i)}{}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}(\gamma^{(i)}{}_{cd}\,\eta^{cd}), \qquad \text{where } i = 1, 2\,;$$

and

$$\frac{1}{2}\partial^\alpha\partial_\alpha \bar{\gamma}^{(1)}{}_{\mu\nu} = K\,T(m)_{\mu\nu}\,. \qquad (14)$$

Then $\bar{\gamma}^{(1)}{}_{\mu\nu}$ is of a first-order; and $\gamma^{(2)}{}_{\mu\nu}$ is finite. On the other hand, from the Einstein equation (1), one has

$$\frac{1}{2}\partial^\alpha\partial_\alpha \bar{\gamma}^{(2)}{}_{\mu\nu} + H^{(1)}{}_{\mu\nu} + G^{(2)}{}_{\mu\nu} = 0 \qquad (15)$$

Note that, for a dynamic case, equation (15) may not be satisfied. If (14) is a first-order approximation, $G^{(2)}{}_{\mu\nu}$ has a nonzero time-average of $O(K^2/r^2)$ [1] (but $[\partial^\alpha\partial_\alpha \bar{\gamma}^{(2)}{}_{\mu\nu}/2 + H^{(1)}{}_{\mu\nu}]$ would have zero time-average); and thus $\bar{\gamma}^{(2)}{}_{\mu\nu}$ cannot have a solution.

However, if $\bar{\gamma}^{(2)}{}_{\mu\nu}$ is also of the first-order of K, one cannot estimate $G^{(2)}{}_{\mu\nu}$ by assuming that $\bar{\gamma}^{(1)}{}_{\mu\nu}$ provides a first-order approximation. For example, equation (11) does not provide the first approximation for the static Schwarzschild solution, although it can be transformed to a form such that (11) provides a first-order approximation [15]. According to eq.(9), $\bar{\gamma}^{(2)}{}_{\mu\nu}$ will be a second order term if the sum $H^{(1)}{}_{\mu\nu}$ is of second order. From (9), this would require $\partial^\mu \bar{\gamma}_{\mu\nu}$ being of second order. For weak gravity, it is known that a coordinate transformation would turn $\partial^\mu \bar{\gamma}_{\mu\nu}$ to a second order term [17, 18, 21]. (Eq. [15] implies that $\partial^c\partial_c \bar{\gamma}^{(2)}{}_{\mu\nu}$ - $\partial^c[\partial_\nu \bar{\gamma}_{\mu c} + \partial_\mu \bar{\gamma}_{\nu c}] + \eta_{\mu\nu}\partial^\alpha\partial^\beta \bar{\gamma}_{\alpha\beta}$ would be of second order.) Thus, it is possible to turn (14) to become an equation for a first-order approximation for weak gravity.

Since it has been proven that (11) necessarily gives a first-order approximation [15], a failure of such a coordinate transformation means only that such a solution is not valid in physics. Moreover, for the dynamic of massive matter, experiment [22] supports the fact that Maxwell-Newton Approximation (11) is related to a dynamic solution of weak gravity [16]. Thus, theoretical considerations as well as experiments eliminate other unverified speculations thought to be possible since 1957.

As shown, the difficulty comes from the assumption of boundedness, which allows the existence of a bounded first-order approximation, which in turn implies that a time-average of the radiative part of $G^{(2)}_{\mu\nu}$ is non-zero. The present method has an advantage over Fock's approach to obtaining logarithmic divergence [18] for being simple and clear.

In short, according to Einstein's radiation formula, a time average of $G^{(2)}_{\mu t}$ is non-zero and of $O(K^2/r^2)$ [17]. Although (11) implies $G^{(1)}_{\mu t}$ is of order K^2, its terms of $O(1/r^2)$ can have a zero time average because $G^{(1)}_{\mu t}$ is linear on the metric elements. Thus, the Einstein equation (1) in vacuum cannot be satisfied. Nevertheless, a static metric can satisfy (1), since both $G^{(1)}_{\mu\nu}$ and $G^{(2)}_{\mu\nu}$ are of $O(K^2/r^4)$ in vacuum. Note that $G_{\mu t} = KT(m)_{\mu t}$ are constraints on the initial data.

In conclusion, assuming the existence of dynamic solutions of weak gravity for Einstein equation (1) [8, 18, 21, 23-28] is invalid. This means that general relativity has not yet totally superseded Newtonian gravity [12]. This illustrates also that theorists should not carelessly follow the erroneous and groundless claims of Christodoulou and Klainerman (see Section 6).

Nevertheless, because of inadequacy in mathematics, many theorists following Einstein's error, claimed or believed that there are dynamic solutions. A good example is that Misner, Thorne and Wheeler [19] claimed they have an explicit bounded dynamic solution of the following form,

$$-ds^2 = c^2dt^2 - dx^2 - L^2\left(e^{2\beta}dy^2 + e^{-2\beta}dz^2\right) \qquad (16)$$

where $L = L(u)$, $\beta = \beta(u)$, $u = ct - x$, and c is the light speed. Then, the Einstein equation $G_{\mu\nu} = 0$ becomes

$$\frac{d^2L}{du^2} + L\left(\frac{d\beta}{du}\right)^2 = 0 \qquad (17)$$

Misner et al. [19] claimed that Eq. (17) has a bounded approximate solution, compatible with a linearization of metric (16). However, it has been shown with undergraduate mathematics [29] that Misner et al. are incorrect and Eq. (17) does not have a physical solution that satisfies Einstein's requirement on weak gravity. In fact, $L(u)$ is unbounded even for a very small $\beta(u)$.

On the other hand, from the Maxwell-Newton approximation in vacuum, Einstein [30] obtained a solution as follows:

$$-ds^2 = c^2dt^2 - dx^2 - (1+2\phi)dy^2 - (1-2\phi)dz^2 \qquad (18)$$

where ϕ is a bounded function of u $(= ct - x)$. Note that metric (18) is the linearization of metric (16) if $\phi = \beta(u)$.

Thus, the waves illustrate that the linearization is not valid for the dynamic case when gravitational waves are involved.

Moreover, Misner et al. [19] also make other serious errors in physics as shown in their eq. (40.14) for the proper time measured by an earth-based clock, but other theorists such as Wald [1] and Weinberg [17] did not make the same mistake.

IV. The Anti-Gravity Coupling and Invalidity of the Space-Time Singularity Theorems to Physics

From the above analysis, there is a conflict between the Einstein equation, which has no dynamic solution and its linearized equation, which has a dynamic solution. The conflict is due to that the second order terms $G^{(2)}_{\mu\nu}$ cannot be eliminated in the Non-linear Einstein equation. Thus, a simple solution is the 1995 update of the Einstein equation [10] as follows:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = K\,[T(m)_{\mu\nu} - t(g)_{\mu\nu}], \qquad (19)$$

where $t(g)_{\mu\nu}$ is the energy-stress tensors for gravity. Then, from (19), the equation in vacuum is

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -K\,t(g)_{\mu\nu} \qquad (19')$$

Note that $t(g)_{\mu\nu}$ is equivalent to $G^{(2)}_{\mu\nu}$ (and Einstein's gravitational pseudotensor) in terms of his radiation formula.

When gravitational wave is present, the gravitational energy-stress tensor $t(g)_{\mu\nu}$ is non-zero. Thus, a radiation does carry energy-momentum as physics requires. This explains also that the absence of an anti-gravity coupling which is determined by Einstein's radiation formula, is the physical reason that the 1915 Einstein equation (1) is incompatible with radiation.

Note that the radiation of the binary pulsar can be calculated without detailed knowledge of $t(g)_{\mu\nu}$. From (19'), the approximate value of $t(g)_{\mu\nu}$ at vacuum can be calculated through $G_{\mu\nu}/K$ as before since the first-order approximation of $g_{\mu\nu}$ can be calculated through (11). In view of the facts that $Kt(g)_{\mu\nu}$ is of the fifth order in a post-Newtonian approximation, that the deceleration due to radiation is of the three and a half order in a post-Newtonian approximation [17] and that the perihelion of Mercury was successfully calculated with the second-order approximation from (1), the orbits of the binary pulsar can be calculated with the second-order post-Newtonian approximation of (19) by using (1). Thus, the calculation approaches of Damour and Taylor [31, 32] would be essentially valid except that they did not realize the crucial fact that (11) is actually an approximation of the updated equation (19) [33].

In light of the above, the Hulse-Taylor experiments support the anti-gravity coupling being crucial to the existence of the gravitational wave [34], and (11) being an approximation of weak waves generated by massive matter. Thus, it has been experimentally verified that Einstein equation (1) is not compatible with radiation, but the updated Einstein equation is.

The 1995 updated Einstein equation actually was first proposed by Lorentz [35] and Levi-Civita [36] as follows;

$$\kappa t(g)_{ab} = G_{ab} + \kappa T_{ab} \qquad (20)$$

where T_{ab} is the sum of other massive energy-stress tensors. Then, the gravitational energy-stress tensor takes a covariant form, However, Einstein [37] objected to this form on the grounds that his field equation implies $t(g)_{ab} = 0$. Now, Einstein is wrong since his equation is proven invalid for the dynamic case. An independent evidence for unboundedness is that the calculated radiation depends on the approached chosen [38]. Thus, eq. (19) should be called the Lorentz-Levi-Einstein equation.

While eq. (19) is consistent with the linearized equation for the massive case and can do an approximate calculation for the gravitational radiation, it is still not clear that it is the exact equation. For this, our position is that this is the best we can get so far. Further verification can be done only after the exact form of the gravitational energy-stress tensor $t(g)_{ab}$ is known. Moreover, if the unique sign for couplings could be attributed to a general $E = mc^2$,[7)] the non-unique signs of coupling would suggest that $E = mc^2$ is only conditionally valid. It has been shown explicitly that the electromagnetic energy is not equivalent to mass.

It should be noted that the anti-gravity coupling is a general feature that would appear in where the gravitational wave is present. For instance, it is necessary to appear in the Einstein equation for the gravitational waves generated by an electromagnetic wave [39, 40]. For the validity of the calculation on light bending, it is necessary that an electromagnetic wave would generate a negligible gravitational wave because this was implicitly assumed in such a calculation. For this case the related equation is the following:

$$G_{ab} = K[T(E)_{ab} - T(p)_{ab}], \quad \text{and} \quad T_{ab} = - T(g)_{ab} = T(E)_{ab} - T(P)_{ab}, \qquad (21)$$

where $T(E)_{ab}$ and $T(P)_{ab}$ are the energy-stress tensors for the electromagnetic wave and the related photons.[8)] The photonic energy-stress is necessary; otherwise there is no bounded gravitational wave solution for equation (21) [39, 40]. Thus, the anti-gravity coupling must be present for any dynamic case and Einstein's understanding in general relativity was still incomplete.

In Einstein's initial assumption, the photons consist of only electromagnetic energy. If the photons consist of only electromagnetic energy, there is a conflict since the photonic energy can be equivalent to mass and the electromagnetic energy-stress tensor is traceless. Now, this conflict is resolved since the photonic energy is the sum of electro-magnetic energy and gravitational energy. Both quantum theory and relativity are based on the phenomena of light. It is gravity that makes the notion of photons compatible with electromagnetic waves. Einstein probably would smile heartily for this.

It should be noted that the existence of an anti-gravity coupling means the energy conditions in the singularity theorems [1] are not valid for a dynamic situation. Thus, the existence of singularity is not certain, and the claim of inevitably breaking of general relativity is baseless since these singularity theorems have been proven to be unrealistic in physics. In other words, the contributions of Hawking and Penrose to physics in terms of those theorems are essentially zero if not negative. Apparently, both Hawking and Penrose also did not understand the principle of causality adequately, and therefore they accepted unbounded solutions as valid in physics. Moreover, Penrose even accepted solutions that include unphysical parameters. [34]

V. E = mc², the Reissner-Nordstrom Metric, and the Question of Black Holes

The existence of the anti-gravity coupling raised a question whether the formula $E = mc^2$ is unconditionally valid. It is found that this is only a speculation that Einstein failed to prove (1905-1909) [41]. Moreover, since the electromagnetic energy-stress tensor is traceless, an electromagnetic energy-stress tensor would generate gravitation which does not change the Ricci curvature R in the Einstein equation. However, nobody seriously studied gravitation generated by the electromagnetic energy, although the Riessner-Nordstrom metric [19] for a charged particle would answer the above issues.[9)]

Now, let us examine the Reissner-Nordstrom metric [19] (with c =1) as follows:

$$-ds^2 = \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)dt^2 - \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2 \qquad (22)$$

where q and M are the charge and mass of a particle, and r is the radial distance (in terms of the Euclidean-like structure [42]) from the particle center. In this metric (22), the gravitational components generated by electricity have not only a very different radial coordinate dependence but also a different sign that makes it a new repulsive gravity in general relativity [43]. Thus, general relativity must be extended to include the unification of electromagnetism and gravitation [43].

Nevertheless, some argued that the effective mass could be considered as

$$M - q^2/2r, \tag{23}$$

because the total electric energy outside a sphere of radius r is $q^2/2r$, and thus (23) could be interpreted as supporting $m = E/c^2$. If the electric energy has a mass equivalence, an increase of such energy should lead to an increment of gravitational strength. However, from metric (22), the strength of a gravitational force decreases everywhere after an increase of the electric energy.

Moreover, the gravitational forces would be different from the force created by the "effective mass" $M - q^2/2r$ because

$$-\frac{1}{2}\frac{\partial}{\partial r}\left(1-\frac{2M}{r}+\frac{q^2}{r^2}\right) = -\left(\frac{M}{r^2}-\frac{q^2}{r^3}\right) > -\frac{1}{r^2}\left(M-\frac{q^2}{2r}\right) \tag{33}$$

Thus Will was defeated because he could not defend his interpretation of $m = E/c^2$ [7].

The validity of $E = mc^2$ was questioned because for the binary pulsars experiment the coupling constants necessarily have different signs [11]. Nevertheless, with supports from editorials of Nature, the Physical Review D, and Science, Will continued to misinterpret the formula. Also, some theorists [44, 45] argued that M in (22) includes the external electric energy.

For instance, Herrera, Santos, & Skea [45], also argued that M in (31) involves the electric energy. They follow the error of Whittaker [46] and Tolman [47] who believed the equivalence of mass and electric energy. Then they obtained a metric that would imply a charged ball would increase its weight as the charge Q increased, in disagreement with experiments [48].

The above approach is essentially the same as that of Pekeris [44], who gets a similar metric in 1982. The difference is due to that Pekeris requires that $|g_{\mu\nu}| = g = -1$. Thus, the approach of Herrera et al. [45] is essentially what Pekeris had done. Apparently, theorists have run out of ways that can be used against the repulsive force. Nevertheless, Nobel Laureate 't Hooft even claimed incorrectly that the electric energy of an electron contributed to the inertial mass of an electron [49].

On the other hand, if the mass M is just the inertial mass of the particle, the weight of a charged metal ball can be reduced [50]. Thus, as Lo expected [7], experiments of Tsipenyuk and Andreev on two metal balls [48] rejects the claims of Herrera et al. [45] since the charged ball has reduced weight. This is an experimental direct proof. We recommend that the detailed investigation of such experiments [50] should be continued such that this static case of general relativity is fully verified.

Note that the appearing of the repulsive gravitation is important because it would solve a puzzle as to why we have never seen a black hole. If gravity is always attractive to mass, Wheeler simulation convinces him that a black hole must be formed [2]. [10)] Another piece of information for the existence of black holes is the existence of space-time singularities, proven by Hawking and Penrose [1]. Now, because the necessity of the existence of the anti-gravity coupling the energy conditions of their theorems cannot be satisfied. Thus, their space-time singularity theorems are actually irrelevant to physics.

More important, this repulsive force is crucial for establishing the unification of gravitation and electromagnetism [43].

VI. Discussions and Conclusions

In current theory of general relativity, essentially only the case of massive sources is studied, due to Einstein's speculation on mass and energy, $E = mc^2$ being unconditionally true. Moreover, because of such speculation, physicists accepted the assumption that all the coupling constants have the same sign. For instance, such assumption was implicitly included in the energy-conditions used in the space-time singularity theorems of Hawking and Penrose [1]. In turn, such theorems would imply that general relativity is inapplicable for microscopic phenomena. The problem actually is due to a lack of understanding the non-linear mathematics. However, many incorrectly blame the problem as due to the classical nature of general relativity. Furthermore, this leads to their speculation that there was a conflict between general relativity and quantum mechanics.

In spite of the efforts of generations of physicists, there is no experimental evidence to support their claims (see Appendix A). Nevertheless, the Spacetime singularity theorems provide a convenient excuse for the Big Bang Theory of the expanding universe and the notion of black holes. However, although Penrose has won his arguments against the theorist, E. M. Lifshitz [2], the problem is, however, not in mathematics. A time tested practice in physics is that if the conclusion is unusual, one should go back to examine their assumptions. However, the singularity theorems have been treated as exceptions.

Consequently, many believed that there was a conflict between general relativity and quantum mechanics. The fact is, however, that not only this is not true but also general relativity actually necessitates the existence of photons [39, 40].

Nevertheless, Einstein has commanded such a faith and thus nobody ever questions his speculation $E = mc^2$ as unconditional although Einstein has failed to prove it for years (1905-1909) [41]. Moreover, people over-looked that $E = mc^2$ being unconditionally true is in direct conflict with general relativity because of the fact that the electromagnetic energy-momentum tensor is traceless [43]. Some theorists even supply their own errors that lead to further confusion [34].

According to their theorems, Hawking and Penrose claimed that general relativity is unsuitable for microscopic phenomena [1]. However, they were not aware that *the assumption of unique coupling sign also implies that general relativity is also not suitable for macroscopic phenomena* [39, 40]. This is so because in the light bending, it is implicitly assumed that the gravity due to the light ray is negligible [15, 16]. Since the energy-stress tensor of an electromagnetic wave can be a source term, gravitational waves would be generated [51]. However, if such gravity is not negligible or having no solution due to the assumption of unique coupling sign, then general relativity would not be valid even for macroscopic phenomena. Since Einstein was not aware of this, his understanding in general relativity also needs to be improved.

A major problem is that Einstein and his followers do not understand non-linear mathematics. As Gullstrand [52] pointed out, Einstein's calculation on the perihelion of Mercury is problematic. Nevertheless, the coincidence between his calculation and observation was the source of Einstein's confidence in his theory. Understandably, almost the whole physics community was against Gullstrand. Ironically, Gullstrand is right. In fact, many failed to understand that the Einstein equation does not have a bounded solution for a two-body problem [13] as in Newtonian theory. [11)] Moreover, for the dynamic case, the Einstein equation and the linearzied equation are actually not compatible, but independent equations [14].[12)] Another problem was Einstein's partially verified formula $E = mc^2$ which turns out to be invalid for the electromagnetic energy.

Einstein's false confidence leads to serious problems. One of them is the acceptance of space-time singularity theorems of Hawking and Penrose. [13)] They serve as the justification for the Big Bang Theory that has been known to have many problems [53, 54], and black holes that have not been observed. Hawking and Penrose invalidly blame their claim of inapplicability to microscopic problem as due to being a classical theory. Moreover, the Wheeler School, in particular the errors of Christodoulou [55],[14)] have been proven as creating further confusions [34] that even misled the 1993 Nobel Prize Committee for physics [56] to claim erroneously that the Einstein equation has bounded dynamic solutions [10-13].[5)]

The formula $E = mc^2$ was proved for the case of light rays [15]. However, it remains to reconcile that the electromagnetic energy-stress tensor is traceless. Now, the notion of a photonic energy-stress tensor of massless particles solves this puzzle since the photonic energy can be equivalent to mass because photons are massless particles, but the electromagnetic energy alone cannot. Note that both quantum theory and relativity are based on the phenomena of light. Now, it is gravity in general relativity that makes photons necessary for electromagnetic waves, and thus Einstein's photonic proposal is inadequate.

The important results from this analysis are: 1) The invalid covariance principle confuses mathematics and physics. 2) The electromagnetic energy is not equivalent to mass. 3) The photons include energy from its gravitational components. [9)] 4) Einstein's general relativity is invalid for the dynamic case, for which it remains to be rectified and completed in at least two aspects: a) The exact form of the gravitational energy-stress tensor is not known; and b) The radiation reaction force is also not known. Since the photons include gravitational energy, general relativity is clearly compatible with quantum theory.

Thus, the space-time singularity theorems of Hawking and Penrose are actually irrelevant to physics. Using the same invalid unique sign assumption, the positive energy theorem of Schoen and Yau actually does not include the case of the dynamic solutions [57, 58]. [15)] Thus, such a theorem is misleading because it is based on an invalid assumption.[16),] In fact, recognizing $E = mc^2$ as only conditionally valid [6], is crucial to identify the charge-mass interaction. Moreover, such a force is coupled to the charge square, and thus such a force exists naturally in a five-dimensional theory of Lo et al. [59].

Einstein failed to show such unification because of his three shortcomings: 1) He failed to see, as Maxwell showed, that unification is necessary to have new interactions. 2) He has mistaken that $E = mc^2$ was unconditional. 3) Einstein invalidly rejected the repulsive gravitation [7]. Einstein's invalid covariance principle also added confusions [9].[17)] However, this new repulsive force can also be detected from a charged capacitor. Thus, unification of electromagnetism and gravitation beyond Einstein is confirmed [6]. Hence, Einstein turns out to be the biggest winner from the rectification of his errors. [18)]

The earliest error of Einstein started from his inadequate notion of photons which include only electromagnetic energy. This led him to believe erroneously that $E = mc^2$ could be generally valid [15]. Consequently, he rejected repulsive gravitation [7] and believed all the coupling constants had the same sign.

Einstein incorrectly believed that his test particle approach could be derivable as a limit of a dynamic solution [81] because he has never derived one with nonlinear mathematics. [19] A serious problem in general relativity is that many rush to obtain new conclusions without careful deliberations [10].

Clearly, the errors of Einstein and his followers are the obstacle of progress in physics. However, if one has any doubt on my claims, it would be beneficial for him to find examples to support his objections.[20] Note also that the outstanding work on repulsive gravitation of Musha [5] was not recognized for a long time because of the theoretical errors. Nevertheless, the work of Liu [4] makes it clear that the weight reduction of charged capacitor is due to the repulsive gravitation [6].

Journals such as the Physical Review, the Proceedings of the Royal Society A, the Annals of Physics, and the Chinese Physics accepted the singularity theorems because theorists make mistakes in non-linear mathematics and related physics. Experimental supports and explicit calculations of the present derivation make clear that they are wrong.

VII. Acknowledgments

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Appendix A: On Interpretations of Hubble's Law and Einstein's Theory of Measurement

Hubble's law is often considered as the observational evidence of an expanding universe. Then, the further a galaxies is from the Milky Way, the faster it appears to recede. However, Hubble himself concluded in 1936 that the Galaxies are stationary [60]. In fact, such a receding velocity is incompatible with the local light speeds used in deriving the light bending. Thus, such a questionable assumption that has been pointed out by Whitehead [61] and also proven as theoretically invalid.

Note that the Doppler redshifts of the light from receding Galaxies is based on an implicit assumption that there is no expansion for the space coordinates. Moreover, the receding velocity is incompatible with the light speeds used in deriving the light bending. In short, the notion of expanding universe is a production due to an inadequate understanding of a physical space. Thus, it is questionable that such a universe is related to the reality.

A.1. Hubble's Law

Hubble observed from light emitted by galaxies that the redshifts S are linearly proportion to the distance L from the Milky Way as,

$$S = H L \qquad \text{(A1)}$$

where H is the Hubble constant although the redshifts of distant galaxies will deviate from this linear law slightly.

A.2. The Redshifts

In terms of a theory from general relativity, it is well known that this law can be derived with the following metric [1],

$$ds^2 = -d\tau^2 + a^2(\tau)\{dx^2 + dy^2 + dz^2\}, \qquad \text{(A2)}$$

since

$$S = \frac{\lambda_2 - \lambda_1}{\lambda_1} = \frac{\omega_1}{\omega_2} - 1 = \frac{a(\tau_2)}{a(\tau_1)} - 1 , \qquad \text{(A3)}$$

where ω_1 is the frequency of a photon emitted at event P_1 at time τ_1, and ω_2 is the frequency of the photon observed at P_2 at time τ_2. Furthermore, for nearby galaxies, one has

$$a(\tau_2) \approx a(\tau_1) + (\tau_2 - \tau_1)\dot{a} \text{ and } \tau_2 - \tau_1 \approx R \qquad \text{(A4)}$$

Thus,

$$S = \frac{\dot{a}}{a} L = H L, \text{ and } H = \frac{\dot{a}}{a} \qquad \text{(A5)}$$

Note that Hubble's Law need not be related to the Doppler redshifts. In fact, Hubble rejected such an interpretation [61].

A.3. Hubble's Law and the Doppler Redshifts

If one chooses to define the distance between two points as

$$R = \int_1^2 a(\tau)\sqrt{dx^2 + dy^2 + dz^2} = a(\tau)L, \quad \text{where} \quad L = \int_1^2 \sqrt{dx^2 + dy^2 + dz^2} \qquad \text{(A6)}$$

Then

$$v = \frac{dR}{d\tau} = \frac{da}{d\tau}L + \frac{dL}{d\tau}a = \frac{da}{d\tau}\frac{R}{a} = HR , \qquad \text{(A7)}$$

Thus,

$$v = S. \qquad \text{(A8)}$$

This means that the redshifts could be superficially considered as a Doppler effect.

A.4. Remarks

However, if we define the distance as L, there is actually no receding velocity since L is fixed (i.e., $dL/d\tau = 0$).

Thus, whether Hubble's Law represents the effects of an expanding universe is a matter of the interpretation of the local distance. From the above analysis, the crucial point is what a valid physical velocity in a physical space is.

It should be noted that $dL/d\tau = 0$ means that the space coordinates are independent of physics. In other words, the physical space has a Euclidean-like structure, which is independent of time. However, since L between any two space-points is fixed, the notion of an expanding universe, if it means anything, is just an illusion. Moreover, the validity of (A6) as the physical distance has no known experimental supports since it is not even clearly measurable. Also, a problem is that the notion of velocity in (A7) would be incompatible with the light speeds in the calculation of light bending experiment.

A.5. The Coordinates of an Einstein Physical Space, and Definition of Velocity

If the Riemannian space is embedded in a higher dimensional flat space [62], then the coordinates dx^μ are determined by

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu, \qquad \text{or } -g_{tt}dt^2 + g_{ij}dx^i dx^j \qquad \text{(A9)}$$

such as the surface of a sphere in a three dimensional Euclidean space. For a physical space since the metric is a variable function, it is impossible to determine the coordinates with the metric. Moreover, it has been proven [42] that a frame of reference with the Euclidean-like structure must exist for a physical space.

For a spherical mass distribution with the center at the origin, the metric with the isotropic gauge is,

$$-ds^2 = -[(1 - M\kappa/2r)^2/(1 + M\kappa/2r)^2]\, c^2dt^2 + (1 + M\kappa/2r)^4\, (dx^2 + dy^2 + dz^2) \qquad \text{(A10)}$$

where $\kappa = G/c^2$ ($G = 6.67 \times 10^{-8}$ erg cm/gm^2), M is the total mass, and $r = \sqrt{x^2+y^2+z^2}$. Then, if the equivalence principle is satisfied, the light speeds are determined by $ds^2 = 0$ [12, 13], i.e.,

$$\frac{\sqrt{dx^2+dy^2+dz^2}}{dt} = c\frac{1 - M\kappa/2r}{(1+M\kappa/2r)^3} \qquad \text{(A11)}$$

However, such a definition of light speeds is incompatible with the definition of velocity (A7). *Since this light speed is supported by observations, definition (A7) is invalid in physics.*

Nevertheless, Liu [63] has defined light speeds, which is more compatible with (A7), as

$$\frac{\sqrt{g_{ij}dx^i dx^j}}{dt} = c\frac{1 - M\kappa/2r}{1+M\kappa/2r} \qquad \text{(A12)}$$

for metric (A10). However, (A12) implies only half of the deflection implied by (A11) [15].

The above analysis also explains why many current theorists insist on that the light speeds are not defined even though Einstein defined them clearly in his 1916 paper [15] as well as in his book [16]. The light speeds are well defined although diffeomorphic metrics give different sets of light speeds for the same frame of reference. However, Einstein defines light speeds after the assumption that his equivalence principle is satisfied. Thus, at most only one of such metrics is valid in physics.

Moreover, it has been proven that the Maxwell-Newton Approximation gives the valid first order approximation of the physical metric [11-13]. Since metric (A10) is compatible with the Maxwell-Newton approximation, the first order of light speed (A11) is valid in physics. Thus, the speculation that local light speeds are not well defined is proven incorrect. In essence, the velocity definition (A7), which leads to the notion of the Doppler redshifts, has been rejected by experiments.

a) Discussions

One may ask what causes such redshifts that are roughly proportional to the distances from the observer. One possibility is that the scatterings of a light ray along its path to the observer. In physics, it is known that different scatterings are common causes for losing energy of a particle, and for the case of photons it means redshifts. Unfortunately, to test such a conjecture is not possible because no current theory of gravity is capable of handling the inelastic scatterings of lights.

Nevertheless, the assumption that observed redshifts could be due to inelastic scatterings may help to explain some puzzles of observed facts [64]. For instance, younger objects such as star forming galaxies have higher intrinsic redshifts, and objects with the same path length to the observer have much different redshifts while all parts of the object have about the same amount of redshifts. For those interested in alternative cosmology theories, there are the plasma universe model [54] and others [65].

Appendix B: The Principle of Causality and the Physics of Plane-Waves

There are two aspects in causality: its relevance and its time ordering. In time ordering, a cause event must happen before its effects. This is further restricted by relativistic causality that no cause event can propagate faster than the light speed in vacuum. The time-tested assumption that phenomena can be explained in terms of identifiable causes will be called the principle of causality. This is the basis of relevance for all scientific investigations.

Thus, the principle of causality implies that any parameter in a solution for physics must be related to some physical causes. Moreover, Einstein's notion of weak gravity is also based on the principle of causality that implies a weak source would produce a weak gravity. Here this principle will be elucidated first in connection with symmetries of a field, the boundedness of a field solution, and consequently in the validity of a field equation in physics.

In practice, when the considered field is absent, physical properties are ascribed to the space-time as in a "normal" state. For example, the electromagnetic field is zero in a normal state. Then, any deviation from the normal state must have physically identifiable causes. Thus, the principle of causality implies that the symmetry must be preserved if no cause breaks it. The implication of causality to symmetry has been used in deriving the inverse square law from Gauss's law.

The normal state of a space-time metric is the flat metric in special relativity. Thus, if a metric does not possess a symmetry, then there must be physical cause(s) which has broken such a symmetry. For a spherically symmetric mass, causality requires that the metric is spherically symmetric and asymptotically flat. Also, since gravity must have a cause, a weak cause can lead to only weak gravity. Therefore, Einstein's notion of weak gravity is also a consequence of the principle of causality.

However, the physical cause(s) should not be confused with the mathematical source term in the field equation. In general relativity, the cause of gravity is the physical matter itself, but not its energy tensors in the source term of Einstein's field equation. The energy-stress tensors (for example the perfect fluid model) may explicitly depend on the metric. Since nothing should be a cause of itself, such a source tensor does not represent the cause of a metric. For the accompanying gravitational wave of an electromagnetic wave, the physical cause is the electromagnetic wave. Thus, one should not infer the symmetries of the metric based on the source term (instead of its causes) although their symmetries are not unrelated.

Moreover, inferences based on the source term can be misleading. The source term may have higher symmetries than those of the cause and the metric. For instance, a transverse electromagnetic plane-wave (1) is not rotationally invariant with respect to the z-direction of propagation. But the related electromagnetic energy-stress tensor component $T(E)_{tt}$ for a circularly polarized wave is rotationally invariant. This assumption violates causality and results in theoretical difficulties.

Classical electrodynamics implies that the flat metric is an accurate approximation, caused by the presence of weak electromagnetic waves. This physical requirement is supported by the principle of causality which implies such a metric to be a bounded periodic function. However, this required boundedness is not satisfied by solutions in the literature [66-68]. These solutions also violate causality directly since they involve parameters without any physical cause [67]. They also do not satisfy the equivalence principle [69, 70] although they are Lorentz manifolds.

A necessary and sufficient condition for satisfying the equivalence principle [21)] is that a time-like geodesic represents a physical free falling; but the mathematical existence of local Minkowski spaces is only necessary. A major problem in general relativity is that many theorists and journals do not understand related physics, in particular, the principle of causality adequately.

Endnotes

1. Although a Nobel Prize in Physics has been awarded to S. Perlmutter, B. PSchmidt, and A. G. Riess for the accelerating expansion of the universe in 2011, the expanding universe actually has not been verified (See Appendix A).
2. The existence of anti-gravity coupling has been considered by Lorentz [35], Levi-Civita [36] and Pauli [71].
3. Thus, the observation of Galileo that all neutral matter falls in the same rate under gravitation is actually incomplete [72].
4. Wald has never provided a dynamic solution [73].
5. The proof on the non-existence of dynamic solution [11] was published in 1995 Astrophys. J. when S. Chandrasekhar, a Nobel Laureate, was the editor-in-chief. This was two years after the 1993 Nobel Prize was awarded to Hulse and Taylor. Thus, Chandrasekhar also agreed that there are problems in the 1993 Nobel prize in physics.
6. Prof. S. Weinberg taught us that general relativity must be understood in terms of physics, and thus summarized the viewpoints and tradition of M.I.T on general relativity. This tradition has a long history, starting from N. Rosen and A. Einstein's paper of 1937, followed by H. Yilmaz, advocated by V. F. Weisskopf and P. Morrison, and so on. It is a pleasure to be able to contribute to such an outstanding tradition. However, to repair such a tradition is urgently needed since it has been broken by the Wheeler School [34] after Prof. Morrison passed away. This would not be an easy task for MIT since Einstein also has mistakes. And Harvard University as well as Princeton Advanced Studies were also sources of errors [57].
7. For a thorough discussion on the relation between the mass and the total energy of a particle, one can read the paper of L. B. Okun [74]. However, since Okun believed that $E= mc^2$ is generally true [75], Okun did not understand that the electromagnetic energy is not equivalent to mass [75]. Nevertheless,

Okun has some interesting thought [76] about the energy of photon related to gravity, and would question to interpret the Hubble's law as due to receding of the stars.

8. Now, we understand that Einstein's proposal of the photons is inadequate, and moreover it is a necessary consequence of general relativity. It is interesting that this is a major triumph of general relativity, but shows also a major error of Einstein in general relativity [77]. The Chinese Physics also mistaken that a bounded solution can be obtained by perturbation.
9. I have reported these to MIT President Hockfield and the subsequent President Reif. They have promised to up-grade the related education in gravitation.
10. The unconditional validity of $E = mc^2$ is responsible for the invalid notion that energy always produces attractive gravity [2; p.488]. Thus, the notion of black holes could be considered as due to the errors of Einstein.
11. Because of inadequacy in non-linear mathematics, theorists such as Hod [78], like many other "theorists", carelessly failed in recognizing the problem is that there is no bounded dynamic solution [10-13] since linearization for the static case is valid. Thus, they claimed that general relativity has superseded Newtonian theory in every aspects. Unfortunately, this is simply not true although Einstein also wrongly believed this as Gullstrand [52] pointed out.
12. Bertschinger [79} also does not know that, for the dynamic case, the linearized equation and the non-linear Einstein equation have only unrelated solutions [14]. Because he does not understand non-linear mathematics, Bertschinger erroneously [14, 34] believed that the linearization of Einstein equation was in general mathematically valid [79].
13. The space-time singularity theorems made sense only if the Einstein equation has bounded dynamic solutions. This is why Hawking and Penrose and their followers had to believe the existence of bounded dynamic solutions.
14. The Ph. D. degree advisor of D. Christodoulou is J. A. Wheeler, whose mathematics has been shown also in his book Gravitation [19] having errors in crucial arguments and unreliable at the undergraduate level. In fact, mathematician Perlick, [80, 81] has pointed out the book of Christodoulou and Klainerman is incomprehensible. Accordingly, the honors awarded to Christodoulou, in fact, reflected, the blind faith toward Einstein and accumulated errors in general relativity [82]. These expose that many theorists just do not understand non-linear mathematics. In fact, Christodoulou has never completed the construction of dynamics solutions [83]. In short, the contributions of Christodoulou to general relativity are just errors.
15. Michael Francis Atiyah has been president of the Royal Society (1990-1995), and President of the Royal Society of Edinburgh (2005-2008). Since 1997, he has been an honorary professor at the University of Edinburgh (Wikipedia). However, like many mathematicians, clearly he does not understand general relativity just as Hilbert [84] did not and thus, Yau and Witten were awarded the Fields Medal for their errors [58].
16. Due to making the same erroneous assumptions in physics (claiming the existence of dynamic solution for the Einstein equation), S. T. Yau had organized seminars for Hawking in Hong Kong and other cities of China. Now, he claimed, however, that he is no longer interested in general relativity. Thus, he ignores that his positive mass theorem [85] is misleading in physics. [57, 58]. Moreover, later the same error was made by Witten [86]. This error may also explain the fact that there is little progress in the string theory. Thus, it seems, many mathematicians at the top as well as physicists "at the top" have made errors in general relativity.
17. Due to the influence of L. Z. Fang and C. N. Yang, currently in China few understand general relativity after P. Y. Zhou [87]. Yang is against Zhou, who correctly pointed out that Einstein's covariance principle is invalid [34]. However, as Weinberg [88] pointed out, Yang's understanding on the gauge theories is incorrect [34]. Moreover, Yang is also proven wrong by explicit examples [89].
18. Apparently, Einstein did not know that his unification was that close to confirmation. If he had known this, he may not be that willing to go by rejecting the modern medicine to prolong his life [90].
19. It has never occurred to Einstein that his field equation is invalid for the dynamic case, and this is a main reason that he failed to see the necessary unification of gravitation and electromagnetism.
20. I claim a statement in physics is wrong only under two conditions: 1) it is logically not self-consistent; and 2) it is against by experiments or observations. Otherwise, I claim them as disagreements only.
21. Eric J. Weinberg, editor of the Physical Review D, invalidly insists that there is no difference in physics between Einstein's equivalence principle and Pauli's version [71] although Einstein pointed out that Pauli's is a misinterpretation [91]. Eric does not understand the functional analysis related to Einstein's equivalent principle [34], and the non-linear mathematics with related physics. His incompetence is a main reason that APS is lack behind in the physics of gravitation [92].

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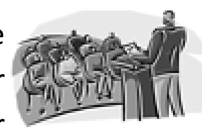

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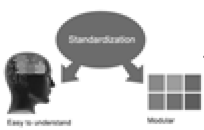

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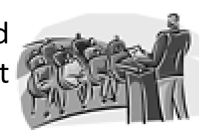

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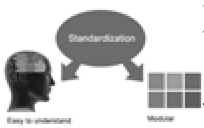

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3. Submission of Manuscripts,

4. Manuscript’s Category,

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6. After Acceptance.

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- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
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- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

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Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

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- Leave out information that is immaterial to a third party.

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The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
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Approach

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Approach:

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References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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