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Mathematics and Decision Sciences

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Magic Squares Semigroups

Ground Handling Management

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Dissemination Sinusoidal Waves

Discovering Thoughts, Inventing Future

VOLUME 15 ISSUE 1 VERSION 1.0

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The Axisymmetric Slow Viscous Flow About A Shear Stress Free Sphere

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Abstract- Harper's sphere theorem for the axisymmetric slow viscous flow exterior to a shear stress-free sphere is established in an alternative way and then given an extension of the theorem for the flow interior to the same sphere.

Keywords: harper's theorem, viscous flow, shear stress, circle theorem.

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The Axisymmetric Slow Viscous Flow About A Shear Stress Free Sphere

S. K. Sen [α], M. Kamran Chowdhury [σ] & M. Jalal Ahammad [ρ]

Abstract- Harper's sphere theorem for the axisymmetric slow viscous flow exterior to a shear stress-free sphere is established in an alternative way and then given an extension of the theorem for the flow interior to the same sphere.

Keywords: harper's theorem, viscous flow, shear stress, circle theorem.

I. Introduction

In Harper [1] it is stated that sphere on which there is no shear stress are found as boundaries in slow viscous fluid flow in two important contexts. The earth's core is, to a good approximation, such a boundary for the convection in its mantle, and the surface of a gas bubble is such a boundary for the flow outside it. In the literature corresponding to Harper's sphere theorem [1] for the axisymmetrical slow viscous flow past a shear stress-free sphere, there are the sphere theorems for axisymmetrical potential flows outside or inside a rigid sphere due to Butler [2] in terms of Stokes stream function [3]. Again there are exterior sphere theorem due to Weiss [4] and the interior sphere theorem of Ludford et al. [5] each for a general irrotational motion of inviscid fluid, both being expressed in terms of the potential function. Furthermore, for axisymmetrical slow viscous fluid motion outside or inside a rigid sphere there are sphere theorems in terms of the Stokes stream function, which are due to Collins [6, 7].

The two dimensional analogue of Harper's theorem [1] referred above is the circle theorem due to Usha et al. [8] for the slow viscous flow past a shear free circular boundary. Relevantly, there is a circle theorem for potential flow past a circular boundary, which is due to Milne - Thomson [3, 9]. Further, in the two-dimensional viscous flow theory similar theorems are found in Avudainayagon et al. [10] and Sen [11] for solving the problems of slow viscous flow past a rigid circular boundary with shear stress.

Following Batchelor [12], we may that when a body of small size moves through fluid, it generates a flow problem which is important in a variety of physical contexts, such as setting of sediment in liquid and fall of mist droplets in air. The matter of great practical interest is the drag force exerted by the fluid on the body. Except in a few simple bodies, such as spherical ones exact solutions for arbitrary body shapes in viscous fluid motions are, in general, not found in the literature.

Our main interest lies in studying the viscous flow about arbitrary rigid bodies which are shear stress-free. With this object in mind first we derive Harper's theorem for a shear stress-free sphere by an analytic technique; and this is done in section 3. For this purpose we need some relevant mathematical results, which are established in the following section.

Ref

5. G.S.S. Ludford, J. Martinek and G.C.K.Yeh (1955), The sphere theorem in rotational theory, Proc. Camb. Phil. Soc. 51, 389-393.

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II. Mathematical Theory

In this section, we first derive Stokes' equation in terms of the Stokes stream function $\psi=\psi(r,\theta)$ for the axisymmetrical motion about axisymmetrical bodies, such as a rigid sphere and then the condition of no shear stress on the sphere, due to an axisymmetrical fluid motion.

When the inertia force in a steady viscous flow field is negligibly small, the Navier-Stokes equations, governing of the flow become

$$\text{grad } p = \mu \nabla^2 q, \tag{1}$$

and

$$\text{div } q = 0, \tag{2}$$

where q is the fluid velocity, p the pressure, and μ the coefficient of viscosity of the fluid.

In the present paper it is convenient to derive the scalar expression of the vector equations (1) and (2) in spherical polar coordinates (r, θ, ϕ) and then to express them in terms of Stokes stream function $\psi = \psi(r, \theta)$ as a dependent variable for the differential equations for an axis-symmetrical slow fluid motion in a viscous fluid.

The scalar expressions of equations (1) and (2) can be derived with the help of the relevant results in Batchelor [12, appendix 2] as

$$-\frac{\partial p}{\partial r} = \mu\left\{ \nabla^2 q_r - \frac{2q_r}{r^2} - \frac{2}{r^2 \sin\theta}\frac{\partial}{\partial\theta}(q_\theta \sin\theta) - \frac{2}{r^2\sin\theta}\frac{\partial q_\phi}{\partial\phi} \right\}, \tag{3}$$

$$\frac{1}{r}\frac{\partial p}{\partial\theta} = \mu\left\{ \nabla^2 q_\theta + \frac{2}{r^2}\frac{\partial q_r}{\partial\theta} - \frac{q_\theta}{r^2\sin^2\theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial q_\phi}{\partial\phi} \right\}, \tag{4}$$

$$\frac{1}{r\sin\theta}\frac{\partial p}{\partial\phi} = \mu\left\{ \nabla^2 q_\phi + \frac{2}{r^2\sin\theta}\frac{\partial q_r}{\partial\phi} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial q_\theta}{\partial\phi} - \frac{q_\phi}{r^2\sin^2\theta} \right\}, \tag{5}$$

and

$$\frac{\partial (r^2 q_r)}{\partial r} + \frac{r}{\sin\theta}\frac{\partial}{\partial\theta}(q_\theta\sin\theta) + \frac{r}{\sin\theta}\frac{\partial q_\phi}{\partial\phi} = 0. \tag{6}$$

If the fluid motion is axisymmetrical about the z-axis, the fluid velocity everywhere in the flow field becomes independent of the azimuthal coordinate ϕ and the azimuthal velocity component $q_\phi = 0$. Thus equations (3) to (5) appear as

$$-\frac{\partial p}{\partial r} = \mu\left\{\nabla^2 q_r - \frac{2q_r}{r^2} - \frac{2}{r^2\sin\theta}\frac{\partial}{\partial\theta}(q_\theta\sin\theta)\right\}, \tag{7}$$

$$\frac{1}{r}\frac{\partial p}{\partial\theta} = \mu\left\{\nabla^2 q_\theta + \frac{2}{r^2}\frac{\partial q_r}{\partial\theta} - \frac{q_\theta}{r^2\sin^2\theta}\right\}, \tag{8}$$

And

$$\frac{\partial}{\partial r}(r^2 q_r) + \frac{r}{\sin\theta}\frac{\partial}{\partial\theta}(q_\theta\sin\theta) = 0, \tag{9}$$

where ∇^2 is the three – dimensional Laplace's operator.

Notes

These equations can be further simplified with the help of the formulae for velocity components q_r and q_θ, defined in terms of Stokes stream function $\psi = \psi(r, \theta)$ for the axisymmetrical fluid motion; and these formulae are

$$q_r = -\frac{1}{r^2 \sin\theta}\frac{\partial\psi}{\partial\theta} \text{ and } q_\theta = \frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r}, \tag{10}$$

which clearly satisfy the mass conservation equation (9).

Now eliminating q_r and q_θ from equations (7) and (8) by using the relations (10), yields

$$\frac{\partial p}{\partial r} = \mu\frac{1}{r^2\sin\theta}\left[\frac{\partial^2\psi}{\partial\theta\partial r} + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(-\cot\theta\frac{\partial\psi}{\partial\theta} + \frac{\partial^2\psi}{\partial\theta^2}\right)\right], \tag{11}$$

$$\frac{1}{r}\frac{\partial p}{\partial\theta} = \frac{1}{r^2\sin\theta}\left[r\frac{\partial^3\psi}{\partial r^3} - \frac{\sin\theta}{r}\left(\frac{\partial^2}{\partial r\partial\theta}\left(\operatorname{cosec}\theta\frac{\partial\psi}{\partial\theta}\right)\frac{2}{r^2}\frac{\partial}{\partial\theta}\left(\operatorname{cosec}\theta\frac{\partial\psi}{\partial\theta}\right)\right]. \tag{12}$$

On using the operator, $E^2 = \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right)$ for treating the axisymmetrical fluid motion [3], two concise forms of equations (11) and (12) are easily obtained as

$$\frac{\partial p}{\partial r} = \mu\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(E^2\psi) , \tag{13}$$

$$\frac{\partial p}{\partial\theta} = -\mu\frac{1}{\sin\theta}\frac{\partial}{\partial r}(E^2\psi) . \tag{14}$$

Next eliminating the pressure p from these equations results in

$$E^4\psi = 0, \tag{15}$$

which is obtained by different methods in Milne-Thomson [3]. The last equation may be called Stokes equation for the stream function of axisymmetrical and slow viscous fluid motion. We note that the stream function $\psi = \psi(r, \theta)$ for a slow viscous fluid motion past a axisymmetrical rigid body must satisfy the differential equation (15).

A general solution for the stream function ψ in spherical polar coordinates is given in [13]. For the convenience of the reference in our present study for a viscous flow past a shear stress-free sphere, we only quote here the relevant part of the general solution and this is

$$\psi(r,\theta) = \sum_{n=2}^{\infty}(a_n r^n + b_n r^{n+1} + c_n r^{n+2} + d_n r^{-n+3})\phi_n(\xi), \tag{16}$$

where a_n, b_n, c_n and d_n are arbitrary real constants, $\xi = \cos\theta$ and $\phi_n(\xi)$ is the Gegenbauer function of the first kind defined by

$$\phi_n(\xi) = \frac{P_{n-2}(\xi) - P_n(\xi)}{2n-1}, n \geq 2, \tag{17}$$

where $P_n(\xi)$ is the Legendre function of the first kind.

Ref

13. J. Happel and H. Brenner ((1986)), Low Reynolds number hydrodynamics, 4th Print, Martinus Nijhoff Publ. Dordrecht.

Now we are interested in deriving the condition for no shear stress on a rigid sphere in axisymmetrical viscous fluid motion. Here on the surface of a sphere $r = a$, out of the six components of the stress tensor [12, appendix 2] only three exist, which are

$$\sigma_{rr} = 2\mu\left(\frac{\partial q_r}{\partial r}\right)_{r=a},$$

$$\sigma_{\phi r} = 2\mu\left(\frac{1}{r\sin\theta}\frac{\partial q_r}{\partial\phi} + \frac{r}{2}\frac{\partial}{\partial r}\left(\frac{q_\phi}{r}\right)\right)_{r=a}$$

and

$$\sigma_{\theta r} = \left(\frac{r}{2}\frac{\partial}{\partial r}\left(\frac{q_\theta}{r}\right) + \frac{1}{2r}\frac{\partial q_r}{\partial\theta}\right)_{r=a}. \tag{18}$$

We first evaluate the velocity components q_r and q_θ by substituting Stokes' stream function (16) in the formulae (10) for the axisymmetrical flow past the shear stress- free sphere $r = a$.

On using the stress components (18), we then find that, on $r = a$, the normal stress $\sigma_{rr} \neq 0$, the shearing stress $\sigma_{\phi r} = 0$ in the ϕ - direction, since $q_\phi = 0$; and q_r is independent of ϕ and the shear stress in the θ - direction $\sigma_{\theta r} \neq 0$.

Finally, it is easy to calculate that

$$\sigma_{\theta r} = 0 \text{ on } r = a \quad \text{when } \frac{\partial}{\partial r}\left(\frac{1}{r^2}\frac{\partial\psi}{\partial r}\right) = 0, \tag{19}$$

and $q_r = 0$ on $r = a$ when $\psi = 0.$ (20)

Therefore, the results (19) and (20) are the required conditions for shear stress free-sphere $r = a$ for axisymmetrical fluid motion past the same sphere.

Our future aim is to solve the problems of axisymmetrical flows past arbitrary symmetrical body shapes which are shear stress free e.g. oblate and prolate spheroids, etc.

With this object in mind, we now present a relatively different analysis to establish the Harper's theorem [1] for the slow axisymmetrical viscous flow exterior to a shear stress-free sphere, and finally we also add an extension of the same theorem for the flow interior to the same sphere.

III. Harper's Theorem

In an unlimited incompressible viscous fluid there is a steady and slow axisymmetrical motion and the motion is characterized by the Stokes stream function $\psi_0 = \psi_0(r,\ \theta)$, whose singularities are all at a distance greater than 'a' from the origin and $\psi_0(r,\ \theta) \sim O(r^2)$ near the origin. Then if a shear stress-free sphere is introduced into the flow, Stokes stream function for the new flow outside the same sphere become

$$\psi(r,\theta) = \psi_0(r,\theta) - \frac{r^3}{a^3}\psi_0\left(\frac{a^2}{r},\theta\right). \tag{21}$$

Ref

1. J.F. Harper(1983), Axisymmetric Stokes flow images in Spherical free surfaces with application to rising bubbles, J. Australian. Math. Soc. Ser B 25 , 217-231.

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Proof : Since the singularities of $\psi_0 = \psi_0(r, \theta)$ are at a distance greater than '*a*' from the origin, $\psi_0(r, \theta)$ is regular at the origin. Then we suppose ψ_0 in the absence of any boundary, has an expression of the from

$$\psi_0(r,\theta) = \sum_{n=2}^{\infty} (A_n r^n + B_n r^{n+2})\phi(\xi), \tag{22}$$

where A_n, B_n are all known constants and $\phi_n(\xi)$ is the Gegenbauer function of, $\xi = \cos\theta$, of the first kind.

If a shear stress free sphere $r = a$ is now introduced into the viscous flow, the Stokes stream function for a possible new fluid motion must be obtained from the general expression (16), that is,

$$\psi(r,\theta) = \sum_{n=2}^{\infty} (A_n r^n + B_n r^{n+2} + C_n r^{-n+1} + D_n r^{-n+3})\phi_n(\xi), \tag{23}$$

where the last two terms constitute the perturbation stream function of the flow due to the presence of the sphere, and where C_n and D_n are the constants to be determined.

Here the conditions for the flow to be possible are

on $r = a$, $\psi(r, \theta) = 0$,

and on $r = a$, $\frac{\partial}{\partial r}\left(\frac{1}{r^2}\frac{\partial \psi}{\partial r}\right) = 0.$

Thus, by using these conditions, we have

$$A_n a^n + B_n a^{n+2} + C_n a^{-n+1} + D_n a^{-n+3} = 0, \tag{24}$$

$$A_n n(n-3) a^{n-4} + B_n (n-1)(n+2) a^{n-2} + C_n(-n+1)(-n-2) a^{-n-3} + D_n(-n+3)(-n) a^{-n-1} = 0. \tag{25}$$

Solving (24) and (25) for C_n and D_n, we obtain

$$C_n = -B_n a^{2n+1}, \tag{26}$$

$$D_n = -A_n a^{2n-3}. \tag{27}$$

Next, we adopt the following analysis to obtain the result (21). Using the basic stream function (22) in (23) gives

$$\psi(r, \theta) = \psi_0(r, \theta) + \sum_{n=2}^{\infty} (C_n r^{-n+1} + D_n r^{-n+3})\, \phi_n(\xi). \tag{28}$$

Substituting (26) and (27) in this expression yields

$$\psi(r, \theta) = \psi_0(r, \theta) - \sum_{n=2}^{\infty} (B_n a^{2n+1} r^{-n+1} + A_n a^{2n-3} r^{-n+3})\, \phi_n(\xi). \tag{29}$$

Now our object is to give the expression (29) a closed form and this is done as follows. From the expression (22) one gets

$$\sum_{n=2}^{\infty} (A_n a^{2n} r^{-n} + B_n a^{2n+4} r^{-n-2})\phi_n(\xi) = \psi_0\left(\frac{a^2}{r}, \theta\right)$$

Multiplying both sides by $\frac{r^3}{a^3}$, gives

$$\sum_{n=2}^{\infty}(A_n a^{2n-3} r^{-n+3} + B_n a^{2n+1} r^{-n+1})\phi_n(\xi) = \frac{a^3}{r^3}\psi_0\left(\frac{a^2}{r},\theta\right). \quad (30)$$

Finally, substituting (30) in (29) yields the Stokes stream function for the slow viscous fluid motion exterior to the shear stress-free sphere $r = a$ as

$$\psi\ (r,\ \theta) = \psi_0\ (r,\ \theta) - \left(\frac{r^3}{a^3}\right)\psi_0\left(\frac{a^2}{r},\theta\right),$$

which is in agreement with Harper's result [1]. We then show that the perturbation velocity due to the last term in (3.14) vanishes at infinity. Since $\psi_0(r,\ \theta)$ is $O(r^2)$ near the origin the perturbation stream function $\left(\frac{r^3}{a^3}\right)\psi_0\left(\frac{a^2}{r},\theta\right)$ is clearly $O(r)$ at infinity which implies a vanishing velocity at infinity. Hence the theorem is established.

IV. Extension of Harper's Theorem

We now extend Harper's sphere theorem for the viscous flow exterior to a shear stress-free sphere, to case of the flow interior to the same sphere. This extension corresponds to the Butler's interior sphere theorem [2] for the axi-symmetric and irrotational inviscid fluid flow within a sphere.

a) An Extension of Harper's Sphere Theorem

Let an axi-symmetric slow flow in an incompressible viscous fluid in the absence of rigid boundaries be characterized by Stokes steam $\psi_0 = \psi_0(r,\ \theta)$, whose singularities are all at a distance less '*a*' from the origin. Let $\psi_0 \sim O\left(\frac{1}{r^k}\right)$, $k \geq 1$ as $r \to \infty$. Now if a shear stress-free rigid sphere be introduced into the flow, the resultant flow interior to the sphere becomes

$$\psi = \psi_0(r,\ \theta) - \frac{r^3}{a^3}\,\psi_0(\frac{a^2}{r},\ \theta). \quad (31)$$

Proof : Since the singularities of the Stokes stream function $\psi_0(r,\ \theta)$ are all at a distance less than '*a*' from the origin, the function is regular everywhere in the region outside the sphere $r = a$, i.e., the region $r \geq a$.

Therefore a relevant expansion of $\psi_0(r,\theta)$ must be an expansion of the from

$$\psi_0(r,\theta)=\sum_{n=2}^{\infty}\left(A_n\frac{1}{r^{n-1}}+B_n\frac{1}{r^{-n+3}}\right)\phi_n(\xi)\ , \quad (32)$$

where A_n and B_n are all known constants, and $\phi_n\ (\xi)$ is the Gegenbauer function of the first kind already referred above.

If the shear free rigid sphere $r = a$ now is introduced into the basic flow characterized by the stream function (32), the Stokes stream function for the disturbed fluid motion may be given by

$$\psi(r,\theta) = \sum_{n=2}^{\infty}\left(A_n\frac{1}{r^{n-1}} + B_n\frac{1}{r^{n-3}} + \mathrm{C_n\ r^n + D_n\ r^{n+2}}\right)\phi_n(\xi), \quad (33)$$

Ref

2. Butler. S.F.J. (1954), A note on Stokes stream function for motion with a spherical boundary, Proc. Camb phil. Soc, Vol. 49, 169-174.

where the last two terms constitute the perturbation stream function with the undetermined constants C_n and D_n.

First we determine the constants C_n and D_n as follows . On the shear stress-free sphere $r = a$, the Stokes stream function (33) must satisfy the boundary conditions on $r = a$, $\psi = 0$,
and

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$$r = a, \quad \frac{\partial}{\partial r}\left(\frac{1}{r^2}\frac{\partial \psi}{\partial r}\right) = 0 .$$

Using these boundary conditions one obtains

$$A_n\, a^{-n+1} + B_n\, a^{-n+3} + C_n\, a^n + D_n\, a^{n+2} = 0, \tag{34}$$

$$A_n(n-1)\,(n+2)a^{-n-3} + B_n\, n(n-3)\, a^{-n-1} + C_n\, n(n-3)a^{n-4} + D_n(n-1)(n+2)a^{n-2} = 0. \tag{35}$$

Solving (34) and (35) for the values of C_n and D_n, we get very simple results as

$$C_n = -\, B_n a^{-2n+3} \quad \text{and} \quad D_n = -\, A_n a^{-2n-1}. \tag{36}$$

We now give a closed form of the stream function (33) in the following way. At the outset we note that the first two terms of (33) may be replaced by the stream function $\psi_0(r, \theta)$ referred to the expansion (32). Thus we have

$$\psi(r, \theta) = \psi_0(r, \theta) - \sum_{n=2}^{\infty} \left(B_n\, a^{-2n+3}\, r^n + A_n\, a^{-2n-1}\, r^{n+2}\right) \phi_n(\xi). \tag{37}$$

By using the relation (32) we at once have

$$\sum_{n=2}^{\infty} \left(A_n\, a^{-2n-1}\, r^{n+2} + B_n\, a^{-2n+3}\, r^n\right) = \frac{r^3}{a^3}\psi_0\left(\frac{a^2}{r}, \theta\right). \tag{38}$$

Finally, the use of this relation in (37), yields the Stokes stream function in closed form for the flow within a shear stress-free sphere as

$$\psi(r,\theta) = \psi_0(r, \theta) - \left(\frac{r^3}{a^3}\right)\psi_0\left(\frac{a^2}{r}, \theta\right). \tag{39}$$

Next we show that the stream function $\psi(r, \theta)$ gives a finite value at the origin.

Since $\psi_0(r,\theta) \sim O\left(\frac{1}{r}\right)$ for large r, the last term on the right hand side of the stream function (39) is $O(r^4)$ near the origin so that the same term gives the vanishing velocity at the origin. Thus the theorem is established.

Example : A source and sink interior to a shear stress free sphere.

Let us consider, there be a source of strength m at the point $A_1(-c,0,\ 0)$ and a sink of strength $-m$ at the point $A_2(c,\ 0,\ 0)$ on the axis of symmetry, z-axis. The Stokes stream due to their combination is given by

$$\psi_0(r,\theta) = m\cos\theta_1 - m\cos\theta_2 . \tag{40}$$

To find out the Stokes stream function for the flow within the sphere $r = a$, first we are to show that $\psi_0(r,\theta) \approx O\left(\frac{1}{r}\right)$ for large r.

Since the source and the sink lie within the sphere $r = a$, we see that c is less than a, i.e., $c < a$, then we can rewrite the stream function (40) as

$$\psi_0(r,\theta) = \frac{m(r\cos\theta + c)}{\sqrt{r^2 + c^2 + 2rc\cos\theta}} - \frac{m(r\cos\theta - c)}{\sqrt{r^2 + c^2 - 2rc\cos\theta}}, \tag{41}$$

which can be expended as

$$\psi_0(r,\theta) = m(r\cos\theta + c)\sum_{n=0}^{\infty}\frac{(-1)^n c^n}{r^{n+1}}P_n(\cos\theta)$$
$$- m(r\cos\theta - c)\sum_{n=0}^{\infty}\frac{c^n}{r^{n+1}}P_n(\cos\theta). \tag{42}$$

After the reduction we see that $\psi_0(r,\theta) \approx 0\left(\frac{1}{r}\right)$ for large *r*.

Therefore here the extension of Harper's theorem applies and yields the Stokes stream function for the flow within the sphere as

$$\psi(r,\theta) = m\cos\theta_1 - m\cos\theta_2 + \frac{m}{ac}\cos\theta_3, - \frac{m}{ac}\cos\theta_4 + \frac{m}{ac}r^2\left(R_{01}^2 - R_{02}^2\right) \tag{43}$$

where the last four terms constitute the image system outside the sphere *r=a,* and where

$$R_{01}^2 = r^2 + \frac{a^4}{c^2} + 2\left(\frac{a^2}{c}\right)r\cos\theta, \text{ and } \; R_{02}^2 = r^2 + \frac{a^4}{c^2} - 2\left(\frac{a^2}{c}\right)r\cos\theta \; .$$

V. Conclusion

We have shown an alternative way of the proof of Harper's theorem. In addition, we have extended the theorem for the flow interior to the same sphere and illustrate with an example. Numerical solutions of the problem can be useful for bubble rising research. This type of the problem has a great interest in geophysical applications.

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Notes

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A Unified Integral Associated with the Aleph Function

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Abstract- In this note we obtain a unified new integral whose integrand contains product of Aleph function and generalized multivariable polynomials having general arguments. Several integrals containing many simpler functions follow as special cases of this integral.

Keywords: aleph function, generalized polynomials, hypergeometric function, fox's H- function.

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A Unified Integral Associated with the Aleph Function

Harshita Garg [α] & Ashok Singh Shekhawat [σ]

Abstract- In this note we obtain a unified new integral whose integrand contains product of Aleph function and generalized multivariable polynomials having general arguments. Several integrals containing many simpler functions follow as special cases of this integral.

Keywords: aleph function, generalized polynomials, hypergeometric function, fox's H- function.

I. Introduction

The Aleph function introduced by Südland et al [10] is defined as Mellin-Barnes type contour integrals as following:

$$\aleph(x)=\aleph^{e,f}_{p_i,q_i,c_i;r}\left[x\left|\begin{matrix}(a_j,A_j)_{1,f},[c_i(a_{ji},A_{ji})]_{f+1,p_i;r}\\(b_j,B_j)_{1,e},[c_i(b_{ji},B_{ji})]_{e+1,q_i;r}\end{matrix}\right.\right]$$

$$=\frac{1}{2\pi i}\int_L \Omega^{e,f}_{p_i,q_i,c_i;r}(\xi)\,x^{-\xi}\,d\xi \qquad \dots (1.1)$$

For all x ≠ 0, where $i=\sqrt{-1}$ and

$$\Omega^{e,f}_{p_i,q_i,c_i;r}(\xi)=\frac{\prod_{j=1}^{e}\Gamma(b_j+B_j\xi)\prod_{j=1}^{f}\Gamma(1-a_j-A_j\xi)}{\sum_{i=1}^{r}c_i\prod_{j=f+1}^{p_i}\Gamma(a_{ji}+A_{ji}\xi)\prod_{j=e+1}^{q_i}\Gamma(1-b_j-B_{ji}\xi)} \qquad \dots (1.2)$$

The $L=L_{i\gamma\infty}$ is a suitable contour of the Mellin-Barnes type which runs from γ–i∞ to γ+i∞ with γ ∈ R, the integers e, f, p_i , q_i satisfy the inequality 0 ≤ f ≤ p_i, 1 ≤ e ≤ q_i, c_i ¿ 0; i = 1,…,r. The parameters A_j, B_j, A_{ji}, B_{ji} are positive real numbers and a_j, b_j, a_{ji}, b_{ji} are complex numbers, such that the poles of $\Gamma(b_j+B_j\xi)$, j = 1,2,…..,e separating from those of $\Gamma(1-a_j-A_j\xi)$, j = 1,…,f. All the poles of the integrand (1.2) are supposed to be easy and empty products are considered as unity. The existence conditions [4] for the Aleph function (1.2) are given below:

$$\psi_k>0,|\arg(x)|<\frac{\pi}{2}\psi_k;k=1,...,r, \qquad \dots (1.3)$$

$$\psi_k\ge 0,|\arg(x)|<\frac{\pi}{2}\psi_k \text{ and } R\{\Lambda_k\}+1<0 \qquad \dots (1.4)$$

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Ref

10. Srivastava H.M., Gupta K.C., and Goyal S.P., The H functions of one and two variables with applications (New Delhi and Madras: South Asian publ.)1982 p.11, 18-19.

Where

$$\psi_k = \sum_{j=1}^{f} A_j + \sum_{j=1}^{e} B_j - C_k\left(\sum_{j=f+1}^{p_k} A_{jk} + \sum_{j=e+1}^{q_k} B_{jk}\right) \qquad \ldots (1.5)$$

$$\Lambda_k = \sum_{j=1}^{e} b_j - \sum_{j=1}^{f} a_k + C_k\left(\sum_{j=1}^{q_k} b_{jk} - \sum_{j=f+1}^{p_k} a_{jk}\right) + \frac{1}{2}(p_k - q_k) \qquad \ldots (1.6)$$

The generalized polynomial defined by Srivastava [5] is as follows:

$$S_{f_1,\ldots,f_s}^{e_1,\ldots,e_s}[z_1,\ldots,z_s] = \sum_{\beta_1=0}^{[f_1/e_1]} \ldots \sum_{\beta_s=0}^{[f_s/e_s]} \frac{(-f_1)_{e_1\beta_1}}{\beta_1!} \ldots \frac{(-f_s)_{e_s\beta_s}}{\beta_s!} \cdot A[f_1,\beta_1;\ldots;f_s,\beta_s] z_1^{\beta_1} \ldots z_s^{\beta_s} \qquad \ldots (1.7)$$

Where $f_i = 0, 1, 2\ldots \ \forall i = (1,\ldots,s)$, $e_1,\ldots,e_s$ are arbitrary positive integers and the coefficients $[f_1,\beta_1;\ldots;f_s,\beta_s]$ are arbitrary constants, real or complex.

II. The Main Integral

We derive the following result:

$$\int_0^\infty z^{\delta-1}\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\mu} . \aleph_{p_i,q_i,c_i;r}^{e,f}\left[t\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\lambda}\right]$$

$$S_{f_1,\ldots,f_s}^{e_1,\ldots,e_s}\left[x_1\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-a_1} \ldots . x_2\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-a_s}\right]dz$$

$$= 2\Gamma(2\delta)\left(\frac{\alpha}{2}\right)^{\delta} \sum_{\beta_1=0}^{[f_1/e_1]} \ldots \sum_{\beta_s=0}^{[f_s/e_s]} \frac{(-f_1)_{e_1\beta_1}}{\beta_1!} \ldots \frac{(-f_s)_{e_s\beta_s}}{\beta_s!} A[f_1,\beta_1;\ldots;f_s,\beta_s] x_1^{\beta_1} \ldots x_s^{\beta_s} \ \alpha^{\left(-\mu-\sum_{i=1}^{s} a_i\beta_i\right)}$$

$$\aleph_{p_i+2,q_i+2,c_i;r}^{e,f+2}\left[t\alpha^{-\lambda}\left|\begin{matrix}\left(-\mu-\sum_{i=1}^{s} a_i\beta_i;\lambda\right),\left(1+\delta-\mu-\sum_{i=1}^{s} a_i\beta_i;\lambda\right)(a_j,A_j)_{1,f},[C_i(a_{ji},A_{ji})]_{f+1,p_i;r} \\ (b_j,B_j)_{1,e},[C_i(b_{ji},B_{ji})]_{e+1,q_i;r}\left(-\mu-\sum_{i=1}^{s} a_i\beta_i-\delta;\lambda\right),\left(1-\mu-\sum_{i=1}^{s} a_i\beta_i;\lambda\right)\end{matrix}\right.\right] \qquad \ldots (2.1)$$

Where

(i) Where $\lambda > 0$, Re $(\delta, \mu, a) > 0$

(ii) $\mathrm{Re}(\delta) - \mathrm{Re}(\mu) - \lambda \min_{1\le j\le e} \mathrm{Re}\left(\frac{b_j}{\beta_j}\right) < 0$ and

(iii) $e_1,\ldots,e_s$ are arbitrary positive integers and the coefficients $[f_1,\beta_1;\ldots;f_s,\beta_s]$ are arbitrary constants, real or complex.

PROOF: The integral in (2.1) can be obtained by using the Aleph function in terms of Mellin-Barnes contour integral given by (1.1) and the definition of a generalized polynomials given by (1.7), then interchanging the order of summation and integration (which is permissible under the conditions stated with (2.1)) and evaluating the inner integral by using a result given by Oberthettinger F. [3] and we get the desired result.

III. Special Cases

(1) Taking general class of polynomials in our main integral(2.1), we have

Ref

3. Oberthettinger F, Tables of Mellin transforms (Berlin, Heidelberg, New York; Springer-verlag) p.22, 1974.

$$\int_0^\infty z^{\delta-1}\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\mu} .\,\aleph^{e,f}_{p_i,q_i,c_i;r}\left[t\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\lambda}\right]$$

$$S^{e_1,\dots,e_s}_{f'}.\left[x_1\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-a_1}\dots .x_2\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-a_s}\right]dz$$

$$=2\Gamma(2\delta)\left(\frac{\alpha}{2}\right)^{\delta}\sum_{\beta_1,\dots,\beta_s=0}^{e_1\beta_1+\dots+e_s\beta_s\le f}\frac{(-f')_{e_1\beta_1+\dots+e_s\beta_s}}{\beta_1!\dots\beta_s!}A[f';\beta_1;\dots;\beta_s]\; x_1^{\beta_1}\dots x_s^{\beta_s}\,\alpha^{\left(-\mu-\sum_{i=1}^{s}a_i\beta_i\right)}$$

$$\aleph^{e,f+2}_{p_i+2,q_i+2,c_i;r}\left[t\alpha^{-\lambda}\left|\begin{matrix}\left(-\mu-\sum_{i=1}^{s}a_i\beta_i;\lambda\right),\left(1+\delta-\mu-\sum_{i=1}^{s}a_i\beta_i;\lambda\right)(a_j,A_j)_{1,f},[C_i(a_{ji},A_{ji})]_{f+1,p_i;r}\\(b_j,B_j)_{1,e},[C_i(b_{ji},B_{ji})]_{e+1,q_i;r}\left(-\mu-\sum_{i=1}^{s}a_i\beta_i\text{-}\delta;\lambda\right),\left(1-\mu-\sum_{i=1}^{s}a_i\beta_i;\lambda\right)\end{matrix}\right.\right] \quad \dots (3.1)$$

Where $e_1,\dots,e_s$ are arbitrary positive integers and the coefficients $[f',\beta_1;\dots;\beta_s]$ are arbitrary constants, real or complex and valid sufficient conditions (i), (ii) and of (2.1).

(2) If we take $s\to 1$, $e_1= 2$, $A_{f_1,\beta_1}=(-1)^{\beta_1}$ then by applying our results given in (2.1) to the case of Hermite polynomial [7] and [12] and by taking

$$S^2_{f_1}(x)\to x^{f_1/2}H_{f_1}\left[\frac{1}{2\sqrt{x}}\right]$$

We have the following result

$$\int_0^\infty z^{\delta-1}\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\mu}\left[x_1\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-a_1}\right]^{\frac{f_1}{2}}$$

$$.H_{f_1}\left[\frac{1}{2\sqrt{x_1\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-a_1}}}\right]\aleph^{e,f}_{p_i,q_i,c_i;r}\left[t\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\lambda}\right]dz$$

$$=2\Gamma(2\delta)\left(\frac{\alpha}{2}\right)^{\delta}\sum_{\beta_1=0}^{[f_1/2]}\frac{(-f_1)_{2\beta_1}}{\beta_1!}(-1)^{\beta_1}\,x_1^{\beta_1}\,\alpha^{(-\mu-a_1\beta_1)}$$

$$\aleph^{e,f+2}_{p_i+2,q_i+2,c_i;r}\left[t\alpha^{-\lambda}\left|\begin{matrix}(-\mu-a_1\beta_1;\lambda),(1+\delta-\mu-a_1\beta_1;\lambda)(a_j,A_j)_{1,f},[C_i(a_{ji},A_{ji})]_{f+1,p_i;r}\\(b_j,B_j)_{1,e},[C_i(b_{ji},B_{ji})]_{e+1,q_i;r}\,(-\mu-a_1\beta_1\text{-}\delta;\lambda),(1-\mu-a_1\beta_1;\lambda)\end{matrix}\right.\right] \quad \dots (3.2)$$

Valid under the set of sufficient conditions (i) and (ii) of (2.1)

(3) For the Laguerre polynomials ([7] and [12]) setting $s\to 1, S^{'}_{f_1}(x)\to L^{(\alpha')}_{f_1}(x)$ in which case $e_1 = 1$, $A_{f_1,\beta_1}=\binom{f_1+\alpha'}{f_1}\frac{1}{(\alpha'+1)_{\beta_1'}}$ the results (2.1) reduce to the following formulae:

Ref

7. Srivastava H.M. and Daoust M.C., Certain generalized Neumann expansions associated with the Kampé de Fériet function, Nederal. Akad. Wetensch. Indag. Math. 31 (1969), 449-457.

$$\int_0^\infty z^{\delta-1}\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\mu} L_{f_1}^{(\alpha')}\left(x_1\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-a_1}\right)$$

$$.\aleph_{p_i,q_i,c_i;r}^{e,f}\left[t\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\lambda}\right]dz$$

$$=2\Gamma(2\delta)\left(\frac{\alpha}{2}\right)^{\delta}\sum_{\beta_1=0}^{[f_1/2]}\frac{(-f_1)_{2\beta_1}}{\beta_1!}\binom{f_1+\alpha'}{f_1}\frac{1}{(\alpha'+1)_{\beta_1'}}\; x_1^{\beta_1}\,\alpha^{(-\mu-a_1\beta_1)}$$

$$\aleph_{p_i+2,q_i+2,c_i;r}^{e,f+2}\left[t\alpha^{-\lambda}\left|\begin{matrix}(-\mu-a_1\beta_1;\lambda),(1+\delta-\mu-a_1\beta_1;\lambda)(a_j,A_j)_{1,f},[C_i(a_{ji},A_{ji})]_{f+1,p_i;r}\\ (b_j,B_j)_{1,e},[C_i(b_{ji},B_{ji})]_{e+1,q_i;r}\,(-\mu-a_1\beta_1\text{-}\delta;\lambda),(1-\mu-a_1\beta_1;\lambda)\end{matrix}\right.\right] \quad \ldots (3.3)$$

Valid under the set of sufficient conditions (i) and (ii) of (2.1)

(4) Taking $c_i \to 1$, Aleph function reduces to I-function given by Saxena [5], then our main integral (2.1) reduces to the following form:

$$\int_0^\infty z^{\delta-1}\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\mu}.I_{p_i,q_i;r}^{e,f}\left[t\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\lambda}\right]$$

$$.S_{f_1,\ldots,f_s}^{e_1,\ldots,e_s}\left[x_1\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-a_1}\ldots.x_2\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-a_s}\right]dz$$

$$=2\Gamma(2\delta)\left(\frac{\alpha}{2}\right)^{\delta}\sum_{\beta_1=0}^{[f_1/e_1]}\ldots\sum_{\beta_s=0}^{[f_s/e_s]}\frac{(-f_1)_{e_1\beta_1}}{\beta_1!}\ldots\frac{(-f_s)_{e_s\beta_s}}{\beta_s!}\,A[f_1,\beta_1;\ldots;f_s,\beta_s]\,x_1^{\beta_1}\ldots x_s^{\beta_s}$$

$$.\alpha^{\left(-\mu-\sum_{i=1}^{s}a_i\beta_i\right)}.I_{p_i+2,q_i+2;r}^{e,f+2}\left[t\alpha^{-\lambda}\left|\begin{matrix}\left(-\mu-\sum_{i=1}^{s}a_i\beta_i;\lambda\right),\left(1+\delta-\mu-\sum_{i=1}^{s}a_i\beta_i;\lambda\right)(a_j,A_j)_{1,f},[a_{ji},A_{ji}]_{f+1,p_i;r}\\ (b_j,B_j)_{1,e},[b_{ji},B_{ji}]_{e+1,q_i;r}\left(-\mu-\sum_{i=1}^{s}a_i\beta_i\text{-}\delta;\lambda\right),\left(1-\mu-\sum_{i=1}^{s}a_i\beta_i;\lambda\right)\end{matrix}\right.\right] \quad \ldots (3.4)$$

Valid under the set of sufficient conditions (i), (ii) and (iii) of (2.1)

(5) Taking $c_i \to 1$ and r=1 Aleph function reduces to Fox's H-function[1], then our main integral (2.1) reduces to the following form:

$$\int_0^\infty z^{\delta-1}\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\mu}.H_{p,q}^{e,f}\left[t\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\lambda}\right]$$

$$.S_{f_1,\ldots,f_s}^{e_1,\ldots,e_s}\left[x_1\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-a_1}\ldots.x_2\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-a_s}\right]dz$$

$$=2\Gamma(2\delta)\left(\frac{\alpha}{2}\right)^{\delta}\sum_{\beta_1=0}^{[f_1/e_1]}\ldots\sum_{\beta_s=0}^{[f_s/e_s]}\frac{(-f_1)_{e_1\beta_1}}{\beta_1!}\ldots\frac{(-f_s)_{e_s\beta_s}}{\beta_s!}\,A[f_1,\beta_1;\ldots;f_s,\beta_s]\,x_1^{\beta_1}\ldots x_s^{\beta_s}$$

Ref

5. Saxena V.P., The I-function, *Anamaya Publishers*, New Delhi, (2008).

$$.\alpha^{\left(-\mu-\sum_{i=1}^{s} a_i\beta_i\right)} .H^{e,f+2}_{p+2,q+2}\left[t\alpha^{-\lambda}\left|\begin{matrix}\left(-\mu-\sum_{i=1}^{s} a_i\beta_i;\lambda\right),\left(1+\delta-\mu-\sum_{i=1}^{s} a_i\beta_i;\lambda\right)(a_j,A_j)_{1,p}\\(b_j,B_j)_{1,q}\left(-\mu-\sum_{i=1}^{s} a_i\beta_i\text{-}\delta;\lambda\right),\left(1-\mu-\sum_{i=1}^{s} a_i\beta_i;\lambda\right)\end{matrix}\right.\right] \quad \ldots (3.5)$$

Valid under the set of sufficient conditions (i), (ii) and (iii) of (2.1)

(6) If we take $c_i \to 1$, r=1and $e_1,\ldots,e_s \to e$ and $f_1,\ldots,f_s \to f$ i.s. (1,..,s→1) in the integral (2.1), we arrive at the following result which is obtained by Garg and Mittal [2] .

$$\int_0^\infty z^{\delta-1}\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\mu} .H^{e,f}_{p,q}\left[t\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\lambda}\right]$$

$$S^e_f\left[x\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-a}\right]dz$$

$$=2\Gamma(2\delta)\left(\frac{\alpha}{2}\right)^{\delta}\sum_{\beta=0}^{[f/e]}\frac{(-f)_{e\beta}}{\beta!}A[f,\beta]x^{\beta}\ \alpha^{(-\mu-a\beta)}$$

$$.H^{e,f+2}_{p+2,q+2}\left[t\alpha^{-\lambda}\left|\begin{matrix}(-\mu-a\beta;\lambda),(1+\delta-\mu-a\beta;\lambda),(a_1,A_1),(a_p,A_p)\\(b_1,B_1),(b_q,B_q),(-\mu-a\beta\text{-}\delta;\lambda),(1-\mu-a\beta;\lambda)\end{matrix}\right.\right] \quad \ldots (3.6)$$

Where

(i) $\lambda>0$, Re (δ, μ, a) >0

(ii) Re (δ) - Re (μ) − $\lambda \min_{1\le j\le e} \mathrm{Re}\left(\frac{b_j}{\beta_j}\right)<0$ and

(iii) e is are arbitrary positive integers and the coefficients $[f,\beta]$ is arbitrary constant, real or complex.

(7) If we take $S^{e_1,\ldots,e_s}_{f_1,\ldots,f_s}\to 1$ and reduce Aleph function to Gauss hypergeometric function [9] in the integral (2.1), we arrive at the following result after some simplifications:

$$\int_0^\infty z^{\delta-1}\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-\mu} {}_2F_1(l,m;n;t\left[z+\alpha+(z^2+2\alpha z)^{\frac{1}{2}}\right]^{-1})dx$$

$$=2^{1-\delta}\mu\Gamma(2\delta)(\alpha)^{\delta-\mu}\frac{\Gamma(\mu-\delta)}{\Gamma(\mu+\delta+1)}\,{}_4F_3(\ell,m,\mu-\delta,\mu+1;n,\mu,\mu+\delta+1;\frac{t}{\alpha}) \quad \ldots (3.7)$$

Where 0<Re (δ) < Re (μ); $|t|<|\alpha|$

IV. Conclusion

The result so established may be found useful in several interesting situation appearing in the literature on mathematical analysis. The result (3.1) not only gives the value of the integral but also 'augments' the coefficients in the series in the integrand to give a ${}_4F_3$ series as the integrated series.

Ref

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Notes

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Integrated Decision Making for Ground Handling Management

By Salma Fitouri-Trabelsi, Felix Mora-Camino, Carlos Alberto N. Cosenza & Li Weigang

Coppe/Ufrj, Brasil

Summary- In this paper a hierarchical structure for the management of airport ground handling activities is proposed. The main decision making processes in charge of the managerial units composing a proposed ground handling management organization are considered. The global objective is to turn available the ground handling resources so that arriving and departing flight are serviced with as little delay as possible. Two operational situations are considered: a normal one where small delays are coped with when arriving and departing traffic is globally on schedule, and a disrupted situation where arriving or departing traffic suffer very large delays.

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INTEGRATEDDECISIONMAKINGFORGROUNDHANDLINGMANAGEMENT

Strictly as per the compliance and regulations of :

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Integrated Decision Making for Ground Handling Management

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Summary- In this paper a hierarchical structure for the management of airport ground handling activities is proposed. The main decision making processes in charge of the managerial units composing a proposed ground handling management organization are considered. The global objective is to turn available the ground handling resources so that arriving and departing flight are serviced with as little delay as possible. Two operational situations are considered: a normal one where small delays are coped with when arriving and departing traffic is globally on schedule, and a disrupted situation where arriving or departing traffic suffer very large delays.

I. Introduction

The sustained global economic growth of the last decades has been possible with the development of improved means of communication and of transportation of people and goods. It has been particularly the case with air transportation where, during the last forty years, the number of passengers has been multiplied by seven. This increase of passenger volume has generated a permanent challenge for civil aviation authorities, airlines and airports to supply sufficient capacity to provide a safe transportation service with acceptable quality standards (Santos et al., 2010). In the last decade, new traffic management practices, such as Airport Collaborative Decision Making (A-CDM) (Eurocontrol, 2011), based on multi-agent and collaborative decision making concepts have been introduced at airports. Among the many activities which contribute to the safety and efficiency of air transportation, airport ground handling plays an important role even if it has remained in the shadow of other traffic activities in the Operations Research literature. While among the overall airport operations costs, ground handling costs represent a rather small portion, their dysfunction can generate huge extra costs for airlines and airports as well as high discomfort for passengers (Pestana, 2008).

In this study a hierarchical structure for the management of airport ground handling activities is considered. The global objective is to turn available the ground handling resources so that arriving and departing flight are serviced with as little delay as possible. Two operational situations are considered: a normal one where small delays are coped with when arriving and departing traffic is globally on schedule, and a disrupted situation where arriving or departing traffic suffer very large delays.

In the first situation a ground handling coordinator produces an estimate of the necessary resources from each ground handling service provider while these service providers assign the available resources to the scheduled ground handling activities. At both levels, the formulation of corresponding optimization problems leads to NP-complete problems while a new solution should be at hand whenever new operations conditions appear. So, heuristic approaches have been developed to generate working solutions to this overall problem. While in the case of normal operations these heuristics consider the flights according to their nominal schedule, in the disrupted operations, flights are treated in accordance with an estimated degree of criticity computed by the

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ground handling coordinator. The proposed approach is illustrated with traffic data from a large European airport.

II. Hierarchical Structure for the Management of Ground Handling at Airports

When considering ground handling organization in different airports, it appears that this organization depends strongly on the size and the physical organization of the airside as well as on the volume and composition of traffic. Then, a large diversity of actual ground handling organizations is found in major and medium size airports. Then it does not appear desirable to propose a general paradigm to organize airport ground handling since the resulting efficiency can be quite unequal from an airport to the next. However, when some key characteristics are met, delimiting a specific class of ground handling situations, common organizing principles can be of interest.

Here some assumptions with respect to airport ground handling characteristics, which are frequently encountered in medium to large airports, are adopted. They are the following:

Here is considered the case of airports in which ground handling is performed by a set of specialized operators working in parallel under the management of the airport authorities.

The ground handling process is supposed to follow pre-established sequencings and to be performed at the parking stands. It is supposed that the parking stands are assigned to arriving flights by the airport and communicated through ATC, while the status of the parking stands is monitored by ATC which is in charge of driving the aircraft out of the parking position. It is also supposed that the arriving parking position is its departure parking position for the next flight. This last assumption introduces constraints on the ground handling activities.

From the considerations developed in the previous paragraph, it appears interesting to consider that the airport ground handling operators do not interact directly within the A-CDM framework (Eurocontrol,2011) , but through a ground handling coordinator.

The introduction of the GHC led to a hierarchical structure for the ground handling management as it showed in Fig.1.

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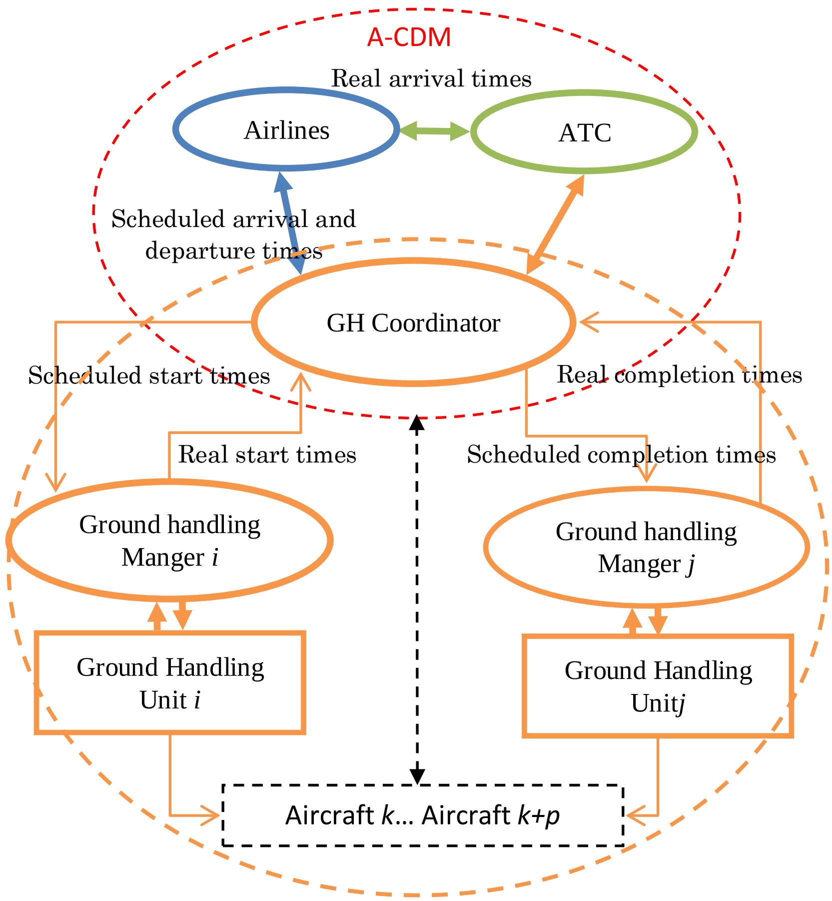

Figure 1 : Connection of A-CDM with Ground Handling

a) Ground Handling Coordinator

This coordinator will be a communication interface between the other A-CDM partners and the ground handling managers.

The principal functions of the GHC are:

- To provide to the other airport partners:
  - predictions of ground handling delays
  - Generation of milestones
- To provide to the ground handling managers:
  - Predictions about activity levels
  - required ground handling resources per period

i. *Ground handling milestones monitoring*

The ground handling activities around an aircraft can be divided in two set of operation:

- The set of arrival ground handling operations, A_i^{gh} , which includes all the ground handling activities which must be performed to conclude properly the current commercial flight. The main arrival ground handling activities are de-boarding passengers, unloading baggage, performing cleaning and sanitation.
- The set of departure ground handling operations, D_i^{gh}, which gathers the ground handling activities which must be performed to prepare the next commercial flight. The main departure activities are passengers boarding, baggage loading, fuelling, catering.

The possible milestones monitored by the ground handling coordinator are:

- time of start of arrival ground handling activities :

$$T_i^{agh} = \min_{k \in A_i^{gh}} \left\{ t_{ik}^{agh} \right\} \qquad (1)$$

- time of completion of arrival ground handling activities :

$$\tau_i^{agh} = \max_{k \in A_i^{gh}} \left\{ t_{ik}^{agh} + d_{ik}^{agh} \right\} \qquad (2)$$

- time of start of departure ground handling activities :

$$T_i^{dgh} = \min_{k \in D_i^{gh}} \left\{ t_{ik}^{dgh} \right\} \qquad (3)$$

- time of completion of departure ground handling activities :

$$\tau_i^{dgh} = \max_{k \in D_i^{gh}} \left\{ t_{ik}^{dgh} + d_{ik}^{dgh} \right\} \qquad (4)$$

Here t_{ik}^{dgh} is the start time of ground handling activity k on departing aircraft i, d_{ik}^{dgh} is the duration of the ground handling activity k on aircraft i. All these time related variables and parameter adopt two values: their estimated value which can evolve and their effective value at completion.

ii. *Global planning of ground handling resources*

The planning of ground handling resources should be performed at start for a whole day of operation by considering as basic input information:

- the time schedule of arriving and departure flight,
- the operational characteristics of these flights.

The prediction of the necessary GH resources (vehicles and work force) over the operations period is performed in three steps:

- *a global ground handling assignment* (GGHA) problem is solved for a nominal schedule of flights. A fast heuristic solution is proposed (greedy approach)
- *totalization of necessary resources* is performed for each time interval. Here a time interval within the operating period is chosen for the resources used by task t :

$$u_t = \max\left\{ \text{Timing}, \min_{j \in K} s_j^t \right\} \qquad (5)$$

- *margins are added to the estimation of necessary resources:*

For arrival ground handling activities:

$$r_i^k = n_i^k + p_A^k A_i^k \qquad (6)$$

For departure ground handling activities:

$$r_i^k = n_i^k + p_D^k D_i^k \qquad (7)$$

where: n_i^k is the nominal number of teams (vehicle and staff) of type i necessary at period k to process scheduled arrivals/departures, r_i^k is the computed required number of teams of type i necessary at period k, to process schedules arrivals/departures, included reserve, A_i^k is the number of teams of type i necessary to handle flight arrivals at parking stands during the previous half an hour which are supposed to be processed before period k , D_i^k is the number of teams of type i necessary to handle flight departures at parking stands during the previous half an hour which are supposed to be

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processed before period k and, p_A^k is the probability that an arrival scheduled within half an hour before period k is delayed and should be processed at period k and p_D^k is the probability that a departure scheduled within half an hour before period *k* is delayed and should be processed at period *k*.

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b) Ground Handling Manager

The ground handling manager has two principal functions:

- Planning operations
- Managing operations

i. *Planning operations*

To achieve this function the ground handling manager has to:

- Solve its *pairing problem* to cover all planned demands for its services: during the current operations period. Result: list of duties which will be performed by its GHU's.
- Create the ground handling units by *assigning* its resources to its duties (a resource roastering problem).

ii. *Managing operations*

Managing operations consists in the first time to update the assignment of his ground handling resources to aircraft considering the information received from the GHC has in case of:

- perturbation at the level the aircraft's arrival times
- perturbation at the level of the duration of performing of the tasks
- weather conditions (strong rain, snow, strong wind, etc.)

It consists also in monitoring the GHUs. A ground handling unit can be in the following states:

- *deactivated:* either the equipment is not ready (under repair or maintenance) or the operators are not available,
- *waiting for assignment:* the unit is enabled but has not been assigned to flights,
- *assigned:* the unit has been assigned to one or more flights, but the realization of the activity on the first of these flights is planned far in the time horizon,
- *made ready to perform its next activity:* this happens when the planned time to perform a ground handling activity is near. This corresponds either to the time necessary to adapt the resource to the flight to be served or to a minimum time delay to inform the operators of the next operation,
- *operating:* the unit is performing the activity (transfer operations and processing at aircraft or terminal).

III. Nominal Decision Making Processes with the Proposed Approach

a) The ground handling coordinator level

The decision making considered at this level is to solve the global ground handling assignment which is the first step of the global planning of ground handling resources.

A fast heuristic solution is proposed (greedy approach) which consists in. this approach will ensure the feasibility of all ground handling operations. The idea of the porposed heuristic is to rank arriving and departing aircraft according to their planned start time of the corresponding ground operations (either arrival ground handling tasks or departure grand handling tasks). Then the GHC will process in this order each aircraft ground handling activity by linking each task to a route to build a ground handling duty:

- To cover task j at aircraft k it will search between the already created routes of type j , which one can cope with it, within the planned interval and at lower transportation cost.
- If none of the existing route provides a feasible solution
  - and there are remaining capacity of type j at the corresponding base, a new route of type j starting at this base is created with first stop at aircraft k.
  - and there are no remaining transport capacity at base of type j, add this task at the route of type j which minimizes the mix of resulting delay for aircraft k and of distance travelled to reach it with the weight λ.

Then repeat with all the expected ground handling tasks j at an arriving or departing aircraft.

This will produce feasible sets of duties (routes) to be performed by the different ground handling fleets and workforce. Then this data will be used by the ground handling coordinator to compute, according to the process proposed in the previous chapter, the level of resources that each ground handling manager must provide at each time period. These resources will be afterwards either effectively used to process aircraft and passengers or will remain as a warm reserve to face perturbations and incidents.

b) The ground handling manager level

In a nominal situation, the ground handler fleet managers will assign a vehicle and a work team to each route. This vehicle may be changed by another to pursue the duty in accordance with operational considerations (refueling need, mechanical failure, etc) while work teams will be shifted according to labor and safety regulations.

Here it is supposed that there are enough spare vehicles and work teams to meet operational perturbations.

The proposed heuristic consists in:

- For each ground handling manager:
  - Order the aircraft in accordance with their arrival/ departure time, depending on the type of the ground handing fleet service.
  - Assign to each aircraft taken in order a vehicle considering:
    - Availability of all vehicles of the fleet.
    - The distance from its current position to the considered aircraft

This is a rather simple greedy heuristic which provides for each fleet facing the current service demand a complete solution through a reduced computational effort. So there is no limitation in calling back this solution process any time a significant perturbation occurs.

In the case of ground handling fleets involved in unloading/loading activities at parked aircraft, aircraft will be duplicated considering their current scheduled arrival time at the parking position and their current scheduled departure time from the same parking position. Then each duplicate will be ordered according to increasing time.

From the solutions of the assignment problems solved by each ground handling manager , the ground handling coordinator forward the milestones corresponding to the completion of ground handling activities to the airlines and the ATC to produce if necessary new estimates for the departure schedule of the aircraft.

c) Case of study

To validate the proposed ground handling organization and the associated decision making processes real traffic data from Palma de Mallorca Airport was considered. Palma de Mallorca Airport is, with respect to aircraft and passengers traffic, the third largest Spanish airport. During the summer period it is one of the busiest airports in Europe, and was used by 22.7 million passengers in 2011. The airport is the main base for the Spanish carrier Air Europa and also a focus airport for German carrier Air Berlin. It occupies an area of 6.3 km2 (2.4 sq mi). Due to rapid growth of aircraft traffic and passenger numbers, additional infrastructure has been added to the

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two first terminals A (1965) and B (1972). It is composed now of two runways, four terminals and 180 parking stand (27 of them at aprons) (PDM, 2012). It can handle up to 25 million passengers per year, with a capacity to dispatch 12,000 passengers per hour.

To evaluate the proposed approach, we tested it using aircraft traffic for a 24h period (01/08/2007) with 690 arrivals and departures distributed between the four parking areas related with the four terminals of Palma de Mallorca Airport. Except for aircraft staying at night at the airport, all ground handling operations are done in the context of fast turnaround operations. Different sizes of ground handling fleets have been considered. The resulting earliest departure time for aircraft have been compared with the real time departure data, showing that with rather reduced ground handling fleets at each terminal, the proposed heuristic, coded in Java, does not generate additional delays. Fig.2 displays the hourly traffic of arriving and departing aircraft on a typical summer day at this airport. It appears that aircraft traffic remains intense from early morning until the beginning of night hours.

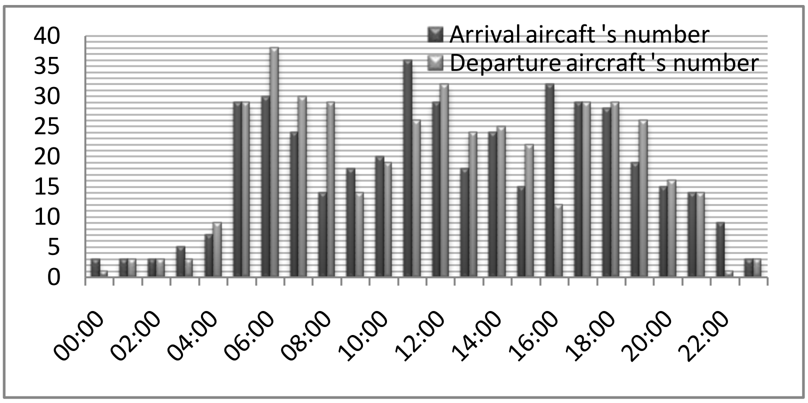

Figure 2 : 01/08/2007 PDM Airport aircraft hourly traffic

The proposed heuristic approach has been tested for the aircraft traffic with the ground handling fleets of Fig.3.

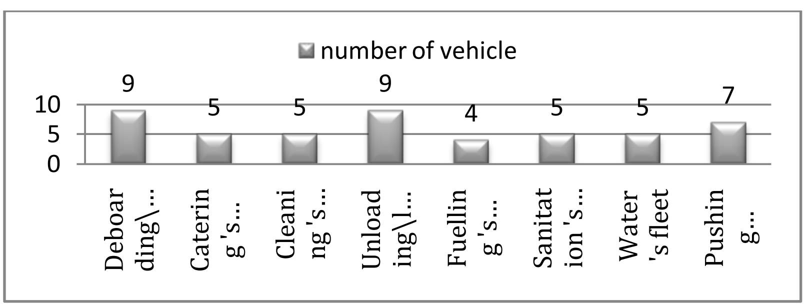

Figure 3 : Nominal composition of ground handling fleets

i. *Implementing the global planning of ground handling resources*

This approach is proposed to calculate the nominal number of resources required for each ground handling manager during a day of traffic. The solution of this approach is given in the Table 1. It represents the number of the aircraft which will be performed by each ground handling unit of each ground handling service provider.

Ground handling activity	**GHU1**	**GHU2**	**GHU3**	**GHU4**	**GHU5**	**GHU6**	**GHU7**	**GHU8**	**GHU9**
De-boarding/ Boarding	71	58	43	38	32	25	19	12	6

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passengers									
Unloading/ Loading baggage	133	95	93	85	66	79	60	51	28
Catering	86	80	66	58	55				
Cleaning	97	77	60	61	50				
Refuelling	103	92	84	66					
Sanitation	144	94	59	34	14				
Water Supply	103	82	66	53	41				
Push back	118	112	84	37	31				

Table 1 : Solution of hierarchical approach

Using this solution, only 14 aircraft will have a delay at the level of the departure times with a maximum delay of 14 minutes. The 14 aircraft that would leave their parking stand later that which it had been predicted their departure times match with busiest flight traffic period.

This global planning of ground handling resources as it has been described is composed of three steps:

For the first step, it has been supposed that the nominal number of each ground handling resources is presented in the figure.

In the second step, the unit time period which has been considered has been taken equal to the maximum between 5 minutes and the smallest duration of a ground handling operation, including transfer time according to the formula (5).

Ground handling activity	Duration (min)
De-boarding passengers	5
Catering	5
Cleaning	5
Boarding passengers	5
Unloading baggage	5
Fuelling	5
Loading baggage	5
Sanitation	5
Potable water supply	5
Push-back	5

Table 2 : The unit time period of each ground handling operation results

The third step of the estimation of the necessary resources at a given time for all ground handling managers is performed by adding margins to the nominal level of demand of scheduled arrival and departure flights. This is done according to formula (6) and (7).

The figures presented below provide the size of the resources required for each ground handling manager to perform their corresponding ground handling tasks in case of perturbations that can occur during the day. As it can be seen, the number of reserved resources increases in the busiest flight traffic period (arrival/departure aircraft) according to the Fig-4.

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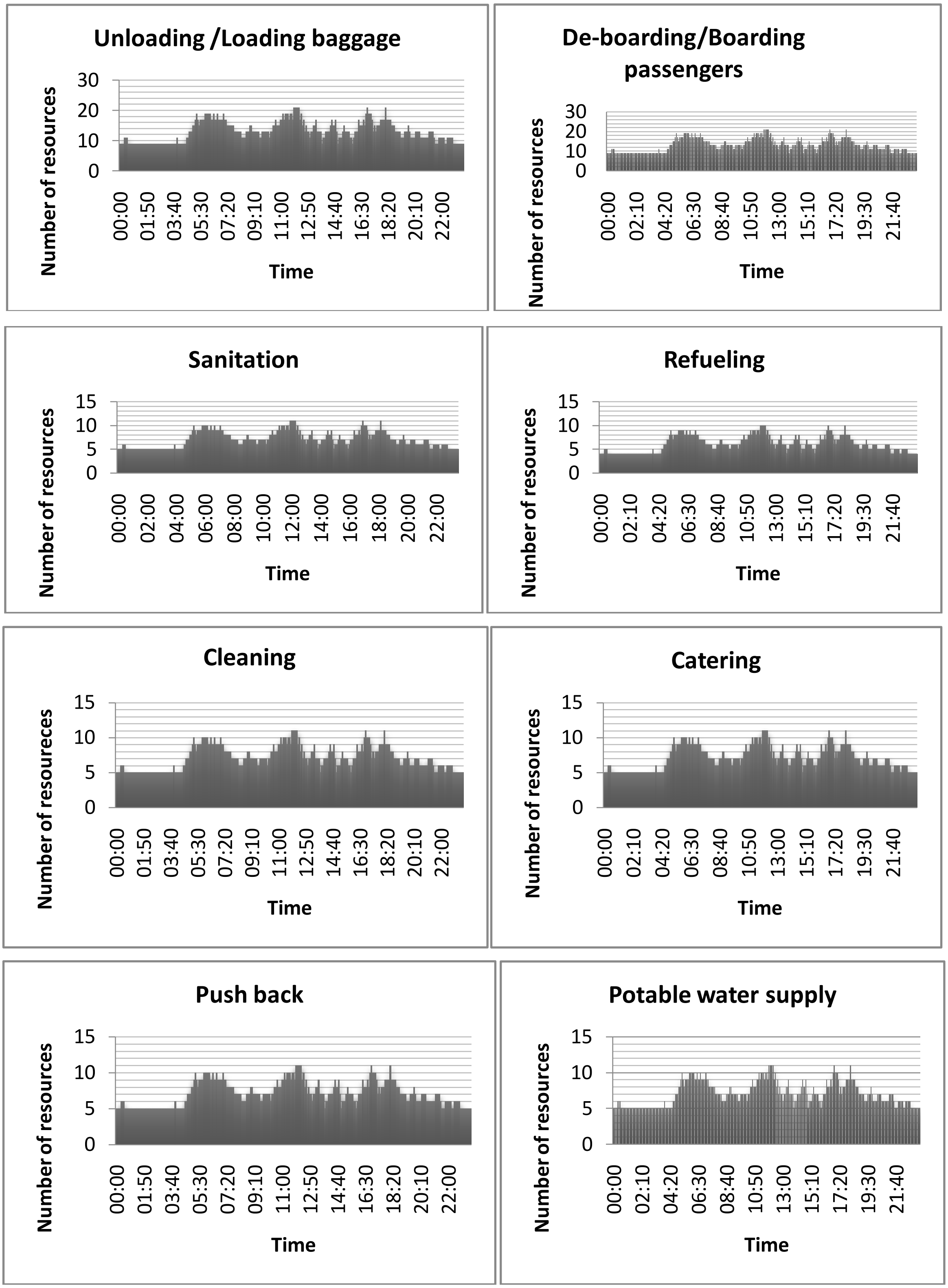

Figure 4 : Number of the resources required for each ground handling activities each of period of time

ii. *Implementing the heuristics for on-line GHFA*

To test the efficiency of this approach, the accurate arrival times of each considered flights are supposed to be communicated to the ground handling managers thirty minutes before the effective landing. Here, this allows the ground handling

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managers to reassign the ground handling resources by considering the updated arrival times at the parking stands of the flights announced to land within the next half hour. Aircraft within five minutes to land have been supposed to maintain the previous assignment solution. No flight directed towards the considered airport has duration less than forty minutes. Then the real departure times where compared with the ones obtained through the proposed heuristic approach. The considered ground handling resources were the ones effectively existing at that airport.

The application of the proposed heuristic approach to the nominal schedule of arrivals during the considered reference day provided a feasible assignment for each ground handling manager in at most 0.3 seconds. These solutions led to delays with respect to scheduled departure schedule involving only 36 aircraft, with a maximum delay of 16 minutes. The average delay among delayed aircraft has been of 7 minutes. Fig.5 displays the hourly distribution of delayed aircraft at departure resulting from the application of the proposed decentralized approach. Clearly, the occurrence of these delays corresponds to the busiest aircraft traffic periods at the airport where ground handling resources become short. The proposed heuristic could be restarted using higher ground handling resource levels provided by the ground handling coordinator to improve the expected delay performance of the system.

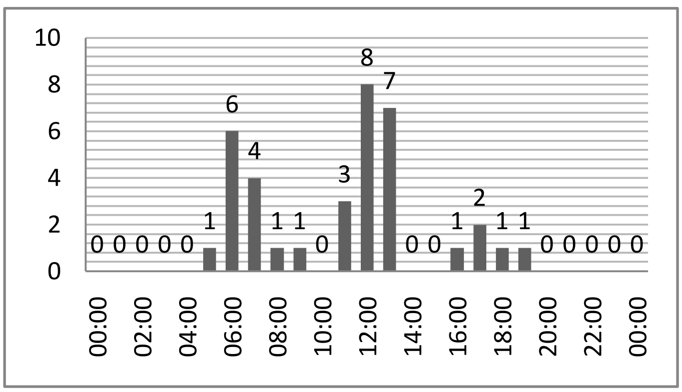

Figure 5 : Hourly delays distribution resulting from the proposed heuristic

Historical data from 01/08/2007 at Palma de Mallorca Airport indicate that about 244 aircraft departures where delayed for multiple reasons, including one of the main reasons, ground handling delays. The maximum observed delay is about 520 minutes and the average delay among delayed aircraft has been of 30 minutes. There is information about the use of a particular system to manage ground handling at that airport.

It is clear, that in theory, the proposed heuristic approach provide significantly improved results with respect to departure delays. Then it can be expected for this particular airport that, even if the implementation of the proposed heuristic approach is not perfectly performed, some noticeable improvement with respect to the current practice will take effect. This is quite noteworthy since the proposed heuristic has not been particularly improved with respect to a basic greedy approach.

IV. Ground Handling Management Under Disruption

To our knowledge there exists no specific definition for airport disruption while some recent works refer to this situation (Ploog, 2005) and (Tanger and al, 2013) without providing any definition. According to the British Standards Institute (Business continuity management, 2006), "a disruption is an event which causes an unplanned, negative deviation from the expected delivery according to the organization's objectives". According to this definition, the term disruption could be perceived as equivalent to the term perturbation. The ground handling services are delivered in a changing environment with many operational uncertainties. For example, the expected arrival times for flights are subject to frequent delays, the duration of ground handling tasks is sensitive to unexpected events such as additional travel time due to traffic congestion on airside service ways or machine breakdowns. Then it could be considered that ground handling management tackles in permanence disrupted situations.

Notes

a) *fuzzy heuristic for on-line ground handling management problem*

The problem for each ground handling fleet is here to assign ground handling vehicles to arriving or departing aircraft so that each aircraft is serviced by a vehicle while, according to the current operational situation, no delay or a minimum delay is produced. For that, the airline ground station managers generate resources requests to the ground handling fleet managers. The produced schedules are based on the predicted arrival times as well as the scheduled departure times. These schedules take not only into consideration the possible variation of the ground handling tasks durations by using a fuzzy dual formalism (Cosenza, 2011; Cosenza, 2012), but consider also the criticality of the flight. This criticality depends on the current predicted delay as well as the operational consequences on other flights. Then more critical flights may get their ground handling solution treated before earlier less critical scheduled flights. The following notations are adopted: Each task of the turnaround process $t \in \{1,\ldots,T\}$ is carried out on an aircraft a(i) associated to a flight i, i∈I, (I=IA∪ID, IA is the set of arriving flights and ID is the set of departing flights) by a specific service provider $k \in \{1,\ldots,K\}$.

b) *Fuzzy-based ranking of flights*

The first step of the proposed heuristic consists in performing an initial ordering of the flights in accordance with their current predicted arrival time $\hat{t}_i^a$ at their assigned parking amended by considering their criticality. To each arriving flight i ∈ I, can be assigned the difference $\Delta t_i^a = \hat{t}_i^a - \bar{t}_i^a$ between the predicted arrival time $\hat{t}_i^a$ and the scheduled arrival time $\bar{t}_i^a$. Here $\hat{t}_i^a$ and $\bar{t}_i^a$ can be either real numbers or fuzzy dual numbers, where $\hat{t}_i^a$ is provided by the ATC. Each arriving flight must cope with two types of operational constraints:

Connection constraints when arriving passengers must reach without delay another departing flight.

Departure schedule when the arriving aircraft must be ready to start a new flight with a tight schedule..

When considering connection constraints, let C_i be the set of departing flights connected to arriving flight i. The time margin between fight i and each flight j in C_i is given by:

$$\tilde{m}_{ij}^a = \bar{t}_j^d - \hat{t}_i^a - \max\left\{ \tilde{d}_{db}^{\,i} + \tilde{T}_{ij}, \tilde{d}_{ul}^{\,i} + \tilde{\theta}_{ij} \right\} \quad j \in C_i \tag{8}$$

Here $\tilde{T}_{ij}$ and $\tilde{\theta}_{ij}$ are respectively the connecting delay for passengers and luggage between flights i and j. The margin between arrival flight i and departure flight j serviced in immediate succession by the same aircraft is:

$$\tilde{m}_{ij}^a = \bar{t}_j^d - \hat{t}_i^a - \tilde{D}_{ij} \text{ with } j = \sigma(i) \tag{9}$$

where $\tilde{D}_{ij}$ is the minimum fuzzy dual duration of ground handling around arrival of flight i and departure of flight j. Here $\sigma(i)$ provides the number of the next flight serviced by the aircraft operating flight i. Then:

$$\tilde{D}_{ij} = \max\begin{Bmatrix} \tilde{d}_{ul} + \tilde{d}_{fu} + \tilde{d}_{ll} \\ \tilde{d}_{db} + \tilde{d}_{ca} + \tilde{d}_{bd} \\ \tilde{d}_{db} + \tilde{d}_{cl} + \tilde{d}_{bd} \\ \tilde{d}_{sa} + \tilde{d}_{wa} \end{Bmatrix} + \tilde{d}_{pb} \tag{10}$$

Then, the fuzzy margin of arriving aircraft i is given by:

$$\tilde{m}_i^a = \min_{j \in C_i \cup \sigma(i)} \tilde{m}_{ij}^a \tag{11}$$

The amended arrival time for flight i is then given by:

$$\tilde{\tilde{t}}_i^a = \hat{t}_i^a + \tilde{m}_i^a \tag{12}$$

To each departing flight I ∈ ID, can be assigned the difference $\Delta t_i^d = \hat{t}_i^d - \bar{t}_i^d$ between the predicted departure time $\hat{t}_i^d$ and the scheduled departure time $\bar{t}_i^d$. Here also, $\hat{t}_i^d$ and $\bar{t}_i^d$ can be either real numbers or fuzzy dual numbers. Symmetrically, each departing flight must cope with operational constraints related with successive flights by the same aircraft and flight connections for passengers and cargo.

In the case in which the ground handling tasks are relative to a departing flight j, the amended predicted time to start grand handling activities at the corresponding parking position is now given by:

$$\tilde{\tilde{t}}_j^d = \bar{t}_j^d - \min_{i \mid j \in C_i and\ i = \sigma^{-1}(j)} \tilde{m}_{ij}^a \tag{13}$$

with

$$\tilde{m}_{i\sigma(i)}^a = \max \begin{Bmatrix} \tilde{d}_{fu} + \tilde{d}_{ll} \\ \tilde{d}_{ca} + \tilde{d}_{bd} \\ \tilde{d}_{wa} \end{Bmatrix} + \tilde{d}_{pb} \tag{14}$$

Then, to each flight *i*, either arriving or departing, is assigned a time parameter τ_i such as:

For arriving flights:

$$\tau_i = \left\| \tilde{\tilde{t}}_i^a \right\| \tag{15}$$

For departing flights:

$$\tau_i = \left\| \tilde{\tilde{t}}_i^d \right\| \tag{16}$$

where $\| \|$ is the fuzzy dual pseudo norm. Then the flights, either arriving or departing, present in the considered period of operation can be ranked according to an increasing τ_i index. Let the integer ra (i) be the amended rank of flight i.

c) Ground Handling Fleets assignment to flights

Then flights are processed in the produced order ra(i) where ground handling vehicles are assigned to the corresponding aircraft. In the case of an arriving flight, ground handling arrival tasks (unloading luggage, de-boarding, cleaning and sanitation) are coped with by assigning the corresponding vehicles in accordance to their previous assigned tasks with other aircraft, their current availability, and their current distance to the considered aircraft. Here the common reference time schedule for the ground handling arrival tasks is $\hat{t}_i^a, i \in I_A$. In the case of a departing flight, ground handling departure tasks (fuelling, catering, luggage loading, boarding, water and push back) are also coped with by assigning the corresponding vehicles in accordance to their previous assigned tasks with other aircraft, their current availability, and their current distance to the considered aircraft. Here the common reference time schedule for the ground handling departure tasks is $B^{low}(\tilde{\tilde{t}}_i^d), i \in I_D$.

In both cases it is considered that the whole set of different ground handling vehicles necessary at arrival or departure is assigned by considering the common reference time schedule. This assignment of vehicles to flights either arriving or departing is performed on a greedy base by considering the closest vehicle available to

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perform the required task. This will make that at the start of ground handling activities for an arrival or departure flight, all necessary resources will be nearby the parking place and that scheduling constraints between elementary ground handling tasks will be coped with locally without need of communication between the different ground handling fleet managers. This is a rather simple greedy heuristic which provides for each fleet facing the current service demand a complete solution through a reduced computational effort. So there is no limitation in calling back this solution process any time a significant perturbation occurs.

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d) Illustration of the proposed approach

To evaluate the proposed approach, the data used on the case of study of the previous part has been modified to create artificially a disruption situation. Here it has been considered that for any external reason, for exemple some severe weather conditions, a part of earlier scheduled arriving flights in the morning have been delayed and the airport operates under a concentrated arriving traffic at capacity between 11a.m. and 1 p.m.. Then, the effective arrivals and scheduled departures are those of Table.3.

It is considered that during and after this period the airside capacity of the airport is insufficient, including taxiing capacity with the appearence of queues of taxiing aircraft, parking positions with apron congestion and saturated ground handling capacity. In that conditions, transfer times for aircraft and ground handling units activities durations are subject to large uncertainties. Here it has been considered two scenarios for the uncertainty: in the first one additional delays are between 0% and 40% of the original duration between 11a.m. and 2 p.m. with return to nominal situation afterwards, in the second scenario additional delays are between 0% and 40% of the original duration between 11a.m. and noon, between 20% and 60% of the original duration between noon and 1:30 p.m., between 0% and 40% of the original duration between 1:30 p.m. and 2:30 p.m. with return to nominal situation afterwards.

	10h→11h	11h→12h	12h→13h	13h→14h	14h→15h	15h→16h
Arrival traffic	20 + 30	34 +15	25	7	15	15
Scheduled departures	17	19	28+15	17+20	17+10	17

Table 3 : Effective arrivals and scheduled departures

In the case of this airport, there are no connections between the flights since in general this airport is a final destination for most of the passengers, so the arrival and the departure priority lists coincide. The priority list is calculated here by taking into account the predicted departure date of the flight j, which is the flight serviced by the same aircraft than for flight i. Here $\tilde{D}_{ij}$ is the minimum fuzzy dual duration of ground handling around arrival of flight i and departure of flight j and the real arrival date of the flight i respecting the considering degree of uncertainty. This duration $\tilde{\Delta}_{ij}$, which is a fuzzy dual number, can be expressed by:

$$\tilde{\Delta}_{ij} = \left(\tilde{D}_{ij} + \hat{t}_i^a - \bar{t}_j^d\right) \tag{17}$$

This application provided a feasible assignment for each ground handling manager in at most 0.4 seconds each updating of the priority lists.

The numerical results show that the delayed aircraft get in general the highest priority on the list. During the period of time between 11a.m and 2:30 p.m. ground handling achieves to serve 200 flights (arrival and departure of aircraft). The main numerical results are displayed in Table.4.

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	Scenario 1	Scenario 2
Mean delay for GH processing at arrival	7.36 min	8.86 min
Maximum delay for GH processing at arrival	27 min	30 min
Mean delay for GH processing at departure	45.1 min	59.4 min
Maximum delay for GH processing at departure	195 min	197 min

Table 4 : Statistical results for disruption scenarios

Notes

Fig.6 displays the hourly distribution of delayed aircraft at departure resulting from the application of the proposed approach for the two scenarios. It appears that the impact of arriving traffic delays has resulted in an airport disruption situation which has extended in the afternoon. In the first scenario it can be considered that the disruption situation ends around 5 p.m. and in the other case it ends around 8 p.m. It appears then, that the more uncertainty about airside operations delays, the less the available ground handling capacity is able to cope with this disruption situation. Then insuring predictability of airside delays through fluidity of operations even in heavy activity levels situations emerge as an important objective.

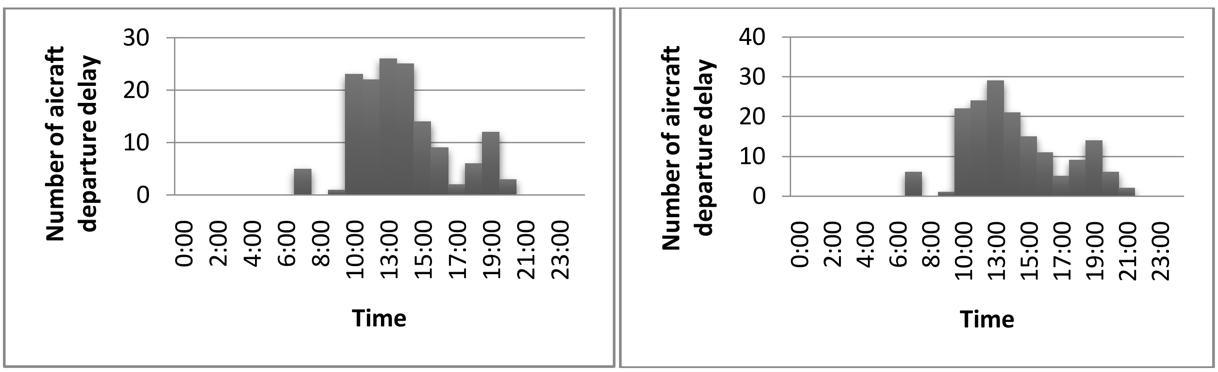

Figure 6 : The hourly distribution of delayed aircraft at departure for the scenario 1 and the scenario 2

V. Conclusion

In this paper, an organization for the ground handling management has been proposed. This proposed organization is based on the introduction of a ground handling coordination which has the role of a communication interface between the ground handling manager and the other airport partners. The solution of the different assignment problems solved by the ground handling coordinator and ground handling managers has been considered. A heuristic approaches has been developed in that case. In the case of the pairing problems faced by the ground handling managers, a heuristic approach has been developed.

The whole process has been illustrated by considering a case study with real traffic where it has been assumed that flight arrival times are perfectly known half an hour in advance. Even if scheduled and effective arrival times are different, the adopted traffic situation can be considered as normal. Also the ground handling management has been considered in the case of a huge traffic perturbation characterizing an airport disruption. The operations planning procedures performed within the proposed management structure of ground handling have been revised by adopting temporary new objectives and taking into account the uncertainty with respect to activity delays in this situation. During the disruption period, the ground handling coordinator takes over the direction of the ground handling management by imposing to the ground handling managers, priority lists of flights to be processed. The computation of these priority lists makes use of fuzzy dual calculus to take into account delays uncertainty. The feasibility of the proposed approach is displayed by considering the case of a disruption at Palma de Mallorca airport.

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Special Pairs of Pythagorean Triangles and Dhuruva Number

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Abstract- We present pairs of Pythagorean triangles, such that in each pair, the difference between their perimeters is two times the Dhuruva number. Also we present the number of pairs of primitive and non-primitive Pythagorean triangles.

Keywords: pairs of pythagorean triangles, dhuruva number, primitive and non-primitive pythagorean triangles.

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Special Pairs of Pythagorean Triangles and Dhuruva Number

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Abstract- We present pairs of Pythagorean triangles, such that in each pair, the difference between their perimeters is two times the Dhuruva number. Also we present the number of pairs of primitive and non-primitive Pythagorean triangles.

Keywords: pairs of pythagorean triangles, dhuruva number, primitive and non-primitive pythagorean triangles.

I. Introduction

The fascinating branch of mathematics is the theory of numbers where in Pythagorean triangles have been a matter of interest to various mathematicians and to the lovers of mathematics, because it is a treasure house in which the search for many hidden connection is a treasure hunt. For a rich variety of fascinating problems one may refer [1-17].A careful observer of patterns may note that there is a one to one correspondence between the polygonal numbers and the number of sides of the polygon. Apart from the above patterns we have some more fascinating patterns of numbers namely Jarasandha numbers, Nasty numbers and Dhuruva numbers. These numbers have been presented in [18-21].

In [22-24], special Pythagorean triangles connected with polygonal numbers and Nasty numbers are obtained. Recently in [25], special Pythagorean triangles in connection with Hardy Ramanujan number 1729 are exhibited. In [26], Pythagorean triangles in connections with 5-digit Dhuruva numbers are presented.

In this communication, we search for pairs of Pythagorean triangles, such that in each pair, the difference between their perimeters is two times the Dhuruva number.

II. Basic Definitons

Definition 2.1

The ternary quadratic Diophantine equation given by $x^2 + y^2 = z^2$ is known as Pythagorean equation where x, y, z are natural numbers. The above equation is also referred to as Pythagorean triangle and denote it by T(x,y,z).

Also, in Pythagorean triangle T(x,y,z) : $x^2 + y^2 = z^2$, x and y are called its legs and z its hypotenuse.

Definition 2.2

Most cited solution of the Pythagorean equation is $x = m^2 - n^2, y = 2mn, z = m^2 + n^2$, where $m > n > 0$. This solution is called primitive, if m,n are of opposite parity and gcd(m,n)=1.

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Definition 2.3: Dhuruva numbers

The numbers which do not change when we perform a single operation or a sequence of operations are known as Dhuruva numbers.

III. Method of Analysis

Let PT_1, PT_2 be two distinct Pythagorean triangles with generators m, q (m > q > 0), and p, q (p > q > 0) respectively.Let P_1, P_2 be the perimeters of PT_1, PT_2 such that $P_1 - P_2 = 2$ times the 3-digit Dhuruva numer 495.

The above relation leads to the equation

$$(2m+q)^2-(2p+q)^2=1980 \tag{1}$$

After performing numerical computations, it is noted that there are 82 distinct values for m, p and q satisfying (1). For simplicity and clear understanding, we have presented below in table1 the values of m, p, q, P_1 and P_2.

S.no	m	q	p	P_1	P_2	$(P_1-P_2)/2$
1	166	164	165	109560	108570	495
2	167	162	166	109886	108896	495
3	168	160	167	110208	109218	495
4	169	158	168	110526	109536	495
5	170	156	169	110840	109850	495
6	171	154	170	111150	110160	495
7	172	152	171	111456	110466	495
8	173	150	172	111758	110768	495
9	174	148	173	112056	111066	495
10	175	146	174	112350	111360	495
11	176	144	175	112640	111650	495
12	177	142	176	112926	111936	495
13	178	140	177	113208	112218	495
14	179	138	178	113486	112496	495
15	180	136	179	113760	112770	495
16	181	134	180	114030	113040	495
17	182	132	181	114296	113306	495
18	183	130	182	114558	113568	495
19	184	128	183	114816	113826	495
20	185	126	184	115070	114080	495
21	186	124	185	115320	114330	495
22	187	122	186	115566	114576	495
23	188	120	187	115808	114818	495
24	189	118	188	116046	115056	495
25	190	116	189	116280	115290	495
26	191	114	190	116510	115520	495
27	192	112	191	116736	115746	495
28	193	110	192	116958	115968	495
29	194	108	193	117176	116186	495
30	195	106	194	117390	116400	495

Notes

Notes

31	196	104	195	117600	116610	495
32	197	102	196	117806	116816	495
33	198	100	197	118008	117018	495
34	199	98	198	118206	117216	495
35	200	96	199	118400	117410	495
36	201	94	200	118590	117600	495
37	202	92	201	118776	117786	495
38	203	90	202	118958	117968	495
39	204	88	203	119136	118146	495
40	205	86	204	119310	118320	495
41	206	84	205	119480	118490	495
42	207	82	206	119646	118656	495
43	208	80	207	119808	118818	495
44	209	78	208	119966	118976	495
45	210	76	209	120120	119130	495
46	211	74	210	120270	119280	495
47	212	72	211	120416	119426	495
48	213	70	212	120558	119568	495
49	214	68	213	120696	119706	495
50	215	66	214	120830	119840	495
51	216	64	215	120960	119970	495
52	217	62	216	121086	120096	495
53	218	60	217	121208	120218	495
54	219	58	218	121326	120336	495
55	220	56	219	121440	120450	495
56	221	54	220	121550	120560	495
57	222	52	221	121656	120666	495
58	223	50	222	121758	120768	495
59	224	48	223	121856	120866	495
60	225	46	224	121950	120960	495
61	226	44	225	122040	121050	495
62	227	42	226	122126	121136	495
63	228	40	227	122208	121218	495
64	229	38	228	122286	121296	495
65	230	36	229	122360	121370	495
66	231	34	230	122430	121440	495
67	232	32	231	122496	121506	495
68	233	30	232	122558	121568	495
69	234	28	233	122616	121626	495
70	235	26	234	122670	121680	495
71	236	24	235	122720	121730	495
72	237	22	236	122766	121776	495
73	238	20	237	122808	121818	495
74	239	18	238	122846	121856	495

75	240	16	239	122880	121890	495
76	241	14	240	122910	121920	495
77	242	12	241	122936	121946	495
78	243	10	242	122958	121968	495
79	244	8	243	122976	121986	495
80	245	6	244	122990	122000	495
81	246	4	245	123000	122010	495
82	247	2	246	123006	122016	495

Notes

Thus it is seen that there are 82 pairs of Pythagorean triangles such that for each pair the difference in the perimeters is twice the 3- digit Dhuruva number 495.

Out of these 82 pairs of Pythagorean triangles 6-pairs are non-primitive and in each of the remaining pairs, one of the triangles is primitive and the other is non-primitive triangle.

A similar observation, regarding 5- digit and 6- digit dhuruva numbers are exhibited in the table2 below.

Dhuruva number	**pairs of Pythagorean triangles**	**pairs of non-primitve Pythagorean triangles**	**pairs of primitve and non-primitve Pythagorean triangles**
53955	8992	908	8084
59995	9998	2111	7887
549945	91657	1	91656

IV. Conclusion

One may search for the connections between the pairs of Pythagorean triangles and other Dhuruva numbers of higher order.

V. Acknowledgement

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Dissemination Sinusoidal Waves in of A Viscoelastic Strip

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Abstract- In this paper we consider the spectral problem for the wave propagation in extended plates of variable thickness. Describes how to solve problems and numerical results of wave propagation in infinitely large plates of variable thickness. Viscous properties of the material are taken into account by means of an integral operator Voltaire. The study is part of the spatial theory of visco elastic. The technique is based on the separation of spatial variables and formulating boundary eigenvalues problem to be solved by the method of orthogonal sweep Godunov. Numerical values obtained for the real and imaginary parts of phase velocity as a function of wave number. When this coincidence numerical results obtained with the known data.

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GJSFR-F Classification : *FOR Code : MSC 2010: 15B10*

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Dissemination Sinusoidal Waves in of A Viscoelastic Strip

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Annotation- In this paper we consider the spectral problem for the wave propagation in extended plates of variable thickness. Describes how to solve problems and numerical results of wave propagation in infinitely large plates of variable thickness. Viscous properties of the material are taken into account by means of an integral operator Voltaire. The study is part of the spatial theory of visco elastic. The technique is based on the separation of spatial variables and formulating boundary eigenvalues problem to be solved by the method of orthogonal sweep Godunov. Numerical values obtained for the real and imaginary parts of phase velocity as a function of wave number. When this coincidence numerical results obtained with the known data.

Keywords: plate, spectral problem, frequency, variable thickness, orthogonal sweep.

I. Introduction

Known [1,2] that in normal wave deformable layer (Lamb wave) is not orthogonal thickness, i.e. the integral of the scalar product of vectors of displacements of two different waves, considered as functions of the coordinate perpendicular to the surface layer is not zero. They also are not orthogonal conjugate waves is obtained by considering the dual problem. This introduces additional difficulties in solving practical problems [3,4,8]. In this paper, we present spectral problem formulation and methods of its tasks.

II. Statement of the Wave Problem and the Basic Relations for the Plate Kirchhoff - Love Variable Thickness

Derive the fundamental relationships of the classical theory of plates with variable thickness on the basis of the principle of virtual displacements. In the three-dimensional formulation of the elasticity problem reduces to the solution of the variation equation, which has the form:

$$\delta A_F + \delta A_I = 0 \tag{1}$$

For virtual work (δA_F) internal forces, we have:

$$\delta A_F = -\delta \Pi = -\int_V \sigma_{ij} \delta \varepsilon_{ij} dV \tag{2}$$

where Π – potential energy; σ_{ij} – components of the stress tensor; ε_{ij} – components the deformation tensor; V – the volume occupied by the body.

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The physical properties of the plastic material describes the relationship

$$\sigma_{ij} = \bar{\lambda}\varepsilon_{kk}\delta_{ij} + 2\bar{\mu}\varepsilon_{ij} \quad (i, j, k = 1,2,3) \tag{3}$$

Where $\sigma_{ij}, \varepsilon_{ij}$ - components of the stress and strain tensors.

$$\lambda = \frac{E\nu(1+i\eta)}{(1+\nu)(1-2\nu)}; \qquad \mu = \frac{E(1+i\eta)}{2(1+\nu)} \tag{4}$$

Integrated in the case the Young's modulus of the viscoelastic material $E^* = E' + iE'' = E'(1+i\eta_e)$ an analogue of the classical Young modulus [85]. Using a complex representation for the elastic modulus (Young's modulus) for the polymeric material can be written as

$$E^*(\omega) = E(\omega)[1+i\eta\omega] \tag{5}$$

Where two functions of vibration frequency $E(\omega)$ and $\eta(\omega)$ may be represented by analytical variety of ways [1,2].

For virtual work of inertial forces (δA_I) we can write the following relation:

$$\delta A_I = -\int_V \rho\ddot{u}_i\delta u_i dV\,, \tag{3}$$

where ρ - body density; u_i – displacement components; $\ddot{u}_i = \partial^2 ui/\partial t^2$; t - time. Here and below, summation over repeated indices. Consider the wedge plate shown in Fig. 1, along the axis of an infinite x_2. In accordance with the hypotheses of Kirchhoff - Love have:

$$\sigma_{13} = \sigma_{23} = \sigma_{33} = 0;$$

$$u_i = -x_3\frac{\ddot{a}W}{\ddot{a}x_i}; \tag{4}$$

$$W(x_3) \equiv W\,,$$

where W – deflection of the middle plane of the plate.

Neglecting in (3) members to take account of the inertia of rotation normal to the median plane we obtain:

$$-\int_s ds \int_{-h/2}^{h/2} (\sigma_{11}\delta\varepsilon_{11} + 2\sigma_{12}\delta\varepsilon_{12} + \sigma_{22}\delta\varepsilon_{22})dx_3 -$$

$$-\int_s ds \int_{-h/2}^{h/2} \rho\frac{\partial^2 W}{\partial t^2}\delta W dz = 0 \tag{5}$$

The expressions for the components of strain and stress tensors are determined from the geometric relationships and relations generalized Hooke's law, which, taking into account the kinematic hypotheses (4) takes the form:

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Notes

$$\begin{cases} \varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - x_3 \frac{\partial^2 W}{\partial x_i \partial x_j}; \qquad i, j = 1,2 \\ \sigma_{11} = \frac{E}{1-v}\left(\varepsilon_{11} + v\varepsilon_{22}\right)\Gamma_\kappa; \\ \sigma_{22} = \frac{E}{1-v}\left(\varepsilon_{22} + v\varepsilon_{11}\right)\Gamma_\kappa \\ \sigma_{12} = \frac{E}{1-v}\varepsilon_{12}\Gamma_\kappa, \end{cases} \tag{6}$$

Where E - Young's modulus; v - Poisson's ratio of the plate material.
Introducing the following notation:

$$M_{11} = -D\left(\frac{\partial^2 W}{\partial x_1^2} + v\frac{\partial^2 W}{\partial x_2^2}\right);$$

$$M_{22} = -D\left(\frac{\partial^2 W}{\partial x_2^2} + v\frac{\partial^2 W}{\partial x_1^2}\right); \tag{7}$$

$$M_{12} = -D(1-v)\frac{\partial^2 W}{\partial x_1 \partial x_2};$$

$$\overline{D} = \frac{\overline{E}h^3}{12(1-v^2)} = D_1\Gamma_k; \quad D_1 = \frac{Eh^3}{12(1-v^2)}; \ \Gamma_\kappa = 1 + i\eta\,(\omega)$$

and integrating over the thickness of the plate, let (2.5) to the following form

$$\int_s \left(M_{11}\frac{\partial^2 \delta W}{\partial x_1^2} + 2M_{12}\frac{\partial^2 \delta W}{\partial x_1 \partial x_2} + M_{22}\frac{\partial^2 \delta W}{\partial x_2^2}\right) dS - \tag{8}$$

$$-\int_s \rho h \frac{\partial^2 W}{\partial t^2}\delta W ds = 0$$

Converting the first integral (8) twice by parts and equating to zero the coefficients of variation δW inside the body and on its borders obtain the following differential equation:

$$\frac{\partial^2 M_{11}}{\partial x_1^2} + 2\frac{\partial^2 M_{12}}{\partial x_1 \partial x_2} + \frac{\partial^2 M_{22}}{\partial x_2^2} - \rho h \frac{\partial^2 W}{\partial t^2} = 0 \tag{9}$$

with natural boundary conditions

$$M_{11}(0, l_1) = 0;$$

$$\frac{\partial M_{11}}{\partial x_1} + 2\frac{\partial M_{12}}{\partial x_2} = 0, \qquad x_1 = 0, l_1$$

the main alternative, which will be the following:

$$\begin{cases} \dfrac{\partial W}{\partial x_1} = 0, \\ W = 0, \;\; x_1 = 0, \;\; l_1 \end{cases},$$

Introducing new variables

$$W, \varphi_1 = \frac{\partial W}{\partial x_1}, \;\; M_{11}, Q_1 = \frac{\partial M_{11}}{\partial x_1} + 2\frac{\partial M_{12}}{\partial x_2}$$

and express through them M_{22} with the help of (2.7). Then

$$M_{22} = -D\frac{\partial^2 W}{\partial x_2^2} + \nu M_{11} + \nu^2 D\frac{\partial^2 W}{\partial x_2^2},$$

or

$$M_{22} = -\frac{Eh^3}{12}\frac{\partial^2 W}{\partial x_2^2} + \nu M_{11} \tag{10}$$

We note that M_{11} and M_{22} are bending moments, at M12 the torque.

Thus, we arrive at the following system of equations:

$$\begin{cases} \dfrac{\partial W}{\partial x_1} = \varphi_1; \\ \dfrac{\partial \varphi}{\partial x_1} = -\dfrac{M_{11}}{D} - \nu\dfrac{\partial^2 W}{\partial x_2^2}; \\ \dfrac{\partial M_{11}}{\partial x_1} = Q_1 + \dfrac{\bar{E}h^3}{6(1+\nu)}\dfrac{\partial^2 \varphi}{\partial x_2^2}; \\ \dfrac{\partial Q_1}{\partial x_1} = -\nu\dfrac{\partial^2 M_{11}}{\partial x_2^2} + \dfrac{\bar{E}h^3}{12}\dfrac{\partial^2 W}{\partial x_2^2} + \rho h\dfrac{\partial^2 W}{\partial t^2}, \end{cases} \tag{11}$$

Or

$$\begin{cases} \dfrac{\partial W}{\partial x_1} = \varphi_1; \\ \dfrac{\partial \varphi}{\partial x_1} = -\dfrac{6(1-\nu)}{h^3}\dfrac{M_{11}\cdot 2(1-\nu)}{\bar{E}} - \nu\dfrac{\partial^2 W}{\partial x_2^2}; \\ \dfrac{2(1+\nu)}{\bar{E}}\dfrac{\partial M_{11}}{\partial x_1} = \dfrac{2(1+\nu)}{\bar{E}}Q_1 + \dfrac{h^3}{3}\dfrac{\partial \varphi}{\partial x_2}; \\ \dfrac{2(1+\nu)}{\bar{E}}\dfrac{\partial Q_1}{\partial x_1} = -\nu\dfrac{2(1+\nu)}{\bar{E}}\dfrac{\partial^2 M_{11}}{\partial x_2^2} + \dfrac{(1+\nu)h^3}{6}\dfrac{\partial^4 W}{\partial x_2^4} + \dfrac{2(1+\nu)}{\bar{E}}\rho h\dfrac{\partial^2 W}{\partial t^2}, \end{cases}$$

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Or

$$\begin{cases} \frac{\partial y_1}{\partial x_1} = y_2; \\ \frac{\partial y_2}{\partial x_1} = -\frac{6(1-v)}{h^3} y_3 - v\frac{\partial^2 y_1}{\partial x_2^2}; \\ \frac{\partial y_3}{\partial x_1} = y_4 + \frac{h^3}{3}\frac{\partial y_2}{\partial x_2}; \\ \frac{\partial y_4}{\partial x_1} = -v\frac{\partial^2 y_3}{\partial x_2^2} + \frac{(1+v)h^3}{6}\frac{\partial^4 y_1}{\partial x_2^4} + \frac{h}{C_s^2}\frac{\partial^2 y_1}{\partial t^2}, \end{cases} \tag{12}$$

Where $y_1 = W$, $y_2 = \varphi_1$, $y_3 = \frac{2(1+v)}{E} M_{11}$, $y_4 = \frac{2(1+v)}{E} Q$, $\tilde{N}_s^2 = \frac{E}{2(1+v)\rho}$, C_s – shear wave Velocity

Among the many solutions of (12) we choose those that describe harmonic plane waves propagating along the axis x_2

$$y_i = z_i(x_1)e^{i(\hat{e}\tilde{o}_2 - \omega t)} \tag{13}$$

Substituting the solution (13) in the system of differential equations (12), we obtain a system of ordinary differential equations of the first order, solved for the derivative:

$$\begin{cases} z_1' = z_2; \\ z_2' = -\frac{6(1-v)}{h^3} z_3 + v\kappa^2 z_1; \\ z_3' = z_4 - \frac{h^3 \Gamma_k}{3}\kappa^2 z_2; \\ z_4' = v\kappa^2 z_3 + \frac{(1+v)h}{6}\kappa^4 z_1 - h\left(\frac{\omega}{C_s}\right)^2 \Gamma_k z_1; \end{cases} \tag{14}$$

The boundary conditions for this system can be written as follows:

а) free left edge of the plate:

$$z_3(0) = z_4(0) = 0 \tag{15}$$

б) free right edge of the plate:

$$z_3(l_1) = z_4(l_1) = 0 \tag{16,a}$$

в) pinched right edge of the plate:

$$z_1(l_1) = z_2(l_1) = 0 \tag{16,б}$$

T Thus formed the spectral problem (14-16) in the parameter ω, describing the propagation of flexural waves in a flat edge plate Kirchhoff-Love.

III. Basic Relations for Timoshenko Plates of Variable Thickness. Statement of the Wave Problem

Applying the principle of virtual displacements (1-3), replacing the Kirchhoff-Love hypotheses (2.4) on the hypothesis Timoshenko:

$$\sigma_{33} = 0;\ \sigma_{3i} = \frac{\chi \bar{E}}{2(1+v)}\left(\frac{\partial W}{\partial x_i} - \theta_i\right); \qquad (17)$$

$$u_i^{(x_3)} = x_3\theta_i;\ W^{(x_3)} = W; \qquad i = 1, 2,$$

where $\theta_i -$ normal rotation angles (Fig. 2) $\chi -$ correction factor that takes into account the distribution of shear stresses across the thickness.

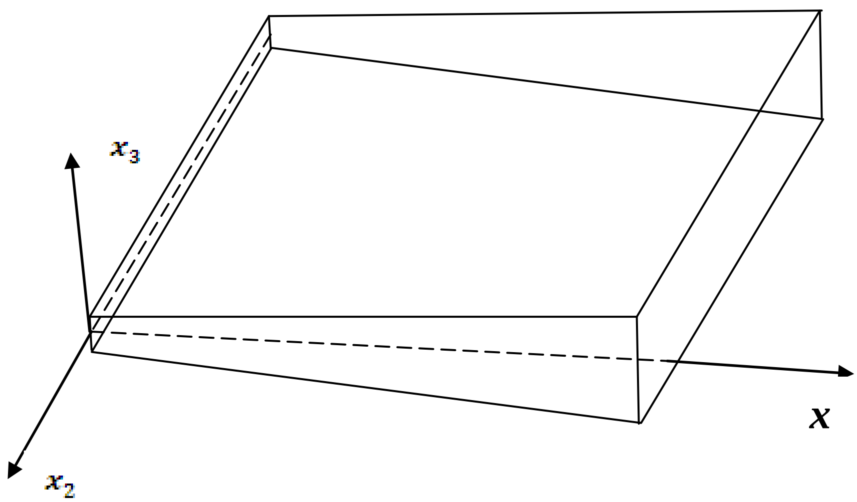

Figure 1 : Design scheme

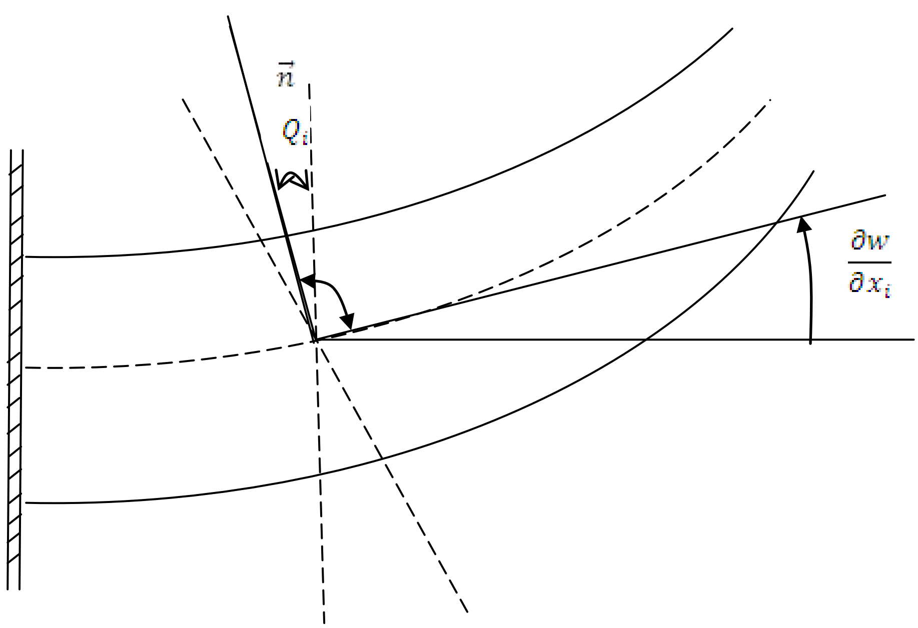

Figure 2 : shows the angle of rotation of the normal

In this case, the tensor components of strain and stress take the form:

$$\varepsilon_{ij} = -\frac{1}{2}x_3\left(\frac{\partial\theta_i}{\partial x_j} + \frac{\partial\theta_j}{\partial x_i}\right);$$

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$$\varepsilon_{3i} = \frac{1}{2}\left(\frac{\partial W}{\partial x_i} - \theta_i\right);$$

$$\sigma_{11} = -\frac{E\Gamma_k}{1-v^2} x_3 \left(\frac{\partial \theta_1}{\partial x_1} + v\frac{\partial \theta_2}{\partial x_2}\right); \quad (18)$$

$$\sigma_{22} = -\frac{E\Gamma_k}{1-v^2} x_3 \left(\frac{\partial \theta_2}{\partial x_2} + v\frac{\partial \theta_1}{\partial x_1}\right);$$

$$\sigma_{12} = -\frac{E\Gamma_k}{2(1+v)} x_3 \left(\frac{\partial \theta_1}{\partial x_2} + v\frac{\partial \theta_2}{\partial x_1}\right);$$

$$\sigma_{3i} = \frac{\chi E\Gamma_k}{2(1+v)}\left(\frac{\partial W}{\partial x_i} - \theta_i\right), \qquad i, j=1, 2$$

Substitute the expression for the work on virtual displacements, we obtain:

$$\delta A = \int_{-h/s}^{h/2}\int_{s}\left[-\sigma_{ij}\frac{x^3}{2}\left(\frac{\partial \delta\theta_i}{\partial x_j} + \frac{\partial \delta\theta_j}{\partial x_i}\right) + \sigma_{3i}\left(\frac{\partial \delta W}{\partial x_i} - \delta\theta_i\right) + \right.$$

$$\left. + \rho\ddot{W}\delta W + \rho x_3^2\ddot{\theta}_i\delta\theta_i\right] dSdx_3 = 0 \quad (19)$$

Or by introducing a notation for the corresponding moments:

$$\overline{M}_{11} = D_1\Gamma_k\left(\frac{\partial \theta_1}{\partial x_1} + v\frac{\partial \theta_2}{\partial x_2}\right) = \Gamma_k M_{11};$$

$$\overline{M}_{22} = D_1\Gamma_k\left(\frac{\partial \theta_2}{\partial x_2} + v\frac{\partial \theta_1}{\partial x_1}\right) = \Gamma_k M_{22}; \quad (20)$$

$$\overline{M}_{12} = D_2\Gamma_k\left(\frac{\partial \theta_1}{\partial x_2} + \frac{\partial \theta_2}{\partial x_1}\right) = \Gamma_k M_{12}$$

where $D_2 = \frac{1}{2}D_1$

$$M_{22} = -D_1\left(\frac{\partial \theta_2}{\partial x_2} + v\frac{\partial \theta_1}{\partial x_1}\right)$$

$$M_{11} = -D_1\left(\frac{\partial \theta_1}{\partial x_1} + v\frac{\partial \theta_2}{\partial x_2}\right)$$

$$M_{12} = D_2\left(\frac{\partial\theta_1}{\partial x_2} + \frac{\partial\theta_2}{\partial x_1}\right)$$

and integrating over x_3 we have

$$\delta A = -\int_s \left[-\frac{\partial}{\partial x_j}\left(\overline{M}_{ij}\delta\theta_i\right) + \frac{\partial}{\partial x_j}\left(h\delta_{3j}\delta W\right)\right]dS +$$

$$+\int_s \left(-\frac{\partial\overline{M}_{ij}}{\partial x_j}\delta\theta_i + \frac{\partial\left(h\overline{\sigma}_{3j}\right)}{\partial x_j}\delta W + h\overline{\sigma}_{3i}\delta\theta_i - \right.$$

$$\left. -\rho h\ddot{W}\delta W - \frac{\rho h^3}{12}\ddot{\theta}_i\delta\theta_i\right)dS = 0 \qquad (21)$$

Integrating (21) by parts and equating to zero the coefficients of variation δW and $\delta\theta_i$ inside the body and on its borders obtain the following system of differential equations

$$\begin{cases} -\dfrac{\partial M_{12}}{\partial x_2} - \dfrac{\partial M_{11}}{\partial x_1} + h\sigma_{31} - \dfrac{\rho h^3}{12\Gamma_k}\ddot{\theta}_1 = 0; \\ -\dfrac{\partial M_{22}}{\partial x_2} - \dfrac{\partial M_{12}}{\partial x_1} + h\sigma_{32} - \dfrac{\rho h^3}{12\Gamma_k}\ddot{\theta}_2 = 0; \\ \dfrac{\partial\left(h\sigma_{32}\right)}{\partial x_2} + \dfrac{\partial\left(h\sigma_{31}\right)}{\partial x_1} - \dfrac{\rho h\ddot{W}}{\Gamma_k} = 0 \end{cases} \qquad (22)$$

With natural boundary conditions:

$$\begin{cases} M_{12} = 0; \\ M_{11} = 0; \\ h\sigma_{31} = 0, \; x_1 = 0, \; l_1 \end{cases}$$

The main alternative, which will be the following:

$$\begin{cases} \theta_1 = 0; \\ \theta_2 = 0; \\ W = 0, \; x_1 = 0, \; l_1 \end{cases}$$

Equation (22) is a differential complex coefficients, it is possible to write in the following form

$$\begin{pmatrix} -\dfrac{\partial M_{12}}{\partial x_2} - \dfrac{\partial M_{11}}{\partial x_1} + h\tau_{31} - \dfrac{sh^3}{12\Gamma_{KR}}\theta_1'' \\ \dfrac{-\partial M_{22}}{\partial x_2} - \dfrac{\partial M_{12}}{\partial x_1} + h\tau_{32} - \dfrac{sh^3}{12\Gamma_{KR}}\theta'' \\ \dfrac{\partial\left(h\tau_{32}\right)}{\partial x_2} + \dfrac{\partial\left(h\tau_{31}\right)}{\partial x_1} - \dfrac{sh^3}{\Gamma_{K12}}\ddot{W} \end{pmatrix} + i\Gamma_{KI}\begin{pmatrix} -\dfrac{\partial M_{12}}{\partial x_2} - \dfrac{\partial M_{11}}{\partial x_1} + h\tau_{31} \\ \dfrac{-\partial M_{22}}{\partial x_2} - \dfrac{\partial M_{12}}{\partial x_1} + h\tau_{32} \\ \dfrac{\partial\left(h\tau_{32}\right)}{\partial x_2} + \dfrac{\partial\left(h\tau_{31}\right)}{\partial x_1} \end{pmatrix} = 0$$

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The main variables in this system, we assume: W_1, θ_1, θ_2, M_{12}, M_{11}, $Q_1=h\sigma_{31}$. Out of the equation variables M_{22} and Q_2.

$$M_{22}=-\frac{Eh^3}{12}\frac{\partial\theta_2}{\partial x_2}+\nu M_{11};\ Q_2=h\sigma_{32}=\frac{\chi Eh}{2(1+\nu)}\left(\frac{\partial W}{\partial x_2}-\theta_2\right).$$

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Thus we arrive at the following system of equations:

$$\begin{cases}\dfrac{\partial W}{\partial x_1}=\theta_1+\dfrac{2(1+\nu)}{\chi Eh}Q_1;\\ \dfrac{\partial\theta_2}{\partial x_1}=-\dfrac{\partial\theta_1}{\partial x_2}-\dfrac{24(1+\nu)}{Eh^3}M_{12};\\ \dfrac{\partial\theta_1}{\partial x_1}=-\nu\dfrac{\partial\theta_2}{\partial x_2}-\dfrac{12(1-\nu^2)}{Eh^2}M_{12};\\ \dfrac{\partial M_{11}}{\partial x_1}=-\dfrac{\partial M_{12}}{\partial x_2}+Q_1-\dfrac{ph^3}{12\Gamma_k}\ddot{\theta}_1;\\ \dfrac{\partial M_{22}}{\partial x_1}=-\dfrac{Eh^3}{12}\dfrac{\partial^2\theta_2}{\partial x_2^2}-\nu\dfrac{\partial M_{11}}{\partial x_2}+\dfrac{\chi Eh}{2(1+\nu)}\left(\dfrac{\partial W}{\partial x_2}-\theta_2\right)-\dfrac{ph^3}{12\Gamma_k}\ddot{\theta}_2;\\ \dfrac{\partial Q_1}{\partial x_1}=-\dfrac{\chi Eh}{2(1+\nu)}\left(\dfrac{\partial^2 W}{\partial x_2^2}-\dfrac{\partial\theta_2}{\partial x_2}\right)+\dfrac{\rho h\ddot{W}}{\Gamma_k}.\end{cases}\tag{23}$$

or

$$\begin{aligned}&\frac{\partial y_1}{\partial x_1}=y_2+\frac{y_4}{\chi h};\qquad \frac{\partial y_2}{\partial x_1}=-\nu\frac{\partial y_3}{\partial x_2}-\frac{6(1-\nu)}{h^3}y_5;\\ &\frac{\partial y_3}{\partial x_1}=-\frac{\partial y_2}{\partial x_2}-\frac{12}{h^3}y_6;\\ &\frac{\partial y_4}{\partial x_1}=\chi h\frac{\partial}{\partial x_2}\left(y_3-\frac{\partial y_1}{\partial x_2}\right)+\frac{h}{\Gamma_k}\frac{\partial^2 y_2}{\partial\tilde{t}^2};\\ &\frac{\partial y_5}{\partial x_1}=-\frac{\partial y_6}{\partial x_2}+y_4-\frac{h^3}{12\Gamma_k}\frac{\partial^2 y_2}{\partial\tilde{t}^2};\\ &\frac{\partial y_6}{\partial x_1}=\frac{\partial}{\partial x_2}\left(\frac{(1+\nu)h^3}{6}\cdot\frac{\partial y_3}{\partial x_2}-\nu y_5\right)+\chi h\left(\frac{\partial y_1}{\partial x_2}-y_3\right)-\frac{h^3}{12\Gamma_k}\frac{\partial^2 y_3}{\partial\tilde{t}^2}.\end{aligned}\tag{24}$$

Where

$$y_1=W;\quad y_2=\theta_2;\quad y_3=\theta/\nu;\quad y_4=\frac{2(1+\nu)}{E}Q_1;$$

$$y_5=\frac{4(1+\nu)}{1-\nu}M_{12};\quad y_6=\frac{h(1-\nu^2)}{E\nu}M_{12}$$

$$M_{22}=-D\left(\frac{\partial\theta_2}{\partial x_2}+\nu\frac{\partial\theta_1}{\partial x_1}\right)+\nu M_{11}-\nu M_{11}=$$

$$= -D(1-\nu^2)\frac{\partial \theta_2}{\partial x_2} + \nu M_{11} = -\frac{Eh^3}{12(1-\nu^2)}(1-\nu^2)\frac{\partial \theta_2}{\partial x_2} + \nu M_{11} =$$

$$= -\frac{Eh^3}{12}\frac{\partial \theta_2}{\partial x_2} + \nu M_{11}$$

Finding, as before, the solutions described by a plane harmonic waves propagating along the axis x_1, we seek a solution of (24) in the form

$$\begin{cases} y_1 = z_1(x_1)\cos(\kappa x_2 - \omega t); \\ y_2 = z_2(x_1)\cos(\kappa x_2 - \omega t); \\ y_3 = z_3(x_1)\sin(\kappa x_2 - \omega t); \\ y_4 = z_4(x_1)\cos(\kappa x_2 - \omega t); \\ y_5 = z_5(x_1)\cos(\kappa x_2 - \omega t); \\ y_6 = z_6(x_1)\sin(\kappa x_2 - \omega t). \end{cases} \quad (25)$$

Substituting relation (25) in the system of differential equations (24) we obtain a system of ordinary differential equations of the first order, solved for the derivative:

$$\begin{cases} z_1' = z_2 + \dfrac{z_n}{\chi h}; \\ z_2' = -\nu\kappa z_3 - \dfrac{6(1-\nu)}{3} z_5; \\ z_3' = \kappa z_2 - \dfrac{12}{h^3} z_6; \\ z_4' = \chi h\kappa z_3 + \kappa^2\left(\chi h - \dfrac{hc^2}{\Gamma_л}\right) z_1; \\ z_5 = -\kappa z_6 + z_4 + \dfrac{h^3}{12\Gamma_л}\omega^2 z_2; \\ z_6' = -\chi h\kappa z_1 - \left[\chi h + \dfrac{\kappa^2 h^3}{12\Gamma_л}\left(2(1+\nu) - \dfrac{c^2}{\Gamma_л}\right)\right] z_3 + \nu\kappa z_5. \end{cases} \quad (26)$$

The boundary conditions for this system can be written as follows:

а) free left edge of the plate:

$$z_4 = z_5 = z_6 = 0, \qquad x_1 = 0; \quad (27)$$

б) free right edge of the plate:

$$z_4 = z_5 = z_6 = 0, \qquad x_1 = l_1; \quad (28,а)$$

в) pinched right edge of the plate:

$$z_1 = z_2 = z_3 = 0, \qquad x_1 = l_1; \quad (28, б)$$

Thus formulated spectral problem (26-28) in the parameter ω, describing the propagation of flexural waves in a flat edge plate Timoshenko.

Notes

IV. Numerical Analysis of the Dispersion of the Edge Waves in the Wedge-Shaped Plates

The decision stated above spectral boundary-value problems (14), (15), (16) and (26), (27), (28) was performed by the method of orthogonal sweep Godunov [4]. Numerical implementation of this method was carried out on a computer using software package MAPLE. To test the method and the program was designed version of the album with the boundary conditions can be solved analytically in terms of trigonometric functions.

For resolving the system of equations (14) Kirchhoff-Love plate, these boundary conditions of the form:

$$X_1=0,1; \qquad z_2=z_4=0 \tag{29}$$

Here and below we use the dimensionless system of units in which the bandwidth *l*, shear modulus *G* and bulk density equal to unity.
In this case, the waveform is given by the expression W

$$z_1 = z_o \cos 2\pi\, n\, x_1 \tag{30}$$

$$z = -(2\pi n)^2 z_1 = A_2 z_1$$

$$z_3 = \frac{\left(\nu K^2 + (2\pi\, n)^2\right)h^3}{6(1-\nu)} z = A_3 z_1$$

$$z_4 = \left[\nu k^2 \frac{\left(\nu k^2 + (2\pi n)^2\right)h^3}{6(1-\nu)} + \frac{(1+\nu)}{6} k^4 - h\left(\frac{\omega}{g*}\right)^2\right] z_1 = A_4 z_1$$

$$z_3^1 = -\frac{\left(\nu k^2 + (2\pi n)^2\right)h^3}{6(1-\nu)}(2\pi n) z_1 \qquad z_2^1 = (2\pi n) z_2$$

$$z_4^1 = -A_4 (2\pi n) z_1 = -(2\pi n) z_4$$

Where z_o – arbitrary constant; c_n - The real part of the complex frequency; successively substituting the expression (30) into equation (26) we obtain the dispersion equation

$$\begin{vmatrix} 2\pi n & 1 & 0 & 0 \\ \nu k^2 & -(2\pi n) & -\frac{6(1-\nu)}{n^3} & 0 \\ 0 & -\frac{n^3}{3}k^2 & -(2\pi n) & 1 \\ B_1 & 0 & \nu k^2 & 2\pi n \end{vmatrix} = 0, \tag{31}$$

Where $B_1 = \frac{(1+\nu)h}{6} k^4 - n\left(\frac{\omega}{C_K + iC_I}\right)^2$

Ref
4. Grinchenko V.T., V.V. Myaleshka Harmonic oscillations and waves in elastic bodies, K .: Science Dumka, 1981, -283 p.

Similarly, choosing the boundary conditions for the resolution of the system (22) in the form of plates Timoshenko

$$x=0,1; \quad z_4=z_5=z_6=0 \tag{32}$$

Find the expression for the wave form

$$\begin{aligned} z_1 &= A_1 \cos 2\pi n x_2; & z_4 &= A_4 \sin 2\pi n x_2; \\ z_2 &= A_2 \cos 2\pi n x_2; & z_5 &= A_5 \sin 2\pi n x_2; \\ z_3 &= A_3 \sin 2\pi n x_2; & z_6 &= A_6 \cos 2\pi n x_2. \end{aligned} \tag{33}$$

Notes

In (33) permanent A_i $(i=1,2, 3,4,5, 6)$ are determined by solving the system of equations

$$\begin{cases} A_3 + \dfrac{A_4}{\chi h} = 0; \\ \kappa A_3 - \dfrac{12}{h^3} A_5 = 0; \\ -\nu\kappa\kappa_2 - \dfrac{6(1-\nu)}{h^3} A_6 = 0; \\ \chi h\kappa\kappa_2 + \kappa^2 (\chi h - hc^2) A_1 = 0; \\ -\chi h\kappa\kappa_1 - \left[\dfrac{(1-\nu)h^3}{6}\kappa^2 + \chi h - \dfrac{h^3}{12}\omega^2 \right] A_2 + \nu\kappa\kappa_6 = 0; \\ -\kappa A_5 + A_4 + \dfrac{h^3}{12}\omega^2 A_3 = 0. \end{cases} \tag{34}$$

The system of equations (34) is obtained by substituting (33) in the resolution of the system of differential equations (22). Condition vanishing of the determinant of the system (34) is the dispersion equation boundary value problem (22), (33). The values of the phase velocities found from the above dispersion equations and solving the corresponding test problems (14). (29). (26) and (32) coincide with each other up to the fourth decimal place in the wave number range from 0.1 to 15 for the first two modes (n = 0.1). For the Kirchhoff-Love plates of variable thickness were investigated first five modes with minimum phase velocity of the complexes. Where $C = C_R + iC_I$, C_R - the phase velocity of wave propagation; C_I - speed damping. Figure 3a shows the dispersion curves of the first mode, depending on the thickness varies linearly. Here we assume that the two edges of the plate are free. The straight line I corresponds to a constant thickness $h_1=h_2=0,1$. In this case, the plate varies as a rod. Curve II - variant $h_1=h_2/2=0,05$; curve III - variant $h_1=h_2/100=0,001$, curve IV $h_1 = h_2/1000 = 0,0001$ and $E_{\min} = 6,9\cdot10^6\ \kappa/\textit{м}^2$, $E_{\max s} = 6,9\cdot10^8\ \kappa/\textit{м}^2$, $\beta = 10^{-4}$. Found that $\kappa > 9$ speed damping increase depending on k. For plastics constant thickness C_Γ on the segment $10^{-4} < C < 70$ decreases in a straight line. It can be seen, the dependence of the damping of the wave number starts on the wave number 3-6. With enthusiasm wave number damping factor tends to reduce hand. It can be seen that for a plate of constant thickness, the phase velocity tends to infinity, and for acute wedge plate there is a finite limit as $\kappa \to \infty$, i.e. the bending edge wave length sufficiently small (compared with the width of the plate) are distributed without dispersion. This fact is evident physical, since the edge of the wedge is no characteristic linear dimension. Land without dispersive waveguide movement begins with a wave of 3-9, which corresponds to the length of the waves, is less than 1.

It should be noted one fundamental point. Strictly speaking, this study did not consider the case of theoretical $h_1=0$ or $\kappa \to \infty$. All the numerical results obtained by the simulation of wave processes on a computer that can not operate with an infinitely small and infinitely large numbers. However, the numerical stability can check the result in sufficiently large range of parameters h_1 or κ. Despite the lack of theoretical basis, this verification sufficiently suggests that a known controlled precision found the limit value of any quantity at $h_1 \to 0$ or $\kappa \to \infty$. Physically, it is obvious that the parameters h_1 and κ must be coordinated so that the wavelength was substantially greater than the width edge h_1.

The numerical experiments show that the maximum dimensionless phase velocity (the real part of the complex frequency) during the first mode $K \to \infty$ largest coincides with the dimensionless thickness h_2. In dimensional terms, this corresponds to the following changes in the law (the actual number of complex frequency) limit the phase velocity C_{Ro} the angle of the wedge φ_o.

$$C_{Ro} = C_s tg \frac{\varphi_o}{2} \quad (35)$$

Coincides with the results of $(C_{Ro} = C_o)$ [6]. Numerical experiment also showed that the family of the dispersion curves with different angles at the vertex of the wedge has a certain similarity property, namely: the ratio of the phase velocity to the speed C_o (35) does not depend on the angle φ_o. For constant thickness form varies only slightly, while the wedge-shaped plate with increasing K, observed near the localization own form an acute angle. Figure 4 shows the dispersion curves and the second oscillation mode, depending on wavelength in distinguishing values of the thickness of the plate. When $K=0$, the phase velocity is finite. Localization waveform and a limited range of the phase velocity with valid for this mode. Figure 4 b shows the imaginary part of the complex, depending on the speed of the wave hours for different thicknesses. It is seen that the rate of 3-4 imaginary second mode at $K > 5$ does not tolerate dispersion. Figure 4 b shows the evolution of the dispersion curves as a function of the wedge angle and thickness h_2. For small to form close to the line that corresponds to the torsion vibration at large to the observed localization. In contrast to the first mode is available hotspot. Fig. 5 a, b shows the dispersion curves and mode shapes for III and IV of fashion. With integrated small wave numbers phase velocity tends to infinity, and for large - to a finite limit. Also observed localization forms. The number of nodal points two and three, respectively modes (Figure 5).

Figure 6 shows the dependence of the real part of the phase velocities of the first four vibration modes acute wedge plate with different Poisson's ratio. As can be seen from the figure, the maximum phase velocity C_o the first mode is virtually independent of Poisson's ratio. In the last phase velocity modes C_o increases with increasing v, where the effect of Poisson's ratio is more pronounced at the higher-order modes (real part of complex velocity). Fig. 7 shows the dispersion curves of phase velocities of the four vibration modes for two variants of the legal termination edge of the plate: the free edge (dashed) and fixed (solid). Unlike these options significantly at small wave numbers and virtually absent at large, that is, as one would expect, the maximum phase velocity is independent of the conditions of securing the plate away from the edge of the wedge.

In [7], the distribution of the bending edge waves in the wedge-shaped waveguides in the framework of the linear theory of elasticity. We used the finite element method, based on which the empirical relation for the phase velocities of the normal modes of oscillation depending on the angle of the wedge φ:

$$C_o = C_r \sin(m\varphi); \quad m = 1,2,\ldots, \quad m\varphi¡90^\circ, \quad (36)$$

Where: C_r – Rayleigh wave speed for a half; m – mode number. It is easy to see that the relations (35) and (36) do not agree with each other at small angles φ. It is therefore of interest to find out what the limiting phase velocities obtained in the

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7. Sunchaliev R.M., Filatov A. On some methods for the study of nonlinear problems in the theory of viscoelasticity // Dokl, 1972.206, №1. p. 201-203.

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framework of a more general theory of plates Timoshenko. The spectral problem (26-28), which describes the distribution of edge waves plate Timoshenko was solved numerically orthogonal sweep method of Godunov. To control the numerical convergence of the method, the number of points equal to the orthogonalization taken from 10 to 100. In parallel redundant calculations were carried out in double precision. The result is considered satisfactory if the doubling of the number of points did not change the orthogonalization four significant digits in the phase velocity. Limiting the phase velocity of the first mode for thickness $h_2=0, 2$ equal to 0.1945 and is independent of Poisson's ratio. Compared with the same result obtained in the theory of Kirchhoff-Love, in this case the difference is less than 3%. Figure 8 shows the first three modes indeed part of the complex phase velocity of the plate Timoshenko (b) compared to the corresponding modes of Kirchhoff-Love plate (a) in the Poisson's ratio of 0.25. In the case of the Kirchhoff-Love plate limit above the phase II and III modes, and with increasing mode number increases contrast. The comparative analysis of the propagation of the edge waves on the basis of these theories plates shows a satisfactory agreement for the first vibration mode. The resulting discrepancy with the results in [5] indicates the need for more detailed research into the general theory of elasticity. Overall, however, conducted a numerical analysis of edge waves in the Kirchhoff-Love plates and Timoshenko suggests that the Kirchhoff-Love hypotheses are justified in the calculation of wave processes in the wedge-shaped plates, including frequencies with a wavelength of the order of the thickness of the plate. This discrepancy with the classical results of the theory of Kirchhoff-Love plates of constant thickness above phenomenon is explained by established localization waveforms with increasing frequency, which occurs only in the plates of variable thickness. At the same time, the relative simplicity of the mathematical apparatus of the theory of Kirchhoff-Love plates, allows us to investigate the dispersion characteristics of the waveguides with a more complex configuration section, which is very difficult to build as part of three-dimensional theory. Consider a plate, whose Thickness varies in accordance

$$h(x_1) = h_o /x_1/, \qquad -b \le x_1 \le b.$$

It is clear that such a plate vibrations are reduced to fluctuations in the wedge plate with boundary conditions at $x_1=0$, corresponds to the case of symmetry

$$\varphi = 0, \qquad Q = 0 \tag{37}$$

and of ant symmetry

$$W = 0, \qquad M = 0 \tag{38}$$

Figure 9.a. and 9.b. (solid lines) shows the dispersion curves of phase velocities of the first three modes in the Kirchhoff-Love plate with a linear variation of thickness.

$$h(x_1) = h_0 x_1^p, \qquad o < x_1 \le b,$$

where the parameter p taken equal to 1.5; 2; 2.5; 3 in accordance with designations of curves 1, 2, 3 and 4. For comparison, the dashed lines indicate similar curves discussed above relating to the wedge plate with a thickness $h\ (1) = h_o = 0,\ 2$. Note the qualitative difference in the behavior of solid and dotted lines. When p = 1, as mentioned above, the phase velocities approaching asymptoticity nonzero limits, the curve of the first mode increases monotonically. For p>1, the curve of the first mode is not monotonic and has a characteristic maximum in the medium range. Starting with a certain wave number of the phase velocities of all modes decrease monotonically without entering the asymptote nonzero. With increasing p the maximum curve of the first mode is shifted to lower frequencies, and shortwave phase velocities decrease more rapidly. Thus, summarizing the results obtained earlier in the event of a non-linear law of variation of the thickness of the plate, it can be argued that the phase velocity of the first mode in the wedge plate at high frequencies is determined by the rate of change of the thickness in the vicinity of the sharp edge.

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5. Koltunov M.A. Creep and relaxation. - Publishing Moscow, 1976.- 276 p.

Notes

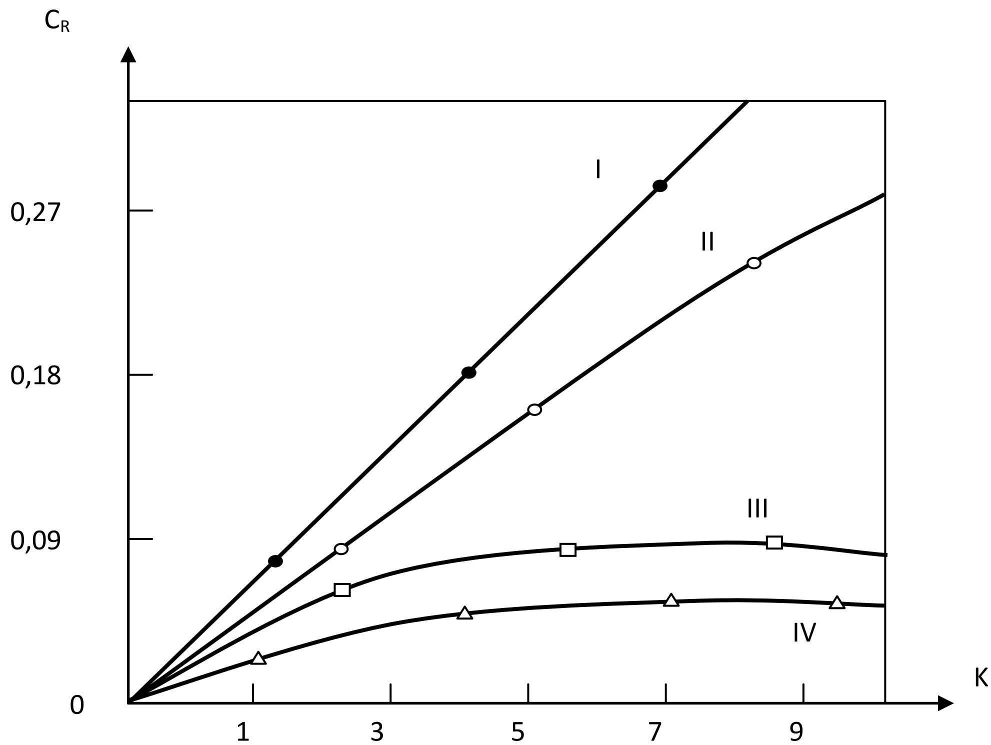

Figure 3 : The dispersion curves of the first mode

I. $h_1=h_2=0{,}1$; *II.* $h_1=h_{2/2}=0{,}05$; *III.* $h_{2/100} = 0{,}001$; *IV.* $h_1=h_{2/1000}=0{,}001$

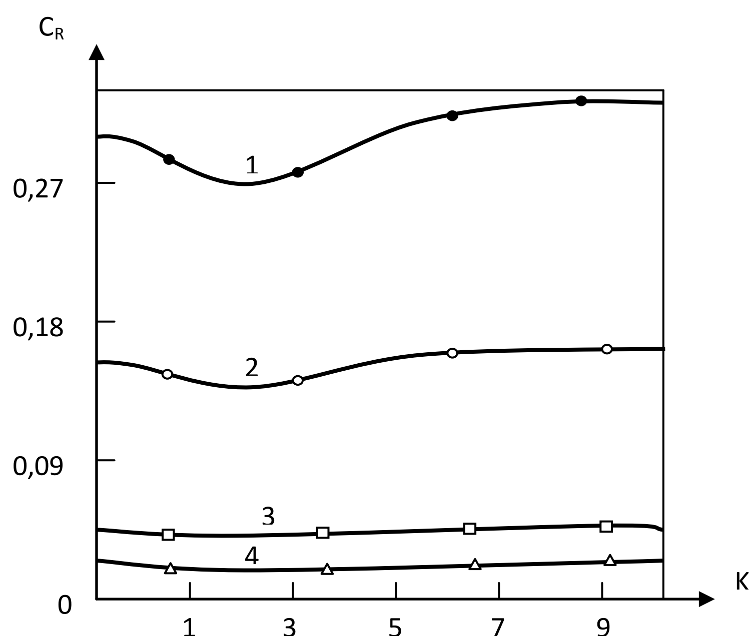

Figure 4a : The dispersion curves of the second mode

1. $h_2=0{,}002$, $h_2=0{,}2$; *2.* $h_1=0{,}001$, $h_2=0{,}1$; *3.* $h_1=0{,}0002$, $h_2=0{,}02$

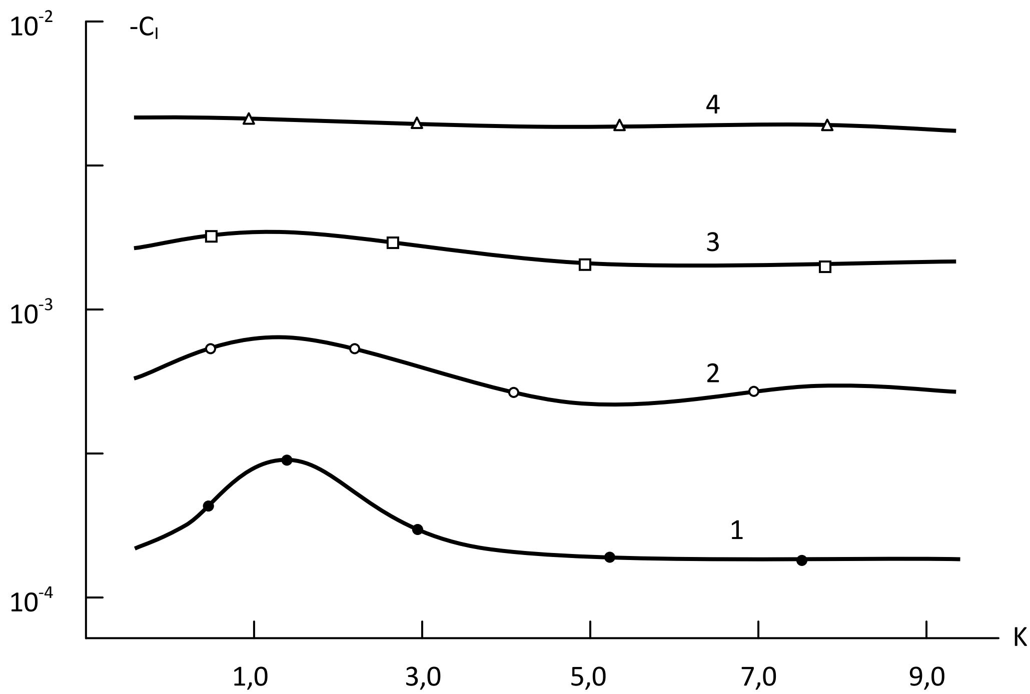

Figure 4 b : The dependence of the damping rate κ

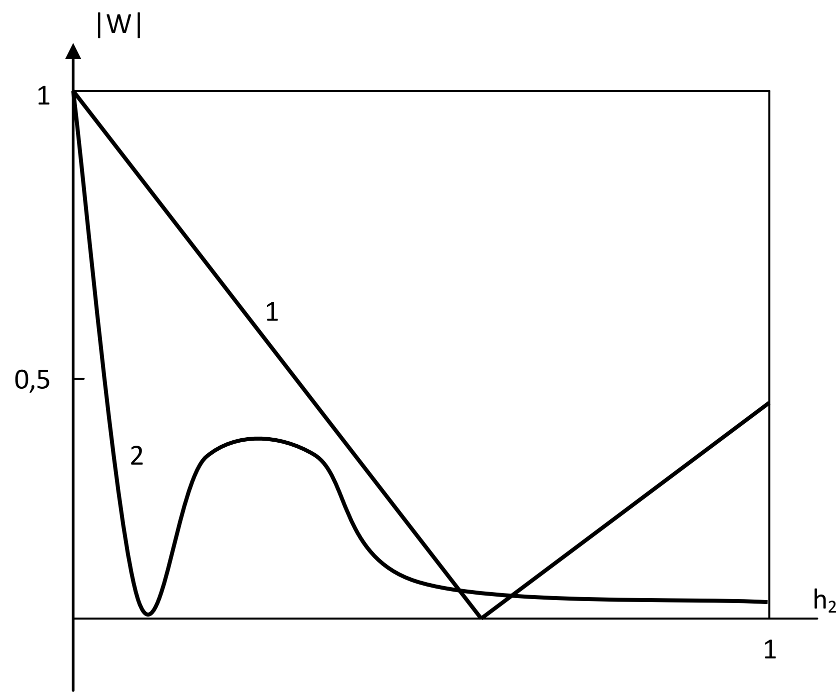

Figure 4 v. : Wave form corresponding dispersion curves of the second mode *1.K=1; 2.K=10*

Notes

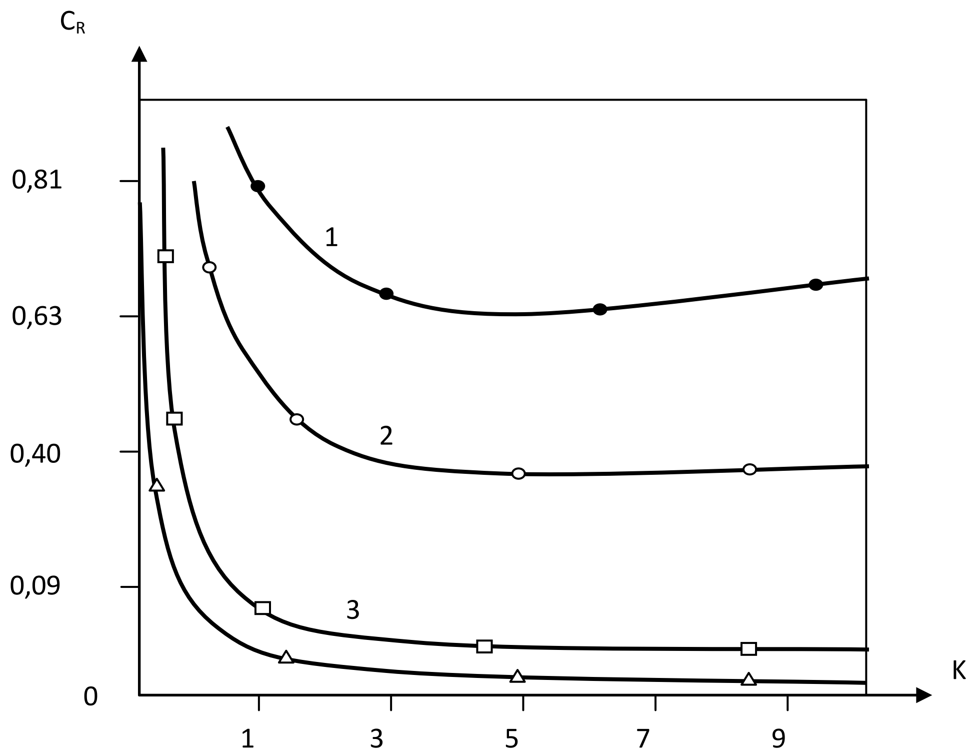

Figure 5a : The dispersion curves of the third mode

h₂=0,002, h₂=0,2; 2. h₁=0,001, h₂=0,1; 3. h₁=0,0002, h₂=0,02

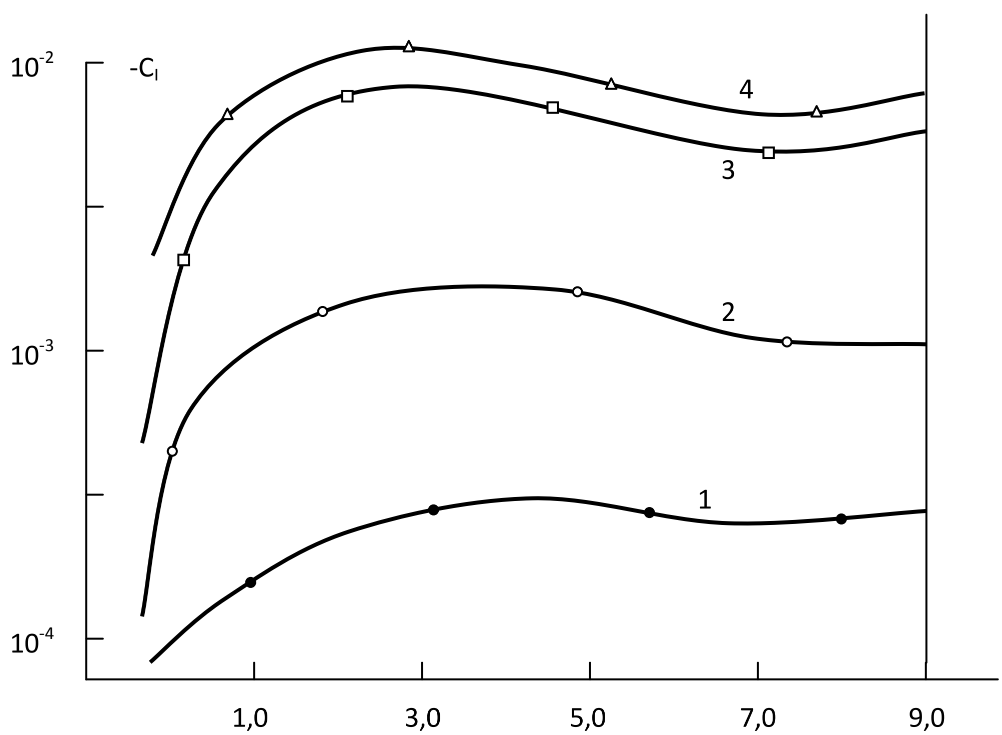

Figure 5 b : The dependence of the damping rate κ

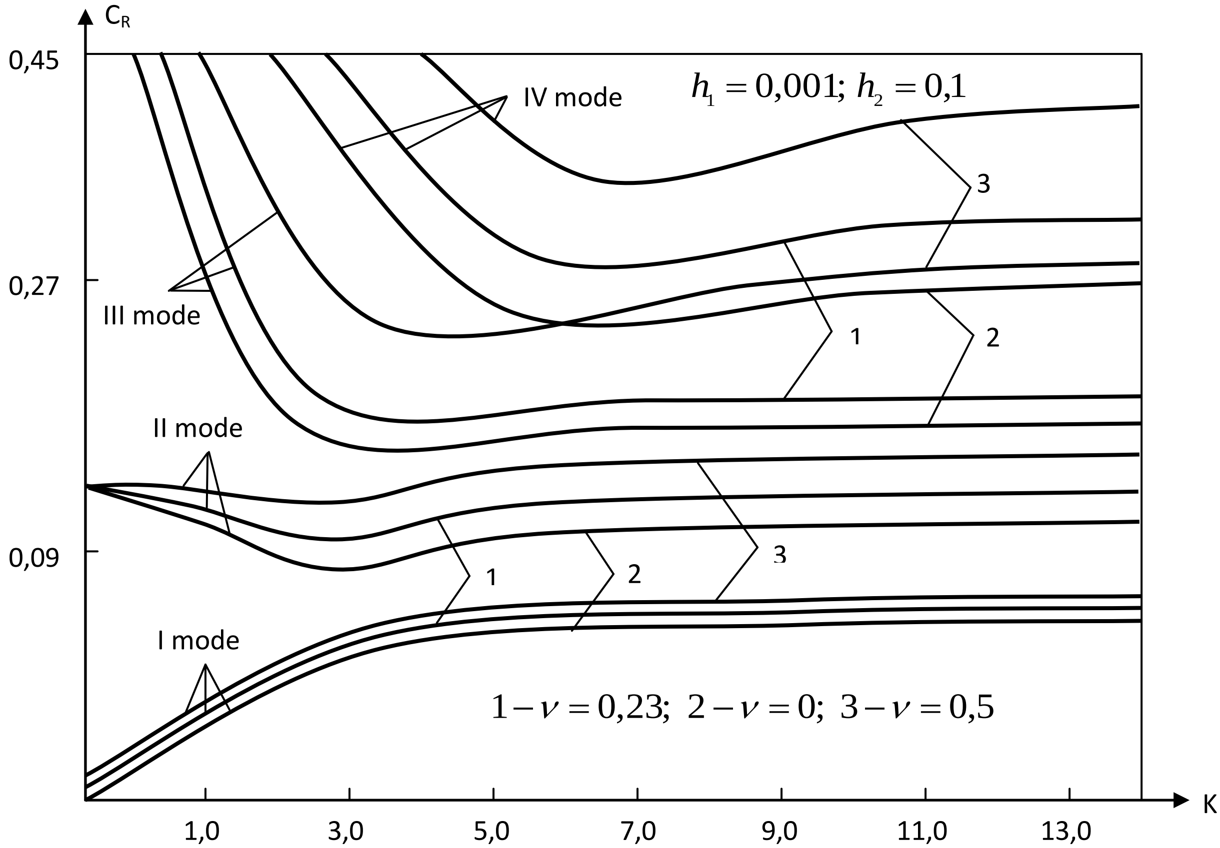

Figure 6 : The dependence of the phase velocity of the first four modal wedge plates with different Poisson's ratios

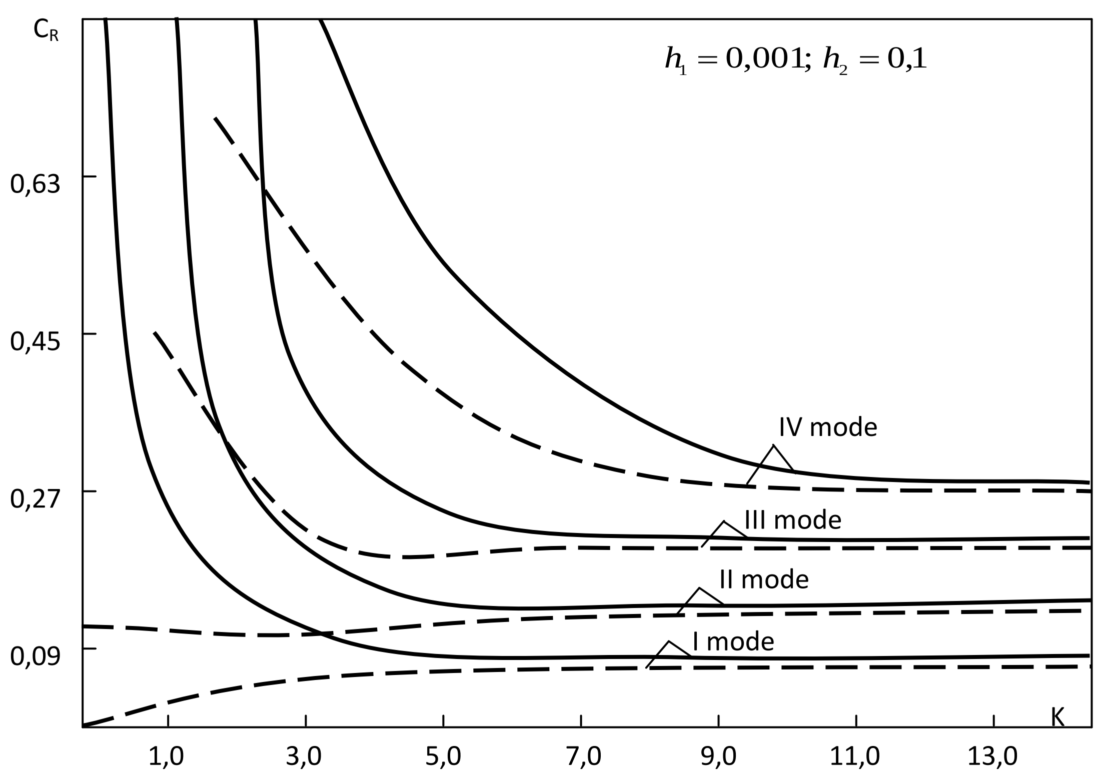

Figure 7 : The dispersion curves of phase velocities of the four modes for the two variants of the legal termination edge of the plate

——— free edge, -------- rigidly fixed

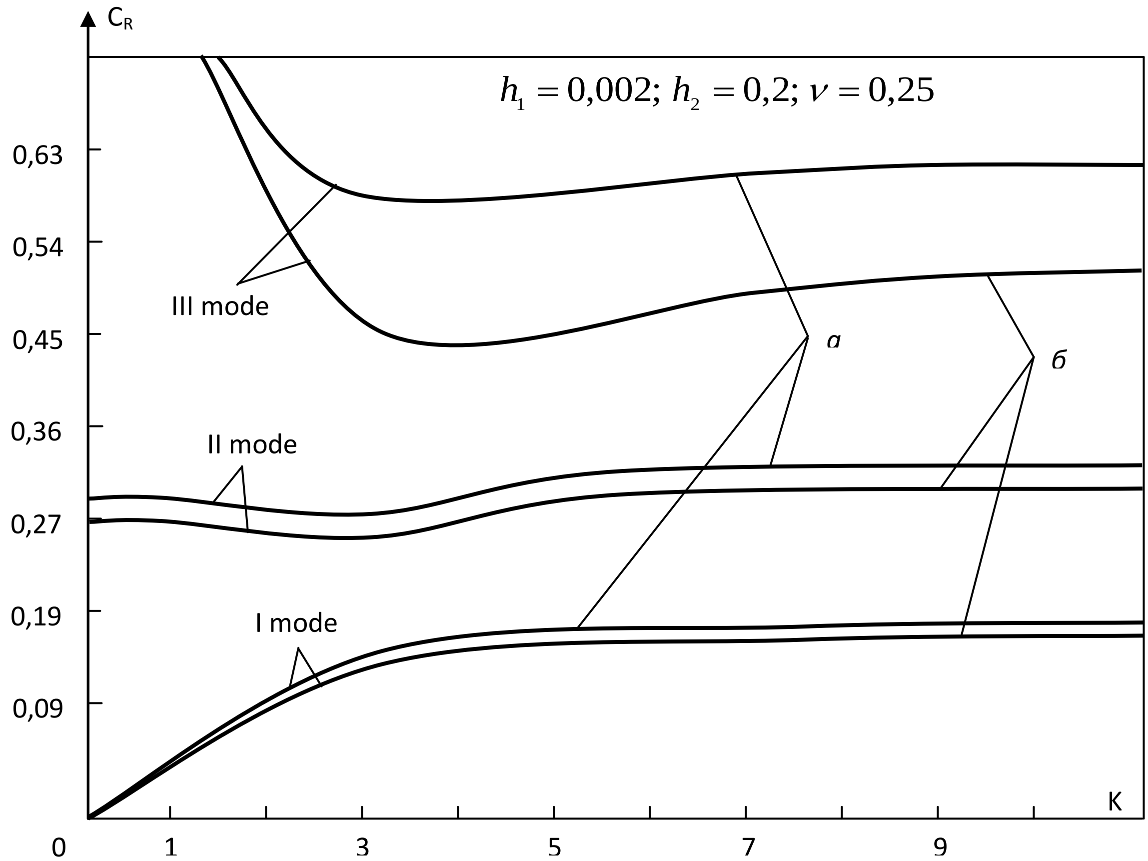

Figure 8 : The dispersion curves of phase velocities a Kirchhoff - Love; **б** - Timoshenko

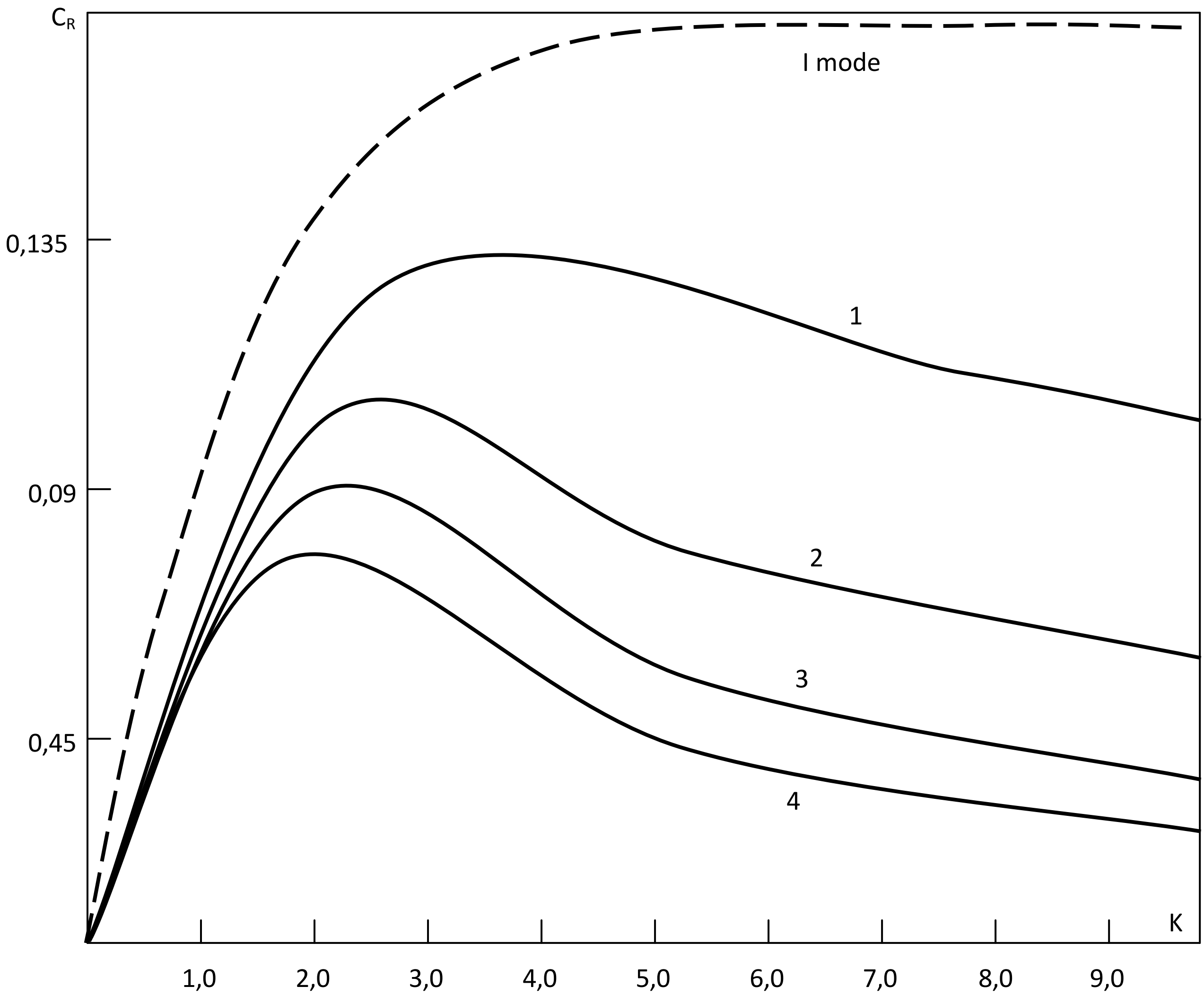

Figure 9a : The dispersion curves of phase velocities at different values P

1. P=1,5; 2. P=2; 3. P=2,5; ;. P=3

Notes

Notes

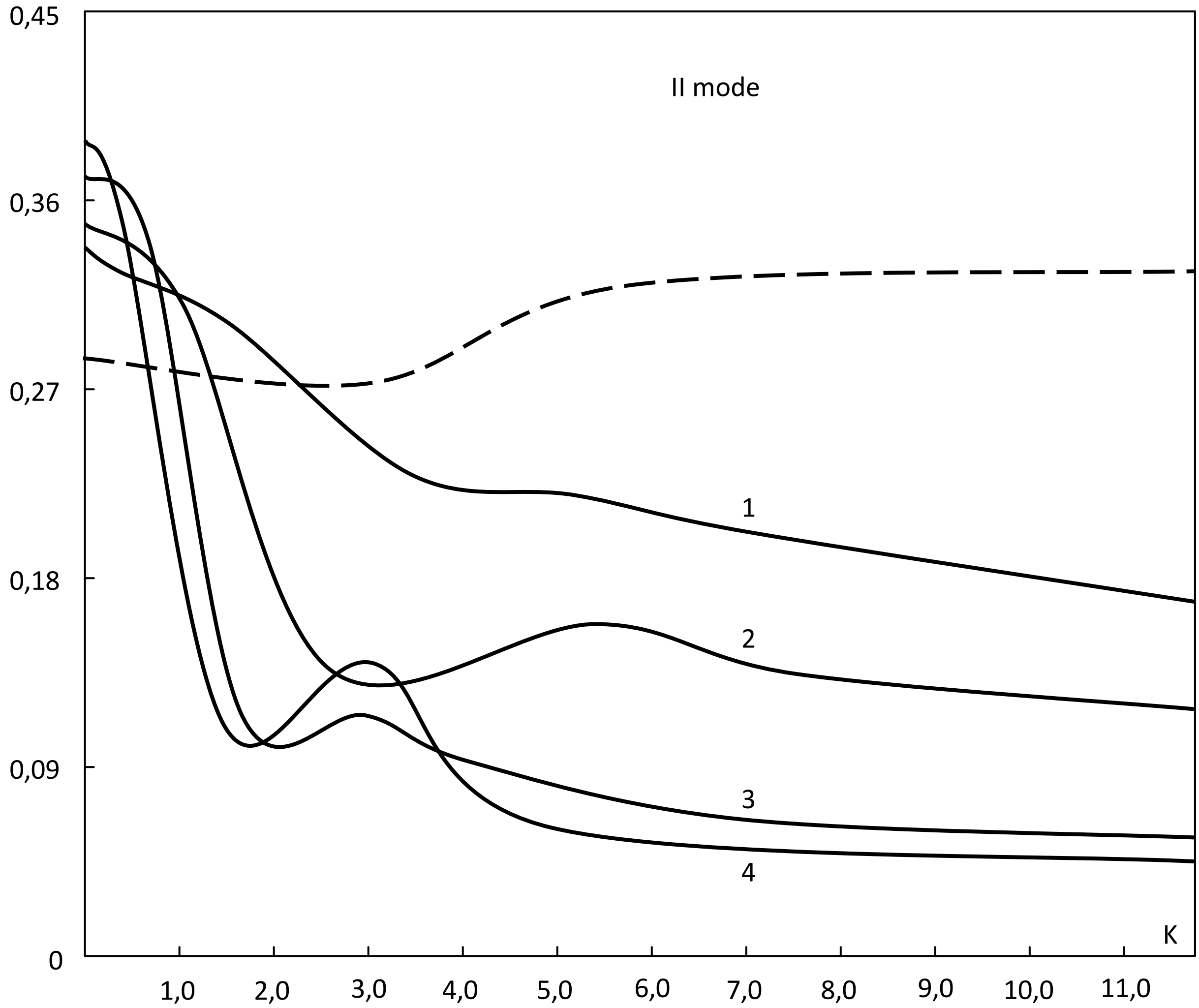

Figure 9.v. : The dispersion curves of phase velocities of the second mode at different values P.

1. P=1,5; 2. P=2; 3. P=2,5; ;. P=3

On the basis of these results the following conclusions:

- With increasing wave number of the velocity of propagation is real and the imaginary part of the normal modes in a wedge-shaped (plate) band Kirchhoff-Love and Timoshenko tend to constant values. At the same time there is the localization movement near the sharp edge of the waveguide.
- For small wedge angles comparison of the results obtained by the Kirchhoff-Love theory and Timoshenko, shows satisfactory agreement.
- Valid and imaginary parts of the complex phase velocity of the first mode in the wedge plate practically does not depend on the Poisson ratio (change within 0.5%).
- In the short-range limit value is valid and the imaginary part of the phase velocity of the first mode in the tapered waveguide is determined by the rate of change of the thickness in the vicinity of the sharp edge.
- In wedge-shaped plates with a small angle at the apex of a no dispersive waves propagate with a length not exceeding bandwidth.

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Loubéré Magic Squares Semigroups and Groups

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Abstract- This work is a pioneer investigation of semigroups and groups over the Loubéré Magic Squares. By the Loubéré Magic Squares, we understand the magic squares formed by the De La Loubéré Procedure. The set of the Loubéré Magic Squares equipped with the matrix binary operation of addition forms a semigroup if the underlining set so considered is the multi set of natural numbers; and if we consider the multi set of integer numbers as the underlined set of entries of the square, the set of the squares enclosed with the aforementioned operation forms an abelian group. The Loubéré Magic Squares are always recognized with centre piece C and magic sum M(S). We showcase that the set of the centre pieces and the set of the magic sums form respective abelian groups if both are equipped with integer numbers operation of addition. We also explicate that the set of the eigen values of the squares enclosed with the integer addition (operation) forms an abelian group. We reveal that the subelement (a terminology we introduced) Magic Squares of the Loubéré Magic Squares forms a semigroup and the Subelement Magic Squares of the Loubéré Magic Squares Group forms a group, with respect to the matrix binary operation of addition.

Keywords: semigroup, group, centre piece, eigen values, subelement, magic sum.

GJSFR-F Classification : FOR Code : MSC 2010: 16W22

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Babayo A. M. [α] & G. U. Garba [σ]

Abstract- This work is a pioneer investigation of semigroups and groups over the Loubéré Magic Squares. By the **Loubéré** Magic Squares, we understand the magic squares formed by the De La Loubéré Procedure. The set of the **Loubéré** Magic Squares equipped with the matrix binary operation of addition forms a semigroup if the underlining set so considered is the multi set of natural numbers; and if we consider the multi set of integer numbers as the underlined set of entries of the square, the set of the squares enclosed with the aforementioned operation forms an abelian group. The Loubéré Magic Squares are always recognized with centre piece C and magic sum M(S). We showcase that the set of the centre pieces and the set of the magic sums form respective abelian groups if both are equipped with integer numbers operation of addition. We also explicate that the set of the eigen values of the squares enclosed with the integer addition (operation) forms an abelian group. We reveal that the subelement (a terminology we introduced) Magic Squares of the Loubéré Magic Squares forms a semigroup and the Subelement Magic Squares of the Loubéré Magic Squares Group forms a group, with respect to the matrix binary operation of addition.

Keywords: semigroup, group, centre piece, eigen values, subelement, magic sum.

I. Introduction

This pioneering work disclosed a new realm of semigroup and group, the Loubéré Magic Squares Semigroup and Group. The set of the Loubéré Magic Squares of the arithmetic sequence of the set of the natural numbers or of its multi set form a semigroup which by analogy we refer to as the Loubéré Magic Squares Semigroup; and the set of the **Loubéré** Magic Squares of the arithmetic sequence of the set of integer numbers or of the multi set of the integer numbers form a group which by analogy we refer to as the **Loubéré** Magic Squares Group. The aforementioned semigroup [3] and group [4]are both with respect to the matrix binary operation of addition, thus they are both additive.

The collection of the centre pieces with formula $c_n = a_n + \left(\frac{m-1}{2}\right) j_n$ equipped with the integer addition forms an abelian group and the set of all the magic sums with formula $M(S_n) = \frac{m}{2}[2a_n + (m-1)j_n]$ equipped with the integer numbers binary operation of addition form an abelian group also, where $n = 1, 2, 3, \ldots$ and a_n , j_n are the corresponding first term and common difference along the main column respectively of $m \times m$ Loubéré Magic Squares.

We also showcase that the set of eigen values of the Loubéré Magic Squares enclosed with integer numbers operation of addition forms an abelian group. This is meaningful for the principal value of the eigen value corresponds to the magic sum [1].

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Definition 1.1.

A magic square $n-1\times n-1$ formed by removing the border cells of an $n\times n$ Loubéré Magic Square is called the subelement magic square of the $n\times n$ Loubéré Magic Square.

Remarks 1.1.

We have interest in the least subelement which is a subset of 3×3 Pancolumn Magic Squares. Purposefully, the 3×3 Loubéré Magic Square has no subelement for it is not a pancolumn. We explicate that the subelement magic squares of the Loubéré Magic Squares Semigroup forms a semi group and the subelement magic squares of the Loubéré Magic Squares Group forms a group with respect to the same underlining set and operation.

II. PRELIMINARIES

A basic magic square of order n is an arrangement of arithmetic sequence of common difference of 1 from 1 to n^2 in an $n\times n$ square grid of cells such that every row, column and diagonal add up to the same number, called the magic sum M(S) expressed as $M(S)=\frac{n^3+n}{2}$ and a centre piece C as $C=\frac{M(S)}{n}$.

a) Loubéré Procedure (NE-W-S or NW-E-S, the cardinal points)

Consider an empty $n\times n$ square of grids (or cells). Start, from the central column or row at a position $\left\lceil\frac{n}{2}\right\rceil$ where $\lceil\,\rceil$ is the greater natural number less than or equal to, with the number 1. The fundamental movement for filling the square is diagonally up, right (clock wise or NE or SE) or up left (anti clock wise or NW or SW) and one step at a time. If a filled cell (grid) is encountered, then the next consecutive number moves vertically down ward one square instead. Continue in this fashion until when a move would leave the square, it moves due N or E or W or S (depending on the position of the first term of the sequence) to the last row or first row or first column or last column.

Definition 2.1

Main Row or Column is the column or row of the Loubéré Magic Squares containing the first term and the last term of the arithmetic sequence in the square.

b) The Proof of the $\left\lceil\frac{m^2}{2}\right\rceil=a+\left(\frac{m-1}{2}\right)j$ and of the $M(S)=\frac{m}{2}[2a+(m-1)j]$, where $j=\frac{l-a}{m-1}$

Theorem 2.1.

Let the arithmetic sequence $a,a+d,\ldots,l=a+(n-1)d$ be arranged in an $m\times m$ Loubéré Magic Square. Then the magic sum of the square is expressed as $M(S)=\frac{m}{2}[\,2a+(m-1)j\,]$ and the middle term of the sequence (centre piece of the square) is expressed as $C=a+\left(\frac{m-1}{2}\right)j$ where j denotes the common difference of entries along the main column or row and is given as $j=\frac{l-a}{m-1}$.

Proof.

Consider any arbitrary General Loubéré Magic Square (here we consider 3×3) as follows:

$c+b$	$c-b-d$	$c+d$
$c-b+d$	c	$c+b-d$
$c-d$	$c+b+d$	$c-b$

Notes

Let $a = c - b - d$ and $l = c + b + d$. Then we have (from the square) an arithmetic sequence: $c - b - d, c - b, \dots, c + b + d$ having the sums S as

$$S = (c - b - d) + (c - b) + \cdots + (c + b) + (c + b + d) \rightarrow (1)$$
$$+$$
$$S = (c + b + d) + (c + b) + \cdots + (c - b) + (c - b - d) \rightarrow (2)$$

Adding (1)and (2), $2s = 2c + 2c + \cdots$ n times

i.e. $2s = 2nc \Longrightarrow c = \frac{s}{n} \dots (3)$ and $s = \frac{n}{2}(a + l) \dots (4)$ from the Gaussian High School (Elementary) Method. Since our square is $m \times m$, m number of cells (terms) are on the main column whence $a = c - b - d$. Thus, (3) and (4) become $C = \frac{M(S)}{m}$ … (5) and $M(S) = \frac{m}{2}[a + l] \dots (6)$ respectively. And, $l = a + (m - 1)j \dots (7)$ where j is along the main column. Substituting (7) in (6), we have: $M(S) = \frac{m}{2}[2a + (m - 1)j]$ … (8). Substituting (8) in (5), we get: $C = a + \left(\frac{m-1}{2}\right)j \dots (9)$ From (3) and (4), $C = \frac{1}{2}(a + l) = \left(a - \frac{a}{2}\right) + \frac{l}{2} = a + \frac{(l-a)}{2} = a + \frac{l-a}{m-1}\frac{m-1}{2}$, i.e. $C = a + \left(\frac{m-1}{2}\right)\frac{l-a}{m-1}$ … (10). Comparing (9) and (10), we have: $j = \frac{l-a}{m-1}$ … (11).

Definition 2.2.

A non empty set S equipped with a binary operation * is said to be a Semigroup (S,*) if it satisfies the following axioms:

i. $a, b \in S \Longrightarrow a * b \in S;$ *and*

ii. $a, b, c \in S \Rightarrow a * (b * c) = (a * b) * c$.

If in addition to the 2 axioms above, the following axioms are satisfied; then we call the algebraic structure a group $(G,*)$.

iii. $\exists e \in S \ni a * e = e * a \ \forall a \in S;$ and

iv. $\forall a \in S, \exists a^{-1} \in S \ni a * a^{-1} = a^{-1} * a = e \in S$.

If in addition to the above 4 axioms: I; ii; iii; and iv; the following axiom is satisfied; then we call (G,*) an abelian group.

v. $\forall a, b \in S, a * b = b * a$

Remark 2.1. The shift in notations from the use of S to G is intentional by the respective specialists.

III. The Loubéré Magic Squares Semigroup and Groups

We hereby present that the set of Loubéré Magic Squares L over the set of natural numbers equipped with the matrix binary operation of addition $\oplus$ forms a semigroup, and over the set of integer numbers forms a group-enclosed with the same operation.

a) Definition 3.1.

The square of grid of cells $[a_{ij}]_{n\times n}$ is said to be Loubéré Magic Square if the following conditions are satisfied.

i. $\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} = k$

ii. $\mathrm{trace}[a_{ij}]_{n\times n} = \mathrm{trace}[a_{ij}]^{T}_{n\times n} = k$

iii. $a_{1,\lceil\frac{n}{2}\rceil}$, $a_{\lceil\frac{n}{2}\rceil,\lceil\frac{n}{2}\rceil}$, $a_{n,\lceil\frac{n}{2}\rceil}$ are on the same main column or row and $a_{\lceil\frac{n}{2}\rceil,n}$, $a_{\lceil\frac{n}{2}\rceil\lceil\frac{n}{2}\rceil}$, $a_{\lceil\frac{n}{2}\rceil,1}$ are on the same main column or row, where $[\square]$ is the greater integer less or equal to, T is the transpose (of the square), k is the magic sum (magic product is defined analogously) usually expressed as $k=\frac{n}{2}[2a+(n-1)j]$ – from the sum of arithmetic sequence, where j is the common difference along the main column or row and a is the first term of the sequence– and $a_{\left[\frac{n}{2}\right]\left[\frac{n}{2}\right]}=\frac{k}{n}$.

b) *Theorem 3.2.*

$(L,\oplus)$ forms an Infinite Commutative Semigroup if the underlining multi set is of natural numbers and it forms an Infinite Additive Abelian Group if the underlining multi set is of integer numbers.

Proof. Let $[a_{i,j}]_{n\times n}$ and $[b_{ij}]_{n\times n}\in L$. Then, by Definition 3.1, $\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}=k$, $trace[a_{ij}]_{n\times n}=trace[a_{ij}]_{n\times n}^{T}=k$, and $a_{1,\left[\frac{n}{2}\right]}, a_{\left[\frac{n}{2}\right],\left[\frac{n}{2}\right]}, a_{n,\left[\frac{n}{2}\right]}$ are on the same main column or row and $a_{\left[\frac{n}{2}\right],n}, a_{\left[\frac{n}{2}\right]\left[\frac{n}{2}\right]}, a_{\left[\frac{n}{2}\right],1}$ are on the same main column or row, and $\sum_{i=1}^{n}\sum_{j=1}^{n}b_{ij}=l$, $trace[b_{ij}]_{n\times n}=trace[b_{ij}]_{n\times n}^{T}=l$, and $b_{1,\left[\frac{n}{2}\right]}, b_{\left[\frac{n}{2}\right],\left[\frac{n}{2}\right]}, b_{n,\left[\frac{n}{2}\right]}$ are on the same main column or row and $b_{\left[\frac{n}{2}\right],n}, b_{\left[\frac{n}{2}\right]\left[\frac{n}{2}\right]}, b_{\left[\frac{n}{2}\right],1}$ are on the same main column or row.

Then,

$$\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}+\sum_{i=1}^{n}\sum_{j=1}^{n}b_{ij}=k+l=\mathrm{trace}[\mathrm{a_{ij}}]_{n\times n}+\mathrm{trace}[\mathrm{b_{ij}}]_{n\times n}=\mathrm{trace}[\mathrm{a_{ij}}+\mathrm{b_{ij}}]_{n\times n}$$

$\mathrm{trace}[\mathrm{a_{ij}}]_{n\times n}^{T}+\mathrm{trace}[\mathrm{b_{ij}}]_{n\times n}^{T}=\mathrm{trace}[\mathrm{a_{ij}}+\mathrm{b_{ij}}]_{n\times n}^{T}=\mathrm{k}+\mathrm{l}$, and $a_{1,\left[\frac{n}{2}\right]}+b_{1,\left[\frac{n}{2}\right]}$ $a_{\left[\frac{n}{2}\right],\left[\frac{n}{2}\right]}+b_{\left[\frac{n}{2}\right],\left[\frac{n}{2}\right]}$, $a_{n,\left[\frac{n}{2}\right]}+b_{n,\left[\frac{n}{2}\right]}$ are on the same main column or row and $a_{\left[\frac{n}{2}\right],n}+b_{\left[\frac{n}{2}\right],n}$, $a_{\left[\frac{n}{2}\right]\left[\frac{n}{2}\right]}+b_{\left[\frac{n}{2}\right]\left[\frac{n}{2}\right]}$, $a_{\left[\frac{n}{2}\right],1}+b_{\left[\frac{n}{2}\right],1}$ since $a_{1,\left[\frac{n}{2}\right]}, a_{\left[\frac{n}{2}\right],\left[\frac{n}{2}\right]}, a_{n,\left[\frac{n}{2}\right]}$ are on the same main column or row and $a_{\left[\frac{n}{2}\right],n}, a_{\left[\frac{n}{2}\right]\left[\frac{n}{2}\right]}, a_{\left[\frac{n}{2}\right],1}$ are on the same main column or row, and $b_{1,\left[\frac{n}{2}\right]}, b_{\left[\frac{n}{2}\right],\left[\frac{n}{2}\right]}, b_{n,\left[\frac{n}{2}\right]}$ are on the same main column or row and $b_{\left[\frac{n}{2}\right],n}, b_{\left[\frac{n}{2}\right]\left[\frac{n}{2}\right]}, b_{\left[\frac{n}{2}\right],1}$ are on the same main column or row.

i. *Associativity*

Let $[a_{i,j}]_{n\times n}, [b_{i,j}]_{n\times n}$ *and* $[c_{i,j}]_{n\times n}\in L$. Then, by Definition 3.1, $\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}=k$, $\mathrm{trace}[\mathrm{a_{ij}}]_{n\times n}=\mathrm{trace}[\mathrm{a_{ij}}]_{n\times n}^{T}=\mathrm{k}$, and $a_{1,\left[\frac{n}{2}\right]}, a_{\left[\frac{n}{2}\right],\left[\frac{n}{2}\right]}, a_{n,\left[\frac{n}{2}\right]}$ are on the same main column or row and $a_{\left[\frac{n}{2}\right],n}, a_{\left[\frac{n}{2}\right]\left[\frac{n}{2}\right]}, a_{\left[\frac{n}{2}\right],1}$ are on the same main column or row, $\sum_{i=1}^{n}\sum_{j=1}^{n}b_{ij}=l$, $\mathrm{trace}[\mathrm{b_{ij}}]_{n\times n}=\mathrm{trace}[\mathrm{b_{ij}}]_{n\times n}^{T}=\mathrm{l}$ and $b_{1,\left[\frac{n}{2}\right]}, b_{\left[\frac{n}{2}\right],\left[\frac{n}{2}\right]}, b_{n,\left[\frac{n}{2}\right]}$ are on the same main column or row and $b_{\left[\frac{n}{2}\right],n}, b_{\left[\frac{n}{2}\right]\left[\frac{n}{2}\right]}, b_{\left[\frac{n}{2}\right],1}$ are on the same main column or row.

and $\sum_{i=1}^{n}\sum_{j=1}^{n}c_{ij}=m$, $\mathrm{trace}[\mathrm{c_{ij}}]_{n\times n}=\mathrm{trace}[\mathrm{c_{ij}}]_{n\times n}^{T}=\mathrm{m}$ and $c_{1,\left[\frac{n}{2}\right]}, c_{\left[\frac{n}{2}\right],\left[\frac{n}{2}\right]}, c_{n,\left[\frac{n}{2}\right]}$ are on the same main column or row and $c_{\left[\frac{n}{2}\right],n}, c_{\left[\frac{n}{2}\right]\left[\frac{n}{2}\right]}, c_{\left[\frac{n}{2}\right],1}$ are on the same main column or row

Then,

Notes

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} + \left(\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\right) = \mathrm{trace}[\mathrm{a_{ij}}]_{\mathrm{n\times n}} + \left(\mathrm{trace}[\mathrm{b_{ij}}]_{\mathrm{n\times n}} + \mathrm{trace}[\mathrm{c_{ij}}]_{\mathrm{n\times n}}\right)$$
$$= \mathrm{trace}[\mathrm{a_{ij}}]_{\mathrm{n\times n}} + \mathrm{trace}[\mathrm{b_{ij}}]_{\mathrm{n\times n}} + \mathrm{trace}[\mathrm{c_{ij}}]_{\mathrm{n\times n}}$$
$$= \left(\mathrm{trace}[\mathrm{a_{ij}}]_{\mathrm{n\times n}} + \mathrm{trace}[\mathrm{b_{ij}}]_{\mathrm{n\times n}}\right) + \mathrm{trace}[\mathrm{c_{ij}}]_{\mathrm{n\times n}} = \left(\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}\right) + \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}$$
$$\mathrm{trace}[\mathrm{a_{ij}}]^{\mathrm{T}}_{\mathrm{n\times n}} + \left(\mathrm{trace}[\mathrm{b_{ij}}]^{\mathrm{T}}_{\mathrm{n\times n}} + \mathrm{trace}[\mathrm{c_{ij}}]^{\mathrm{T}}_{\mathrm{n\times n}}\right) = \mathrm{trace}[\mathrm{a_{ij}} + \mathrm{b_{ij}}]^{\mathrm{T}}_{\mathrm{n\times n}} =$$

$\mathrm{trace}[\mathrm{a_{ij}}]^{\mathrm{T}}_{\mathrm{n\times n}} + \left(\mathrm{trace}[\mathrm{b_{ij}}]^{\mathrm{T}}_{\mathrm{n\times n}} + \mathrm{trace}[\mathrm{c_{ij}}]^{\mathrm{T}}_{\mathrm{n\times n}}\right) = \mathrm{k + l + m}$, and $a_{1,\lceil\frac{n}{2}\rceil} + \left(b_{1,\lceil\frac{n}{2}\rceil} + c_{1,\lceil\frac{n}{2}\rceil}\right)$, $a_{\lceil\frac{n}{2}\rceil,\lceil\frac{n}{2}\rceil} + \left(b_{\lceil\frac{n}{2}\rceil,\lceil\frac{n}{2}\rceil} + c_{\lceil\frac{n}{2}\rceil,\lceil\frac{n}{2}\rceil}\right)$, $a_{n,\lceil\frac{n}{2}\rceil} + \left(b_{n,\lceil\frac{n}{2}\rceil} + c_{n,\lceil\frac{n}{2}\rceil}\right)$ are on the same main column or row and $a_{\lceil\frac{n}{2}\rceil,n} + \left(b_{\lceil\frac{n}{2}\rceil,n} + c_{\lceil\frac{n}{2}\rceil,n}\right)$, $a_{\lceil\frac{n}{2}\rceil\lceil\frac{n}{2}\rceil} + \left(b_{\lceil\frac{n}{2}\rceil\lceil\frac{n}{2}\rceil} + c_{\lceil\frac{n}{2}\rceil\lceil\frac{n}{2}\rceil}\right)$, $a_{\lceil\frac{n}{2}\rceil,1} + \left(b_{\lceil\frac{n}{2}\rceil,1} + c_{\lceil\frac{n}{2}\rceil,1}\right)$, then $\left(a_{\lceil\frac{n}{2}\rceil,n} + b_{\lceil\frac{n}{2}\rceil,n}\right) + c_{\lceil\frac{n}{2}\rceil,n}$, $\left(a_{\lceil\frac{n}{2}\rceil\lceil\frac{n}{2}\rceil} + b_{\lceil\frac{n}{2}\rceil\lceil\frac{n}{2}\rceil}\right) + c_{\lceil\frac{n}{2}\rceil\lceil\frac{n}{2}\rceil}$, $\left(a_{\lceil\frac{n}{2}\rceil,1} + b_{\lceil\frac{n}{2}\rceil,1}\right) + c_{\lceil\frac{n}{2}\rceil,1}$ *and* $a_{1,\lceil\frac{n}{2}\rceil}$, $a_{\lceil\frac{n}{2}\rceil,\lceil\frac{n}{2}\rceil}$, $a_{n,\lceil\frac{n}{2}\rceil}$ are on the same main column or row and $a_{\lceil\frac{n}{2}\rceil,n}$, $a_{\lceil\frac{n}{2}\rceil\lceil\frac{n}{2}\rceil}$, $a_{\lceil\frac{n}{2}\rceil,1}$ are on the same main column or row , and $b_{1,\lceil\frac{n}{2}\rceil}$, $b_{\lceil\frac{n}{2}\rceil,\lceil\frac{n}{2}\rceil}$, $b_{n,\lceil\frac{n}{2}\rceil}$ are on the same main column or row and $b_{\lceil\frac{n}{2}\rceil,n}$, $b_{\lceil\frac{n}{2}\rceil\lceil\frac{n}{2}\rceil}$, $b_{\lceil\frac{n}{2}\rceil,1}$ are on the same main column or row, and $c_{1,\lceil\frac{n}{2}\rceil}$, $c_{\lceil\frac{n}{2}\rceil,\lceil\frac{n}{2}\rceil}$, $c_{n,\lceil\frac{n}{2}\rceil}$ are on the same main column or row and $c_{\lceil\frac{n}{2}\rceil,n}$, $c_{\lceil\frac{n}{2}\rceil\lceil\frac{n}{2}\rceil}$, $c_{\lceil\frac{n}{2}\rceil,1}$ are on the same main column or row.

ii. *Identity Element*

$\exists[a_{ij}]_{n\times n} \in L$ *and* is said to be Loubéré Magic Square if the following conditions are satisfied.

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} = 0$$
$$\mathrm{trace}[\mathrm{a_{ij}}]_{\mathrm{n\times n}} = \mathrm{trace}[\mathrm{a_{ij}}]^{\mathrm{T}}_{\mathrm{n\times n}} = 0$$

$a_{1,\lceil\frac{n}{2}\rceil} = 0$, $a_{\lceil\frac{n}{2}\rceil,\lceil\frac{n}{2}\rceil} = 0$, $a_{n,\lceil\frac{n}{2}\rceil} = 0$ are on the same main column or row and $a_{\lceil\frac{n}{2}\rceil,n} = 0$, $a_{\lceil\frac{n}{2}\rceil\lceil\frac{n}{2}\rceil} = 0$, $a_{\lceil\frac{n}{2}\rceil,1} = 0$ are on the same main column or row, whence $\mathrm{a_{ij}} = 0, \forall \mathrm{i,j} \Rightarrow$ the identity is $[0]_{\mathrm{n\times n}} \in \mathrm{L}$

iii. *Inverse Element Property*

Given $[a_{i,j}]_{n\times n} \in L \ni \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} = k$,
$\mathrm{trace}[\mathrm{a_{ij}}]_{\mathrm{n\times n}} = \mathrm{trace}[\mathrm{a_{ij}}]^{\mathrm{T}}_{\mathrm{n\times n}} = \mathrm{k}$, and

$a_{1,\lceil\frac{n}{2}\rceil}$, $a_{\lceil\frac{n}{2}\rceil,\lceil\frac{n}{2}\rceil}$, $a_{n,\lceil\frac{n}{2}\rceil}$ are on the same main column or row and $a_{\lceil\frac{n}{2}\rceil,n}$, $a_{\lceil\frac{n}{2}\rceil\lceil\frac{n}{2}\rceil}$, $a_{\lceil\frac{n}{2}\rceil,1}$ are on the same main column or row, there exists $[-a_{i,j}]_{n\times n} \in L$ such that $\sum_{i=1}^{n}\sum_{j=1}^{n} -a_{ij} = -k$, $\mathrm{trace}[-\mathrm{a_{ij}}]_{\mathrm{n\times n}} = \mathrm{trace}[-\mathrm{a_{ij}}]^{\mathrm{T}}_{\mathrm{n\times n}} = -\mathrm{k}$, and $-a_{1,\lceil\frac{n}{2}\rceil}$, $-a_{\lceil\frac{n}{2}\rceil,\lceil\frac{n}{2}\rceil}$, $-a_{n,\lceil\frac{n}{2}\rceil}$ are on the same main column or row and $-a_{[\frac{n}{2}],n}$, $-a_{[\frac{n}{2}][\frac{n}{2}]}$, $-a_{[\frac{n}{2}],1}$ are on the same main column or row. Thus, $[a_{ij}]_{n\times n} + [-a_{ij}]_{n\times n} = [-a_{ij}]_{n\times n} + [a_{ij}]_{n\times n} = [0]_{n\times n}$.

iv. *Commutativity*

$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij} = k + l = l + k = \sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}$ and $\mathrm{trace}[a_{ij}]_{n\times n} + \mathrm{trace}[b_{ij}]_{n\times n} = \mathrm{trace}[a_{ij} + b_{ij}]_{n\times n} = k + l = l + k = \mathrm{trace}[b_{ij} + a_{ij}]_{n\times n} = \mathrm{trace}[b_{ij}]_{n\times n} + \mathrm{trace}[a_{ij}]_{n\times n}$

$\mathrm{trace}[a_{ij}]_{n\times n}^{T} + \mathrm{trace}[b_{ij}]_{n\times n}^{T} = \mathrm{trace}[a_{ij} + b_{ij}]_{n\times n}^{T} = k + l = l + k = \mathrm{trace}[b_{ij} + a_{ij}]_{n\times n}^{T} = \mathrm{trace}[b_{ij}]_{n\times n}^{T} + \mathrm{trace}[a_{ij}]_{n\times n}^{T}$, and $a_{1,[\frac{n}{2}]} + b_{1,[\frac{n}{2}]}$, $a_{[\frac{n}{2}],[\frac{n}{2}]} + b_{[\frac{n}{2}],[\frac{n}{2}]}$, $a_{n,[\frac{n}{2}]} + b_{n,[\frac{n}{2}]}$ are on the same main column or row and $a_{[\frac{n}{2}],n} + b_{[\frac{n}{2}],n}$, $a_{[\frac{n}{2}][\frac{n}{2}]} + b_{[\frac{n}{2}][\frac{n}{2}]}$, $a_{[\frac{n}{2}],1} + b_{[\frac{n}{2}],1}$ as well as $b_{1,[\frac{n}{2}]} + a_{1,[\frac{n}{2}]}$, $b_{[\frac{n}{2}],[\frac{n}{2}]} + a_{[\frac{n}{2}],[\frac{n}{2}]}$, $b_{n,[\frac{n}{2}]} + a_{n,[\frac{n}{2}]}$ are on the same main column or row and $b_{[\frac{n}{2}],n} + a_{[\frac{n}{2}],n}$, $b_{[\frac{n}{2}][\frac{n}{2}]} + a_{[\frac{n}{2}][\frac{n}{2}]}$, $b_{[\frac{n}{2}],1} + a_{[\frac{n}{2}],1}$.

We can now consider the general multi set. The Loubéré Magic Squares over the multi set of integer numbers, since multi set of natural numbers is its subset, is a semi pandiagonal. By semi pandiagonal, we mean in $n \times n$ square, n elements repeats on every row, column and on a diagonal. Though the sum of the numbers on the rows, the columns and the diagonals add up to the magic sum; yet one diagonal has an n repetition of one element. To change the orientation (from left to right or the reverse) of the pandiagonal of the 3×3, use the sequence $a, a, a, b, b, b, c, c, c$ rather than $a, b, c, a, b, c, a, b, c$.

We can now show that they form a group as in the above. Consider 3 arbitrary elements of the set of Lefty Semi Pandiagonal Loubéré Magic Squares,

b	a	c
c	b	a
a	c	b

,

e	d	f
f	e	d
d	f	e

and

h	g	i
i	h	g
g	i	h

Then,

b	a	c
c	b	a
a	c	b

+

e	d	f
f	e	d
d	f	e

=

b+e	a+d	c+f
c+f	b+e	a+d
a+d	c+f	b+e

Notes

Notes

i. is also a Lefty Semi Pandiagonal 3×3 Loubéré Magic Squares, hence *closure property* is satisfied.

ii. *Associativity.* It is clear (even from inherited property of the underlining set) that

b	a	c
c	b	a
a	c	b

$+\Big($

e	d	f
f	e	d
d	f	e

$+$

h	g	i
i	h	g
g	i	h

$\Big) =\Big($

b	a	c
c	b	a
a	c	b

$+$

e	d	f
f	e	d
d	f	e

$\Big)+$

h	g	i
i	h	g
g	i	h

iii. The *identity* is −as in the above−

0	0	0
0	0	0
0	0	0

iv. Let the following be an arbitrary Lefty Semi Pandiagonal Magic Square.

V	U	W
W	V	U
U	W	V

Clearly, its *inverse* is

-V	-U	-W
-W	-V	-U
-U	-W	-V

v. Every 2 Loubéré Magic Squares (whether semi pancolumn or not) over multi set of naturals or over multi set of integer numbers *commute* since natural and integer numbers commutes.

Thus the group and the semigroups of the Loubéré Magic Squares are commutative.

IV. Centre Pieces and Magic Sums Abelian Groups

a) Centre Pieces Abelian Group

The set of the centre pieces $c_1, c_2, c_3, \dots$ of $m \times m$ Loubéré Magic Squares equipped with the integer number binary operation of addition forms an infinite abelian group. Given the centre pieces $c_1, c_2, c_3, \dots$ of $m \times m$ Loubéré Magic Squares with corresponding formula

Notes

$$c_1 = a_1 + \left(\frac{m-1}{2}\right) j_1, c_2 = a_2 + \left(\frac{m-1}{2}\right) j_2,\ c_3 = a_3 + \left(\frac{m-1}{2}\right) j_3, \dots; \text{then}$$

i. $c_1 + c_2 = (a_1 + a_2) + \left(\frac{m-1}{2}\right)(j_1 + j_2)$ is the centre piece of the $m \times m$ Loubéré Magic Square with first term $a_1 + a_2$ and common difference along the main column $j_1 + j_2$. Hence, the set is closed.

ii. This is an inherited property of the set of integer numbers:

$$c_1 + (c_2 + c_3) = (a_1 + a_2 + a_3) + \left(\frac{m-1}{2}\right)(j_1 + j_2 + j_3) = (c_1 + c_2) + c_3$$

iii. The identity element is the zero centre piece e.g.

C	-D	A
-B	0	B
-A	D	-C

iv. Given an arbitrary centre piece $c_n = a_n + \left(\frac{m-1}{2}\right) j_n$ of the $m \times m$ Loubéré Magic Square, there exists another centre piece c_{-n} of another $m \times m$ Loubéré Magic Square having first term as $-a_n$ and common difference along the main column or row as $-j_n$, thus its formula is $c_{-n} = -a_n + \left(\frac{m-1}{2}\right)(-j_n)$ such that $c_n + c_{-n} = c_{-n} + c_n = (a_n - a_n) + \left(\frac{m-1}{2}\right)[j_n - j_n] = 0 = c_i$, the identity centre piece.

v. Clearly $c_1 + c_2 = a_1 + a_2 + \left(\frac{m-1}{2}\right)(j_1 + j_2) = a_2 + a_1 + \left(\frac{m-1}{2}\right)(j_2 + j_1) = c_2 + c_1$

The set equipped with the operation is an abelian group.

b) Magic Sum Abelian Groups

The set of the magic sums $M(s_1), M(s_2), M(s_3), \dots$ of $m \times m$ Loubéré Magic Squares equipped with the integer binary operation of addition form an infinite abelian group. Given the magic sums $M(s_1), M(s_2), M(s_3), \dots$ of $m \times m$ Loubéré Magic Squares with corresponding formula

$$M(s_1) = \frac{m}{2}\left[2a_1 + (m-1)j_1, M(s_2) = \frac{m}{2}[2a_2 + (m-1)j_2], M(s_3)\right) = \frac{m}{2}[2a_3 + (m-1)j_3, \dots;$$

then (as in the above):

i. $M(s_1) + M(s_2) = M(s_?)$ where $M(s_?)$ is a magic sum of another $m \times m$ Loubéré Magic Square with first term $a_1 + a_2$ and common difference along the main column as $j_1 + j_2$.

The axioms: ii, iii, iv and v follow, by analogy to the centre piece abelian group, immediately.

Notes

V. Eigen Values Abelian Group

The Eigen values computation in the magic squares is what is zealotly prophesized that magic squares are special type of matrices, hence the definition of the magic squares, we do not love to like such a sudden conclusion if loving to liking forces choosing the definitions in terms of just the square grids (or cells).

We want to show through concrete examples that the set of Eigen Values of the Loubéré Magic Squares with the usual integer numbers binary operation of addition forms a group. Consider the following arbitrary two 3×3 Loubéré Magic Squares – which we let

a =

4	3	2
-1	1	3
0	5	-2

, b =

2	-5	0
-3	-1	1
-2	3	-4

and their sum c =

6	-8	2
-4	0	4
-2	8	-6

We compute the eigen values for a as follows: The corresponding matrix of a is $(a)=\begin{pmatrix}4 & -3 & 2\\ -1 & 1 & 3\\ 0 & 5 & -2\end{pmatrix}$, its eigen vector is $|a-\lambda I| = \begin{vmatrix}4-\lambda & -3 & 2\\ -1 & 1-\lambda & 3\\ 0 & 5 & -2-\lambda\end{vmatrix} = 0$, i.e. $\lambda^3 - 3\lambda^2 - 24\lambda - 72 = (\lambda-3)(\lambda^2-24) = 0$ having characteristic equation as $\lambda_{a_1}=3$, $\lambda_{a_2}=4.9$ and $\lambda_{a_3}=-4.9$.

We compute the eigen values for b as follows: The corresponding matrix of b is $(b)=\begin{pmatrix}2 & -5 & 0\\ -3 & -1 & 1\\ -2 & 3 & -4\end{pmatrix}$, its characteristic equation is $|b-\lambda I| = \begin{vmatrix}2-\lambda & -5 & 0\\ -3 & -1-\lambda & 1\\ -2 & 3 & -4-\lambda\end{vmatrix} = 0$ i.e. $\lambda^3+3\lambda^2-24\lambda-72=(\lambda+3)(\lambda^2-24)=0$ with eigen values $\lambda_{b_1}=-3, \lambda_{b_2}=4.9$ and $\lambda_{b_3}=-4.9$.

We compute the eigen values for c as follows: The corresponding matrix of c is $(c)=\begin{pmatrix}6 & -8 & 2\\ -4 & 0 & 4\\ -2 & 8 & -6\end{pmatrix}$, its characteristic equation is $|c-\lambda I| = \begin{vmatrix}6-\lambda & -8 & 2\\ -4 & -\lambda & 4\\ -2 & 8 & -6-\lambda\end{vmatrix} = 0$, i.e. $\lambda^3 - 96\lambda = 0$ with corresponding eigen values $\lambda_{c_1}=0, \lambda_{c_2}=9.8$ and $\lambda_{c_3}=-9.8$.

We now conclude this session by showing that the set of eigen values satisfies *The Properties of a Group* as follows:

Closure Property. Consider any 3 arbitrary Loubéré Magic Squares a, b, c; such that $a+b=c$; then from the example above, the corresponding eigen values of a; $\lambda_{a_1}, \lambda_{a_2}\lambda_{a_3}$; the corresponding eigen values of b; $\lambda_{b_1}, \lambda_{b_2}, \lambda_{b_3}$; are such that $\lambda_{a_1}+\lambda_{b_1}=\lambda_{c_1}$, $\lambda_{a_2}+\lambda_{b_2}=\lambda_{c_2}$, and $\lambda_{a_3}+\lambda_{b_3}=\lambda_{c_3}$ where $\lambda_{c_1}, \lambda_{c_2}, \lambda_{c_3}$ are the corresponding eigen values of c.

Associativity Property. Since Loubéré Magic Squares are a semigroup (which is easy to observe), the eigen values are associative.

Identity Element Property. The eigen value 0 is the identity element that corresponds to the sum of the Loubéré Magic Squares of opposite eigen values as in the above.

Inverse Elements Property. For any arbitrary eigen value λ_m corresponding to a Loubéré Magic Square m, there exist a $-\lambda_m$ eigen value corresponding to another Loubéré Magic Square such that $\lambda_m + (-\lambda_m)$ gives the identity element which is formed as a result of matrix addition of the aforementioned Loubéré Magic Squares.

Notes

Commutativity. Consider any 2 arbitrary Loubéré Magic Squares a, b, ; such that $a + b = b + a$; then from the example above, the corresponding eigen values of a; $\lambda_{a_1}, \lambda_{a_2}\lambda_{a_3}$; the corresponding eigen values of b; $\lambda_{b_1}, \lambda_{b_2}, \lambda_{b_3}$; are such that $\lambda_{a_1} + \lambda_{b_1} = \lambda_{b_1} + \lambda_{a_1}$, $\lambda_{a_2} + \lambda_{b_2} = \lambda_{b_2} + \lambda_{a_2}$, and $\lambda_{a_3} + \lambda_{b_3} = \lambda_{b_3} + \lambda_{a_3}$.

The idea of eigen values computation of a magic square is conceived from the work of [1].

VI. The Subelement Magic Squares Semigroup and Group

The set of least subelement of Loubéré Magic Squares is a subset of pancolumn 3×3 Magic Squares. By convention, the 3×3 Loubéré Magic Square(since not pancolumn) is not a self subelement. The sum of two arbitrary subelements of $m \times m$ Loubéré Magic Squares is a subelement of $m \times m$ Loubéré Magic Square, hence closure property is exhibited. Associativity , Identity, Inverse and Commutativity Properties are inherited from the super elements, the Loubéré Magic Squares. Both the binary and the unary operations of the super elements and of the subelements are equal.

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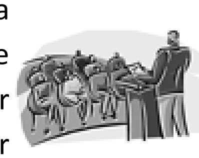
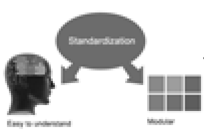


You may join as member of the Editorial Board of Global Journals Incorporation (USA) after successful completion of three years as Fellow and as Peer Reviewer. In addition, it is also desirable that you should organize seminar/symposium/conference at least once.

We shall provide you intimation regarding launching of e-version of journal of your stream time to time.This may be utilized in your library for the enrichment of knowledge of your students as well as it can also be helpful for the concerned faculty members.

The FARSS can go through standards of OARS. You can also play vital role if you have any suggestions so that proper amendment can take place to improve the same for the benefit of entire research community.

As FARSS, you will be given a renowned, secure and free professional email address with 100 GB of space e.g. johnhall@globaljournals.org. This will include Webmail, Spam Assassin, Email Forwarders,Auto-Responders, Email Delivery Route tracing, etc.



The FARSS will be eligible for a free application of standardization of their researches. Standardization of research will be subject to acceptability within stipulated norms as the next step after publishing in a journal. We shall depute a team of specialized research professionals who will render their services for elevating your researches to next higher level, which is worldwide open standardization.

The FARSS member can apply for grading and certification of standards of their educational and Institutional Degrees to Open Association of Research, Society U.S.A. Once you are designated as FARSS, you may send us a scanned copy of all of your credentials. OARS will verify, grade and certify them. This will be based on your academic records, quality of research papers published by you, and some more criteria. After certification of all your credentials by OARS, they will be published on your Fellow Profile link on website https://associationofresearch.org which will be helpful to upgrade the dignity.

The FARSS members can avail the benefits of free research podcasting in Global Research Radio with their research documents. After publishing the work, (including published elsewhere worldwide with proper authorization) you can upload your research paper with your recorded voice or you can utilize chargeable services of our professional RJs to record your paper in their voice on request.

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The FARSS is eligible to earn from sales proceeds of his/her researches/reference/review Books or literature, while publishing with Global Journals. The FARSS can decide whether he/she would like to publish his/her research in a closed manner. In this case, whenever readers purchase that individual research paper for reading, maximum 60% of its profit earned as royalty by Global Journals, will be credited to his/her bank account. The entire entitled amount will be credited to his/her bank account exceeding limit of minimum fixed balance. There is no minimum time limit for collection. The FARSS member can decide its price and we can help in making the right decision.

The FARSS member is eligible to join as a paid peer reviewer at Global Journals Incorporation (USA) and can get remuneration of 15% of author fees, taken from the author of a respective paper. After reviewing 5 or more papers you can request to transfer the amount to your bank account.

MEMBER OF ASSOCIATION OF RESEARCH SOCIETY IN SCIENCE (MARSS)

The ' MARSS ' title is accorded to a selected professional after the approval of the Editor-in-Chief / Editorial Board Members/Dean.

The “MARSS” is a dignified ornament which is accorded to a person’s name viz. Dr. John E. Hall, Ph.D., MARSS or William Walldroff, M.S., MARSS.

MARSS accrediting is an honor. It authenticates your research activities. After becoming MARSS, you can add 'MARSS' title with your name as you use this recognition as additional suffix to your status. This will definitely enhance and add more value and repute to your name. You may use it on your professional Counseling Materials such as CV, Resume, Visiting Card and Name Plate etc.

The following benefitscan be availed by you only for next three years from the date of certification.

MARSS designated members are entitled to avail a 25% discount while publishing their research papers (of a single author) in Global Journals Inc., if the same is accepted by our Editorial Board and Peer Reviewers. If you are a main author or co-author of a group of authors, you will get discount of 10%.

As MARSS, you will be given a renowned, secure and free professional email address with 30 GB of space e.g. johnhall@globaljournals.org. This will include Webmail, Spam Assassin, Email Forwarders,Auto-Responders, Email Delivery Route tracing, etc.

We shall provide you intimation regarding launching of e-version of journal of your stream time to time.This may be utilized in your library for the enrichment of knowledge of your students as well as it can also be helpful for the concerned faculty members.

The MARSS member can apply for approval, grading and certification of standards of their educational and Institutional Degrees to Open Association of Research, Society U.S.A.

Once you are designated as MARSS, you may send us a scanned copy of all of your credentials. OARS will verify, grade and certify them. This will be based on your academic records, quality of research papers published by you, and some more criteria.

It is mandatory to read all terms and conditions carefully.

Auxiliary Memberships

Institutional Fellow of Global Journals Incorporation (USA)-OARS (USA)

Global Journals Incorporation (USA) is accredited by Open Association of Research Society, U.S.A (OARS) and in turn, affiliates research institutions as “Institutional Fellow of Open Association of Research Society” (IFOARS).

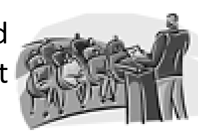
The “FARSC” is a dignified title which is accorded to a person’s name viz. Dr. John E. Hall, Ph.D., FARSC or William Walldroff, M.S., FARSC.

The IFOARS institution is entitled to form a Board comprised of one Chairperson and three to five board members preferably from different streams. The Board will be recognized as “Institutional Board of Open Association of Research Society”-(IBOARS).

The Institute will be entitled to following benefits:

The IBOARS can initially review research papers of their institute and recommend them to publish with respective journal of Global Journals. It can also review the papers of other institutions after obtaining our consent. The second review will be done by peer reviewer of Global Journals Incorporation (USA) The Board is at liberty to appoint a peer reviewer with the approval of chairperson after consulting us.

The author fees of such paper may be waived off up to 40%.



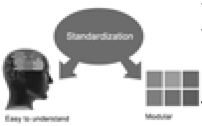
The Global Journals Incorporation (USA) at its discretion can also refer double blind peer reviewed paper at their end to the board for the verification and to get recommendation for final stage of acceptance of publication.

The IBOARS can organize symposium/seminar/conference in their country on behalf of Global Journals Incorporation (USA)-OARS (USA). The terms and conditions can be discussed separately.

The Board can also play vital role by exploring and giving valuable suggestions regarding the Standards of “Open Association of Research Society, U.S.A (OARS)” so that proper amendment can take place for the benefit of entire research community. We shall provide details of particular standard only on receipt of request from the Board.

The board members can also join us as Individual Fellow with 40% discount on total fees applicable to Individual Fellow. They will be entitled to avail all the benefits as declared. Please visit Individual Fellow-sub menu of GlobalJournals.org to have more relevant details.

We shall provide you intimation regarding launching of e-version of journal of your stream time to time. This may be utilized in your library for the enrichment of knowledge of your students as well as it can also be helpful for the concerned faculty members.



After nomination of your institution as "Institutional Fellow" and constantly functioning successfully for one year, we can consider giving recognition to your institute to function as Regional/Zonal office on our behalf.
The board can also take up the additional allied activities for betterment after our consultation.

The following entitlements are applicable to individual Fellows:

Open Association of Research Society, U.S.A (OARS) By-laws states that an individual Fellow may use the designations as applicable, or the corresponding initials. The Credentials of individual Fellow and Associate designations signify that the individual has gained knowledge of the fundamental concepts. One is magnanimous and proficient in an expertise course covering the professional code of conduct, and follows recognized standards of practice.

Open Association of Research Society (US)/ Global Journals Incorporation (USA), as described in Corporate Statements, are educational, research publishing and professional membership organizations. Achieving our individual Fellow or Associate status is based mainly on meeting stated educational research requirements.

Disbursement of 40% Royalty earned through Global Journals : Researcher = 50%, Peer Reviewer = 37.50%, Institution = 12.50% E.g. Out of 40%, the 20% benefit should be passed on to researcher, 15 % benefit towards remuneration should be given to a reviewer and remaining 5% is to be retained by the institution.

We shall provide print version of 12 issues of any three journals [as per your requirement] out of our 38 journals worth $ 2376 USD.

Other:

The individual Fellow and Associate designations accredited by Open Association of Research Society (US) credentials signify guarantees following achievements:

- The professional accredited with Fellow honor, is entitled to various benefits viz. name, fame, honor, regular flow of income, secured bright future, social status etc.

- In addition to above, if one is single author, then entitled to 40% discount on publishing research paper and can get 10%discount if one is co-author or main author among group of authors.
- The Fellow can organize symposium/seminar/conference on behalf of Global Journals Incorporation (USA) and he/she can also attend the same organized by other institutes on behalf of Global Journals.
- The Fellow can become member of Editorial Board Member after completing 3yrs.
- The Fellow can earn 60% of sales proceeds from the sale of reference/review books/literature/publishing of research paper.
- Fellow can also join as paid peer reviewer and earn 15% remuneration of author charges and can also get an opportunity to join as member of the Editorial Board of Global Journals Incorporation (USA)
- • This individual has learned the basic methods of applying those concepts and techniques to common challenging situations. This individual has further demonstrated an in–depth understanding of the application of suitable techniques to a particular area of research practice.

Note :

"

- In future, if the board feels the necessity to change any board member, the same can be done with the consent of the chairperson along with anyone board member without our approval.
- In case, the chairperson needs to be replaced then consent of 2/3rd board members are required and they are also required to jointly pass the resolution copy of which should be sent to us. In such case, it will be compulsory to obtain our approval before replacement.
- In case of "Difference of Opinion [if any]" among the Board members, our decision will be final and binding to everyone.

"

Process of submission of Research Paper

The Area or field of specialization may or may not be of any category as mentioned in 'Scope of Journal' menu of the GlobalJournals.org website. There are 37 Research Journal categorized with Six parental Journals GJCST, GJMR, GJRE, GJMBR, GJSFR, GJHSS. For Authors should prefer the mentioned categories. There are three widely used systems UDC, DDC and LCC. The details are available as 'Knowledge Abstract' at Home page. The major advantage of this coding is that, the research work will be exposed to and shared with all over the world as we are being abstracted and indexed worldwide.

The paper should be in proper format. The format can be downloaded from first page of 'Author Guideline' Menu. The Author is expected to follow the general rules as mentioned in this menu. The paper should be written in MS-Word Format (*.DOC,*.DOCX).

The Author can submit the paper either online or offline. The authors should prefer online submission.Online Submission: There are three ways to submit your paper:

(A) (I) First, register yourself using top right corner of Home page then Login. If you are already registered, then login using your username and password.

(II) Choose corresponding Journal.

(III) Click 'Submit Manuscript'. Fill required information and Upload the paper.

(B) If you are using Internet Explorer, then Direct Submission through Homepage is also available.

(C) If these two are not conveninet , and then email the paper directly to dean@globaljournals.org.

Offline Submission: Author can send the typed form of paper by Post. However, online submission should be preferred.

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Page Size: 8.27" X 11'"

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- Font type of all text should be Swis 721 Lt BT.
- Paper Title should be of Font Size 24 with one Column section.
- Author Name in Font Size of 11 with one column as of Title.
- Abstract Font size of 9 Bold, "Abstract" word in Italic Bold.
- Main Text: Font size 10 with justified two columns section
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- First Character must be three lines Drop capped.
- Paragraph before Spacing of 1 pt and After of 0 pt.
- Line Spacing of 1 pt
- Large Images must be in One Column
- Numbering of First Main Headings (Heading 1) must be in Roman Letters, Capital Letter, and Font Size of 10.
- Numbering of Second Main Headings (Heading 2) must be in Alphabets, Italic, and Font Size of 10.

You can use your own standard format also.
Author Guidelines:

1. General,

2. Ethical Guidelines,

3. Submission of Manuscripts,

4. Manuscript's Category,

5. Structure and Format of Manuscript,

6. After Acceptance.

1. GENERAL

Before submitting your research paper, one is advised to go through the details as mentioned in following heads. It will be beneficial, while peer reviewer justify your paper for publication.

Scope

The Global Journals Inc. (US) welcome the submission of original paper, review paper, survey article relevant to the all the streams of Philosophy and knowledge. The Global Journals Inc. (US) is parental platform for Global Journal of Computer Science and Technology, Researches in Engineering, Medical Research, Science Frontier Research, Human Social Science, Management, and Business organization. The choice of specific field can be done otherwise as following in Abstracting and Indexing Page on this Website. As the all Global

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The Global Journals Inc. (US) follows the definition of authorship set up by the Global Academy of Research and Development. According to the Global Academy of R&D authorship, criteria must be based on:

1) Substantial contributions to conception and acquisition of data, analysis and interpretation of the findings.

2) Drafting the paper and revising it critically regarding important academic content.

3) Final approval of the version of the paper to be published.

All authors should have been credited according to their appropriate contribution in research activity and preparing paper. Contributors who do not match the criteria as authors may be mentioned under Acknowledgement.

Acknowledgements: Contributors to the research other than authors credited should be mentioned under acknowledgement. The specifications of the source of funding for the research if appropriate can be included. Suppliers of resources may be mentioned along with address.

Appeal of Decision: The Editorial Board’s decision on publication of the paper is final and cannot be appealed elsewhere.

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Manuscripts should be uploaded via this online submission page. The online submission is most efficient method for submission of papers, as it enables rapid distribution of manuscripts and consequently speeds up the review procedure. It also enables authors to know the status of their own manuscripts by emailing us. Complete instructions for submitting a paper is available below.

Manuscript submission is a systematic procedure and little preparation is required beyond having all parts of your manuscript in a given format and a computer with an Internet connection and a Web browser. Full help and instructions are provided on-screen. As an author, you will be prompted for login and manuscript details as Field of Paper and then to upload your manuscript file(s) according to the instructions.

To avoid postal delays, all transaction is preferred by e-mail. A finished manuscript submission is confirmed by e-mail immediately and your paper enters the editorial process with no postal delays. When a conclusion is made about the publication of your paper by our Editorial Board, revisions can be submitted online with the same procedure, with an occasion to view and respond to all comments.

Complete support for both authors and co-author is provided.

4. MANUSCRIPT'S CATEGORY

Based on potential and nature, the manuscript can be categorized under the following heads:

Original research paper: Such papers are reports of high-level significant original research work.

Review papers: These are concise, significant but helpful and decisive topics for young researchers.

Research articles: These are handled with small investigation and applications

Research letters: The letters are small and concise comments on previously published matters.

5.STRUCTURE AND FORMAT OF MANUSCRIPT

The recommended size of original research paper is less than seven thousand words, review papers fewer than seven thousands words also.Preparation of research paper or how to write research paper, are major hurdle, while writing manuscript. The research articles and research letters should be fewer than three thousand words, the structure original research paper; sometime review paper should be as follows:

Papers: These are reports of significant research (typically less than 7000 words equivalent, including tables, figures, references), and comprise:

(a)Title should be relevant and commensurate with the theme of the paper.

(b) A brief Summary, "Abstract" (less than 150 words) containing the major results and conclusions.

(c) Up to ten keywords, that precisely identifies the paper's subject, purpose, and focus.

(d) An Introduction, giving necessary background excluding subheadings; objectives must be clearly declared.

(e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition; sources of information must be given and numerical methods must be specified by reference, unless non-standard.

(f) Results should be presented concisely, by well-designed tables and/or figures; the same data may not be used in both; suitable statistical data should be given. All data must be obtained with attention to numerical detail in the planning stage. As reproduced design has been recognized to be important to experiments for a considerable time, the Editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned un-refereed;

(g) Discussion should cover the implications and consequences, not just recapitulating the results; conclusions should be summarizing.

(h) Brief Acknowledgements.

(i) References in the proper form.

Authors should very cautiously consider the preparation of papers to ensure that they communicate efficiently. Papers are much more likely to be accepted, if they are cautiously designed and laid out, contain few or no errors, are summarizing, and be conventional to the approach and instructions. They will in addition, be published with much less delays than those that require much technical and editorial correction.

The Editorial Board reserves the right to make literary corrections and to make suggestions to improve briefness.

It is vital, that authors take care in submitting a manuscript that is written in simple language and adheres to published guidelines.

Format

Language: The language of publication is UK English. Authors, for whom English is a second language, must have their manuscript efficiently edited by an English-speaking person before submission to make sure that, the English is of high excellence. It is preferable, that manuscripts should be professionally edited.

Standard Usage, Abbreviations, and Units: Spelling and hyphenation should be conventional to The Concise Oxford English Dictionary. Statistics and measurements should at all times be given in figures, e.g. 16 min, except for when the number begins a sentence. When the number does not refer to a unit of measurement it should be spelt in full unless, it is 160 or greater.

Abbreviations supposed to be used carefully. The abbreviated name or expression is supposed to be cited in full at first usage, followed by the conventional abbreviation in parentheses.

Metric SI units are supposed to generally be used excluding where they conflict with current practice or are confusing. For illustration, 1.4 l rather than 1.4 × 10-3 m3, or 4 mm somewhat than 4 × 10-3 m. Chemical formula and solutions must identify the form used, e.g. anhydrous or hydrated, and the concentration must be in clearly defined units. Common species names should be followed by underlines at the first mention. For following use the generic name should be constricted to a single letter, if it is clear.

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All manuscripts submitted to Global Journals Inc. (US), ought to include:

Title: The title page must carry an instructive title that reflects the content, a running title (less than 45 characters together with spaces), names of the authors and co-authors, and the place(s) wherever the work was carried out. The full postal address in addition with the e-mail address of related author must be given. Up to eleven keywords or very brief phrases have to be given to help data retrieval, mining and indexing.

Abstract, used in Original Papers and Reviews:

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Many researchers searching for information online will use search engines such as Google, Yahoo or similar. By optimizing your paper for search engines, you will amplify the chance of someone finding it. This in turn will make it more likely to be viewed and/or cited in a further work. Global Journals Inc. (US) have compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

Key Words

A major linchpin in research work for the writing research paper is the keyword search, which one will employ to find both library and Internet resources.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy and planning a list of possible keywords and phrases to try.

Search engines for most searches, use Boolean searching, which is somewhat different from Internet searches. The Boolean search uses "operators," words (and, or, not, and near) that enable you to expand or narrow your affords. Tips for research paper while preparing research paper are very helpful guideline of research paper.

Choice of key words is first tool of tips to write research paper. Research paper writing is an art.A few tips for deciding as strategically as possible about keyword search:

- One should start brainstorming lists of possible keywords before even begin searching. Think about the most important concepts related to research work. Ask, "What words would a source have to include to be truly valuable in research paper?" Then consider synonyms for the important words.
- It may take the discovery of only one relevant paper to let steer in the right keyword direction because in most databases, the keywords under which a research paper is abstracted are listed with the paper.
- One should avoid outdated words.

Keywords are the key that opens a door to research work sources. Keyword searching is an art in which researcher's skills are bound to improve with experience and time.

Numerical Methods: Numerical methods used should be clear and, where appropriate, supported by references.

Acknowledgements: Please make these as concise as possible.

References

References follow the Harvard scheme of referencing. References in the text should cite the authors' names followed by the time of their publication, unless there are three or more authors when simply the first author's name is quoted followed by et al. unpublished work has to only be cited where necessary, and only in the text. Copies of references in press in other journals have to be supplied with submitted typescripts. It is necessary that all citations and references be carefully checked before submission, as mistakes or omissions will cause delays.

References to information on the World Wide Web can be given, but only if the information is available without charge to readers on an official site. Wikipedia and Similar websites are not allowed where anyone can change the information. Authors will be asked to make available electronic copies of the cited information for inclusion on the Global Journals Inc. (US) homepage at the judgment of the Editorial Board.

The Editorial Board and Global Journals Inc. (US) recommend that, citation of online-published papers and other material should be done via a DOI (digital object identifier). If an author cites anything, which does not have a DOI, they run the risk of the cited material not being noticeable.

The Editorial Board and Global Journals Inc. (US) recommend the use of a tool such as Reference Manager for reference management and formatting.

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Tables: Tables should be few in number, cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g. Table 4, a self-explanatory caption and be on a separate sheet. Vertical lines should not be used.

Figures: Figures are supposed to be submitted as separate files. Always take in a citation in the text for each figure using Arabic numbers, e.g. Fig. 4. Artwork must be submitted online in electronic form by e-mailing them.

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Even though low quality images are sufficient for review purposes, print publication requires high quality images to prevent the final product being blurred or fuzzy. Submit (or e-mail) EPS (line art) or TIFF (halftone/photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Do not use pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings) in relation to the imitation size. Please give the data for figures in black and white or submit a Color Work Agreement Form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

For scanned images, the scanning resolution (at final image size) ought to be as follows to ensure good reproduction: line art: >650 dpi; halftones (including gel photographs) : >350 dpi; figures containing both halftone and line images: >650 dpi.

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Upon approval of a paper for publication, the manuscript will be forwarded to the dean, who is responsible for the publication of the Global Journals Inc. (US).

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Proofs must be returned to the dean at dean@globaljournals.org within three days of receipt.

As changes to proofs are costly, we inquire that you only correct typesetting errors. All illustrations are retained by the publisher. Please note that the authors are responsible for all statements made in their work, including changes made by the copy editor.

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Please note that if not specifically requested, publisher will dispose off hardcopy & electronic information submitted, after the two months of publication. If you require the return of any information submitted, please inform the Editorial Board or dean as soon as possible.

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Before start writing a good quality Computer Science Research Paper, let us first understand what is Computer Science Research Paper? So, Computer Science Research Paper is the paper which is written by professionals or scientists who are associated to Computer Science and Information Technology, or doing research study in these areas. If you are novel to this field then you can consult about this field from your supervisor or guide.

TECHNIQUES FOR WRITING A GOOD QUALITY RESEARCH PAPER:

1. Choosing the topic: In most cases, the topic is searched by the interest of author but it can be also suggested by the guides. You can have several topics and then you can judge that in which topic or subject you are finding yourself most comfortable. This can be done by asking several questions to yourself, like Will I be able to carry our search in this area? Will I find all necessary recourses to accomplish the search? Will I be able to find all information in this field area? If the answer of these types of questions will be "Yes" then you can choose that topic. In most of the cases, you may have to conduct the surveys and have to visit several places because this field is related to Computer Science and Information Technology. Also, you may have to do a lot of work to find all rise and falls regarding the various data of that subject. Sometimes, detailed information plays a vital role, instead of short information.

2. Evaluators are human: First thing to remember that evaluators are also human being. They are not only meant for rejecting a paper. They are here to evaluate your paper. So, present your Best.

3. Think Like Evaluators: If you are in a confusion or getting demotivated that your paper will be accepted by evaluators or not, then think and try to evaluate your paper like an Evaluator. Try to understand that what an evaluator wants in your research paper and automatically you will have your answer.

4. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

5. Ask your Guides: If you are having any difficulty in your research, then do not hesitate to share your difficulty to your guide (if you have any). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work then ask the supervisor to help you with the alternative. He might also provide you the list of essential readings.

6. Use of computer is recommended: As you are doing research in the field of Computer Science, then this point is quite obvious.

7. Use right software: Always use good quality software packages. If you are not capable to judge good software then you can lose quality of your paper unknowingly. There are various software programs available to help you, which you can get through Internet.

8. Use the Internet for help: An excellent start for your paper can be by using the Google. It is an excellent search engine, where you can have your doubts resolved. You may also read some answers for the frequent question how to write my research paper or find model research paper. From the internet library you can download books. If you have all required books make important reading selecting and analyzing the specified information. Then put together research paper sketch out.

9. Use and get big pictures: Always use encyclopedias, Wikipedia to get pictures so that you can go into the depth.

10. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right! It is a good habit, which helps to not to lose your continuity. You should always use bookmarks while searching on Internet also, which will make your search easier.

11. Revise what you wrote: When you write anything, always read it, summarize it and then finalize it.

12. Make all efforts: Make all efforts to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in introduction, that what is the need of a particular research paper. Polish your work by good skill of writing and always give an evaluator, what he wants.

13. Have backups: When you are going to do any important thing like making research paper, you should always have backup copies of it either in your computer or in paper. This will help you to not to lose any of your important.

14. Produce good diagrams of your own: Always try to include good charts or diagrams in your paper to improve quality. Using several and unnecessary diagrams will degrade the quality of your paper by creating "hotchpotch." So always, try to make and include those diagrams, which are made by your own to improve readability and understandability of your paper.

15. Use of direct quotes: When you do research relevant to literature, history or current affairs then use of quotes become essential but if study is relevant to science then use of quotes is not preferable.

16. Use proper verb tense: Use proper verb tenses in your paper. Use past tense, to present those events that happened. Use present tense to indicate events that are going on. Use future tense to indicate future happening events. Use of improper and wrong tenses will confuse the evaluator. Avoid the sentences that are incomplete.

17. Never use online paper: If you are getting any paper on Internet, then never use it as your research paper because it might be possible that evaluator has already seen it or maybe it is outdated version.

18. Pick a good study spot: To do your research studies always try to pick a spot, which is quiet. Every spot is not for studies. Spot that suits you choose it and proceed further.

19. Know what you know: Always try to know, what you know by making objectives. Else, you will be confused and cannot achieve your target.

20. Use good quality grammar: Always use a good quality grammar and use words that will throw positive impact on evaluator. Use of good quality grammar does not mean to use tough words, that for each word the evaluator has to go through dictionary. Do not start sentence with a conjunction. Do not fragment sentences. Eliminate one-word sentences. Ignore passive voice. Do not ever use a big word when a diminutive one would suffice. Verbs have to be in agreement with their subjects. Prepositions are not expressions to finish sentences with. It is incorrect to ever divide an infinitive. Avoid clichés like the disease. Also, always shun irritating alliteration. Use language that is simple and straight forward. put together a neat summary.

21. Arrangement of information: Each section of the main body should start with an opening sentence and there should be a changeover at the end of the section. Give only valid and powerful arguments to your topic. You may also maintain your arguments with records.

22. Never start in last minute: Always start at right time and give enough time to research work. Leaving everything to the last minute will degrade your paper and spoil your work.

23. Multitasking in research is not good: Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. Think technically: Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page
- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.

- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

The Administration Rules

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- Do not give permission to anyone else to "PROOFREAD" your manuscript.
- Methods to avoid Plagiarism is applied by us on every paper, if found guilty, you will be blacklisted by all of our collaborated research groups, your institution will be informed for this and strict legal actions will be taken immediately.)
- To guard yourself and others from possible illegal use please do not permit anyone right to use to your paper and files.

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Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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