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Highlights

Field Theory of Gravitation

Barotropic Cosmological Models

Discovering Thoughts, Inventing Future

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Structure of Regular Semigroups

By P. Sreenivasulu Reddy & Mulugeta Dawud
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Abstract- This paper concerned with basic concepts and some results on (idempotent) semigroup satisfying the identities of three variables. The motivation of taking three for the number of variables has come from the fact that many important identities on idempotent semigroups are written by three or fewer independent variables. We consider the semigroup satisfying the property abc = ac and prove that it is left semi-normal and right quasi-normal. Again an idempotent semigroup with an identity aba = ab and aba = ba (ab = a, ab = b) is always a semilattices and normal. An idempotent semigroup is normal if and only if it is both left quasi-normal and right quasi-normal. If a semigroup is rectangular then it is left and right semi-regular.

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STRUCTUREOFREGULARSEMIGROUPS

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Notes

Structure of Regular Semigroups

P. Sreenivasulu Reddy [α] & Mulugeta Dawud [σ]

Abstract- This paper concerned with basic concepts and some results on (idempotent) semigroup satisfying the identities of three variables. The motivation of taking three for the number of variables has come from the fact that many important identities on idempotent semigroups are written by three or fewer independent variables. We consider the semigroup satisfying the property abc = ac and prove that it is left semi-normal and right quasi-normal. Again an idempotent semigroup with an identity aba = ab and aba = ba (ab = a, ab = b) is always a semilattices and normal. An idempotent semigroup is normal if and only if it is both left quasi-normal and right quasi-normal. If a semigroup is rectangular then it is left and right semi-regular.

I. Preliminaries and Basic Properties of Regular Semigroups

In this section we present some basic concepts of semigroups and other definitions needed for the study of this chapter and the subsequent chapters.

- *Definition:* A semigroup (S, .) is said to be left(right) singular if it satisfies the identity ab = a (ab = b) for all a,b in S
- *Definition:* A semigroup (S, .) is rectangular if it satisfies the identity aba = a for all a, b in S.
- *Definition:* A semigroup (S, .) is called left(right) regular if it satisfies the identity aba = ab (aba = ba) for all a,b in S.
- *Definition:* A semigroup (S, .) is called regular if it satisfies the identity abca = abaca for all a,b,c in S
- *Definition:* A semigroup (S, .) is said to be total if every element of S can be written as the product of two elements of S. i.e, $S^2 = S$.
- *Definition:* A semigroup (S, .) is said to be left(right) normal if abc = acb (abc = bac) for all a,b,c in S.
- *Definition:* A semigroup (S, .) is said to be normal if satisfies the identity abca = acba for all a,b,c in S.
- *Definition:* A semigroup (S, .) is said to be left(right) quasi-normal if it satisfies the identity abc = acbc (abc = abac) for all a,b,c in S.
- *Definition:* A semigroup (S, .) is said to be left (right) semi-normal if it satisfies the identity abca = acbca (abca =abcba) for all a,b,c in S.
- *Definition:* A semigroup (S, .) is said to be left(right) semi-regular if it satisfies the identity abca =abacabca (abca = abcabaca) if for all a,b,c in S.
- *Result:* [11,12] A left (right) singular semigroup is rectangular.
- *Theorem:* Every left(right) singular semigroup is total.

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Proof: Let (S, .) be a left(right) singular semigroup.

Then $ab = a$ for any a,b in S

To prove that S is total we have to prove $S^2 = S$

We know that $S \subseteq S^2$

To prove $S^2 \subseteq S$

Let $x \in S^2 \Rightarrow x = a.b$ for any a,b in S

$x = a$

$x \in S$

$S^2 \subseteq S$

$\therefore S = S^2$

Hence (S, .) is total.

- *Note:* From the result 2.1.11 every left(right) singular semigroup is rectangular and again from theorem 2.1.12 it is total. Hence every rectangular semigroup is total.
- *Theorem:* A semigroup satisfying the singular properties is always a semilattice.
- *Theorem:* Let (S, .) be a semigroup. If S is left and right regular then S is a semilattice.

Proof: Let (S, .) be a semigroup with left regular and right regular then

$aba = ab$ (left regular)

$aba = ba$ (right regular) for all a,b in S.

From the above we have $ab = ba$

$\therefore$ (S, .) is a commutative.

To prove that (S, .) is band, let $aba = ab \Rightarrow a(ba) = ab \Rightarrow abab = ab$ ($ba = bab$)

$\Rightarrow (ab)(ab) = ab \Rightarrow (ab)^2 = ab$ put $a = b \Rightarrow (a.a)^2 = a.a \Rightarrow (a.a)^2 = a^2$ $a.a = a$

$\therefore$ (S, .) is a band

So, if S is left and right regular then (S, .) is commutative band

Hence (S, .) is a semilattice.

- *Lemma:* A left (right) regular semigroup is regular.
- *Note:* Every regular semigroup need not be a left (right) regular.
- *Lemma:* A right (left) singular semigroup is regular.
- *Lemma:* A left (right) normal semigroup is normal.
- *Theorem:* Let (S, .) be a semigroup. If S is both left and right regular then S is normal.
- *Lemma:* A left (right) regular semigroup is right (left) quasi-normal.
- *Lemma:* A left (right) regular semigroup is right (left) semi-normal.
- *Theorem:* A left (right) regular semigroup is left (right) semi-regular
- *Theorem:* If an idempotent semigroup satisfies the right (left) quasi-normal then it is right(left) semi-regular.
- *Lemma:* A semigroup (S, .) with left (right) quasi-normal is right (left) semi-normal.
- *Theorem:* If a semigroup (S, .) is rectangular then (S, .)is right (left) semi-regular.
- *Note:* Similarlly, we prove that,

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1. A semigroup(S, .) is regular then (S, .) is left (right) semi-regular.
2. A semigroup (S, .) is satisfies the left (right) semi-normal property is right (left) semi-regular.

- *Theorem:* An idempotent semigroup (S, .) is normal if and only if it is both left and right quasi-normal.

Proof: Let (S, .) be an idempotent semigroup.

Assume that (S, .) is both left and right quasi-normal

then abc = acbc (leftquasi-normal)

abc = abac (right quasi-normal)

To prove that (S, .) is normal. Since S is right quasi-normal, we have

abc = abac ⇒abc= a(bac) ⇒ abc=abcac (left quasi-normal)

⇒ abc=(abc)ac ⇒ abc=acbcac (left quasi normal)

⇒ abc=ac(bcac) ⇒ abc = acbac (left quasi normal)

⇒ abca = acbaca⇒abc= ac(baca) ⇒abc = acbca ⇒abc = a(cbca) ⇒ abc = acba

⇒ abca = acba.

Hence (S, .) is normal.

Conversely; let (S, .) be normal then abca = acba

we show that (S, .) is both left quasi-normal and right quasi-normal.

Consider abc = abc.c ⇒abc = a(bc)c⇒abc= acbc ⇒ abc = acbc.

(S, .) is left quasi-normal.

Symilarlly, abc = a.abc ⇒abc = a(ab)c⇒abc = abac ⇒ abc = abac.

Therefore (S, .) is right quasi-normal.

II. SEMIGROUP SATISFYING THE IDENTITY ABC = AC.

In this section we discuss if a semigroup S satisfying the identity abc = ac for any three variables a,b,c∈S, then the following conditions are equivalent to one another.

a) left semi-normal
b) left semi-regular
c) right semi-normal
d) right semi-regular
e) regular
f) normal
g) left quasi-normal
h) right quasi-normal

- *Theorem:* A semigroup S with an identity abc = ac, for any a,b,c∈S is left (right) semi-normal if and only if it is left (right) semi-regular.

Proof: Let S be a semigroup with an identity abc = ac for any a,b,c∈S.

Assume that S be a left semi-normal.

Now we show that S is left semi-regular

Since S is left semi-normal we have abca = acbca ⇒abca = (ac)bca

⇒abca= abcbca (ac = abc) ⇒abca = a(bc)bca ⇒abca = abacbca (bc = bac)

⇒abca = aba(cb)ca ⇒abca = abacabca (cb = cab) ⇒ abca = abacabca.

∴S is left semi-regular.

Conversely, let S be left semi regular then abca = abacabca

⇒abca = a(bac)abca ⇒abca = abcabca (bac = bc) ⇒abca = (abc)abca

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$\Rightarrow$abca = acabca (abc = ac) $\Rightarrow$abca = a(cab)ca $\Rightarrow$abca = acbca (cab = cb) abca = acbca.

Hence S is left semi-normal.

- *Theorem:* An idempotent semigroup S with an identity abc = ac for any a,b,c in S is left (right) semi-regular if and only if it is regular.

Proof: Let S be an idempotent semigroup with an identity abc = ac for any a,b,c in S. Assume that S be a regular semigroup then abca = abaca

$\Rightarrow$ abca = ab(ac)a $\Rightarrow$abca = ababca (ac = abc) $\Rightarrow$abca = a(ba)bca $\Rightarrow$abca = abcabca (ba = bca)

$\Rightarrow$abca = a(bc)abca $\Rightarrow$abca = abacabca (bc = bac) $\Rightarrow$ abca = abacabca

Hence S is left semi-regular.

Conversely, let S be left semi-regular then abca = abacabca$\Rightarrow$abca = abac(abc)a

$\Rightarrow$abca = abacaca (abc = ac) $\Rightarrow$abca = abac(aca) $\Rightarrow$abca=abaca.a (aca = a.a)

$\Rightarrow$abca = abaca (a.a = a) $\Rightarrow$ abca = abaca

Hence S is regular

- *Theorem:* A semigroup S with an identity abc = ac for any a,b,c in S is left (right) semi-regular if and only if it is normal.

Proof: Let S be a semigroup with an identity abc = ac for any a,b,c∊S and assume that S be normal then abca = acba $\Rightarrow$abca = (ac)ba $\Rightarrow$abca = abcba (ac=abc) $\Rightarrow$abca = abc(ba)

$\Rightarrow$abca = abcbca (ba=bca) $\Rightarrow$abca = a(bc)bca $\Rightarrow$abca = abacbca (bc = bac)

$\Rightarrow$abca = aba(cb)ca $\Rightarrow$ abca = abacabca (cb = cab) $\Rightarrow$abca = abacabca

S is left semi-regular

Conversely, assume that S is left semi-regular then abca = a(bac)abca

$\Rightarrow$abca = abcabca (bac=bc) $\Rightarrow$abca =(abc)abca $\Rightarrow$abca = acabca (abc = ac)

$\Rightarrow$abca = a(cab)ca $\Rightarrow$ abca = acbca (cab=cb) $\Rightarrow$abca = ac(bca) $\Rightarrow$abca = acba (bca = ba)

$\Rightarrow$ abca = acba

Hence S is normal.

- *Theorem:* A semigroup S with an identity abc = ac where a,b,c in S is regular if and only if it is left (right) semi-normal

Proof: Let S be a semigroup with an identity abc = ac for all a,b,c in S.

Assume that S is regular then abca= (ab)aca $\Rightarrow$abca = acbaca (ab=acb) $\Rightarrow$abca = ac(bac)a

$\Rightarrow$abca =acbc a (bac=bc) $\Rightarrow$ abca = acbca

S is left semi-normal

Conversely, let S be left semi-normal. Then abca = (ac)bca $\Rightarrow$abca = abcbca (ac = abc) $\Rightarrow$abca = a(bc)bca $\Rightarrow$abca = abacbca (bc= bac) $\Rightarrow$abca = abac(bca) $\Rightarrow$abca = abacba (bca=ba)

$\Rightarrow$abca = aba(cba) $\Rightarrow$ abca = abaca (cba = ca)

Hence S is regular.

- *Lemma:* A semigroup S satisfying the property abc = ac for any a,b,c ∊ S is left (right) semi-regular if and only if it is right(left) semi-regular.

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- *Lemma:* A semigroup S with an identity abc = ac is left(right) semi-normal if and only if it is right (left) semi-normal
- *Note:* A semigroup S with an identity abc = ac where a,b,c∈S is left(right) quasi-normal if and only if it is right (left) quasi-normal.
- *Lemma:* A semigroup S with the property abc = ac for all a,b,c∈S is normal if and only if it is left(right) semi-normal
- *Theorem:* A semigroup with an identity abc = ac, for any a,b,c∈ S is normal if and only if it is left(right) quasi-normal.

Proof: Let S be a semigroup with an identity abc = ac for any a,b,c ∈S

Let S be a left quasi-normal. Then, abc = acbc ⇒ abca = acbca ⇒abca = ac(bca)

⇒abca = acba (bca = ba) ⇒ abca = acba

S is normal.

Conversely, let S be normal. Then, abca = acba ⇒ abcac = acbac

⇒ a(bca)c = acbac ⇒ abac = acbac (bca = ba) ⇒ a(bac) = acbac

⇒ abc = acbac (bac = bc) ⇒ abc = ac(bac) ⇒ abc = acbc.

∴S is a left quasi-normal.

Similarlly, we can prove that a semigroup S with an identity abc = ac for any a,b,c∈ S is regular if and only if it is left (right) quasi -normal.

- *Theorem:* A semigroup S with an identity abc = ac for all a,b,c ∈S is regular if and only if it is normal.

Proof: Let s be a semigroup with an identity abc = ac for any a,b,c∈S.

Let S be a normal semigroup then abca = acba ⇒abca = (ac)ba

⇒abca = abcba (ac = abc) ⇒abca = abc(ba) ⇒abca = abcbca (ba = bca)

⇒abca = a(bc)bca ⇒abca = abacbca ⇒abca = abac(bca) ⇒abca = abacba (bca = ba)

⇒abca = aba(cba) ⇒abca = abaca (cba = ca) ⇒ abca = abaca.

Hence S is a regular.

Conversely, let S be a regular semigroup then abca = abaca ⇒abca = (ab)aca

⇒abca = acbaca (ab = acb) ⇒abca = ac(bac)a ⇒abca = acbca (bac = bc)

⇒abca = ac(bca) ⇒abca = acba (bca = ba) ⇒ abca = acba.

∴S is normal.

III. Semigroup Satisfies the Identity ab = a (ab = b)

we present some results on semigroup with an identity ab = a (ab = b) for all a,b in a semigroup S. We prove that the necessary and sufficient conditions for a semigroup S to be regular, normal, left (right) normal, left (right) semi-normal, right(left) semi-regular, left (right) regular, left (right) quasi-normal.

- *Theorem:* A semigroup S with an identity ab = a for any a,b∈S is normal if and only if it is regular.

Proof: Let S be a semigroup satisfying the identity ab = a for all a,b∈S.

Assume that S is normal. Then abca = (a)cba ⇒abca = abcba (a = ab)

⇒abca = a(b)cba ⇒abca = abacba (b = ba) ⇒abca = aba(cb)a ⇒abca = abaca (cb = c)

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$\Rightarrow$ abca = abaca.

Therefore S is regular.

Conversely, let S be a regular semigroup then abca = abaca $\Rightarrow$abca = (ab)aca

$\Rightarrow$abca = aaca (ab = a) $\Rightarrow$abca = (aa)ca $\Rightarrow$abca = aca (aa = a) $\Rightarrow$abca = a(c)a

$\Rightarrow$abca = acba (c = cb) $\Rightarrow$ abca = acba.

$\therefore$S is normal.

- *Theorem:* A semigroup S satisfying the identity ab = a for any a,b in S is left (right) regular if and only if it is regular.
- *Theorem:* A semigroup S with an identity ab = a for all a,b in S is left(right) semi-normal if and only if it is right(left) semi-normal.
- *Theorem:* A semigroup S with an identity ab = a for any a,b in S is left (right) semi-normal if and only if it is regular.

Proof: Let S be a semigroup with an identity ab = a for any a,b$\in$S.

Let S be left semi-normal then abca = acbca $\Rightarrow$abca = ac(bc)a $\Rightarrow$abca = acba (bc = b)

$\Rightarrow$abca = (a)cba $\Rightarrow$abca = abcba (a = ab) $\Rightarrow$abca = a(b)cba $\Rightarrow$abca = abacba (b = ba)

$\Rightarrow$abca = aba(cb) a $\Rightarrow$abca = abaca (cb = c) $\Rightarrow$ abca = abaca

Hence S is regular.

Conversely, let S be regular then abca = abaca $\Rightarrow$abca = (ab)aca $\Rightarrow$abca = aaca (ab = a)

$\Rightarrow$abca = (aa)ca (aa = a) $\Rightarrow$abca = aca $\Rightarrow$abca = a(c)a $\Rightarrow$abca = acba (c = cb)

$\Rightarrow$abca = ac(b)a

$\Rightarrow$abca = acbca (b = bc) $\Rightarrow$ abca = acbca

$\therefore$S is left semi-normal.

- *Lemma:* A semigroup S with an identity ab = a for any a,b in S is left(right) semi-normal if and only if it is normal.
- *Theorem:* Let S be a semigroup and assume that S satisfies the identity ab = a then S is left(right) normal if and only if it is normal
- *Theorem:* A semigroup S satisfying the identity ab = a for any a,b in S is left(right) semi-regular if and only if it is right(left) semi regular
- *Theorem:* A semigroup S with an identity ab = a, for any a,b$\in$S is left(right) semi-regular if and only if it is normal.

Proof: Let S be a semigroup with an identity ab = a for any a,b in S

Assme that S be left semi-regular then abca = abacabca $\Rightarrow$abca = (ab)acabca

$\Rightarrow$abca = aacabca (ab = a) $\Rightarrow$abca = (aa)cabca $\Rightarrow$abca = acabca (aa = a) $\Rightarrow$abca = a(ca)bca

$\Rightarrow$abca = acbca (ca = c) $\Rightarrow$abca = ac(bc)a $\Rightarrow$abca = acba (bc = b) $\Rightarrow$ abca = acba.

$\therefore$S is normal.

Conversely, let S be normal then abca = acba $\Rightarrow$abca = (a)cba $\Rightarrow$abca = abcba (a = ab)

$\Rightarrow$abca = a(b)cba $\Rightarrow$abca = abacba (b = ba) $\Rightarrow$abca = aba(c)ba $\Rightarrow$abca = abacaba (c = ca)

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⇒abca = abaca(b)a ⇒abca = abacabca (b = bc) ⇒ abca = abacabca.
Therefore S is left semi-regular.

- *Theorem:* A semigroup S with an identity ab = a for all a,b in S is left(right) semi-regular if and only if it is left(right) semi-normal

Proof: Let S be a semigroup with the identity ab = a for any a,b∈S.
Assume that S be a left semi-regular semigroup then, we have abca = abacabca

⇒abca = (ab)acabca ⇒abca = aacabca (a = ab) ⇒abca = (aa)cabca ⇒abca = acabca (aa = a)

⇒abca = a(ca)bca ⇒abca = acbca (ca = c) ⇒ abca = acbca.
Hence S is left semi-normal.

Conversely, let S be left semi-normal then abca = acbca ⇒abca = (a)cbca

⇒abca = abcbca (a = ab) ⇒abca = a(b)cbca ⇒abca = abacbca (b = ba) ⇒abca = aba(c)bca

⇒abca = abacabca (c = ca) ⇒ abca = abacabca.
Hence S is left semi-regular.

- *Theorem:* Let S be a semigroup and assume that S satisfy the left singular property then S is left(right) quasi-normal if and only if it is normal
- *Theorem:* A semigroup S with an identity ab = a for any a,b in S is left (right) quasi-normal if and only if it is left(right) semi-regular.

Proof: Let S be a semigroup and it satisfies the identity ab = a for all a,b ∈S

Assume that S be left quasi-normal then abc = acbc ⇒ ab(c) = acb(c)

⇒ abca = acbca (c = ca) ⇒ abca = (a)cbca ⇒abca = abcbca (a = ab) ⇒abca = a(b)cbca

⇒abca = abacbca (b = ba) ⇒abca = aba(c)bca ⇒abca = abacabca (c = ca)

⇒abca = abacabca

∴S is left semi-regular.

Conversely, let S be left semi-regular then abca = abacabca ⇒ ab(ca) = abacab(ca)

⇒ abc = abacabc (ca = a) ⇒abc = (ab)acabc⇒abc = aacabc (ab = a) ⇒abc = (aa)cabc

⇒abc = acabc (aa = a) ⇒abc = a(ca)bc ⇒abc = acbc ⇒ abc = acbc.
Hence S is left quasi-normal.

- *Note:* Similarly, we can prove that,

a) a semigroup S with an identity ab = a for any a,b, in S is left(right) semi-regular if and only if it is right(left) semi-normal.

b) a semigroup S satisfies the identity ab = a , for any a,b∈S is left(right) quasi-normal if and only if it is any one of the following:

(1) Regular.

(2) Left(right semi-normal.

(3) Left(right) semi-regular.

(4) Left(right) regular.

(5) Left(right) normal.

Notes

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References Références Referencias

1. A. H. Clifford and G. B. Preston :"The algebraic theory of semigroups" Math.surveys7; vol .I Amer.math. soc 1961.
2. David. McLean "idempotent semigroups" Amer.math.monthly; 61;110-113.
3. J. M. Howie " An introduction to semigroup theory" Academic Press (1976).
4. Miyuki Yamada and Naoki Kimura " Note on idempotent semigroups.II" Proc.Japan Acad. 34;110 (1958).
5. Naoki. Kimura "The structure of idempotent semigroups(1) Proc.Japan.Acad., 33,(1957) P.642.
6. Naoki. Kimura "Note on idempotent semigroups I".Proc. Japan Acad.,33,642 (1957).
7. Naoki Kimura "Note on idempotent semigroups III". Proc.Japan Acad.;34;113 (1958).
8. Naoki Kimura "Note on idempotent semigroups IV identities of three variables". Proc.Japan Acad.,34,121-123(1958).

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Linear Elliptic Systems with Nonlinear Boundary Conditions without Landesman-Lazer Conditions

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Abstract- The boundary value problem is examined for the system of elliptic equations of from $-\Delta u+A(x)u=$ in Ω , where $A(x)$ is positive semidefinite matrix on $\mathbb{R}^{k\times k}$, and $\frac{\partial u}{\partial \nu}+g(u)=h(x)$ on $\partial\Omega$. It is assumed that $g\in C(\mathbb{R}^k,\mathbb{R}^k)$ is a bounded function which may vanish at infinity. The proofs are based on Leray-Schauder degree methods.

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Linear Elliptic Systems with Nonlinear Boundary Conditions without Landesman-Lazer Conditions

Alzaki Fadlallah

Abstract- The boundary value problem is examined for the system of elliptic equations of from $-\Delta u + A(x)u =$ in Ω, where $A(x)$ is positive semidef inite matrix on $\mathbb{R}^{k\times k}$ and $\frac{\partial u}{\partial \nu}+g(u)=h(x)$ on $\partial\Omega$. It is assumed that $g \in C(\mathbb{R}^k, \mathbb{R}^k)$ is a bounded function which may vanish at infinity. The proofs are based on Leray-Schauder degree methods.

I. Introduction

Let $\mathbb{R}^k$ be real $k-$dimensional space, if $w \in \mathbb{R}^k$, then $|w|_E$ denotes the Euclidean norm of w. Let $\Omega \subset \mathbb{R}^N$, $N \geq 2$ is a bounded domain with boundary $\partial\Omega$ of class C^∞. Let $g \in C^1(\mathbb{R}^k, \mathbb{R}^k)$, $h \in C(\partial\Omega, \mathbb{R}^k)$, and the matrix

$$A(x) = \begin{bmatrix} a_{11}(x) & a_{12}(x) & \cdots & a_{1k}(x) \\ a_{21}(x) & a_{22}(x) & \cdots & a_{2k}(x) \\ \vdots & \vdots & \ddots & \vdots \\ a_{k1}(x) & a_{k2}(x) & \cdots & a_{kk}(x) \end{bmatrix}.$$

Verifies the following conditions:

(A1) The functions $a_{ij} : \Omega \to \mathbb{R}$, $\forall\, i,\ j \in \{1, \cdots, k\}$.

(A2) $A(x)$ is positive semidefinite matrix on $\mathbb{R}^{k\times k}$, almost everywhere $x \in \Omega$, and $A(x)$ is positive definite on a set of positive measure with $a_{ij} \in L^p(\Omega)\ \forall\, i,\ j \in \{1, \cdots, k\}$ for $p > \frac{N}{2}$ when $N \geq 3$, and $p > 1$ when $N = 2$.

We will study the solvability of

$$\begin{aligned} -\Delta u + A(x)u &= 0 \quad \text{in } \Omega, \\ \frac{\partial u}{\partial \nu} + g(u) &= h(x) \quad \text{on } \partial\Omega. \end{aligned} \tag{1.1}$$

The interest in this problem is the resonance case at the boundary with a bounded nonlinearity, we will assume that g a bounded function, and there is a constant $R > 0$ such that

$$|g(w(x))|_E \leq R \quad \forall\, w \in \mathbb{R}^k \;\&\; x \in \partial\Omega. \tag{1.2}$$

Our assumptions allow that g is not only bounded, but also may be vanish at infinity i.e.;

$$\lim_{|w|_E\to\infty} g(w) = 0 \in \mathbb{R}^k. \tag{1.3}$$

Condition (1.3) is not required by our assumptions, but allowing for it is the main result of this paper.

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In case of the scalar equation $i.e.$; $k = 1$ and g doesn't satisfy condition (1.3) but satisfying the Landesman-Lazer condition

$$g_- < \bar{h} < g^+$$

where $\lim_{w\to-\infty} g(w) = g_-$, $\bar{h} = \frac{1}{|\partial\Omega|}\int_{\partial\Omega} h\,dx$, $\lim_{w\to\infty} g(w) = g^+$, and $A(x) = 0 \in \mathbb{R}^{k\times k}$. Then it is well know that there is a solution for (1.1). The first results when the nonlinearity in the equation in scalar case was done by Landesman and Lazaer [1] in 1970. Their work led to great interest and activity on boundary value problems at resonance which continuous to this day. A particularly interesting extension of Landesman and Lazer's work to systems was done by Nirenberg [2], [3] in case of system and the nonlinearity in the equation was done by Ortega and Ward [4], in the scalar case without Landesman-Lazer condition was done by Iannacci and Nkashama [5], Ortega and Sánchez [6], more completely the case for periodic solutions of the system of ordinary differential equations with bounded nonlinear g satisfying Nirenberg's condition. They studied periodic so solutions

$$u'' + cu' + g(u) = p(t),$$

for $u \in \mathbb{R}^k$.

In case $c = 0$ was done by Mawhin [7]. In case the nonlinear terms vanish at infinity, as in (1.3), the Landesman-Lazer conditions fail. We would like to know what we can do in this case, and what conditions on a bounded nonlinearity that vanishes at infinity might replace that ones of the Landesman-Lazer type. Several authors have considered the case when the nonlinearity $g\colon \partial\Omega\times\mathbb{R}\to\mathbb{R}$ is a scalar function satisfies Carathéodory conditions i,e.;

- **i:** $g(.,u)$ is measurable on $\partial\Omega$, for each $u \in \mathbb{R}$,
- **ii:** $g(x,.)$ is continuous on $\mathbb{R}$, for $a.e.x \in \partial\Omega$,
- **iii:** for any constant $r > 0$, there exists a function $\gamma_r \in L^2(\partial\Omega)$, such that

$$|g(x,u)| \le \gamma_r(x), \tag{1.4}$$

for $a.e.x \in \Omega$, and all $u \in \mathbb{R}$ with $|u| \le r$,

was done by Fadlallah [8] and the others have considered the case when the nonlinearity does not decay to zero very rapidly. For example in case the nonlinearity in the equation if $g = g(t)$ is a scalar function, the condition

$$\lim_{|t|\to\infty} tg(t) > 0. \tag{1.5}$$

and related ones were assumed in [9], [10], [11], [12], [13], [14], [15], [16], [17]. These papers all considered scalar problem, but also considered the Dirichlet (Neumann) problem at resonance (non-resonance) at higher eigenvalues (Steklov-eigenproblems). The work in some of these papers makes use of Leray-Schauder degree arguments, and the others using critical point theory both the growth restrictions like (1.5) and Lipschitz conditions have been removed (see [15], [17]). In this paper we study systems of elliptic boundary value problems with nonlinear boundary conditions Neumann type and the nonlinearities at boundary vanishing at the infinity. We do not require the problem to be in variational from.

Let S^{k-1} be the unit sphere in $\mathbb{R}^k$. We will assume that $S^{k-1}\cap\partial\Omega \neq \emptyset$ and Let $\mathbb{S} = S^{k-1}\cap\partial\Omega$.

1.1. *Assumptions*

- $G1$**:** $g \in C^1(\mathbb{R}^k,\mathbb{R}^k)$ and g is bounded with $g(w) \neq 0$ for $|w|_E$ large.
- $G2$**:** For each $z \in \mathbb{S}$ the $\lim_{r\to\infty} \dfrac{g(rz)}{|g(rz)|_E} = \varphi(z)$ exists, and the limits is uniform for $z \in \mathbb{S}$. It follows that $\varphi \in C(\mathbb{S},\mathbb{S})$ and the topological degree of φ is defined.
- $G3$**:** $deg(\varphi) \neq 0$

Ref

1. E. Landesman and A. Lazer: *Nonlinear perturbations of linear elliptic boundary value problems at resonance, J. Math. Mech. 19 (1970), 609-623.*

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1.2. *Notations*

- Let $\langle .,. \rangle_{L^2}$ denote the inner product in $L^2 := L^2(\Omega, \mathbb{R}^k)$ where L^2 is Lebesgue space
- Let $\langle .,. \rangle_E$ denote the standard inner product in $\mathbb{R}^k$
- Assume that $((A1)$-$(A2))$ holds, then define

$$E(u,v) := \sum_{i=1}^{k} \langle \bigtriangledown u_i, \bigtriangledown v_i \rangle_{L^2} + \langle a_{ij}(x) u_i, v_i \rangle_{L^2}, \ j = 1, \ldots, k,$$

for $u, v \in H^1 = H^1(\Omega, \mathbb{R}^k)$ where H^1 the Sobolev space.

We note that it follows from the assumptions $G1:$ - $G3:$ that on large balls

$$B(R) := \{y : |y|_E \leq R\},$$

the $deg(g, B(R), 0) \neq 0$ see [18],[19].

We modify the Lemma 1 and Theorem 1 in [4] to fit our problem.

Lemma 1.1. Assume that $G1:$ and $G2:$ hold and $C > 0$ is a given constant. Then there exists $R > 0$ such that

$$\int_{\partial\Omega} g(u(x))\, dx \neq 0,$$

for each function $u \in C(\partial\Omega, \mathbb{R}^k)$ (we can write $u = \bar{u} + \tilde{u}$ where $\bar{u} = \int_{\partial\Omega} u(x) dx = 0$, and $\bar{u} \perp \tilde{u}$) with $|\bar{u}|_E \geq R$ and $||u - \bar{u}||_{L^\infty(\partial\Omega)} \leq C$

Proof. By the way of contradiction. Assume that for some $C > 0$ there is exist a sequence of functions $\{u_n\}_{n=1}^{\infty} \in C(\bar{\Omega}, \mathbb{R}^k)$, with

$$|\bar{u}_n|_E \to \infty, \ ||u_n - \bar{u}_n||_{L^\infty(\partial\Omega)} \leq C$$

and

$$\int_{\partial\Omega} g(u_n(x))\, dx = 0. \tag{1.6}$$

We constructed a subsequence of u_n one can assume that $\bar{z}_n = \frac{\bar{u}_n}{|\bar{u}_n|_E}$ converges to some point $z \in \mathbb{S}$. The uniform bound on $u_n - \bar{u}_n$ implies that also $\frac{u_n}{|u_n|_E}$ converges to z and this convergence is uniform with respect to $x \in \bar{\Omega}$. It follows from the assumption $G2:$ that

$$\lim_{n\to\infty} \frac{g(u_n(x)}{|g(u_n(x))|_E} = \varphi(z)$$

uniformly in $\bar{\Omega}$. Since $\varphi(z)$ is in the unit sphere one can find an integer n_0 such that if $n \geq n_0$ and $x \in \bar{\Omega}$, then

$$\langle \frac{g(u_n(x)}{|g(u_n(x))|_E}, \varphi(z) \rangle_E \geq \frac{1}{4}$$

Define

$$\gamma_n(x) = |g(u_n(x))|_E.$$

By $G1:$ clearly $\gamma_n > 0$ everywhere. For $n \geq n_0$

$$\langle \int_{\partial\Omega} g(u_n(x))\, dx, \varphi(z) \rangle_E = \int_{\partial\Omega} \langle g(u_n(x)), \varphi(z) \rangle_E\, dx$$

$$= \int_{\partial\Omega} \gamma_n(x) \langle \frac{g(u_n(x))}{\gamma_n(x)}, \varphi(z) \rangle_E\, dx \geq \frac{1}{4} \int_{\partial\Omega} \gamma_n(x)\, dx > 0$$

Therefore, $\int_{\partial\Omega} g(u_n(x))\, dx > 0$. Now we have contradiction with (1.6)

The proof completely of the lemma. □

Ref

4. R. Ortega, and J.R. Ward, *A Semilinear elliptic system with vanishing nonlinearities, proceedings of the fourth international conference on dynamical systems and differential equations May 24 – 27, 2002, Wilmington, NC.*

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II. Main Result

Let

$$Qu = Nu. \tag{2.1}$$

Be linear elliptic equation with nonlinear boundary condition. Suppose N is continuous and bounded (i.e.; $|Nu|_E \leq C$ for all u). If Q has a compact inverse Q^{-1} then by Leray-Schauder theory (2.1) has a solution. On the other hand if Q is not invertible the existence of a solution depends on the behavior of N and its interaction with the null space of Q see [19].

Theorem 2.1. *Suppose $g \in C^1(\mathbb{R}^k, \mathbb{R}^k)$ satisfies $G1$:, $G2$:, and $G3$:. If $h \in C(\partial\Omega, \mathbb{R}^k)$, satisfies $\bar{h} = 0$. Then, (1.1) has at least one solution.*

Proof. Define

$$J : H^1(\Omega) \to R$$

be continuous map in $H^1(\Omega)$ with the $L^2(\Omega)$ norm

$$J(v) = E(u, v)$$

Define

$$Dom(L) := \{u \in H^1(\Omega) : -\Delta u + A(x)u = 0\}$$

Define an operator L on $L^2 = L^2(\Omega, \mathbb{R}^k)$ for $u \in Dom(L)$ and each $v \in H^1(\Omega)$ by

$$E(u, v) =< Lu, v >_{L^2(\Omega)},$$

we use the embedding theorem see [20] since you know that $H^1(\Omega) \hookrightarrow L^2(\Omega)$ and the trace theorem $(H^1 \to L^2(\partial\Omega))$. Thus, $L : Dom(L) \subset L^2(\partial\Omega) \to L^2(\partial\Omega)$ then the equation

$$E(u, v) =< h, v >_{L^2(\partial\Omega)} \;\; \forall\, v \in H^1(\partial\Omega),$$

if and only if

$$Lu = h.$$

The latter equation is solvable if and only if

$$Ph := \frac{1}{|\partial\Omega|} \int_{\partial\Omega} h = 0.$$

Now if $h \in L^\infty(\partial\Omega, \mathbb{R}^k)$ and $Ph = 0$. Then, each solution $u \in H^1(\Omega)$ is Hölder continuous, so $u \in C^\gamma(\bar{\Omega}, \mathbb{R}^k)$ for some $\gamma \in (0, 1)$. Since we know that there is constant $r_1 > 0$ such that

$$||u||_\gamma \leq r_1 \left(||u||_{L^2(\partial\Omega)} + ||h||_{L^\infty(\partial\Omega)} \right).$$

When $Ph = 0$ there is a unique solution $Kh = \tilde{u} \in H^1(\Omega)$ with $P\tilde{u} = 0$ to

$$Lu = h,$$

and if $h \in C(\partial\Omega) = C(\partial\Omega, \mathbb{R}^k)$ then

$$||Kh||_\gamma \leq r_1 \left(||Kh||_{L^2(\partial\Omega)} + ||h||_{L^\infty(\partial\Omega)} \right) \leq r_2 ||h||_{C(\partial\Omega)}$$

and K maps $C(\partial\Omega)$ into itself take compact set to compact set i.e.; compactly. Let Q be the restriction of L to $L^{-1}(C(\partial\Omega)) = KC(\partial\Omega) + \mathbb{R}^k$. We define $N : C(\partial\Omega) \to C(\partial\Omega)$ by

$$N(w)(x) := h(x) - g(w(x)) \; \forall w \in C(\partial\Omega)$$

is continuous. Now (1.1) can be written as

$$Qu = Nu$$

Ref

19. J. Mawhin; *Topological degree methods in nonlinear boundary-value problems, in NSFCBMS Regional Conference Series in Mathematics, American Mathematical Society, Providence, RI, 1979.*

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and $\ker Q = ImP$, $ImQ = \ker P$. The linear map Q is a Fredholm map (see [16]) and N is $Q-$compact (see [19]). Now we define the Homotopy equation as follows Let $\lambda \in [0,1]$ such that

$$Qu = \lambda Nu. \tag{2.2}$$

The a priori estimates (i.e.; the possible solutions of (2.2) are uniformly bounded in $C(\partial\Omega)$). Now we show that the possible solutions of (2.2) are uniformly bounded in $C(\partial\Omega)$ independent of $\lambda \in [0,1]$ Since we know that $u = \bar{u} + \tilde{u}$ where $\bar{u} = Pu$. Then

$$||\tilde{u}||_\gamma = ||\lambda KNu||_\gamma \leq r_2||Nu||_{C(\partial\Omega)} \leq R_1,$$

where R_1 is a constant (g is abounded function). It remains to show that $\bar{u} \in \mathbb{R}^k$ is bounded, independent of $\lambda \in [0,1]$. By the way of contradiction assume is not the case (i.e.; $\bar{u}$ unbounded). Then there are sequence $\{\lambda_n\} \subset [0,1]$, and $\{u_n\} \subset Dom(Q)$ with $||\tilde{u}_n||_\gamma \leq R_1$,

$$Qu_n = \lambda_n Nu_n \text{ and } |\bar{u}_n|_E \to \infty,$$

we get that

$$PNu_n = PN(\tilde{u}_n + \bar{u}_n) = -\int_{\partial\Omega} g(\tilde{u}_n(x) + \bar{u}_n(x))\,dx = 0.$$

Now $u_n = \tilde{u}_n + \bar{u}_n$ so $||u_n - \bar{u}_n||_{L^\infty(\partial\Omega)} = ||\tilde{u}_n||_{L^\infty(\partial\Omega)} \leq R_1$ and $||\bar{u}_n||_{L^\infty(\partial\Omega)} \to \infty$. It follows from Lemma1.1 that for all sufficiently large n

$$\int_{\partial\Omega} g(u_n(x))\,dx \neq 0.$$

We have reached a contradiction, and hence all possible solutions of (2.2) are uniformly bounded in $C(\partial\Omega)$ independent of $\lambda \in [0,1]$

Let $\bar{B}(0,r) = \{x : |x|_E \leq r\}$ denote the ball in $C(\partial\Omega, \mathbb{R}^k)$. Now you can apply Leray-Schauder degree theorem see ([18],[19]), the only thing left to show is that

$$deg(PN, \bar{B}(0,r) \cap \ker Q, 0) \neq 0,$$

for large $r > 0$. So $deg(PN, \bar{B}(0,r) \cap \ker Q, 0) = deg(g, \bar{B}_r, 0)$, where $\bar{B}_r$ is the ball in $\mathbb{R}^k$ of radius r. Since for $|x|_E$ large, and $deg(\varphi) \neq 0$ we have that $deg(g, \bar{B}_r, 0) \neq 0$ for large r. Therefore $deg(PN, \bar{B}(0,r) \cap \ker Q, 0) \neq 0$ By Leray-Schauder degree theorem equation (2.2) has a solution when $\lambda = 1$. Therefore, equation (1.1) has at least one solution. This proves the theorem. □

We will give one example.

Example 2.1. Let $\Omega \subset \mathbb{R}^N$, $N \geq 2$ is a bounded domain with boundary $\partial\Omega$ of class C^∞. Let

$$\begin{aligned} -\Delta u + A(x)u = 0 \quad &\text{in } \Omega, \\ \frac{\partial u}{\partial \nu} + \frac{u}{1 + |u|_E^2} = h(x) \quad &\text{on } \partial\Omega \end{aligned} \tag{2.3}$$

where $A(x)$ is positive semidefinite matrix on $\mathbb{R}^{2\times 2}$, and where $u = (u_1, u_2) \in \mathbb{R}^2$ and h real valued function and continuous on $\partial\Omega$, and $\int_{\partial\Omega} h(x)\,dx = 0$ and $g(u) = \frac{u}{1+|u|_E^2}$

$$\lim_{|u|_E \to \infty} g(u) = \lim_{|u|_E \to \infty} \frac{u}{1 + |u|_E^2} = 0$$

$g(u)$ vanishes at infinity, clearly $g \in C^1(\mathbb{R}^2, \mathbb{R}^2)$ and bounded with $g(u) \neq 0$, for $|u|_E$ large. Therefore g satisfies $G1$:.

$$\frac{g(ru_1, ru_2)}{|g(ru_1, ru_2)|} = \frac{g(ru)}{|g(ru)|} = \frac{\frac{ru}{1+|ru|_E^2}}{\left|\frac{ru}{1+|ru|_E^2}\right|} = \frac{u}{|u|_E} = u$$

Ref

16. N. Mavinga, M. N. Nkashama, *Nonresonance on the boundary and strong solutions of elliptic equations with nonlinear boundary conditions. Journal of Applied functional Analysis, Vol.7, NO.3,248-257, copyright 2012 Eudoxus Press, LLC.*

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For all u in $\mathbb{S}$ and $r > 0$. Therefore $G2$: holds.
And $\varphi(u) = u$ so that $deg(\varphi) \neq 0$. Therefore $G3$: holds. By Theorem 2.1. Then, equation (2.3) has at least one solution.

References Références Referencias

1. E. Landesman and A. Lazer: *Nonlinear perturbations of linear elliptic boundary value problems at resonance, J. Math. Mech. 19 (1970), 609-623.*
2. L. Nirenberg, *An application of generalized degree to a class of nonlinear problems. Troisijeme Colloque sur l'Analyse Fonctionnelle (Lijege, 1970), pp. 57–74. Vander, Louvain, 1971.*
3. L. Nirenberg, *Generalized degree and nonlinear problems. Contributions to nonlinear functional analysis (Proc. Sympos., Math. Res. Center, Univ. Wisconsin, Madison, Wis, 1971), pp. 1–9. Academic Press, New York, 1971.*
4. R. Ortega, and J.R. Ward, *A Semilinear elliptic system with vanishing nonlinearities, proceedings of the fourth international conference on dynamical systems and differential equations May 24 – 27, 2002, Wilmington, NC.*
5. R. Iannacci, M. N. Nkashama; *nonlinear two point boundary value problems at resonance without Landesman-lazer condition. Journal of Proceeding of the american mathematical society Vol.106, NO.4, pp 943-952, August 1989.*
6. R. Ortega and L.A. Sjanchez, *Periodic solutions of forced oscillators with several degrees of freedom, Bull. London Math. Soc. 34 (2002) 308–318.*
7. J. Mawhin, *Forced second order conservative systems with periodic nonlinearity, Ann. Inst. Henri-Poincarje Anal. Non Linjeaire 6 suppl. (1989), 415–434.*
8. A. Fadlallah *Linear second order elliptic partial differential equations with Nonlinear Boundary Conditions at Resonance Without Landesman-Lazer Conditions, http://arxiv.org/pdf/1410.7315v2, Submitted on 27 Oct 2014.*
9. A. Ambrosetti and G. Mancini, *Existence and multiplicity results for nonlinear elliptic problems with linear part at resonance the case of the simple eigenvalue,J. Differential Equations 28 (1978), 220–245.*
10. A. Ambrosetti and G. Mancini, *Theorems of existence and multiplicity for nonlinear elliptic problems with noninvertible linear part, Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4) 5 (1978),15–28.*
11. G. Auchmuty, *Steklov eigenproblems and the representation of solutions of elliptic boundary value problems. Numer. Func. Analysis Optim. 25 (2004), 321–348.*
12. S. Fucik and M. Krbec, *Boundary value problems with bounded nonlinearity and general null-space of the linear part,Math. Z. 155 (1977), 129–138.*
13. P. Hess, *A remark on the preceding paper of Fucik and Krbec , Mat. Z. 155 (1977), 139–141.*
14. N. Mavinga and M. N. Nkashama, *Steklov–Neumann eigenproblems and nonlinear elliptic equations with nonlinear boundary conditions. J. Diff. Eqns 248 (2010), 1212–1229.*
15. N. Mavinga and M. N. Nkashama, *Nonresonance on the boundary and strong solutions of elliptic equations with nonlinear boundary conditions. Journal of Applied Functional Analysis, Vol.7,No3,243,257,2011.*
16. N. Mavinga, M. N. Nkashama, *Nonresonance on the boundary and strong solutions of elliptic equations with nonlinear boundary conditions. Journal of Applied functional Analysis, Vol.7, NO.3,248-257, copyright 2012 Eudoxus Press, LLC.*
17. T. Runst, *A unified approach to solvability conditions for nonlinear second-order elliptic equations at resonance , Bull. London Math. Soc. 31 (1999) 385–314.*
18. Lloyd, N.G., *Degree theory, Cambridge, University Press, 1978.*
19. J. Mawhin; *Topological degree methods in nonlinear boundary-value problems, in NSFCBMS Regional Conference Series in Mathematics, American Mathematical Society, Providence, RI, 1979.*

Notes

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20. Brezis, H., *Functional Analysis,Sobolev Spaces and Partial Differential Equations.Springer, 2011.*

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Applications of Semigroups

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Abstract- This section deals with the applications of semigroups in general and regular semigroups in particular. The theory of semigroups attracts many algebraists due to their applications to automata theory, formal languages, network analogy etc. In section 2 we have seen different areas of applications of semigroups. We identified some examples in biology, sociology etc. whose semigroup structures are nothing but regular, E-inversive and inverse semigroup etc.

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APPLICATIONSOFSEMIGROUPS

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Applications of Semigroups

P. Sreenivasulu Reddy [α] & Mulugeta Dawud [σ]

Absract- This section deals with the applications of semigroups in general and regular semigroups in particular. The theory of semigroups attracts many algebraists due to their applications to automata theory, formal languages, network analogy etc. In section 2 we have seen different areas of applications of semigroups. We identified some examples in biology, sociology etc. whose semigroup structures are nothing but regular, E-inversive and inverse semigroup etc.

I. Introduction

The concept of a semigroup is relatively young, the first, often fragmentary, studies were carried out early in the twentieth century. Then the necessity of studying general transformations, rather than only invertible transformations (which played a large role in the development of group theory) became clear. During the past few decades connection in the theory of semigroups and the theory of machines became of increasing importance, both theories enriching each other. In association with the study of machines and automata, other areas of applications such as formal languages and the software use the language of modern algebra in terms of Boolean algebra, semigroups and others. But also parts of other areas, such as biology, psychology, biochemistry and sociology make use of semigroups.

The theory of automata has its origins in the work by Turing (Shannon1948, and Heriken 1994.).Turing developed the theoretical concept of what is now called Turing machines, in order to give computability a more concrete and precise meaning. Hannon investigated the analysis and synthesis of electrical contact circuits using switching algebra. The work of McCullon and pitts centers on neuron models to explain brain functions and neural networks by using finite automata. Their work was continued by Kleene. The development of technology in the areas of electromechanical and machines and particularly computers had a great influence on automata theory which traces back to the mid-1950s. Many different parts of pure mathematicians are used as tools such as abstract algebra, universal algebra, lattice theory, category theory, graph theory, mathematical logic and the theory of algorithms. In turn automata theory can be used in economics, linguistics and learning processes.

The beginning of the study of formal languages can be traced to Chomsky, who introduced the concept of a context-free language in order to model natural languages in 1957. Since then late 1960 there has been considerable activity in the theoretical development of context-free languages both in connection with natural languages and with the programming languages. Chomsky used semi-thue systems to define languages, which can be described as certain subsets of finitely generated free monoids. Chomsky (1957) details a revised approach in the light of experimental evidence and careful consideration of semantic and syntactic structures of sentences. For a common approach to formal languages and the theory of automata we refer to Eilenberg (1974).

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Semigroups can be used in biology to describe certain aspects in the crossing of organisms, in genetics and in consideration of metabolisms. The growth of plants can be described algebraically in Hermann and Rosenberg (1975). Further material on this subject is contained in Holcombe (1982). Details on the use of semiautomata in metabolic pathways and the aid of a computer therein, including a theory of scientific experiments, can be found in Krohn, Langer and Rhodes (1976). Rosen (1973) studies ways in which environmental changes can affect the repair capacity of biological systems and considers carcinogenesis and reversibility problems. Language theory is used in cell – development problems, as introduced by Lindenmayes (1968), Hermann and Rosendalg (1975). Suppes (1969) Kiesras (1976) develop a theory of learning in which a subject is instructed to behave like a semiautomaton.

The study of kinship goes back to a study by A. Weil in response to an inquiry by the anthropologist C. Levi-Strauss in 1949. White (1963), Kim and Breiger (1979) and Breiger, Boorman and Srable (1975) are also developed the elementary structure of kinship. Ballonoff (1974) presents several fundamental papers on kinship. Carlso 1980) gives elementary examples of applications of groups in anthropology and sociology. Rudolf Lidl and Guter Pilz were started with a selected set X of basic relations such that the set of all their relation products yields all remaining kinship relations. In this way they arrive at the concept of a free (hence infinite) semigroups over X. Sociology includes the study of human interactive behavior in group situations, in particular, in underlying structures of societies. Such structures can be revealed by mathematical analysis. This indicates how algebraic techniques may be introduced into studies of this kind.

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II. Semigroups and its Applications

Now a days the theory of semigroups has been expanded greatly due to its applications to computer science and we also finds its usage in biological science and sociology. In this section we discuss some applications of semigroups in different areas.

a) Semigroups –Automaton

The algebraic theory of automata, which uses algebraic concepts to formalize and study certain types of finite-state machines. One of the main algebraic tools used to do this is the theory of semigroups. Automaton is an abstract model of computing device. Using this models different types of problems can be solved. We discuss what is common to all automata by describing an abstract model will be amenable to mathematical treatment and see that there is a close relationship between automata and semigroup. We can establish a correspondence between automata and monoids.

The problem may be identifying or adding two integers etc. i.e., we will be encounting automata in several forms such as calculating machines, computers, money changing devices, telephone switch boards and elevator or left switchings. All the above have one aspect in common namely a “box” which can assume various states. These states can be transformed into other states by outside influence and process “outputs” like results of computations.

i. *Semi automata:* A semi automaton is a triple $Y = (Z, A, \delta)$ consisting of two non empty sets Z and A and a function $\delta : Z \times A \rightarrow Z$. Z is called the set of states, A is the set of input alphabet and δ the “ next – state function “ of Y.

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ii. *Automata:* An automaton is a quintuple Ă = (Z, A, B, δ, λ) where (Z, A, δ) is a semi automaton, B is a non empty set called the output alphabet and $\lambda : Z \times A \rightarrow B$ is the " output function."

If $z \in Z$ and $a \in A$ then we interpret $\delta(z, a) \in Z$ as the next state into which z is transformed by the input a. $\lambda(z, a) \in B$ is the output of z resulting from the input a. Thus if the automaton is in the stage z and receives input a, then it changes to state $\delta(z, a)$ with out put $\lambda(z, a)$. A(semi)-automaton is finite, if all the sets Z, A and B are finite, finite automata are also called mealy automata.

Examples: 1) Let $A = \{a_1, a_2, \ldots\ldots a_n\}$, $B = \{b_1, b_2, \ldots\ldots b_m\}$ and $Z = \{z_1, z_2 \ldots\ldots z_k\}$

Description by tables:

Input table

δ	a_1	a_2……………	a_n
z_1	$\delta(z_1, a_1)$	$\delta(z_1, a_2)$	$\delta(z_1, a_1)$
z_2	$\delta(z_2, a_1)$	$\delta(z_2, a_2)$	$\delta(z_2, a_n)$
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
z_k	$\delta(z_k. a_1)$	$\delta(z_k, a_2)$	$\delta(z_k, a_n)$

Out put table

λ	a_1	a_2……………	a_n
z_1	$\lambda(z_1, a_1)$	$\lambda(z_1, a_2)$	$\lambda(z_1, a_1)$
z_2	$\lambda(z_2, a_1)$	$\lambda(z_2, a_2)$	$\lambda(z_2, a_n)$
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
z_k	$\lambda(z_k. a_1)$	$\lambda(z_k, a_2)$	$\lambda(z_k, a_n)$

Description by graphs

We depict $z_1, z_2, \ldots\ldots z_k$ as "discs" in the plane and draw an arrow labeled a_i from z_r to z_s, if $\delta(z_r, a_i) = z_s$. In case of an automaton we denote the arrow also by $\lambda(z_r, a_i)$.This graph is called the state graph

Example: (marriage Automation)

Let us consider the following situation in a household . The husband is angry or bored or happy: the wife is quite or shouts or cooks his favorite dish. Silence on her part does not change the husband's mood, shouting "lowers" it by one "degree "(if he is already angry, then no change), cooking of his favorite dish creats general happiness for him. We try to describe this situation in terms of a semi-automaton Y = (Z , A ,δ) . We define $Z = (z_1, z_2, z_3)$ and $A = \{a_1, a_2, a_3\}$ with

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z_1 = husband is angry a_1 = wife is quite
z_2 = husband is bored a_2 = wife is shout
z_3 = husband is happy a_3 = wife cooks

The following is the description of δ

δ	a_1	a_2	a_3
z_1	z_1	z_1	z_3
z_2	z_2	z_1	z_3
z_3	z_3	z_2	z_3

For this situation add the output $B = \{b_1, b_2\}$ with the interpretation b_1 = husband shouts, b_2 = husband quite.

Let us assume that the husband is only shouts if he is angry and his wife shouts. Otherwise he is quite even in state z_3. We shall define the ouput function λ by using the following output table.

λ	a_1	a_2	a_3
z_1	b_2	b_1	b_2
z_2	b_2	b_2	b_2
z_3	b_2	b_2	b_2

Let A^1 be the free monoid on A , i.e., $A^1 = F_A^{(1)}$, the monoid of all finite (including the empty) sequences of elements of the set A. Let us denote the identity (the empty sequence) in A^1 be Λ. We shall extend the next state function δ and the output function λ from $Z \times A$ to $Z \times A^1$ as follows:

For any $z \in Z$ and $a_1 a_2, \ldots\ldots\ldots an \in A^1$, define

$$\delta^1(z, a_1a_2\ldots\ldots a_n) = \begin{cases} z, & \text{if } n = 0 \\ \delta(\delta^1(z,a_1a_2\ldots\ldots a_{n-1}), a_n), & \text{if } n > 0. \end{cases}$$

and

$$\lambda^1(z, a_1a_2\ldots\ldots a_n) = \begin{cases} \Lambda, & \text{if } n = 0 \\ \lambda(z,a_1)\, \lambda^1(\delta(z,a_1), a_2\ldots\ldots a_n), & \text{if } n > 0. \end{cases}$$

In other words, $\delta^1: Z \times A^1 \to Z$ and $\lambda^1 : Z \times A^1 \to B^1$ are defined inductively as given above . Note that B^1 is the free monoid on B ; i.e., B^1 is the set of all finite (including the empty) sequences of elements in B.

Now we establish a correspondence between monoids and automata and discuss certain examples. If (S , .) is a monoid and we define $\delta : S \times S \to S$; $\lambda : S \times S \to S$ by $\delta(s, t) = s.t$ and $\lambda(s, t) = s$ $\forall\, s, t \in S$ .Then (S ,S , S , δ , λ) is an automata and if we define $f_a : S \to S$ by $f_a(z) = \delta^1(z, a)$ for any $a \in A$ and $z \in S$, then $\{ f_a / a \in A^1 \}$ is a monoid (under the composition of mappings).

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Let us consider the semi automaton given in above marriage automata. First we construct the table for f_Λ, f_{a1}, f_{a2}, f_{a3} indicating their actions on the states z_1, z_2, z_3.

*	f_Λ	f_{a1}	f_{a2}	f_{a3}
$z_1 \rightarrow$	z_1	z_1	z_1	z_3
$z_2 \rightarrow$	z_2	z_2	z_1	z_3
$z_3 \rightarrow$	z_3	z_3	z_2	z_3

Since $f_\Lambda = f_{a1}$, we delete f_{a1} and check whether $\{f_\Lambda, f_{a2}, f_{a3}\}$ forms a monoid. The operation "o" is given in the following table.

$\circ$	f_Λ	f_{a2}	f_{a3}
f_Λ	f_Λ	f_{a2}	f_{a3}
f_{a2}	f_{a2}	f_{a2a2}	f_{a3a2}
f_{a3}	f_{a3}	f_{a3}	f_{a3}

$\circ$	F_{a2a2}	f_{a3a2}
$z_1 \rightarrow$	z_1	z_2
$z_2 \rightarrow$	z_1	z_2
$z_3 \rightarrow$	z_1	z_2

Therefore $\{ f_\Lambda, f_{a2}, f_{a3}\}$ is not a monoid. Now, we extend it to$\{f_\Lambda, f_{a2}, f_{a3}, f_{a3a2}, f_{a3a2}\}$ and verify that this set is a monoid. This is the monoid M corresponding to the automaton A^1.

b) Semigroups- Formal languages

A formal language is an abstraction of general characterization of programing language. A grammar is model to describe a language. Given an alphabet or vocabulary (constructing of letters, words, symbols……) we have a method (grammar) for constructing meaningful words or sentences from this alphabets. This immediately reminds us of the term " word semigroup" and indeed these free (word) semigroup well play a major role in constructing language. The formal language L constructed will be a subset of the free semigroup A_* or the free monoid A^* on the alphabet A.

We are familiar with notion of natural languages such as English, Telugu. English language consists of sentences like "Ram walked quickly, Gita ate slowly…". The question is how to define a language, we need some basics. A finite non empty set of symbols denoted by A. Finite sequence of symbols from the alphabet, for example abab, abba are strings over alphabet A = {a, b}.

Concatenation of two strings u and v is the string obtained by appending symbols of v to the right end of u.u = abab, v = abba then uv = abababba. The length of a string u is the number of symbols in the string and denoted by $|u|$. The string with no symbols is denoted by Λ and its length is zero.

If u and v are two strings then $|uv| = |u| + |v|$.

If A is an alphabet, A* is the set of all strings obtained by concatenation zero or more symbols from A, and $A^* = A^+ - \{\Lambda\}$.

If A = –a, b″ then A* = {Λ, a, b, ab, aa, ….}.

In general a language is defined as a subset of A^*.

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Grammar: A model to describe language is known as grammar. There are essentially three ways to construct a language.

I) Approach via grammar

II) Approach via automata

III) Algebraic approach

Example: Let A = {a} and G = {g_0}, $g_0 \neq a$ and → = { g_0 →a, g_0 → a g_0 }, G^1 = {A, G, →, g_0}. Then g_0 = $_z$ implies z = a or z = a g_0. Again there is no x with a → x, a g_0 → y is only possible for y = aa or y = aa g_0 . Thus aa g_0 → aaa and aa g_0→ aaa g_0 etc.

g_0 → a
↓
ag_o → aa
↓
aag_o → aaa
↓
aaag_o →.....
.
.
.

Cancellation of elements outside A^* gives us the result L(G) = {a, aa, aaa, ...} = A^*

c) Semigroups in Biology

Semigroup can be used in biology to describe certain aspects in the crossing of organisms, in genetics, and in consideration of metabolisms.

Example: In breeding a strain of cattle, which can be block or brown monochromatic or spotted, it is known that black is dominant and brown receive and that monochromatic is dominant over spotted. Thus there are four possible types of cattle in this herd.

a = Black monochromatic   c = Brown monochromatic
b = black spotted   d = brown spotted.

Due to dominance, in crossing a black spotted one with a brown monochromatic one, we expect a black monochromatic one. This can be symbolized by " b*c = a". The operation '* 'can be studied for all possible pairs to obtain the table.

*	a	b	c	d
a	a	a	a	a
b	a	b	a	b
c	a	a	c	c
d	a	b	c	d

Then S = {a, b, c, d} is a semigroup with identity element d.

In general, the table for breeding operations is more complicated .

We can ask for connections between hereditary laws and the corresponding semigroups. Of course, we need SoS = S for such semigroups S, since s∈S"S^2 would vanish after the first generation and would not even be observed. A different biological problem which leads to algebraic problems is as follows: all genetic information in an organisms is given in the so called deoxy ribonucleic acid(DNA) which consists of two stands which are combined together to form the famous double helix. Each stand is made up as a polymer of four different basics substances, the nucleotides. If the

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nucleotides are denoted by n_1, n_2, n_3 and n_4, then the stand can be regarded as a word. Over $\{n_1, n_2, n_3, n_4\}$. DNA cannot put the genetic information into effect. By means of a messenger ribonucleic acid, the information contained in the DNA is copied ("translation") and then transferred onto the protein chains ("tranlation"). These protein chains are polymers consisting of 21 different basic substances, the amino acids, denoted by a_1, a_2, a_3, a_4 .....a_{21}. As with the DNA each protein chain can be regarded as a word over $\{a_1, a_2, a_3 \ldots a_{21}\}$.

In general, it is assumed that the sequence of nucleotides in the DNA is uniquely detemined by the sequence of amino acids in a protein chain. In otherwords, it is assumed that there is a monomorphism from the free semigroup $F_{21} = \{a_1, a_2, a_3, \ldots a_{21}\}_*$ into the free semigroup $F_4 = \{n_1, n_2, n_4, n_4\}_*$.

d) Semigroups in Sociology

Sociology includes the study of human interactive behavior in group situations, in particular in underlying structures of societies. Such structures can be revealed by mathematical analysis. This indicates how algebraic techniques may be introduced into studies of this kind. So the study of such relations can be elegantly formulated in the language of semigroups.

Rudolf lidl and Gunter Pilz defind relation semigroup as follows.

Definition: (R(M), o) is called the relation semigroup on M. The operation o is called the relation product, where M is a monoid.

It is obvious that $R \in R(M)$ is transitive if and only if $RoR \subseteq R$. The set of their relation products yields all remaining kinship relations. In this way they arrive at the concept of a free semigroup over X. But there are only finite people on the earth and some kinship relations like "daughter of a mother" and " daughter of a father" might be considered to be "the same'.

Definition: A kinship system is a semigroup $S = \{X, R\}$ where R is a relation on X, which express equality of kinship relationships.

Examples:

1) Let X = {"is father of ", "is mother of "} and $R = \phi$. Then the kinship system S is the semigroup { "is father of ", "is mother of", "is grand father on fathers side of "… }.
2) Let F = "is father of", M = " is mother of ", S = " is son of ', D = "is daughter of,' , B = "is brother of", C = "is child of " and Si = "is sister of" then X = –F, M, S, D, B, Si, C″ and R = {(CM, CF), (BS, S), (SiD, D), (CBM, CMM), (MC, FM), (SB, S), (Dsi, D), (MBC, MMC)………….}.

For complete list of R see Boyd, Haehl and Sailer (1972) [6]. The first means that in the semigroup we have CM = CF, children of the mother are the same as the children of the father.

Let G be a society i.e, a non empty set of people and let S(G) be the semigroup of all different kinship relationships of this society.

The semigroup S(G) often have special properties, eg. "is son of " and "is father of" are nearly inverse relations. The framework for investigation of special S(G) would be the theory of inverse semigroups, i.e, semigroups S such that for all $s \in S$ with $ss^1s = s$

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6. Clifford, A. H. and Preston, G. B. "The algebraic theory of semigroups", Math. Surveys No.7,Amer.math.soc., Vol. II, (1967).

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and $s^1ss^1 = s^1$. For instance, (Mn(R), .) is an inverse semigroup for all n∈N. If s denotes "is son of" and s^1 denotes "is father of " then these two equations hold in most societies.

e) Further Study

This section deals with the applications of semigroups in general and regular semigroups in particular. The theory of semigroups attracts many algebraists due to their applications to automata theory, formal languages, network analogy etc. In section 2 we have seen different areas of applications of semigroups. We identified some examples in biology, sociology etc. whose semigroup structures are nothing but regular, E-inversive and inverse semigroup etc.

For consider the following examples:

Examples:

1) Let the kinship system S = {X, R} be defined by X = { P = (" is parent of "), C = ("is child of")}, R = {(PP, P), (PC, CP), (CC, C)}. Let a, b, c be the equivalence classes of P, C and PC respectively. Now "o" is given by

·	**a**	**b**	**c**
a	a	c	c
b	c	b	c
c	c	c	c

Then (S, .) is a semigroup

2) Let X = {F, M} and R = {(FF, F), (MM, M), (FM, FM)} then S = {[F], [M], [FM]} with the following table is semigroup.

·	F	M	FM
F	F	FM	FM
M	FM	M	FM
FM	FM	FM	FM

And it is clearly,

FFF = F, MMM = M, FMFMFM = FM, FMFFM = FM, FMMFM = FM

* So, S is a regular semigroup which is not an inverse semigroup.

3) Let X = {F, M} and R = {(FFF, F), (MM, M), (FM, MF)} with the operatable given below is semigroup.

·	F	FF	M	FM	FFM
F	FF	F	FM	FFM	FM
FF	F	FF	F FM	FM	FFM
M	FM	FFM	M	FM	FFM
FM	FFM	FM	FM	FFM	FM
FFM	FM	FFM	FF	FM	FFM

• 4) Let X = –S, M, F″ and R = –(SS, S), (MM, M), (FF, F), (SM, S), (SF, S), (MF, M), (MS, M)″ then S = –[S], [M], [F]″ is an inverse semigroup with the operation table as shown below

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·	S	M	F
S	S	S	S
M	M	M	M
F	F	F	F

* 5) Let Si = sister and D = daughter then

X = –Si, D″ and R = –(SiD, D), (DSi, D), (SiSi, Si), (DD, D)″ then S = –[Si], [D]″is regular semigroup but not inverse semigroup with the following operation table.

·	Si	D
Si	Si	D
D	D	Si

6) Let X = {F, M} and R = {(FF, F), (MM, M), (FM, MF)} then S = {[F], [M], [MF], [FM]} with the following operation table is semigroup.

·	F	M	FM	MF
F	F	FM	FM	MF
M	MF	M	FM	MF
FM	MF	FM	FM	MF
MF	MF	MF	FM	MF

Further we want to study some more structures of semigroups which will find applications in different areas.

References Références Referencias

1. Arbib,M. A. “Algebraic theory of machines, Languages and Semigroups” New york (1968), Academic Press.
2. Arbib,M. A. “Theories of Abstract Automata”, Englewood Cliffs, N.J.: Prentice Hall.(1969)
3. Boorman, S. A. and White, H. C. “Social structures from multiple networks-II: role Structures.” Amer.J.Sociol.81(1976), 1384-1466.
4. Boyd, J. P., Haehl,J. H. and Sailer,L.D. “Kinship Systems and Inverse semigroups”, J.Math.Sociol.76,(1972), 37-61.
5. Clifford, A. H. and Preston, G. B. “The algebraic theory of semigroups”, Math.Surveys No.7,Amer.math.soc., Vol. I, (1967).
6. Clifford, A. H. and Preston, G. B. “The algebraic theory of semigroups”, Math. Surveys No.7,Amer.math.soc., Vol. II, (1967).

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Kaluza-Klein Barotropic Cosmological Models with Varying Gravitational Constant G in Creation Field Theory of Gravitation

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H. R. Ghate [α] & Sandhya S. Mhaske [σ]

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I. Introduction

From the Astronomical observations in the late eighties, it is concluded that the predictions of FRW type models do not always meet our requirements as was believed earlier (Smooth *et al.* (1992)). Hence alternatives theories were proposed of which the most well-known theory was steady state theory by Bondi and Gold (1948). The main approach of this theory is the universe does not have any singular beginning nor an end on the cosmic time scale where the matter density is constant throughout. They have considered a very slow but continuous creation of the matter in contrast to explosive creation of standard model for maintaining constancy of matter density. But the theory was discarded for not giving any physical justification about continuous creation of matter. To overcome this difficulty Hoyle and Narlikar (1964 a, b, c) adopted a field theoretic approach introducing a massless and charge-less scalar field in the Einstein Hilbert action to account for a matter creation. Bali and Saraf (2013a, b, c) have investigated cosmological models with varying (λ) in Creation field theory of gravitation. Narlikar (1973) stated that introduction of negative energy C-field solved the horizon and flatness problem faced by big bang model. Chatterjee and Banerjee (2004) have extended the study of Hoyle Narlikar theory in higher dimensional space times. Singh and Chaubey (2009) have studied Kantowski-Sachs and Bianchi type universes in creation field theory. Adhav *et al.* (2010) have obtained Bianchi type-I universe with cosmological models in Creation field theory of gravitation with different contexts. Recently Ghate *et al.* (2014 a) have investigated LRS Bianchi type-V dust filled universe with varying $\Lambda(t)$ in creation field theory of gravitation.

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In Einstein's general theory of relativity, the gravitational constant G plays the role of coupling constant between geometry and matter. The concept of gravitational constant G was first proposed by Dirac (1937). Pochoda and Schwarzschild (1963) and Gamow (1967) have studied the solar evolution in the presence of a time varying gravitational constant. Barrow (1978) assumed that $G\alpha t^{-n}$ and obtained from Helium abundance for $5.9\times10^{-13} < n < 7\times10^{-13}$, $\left|\frac{\dot{G}}{G}\right| < (2\pm.93)\times10^{-12}\,yr^{-1}$, by assuming a flat Universe.

Subsequently alternative theories of gravity especially Brans and Dicke (1961), Canuto *et al.* (1977) were developed to generalized Einstein's general theory of relativity by including variable G and satisfying conservation equation. Bali and Kumawat (2011, 12) have investigated cosmological models with variable G in C-field cosmology.

The Kaluza-Klein theory was introduced to unify Maxwell's theory of electromagnetism and Einstein's gravity theory by adding the fifth dimension. Kaluza (1921) has demonstrated that GR when interpreted as a vacuum 5D theory contains four-dimensional GR in the presence of electromagnetic field, together with Maxwell's electromagnetism. To do so, Kaluza supposed that - (i) model should maintain Einstein's vision that nature is purely geometric (ii) GR mathematics is not modified but just extended to five dimensions and (iii) there is no physical dependence on the fifth dimension. Klein (1926) suggested the compactification of fifth dimension. Number of relativists have studied Kaluza-Klein cosmological models with different contexts [Leon (1988), Chi (1990), Fukui (1993), Coley (1994), Liu & Wesson (1994)]. Alvarez *et al.* (1983), Ranjibar– Daemi *et al.,* (1984) and Marciano (1984) suggested that the experimental detection of time variation of fundamental constants could provide strong evidence for the existence of extra dimensions. Recently Ghate *et al.* (2014 b, c) have investigated Kaluza-Klein dust filled universe with time dependent Λ in creation field cosmology.

In this paper, Kaluza-Klein cosmological models with variable G in Hoyle Narlikar's creation field theory of gravitation for barotropic fluid distribution have been investigated. The solution have been obtained by assuming that $G = A^l$, where A is a scale factor and l is a constant $(l=-1)$. The physical properties of the model are studied.

II. Hoyle-Narlikar Theory

The Einstein field equations are modified by Hoyle and Narlikar [3-5] through the introduction of a massless scalar field usually called Creation field *viz.* C-field. The modified field equations are

$$R_i^j - \frac{1}{2}Rg_i^j = -8\pi G\left[\underset{(m)}{T_i^j} + \underset{(c)}{T_i^j}\right], \tag{1}$$

where $\underset{(m)}{T_i^j}$ is a matter tensor for perfect fluid of Einstein's theory given by

$$\underset{(m)}{T_i^j} = (\rho + p)v_i v^j - pg_i^j \tag{2}$$

and $\underset{(c)}{T_i^j}$ is a matter tensor due to C-field given by

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3. Alvarez, E. and Gavelo, M. B.: Phys. Rev. Lett., **51**, 931 (1983).

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$$\underset{(c)}{T_i^{\,j}} = -f\left(C_i C^j - \frac{1}{2} g_i^j C^\alpha C_\alpha\right). \tag{3}$$

Here ρ is the energy density of massive particles and p is the pressure. v_i are co-moving four velocities which obeys the relation $v_i v^j = 1$, $v_\alpha = 0$, $\alpha = 1, 2, 3$. $f > 0$ is the coupling constant between matter and creation field and $C_i = \frac{dC}{dx^i}$.

As T^{00} has negative value (i.e. $T^{00} < 0$), the *C*-field has negative energy density producing repulsive gravitational field which causes the expansion of the universe. Thus the energy conservation law reduces to

$$^{(m)}T^{ij}{}_{;j} = -{}^{(c)}T^{ij}{}_{;j} = fC^i C^j{}_{;j}\,, \tag{4}$$

i.e. the matter creation through a non-zero left hand side is possible while conserving the overall energy and momentum.
The above equation is identical with

$$m g_{ij} \frac{dx^i}{ds} - C_j = 0\,, \tag{5}$$

which indicates the 4-momentum of the created particle is compensated by 4-momentum of the *C*-field. In order to maintain the balance, the *C*-field must have negative energy.
Further, the *C*-field satisfies the source equation

$$f\,C^i{}_{;i} = J^i{}_{;i} \quad \text{and} \quad J^i = \rho \frac{dx^i}{ds} = \rho v^i\,, \tag{6}$$

where ρ is the homogeneous mass density.
The conservation equation for *C*-field is given by

$$\left(8\pi G T_i^{\,j}\right)_{;j} = 0\,. \tag{7}$$

The physical quantities in cosmology are the expansion scalar θ, the mean anisotropy parameter Δ, the shear scalar σ^2 and the deceleration parameter q defined as

$$\theta = 3H\,, \tag{8}$$

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2, \tag{9}$$

$$\sigma^2 = \frac{1}{2}\left(\sum H_i^{\,2} - 3H^2\right) = \frac{3}{2}\Delta H^2, \tag{10}$$

$$q = -\frac{\ddot{R}/R}{\dot{R}^2/R^2}\,, \tag{11}$$

where H is a Hubble parameter.

III. Metric and Field Equations

Kaluza-Klein metric is considered in the form

$$ds^2 = dt^2 - A^2\left[dx^2 + dy^2 + dz^2\right] - B^2 du^2\,, \tag{12}$$

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where A, B are scale factors and are functions of cosmic time t and u is a space-time co-ordinate.

It is assumed that creation field C is a function of time only *i.e.* $C(x,t) = C(t)$ and $\underset{(m)}{T_i^j} = (\rho, -p, -p, -p, -p)$.

The field equations (1) for metric (12) with the help of equations (2) and (3) are given by

$$3\frac{\dot{A}^2}{A^2} + 3\frac{\dot{A}\dot{B}}{AB} = 8\pi G\left(\rho - \frac{1}{2} f\,\dot{C}^2\right) \tag{13}$$

and

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + \frac{\ddot{B}}{B} + 2 = 8\pi G\left(\frac{1}{2} f\,\dot{C}^2 - p\right), \tag{14}$$

where the overdot $\left(\dot{\ }\right)$ denotes partial differentiation with respect to t.

The conservation equation (7) for metric (12) is

$$8\pi\dot{G}\left(\rho - \frac{1}{2} f\dot{C}^2\right) + 8\pi G\left[\dot{\rho} - f\dot{C}\ddot{C} + \rho\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) - f\dot{C}^2\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) + p\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\right] = 0. \tag{15}$$

IV. Solutions of Field Equations

The field equations (13) and (14) are two independent equations in five unknowns A, B, p, ρ and G. Hence two additional conditions may be used to obtain the solution.

We assume that the expansion θ is proportional to shear scalar σ. This condition leads to

$$B = A^n, \quad n \neq 1 \tag{16}$$

where n is a proportionality constant.

The motive behind assuming condition is explained with reference to Thorne (1967), the observations of the velocity red-shift relation for extra galactic sources suggest that Hubble expansion of the universe is isotropic today within ≈ 30 percent (Kantowski and Sachs (1966), Kristian and Sachs (1966)). To put more precisely, red shift place the limit $\frac{\sigma}{\theta} \leq 0.3$ on the ratio of shear σ to Hubble constant H in the neighborhood of our galaxy today. Collin *et al.* (1980) have pointed that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies that the condition $\frac{\sigma}{\theta}$ is constant.

With the help of (16), field equations (13), (14) and (15) take the form

$$3(n+1)\frac{\dot{A}^2}{A^2} = 8\pi G\left(\rho - \frac{1}{2} f\,\dot{C}^2\right), \tag{17}$$

$$(n+2)\frac{\ddot{A}}{A} + \left(n^2 + n + 1\right)\frac{\dot{A}^2}{A^2} = 8\pi G\left(\frac{1}{2} f\,\dot{C}^2 - p\right), \tag{18}$$

and

$$8\pi\dot{G}\left(\rho - \frac{1}{2} f\dot{C}^2\right) + 8\pi G\left(\dot{\rho} - f\dot{C}\ddot{C} + \rho(n+3)\frac{\dot{A}}{A} - f\dot{C}^2(n+3)\frac{\dot{A}}{A} + p(n+3)\frac{\dot{A}}{A}\right) = 0. \tag{19}$$

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Following Hoyle and Narlikar, the source equation of *C*-field: $C^i_{;i}=0$ leads to $C=t$ for large *r*. Thus $\dot{C}=1$.
Using $\dot{C}=1$, equation (17) leads to

$$8\pi G\rho=(3n+1)\frac{\dot{A}^2}{A^2}+4\pi Gf\ . \tag{20}$$

Using $\dot{C}=1$ and barotropic condition $p=\gamma\rho$ in (18), we have

$$(n+2)\frac{\ddot{A}}{A}+(n^2+n+1)\frac{\dot{A}^2}{A^2}=4\pi Gf-8\pi G\gamma\rho\,, \tag{21}$$

where $0\le\gamma\le1$.
Multiplying equation (20) by γ and adding (21) gives

$$(n+2)\frac{\ddot{A}}{A}+\left[(n^2+n+1)+3(n+1)\gamma\right]\frac{\dot{A}^2}{A^2}=(1-\gamma)4\pi Gf\ . \tag{22}$$

To obtain the deterministic solution, we assume

$$G=A^l\,, \tag{23}$$

where l is a constant and A is the scale factor.
Using equation (23) in equation (22), we get

$$2\ddot{A}+2\left[\frac{n^2+(1+3\gamma)n+(1+3\gamma)}{n+2}\right]\frac{\dot{A}^2}{A^2}=\frac{2(1-\gamma)4\pi f}{(n+2)}A^{l+1}\ . \tag{24}$$

Let $\dot{A}=F(A)$,

Thus $\ddot{A}=FF'$ with $F'=\frac{dF}{dA}$.

Using this in equation (24), it reduces to

$$\frac{dF^2}{dA}+2\left[\frac{n^2+n(1+3\gamma)+(1+3\gamma)}{n+2}\right]\frac{F^2}{A}=\frac{2(1-\gamma)4\pi f}{(n+2)}A^{l+1}\,, \tag{25}$$

which on simplification gives

$$F^2=2\frac{4\pi f(1-\gamma)}{(n+2)}\cdot\frac{A^{l+2}}{l+2+2\left[\frac{n^2+n(1+3\gamma)+(1+3\gamma)}{n+2}\right]}\ . \tag{26}$$

The integration constant has been taken zero for simplicity.
Equation (26) leads to

$$\frac{dA}{\sqrt{A^{l+2}}}=\sqrt{\frac{2(1-\gamma)4\pi f}{(n+2)\left[l+2+2\left(\frac{n^2+n(1+3\gamma)+(1+3\gamma)}{n+2}\right)\right]}}\,dt\ . \tag{27}$$

To obtain the determinate value of A in terms of cosmic time *t*, let $l=-1$.
Putting $l=-1$ in (27), we have

$$\frac{dA}{\sqrt{A}}=\sqrt{\frac{2(1-\gamma)4\pi f}{(n+2)\left[1+2\left(\frac{n^2+n(1+3\gamma)+(1+3\gamma)}{n+2}\right)\right]}}\,dt\ . \tag{28}$$

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On integration equation (28) gives

$$A=(at+b)^2, \tag{29}$$

where $a=\frac{1}{2}\sqrt{\frac{2(1-\gamma)4\pi f}{(n+2)\left[1+2\left(\frac{n^2+n(1+3\gamma)+(1+3\gamma)}{n+2}\right)\right]}}.$ (30)

and $b=\frac{N}{2}$. (31)

Here N is a constant of integration.

Thus we have $G=A^{-1}=(at+b)^{-2}$. (32)

Using equations (29) and (32), equation (20) simplifies to

$$8\pi\rho=12a^2(n+1)+4\pi f\ . \tag{33}$$

With the help of (29), the metric (12) leads to

$$ds^2=dt^2-(at+b)^4[dx^2+dy^2+dz^2]-(at+b)^{4n}du^2\ . \tag{34}$$

Using $p=\gamma\rho$, equation (19) leads to

$$8\pi(G\dot{\rho}+\dot{G}\rho)-4\pi\dot{G}f\dot{C}^2-8\pi Gf\dot{C}\ddot{C}-8\pi G(n+3)f\dot{C}^2\frac{\dot{A}}{A}+8\pi G(n+3)\rho\frac{\dot{A}}{A}(1+\gamma)=0\,. \tag{35}$$

With the help of equations (29) and (32), equation (33) yields

$$\frac{d\dot{C}^2}{dt}+(2n+5)\frac{2a}{(at+b)}\dot{C}^2=\frac{a[(2n+4)+(2n+6)\gamma]}{(at+b)}\left[\frac{3a^2(n+1)}{\pi f}+1\right]. \tag{36}$$

To reach the deterministic value of $\dot{C}$, we assume $a=1$ and $b=0$.
Thus (36) leads to

$$\frac{d\dot{C}^2}{dt}+\frac{2(2n+5)}{t}\dot{C}^2=\frac{[(2n+4)+(2n+6)\gamma]}{t}\left[\frac{3(n+1)}{\pi f}+1\right]. \tag{37}$$

On integration equation (37) reduces to

$$\dot{C}^2t^{4n+10}=[(2n+4)+(2n+6)\gamma]\left[\frac{3(n+1)}{\pi f}+1\right]\int\frac{1}{t}t^{4n+10}dt\,. \tag{38}$$

Simplifying equation (38), we get

$$\dot{C}=\sqrt{\frac{(n+2)+(n+3)\gamma}{4(2n+5)}\left[\frac{3(n+1)}{\pi f}+1\right]}. \tag{39}$$

On integration of (39), we get

$$C=t\sqrt{\frac{(n+2)+(n+3)\gamma}{4(2n+5)}\left[\frac{3(n+1)}{\pi f}+1\right]}. \tag{40}$$

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Taking $\pi f = \left[\frac{3(n+1)}{\frac{4(2n+5)}{(n+2)+(n+3)\gamma} - 1} \right]$, we find $\dot{C} = 1$, which agrees with the value used in the source equation. Thus creation field C is proportional to time t and the metric (12) for the constraints mentioned above, leads to

$$ds^2 = dt^2 - t^4\left(dx^2 + dy^2 + dz^2\right) - t^{4n}du^2 . \tag{41}$$

For the model (41), the physical parameters homogeneous mass density (ρ), Gravitational constant (G), the scale factor (A) and the deceleration parameter (q) are given by

Energy density (ρ),

$$8\pi\rho = 12(n+1)\left[1 + \frac{(n+2)+(n+3)\gamma}{(7n+18)-(n+3)\gamma}\right]. \tag{42}$$

Gravitational constant G ,

$$G = t^{-2}, \tag{43}$$

Scale factor A,

$$A = t^2 , \tag{44}$$

Spatial Volume,

$$V = t^{2n+6}$$

Expansion Scalar (θ) ,

$$\theta = \frac{2(n+3)}{t} , \tag{45}$$

Anisotropic parameter (Δ) ,

$$\Delta = \frac{3\left[(n+2)^2 + 3\right]}{4(n+3)^2}, \tag{46}$$

Shear Scalar (σ),

$$\sigma^2 = \frac{6\left[(n+2)^2 + 3\right]}{t^2}, \tag{47}$$

Deceleration parameter (q) ,

$$q = -\frac{1}{2} . \tag{48}$$

Physical Behavior of the Model:

From equations (16) and (29), the spatial scale factors are finite at the initial epoch $t = 0$, hence the model has a point type singularity (MacCallum (1971)). From equations (45) and (46), we observe that the spatial volume is zero and expansion scalar is infinite at $t = 0$ which show that the universe starts evolving with zero volume at $t = 0$ which is big bang scenario.

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In fig. 1, the plot of Gravitational constant versus cosmic time t has shown. We observed that, the gravitational constant G is initially infinite for the model. The Gravitational constant G is a decreasing function of time and approaches to zero for large values of t. In most variable G cosmologies (Weinberg (1972), Norman (1986)) G is a decreasing function of time. But the possibility of an increasing G has also been suggested by several authors (Abdel-Rahaman, A. M. M. (1990), Pradhan *et al.* (2007), Singh *et al.* (2008), Singh and Kale (2009), Singh *et al.* (1998)).

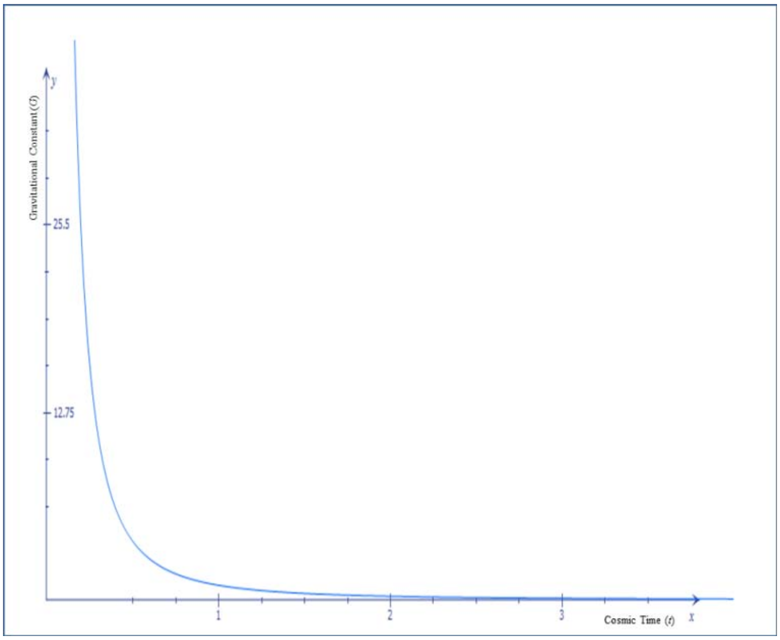

Figure 1 : Plot of Gravitational Constant (G) versus Cosmic Time (t)

From equations (46) and (48), the mean anisotropy parameter Δ is constant and $\frac{\sigma^2}{\theta^2} = constant\ (\neq 0)$ is also constant, hence the model is anisotropic throughout the evolution of the universe (*i.e.* the model does not approach isotropy).

From equation (42), it is observed that the energy density is constant throughout the evolution of the universe hence the model represents steady state universe.

From equation (48), the value of deceleration parameter is $q = -\frac{1}{2}$, which lies between -1 and 0, hence the model is accelerating throughout the evolution.

V. Conclusion

Kaluza-Klein cosmological models have been investigated in Hoyle Narlikar's Creation field theory of gravitation. The source of energy momentum tensor is considered as barotropic fluid with varying gravitational constant G. It is worth to mention that the model obtained is point type singular, expanding, shearing. The mean anisotropic parameter is constant and $\frac{\sigma^2}{\theta^2} = \text{constant}\ (\neq 0)$, hence the model is anisotropic throughout the evolution of the universe. (i.e. the model is not isotropic for the complete evolution). The deceleration parameter is constant $\left(q = -\frac{1}{2}\right)$, which lies between -1 and 0, hence the model is accelerating throughout the evolution.

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Notes

References Références Referencias

1. Abdel-Rahaman, A. M. M.: Gen. Relati. Grav. 24, 665 (1990).
2. Adhav, K. S., Gadodia, P. S., Bansod, A. S., Pund, A. M.: J. Vect. Rel., **5**, 2, pp. 1-11, (2010).
3. Alvarez, E. and Gavelo, M. B.: Phys. Rev. Lett., **51**, 931 (1983).
4. Bali, R. and Saraf, S.: Proc. Natl. Acad. Sci., India, Section A, Phys. Sci., **83** (1), pp. 29-32, (2013 a). doi: 007/s 40010-012-0055-3.
5. Bali, R. and Saraf, S.: Int. J. Theor. Phys., **52**, pp. 1645-1653 (2013 b). doi: 10.1007/s 10773-013-1486-
6. Bali, R. and Saraf, S.:Prespacetime J., **4**, No. 5, pp. 545-553 (2013 c).
7. Bali & Kumawat, M.: Int. J. Theo. Phys., **50**, 27-34 (2011).
8. Bali, R., Kumawat, M.: Ultra Sci., **24** (2) A, 259-268 (2012).
9. Barrow, J. D.: Mon. Not. Roy. Astron. Soc., **184**, 677- 682 (1978).
10. Bondi, H., Gold, T.: Mon. Not. R. Astron. Soc., **108**, 252 (1948).
11. Brans, C. and Dicke, R. H.: Phys. Rev., **124**, 3, 925-935 (1961).
12. Canuto, V., Hsieh, S. H. and Adms, P. J.: Phys. Rev. Lett., **39**, 429 (1977).
13. Chatterjee, S. and Banerjee, A. : Gen. Rel. Grav.. **36**, No. 2, (2004).
14. Chi, L. K.: Gen. Rel. Grav., **22**, 1347 (1990).
15. Coley, A. A.: Astrophys. J., **427**, 585 (1994).
16. Collins, C. B., Glass, E. N., Wilkinson, D. A. : Gen. Relat. Grav., **12**, 10, pp. 805-823 (1980).
17. de Leon, P. J.: Gen. Rel. Grav., **20**, 539 (1988).
18. Dirac, P. A. M.: Nature, **139**, 323-323 (1937).
19. Fukui, T. : Gen. Rel. Grav., **25**, 931 (1993).
20. Gamow, G.: Proc. Nation. Acad. Sci. (USA), **57**, 2, 187 (1967).
21. Ghate, H. R., Salve, S. A.: Prespacetime J., **5**, No.3, pp. 198-206 (2014).
22. Ghate H. R., Salve S. A.: Prespacetime J., **5**, No. 3, pp 198-206 (2014).
23. Ghate H. R., Salve S. A.: Res. J. Recent. Sci.. **3**, (ISC-2013), pp. 53-57 (2014).
24. Hoyle, F. and Narlikar, J. V.: Proc. Roy. Soc. A, Math. Phys. Sci., **282**, 1389, pp 191-207 (1964 a). Doi:10.1098/rspa.1964.0227
25. Hoyle, F. and Narlikar, J. V.: Proc. Roy. Soc. A, Math. Phys. Sci.,, **278**, 1375, pp 465-478 (1964 b). doi:10.1098/rspa.1964.0076
26. Hoyle, F. and Narlikar, J. V. : Proc. Roy. Soc. of Lon.A, Math. Phys. Sci., **282**, 1389, pp 178-183 (1964 c).
27. Kaluza, T., Sitz. Preuss : Acaf. Wiss. Phys. Mth., KI. **966**, (1921).
28. Kantowski, R. K. Sachs : J. of Mathem. Phys., **7**, 3, pp. 443 196.
29. Klein, O.: Phys., **37** (12), 895 (1926).
30. Kristian, R. K. Sachs: Astrophysical J., **143**, pp. 379, (1966)
31. Liu, H, Wesson, P. S.: Int. J. Mod. Phys..D, **3**, 627 (1994).
32. Maccallum, M. H. H.: Commun. Math. Phys., **20**, 57 (1971).
33. Marciano, W. J.: Phys. Rev. Letters., 52, (1984), 489.
34. Narlikar, J. V. : Nat. Phys. Sci., **242**, pp. 135-136 (1973).
35. Norman, E. B.:Am. J. Phys. 54, 317 (1986).
36. Pochoda, P. and Schwarzschild, M.: Astro. Phys. J., **139**, 587 (1963).
37. Pradhan, A., Singh, A. K., Otarod, S.: Rom. J. Phys., **52**, 445 (2007).
38. Ranjibar - Daemi, S., Salam, A., Strathdec, J.: Phys. Lett. B., **135**, 388 (1984).
39. Singh, J. P., Pradhan, A., Singh, A. K. : Astrophys. Space Sci., **314**, 83 (2008).
40. Singh, G. P., Kale, A. Y.: Int. J. Theo. Phys., **48**, 3158 (2009).
41. Singh, T., Beesham, A., Mbokazi, W. S.: Gen. Relat. Grav., **30**, 537 (1998).

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42. Singh, T. and Chaubey, R. : Astro. Spa. Sci., **321**, pp. 5-18 (2009). Doi: 10.1007/s10509-009-9989-6.
43. Smooth, G. F. *et al.* : Astrophys. J., **396**, L1 (1992).
44. Thorne K. S. : Astrophys. J., **148**, pp. 51 (1967).
45. Weinberg, S.:Grav. And Cos., Wiley : New York (1972).

Notes

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General Definition of the Mean for Non-A dditive Variables

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GENERALDEFINITIONOFTHEMEANFORNONADDITIVEVARIABLES

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Ref

3. Billingsley, Patrick (1995), Probability and Measure (Third ed.), John Wiley sons, ISBN 0-471-00710-2.

General Definition of the Mean for Non-Additive Variables

Vaclav Studeny

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I. Introduction

a) *General observation*

If we sum some deterministic variable, we obtain the same result as if we substitute all of the variable by its arithmetical mean. In stochastic case it is true approximately and with a big probability if we sum big enough number of independent and identically distributed variables with finite second moment as central limit theorem show us (see [3], [5]). But in life we do not deal with summing only. If we will multiply them, for instance, and if we sustitue the values by arithmetical mean, the result will differ more if the values of the variables will vary from the mean more. But if we used geometrical mean except of arithmetical ones, the situation will be exactly as in previous case of summation. This is an observation which bring us to our simple, but very useful concept.

There are a lot of other reasons why we need to represent a big set of data by only one number. Expected value or arithmetical mean is the most popular way how to do it, but not the best in all of the cases.

To make the problem of mean value more clear let us consider investment funds. They can measure the profit in different years by the rate of return, but rate of return is not additive variable too, this means, that the sum of the interest rate is not quantity with the sens. What information gives to us the arithmetical mean of the rates of return per different year. The answer is not very heartwarming.

Let us suppose that the number of time periods (say of years to be in the common practical situation) is N, and rate of profit in i-th period is ξ_i. let us suppose, that mean value of rates is equal to K, hence:

$$\frac{1}{N}\sum_{i=1}^{N}\xi_i = K \tag{1}$$

overall rate of profit by all of the N time periodes is

$$\zeta = \prod_{i=1}^{N}(1+\xi_i) - 1 \tag{2}$$

If $N \geq 2$ equations (1) and (2) does not determine ζ — which is the value we are actually interested in — in unique way. On the set of all $(\xi_i)_{i=1}^{N}$ fulfilling (1) take the function ζ different values: its maximum is $(1+K)^N - 1$, and it is not bounded below indeed, hence: *The knowledge of arithmetical mean admit*

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to bounded the rate of the whole return above but not below. With the same arithmetical mean the rate of the whole return can be arbitrarily small.

1.1 Proof: If $N = 2$ then if ξ_1 a ξ_2 are rates of return in a two successive time periods and ζ rate of return per two period and if K is the arithmetical mean of ξ_1 a ξ_2 we have:

$$(1+\xi_1)\cdot(1+\xi_2) = 1+\zeta$$

$$\frac{1}{2}\,\xi_1 + \frac{1}{2}\,\xi_2 = K$$

hence

$$\xi_2 = -\xi_1 + 2\,K$$

$$\zeta = 2\,K - {\xi_1}^2 + 2\,\xi_1\,K \tag{3}$$

The last equation gives attitude toward actual rate of return per two periods and rate of return per one of the period in the case of constant average rate of return K. The dependence is analytical so that we can find the maximum as the zero point of the derivative:

$$\frac{\partial}{\partial \xi_1}\,\zeta = -2\,\xi_1 + 2\,K = 0 \Longleftrightarrow \xi_1 \xi_2 = K \tag{4}$$

and

$$\frac{\partial^2}{\partial {\xi_1}^2}\,\zeta = -2 \tag{5}$$

so that if average rate of return is constant equal to K then:

- rate of return ζ have maximum $\zeta = (K+1)^2 - 1$ if $\xi_1 = \xi_2 = K$, i. e. if all of the rates are equal.
- If one of the rate is equal to zero, $\xi_1 = 0$, and the second is equal to $\xi_2 = 2K$ then the whole rate of return is $\zeta = 2K$.
- If one of the rate $\xi_1 = -1$ i. e. we lost all of the capital, it is enough, to second rate of return be $\xi_2 = 1 + 2\,K$ and the mean value will stay K, while whole rate of return will be $\zeta = -1$. It means that the investor lost all of the invested resources, but arithmetical mean of rates of the return is still K. This fact show the marketing potential of arithmetical mean in the case. You can report the average rate of return 10% while you bereave your clients for all of the money.
- further if ξ_1 goes to $-\infty$, then ξ_2 goes to $-\infty$ and total rate of return goes to $-\infty$ but we can still keep the positive average.

A similar situation occurs also in the case when the number of time periods is greater. If we know arithmetical mean K of the rates of return the rate of return per N time periods is

$$\left(1 + N\,K - \left(\sum_{i=1}^{N-1} \xi_i\right)\right)\left(\prod_{i=1}^{N-1}\,(1+\xi_i)\right) = 1+\zeta \tag{6}$$

and the function ζ, with arguments made of $N-1$ rates of returns ξ_i has again maximum

$$\zeta = (1 + K/N)^N - 1 \tag{7}$$

in the point

$$(\xi_i)_{i=1}^N = (K)_{i=1}^n \tag{8}$$

and ζ is not bounded above.

- Present value of invested capital should be equal to 0 independently on the arithmetical mean of rates of return. It is necessary and sufficient if one of the rates is equal to -1.
- If one of a fund has (arithmetical) average of rate returns higher than other, it does not necessary mean, that it has higher total rate of return per all of the periods (that it brings higher profit to its savers). For the marketing point of view to show mean o rates of returns is better advertisement for the funds which have higher diversity between rates of return.

We can see, that arithmetical mean does not gives to us any substantive information in that case.

Similar situation is, of course, nowadays popular average temperature of the earth (see [4]). What rush people make about it notwithstanding this quantity is not satisfactorily defined. The number, which

Ref

4. Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change.

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should be a result of some integration does not realize themselves anywhere. This fact will goes more clear, if we imagine that we calculated the average temperatures of stars that we see. The result should not be the temperature of any place in the universe. /it There exists average of temperatures, but not average temperature.

In the case of investment funds, one can say, that we only used bad mean. That usage of the geometrical mean, brings to us much more better result. If we should compute the mean using the rule:

$$\zeta = \left(\prod_{i=1}^{N} (1+\xi_i) \right)^{\left(\frac{1}{N}\right)} - 1 \tag{9}$$

expect of

$$\zeta = \frac{1}{N} \sum_{i=1}^{N} \xi_i \tag{10}$$

we should obtain something, connected with the rate of return by the unique and the simple way: $\left(\prod_{i=1}^{N} (1+\xi_i) \right) = (1+\zeta)^N$. Particularly funds with higher mean rate of return should have higher whole return.

In the history there are more cases of attempts to generalize the notion of mean value. Two of them I find the most interesting and the most deep.

II. Two General Conceptions of Mean

With one generalization come János Dezs Aczél, the great Hungarian mathematician living in Canada. As en expert on the functional equations, he watch the question: how big family of possible means of two values one can obtain if he compute aritmetical mean of images of this values by some mapping and then he returned back with inverse mapping. To be more precise on one side we can imagine the mean as a binary operation ∘ an on the other hand as a value

$$(x, y) \mapsto k^{-1} \left(\frac{k(x) + k(y)}{2} \right) \tag{11}$$

General properties of this mean are described in [Azzel p. 229]. Main result, which shows quite general character of this definition is:

2.2. Theorem: There exists a continuous and strictly monotonic function k which gives a value of a mean; (11) holds if, and only if, $\circ: I^2 \mapsto I$ is continuous and strictly increasing in booth variables, idempotent, commutative (symmetric) and medial (bisymetric) which means:

$$(x \circ y) \circ (z \circ w) = (x \circ z) \circ (y \circ w) \tag{12}$$

It is rely beautiful result but not practical enough.

On the other way, in the Book Means and Their Inequalities, P. S. Bullen, D. S. Mitrinović P. M. Vasić (see [2] p. 372) attempt to axiomatic definition of the mean:

- it is symmetric,
- homogeneous of degree 1,
- reflexive
- associative, $\mathrm{f}(a_1, \ldots, a_n) = \mathrm{f}(\mathrm{f}(a_1, \ldots, a_p), \ldots, \mathrm{f}(a_1, \ldots, a_p), a_{p+1}, \ldots, a_n)$
- increasing in each variable.

This definition is very good of generaliyation od the term, but not good enough for us, from a lot of reasons. One of the reason should be that a average interest rate of saving is not symmetrical (and the symmetry is not disrupt only by adding of some weights). By latest interest are interested all of the deposits, but by the first one only the first deposit.

The drawback of these conceptions is, that the mean is related only to the values, it is computed from but not to their meaning, which means to this what we are going to do with the values. This fact has a deep impact. There are lot of monez ion the

(For instance: arithmetical mean has sense only for additive variables it means variables we are going to sum but common definition does not respect this fact.) Our conception is at once more simple,

Ref

2. P.S. Bullen, D.S. Mitrinovic, P.M. Vasic, Means and Their Inequalities, D. Reidel Publishing Com-any, Dordrecht, 1988.

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more general. It is, in some way, the generalization of the Azzel's conception. The central role plays the *objective function* in it. This function characterize what are the values, we deal with, relevant for.

III. General Definition of the Mean

3.3. *Difinition:* Z is the mean value of $(z_i)_{i=1}^n$ *with respect to function* $\mathrm{F}\colon \coprod_{i\in I} X^i \to Y$ if

$$\mathrm{F}\left((z_i)_{i=1}^n\right) = \mathrm{F}\left((Z)_{i=1}^n\right) \tag{13}$$

(i. e. $F(z_1, z_2, \ldots, z_n) = F(Z, Z, \ldots, Z)$).
If F is injection, then the mean value is determined by unique way.
If $\mathrm{Im}\,(f) = \mathrm{Im}\,(f|_\Delta)$, where Δ is diagonal: $\Delta = \{(x)_{i=1}^n \,|x \in X\}$, mean exists for every input values.

3.4. *Example:* If F is sumation, we obtain arithmetical mean. If F is multiplying, we obtain geometrical mean.

In the case of an election, the votes are summed. Arithmetical mean of number of votes corespondents to proportional representation in parliment.
Rates of inflation does multiply to each other. If

$$\iota := [.01,\ .03,\ .02,\ .01,\ .03] \tag{14}$$

is the rate of inflation per 5 time periodes, the rate of inflation per all of the time is

$$\left(\prod_{j=1}^{5} (1+\iota_j)\right) - 1 = .10386857 \tag{15}$$

and if the rate of inflation should be the same per all of the time subintervals an should be equal to

$$\kappa := \left(\prod_{j=1}^{5} (1+\iota_j)\right)^{1/5} - 1 = .019960783 \tag{16}$$

then the rate of inflation per all of the time should be the same:

$$\left(\prod_{j=1}^{5} (1+\kappa)\right) - 1 = .10386857 \tag{17}$$

Hence if the objective function will be

$$\iota \mapsto \left(\prod_{j} (1+\iota_j)\right) - 1 \tag{18}$$

then the mean with respect to this function is (except for some addition of unit) geometrical mean.

IV. Mean Rate of Return of Pension Funds

Let us suppose, that we know rate of return of pension funds. We are looking for mean rate of return, which admits to us compare this funds.

Let us suppose, that deposits of the savers are constant for a long of the time. We can suppose, that, if the deposit per year (together with state grants) are equal to x and if the rate of return, in consequence of dividing of profit in the year i is equal to ξ_i saver should have after the time N:

$$x\left(\sum_{j=1}^{N} \prod_{i=1}^{j} (1+\xi_{N-i+1})\right) \tag{19}$$

For saver is important mean rate of return with respect to the function

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$$(\xi)_{i=1}^{T} \to \left(\sum_{j=1}^{N} \prod_{i=1}^{j} (1+\xi_{N-i+1}) \right) \tag{20}$$

using our definition, this rate of the return is ζ, fulfilling the equation

$$K = \sum_{j=1}^{N} \left(\prod_{i=1}^{j} (1+\xi_{N-i+1}) \right) = \sum_{j=1}^{N} \left(\prod_{i=1}^{j} (1+\zeta) \right) = -\frac{-(1+\zeta)^{(N+1)} + 1 + \zeta}{\zeta} \tag{21}$$

Notes

It is an algebraical equation of the degree N.

If the time period is only one, the mean rate of return is equal to the rate of return per this period:

$$\zeta = -1 + K \tag{22}$$

If the number of time periods is equal to 2, the equation has two solutions, in general.

If the total remittance will be higher than $\frac{1}{4}$ of all saved remittances all of the solution will be real. In the case — it seem to be reasonable — when total remittance will be higher than double of saved remittances one solution will be positive an will be equal to

$$\zeta = -\frac{3}{2} + \frac{1}{2}\sqrt{1+4K} \tag{23}$$

In case of 3 periods, there is the only one real mean rate of return:

$$\zeta = 1/6\,\frac{\left(28+108K+12\sqrt{9+42K+81K^2}\right)^{2/3} - 8 - 8\sqrt[3]{28+108K+12\sqrt{9+42K+81K^2}}}{\sqrt[3]{28+108K+12\sqrt{9+42K+81K^2}}} \tag{24}$$

If the number of time periods is higher than four, we are not able to solve the equation (21) algebraically, and we have to solve it numerically.

4.5. *Example:* There was some pension funds in Czech republic during the five year between the years 1999 and 2003. This was an period with very wildly changed rates of return of the pension funds. We will concentrate onto these:

(1) ČSOB Progres, (2) Zemský PF, (3) PF KB, (4) ING, (5) Credit Suisse, (6) PF ČP, (7) Allianz, (8) Generali, (9) Nový ČP, (10) ČSOB Stabilita, (11) PF ČS, (12) Hornický PF; The following table show us their rate of return:

Funds	**rate of return (in %)**				
	1999	2000	2001	2002	2003
ČSOB Progres	7.7	5.6	3.9	4.3	4.3
Zemský PF	7.0	5.0	4.6	4.1	4.01
PF KB	7.2	4.9	4.4	4.6	3.4
ING	6	4.4	4.8	4	4
Credit Suisse	6.5	4.1	4.3	3.4	3.4
PF ČP	6.6	4.5	3.8	3.2	3.1
Allianz	6	3.8	4.4	3.7	3
Generali	5.3	3.6	4.6	4.1	3
Nový ČP	5.6	3.8	4.1	3.5	3.34
ČSOB Stabilita	6.1	4.2	3.2	3.0	2.34
PF ČS	4.4	4.2	3.8	3.5	2.64
Hornický PF	4.4	2	2.8	3.2	2.44

(25)

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Českomoravská stavební spořitelna published for agents middleman papers for information. They ordered the pension funds by the arithmetical mean of their rate of return, which are, as we have explained, irrelevant (second row of following table, values are in percents). If we computed the mean with respect to objective function (ξ_i^k is rate of return of k-th fund at i-th time period):

$$\Phi\colon \left(\xi^k\right)_{i=1}^5 \longmapsto \sum_{j=1}^{5} \prod_{i=1}^{j} \left(1+\xi_{5-i+1}^k\right)\Bigg) \tag{26}$$

(third column of following table, values are in percents) we can see, that it happens in some cases, that funds with higher rate of return are behind funds with smaller rate of return:

Funds	mean values of the rates of return	
	arithemtical (*inappropriate*)	vith respect to the Φ (*appropriate*)
ČSOB Progres	5.16	4.638
Zemský PF	4.94	4.501
PF KB	4.90	4.394
ING	4.64	4.358
Credit Suisse	4.34	3.895
PF ČP	4.24	† 3.704
Allianz	4.18	† 3.787
Generali	4.12	† 3.857
Nový ČP	4.06	3.757
ČSOB Stabilita	3.76	‡ 3.203
PF ČS	3.70	‡ 3.438
Hornický PF	2.96	2.789

(27)

(†) *We can see, that the fund Allianz, which stay on the 7th place had the rate of return better, than PF ČP which is on the 6th place and better rate of return than this two had thwe fund Generali, which stay behind booth others. Booth are in the papers presented as the funds with the same awarage rate of return equal to 4.2.*

(‡) *The fund ČSOB Stabilita, the papers in question are written for its propagation too, is presented as the fund with mean rate of return (arithmetic mean 3.8) higher than rate of return of fund PF ČS (arithmetic mean 3.7) but it should be presented (with mean 3.2) behind the fund PF ČS (mean 3.4)!*

If we are going to interpreted the result, we have to determine the sensitivity on the change of interest rate. We can substitute the rates of return by mean rates of return we have computed. The relative change of saved money after time T if the interest rate is changed from ξ onto ζ, (i. e.. rate of the profit or loss on the whole saved quontity) is

$$\frac{(1+\zeta)^T\,\xi - \xi - (1+\xi)^T\,\zeta + \zeta}{\left((1+\xi)^T - 1\right)\,\zeta}. \tag{28}$$

In our case is $T = 5$ and the values for differen funds are in the same order as in the previous tables, they are in percents:

0	−0.27	−0.49	−0.56	−1.5	−1.8	−1.7	−1.5	−1.7	−2.8	−2.4	−3.6
0.27	0	−0.21	−0.29	−1.2	−1.6	−1.4	−1.3	−1.5	−2.6	−2.1	−3.4
0.49	0.21	0	−0.072	−0.99	−1.4	−1.2	−1.1	−1.3	−2.4	−1.9	−3.2
0.56	0.29	0.072	0	−0.92	−1.3	−1.1	−1.0	−1.2	−2.3	−1.8	−3.1

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1.5	1.2	1.0	0.93	0	−0.38	−0.21	−0.075	−0.27	−1.4	−0.91	−2.2	
1.9	1.6	1.4	1.3	0.38	0	0.17	0.31	0.11	−1.0	−0.53	−1.8	
1.7	1.4	1.2	1.1	0.21	−0.17	0	0.14	−0.061	−1.2	−0.70	−2.0	
1.6	1.3	1.1	1.0	0.075	−0.31	−0.14	0	−0.20	−1.3	−0.83	−2.1	
1.8	1.5	1.3	1.2	0.28	−0.11	0.061	0.20	0	−1.1	−0.64	−1.9	
2.9	2.6	2.4	2.3	1.4	1.0	1.2	1.3	1.1	0	0.47	−0.83	
2.4	2.1	1.9	1.9	0.92	0.53	0.70	0.84	0.64	−0.47	0	−1.3	
3.8	3.5	3.3	3.2	2.2	1.8	2.0	2.2	2.0	0.83	1.3	0	(29)

In the i-th row j-th column the percentage of how much should be the whole amount higher if we invested into j-th fund except of into i-th fund is written. So in the proper choose of the fund we should have approximately 4 percent more than in the improper choose.

V. Mean Rate of Return of Simultaneous Savings

Suppose, that we have two accounts, each interested with different interest rate ξ_1 and ξ_2. We divide the capital in amount $x_1 + x_2$ between them this way, than we will have in he first capital in amount x_1 and in the second x_2. Objective function — future value in time t is the function $\Psi_{x_1,x_2,t}\colon (\xi_1, \xi_2) \mapsto x_1 (1+\xi_1)^t + x_2 (1+\xi_2)^t$ and it depende on three parameters. Mean interest rate with respect to this function is the solution ζ of the equation:

$$x_1 (1+\xi_1)^t + x_2 (1+\xi_2)^t = (x_1 + x_2)\ (1+\zeta)^t \tag{30}$$

hence

$$\zeta = \left(\frac{x_1\ (1+\xi_1)^t + x_2\ (1+\xi_2)^t}{x_1 + x_2} \right)^{\left(\frac{1}{t}\right)} - 1 \tag{31}$$

And it is generalized exponential mean. Worthy to note is its dependence on the time t, especially the limits $t \to \infty$.

$$\lim_{t\to\infty} \left(\frac{x_1 (1+\xi_1)^t + x_2 (1+\xi_2)^t}{x_1 + x_2} \right)^{\left(\frac{1}{t}\right)} - 1 = \lim_{x\to\infty} e^{\ln\left(\frac{x_1\ (1+\xi_1)^t + x_2\ (1+\xi_2)^t}{x_1+x_2}\right)^{\left(\frac{1}{t}\right)}} - 1 =$$

$$= \lim_{x\to\infty} e^{\dfrac{\ln\left(x_1\ (1+\xi_1)^t + x_2\ (1+\xi_2)^t\right) - \ln(x_1+x_2)}{t}} - 1 \tag{32}$$

using the L'Hospital rule we obtain

$$\lim_{t\to\infty} \frac{\ln\left(x_1\ (1+\xi_1)^t + x_2\ (1+\xi_2)^t\right) - \ln(x_1+x_2)}{t} =$$

$$= \lim_{t\to\infty} \frac{x_1\ (1+\xi_1)^t\ \ln(1+\xi_1) + x_2\ (1+\xi_2)^t\ \ln(1+\xi_2)}{x_1\ (1+\xi_1)^t + x_2\ (1+\xi_2)^t} \tag{33}$$

and after canceling by the term $(1+\xi_2)^t$we obtain:

$$\lim_{t\to\infty} \frac{x_1 \ln(1+\xi_1) \left(\frac{1+\xi_1}{1+\xi_2}\right)^t + x_2 \ln(1+\xi_2)}{x_1 \left(\frac{1+\xi_1}{1+\xi_2}\right)^t + x_2}. \tag{34}$$

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Supposing: $\xi_1 < \xi_2$

$$\lim_{t\to\infty} \left(\frac{x_1 \, (1+\xi_1)^t + x_2 \, (1+\xi_2)^t}{x_1 + x_2} \right)^{\left(\frac{1}{t}\right)} - 1 = \xi_2, \tag{35}$$

we obtain the greatest of both number. It looks mysteriously: If we save divide capital into two parts and save booth with some interest rates per unit of time ξ_1 and ξ_2 respectively , we obtain the same result as to save booth parts with interest rate per unit of time ζ, and if the time goes to infinity the difference bvetween ζ and the maximum of ξ_1 and ξ_2 goes to the zero On the other hand:

$$\lim_{t\to 0} \zeta = t\,(1+\xi_1)^{\frac{x_1}{x_1+x_2}} \,(1+\xi_2)^{\frac{x_2}{x_1+x_2}} - 1 \tag{36}$$

which is generalized geometrical mean.

The same result we obtain also in the case of more than two accounts. If the rates of interest will be (ξ_i) an the initial state of accounts will be (x_i) then the state function will be

$$(\xi_i) \longmapsto \sum_{i=1}^{n} x_i \,(1+\xi_i)^t \tag{37}$$

and mean value (ξ_i) with respect to the function will be solution ζ of the equation

$$\sum_{i=1}^{n} x_i \,(1+\xi_i)^t = \sum_{i=1}^{n} x_i \,(1+\zeta)^t = (1+\zeta)^t \sum_{i=1}^{n} x_i \tag{38}$$

hence

$$\zeta = \left(\frac{\sum_{i=1}^{n} x_i \,(1+\xi_i)^t}{\sum_{i=1}^{n} x_i} \right)^{\frac{1}{t}} - 1 \tag{39}$$

and if $\xi_n = \max_i (\xi_i)$ we can show in the same way, that

$$\lim_{t\to\infty} \zeta = e^{\left(\lim_{t\to\infty} \frac{\left(\sum_{i=1}^{n-1} x_i \ln(1+\xi_i) \left(\frac{1+\xi_i}{1+\xi_n}\right)^t\right) + x_n \ln(1+\xi_n)}{\left(\sum_{i=1}^{n-1} x_i \left(\frac{1+\xi_i}{1+\xi_n}\right)^t\right) + x_n} \right)} - 1 = \xi_n = \max_i (\xi_i) \tag{40}$$

a

$$\lim_{t\to 0} \zeta = \prod_{i=1}^{n} (1+\xi_i)^{\frac{x_i}{\sum_{j=1}^{n} x_j}} \tag{41}$$

Aggregate interest rate per unit of time converges to maximal interest rate per unit of time with time going to infinity.

The similar situation is the saving annuities:

VI. Mean Interest Rate of Saving

Suppose, savings with annuities x_i in the moments $t \in \mathbb{N}$ with constants interest rates ξ_i. Objective function is sum of future values of savings with different interest rates:

$$\Phi_{(x_i)_{i=1}^k, N} := \left((\xi_i)_{i=1}^k \right) \longmapsto \sum_{i=1}^{k} \left(\sum_{j=0}^{N-1} x_i \,(1+\xi_i)^j \right) = \sum_{i=1}^{k} \frac{x_i \left((1+\xi_i)^N - 1 \right)}{\xi_i} \tag{42}$$

saved amount, $(x_i)_{i=1}^k$ and number of savings with different interest rates k are parametrs Φ.

Mean interest rate of savings $\Xi = \Xi((x_i), N)$ with respect to function $\Phi_{(x_i)_{i=1}^k, N}$ is solution of equation:

$$xxx := \frac{\left(\sum_{i=1}^{k} x_i \right) \left((1+\Xi)^N - 1 \right)}{\Xi} = \sum_{i=1}^{k} \frac{x_i \left((1+\xi_i)^N - 1 \right)}{\xi_i} \tag{43}$$

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We will investigate dependence on parametr N, i. e. on the number of saved ammounts. And again:

$$\lim_{t\to\infty} \Xi = \max_i (\xi_i) \tag{44}$$

6.6. Proof: Let

$$\eta = \eta(\xi_i, x_i, n, T) \tag{45}$$

be mean interest rate with respect to the function

Notes

$$\Phi := \xi \to \sum_{\tau=1}^{n} \left(\sum_{j=1}^{k} x_j \, (1+\xi_j)^{(T-\tau)} \right) \tag{46}$$

i. e. solutiobn of the equation

$$\sum_{\tau=1}^{n} \left(\sum_{j=1}^{k} x_j \, (1+\xi_j)^{(T-\tau)} \right) = \left(\sum_{j=1}^{k} x_j \right) \quad \sum_{\tau=1}^{n} (1+\eta)^{(T-\tau)} \Bigg) \tag{47}$$

and let

$$\zeta(\xi_i, x_i, \tau, T) \tag{48}$$

be a mean interest rate with respect to the function

$$\Psi := \xi \to \sum_{j=1}^{k} x_j \, (1+\xi_j)^{(T-\tau)} \tag{49}$$

i. e. solutioon of the equation

$$\sum_{j=1}^{k} x_j \, (1+\xi_j)^{(T-\tau)} = (1+\zeta)^{(T-\tau)} \left(\sum_{j=1}^{k} x_j \right) \tag{50}$$

It is clear, that

$$\min\left(\zeta(\xi_i, x_i, \tau, T)\right)_{\tau=1}^{n} \le \eta(\xi_i, x_i, n, T) \le \max\left(\zeta(\xi_i, x_i, \tau, T)_{\tau=1}^{n}\right) \tag{51}$$

And using (40) we have

$$\lim_{T\to\infty} \min\left(\zeta(\ldots, \tau, T)_{\tau=1}^{n}\right) = \lim_{T\to\infty} \max\left(\zeta(\ldots, \tau, T)_{\tau=1}^{n}\right) = \max_i(\xi) \tag{52}$$

hence

$$\lim_{T\to\infty} \eta(\xi_i, x_i, n, T) \tag{53}$$

that is why

$$\lim_{T\to\infty} \eta(\xi_i, x_i, T, T) = \lim_{n\to\infty} \lim_{T\to\infty} \eta(\xi_i, x_i, n, T) = \lim_{n\to\infty} \max_i(\xi_i) = \max_i(\xi_i) \tag{54}$$

Q. e. d.

Mean interest rate of saving is less than mean iterest rate of interesting, but it has the same limit $t \to 0$ a $t \to \infty$.

The oprevious example vas the mean, which has already a name. Let us finished vith quite eotic mean, the mean which *is not increasing function in all of its arguments*.

6.7. Example: One of the specific bank in Czech republic, which offer mortgages is so called building societies. The common product they ofer has three parts, one used at once (it's happening in order to accommodate certain Czech laws): saving with one interest rate — saved money is then used for the redemption of debt — so called bridging credit with the second interest — this credit is used at the time of saving — and credit with third interest rat, which is used for amortization of the rest of the

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debt. Unnecessarily complicated? Mabe. And now imagine, that someone, who wont to buy a house will comes to the mortgage market try to compare all of the mortgages by their interest rate and here he has three interest rates except of one. Of course, he need to substitute them by only one comparable number and I think that this number should be called, in accordance with tradition, mean of this three. But it is clear, that while customer wishes the interest rate of the loan and bridge loan to be the as low as posiible he wants to have the interest rate of savings by contrast the greatest possible. So the mean will be increasing in the first two interest rates, but in the third will be decrasing.

Let us choose some denotation: suppose, that we pay the dept by payments x_1 with interest rate ξ_1 and at the same time we save money (as saving or insurance,...) by remittance at quantity x_2 with the interest rate ξ_2 both the time of duration N. After that we use the saved money to amortize part of the depth, than we pay the rest of the depth with interest rate ξ_3 by payments x_3 time of the length K.

For the evaluation of an expediency of a product like this is good to know the suitable mean value of the values ξ_1, ξ_2 and ξ_3. Present value of all of the payments, we are going to pay is

$$PV = x_1 \sum_{t=0}^{N-1} (1+\xi_1)^t (1+\xi_2)^{-N} + x_2 \sum_{t=1}^{N} (1+\xi_2)^{-t} + x_3 \sum_{t=1}^{K} (1+\xi_3)^{-t} (1+\xi_2)^{-N} =$$

$$= \frac{(1+\xi_2)^{-N} x_1 \left((1+\xi_1)^N - 1\right)}{\xi_1} - \frac{x_2 \left(\left((1+\xi_2)^{-1}\right)^N - 1\right)}{\xi_2} - \frac{\left(\left((1+\xi_3)^{-1}\right)^K - 1\right)(1+\xi_2)^{-N} x_3}{\xi_3}$$

hence we are interested in mean value with respect to objective function

$$(\xi_1, \xi_2, \xi_3) \mapsto PV$$

and this is the solution of the equation

$$x_1 \sum_{t=0}^{N-1} (1+\xi_1)^t (1+\xi_2)^{-N} + x_2 \sum_{t=1}^{N} (1+\xi_2)^{-t} + x_3 \sum_{t=1}^{K} (1+\xi_3)^{-t} (1+\xi_2)^{-N} =$$

$$= x_1 \sum_{t=0}^{N-1} (1+\zeta)^t (1+\zeta)^{-N} + x_2 \sum_{t=1}^{N} (1+\zeta)^{-t} + x_3 \sum_{t=1}^{K} (1+\zeta)^{-t} (1+\zeta)^{-N}$$

If no one of the interest rate is equal to 0, the equation is equivalent to

$$\frac{(1+\xi_2)^{-N} x_1 \left((1+\xi_1)^N - 1\right)}{\xi_1} - \frac{x_2 \left(\left((1+\xi_2)^{-1}\right)^N - 1\right)}{\xi_2} - \frac{\left(\left((1+\xi_3)^{-1}\right)^K - 1\right)(1+\xi_2)^{-N} x_3}{\xi_3} =$$

$$= \frac{(1+\zeta)^{-N} x_1 \left((1+\zeta)^N - 1\right)}{\zeta} - \frac{x_2 \left(\left((1+\zeta)^{-1}\right)^N - 1\right)}{\zeta} - \frac{\left(\left((1+\zeta)^{-1}\right)^K - 1\right)(1+\zeta)^{-N} x_3}{\zeta}$$

We are not able solve this equation algebraically in general so we cannot write the explicit formula for mean, we are looking for, but for all possible choice of parameters we can solve it numerically.

There is an interesting fact, that this mean is reflective (mean value of (ζ, ζ, ζ) is (ζ)), but *it is not increasing in the first variable* (ξ_1): no other conception mentioned above reckon on that fact!

Literature

1. Aczel, J. and Dhombres, J., Functional Equations in Several Variables, Cambridge University Press, 1989. ISBN-10: 0521352762 — ISBN-13: 9780521352765
2. P.S. Bullen, D.S. Mitrinovic, P.M. Vasic, Means and Their Inequalities, D. Reidel Publishing Com-any, Dordrecht, 1988.

Notes

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3. Billingsley, Patrick (1995), Probability and Measure (Third ed.), John Wiley sons, ISBN 0-471-00710-2.
4. Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change.
5. Levy, Paul: Theorie de I' Addition des Variables ALeatoires, Paul Uvy. GauthierVillars, Paris, 1937.

Notes

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A Class of Regression Estimator with Cum-Dual Product Estimator As Intercept

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Abstract- This paper examines a class of regression estimator with cum-dual product estimator as intercept for estimating the mean of the study variable Y using auxiliary variable X. We obtained the bias and the mean square error (MSE) of the proposed estimator. We also obtained MSE of its asymptotically optimum estimator (AOE). Theoretical and numerical validation of the proposed estimator was done to show it's superiority over the usual simple random sampling estimator and ratio estimator, product estimator, cum-dual ratio and product estimator. It was found that the asymptotic optimal value of the proposed estimator performed better than other competing estimators and performed in exactly the same way as the regression estimator, when compared with the usual simple random estimator for estimating the average sleeping hours of undergraduate students of the department of statistics, Federal University of Technology Akure, Nigeria.

Keywords: *difference estimator, auxiliary variable, cum-dual product estimator, bias, mean square error, efficiency, simple random sampling.*

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ACLASSOFREGRESSIONESTIMATORWITHCUMDUALPRODUCTESTIMATORASINTERCEPT

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A Class of Regression Estimator with Cum-Dual Product Estimator As Intercept

F. B. Adebola [α] & N. A. Adegoke [σ]

Abstract- This paper examines a class of regression estimator with cum-dual product estimator as intercept for estimating the mean of the study variable Y using auxiliary variable X. We obtained the bias and the mean square error (MSE) of the proposed estimator. We also obtained MSE of its asymptotically optimum estimator (AOE). Theoretical and numerical validation of the proposed estimator was done to show it's superiority over the usual simple random sampling estimator and ratio estimator, product estimator, cum-dual ratio and product estimator. It was found that the asymptotic optimal value of the proposed estimator performed better than other competing estimators and performed in exactly the same way as the regression estimator, when compared with the usual simple random estimator for estimating the average sleeping hours of undergraduate students of the department of statistics, Federal University of Technology Akure, Nigeria.

Keywords: difference estimator, auxiliary variable, cum-dual product estimator, bias, mean square error, efficiency, simple random sampling.

I. Introduction

In estimating the mean of the study variable Cochran (1940), used the auxiliary information X at the estimation phase to increase the efficiency of the study variable. To estimate the ratio estimator of the population mean or total of the study variable Y, he used additional knowledge on the auxiliary variable X which was positively correlated with Y. When the relationship between the study variable Y and the auxiliary variable X is linear through the origin and Y proportional to X, the ratio estimator will be more efficient than the normal Simple Random Sampling (SRS) Sanjib Choudhury et al (2012). Also, Robson (1957) proposed product estimator and showed that when the relationship between the study variable Y and the auxiliary variable X is linear through the origin and Y is inversely proportional to X, the product estimator will be more efficient than the usual SRS. Murthy 1964 suggested the use of ratio estimator $\bar{y}_p$ when $\frac{\rho C_y}{C_x} > \frac{1}{2}$ and unbiased estimator $\bar{y}$ when $-\frac{1}{2} \le \rho \frac{C_y}{C_x} \le \frac{1}{2}$, where C_y, C_x and ρ are coefficients of variation of y, x and correlation between y and x respectively.

Suppose that SRSWOR of n units is drawn from a population of N units to estimate the population mean $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$ of the study variable Y. All the sample units are observed for the variables Y and X. Let (y_i, x_i) where $i = 1,2,3,..,n$ denote the set of the

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observation for the study variable Y and X. Let the sample means $(\bar{x}, \bar{y})$ be unbiased of the population means of the auxiliary variable $\bar{X}$ and study variable $\bar{Y}$ based on the n observations.

The usual product estimator of $\bar{Y}$ is given as $\bar{y}_p = \frac{\bar{y}}{\bar{X}}\bar{x}$ and the usual regression estimator is given as $\bar{y}_{reg} = \bar{y} + \hat{\beta}(\bar{X} - \bar{x})$, where $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, and $\hat{\beta} = \frac{s_{xy}}{s_x^{\ 2}}$ is the estimate slope of regression coefficient of Y and X. Cum-Dual Product estimator given as $\bar{y}_{dP} = \bar{y}\frac{\bar{X}}{\bar{x}^*}$, where $\bar{x}^*$ is the un-sampled auxiliary variable in X given as $\bar{x}^* = \frac{N\bar{X} - n\bar{x}}{N-n}$ was obtained by Bandyopadhyay (1980). The use of auxiliary information in sample surveys was extensively discussed in well-known classical text books such as Cochran(1977), Sukhatme and Sukhatme(1970), Sukhatme, Sukhatme, and Asok (1984), Murthy(1967) and Yates(1960) among others. Recent developments in ratio and product methods of estimation along with their variety of modified forms are lucidly described in detail by Singh (2003).

II. The Proposed Class of Estimator

The proposed a class of regression estimator with dual product as the intercept for estimating population mean $\bar{Y}$, given as;

$$\bar{y}^*_{pd} = \bar{y}\frac{\bar{X}}{\bar{x}^*} + \alpha(\bar{X} - \bar{x}^*) \qquad \text{........................ (1)}$$

Where, α is a suitable scalar.

We obtained the bias and MSE of the proposed estimator $\bar{y}^*_{pd}$ up to the first order approximation, this is obtained by substituting $\bar{x}^* = \frac{N\bar{X} - n\bar{x}}{N-n}$ into equation (1),

$$\bar{y}^*_{pd} = \frac{\bar{y}\bar{X}(N-n)}{N\bar{X} - n\bar{x}} + \alpha(\frac{n\bar{x} - n\bar{X}}{N-n})$$

We write $e_0 = \frac{\bar{y} - \bar{Y}}{\bar{Y}}$ and $e_1 = \frac{\bar{x} - \bar{X}}{\bar{X}}$.

The bias $\bar{y}^*_{pd}$ is given as

$$Bias(\bar{y}^*_{pd}) = \frac{(1-f)}{n}(g^2\bar{Y}^2\frac{S_x^2}{\bar{X}^2} + \bar{Y}g\frac{S_{xy}}{\bar{X}\bar{Y}}) \qquad \text{.................... (1.1)}$$

The MSE of $\bar{y}^*_{pd}$ is given as

$$E(\bar{y}^*_{pd} - \bar{Y})^2 = \left(\frac{1-f}{n}\right)\left(S_y^2 + 2g\bar{Y}\frac{\rho S_x S_y}{\bar{X}\bar{Y}}(\bar{Y} + \alpha\bar{X}) + g^2\frac{S_x^2}{\bar{X}^2}(\bar{Y} + \alpha\bar{X})^2\right)$$

$$MSE\ (\bar{y}^*_{pd}) = \left(\frac{1-f}{n}\right)\left(\bar{Y}^2C_y^2 + 2g\rho\bar{Y}C_xC_y(\bar{Y} + \alpha\bar{X}) + g^2C_x^2(\bar{Y} + \alpha\bar{X})^2\right) \qquad \text{......... (2)}$$

The optimum value of the MSE ($\bar{y}^*_{pd}$) is obtained as follows,

$$\frac{\partial}{\partial\alpha}\text{MSE}\ (\bar{y}^*_{pd}) = \left(\frac{1-f}{n}\right)\left(2g\bar{Y}\frac{S_{xy}}{\bar{X}\bar{Y}}(\bar{X}) + g^2C_x^2(2)(\bar{X})(\bar{Y} + \alpha\bar{X})\right) \qquad \text{...... (3)}$$

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Set equation (3) to zero; we have

$$2gS_{xy} + 2g^2C_x^2\bar{X}(\bar{Y} + \alpha\bar{X}) = 0$$

$$\alpha = -\left(R + \frac{\beta}{g}\right)$$

The optimal MSE of $\bar{y}_{pd}^*$.

Notes

$$MSE(\bar{y}_{pd}^{*opt}) = \left(\frac{1-f}{n}\right) S_y^2(1-\rho^2) \qquad \text{... ...... (4)}$$

Equation (4) shows that the MSE $(\bar{y}_{pd}^{*opt})$ is the same as the MSE of Regression Estimate of y on X.

Remark

The bias of $\bar{y}_{pd}^*$ is the same as the bias of the dual product estimator and when α=0, the MSE $(\bar{y}_{pd}^*)$ boils down to product cum estimator proposed by Bandyopadhyay (1980). The bias and MSE of $\bar{y}_p^*$ are given as

$$\text{Bias}(\bar{y}_p^*) = \left(\frac{1-f}{n}\right)(g^2\,\bar{Y}^2C_x^2 + \bar{Y}g\rho C_xC_y)$$

$$\text{MSE}(\bar{y}_p^*) = \left(\frac{1-f}{n}\right)\left(\bar{Y}^2C_y^2 + 2g\rho\bar{Y}^2C_xC_y + \bar{Y}g^2C_x^2\right) \qquad \text{...............(5)}$$

III. The Efficiency Comparisons

In this section, we compared the MSE of $\bar{y}$ with the MSE of $\bar{y}_{pd}^*$. The MSE of $\bar{y}$ under SRS scheme is given as

$$\text{MSE}\,(\bar{y}) = \left(\frac{1-f}{n}\right)\bar{Y}^2C_x^2$$

From equation (2) and (5), $\bar{y}_{pd}^*$ is better than $\bar{y}$

If $\text{MSE}\left(\bar{y}_{pd}^*\right) < \text{MSE}\,(\bar{y})$

That is

$$\left(\frac{1-f}{n}\right)\left(\bar{Y}^2C_y^2 + 2g\rho\bar{Y}C_xC_y(\bar{Y}+\alpha\bar{X}) + g^2C_x^2(\bar{Y}+\alpha\bar{X})^2\right) \le \left(\frac{1-f}{n}\right)\bar{Y}^2C_x^2$$

$$(\bar{Y}+\alpha\bar{X})\left(2g\rho\bar{Y}C_xC_y + g^2C_x^2(\bar{Y}+\alpha\bar{X})\right) < 0$$

This holds if

(1) $\bar{Y}+\alpha\bar{X} < 0$ and $2g\rho\bar{Y}C_xC_y + g^2C_x^2(\bar{Y}+\alpha\bar{X}) > 0$

Or

(2) $\bar{Y}+\alpha\bar{X} > 0$ and $2g\rho\bar{Y}C_xC_y + g^2C_x^2(\bar{Y}+\alpha\bar{X}) < 0$

The range of α under which $\bar{y}_{pd}^*$ is more efficient than usual SRS $\bar{y}$ is

$$min\left\{-R, -R\left(1+\frac{2\rho C_y}{gC_x}\right)\right\}, max\left\{-R, -R\left(1+\frac{2\rho C_y}{gC_x}\right)\right\}.$$

We also compared $\bar{y}_{pd}^*$ with the usual ratio estimator $\bar{y}_R$. The MSE of the $\bar{y}_R$ is given as,

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$$MSE(\bar{y}_R) = \left(\frac{1-f}{n}\right)\left(\bar{Y}^2 C_y^2 - 2\rho\bar{Y}^2 C_x C_y + \bar{Y}^2 C_x^2\right)$$

It is found that $\bar{y}_{pd}^*$ will be more efficient than the usual ratio estimator $\bar{y}_R$ if MSE $(\bar{y}_{pd}^*)$ < MSE $(\bar{y}_R)$. That is,

$$\left(\frac{1-f}{n}\right)\left(\bar{Y}^2 C_y^2 + 2g\rho\bar{Y}C_x C_y(\bar{Y}+\alpha\bar{X}) + g^2 C_x^2(\bar{Y}+\alpha\bar{X})^2\right) \le \left(\frac{1-f}{n}\right)\left(\bar{Y}^2 C_y^2 - 2\rho\bar{Y}^2 C_x C_y + \bar{Y}^2 C_x^2\right)$$

This holds if the following two conditions are satisfied

(1). $(g(\bar{Y}+\alpha\bar{X})+\bar{Y}) < 0$ And $2\rho\bar{Y}C_x C_y + C_x^2(g(\bar{Y}+\alpha\bar{X})-\bar{Y}) > 0$.

or

(2). $(g(\bar{Y}+\alpha\bar{X})+\bar{Y}) > 0$ And $2\rho\bar{Y}C_x C_y + C_x^2(g(\bar{Y}+\alpha\bar{X})-\bar{Y}) < 0$.

This condition holds if $\alpha > -R\left(\frac{N}{n}\right)$ and $\alpha < R\left(\frac{N-2n}{n} - \frac{2\rho C_y}{gC_x}\right)$ or $\alpha < -R\left(\frac{N}{n}\right)$ and $\alpha > R\left(\frac{N-2n}{n} - \frac{2\rho C_y}{gC_x}\right)$

$$\min\left\{-R\left(\frac{N}{n}\right), R\left(\frac{N-2n}{n} - \frac{2\rho C_y}{gC_x}\right)\right\}, \max\left\{-R\left(\frac{N}{n}\right), R\left(\frac{N-2n}{n} - \frac{2\rho C_y}{gC_x}\right)\right\}.$$

We also compared $\bar{y}_{pd}^*$ with the usual product estimator y_P. The MSE of the y_P is given as

$$MSE(\bar{y}_P) = \left(\frac{1-f}{n}\right)\left(\bar{Y}^2 C_y^2 + 2\rho\bar{Y}^2 C_x C_y + \bar{Y}^2 C_x^2\right)$$

It is found that the proposed estimator $\bar{y}_{PD}^*$ will be more efficient than the usual ratio estimator $\bar{y}_P$ if MSE $(\bar{y}_{pd}^*)$ < MSE $(\bar{y}_P)$. That is,

$$\left(\frac{1-f}{n}\right)\left(\bar{Y}^2 C_y^2 + 2g\rho\bar{Y}C_x C_y(\bar{Y}+\alpha\bar{X}) + g^2 C_x^2(\bar{Y}+\alpha\bar{X})^2\right) \le \left(\frac{1-f}{n}\right)\left(\bar{Y}^2 C_y^2 + 2\rho\bar{Y}^2 C_x C_y + \bar{Y}^2 C_x^2\right)$$

This holds if the following two conditions are satisfied

(1). $(g(\bar{Y}+\alpha\bar{X})-\bar{Y}) < 0$ And $2\rho\bar{Y}C_x C_y + C_x^2(g(\bar{Y}+\alpha\bar{X})+\bar{Y}) > 0$.

Or

(2). $(g(\bar{Y}+\alpha\bar{X})-\bar{Y}) > 0$ And $2\rho\bar{Y}C_x C_y + C_x^2(g(\bar{Y}+\alpha\bar{X})+\bar{Y}) < 0$.

This condition holds if $\alpha > R\left(\frac{N-2n}{n}\right)$ and $\alpha < -R\left(\frac{N}{n} + \frac{2\rho C_y}{gC_x}\right)$ or $\alpha < R\left(\frac{N-2n}{n}\right)$ and $\alpha > -R\left(\frac{N}{n} + \frac{2\rho C_y}{gC_x}\right)$

$$\min\left\{R\left(\frac{N-2n}{n}\right), -R\left(\frac{N}{n} + \frac{2\rho C_y}{gC_x}\right)\right\}, \max\left\{R\left(\frac{N-2n}{n}\right), -R\left(\frac{N}{n} + \frac{2\rho C_y}{gC_x}\right)\right\}.$$

We compared the MSE of the proposed estimator with MSE of dual product estimator from equation (2) and (5) it is found that the proposed estimator $\bar{y}_{pd}^*$ will be more efficient than that of Bandyopadhyay (1980) estimator $\bar{y}_p^*$ if MSE$(\bar{y}_{pd}^*)$ < MSE $(\bar{y}_p^*)$

That is

$$\left(\frac{1-f}{n}\right)\left(\bar{Y}^2 C_y^2 + 2g\rho\bar{Y}C_x C_y(\bar{Y}+\alpha\bar{X}) + g^2 C_x^2(\bar{Y}+\alpha\bar{X})^2\right) \le \left(\frac{1-f}{n}\right)\left(\bar{Y}^2 C_y^2 + 2g\rho\bar{Y}^2 C_x C_y + \bar{Y}^2 g^2 C_x^2\right)$$

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This holds if

1. $\alpha\bar{X} < 0$ and $2\rho\bar{Y}C_y + gC_x(2\bar{Y} + \alpha\bar{X}) > 0$

Or

2. $\alpha\bar{X} > 0$ and $2\rho\bar{Y}C_y + gC_x(2\bar{Y} + \alpha\bar{X}) < 0$

The range of α under which the proposed estimator $\bar{y}_{pd}^*$ is more efficient than $\bar{y}_p^*$ is

$$min\left\{0, -2R\left(\frac{\rho C_y}{gC_x} + 1\right)\right\}, max\left\{0, -2R\left(\frac{\rho C_y}{gC_x} + 1\right)\right\}$$

Lastly, we compared MSE of $\bar{y}_{pd}^*$ with that of dual to ratio estimator $\bar{y}_R^*$ proposed Srivenkataramana(1980), $\bar{y}_{pd}^*$ will be more efficient than $\bar{y}_R^*$ if

$$\text{MSE}(\bar{y}_{pd}^*) < \text{MSE}(\bar{y}_R^*)$$

$$\left(\frac{1-f}{n}\right)\left(\bar{Y}^2C_y^2 + 2g\rho\bar{Y}C_xC_y(\bar{Y} + \alpha\bar{X}) + g^2C_x^2(\bar{Y} + \alpha\bar{X})^2\right)$$
$$\leq \left(\frac{1-f}{n}\right)\left(\bar{Y}^2C_y^2 - 2g\rho\bar{Y}^2C_xC_y + \bar{Y}^2g^2C_x^2\right)$$

This holds if

1. $2\bar{Y} + \alpha\bar{X} < 0$ and $2\rho\bar{Y}C_y + gC_x(\alpha\bar{X}) > 0$

Or

2. $2\bar{Y} + \alpha\bar{X} > 0$ and $2\rho\bar{Y}C_y + gC_x(\alpha\bar{X}) < 0$

This condition holds if $-2R > \alpha$ and $\frac{-2R\rho C_y}{gC_x} < \alpha$ or $-2R < \alpha$ and $\frac{-2R\rho C_y}{gC_x} > \alpha$

Therefore, the range of α under which the proposed estimator $\bar{y}_{pd}^*$ is more efficient than dual to ratio estimator $\bar{y}_R^*$ is

$$min\left\{-2R, -2R\left(\frac{\rho C_y}{gC_x}\right)\right\}, max\left\{-2R, -2R\left(\frac{\rho C_y}{gC_x}\right)\right\}$$

Comparison of 'AOE' to $\boldsymbol{\bar{y}_{pd}^{*OPT}}$

$\bar{y}_{pd}^{*OPT}$ is more efficient than the other existing estimators $\bar{y}$, the Ratio estimator $\bar{y}_R$, the product estimator $\bar{y}_p$, the dual to ratio estimator $\bar{y}_R$ and the dual to product estimator $\bar{y}_p^*$ since.

$$MSE(\bar{y}) - MSE(\bar{y}_{pd}^{*OPT}) = \left(\frac{1-f}{n}\right)\left(\bar{Y}^2\rho^2C_y^2\right) > 0$$

$$MSE(\bar{y}_R) - MSE(\bar{y}_{pd}^{*OPT}) = \left(\frac{1-f}{n}\right)\left(\bar{Y}^2C_x^2\left(1 - \frac{\rho C_y}{C_x}\right)^2\right) > 0$$

$$MSE(\bar{y}_P) - MSE(\bar{y}_{pd}^{*OPT}) = \left(\frac{1-f}{n}\right)\left(\bar{Y}^2C_x^2\left(1 + \frac{\rho C_y}{C_x}\right)^2\right) > 0$$

$$MSE(\bar{y}_R^*) - MSE(\bar{y}_{pd}^{*OPT}) = \left(\frac{1-f}{n}\right)\left(\bar{Y}^2C_x^2\left(\frac{\rho C_y}{C_x} - g\right)^2\right) > 0$$

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$$MSE(\bar{y}_p^*) - MSE(\bar{y}_{pd}^{*OPT}) = \left(\frac{1-f}{n}\right)\left(\bar{Y}^2 C_x^2 \left(\frac{\rho C_y}{C_x} + g\right)^2\right) > 0$$

Hence, we conclude that the proposed class of estimator $\bar{y}_{pd}^*$ is more efficient than other estimator in case of its optimality.

IV. Numerical Validation

To illustrate the efficiency of the proposed estimator over the other estimators $\bar{y}$,$\bar{y}_R$,$\bar{y}_p$,$\bar{y}_{R*}$ *and* $\bar{y}_p^*$. Data on the ages and hours of sleeping by the undergraduate students of the Department of Statistics Federal University of Technology Akure, Ondo State, Nigeria were collected. A sample of 150 out of 461 students of the department was obtained using simple random sampling without replacement. The information on the age of the students was used as auxiliary information to increase the precision of the estimate of the average sleeping hours. The estimate of the average hours of sleeping of the students were obtained and also the 95% confidence intervals of the average hours of sleeping were obtained for the proposed estimator and the other estimators. Table 1.0, gives the estimates of the average sleeping hours and the 95% confidence Interval. As shown in Table 1.0, the proposed estimator performed better than the other estimators, also the width of the confidence interval of the proposed estimator is smallest than the other competing estimators.

Table 1.0. : Average Sleeping Hours and 95% confidence intervals for Different Estimators for the undergraduate Students of Department of Statistics, Federals University of Technology Akure. Nigeria

Estimator	Average Sleeping Hours	LCL	UCL	WIDTH
$\bar{y}$	6.08	5.930386531	6.229613469	0.299226939
$\bar{y}_R$	6.210472103	6.042844235	6.378099971	0.335255737
$\bar{y}_p$	5.952268908	5.778821411	6.125716404	0.346894993
$\bar{y}_R^*$	6.141606636	5.988421023	6.294792249	0.306371226
$\bar{y}_p^*$	6.01901342	5.862732122	6.175290562	0.31255844
$\boldsymbol{\bar{y}_{PD}^*}$	**6.072287**	5.927857	**6.216717**	**0.28886**

The proposed estimator performed the same way as the regression estimator when compare with the usual estimator $\bar{y}$. The average Sleeping Hours and 95% confidence intervals for the proposed estimator and the regression estimator is given below, the two estimators have the same confidence Interval width.

Table 2.0. : Average Sleeping Hours and 95% confidence intervals for the proposed estimators and regression estimators for the undergraduate Students of Department of Statistics, Federals University of Technology Akure. Nigeria

Estimator	Average Sleeping Hours	LCL	UCL	WIDTH
$\boldsymbol{\bar{y}_{PD}^*}$	6.072287	5.927857	6.2167177	0.28886
$\boldsymbol{\bar{y}_{REG}^*}$	6.089652737	5.945222793	6.234082681	0.288859888

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To examine the gain in the efficiency of the proposed estimator $\bar{y}_{pd}^{*}$ over the estimator $\bar{y}$,$\bar{y}_R$,$\bar{y}_p$, $\bar{y}_R^{*}$ *and* $\bar{y}_{p,}^{*}$ we obtained the percentage relative efficiency of different estimator of $\bar{Y}$ with respect to the usual unbiased estimator $\bar{y}$ in Table 2.0. The proposed estimator $\bar{y}_{pd}^{*}$ performed better than the other estimators $\bar{y}, \bar{y}_R, \bar{y}_p, \bar{y}_R^{*}$ *and* $\bar{y}_p^{*}$ and performed exactly the same way as regression estimator.

Notes

Table 3.0. : The percentage relative efficiency of different estimator of $\bar{Y}$ with respect to the usual unbiased estimator $\bar{y}$

ESTIMATOR	PERCENATGE RELATIVE FFICIENCY
$\bar{y}$	100
$\bar{y}_R$	79.66158486
$,\bar{y}_p$	74.40554745
$\bar{y}_R^{*}$	95.39056726
$\bar{y}_p^{*}$	91.65136111
$\bar{y}_{REG}^{*}$	107.3067159
$\bar{y}_{pd}^{*}$	107.3067159

V. Conclusion

We have proposed a class of regression estimator with cum-dual product estimator as intercept for estimating the mean of the study variable Y using auxiliary variable X as in equation (1) and obtained 'AOE' for the proposed estimator. Theoretically, we have demonstrated that proposed estimator is always more efficient than other under the effective ranges of α and its optimum values.

Table 1.0 shows that the proposed estimator performed better than the other estimators as the width of the confidence interval of the proposed estimator is smallest than the other competing estimators. The percentage relative efficiency of different estimator of $\bar{Y}$ with respect to the usual unbiased estimator $\bar{y}$ in Table 2.0 shows that the proposed estimator $\bar{y}_{PD}^{*}$ performed better than the other estimators $\bar{y}, \bar{y}_R, \bar{y}_p, \bar{y}_R^{*}$ *and* $\bar{y}_p^{*}$ and performed exactly the same way as regression estimator when compared to the usual estimator $\bar{y}$. Hence, it is preferred to use the proposed class of estimator in practice.

References Références Referencias

1. Bandyopadhyay, S. (1980). Improved ratio and product estimators. Sankhya Series C, 42(2), 45-49.
2. Cochran, W. G. (1940). The estimation of the yields of the cereal experiments by Sampling for the.
3. COCHRAN, W.G. (1977): Sampling Techniques.John Wiley, New York.
4. Murthy, M. N. (1964). Product method of estimation. Sankhya A, 26, 69-74.
5. Murthy, M. N. (1967). Sampling Theory and Methods, Statistical Publishing Society, Calcutta.
6. ROBSON, D .S.(1957): Application of multivariate polykays to the theory of unbiased ratio type estimation. J.Amer.stat. Assoc., 52, 411-422.
7. Sanjib Choudhury and B. K. Singh (2012), An Efficient Class of Dual to Product-Cum-Dual to Ratio Estimators of Finite Population, Mean in Sample Surveys. Global Journal of Science Frontier Research.

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8. SINGH,S. (2003): Advanced sampling theory with applications ,vol. 1, Kluwer Academic Publishers, The Netherlands.
9. Srivenkataramana, T. (1980). A dual to ratio estimator in sample surveys. Biometrika, 67(1), 199-204.
10. SUKHATME, P.V., SUKHATME, B. V. and A SOK, C. (1984): Sampling theory of surveys with applications. Ind. Soc. Ag. Stat., New Delhi.
11. SUKHATME,P V. and SUKHATME, B.V.(1970):Sampling theory of surveys with Applications. Asia Publishing House, New Delhi
12. YATES, F.(1960): Sampling methods in censuses and surveys. Charles Griffin and Co., London:

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On Special Pairs of Pythagorean Triangles

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Abstract- We search for pairs of Pythagorean triangles such that, in each pair, twice the difference between their perimeters is expressed in terms of special polygonal numbers.

Keywords: *pair of pythagorean triangles, special polygonal numbers.*

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Ref

18. A. A. K. Majumdar, On some pseudo smarandache function related triangles, pythagorean triangles hold a special fascination, scientia majna, 4(3), 95-105, 2008

On Special Pairs of Pythagorean Triangles

M. A. Gopalan [α], S. Vidhyalakshmi [σ], N. Thiruniraiselvi [ρ] & R. Presenna [ω]

Abstract- We search for pairs of Pythagorean triangles such that, in each pair, twice the difference between their perimeters is expressed in terms of special polygonal numbers.

Keywords: pair of pythagorean triangles, special polygonal numbers.

Notations used:

$$t_{m,n} = n\left(1 + \frac{(n-1)(m-2)}{2}\right)$$

$$s_n = 6n(n-1) + 1$$

$$cs_n = n^2 + (n-1)^2$$

I. Introduction

Number is the essence of mathematical calculation. Varieties of numbers have variety of range and richness, many of which can be explained very easily but extremely difficult to prove. One of the varieties of numbers that have fascinated mathematicians and the lovers of maths is the pythagorean number as they provide limitless supply of exciting and interesting properties. In other word, the method of obtaining three non-zero integers x,y and z under certain relations satisfying the equation $x^2 + y^2 = z^2$ has been a matter of interest to various mathematicians. For an elaborate review of various properties are may refer [1-17]. In [18], the author proves existence of an infinite family of pairs of dissimilar Pythagorean triangles that are pseudo smarandache related. For problems on pairs of Pythagorean triangles one may refer [1,5].

In this communication concerns with the problem of determining pairs of Pythagorean triangles wherein each pair 2-times the difference between the perimeters is expressed interms of special polygonal numbers.

II. Method of Analysis

Let $T_1(\alpha_1, \beta_1, \gamma_1)$ and $T_2(\alpha_2, \beta_2, \gamma_2)$ be two distinct Pythagorean triangles where $\alpha_1 = 2mq, \beta_1 = m^2 - q^2, \gamma_1 = m^2 + q^2$ and $\alpha_2 = 2pq, \beta_2 = p^2 - q^2, \gamma_2 = p^2 + q^2$, m>q>0;p>q>0

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Let $P_1 = 2m(m+q), P_2 = 2p(p+q)$ be their perimeters respectively. We illustrated below the process of obtaining pair of Pythagorean triangles such that 2 times the difference between their perimeters is expressed in terms of a special polygonal number.

Illustration 1

The assumption $2(P_1 - P_2) = (3\alpha - 1)^2$ (1)

gives $(2m+q)^2 = (2p+q)^2 + (3\alpha - 1)^2$ (2)

which is in the form of well know Pythagorean equation and it is satisfied by

$$3\alpha - 1 = 2RS, 2p + q = R^2 - S^2, 2m + q = R^2 + S^2, R > S > 0 \quad (3)$$

Solving the above system of equations (3), we have

$$\alpha = \left(\frac{2RS+1}{3}\right), m = p + S^2, q = R^2 - S^2 - 2p$$

As our main thrust is on integers, note that α is an integer when

$$R = 3(k_1 + k_2) - 2, S = 3k_2 - 2$$

$$\alpha = (k_1 + k_2)(6k_2 - 4) - 4k_2 + 3$$

and thus $m = p + (3k_2 - 2)^2$ (4)

$$q = 9k_1^2 + 18k_1k_2 - 12k_1 - 2p$$

The conditions m>q>0, p>q>0 lead to

$$2p < 9k_1^2 + 18k_1k_2 - 12k_1 < 3p \quad (5)$$

Choose k_1, k_2, p such that (5) is satisfied. Substituting the corresponding values of k_1, k_2, p in (4) and in the value of m, q, α are obtained. Thus, it is observed that

$$2(P_1 - P_2) = \begin{cases} 3t_{8,\alpha} + 1 \\ s_\alpha + 3t_{4,\alpha} \end{cases}$$

A few examples are given below in Table -1

k_1	k_2	p	m	q	α	$2(P_1 - P_2)$	$3t_{8,\alpha} + 1$	$s_\alpha + 3t_{4,\alpha}$
1	1	7	8	1	3	64	$3(21)+1$	$37+3(3)^2$
1	2	16	32	1	19	3136	$3(1045)+1$	$2053+3(19)^2$
1	2	15	31	3	19	3136	$3(1045)+1$	$2053+3(19)^2$
1	2	14	30	5	19	3136	$3(1045)+1$	$2053+3(19)^2$
1	2	13	29	7	19	3136	$3(1045)+1$	$2053+3(19)^2$
1	2	12	28	9	19	3136	$3(1045)+1$	$2053+3(19)^2$
1	3	25	74	1	47	19600	$3(6533)+1$	$12973+3(47)^2$
1	3	24	73	3	47	19600	$3(6533)+1$	$12973+3(47)^2$

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1	3	23	72	5	47	19600	$3(6533)+1$	$12973+3(47)^2$
1	3	22	71	7	47	19600	$3(6533)+1$	$12973+3(47)^2$
1	3	21	70	9	47	19600	$3(6533)+1$	$12973+3(47)^2$
1	3	20	69	11	47	19600	$3(6533)+1$	$12973+3(47)^2$
1	3	19	68	13	47	19600	$3(6533)+1$	$12973+3(47)^2$
1	3	18	67	15	47	19600	$3(6533)+1$	$12973+3(47)^2$

It is worth to note that there are only finitely many pairs of Pythagorean triangles such that in each pair, twice the difference between their perimeters is expressed in terms of a special polygonal number.

Illustration- 2

Assume $2(P_1 - P_2) = (5\alpha - 2)^2$

Proceeding as in **illustration 1**, it is seen that there are 3 values for α satisfying the equation $\alpha = \left(\frac{2(RS+1)}{5}\right)$ and thus, there are 3 sets of triples (m, q, α) which are represented as follows

SET 1:

$$\alpha = 2(k + k_1)(5k_1 - 4) - 12k_1 + 10$$
$$m = p + (5k_1 - 4)^2$$
$$q = (10k_1 + 5k - 10)(5k - 2) - 2p$$

The condition to be satisfied by k_1, k_2 and p is $2p < (10k_1 + 5k - 10)(5k - 2) < 3p$

SET 2:

$$\alpha = 2(k + k_1)(5k_1 - 3) - 6k_1 + 4$$
$$m = p + (5k_1 - 3)^2$$
$$q = (5k + 10k_1 - 6)(5k) - 2p$$

The condition to be satisfied by k_1, k_2 and p is $2p < (10k_1 + 5k - 6)(5k) < 3p$

SET 3:

$$\alpha = 2(k + k_1)(5k_1 - 2) - 4k_1 + 2$$
$$m = p + (5k_1 - 2)^2$$
$$q = (5k + 10k_1 - 4)(5k) - 2p$$

The condition to be satisfied by k_1, k_2 and p is $2p < (10k_1 + 5k - 4)(5k) < 3p$

From each of the above 3 sets it is observed that $2(P_1 - P_2) = 5t_{12,\alpha} + 4$

Illustration 3

Assume $2(P_1 - P_2) = (\alpha - 1)^2$

For this choice, the values for m,q, α satisfying the equation

$(2m + q)^2 = (2p + q)^2 + (\alpha - 1)^2$ are given by

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$$\alpha = 2rs + 1, m = s^2 + p, q = r^2 - s^2 - 2p$$

The conditions m>q>0,p>q>0 lead to

$$2p < r^2 - s^2 < 3p \tag{6}$$

Choose r,s,p satisfying (6), it is seen that

$$2(P_1 - P_2) = (cs_\alpha - t_{4,\alpha})$$

Notes

III. Conclusion

In this paper, we have obtained finitely many pairs of Pythagorean triangles where each pair connects 2-times the difference between the perimeters with special polygonal numbers. To conclude, one may search for the connections between special numbers and the other characterizations of Pythagorean triangle.

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References Références Referencias

1. L. E. Dickson, History of Theory of numbers, vol.2, Diophantine analysis, New York, Dover, 2005.
2. L. J. Mordell, Diophantine Equations, Academic press, New York, 1969.
3. B. M. Stewart, Theory of Numbers, 2nd edit, Mac Millan Company, New York, 1974.
4. Carl B. Boyer and Utah C.Merzlach, A History of Mathematics, John wiley and sons,1989.
5. Waclaw sierpiriski, Pythagorean triangles, Dover publication, New York, 2003.
6. M. A. Gopalan and A. Gnanam, pairs of pythagorean triangles with equal perimeters, impact J.sci.Tech, 1(2),67-70,2007.
7. M. A. Gopalan and G. Janaki,"Pythagorean triangle with area/perimeter as a special polygonal number", Bulletin of Pure and Applied Science, Vol.27E (No.2), 393-402,2008.
8. M. A. Gopalan and S. Leelavathi, "Pythagorean triangle with area/perimeter as a square integer", International Journal of Mathemtics, Computer sciences and Information Technology, Vol.1, No.2, 199-204, 2008.
9. M. A. Gopalan and S. Leelavathi, "Pythagorean triangle with 2 area/perimeter as a cubic integer", Bulletin of Pure and Applied Science, Vol.26E(No.2), 197-200,2007.
10. M. A. Gopalan and B. Sivakami,"Special Pythagorean triangles generated through the integral solutions of the equation $y^2 = (k^2 + 2k)x^2 + 1$", Diophantus J.Math., Vol 2(1), 25-30, 2013.
11. M. A. Gopalan and A. Gnanam,"Pythagorean triangles and Polygonal numbers", International Journal of Mathematical Sciences, Vol 9, No. 1-2,211-215,2010.
12. K. Meena, S. Vidhyalakshmi, B. Geetha, A. Vijayasankar and M. A. Gopalan,"Relations between special polygonal numbers generated through the solutions of Pythagorean equation", IJISM, Vol 5(2), 15-18, 2008.
13. M. A. Gopalan and G. Janaki,"Pythagorean triangle with perimeter as Pentagonal number", AntarticaJ.Math., Vol 5(2), 15-18,2008.
14. M. A. Gopalan, Manjusomanath and K. Geetha,"Pythagorean triangle with area/perimeter as a special polygonal number",IOSR-JM, Vol.7(3),52-62,2013.

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15. M. A. Gopalan and V. Geetha,"Pythagorean triangle with area/perimeter as a special polygonal number",IRJES,Vol.2(7),28-34,2013.
16. M. A. Gopalan and B. Sivakami,"Pythagorean triangle with hypotenuse minus 2(area/perimeter) as a square integer",ArchimedesJ.Math., Vol 2(2),153-166,2012.
17. M. A. Gopalan and S. Devibala,"Pythagorean triangle with triangular number as a leg", ImpactJ.Sci.Tech., Vol 2(4), 195-199,2008.
18. A. A. K. Majumdar, On some pseudo smarandache function related triangles, pythagorean triangles hold a special fascination, scientia majna, 4(3), 95-105, 2008

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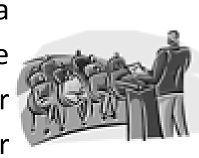

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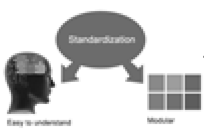

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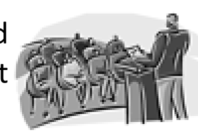
The "FARSC" is a dignified title which is accorded to a person's name viz. Dr. John E. Hall, Ph.D., FARSC or William Walldroff, M.S., FARSC.

The IFOARS institution is entitled to form a Board comprised of one Chairperson and three to five board members preferably from different streams. The Board will be recognized as "Institutional Board of Open Association of Research Society"-(IBOARS).

The Institute will be entitled to following benefits:

The IBOARS can initially review research papers of their institute and recommend them to publish with respective journal of Global Journals. It can also review the papers of other institutions after obtaining our consent. The second review will be done by peer reviewer of Global Journals Incorporation (USA)
The Board is at liberty to appoint a peer reviewer with the approval of chairperson after consulting us.
The author fees of such paper may be waived off up to 40%.



The Global Journals Incorporation (USA) at its discretion can also refer double blind peer reviewed paper at their end to the board for the verification and to get recommendation for final stage of acceptance of publication.

The IBOARS can organize symposium/seminar/conference in their country on behalf of Global Journals Incorporation (USA)-OARS (USA). The terms and conditions can be discussed separately.

The Board can also play vital role by exploring and giving valuable suggestions regarding the Standards of "Open Association of Research Society, U.S.A (OARS)" so that proper amendment can take place for the benefit of entire research community. We shall provide details of particular standard only on receipt of request from the Board.

The board members can also join us as Individual Fellow with 40% discount on total fees applicable to Individual Fellow. They will be entitled to avail all the benefits as declared. Please visit Individual Fellow-sub menu of GlobalJournals.org to have more relevant details.

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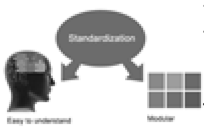

After nomination of your institution as "Institutional Fellow" and constantly functioning successfully for one year, we can consider giving recognition to your institute to function as Regional/Zonal office on our behalf.
The board can also take up the additional allied activities for betterment after our consultation.

The following entitlements are applicable to individual Fellows:

Open Association of Research Society, U.S.A (OARS) By-laws states that an individual Fellow may use the designations as applicable, or the corresponding initials. The Credentials of individual Fellow and Associate designations signify that the individual has gained knowledge of the fundamental concepts. One is magnanimous and proficient in an expertise course covering the professional code of conduct, and follows recognized standards of practice.

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We shall provide print version of 12 issues of any three journals [as per your requirement] out of our 38 journals worth $ 2376 USD.

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- The professional accredited with Fellow honor, is entitled to various benefits viz. name, fame, honor, regular flow of income, secured bright future, social status etc.

- In addition to above, if one is single author, then entitled to 40% discount on publishing research paper and can get 10%discount if one is co-author or main author among group of authors.
- The Fellow can organize symposium/seminar/conference on behalf of Global Journals Incorporation (USA) and he/she can also attend the same organized by other institutes on behalf of Global Journals.
- The Fellow can become member of Editorial Board Member after completing 3yrs.
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- Fellow can also join as paid peer reviewer and earn 15% remuneration of author charges and can also get an opportunity to join as member of the Editorial Board of Global Journals Incorporation (USA)
- • This individual has learned the basic methods of applying those concepts and techniques to common challenging situations. This individual has further demonstrated an in–depth understanding of the application of suitable techniques to a particular area of research practice.

Note :

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- In future, if the board feels the necessity to change any board member, the same can be done with the consent of the chairperson along with anyone board member without our approval.
- In case, the chairperson needs to be replaced then consent of 2/3rd board members are required and they are also required to jointly pass the resolution copy of which should be sent to us. In such case, it will be compulsory to obtain our approval before replacement.
- In case of "Difference of Opinion [if any]" among the Board members, our decision will be final and binding to everyone.

"

Process of submission of Research Paper

The Area or field of specialization may or may not be of any category as mentioned in 'Scope of Journal' menu of the GlobalJournals.org website. There are 37 Research Journal categorized with Six parental Journals GJCST, GJMR, GJRE, GJMBR, GJSFR, GJHSS. For Authors should prefer the mentioned categories. There are three widely used systems UDC, DDC and LCC. The details are available as 'Knowledge Abstract' at Home page. The major advantage of this coding is that, the research work will be exposed to and shared with all over the world as we are being abstracted and indexed worldwide.

The paper should be in proper format. The format can be downloaded from first page of 'Author Guideline' Menu. The Author is expected to follow the general rules as mentioned in this menu. The paper should be written in MS-Word Format (*.DOC,*.DOCX).

The Author can submit the paper either online or offline. The authors should prefer online submission.Online Submission: There are three ways to submit your paper:

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(III) Click 'Submit Manuscript'. Fill required information and Upload the paper.

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You can use your own standard format also.
Author Guidelines:

1. General,

2. Ethical Guidelines,

3. Submission of Manuscripts,

4. Manuscript’s Category,

5. Structure and Format of Manuscript,

6. After Acceptance.

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Complete support for both authors and co-author is provided.

4. MANUSCRIPT'S CATEGORY

Based on potential and nature, the manuscript can be categorized under the following heads:

Original research paper: Such papers are reports of high-level significant original research work.

Review papers: These are concise, significant but helpful and decisive topics for young researchers.

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Research letters: The letters are small and concise comments on previously published matters.

5.STRUCTURE AND FORMAT OF MANUSCRIPT

The recommended size of original research paper is less than seven thousand words, review papers fewer than seven thousands words also.Preparation of research paper or how to write research paper, are major hurdle, while writing manuscript. The research articles and research letters should be fewer than three thousand words, the structure original research paper; sometime review paper should be as follows:

Papers: These are reports of significant research (typically less than 7000 words equivalent, including tables, figures, references), and comprise:

(a)Title should be relevant and commensurate with the theme of the paper.

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(e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition; sources of information must be given and numerical methods must be specified by reference, unless non-standard.

(f) Results should be presented concisely, by well-designed tables and/or figures; the same data may not be used in both; suitable statistical data should be given. All data must be obtained with attention to numerical detail in the planning stage. As reproduced design has been recognized to be important to experiments for a considerable time, the Editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned un-refereed;

(g) Discussion should cover the implications and consequences, not just recapitulating the results; conclusions should be summarizing.

(h) Brief Acknowledgements.

(i) References in the proper form.

Authors should very cautiously consider the preparation of papers to ensure that they communicate efficiently. Papers are much more likely to be accepted, if they are cautiously designed and laid out, contain few or no errors, are summarizing, and be conventional to the approach and instructions. They will in addition, be published with much less delays than those that require much technical and editorial correction.

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It is vital, that authors take care in submitting a manuscript that is written in simple language and adheres to published guidelines.

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Metric SI units are supposed to generally be used excluding where they conflict with current practice or are confusing. For illustration, 1.4 l rather than 1.4 × 10-3 m3, or 4 mm somewhat than 4 × 10-3 m. Chemical formula and solutions must identify the form used, e.g. anhydrous or hydrated, and the concentration must be in clearly defined units. Common species names should be followed by underlines at the first mention. For following use the generic name should be constricted to a single letter, if it is clear.

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Many researchers searching for information online will use search engines such as Google, Yahoo or similar. By optimizing your paper for search engines, you will amplify the chance of someone finding it. This in turn will make it more likely to be viewed and/or cited in a further work. Global Journals Inc. (US) have compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

Key Words

A major linchpin in research work for the writing research paper is the keyword search, which one will employ to find both library and Internet resources.

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Search engines for most searches, use Boolean searching, which is somewhat different from Internet searches. The Boolean search uses "operators," words (and, or, not, and near) that enable you to expand or narrow your affords. Tips for research paper while preparing research paper are very helpful guideline of research paper.

Choice of key words is first tool of tips to write research paper. Research paper writing is an art.A few tips for deciding as strategically as possible about keyword search:

- One should start brainstorming lists of possible keywords before even begin searching. Think about the most important concepts related to research work. Ask, "What words would a source have to include to be truly valuable in research paper?" Then consider synonyms for the important words.
- It may take the discovery of only one relevant paper to let steer in the right keyword direction because in most databases, the keywords under which a research paper is abstracted are listed with the paper.
- One should avoid outdated words.

Keywords are the key that opens a door to research work sources. Keyword searching is an art in which researcher's skills are bound to improve with experience and time.

Numerical Methods: Numerical methods used should be clear and, where appropriate, supported by references.

Acknowledgements: Please make these as concise as possible.

References

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TECHNIQUES FOR WRITING A GOOD QUALITY RESEARCH PAPER:

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2. Evaluators are human: First thing to remember that evaluators are also human being. They are not only meant for rejecting a paper. They are here to evaluate your paper. So, present your Best.

3. Think Like Evaluators: If you are in a confusion or getting demotivated that your paper will be accepted by evaluators or not, then think and try to evaluate your paper like an Evaluator. Try to understand that what an evaluator wants in your research paper and automatically you will have your answer.

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7. Use right software: Always use good quality software packages. If you are not capable to judge good software then you can lose quality of your paper unknowingly. There are various software programs available to help you, which you can get through Internet.

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10. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right! It is a good habit, which helps to not to lose your continuity. You should always use bookmarks while searching on Internet also, which will make your search easier.

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12. Make all efforts: Make all efforts to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in introduction, that what is the need of a particular research paper. Polish your work by good skill of writing and always give an evaluator, what he wants.

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20. Use good quality grammar: Always use a good quality grammar and use words that will throw positive impact on evaluator. Use of good quality grammar does not mean to use tough words, that for each word the evaluator has to go through dictionary. Do not start sentence with a conjunction. Do not fragment sentences. Eliminate one-word sentences. Ignore passive voice. Do not ever use a big word when a diminutive one would suffice. Verbs have to be in agreement with their subjects. Prepositions are not expressions to finish sentences with. It is incorrect to ever divide an infinitive. Avoid clichés like the disease. Also, always shun irritating alliteration. Use language that is simple and straight forward. put together a neat summary.

21. Arrangement of information: Each section of the main body should start with an opening sentence and there should be a changeover at the end of the section. Give only valid and powerful arguments to your topic. You may also maintain your arguments with records.

22. Never start in last minute: Always start at right time and give enough time to research work. Leaving everything to the last minute will degrade your paper and spoil your work.

23. Multitasking in research is not good: Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. Think technically: Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page
- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.

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- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

The Administration Rules

Please carefully note down following rules and regulation before submitting your Research Paper to Global Journals Inc. (US):

Segment Draft and Final Research Paper: You have to strictly follow the template of research paper. If it is not done your paper may get rejected.

- The **major constraint** is that you must independently make all content, tables, graphs, and facts that are offered in the paper. You must write each part of the paper wholly on your own. The Peer-reviewers need to identify your own perceptive of the concepts in your own terms. NEVER extract straight from any foundation, and never rephrase someone else's analysis.
- Do not give permission to anyone else to "PROOFREAD" your manuscript.
- Methods to avoid Plagiarism is applied by us on every paper, if found guilty, you will be blacklisted by all of our collaborated research groups, your institution will be informed for this and strict legal actions will be taken immediately.)
- To guard yourself and others from possible illegal use please do not permit anyone right to use to your paper and files.

Criterion for Grading a Research Paper (Compilation) by Global Journals Inc. (US)

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Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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