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Highlights

- Partial Differential Equations
- The Variational Iteration Method
- Terms of Laguerre Polynomials
- Analytic Treatment of Homogeneous

Discovering Thoughts, Inventing Future

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Certain Sequences and its Integral Representations in Terms of Laguerre Polynomials

By Baghdadi Aloui

Abstract- In this paper, we introduce a connection formula between the monomial basis and the shifted Laguerre basis. As an application, some integral representations in terms of Laguerre polynomials for certain sequences are obtained.

Keywords: *Laguerre polynomials, special functions, integral formulas.*

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Certain Sequences and its Integral Representations in Terms of Laguerre Polynomials

Baghdadi Aloui

Abstract- In this paper, we introduce a connection formula between the monomial basis and the shifted Laguerre basis. As an application, some integral representations in terms of Laguerre polynomials for certain sequences are obtained.
Keywords: Laguerre polynomials, special functions, integral formulas.

I. Introduction and Main Results

By using some special functions and some particular integrals, we recall some integral representations for certain integer (or real) sequences.

a) Some special functions

The Gamma function is defined by the definite integral

$$\Gamma(z)=\int_0^{+\infty} x^{z-1}e^{-x}\,\mathrm{d}x,\quad \Re e(z)>0.$$

We can see directly, that $\Gamma(1)=1$, and using integration by parts, that $\Gamma(z+1)=z\Gamma(z)$. Notice that, for $z=n\in\mathbb{N}\setminus\{0\}$, the following formulas hold

$$\begin{aligned} n! &= \Gamma(n+1) \\ &= \int_0^{+\infty} x^n e^{-x}\,\mathrm{d}x, \qquad (1) \\ \frac{(2n)!}{2^{2n}n!}\sqrt{\pi} &= \Gamma\big(n+\frac{1}{2}\big) \\ &= \int_0^{+\infty} x^n \frac{e^{-x}}{\sqrt{x}}\,\mathrm{d}x. \qquad (2) \end{aligned}$$

The Bêta function is given in terms of the integral

$$B(s,t)=\int_0^1 x^{s-1}(1-x)^{t-1}\,\mathrm{d}x,\quad \Re e(s),\ \Re e(t)>0.$$

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which is symmetric in s and t, i.e., $B(s,t) = B(t,s)$. Notice that, after a change of variable $x = \frac{1}{1+y}$, we get

$$B(s,t) = \int_0^{+\infty} \frac{x^{s-1}}{(1+x)^{s+t}}\, \mathrm{d}x.$$

This function also admits the following representation in terms of the Gamma function [3]

$$B(s,t) = \frac{\Gamma(s)\Gamma(t)}{\Gamma(s+t)}.$$

In particular, if s and t are non-zero integers, then we have

$$\begin{aligned} \frac{n!p!}{(n+p+1)!} &= B(n+1,p+1) \\ &= \int_0^1 x^n(1-x)^p\, \mathrm{d}x, \quad (3)\\ &= \int_0^{+\infty} \frac{x^n}{(1+x)^{n+p+2}}\, \mathrm{d}x, \quad n,\ p \geq 0. \quad (4)\end{aligned}$$

The monic Hermite polynomials $H_n(x)$ are orthogonal in the interval $(-\infty, +\infty)$ with respect to the weight function e^{-x^2} and fulfil the following orthogonality relation [2]

$$\int_{-\infty}^{+\infty} e^{-x^2} H_n(x)H_m(x)\, \mathrm{d}x = \frac{\sqrt{\pi}}{2^m} n!\delta_{n,m}, \quad n,\ m \geq 0,$$

where $\delta_{n,m}$ is the Kronecker delta.

The canonical moments, $(\mathcal{H})_{n\geq 0}$, of the Hermite polynomials have the representation [3]

$$\begin{aligned} \frac{\big(1+(-1)^n\big)n!}{2^{n+1}\Gamma(\frac{n}{2}+1)} &= (\mathcal{H})_n \\ &= \int_{-\infty}^{+\infty} x^n e^{-x^2}\, \mathrm{d}x, \quad n \geq 0. \quad (5)\end{aligned}$$

b) Some other integrals

The Wallis integral is given by

$$I_n = \int_0^{\frac{\pi}{2}} \sin^n x\, \mathrm{d}x, \quad n \geq 0.$$

By a simple integration by parts, we can obtain

$$I_{2n} = \frac{(2n)!\pi}{2^{2n+1}(n!)^2}, \quad I_{2n+1} = \frac{2^{2n}(n!)^2}{(2n+1)!}, \quad n \geq 0.$$

By the change of variable $t = \sin x$, this gives the following formulas

$$\frac{(2n)!\pi}{2^{2n+1}(n!)^2} = \int_0^1 \frac{x^{2n}}{\sqrt{1-x^2}}\, \mathrm{d}x, \quad n \geq 0, \quad (6)$$

$$\frac{2^{2n}(n!)^2}{(2n+1)!} = \int_0^1 \frac{x^{2n+1}}{\sqrt{1-x^2}}\, \mathrm{d}x, \quad n \geq 0. \quad (7)$$

Now, let consider the following integral

Ref

3. P. Maroni, *Fonctions Eulériennes, Polynômes Orthogonaux Classiques.* Techniques de l'Ingénieur, Traité Généralités (Sciences Fondamentales) A 154 Paris, 1994. 1-30.

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$$T_n = \int_0^{\frac{\pi}{4}} \tan^n x \, \mathrm{d}x, \quad n \geq 0.$$

It is easy to see that

$$T_{n+2} + T_n = \frac{1}{n+1}, \quad n \geq 0.$$

We get by iteration the two following formulas

$$T_{2n+1} = \sum_{k=1}^{n} \frac{(-1)^{n+k}}{2k} + (-1)^n T_1, \quad n \geq 0.$$

$$T_{2n+2} = \sum_{k=0}^{n} \frac{(-1)^{n+k}}{2k+1} + (-1)^{n+1} T_0, \quad n \geq 0.$$

Then, by the change of variable $t = \tan x$, we get

$$\frac{(-1)^n}{2}\left(\ln 2 + \sum_{k=1}^{n} \frac{(-1)^k}{k}\right) = \int_0^1 \frac{x^{2n+1}}{1+x^2} \, \mathrm{d}x, \quad n \geq 0, \tag{8}$$

$$(-1)^n\left(\frac{\pi}{4} + \sum_{k=0}^{n-1} \frac{(-1)^{k+1}}{2k+1}\right) = \int_0^1 \frac{x^{2n}}{1+x^2} \, \mathrm{d}x, \quad n \geq 0, \tag{9}$$

with the convention $\sum_{k=1}^{0} = \sum_{k=0}^{-1} = 0$.

We also consider the following integral

$$R_n = \int_0^1 \frac{x^n}{1+x} \, \mathrm{d}x, \quad n \geq 0.$$

It is easy to see that $R_n + R_{n+1} = \frac{1}{n}$, $n \geq 1$, and hence the following formula

$$(-1)^n\left(\ln 2 + \sum_{k=1}^{n} \frac{(-1)^k}{k}\right) = \int_0^1 \frac{x^n}{1+x} \, \mathrm{d}x, \quad n \geq 0. \tag{10}$$

Finally, we consider the integral

$$B_n = \frac{1}{n!} \int_0^1 (1-x)^n e^x \, \mathrm{d}x, \quad n \geq 0.$$

For $n \geq 1$, integration by parts yields $B_n = B_{n-1} - \frac{1}{n!}$, and we obtain the formula

$$e - \sum_{k=0}^{n} \frac{1}{k!} = \frac{1}{n!} \int_0^1 (1-x)^n e^x \, \mathrm{d}x, \quad n \geq 0.$$

This gives, after a change of variable $t = 1 - x$, the following relation

$$n!\left(1 - \frac{1}{e} \sum_{k=0}^{n} \frac{1}{k!}\right) = \int_0^1 x^n e^{-x} \, \mathrm{d}x, \quad n \geq 0. \tag{11}$$

In this paper, we introduce the following connection formula, between the monomial $\{x^n\}_{n\geq 0}$ and the shifted Laguerre polynomials,

$$x^n = \frac{1}{(n+m)!}\int_0^{+\infty} t^m e^{-t} L_n^{(m)}\big(t(x+1)\big)\ \mathrm{d}t, \quad n \geq 0,\ m \in \mathbb{N}\setminus\{0\}.$$

As an application of our formula, we give the integral representations in terms of Laguerre polynomials for the sequences given by the equations (1)−(11).

II. Integral Representations in Terms of Laguerre Polynomials

Let $\{L_n^{(m)}\}_{n\geq 0}$ be the monic Laguerre polynomial sequence, with parameter $m \in \mathbb{N}\setminus\{0\}$, [4]

$$L_n^{(m)}(x) = \sum_{\nu=0}^{n} (-1)^{n-\nu}\binom{n}{\nu}\frac{(n+m)!}{(\nu+m)!}\,x^{\nu}, \quad n \geq 0. \tag{12}$$

For any $c \in \mathbb{C}$ and any polynomial p, let introduce in the space of polynomials the linear isomorphism $\mathfrak{S}_c$, called intertwining operator, and given by [1]:

$$\mathfrak{S}_c(p)(x) = \int_0^{+\infty} te^{-t} p\big(t(x-c)+c\big)\ \mathrm{d}t.$$

The operator $\mathfrak{S}_c$ can be characterized taking into account its linearity as well as the fact

$$\mathfrak{S}_c\big((x-c)^n\big) = (n+1)!(x-c)^n, \quad n \geq 0. \tag{13}$$

By (13) and (12), it is easy to prove that

$$\mathfrak{S}_0\big(x^{m-1}L_n^{(m)}(x)\big) = (n+m)!x^{m-1}(x-1)^n, \quad n \geq 0.$$

Hence, we can obtain the following result.

Theorem 2.1 For every integer $m \in \mathbb{N}\setminus\{0\}$, the following formula holds

$$x^n = \frac{1}{(n+m)!}\int_0^{+\infty} t^m e^{-t} L_n^{(m)}\big(t(x+1)\big)\ \mathrm{d}t, \quad n \geq 0. \tag{14}$$

Now, as an application of the above formula, we can express the sequences given by the equations (1)−(11) by integral representations in terms of Laguerre polynomials. Indeed, substituting expression (14) into (1)−(11), we can state the following theorem.

Theorem 2.2 For every integers $m \geq 1$, and $n,\ p \geq 0$, the following formulas hold

$$n!(n+m)! = \int_0^{+\infty}\int_0^{+\infty} t^m e^{-(x+t)} L_n^{(m)}\big(t(x+1)\big)\ \mathrm{d}t\mathrm{d}x$$

$$\frac{(2n)!m!\sqrt{\pi}}{4^n}\binom{n+m}{n} = \int_0^{+\infty}\int_0^{+\infty} \frac{t^m}{\sqrt{x}}\ e^{-(x+t)} L_n^{(m)}\big(t(x+1)\big)\ \mathrm{d}t\mathrm{d}x$$

$$\frac{n!p!(n+m)!}{(n+p+1)!} = \int_0^{1}\int_0^{+\infty} (1-x)^p t^m e^{-t} L_n^{(m)}\big(t(x+1)\big)\ \mathrm{d}t\mathrm{d}x$$

$$\frac{n!p!(n+m)!}{(n+p+1)!} = \int_0^{+\infty}\int_0^{+\infty} \frac{t^m e^{-t}}{(1+x)^{n+p+2}}\ L_n^{(m)}\big(t(x+1)\big)\ \mathrm{d}t\mathrm{d}x$$

Ref

1. H. Hochstadt, *The Functions of Mathematical Physics.* Dover Publications Inc. New York, 1971.

Notes

$$\frac{(1+(-1)^n)n!(n+m)!}{2^{n+1}\Gamma\left(\frac{n}{2}+1\right)} = \int_{-\infty}^{+\infty}\int_0^{+\infty} t^m e^{-(x^2+t)} L_n^{(m)}\big(t(x+1)\big)\ \mathrm{d}t\mathrm{d}x$$

$$\frac{(2n)!(2n+m)!\pi}{(n!)^2 2^{2n+1}} = \int_0^1\int_0^{+\infty} \frac{t^m e^{-t}}{\sqrt{1-x^2}}\ L_{2n}^{(m)}\big(t(x+1)\big)\ \mathrm{d}t\mathrm{d}x$$

$$\frac{(n!)^2(2n+m+1)!2^{2n}}{(2n+1)!} = \int_0^1\int_0^{+\infty} \frac{t^m e^{-t}}{\sqrt{1-x^2}}\ L_{2n+1}^{(m)}\big(t(x+1)\big)\ \mathrm{d}t\mathrm{d}x$$

$$\frac{(2n+m+1)!(-1)^n}{2}\left(\ln 2+\sum_{k=1}^{n}\frac{(-1)^k}{k}\right) = \int_0^1\int_0^{+\infty} \frac{t^m e^{-t}}{1+x^2}\ L_{2n+1}^{(m)}\big(t(x+1)\big)\ \mathrm{d}t\mathrm{d}x$$

$$(2n+m)!(-1)^n\left(\frac{\pi}{4}+\sum_{k=0}^{n-1}\frac{(-1)^{k+1}}{2k+1}\right) = \int_0^1\int_0^{+\infty} \frac{t^m e^{-t}}{1+x^2}\ L_{2n}^{(m)}\big(t(x+1)\big)\ \mathrm{d}t\mathrm{d}x$$

$$(n+m)!(-1)^n\left(\ln 2+\sum_{k=1}^{n}\frac{(-1)^k}{k}\right) = \int_0^1\int_0^{+\infty} \frac{t^m e^{-t}}{1+x}\ L_{n}^{(m)}\big(t(x+1)\big)\ \mathrm{d}t\mathrm{d}x$$

$$n!(n+m)!\left(1-\frac{1}{e}\sum_{k=0}^{n}\frac{1}{k!}\right) = \int_0^1\int_0^{+\infty} t^m e^{-(t+x)} L_n^{(m)}\big(t(x+1)\big)\ \mathrm{d}t\mathrm{d}x$$

with the convention $\sum_{k=1}^{0}=\sum_{k=0}^{-1}=0$.

Corollary 2.3 *For* $m=1$ *and* $p=0$ *we have, for every integer* $n\geq 0$, *the following special cases*

$$n!(n+1)! = \int_0^{+\infty}\int_0^{+\infty} te^{-(x+t)} L_n^{(1)}\big(t(x+1)\big)\ \mathrm{d}t\mathrm{d}x$$

$$\frac{(2n)!(n+1)\sqrt{\pi}}{2^{2n}} = \int_0^{+\infty}\int_0^{+\infty} \frac{t}{\sqrt{x}}\ e^{-(x+t)} L_n^{(1)}\big(t(x+1)\big)\ \mathrm{d}t\mathrm{d}x$$

$$n! = \int_0^1\int_0^{+\infty} te^{-t} L_n^{(1)}\big(t(x+1)\big)\ \mathrm{d}t\mathrm{d}x$$

$$n! = \int_0^{+\infty}\int_0^{+\infty} \frac{te^{-t}}{(1+x)^{n+2}}\ L_n^{(1)}\big(t(x+1)\big)\ \mathrm{d}t\mathrm{d}x$$

$$\frac{(1+(-1)^n)n!(n+1)!}{2^{n+1}\Gamma\left(\frac{n}{2}+1\right)} = \int_{-\infty}^{+\infty}\int_0^{+\infty} te^{-(x^2+t)} L_n^{(1)}\big(t(x+1)\big)\ \mathrm{d}t\mathrm{d}x$$

$$\frac{(2n+1)!\pi}{2^{2n+1}}\binom{2n}{n} = \int_0^1\int_0^{+\infty} \frac{te^{-t}}{\sqrt{1-x^2}}\ L_{2n}^{(1)}\big(t(x+1)\big)\ \mathrm{d}t\mathrm{d}x$$

$$n!(n+1)2^{2n+1} = \int_0^1 \int_0^{+\infty} \frac{te^{-t}}{\sqrt{1-x^2}} \, L_{2n+1}^{(1)}\big(t(x+1)\big) \, \mathrm{d}t\mathrm{d}x$$

$$\frac{(2n+2)!(-1)^n}{2}\left(\ln 2 + \sum_{k=1}^{n} \frac{(-1)^k}{k}\right) = \int_0^1 \int_0^{+\infty} \frac{te^{-t}}{1+x^2} \, L_{2n+1}^{(1)}\big(t(x+1)\big) \, \mathrm{d}t\mathrm{d}x$$

$$(2n+1)!(-1)^n\left(\frac{\pi}{4} + \sum_{k=0}^{n-1} \frac{(-1)^{k+1}}{2k+1}\right) = \int_0^1 \int_0^{+\infty} \frac{te^{-t}}{1+x^2} \, L_{2n}^{(1)}\big(t(x+1)\big) \, \mathrm{d}t\mathrm{d}x$$

$$(n+1)!(-1)^n\left(\ln 2 + \sum_{k=1}^{n} \frac{(-1)^k}{k}\right) = \int_0^1 \int_0^{+\infty} \frac{te^{-t}}{1+x} \, L_{n}^{(1)}\big(t(x+1)\big) \, \mathrm{d}t\mathrm{d}x$$

$$n!(n+1)!\left(1 - \frac{1}{e}\sum_{k=0}^{n} \frac{1}{k!}\right) = \int_0^1 \int_0^{+\infty} te^{-(t+x)} L_{n}^{(1)}\big(t(x+1)\big) \, \mathrm{d}t\mathrm{d}x$$

with the convention $\sum_{k=1}^{0} = \sum_{k=0}^{-1} = 0$.

Remark 2.1 Note that, if we take n even in the fifth formula, we obtain

$$\frac{(2n)!(2n+1)!}{2^{2n}n!} = \int_{-\infty}^{+\infty} \int_0^{+\infty} te^{-(x^2+t)} L_{2n}^{(1)}\big(t(x+1)\big) \, \mathrm{d}t\mathrm{d}x.$$

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4. G. Szegö, *Orthogonal Polynomials*, Amer. Math. Soc. Colloq. Publ., Vol. 23, Amer. Math. Soc., Providence, RI, 1975.

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The Variational Iteration Method for Analytic Treatment of Homogeneous and Inhomogeneous Partial Differential Equations

By M. O. Olayiwola

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Abstract- In this paper, the author used the variational iteration method (VIM) to find the analytical solution to homogeneous and inhomogeneous partial differential equations. Few numerical examples were presented to show the effectiveness and efficiency of the method. It was observed that by carefully chosen a very good choice of initial guess leads to a solution in a closed form. The method is elegant and reliable.

Keywords: variational iteration method, initial value problems, partial differential equations.

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THEVARIATIONALITERATIONMETHODFORANALYTICTREATMENTOFHOMOGENEOUSANDINHOMOGENEOUSPARTIALDIFFERENTIALEQUATIONS

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The Variational Iteration Method for Analytic Treatment of Homogeneous and Inhomogeneous Partial Differential Equations

M. O. Olayiwola

Abstract- In this paper, the author used the variational iteration method (VIM) to find the analytical solution to homogeneous and inhomogeneous partial differential equations. Few numerical examples were presented to show the effectiveness and efficiency of the method. It was observed that by carefully chosen a very good choice of initial guess leads to a solution in a closed form. The method is elegant and reliable.

Keywords: *variational iteration method, initial value problems, partial differential equations.*

I. Introduction

Partial differential equations; linear or nonlinear, homogenous or inhomogeneous has many applications to real life problems that arise in science, engineering and technology.

There are many numerical methods for the solution of different types of differential equations such as Adomian decomposition method [1-3], homotopy perturbation method [4], variational iteration method [5-9], modified variational iteration method 14-15]. Results by various researchers [1-15] have shown reliability, efficiency and applicability of these methods.

In this paper, a variational iteration method for the solution of homogeneous and inhomogeneous partial differential equations is presented. It is to be noted that the Lagrange multiplier reduces the iteration on integral operator and also minimizes the computational time. The method requires no transformation and or linearisation of any forms. Some numerical problems and results are presented to show the reliability of the method.

II. The Variational Iteration Method

The basic idea of the He's Variational Iteration Method (VIM) [5-9], can be explained by considering the following nonlinear partial differential equations

$$Lu + Nu = g(x) \tag{1}$$

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Ref

4. Hossein Jafari etal (2008): Application of Homotopy Perturbation Method for Solving Gas Dynamic Equation. Applied Mathematical Sciences, Vol.2,2008. No 48,2393-2396.

Where L is the linear operator, N is the nonlinear operator and g(x) is the inhomogeneous term. According to the method, we can construct a correction functional as follows
The corresponding variational iteration method for solving (1) is given as

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda \left[Lu_n(s) + N\bar{u}_n(s) - g(s) \right] ds, \tag{2}$$

where λ is a Lagrange multiplier which can be identified optimally by variational iteration method. The subscript n denote the nth approximation, $\bar{u}_n$ is considered as a restricted variation i.e $\delta\bar{u}_n = 0$. The successive approximation $u_{n+1}, n \geq 0$ of the solution u can be easily obtained by determine the Lagrange multiplier and the initial guess u_0, consequently, the solution is given by $u = \lim_{n\to\infty} u_n$.

III. Application and Numerical Results

In this section, six problems will be presented to illustrate the efficiency of the method.

Example 3.1

Consider the following inhomogeneous equation

$$u_x - u_y = 1 + 2x + 2y, \; u(0,y) = y + y^2, u(x,0) = 2x + 3x^2 \tag{3}$$

The correction functional is given by

$$u_{n+1}(x,y) = y + y^2 + \int_0^x \lambda(s) \left[\frac{\partial u_n(s,y)}{\partial s} - \frac{\partial u_n(s,y)}{\partial y} - 1 - 2s - 2y) \right] ds \tag{4}$$

Making the correction functional stationary to obtain $\lambda(s) = -1$, hence, the iterative formula becomes

$$u_{n+1}(x,y) = y + y^2 - \int_0^x \left[\frac{\partial u_n}{\partial x} - \frac{\partial u_n}{\partial y} - 1 - 2x - 2y) \right] ds \tag{5}$$

Consequently, following approximants are obtained

$$u_1(x,y) = y + y^2 + 4xy + x^2 + 2x \tag{6}$$

$$u_2(x,y) = y + y^2 + 4xy + 3x^2 + 2x \tag{7}$$

$$u_3(x,y) = y + y^2 + 4xy + 3x^2 + 2x \tag{8}$$

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the closed form and exact solution is given as

$$u(x,y) = y + y^2 + 4xy + 3x^2 + 2x \tag{9}$$

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Example 3.2

Consider the following inhomogeneous equation

$$u_x + u_y = 2xy^2 + 2x^2 y,\;\; u(0,y) = 0, u(x,0) = 0 \tag{10}$$

The correction functional and iterative formula becomes:

$$u_{n+1}(x,y) = 0 - \int_0^x \lambda(s)\left[\frac{\partial u_n(s,y)}{\partial s} + \frac{\partial u_n(s,y)}{\partial y} - 2sy^2 - 2s^2 y)\right] ds \tag{11}$$

then, following approximants are obtained

$$u_1(x,y) = x^2 y^2 + \frac{2}{3}x^3 y \tag{12}$$

$$u_2(x,y) = x^2 y^2 + \frac{1}{6}x^4 \tag{13}$$

$$u_3(x,y) = x^2 y^2 \tag{14}$$

$$u_4(x,y) = x^2 y^2 \tag{15}$$

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The exact solution is given by

$$\lim_{n\to\infty} u(x,y) = x^2 y^2 \tag{16}$$

Example 3.3

Consider the following equation

$$u_x + u_y = 2u,\;\; u(0,y) = e^y, u(x,0) = e^x \tag{17}$$

The correction functional is given by

$$u_{n+1}(x,y) = e^y - \int_0^x \left[\frac{\partial u_n(s,y)}{\partial s} + \frac{\partial u_n(s,y)}{\partial y} - 2u(s,y)\right] ds \tag{18}$$

from (18), following iterations are obtained

$$u_1(x,y) = e^y + xe^y \tag{19}$$

$$u_2(x,y) = e^y + xe^y + \frac{x^2}{2}e^y \tag{20}$$

$$u_3(x,y) = e^y + xe^y + \frac{x^2}{2}e^y + \frac{x^3}{6}e^y \tag{21}$$

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The series solution is given by

$$u(x,y) = e^y + xe^y + \frac{x^2}{2!}e^y + \frac{x^3}{3!}e^y + \frac{x^4}{4!}e^y + \frac{x^5}{5!}e^y + \frac{x^6}{6!}e^y + \ldots$$
$$= e^y(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \ldots \quad (22)$$

and the closed form solution is

$$u(x,y) = e^{x+y} \quad (23)$$

which is the exact solution.

Example 3.4

Consider the following equation

$$u_x + u_y + u_z = 3,\ u(0,x,y) = y + z, u(x,0,z) = x + z,\ u(x,y,0) = x + y \quad (24)$$

The correction functional and formula is given by

$$u_{n+1}(x,y,z) = y + z - \int_0^x \left[\frac{\partial u_n(s,y,z)}{\partial s} + \frac{\partial u_n(s,y,z)}{\partial y} + \frac{\partial u_n(s,y,z)}{\partial z} - 3) \right] ds \quad (25)$$

Consequently, following approximants are obtained

$$u_1(x,y) = x + y + z \quad (26)$$

$$u_2(x,y) = x + y + z \quad (27)$$

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$$u(x,y) = x + y + z \quad (28)$$

This is the exact solution to (24).

Example 3.5

Consider the following equation

$$u_x + u_y + u_z = 3u,\ u(0,x,y) = e^{y+z}, u(x,0,z) = e^{x+z},\ u(x,y,0) = e^{x+y} \quad (29)$$

The correction functional is given by

$$u_{n+1}(x,y,z) = e^{y+z} - \int_0^x \left[\frac{\partial u_n(s,y,z)}{\partial s} + \frac{\partial u_n(s,y,z)}{\partial y} + \frac{\partial u_n(s,y,z)}{\partial z} - 3u(s,y,z)) \right] ds \quad (30)$$

therefore,

$$u_1(x,y) = e^{y+z} + xe^{y+z} \quad (31)$$

$$u_2(x,y) = e^{y+z} + xe^{y+z} + \frac{x^2}{2!}e^{y+z} \quad (32)$$

Notes

$$u_3(x,y) = e^{y+z} + xe^{y+z} + \frac{x^2}{2!}e^{y+z} + \frac{x^3}{3!}e^{y+z} + \ldots \tag{33}$$

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Notes

The series solution is given by

$$u(x,y) = e^{y+z}(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \ldots \tag{34}$$

and the closed form solution is

$$u(x,y) = e^{x+y+z} \tag{35}$$

Example 3.6

Consider the following equation

$$u_x - u_y = 0, \quad u(0,y) = \sin y . u(x,0) = \sin x \tag{36}$$

The correction functional is given by

$$u_{n+1}(x,y) = \sin y - \int_0^x \left[\frac{\partial u_n(s,y)}{\partial s} + \frac{\partial u_n(s,y)}{\partial y} \right] ds \tag{37}$$

Consequently, following approximant are obtained

$$u_1(x,y) = \sin y + x\cos y \tag{38}$$

$$u_2(x,y) = \sin y + x\cos y - \frac{x^2}{2!}\sin y \tag{39}$$

$$u_3(x,y) = \sin y + x\cos y - \frac{x^2}{2!}\sin y - \frac{x^3}{3!}\cos y \tag{40}$$

$$u_4(x,y) = \sin y + x\cos y - \frac{x^2}{2!}\sin y - \frac{x^3}{3!}\cos y - \frac{x^4}{4!}\sin y \tag{41}$$

$$u_5(x,y) = \sin y + x\cos y - \frac{x^2}{2!}\sin y - \frac{x^3}{3!}\cos y - \frac{x^4}{4!}\sin y + \frac{x^5}{5!}\cos y \tag{42}$$

The series solution is given by

$$u(x,y) = \sin y(1 - \frac{x^2}{2!} - \frac{x^4}{4!} + \ldots) + \cos y(x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots) \tag{43}$$

and the closed form solution is

$$u(x,y) = \sin x coxy + \sin y \cos x = \sin(x+y) \tag{44}$$

which is the exact solution.

IV. Conclusion

The paper has successfully described and applied the variational iteration method to some partial differential equations of physical significance. The method provides the solutions in terms of rapidly convergent series. It is also clear and remarkable that approximate solutions are in good agreement with analytical solution. The VIM was used in a direct way without using linearization, perturbation or restrictive assumptions. The method is elegant and reliable.

Notes

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Special Pythagorean Triangles and 10 Digit Dhuruva Numbers

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Special Pythagorean Triangles and 10 Digit Dhuruva Numbers

Manju Somanath [α], V. Sangeetha [σ] & M. A. Gopalan [ρ]

Abstract- Pythagorean triangles, each with a leg represented by 10-digit Dhuruva numbers are obtained. A few interesting results are given.

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I. Introduction

The fascinating branch of mathematics is the theory of numbers where in Pythagorean triangles have been a matter of interest to various mathematicians and to the lovers of mathematics, because it is a treasure house in which the search for many hidden connection is a treasure hunt. For a rich variety of fascinating problems one may refer [1-17].A careful observer of patterns may note that there is a one to one correspondence between the polygonal numbers and the number of sides of the polygon. Apart from the above patterns we have some more fascinating patterns of numbers namely Jarasandha numbers, Nasty numbers and Dhuruva numbers. These numbers have been presented in [18-21].

In [22-25], special Pythagorean triangles connected with polygonal numbers and Nasty numbers are obtained. Recently in [26], special Pythagorean triangles in connection with Hardy Ramanujan number 1729 are exhibited. Thus the objective of this paper is to find out the special Pythagorean triangles in connection with 10 digit Dhuruvanumber 9753086421.

In this communication we have presented Pythagorean triangles each with a leg represented by 10 digit Dhuruva numbers 9753086421,9975084201 and 6333176664. Also, a few special Pythagorean triangles in connection with these three numbers are obtained.

II. Basic Definitons

a) Definition 2.1

The ternary quadratic Diophantine equation given by $x^2+y^2=z^2$is known as Pythagorean equation where *x*, *y*, *z* are natural numbers. The above equation is also referred to as Pythagorean triangle and denote it by T(*x*,*y*,*z*).

Also, in Pythagorean triangle T(*x*,*y*,*z*) : $x^2+y^2=z^2$, *x* and *y* are called its legs and *z* its hypotenuse.

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b) Definition 2.2

Most cited solution of the Pythagorean equation is $x = m^2 - n^2; y = 2mn\,; z = m^2 + n^2$, where $m > n > 0$. This solution is called primitive, if *m, n* are of opposite parity and gcd(*m, n*)=1.

c) Definition 2.3: Dhuruva numbers

The numbers which do not change when we perform a single operation or a sequence of operations are known as Dhuruva numbers.

Notes

III. Method of Analysis

In this section, we exhibit Pythagorean triangles, each with a leg represented by the 10-digit Dhuruva number 9753086421 and denote this number by N_1.

To start with, it is noted that the leg *y* cannot be represented by N_1 as *y* is even and N_1 is odd. Also *z* cannot be written as sum of two squares since a positive integer P can be written as a sum of two integer squares iff the canonical prime factorization $P = p_1^{r_1} . p_2^{r_2} \dots p_r^{r_r}$, (where p_i are distinct primes) satisfies the condition if $p_i \equiv 3 \ (mod\ 4)$ then r_i is even. A prime $p \equiv 1 (mod\ 4)$ can be written as $p = a^2 + b^2$.

Now, consider $x = N_1 \Rightarrow x = m^2 - n^2$ which is a binary quadratic Diophantine equation. Solving the above equation for *m,n* we get 15 integer solutions and thus, we have 15 Pythagorean triangles, each having the leg *x* to be represented by the ten digit Dhuruva number $N_1 = 9753086421$ as shown in table 1 below:

Table.1

No.	*m*	*n*	*x*	*y*	*z*
1	1625514405	1625514402	9753086421	5284594151971921620	5284594151971921629
2	541838139	541838130	9753086421	587177127996880140	587177127996880221
3	18062725	18062698	9753086421	652523093464100	652523093464829
4	157307861	257307830	9753086421	4949166438109260	49491516511704221
5	52435995	52435902	9753086421	5499057390184980	5499057390193629
6	17478789	17478510	9753086421	611006376648780	611006376726621
7	375118715	375118702	9753086421	281428090933415860	281428090933416029
8	12100805	12100402	9753086421	292849210047220	292849210209629
9	5826635	5825798	9753086421	67889597059460	67889597760029
10	453611	442730	9753086421	401654396060	401772792221
11	13893461	13893110	9753086421	386046763907420	38604764030621
12	41679915	41679798	9753086421	3474420875714340	3474420875728029
13	4034139	4032930	9753086421	32538800394540	32538801856221
14	1346325	1342698	9753086421	3615415769700	3615428924829
15	4876543211	4876543210	9753086421	47561347367747294620	47561347367747294621

Note that there are 8 primitive and 7 non-primitive triangles. Also the relation $\frac{4A}{P} - y + z$ represents the 10-digit Dhuruva number 9753086421 for each of the above Pythagorean triangles, where A and P represents the area and perimeter of the Pythagorean triangle.

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In a similar manner, it is seen that there are 30 Pythagorean triangles wherein, each of the following expressions $\frac{2A}{P}, \frac{1}{2}(y+x-z)$represent 9753086421 as shown in table 2 below:

Table 2

No	m	n	x	y	z
1	9753086422	1	95122694735494589240	19506172842	95122694735494589242
2	3251028810	3	10569188323450016091	19506172860	10569188323450016109
3	1083676278	9	1174354275499933203	19506173004	1174354275499933365
4	361225450	27	130483825727701771	19506174300	130483825727703229
5	314615722	31	98983052529580323	19506174764	98983052529582245
6	104871990	93	10998134286551451	19506190140	10998134286568749
7	34957578	279	1222032259548243	19506328524	1222032259703925
8	750237430	13	562856201373004731	19506173180	562856201373005069
9	24201610	403	585717926429691	19506497660	585717926754509
10	11653270	837	135798700992331	19506172842	135798702393469
11	907222	10881	822933361123	19742965164	823170153445
12	27786922	351	772113034110883	19506419244	772113034357285
13	83359830	117	6948861257615211	19506200220	6948861257642589
14	8068278	1209	65097108423603	19509096204	65097111346965
15	2692650	3627	7250350867371	19532483100	7250377177629
16	9753086422	9753086421	19506172843	190245389490495351324	190245389490495351325
17	3251028810	3251028807	19506172851	21138376627393859340	21138376627393859349
18	1083676278	1083676269	19506172923	2348708531493693564	2348708531493693645
19	361225450	361225423	19506173571	260967631949230700	260967631949231429
20	314615722	314615691	19506173803	197966085552987804	197966085552988765
21	104871990	104871897	19506181491	21996249066930060	21996249066938709
22	34957578	34957299	19506250683	2444045012923644	2444045013001485
23	750237430	750237417	19506173011	1125712383239836620	1125712383239836789
24	24201610	24201207	19506335251	1171416346686540	1171416346848949
25	11653270	11652433	19506873411	271577895811820	271577896512389
26	907222	896341	19624569003	1626360549404	1626478945565
27	27786922	27786571	19506296043	1544206562048924	1544206562172125
28	83359830	83359713	19506186531	13897703009057580	13897703009071269
29	8068278	8067069	19507634523	130174710674364	130174712136045
30	2692650	2689023	19519327971	14481195561900	14481208717029

Note that there are 15 primitive and 15 non-primitive triangles.

Also,it is observed that there are 15 Pythagorean triangles wherein each of the expressions $x-\frac{2A}{P}, \frac{1}{2}(z+x-y)$ is represented by 9753086421 as shown in table 3 below:

Table 3

No.	m	n	x	y	z
1	3251028807	3251028804	19506172833	2113876588381513656	2113876588381513665
2	1083676269	1083676260	19506172761	2348708492481347880	2348708492481347961
3	361225423	361225396	19506172113	260967592936885016	260967592936885745
4	314615691	314615660	19506171881	197966046540642120	197966046540643081
5	104871897	104871804	19506164193	21996210054584376	21996210054593025
6	34957299	34957020	19506095001	244400600577960	2444006000655801
7	750237417	750237404	19506172673	1125712344227490936	1125712344227491105
8	24201207	24200804	19506010433	1171377334340856	1171377334503265
9	11652433	11651596	19505472273	271538883466136	271538884166705
10	896341	885460	19387776681	1587348203720	1587466599881
11	27786571	27786220	19506049641	1544167549703240	1544167549826441
12	83359713	83359596	19506159153	13897663996711896	13897663996725585
13	8067069	8065860	19504711161	130135698328680	130135699790361
14	2689023	2685396	19493017713	14442183216216	14442196371345
15	9753086421	9753086420	19506172841	190245389451483005640	190245389451483005641

Note that there are 8 primitive and 7 non-primitive triangles.

For simplicity, in table 4, we exhibit the connections between special Pythagorean triangles and the other ten digit Dhuruva numbers N_2 and N_3 in the following table respectively:

Table 4

Dhuruva Number	Expressions	No.of Triangles	Remark
$N_2 = 9975084201$	$\frac{4A}{P} - y + z$; x	4	2-Primitive, 2-Non-primitive
	$x - \frac{2A}{P}$ and $\frac{1}{2}(z + x - y)$	4	2-Primitive, 2-Non-primitive
	$\frac{2A}{P}$; $\frac{1}{2}(y + x - z)$	8	5-Primitive, 3-Non-primitive
$N_3 = 6333176664$	$\frac{4A}{P} - y + z$; x	6	6-Non-primitive
	y	9	4- Primitive 5-Non-primitive
	$x - \frac{2A}{P}$ and $\frac{1}{2}(z + x - y)$	12	2-Primitive 10-Non-primitive
	$\frac{2A}{P}$; $\frac{1}{2}(y + x - z)$	24	6-Primitive 18-Non-primitive

Notes

IV. Conclusion

One may search for the connections between Pythagorean triangles and other Dhuruva numbers.

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Hypergeometric Solutions of Certain Definite Integrals

By Salahuddin & R. K. Khola
Mewar University, India

Abstract- We have developed certain definite integrals involving Hypergeometric functions. This integrals are new.

Keywords: *generalized hypergeometric function, complete elliptic integrals.*

GJSFR-F Classification : *FOR Code : MSC 2010: 81S40.*

HYPERGEOMETRICSOLUTIONSOFCERTAINDEFINITEINTEGRALS

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Hypergeometric Solutions of Certain Definite Integrals

Salahuddin [α] & R. K. Khola [σ]

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I. Introduction

a) Generalized Hypergeometric Function

The generalized hypergeometric function of one variable [Prudnikov, p.437; see also E.D., p.73(2)] is defined as follows:

$$ {}_AF_B\left[\begin{matrix} a_1, a_2, \ldots\ldots, a_A; \\ b_1, b_2, \ldots\ldots, b_B; \end{matrix} z\right] = \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n \ldots\ldots (a_A)_n}{(b_1)_n (b_2)_n \ldots\ldots (b_B)_n} \frac{z^n}{n!} $$

$$ \text{or} \qquad {}_AF_B\left[\begin{matrix} (a_A); \\ (b_B); \end{matrix} z\right] = \sum_{n=0}^{\infty} \frac{[(a_A)]_n}{[(b_B)]_n} \frac{z^n}{n!} \qquad (1.1) $$

where for the sake of convenience (in the contracted notation), (a_A) denotes the array of A number of parameters given by $a_1, a_2, \ldots, a_A$. The denominator parameters are neither zero nor negative integers. The numerator parameters may be zero and negative integers. A and B are positive integers or zero.

b) Kampé De Fériet Double Hypergeometric Function

In 1921 Appell's four double hypergeometric function[Appell, p.296(1); Bailey, p.73(1,2,3,4)] F_1 F_2 F_3 F_4 and their confluents forms[4] $\Phi_1, \Phi_2, \Phi_3, \Psi_1, \Psi_2, \Xi_1, \Xi_2$ were unified and generalized by Kampé de Fériet [5].

We recall the definition of general double hypergeometric function of Kampé de Fériet [Appell, p.150(29)], in slightly modified notation of Srivastava and Panda [Srivastava and Panda, pp.423-424(26,27)]

$$ F^{A:B;D}_{E:G;H}\left[\begin{matrix} (a_A) : (b_B); (d_D); \\ (e_E) : (g_G); (h_H); \end{matrix} x, y\right] = \sum_{m,n=0}^{\infty} \frac{[(a_A)]_{m+n}[(b_B)]_m[(d_D)]_n}{[(e_E)]_{m+n}[(g_G)]_m[(h_H)]_n} \frac{x^m y^n}{m!\, n!} \qquad (1.2) $$

c) Wright's Generalized Hypergeometric Function

Wright's generalized hypergeometric function [Srivastava & Manocha, p.50(1.5.21), p.179(34 iii), p.395(23)], is defined by

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Ref

5. Kampé de Fériet, J., *Les Fonctions Hypergéométriques d'order Supérieur a Deux variable*, C. R. Acad. Sci. Paris, 173(1921), 401-414.

$$ {}_p\Psi_q\left[\begin{matrix}(\alpha_1,A_1),\ldots,(\alpha_p,A_p);\\(\beta_1,B_1),\ldots,(\beta_q,B_q);\end{matrix}\,z\right]=\frac{\Gamma(\alpha_1)\,\Gamma(\alpha_2)\,\ldots\Gamma(\alpha_p)}{\Gamma(\beta_1)\,\Gamma(\beta_2)\,\ldots\Gamma(\beta_q)} $$

$$ \times\ {}_p\Psi_q^*\left[\begin{matrix}(\alpha_1,A_1),\ldots,(\alpha_p,A_p);\\(\beta_1,B_1),\ldots,(\beta_q,B_q);\end{matrix}\,z\right] \tag{1.3} $$

$$ =\sum_{n=0}^{\infty}\frac{\Gamma(\alpha_1+A_1n)\,\Gamma(\alpha_2+A_2n)\,\ldots\Gamma(\alpha_p+A_pn)}{\Gamma(\beta_1+B_1n)\,\Gamma(\beta_2+B_2n)\,\ldots\Gamma(\beta_q+B_qn)}\frac{z^n}{n!} \tag{1.4} $$

$$ {}_p\Psi_q^*\left[\begin{matrix}(\alpha_1,A_1),\ldots,(\alpha_p,A_p);\\(\beta_1,B_1),\ldots,(\beta_q,B_q);\end{matrix}\,z\right]=\sum_{n=0}^{\infty}\frac{(\alpha_1)_{A_1n}(\alpha_2)_{A_2n}\ldots(\alpha_p)_{A_pn}}{(\beta_1)_{B_1n}(\beta_2)_{B_2n}\ldots(\beta_q)_{B_qn}}\frac{z^n}{n!} \tag{1.5} $$

II. Main Integrals

$$ \int_0^{\frac{\pi}{2}}\frac{\sin^{-1}(x\cos^4\phi)\mathrm{d}\phi}{\sqrt{(1-x^2\cos^6\phi)}}=\frac{3}{16}\pi x\ {}_6F_5\left[\begin{matrix}1,\,1,\,\frac{5}{8},\,\frac{7}{8},\,\frac{9}{8},\,\frac{11}{8}\ ;\\ \frac{3}{2},\,\frac{3}{4},\,1,\,\frac{5}{4},\,\frac{3}{2}\ ;\end{matrix}\ x^2\right] \tag{2.1} $$

$$ \int_0^{\frac{\pi}{2}}\frac{\mathrm{d}\phi}{\sqrt{(1-x\cos^6\phi)}}=\frac{\pi}{2}\ {}_4F_3\left[\begin{matrix}\frac{1}{2},\,\frac{1}{6},\,\frac{1}{2},\,\frac{5}{6}\ ;\\ 1,\,\frac{1}{3},\,\frac{2}{3}\ ;\end{matrix}\ x\right] \tag{2.2} $$

$$ \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\frac{\mathrm{d}\theta\mathrm{d}\phi}{\sqrt{(1-x\cos^4\theta)(1-x\cos^4\theta\cos^4\phi)}}=\frac{\pi^2}{4}\ F^{2:1;3}_{2:0;2}\left[\begin{matrix}\frac{1}{4},\,\frac{3}{4}:\frac{1}{2}\ ;\frac{1}{2},\,\frac{1}{4},\,\frac{3}{4}\ ;\\ \frac{1}{2},\,\frac{3}{2}:\,—;\,\frac{1}{2},\,\frac{3}{2}\ ;\end{matrix}\ x,\,x\right] \tag{2.3} $$

III. Use of Series Iteration Technique in Evaluation of Integrals

Derivation of (2.1)

$$ \int_0^{\frac{\pi}{2}}\frac{\sin^{-1}(x\cos^4\phi)\mathrm{d}\phi}{\sqrt{(1-x^2\cos^6\phi)}}=\int_0^{\frac{\pi}{2}}(x\cos^4\phi)\ {}_2F_1\left[\begin{matrix}1,\,1\ ;\\ \frac{3}{2}\ ;\end{matrix}\ x^2\cos^8\phi\right]\mathrm{d}\phi $$

$$ =x\int_0^{\frac{\pi}{2}}\cos^4\phi\sum_{m=0}^{\infty}\frac{(1)_m(1)_m}{(\frac{3}{2})_m\ m!}(x^2\cos^8\phi)^m\mathrm{d}\phi $$

$$ =x\sum_{m=0}^{\infty}\frac{(1)_m(1)_m}{(\frac{3}{2})_m\ m!}(x^2)^m\int_0^{\frac{\pi}{2}}\cos^{8m+4}\mathrm{d}\phi $$

$$ =x\sum_{m=0}^{\infty}\frac{(1)_m(1)_m}{(\frac{3}{2})_m\ m!}(x^2)^m\frac{\Gamma(\frac{8m+5}{2})\Gamma(\frac{1}{2})}{2\ \Gamma(\frac{8m+6}{2})} $$

$$ =\frac{3\pi x}{16}\sum_{m=0}^{\infty}\frac{(1)_m(1)_m(\frac{5}{8})_m(\frac{7}{8})_m(\frac{9}{8})_m(\frac{11}{8})_m}{(\frac{3}{2})_m(\frac{3}{4})_m(1)_m(\frac{5}{4})_m(\frac{3}{2})_m} $$

$$ =\frac{3}{16}\pi x\ {}_6F_5\left[\begin{matrix}1,\,1,\,\frac{5}{8},\,\frac{7}{8},\,\frac{9}{8},\,\frac{11}{8}\ ;\\ \frac{3}{2},\,\frac{3}{4},\,1,\,\frac{5}{4},\,\frac{3}{2}\ ;\end{matrix}\ x^2\right] $$

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Derivation of (2.2)

$$\int_0^{\frac{\pi}{2}} \frac{d\phi}{\sqrt{(1-x\cos^6\phi)}} = \int_0^{\frac{\pi}{2}} \left(1-x\cos^6\phi\right)^{-\frac{1}{2}} d\phi = \int_0^{\frac{\pi}{2}} \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m x^m \cos^{6m}\phi}{m!} d\phi$$

$$= \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m x^m}{m!} \left(\frac{\Gamma(\frac{6m+1}{2})\Gamma(\frac{0+1}{2})}{2\Gamma(\frac{6m+2}{2})} \right) = \frac{\pi}{2} \sum_{m=0}^{\infty} \frac{(\frac{1}{2})_m (\frac{1}{6})_m (\frac{3}{6})_m (\frac{5}{6})_m x^m}{(\frac{1}{3})_m (\frac{2}{3})_m (1)_m m!}$$

$$= \frac{\pi}{2}\, {}_4F_3 \left[\begin{matrix} \frac{1}{2}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6} & ; & \\ & & x \\ 1, \frac{1}{3}, \frac{2}{3} & ; & \end{matrix} \right]$$

Derivation of (2.3)

$$\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \frac{d\theta d\phi}{\sqrt{(1-x\cos^4\phi)(1-x\cos^4\theta\cos^4\phi)}}$$

$$= \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} (1-x\cos^4\theta)^{-\frac{1}{2}} (1-x\cos^4\phi\cos^4\theta)^{-\frac{1}{2}} d\theta d\phi$$

$$= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(\frac{1}{2})_m (\frac{1}{2})_n\, x^{m+n}}{m!\, n!} \left. \int_0^{\frac{\pi}{2}} \cos^{4m+4n}\theta d\theta \right) \left(\int_0^{\frac{\pi}{2}} \cos^{4n}\phi d\phi \right)$$

$$= \frac{\pi}{4} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(\frac{1}{2})_m (\frac{1}{2})_n}{m!\, n!} x^{m+n} \frac{\sqrt{\pi}(\frac{1}{2})_{2m+2n}}{(1)_{2m+2n}} \frac{\sqrt{\pi}(\frac{1}{2})_{2n}}{(1)_{2n}}$$

$$= \frac{\pi^2}{4} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(\frac{1}{2})_m\, (\frac{1}{2})_n\, (\frac{1}{4})_{m+n}\, (\frac{3}{4})_{m+n}\, (\frac{1}{4})_m\, (\frac{3}{4})_n}{(\frac{1}{2})_{m+n}\, (\frac{3}{2})_{m+n}\, (\frac{1}{2})_n\, (\frac{3}{2})_n\, m!\, n1} x^m\, x^n$$

$$= \frac{\pi^2}{4} F^{2:1;3}_{2:0;2} \left[\begin{matrix} \frac{1}{4}, \frac{3}{4} : \frac{1}{2} & ; \frac{1}{2}, \frac{1}{4}, \frac{3}{4} & ; & \\ & & & x, x \\ \frac{1}{2}, \frac{3}{2} : \text{—} & ; \frac{1}{2}, \frac{3}{2} & ; & \end{matrix} \right]$$

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Notes

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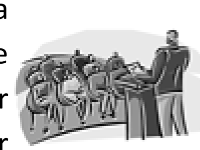

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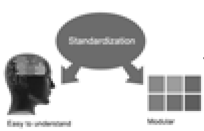

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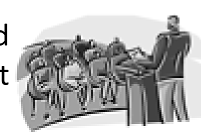

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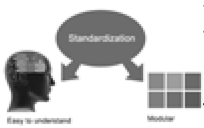

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- Left Margin: 0.65
- Right Margin: 0.65
- Top Margin: 0.75
- Bottom Margin: 0.75
- Font type of all text should be Swis 721 Lt BT.
- Paper Title should be of Font Size 24 with one Column section.
- Author Name in Font Size of 11 with one column as of Title.
- Abstract Font size of 9 Bold, "Abstract" word in Italic Bold.
- Main Text: Font size 10 with justified two columns section
- Two Column with Equal Column with of 3.38 and Gaping of .2
- First Character must be three lines Drop capped.
- Paragraph before Spacing of 1 pt and After of 0 pt.
- Line Spacing of 1 pt
- Large Images must be in One Column
- Numbering of First Main Headings (Heading 1) must be in Roman Letters, Capital Letter, and Font Size of 10.
- Numbering of Second Main Headings (Heading 2) must be in Alphabets, Italic, and Font Size of 10.

You can use your own standard format also.
Author Guidelines:

1. General,

2. Ethical Guidelines,

3. Submission of Manuscripts,

4. Manuscript's Category,

5. Structure and Format of Manuscript,

6. After Acceptance.

1. GENERAL

Before submitting your research paper, one is advised to go through the details as mentioned in following heads. It will be beneficial, while peer reviewer justify your paper for publication.

Scope

The Global Journals Inc. (US) welcome the submission of original paper, review paper, survey article relevant to the all the streams of Philosophy and knowledge. The Global Journals Inc. (US) is parental platform for Global Journal of Computer Science and Technology, Researches in Engineering, Medical Research, Science Frontier Research, Human Social Science, Management, and Business organization. The choice of specific field can be done otherwise as following in Abstracting and Indexing Page on this Website. As the all Global

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2. ETHICAL GUIDELINES

Authors should follow the ethical guidelines as mentioned below for publication of research paper and research activities.

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Authorship: The authors and coauthors should have active contribution to conception design, analysis and interpretation of findings. They should critically review the contents and drafting of the paper. All should approve the final version of the paper before submission

The Global Journals Inc. (US) follows the definition of authorship set up by the Global Academy of Research and Development. According to the Global Academy of R&D authorship, criteria must be based on:

1) Substantial contributions to conception and acquisition of data, analysis and interpretation of the findings.

2) Drafting the paper and revising it critically regarding important academic content.

3) Final approval of the version of the paper to be published.

All authors should have been credited according to their appropriate contribution in research activity and preparing paper. Contributors who do not match the criteria as authors may be mentioned under Acknowledgement.

Acknowledgements: Contributors to the research other than authors credited should be mentioned under acknowledgement. The specifications of the source of funding for the research if appropriate can be included. Suppliers of resources may be mentioned along with address.

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3. SUBMISSION OF MANUSCRIPTS

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Manuscript submission is a systematic procedure and little preparation is required beyond having all parts of your manuscript in a given format and a computer with an Internet connection and a Web browser. Full help and instructions are provided on-screen. As an author, you will be prompted for login and manuscript details as Field of Paper and then to upload your manuscript file(s) according to the instructions.

To avoid postal delays, all transaction is preferred by e-mail. A finished manuscript submission is confirmed by e-mail immediately and your paper enters the editorial process with no postal delays. When a conclusion is made about the publication of your paper by our Editorial Board, revisions can be submitted online with the same procedure, with an occasion to view and respond to all comments.

Complete support for both authors and co-author is provided.

4. MANUSCRIPT'S CATEGORY

Based on potential and nature, the manuscript can be categorized under the following heads:

Original research paper: Such papers are reports of high-level significant original research work.

Review papers: These are concise, significant but helpful and decisive topics for young researchers.

Research articles: These are handled with small investigation and applications

Research letters: The letters are small and concise comments on previously published matters.

5.STRUCTURE AND FORMAT OF MANUSCRIPT

The recommended size of original research paper is less than seven thousand words, review papers fewer than seven thousands words also.Preparation of research paper or how to write research paper, are major hurdle, while writing manuscript. The research articles and research letters should be fewer than three thousand words, the structure original research paper; sometime review paper should be as follows:

Papers: These are reports of significant research (typically less than 7000 words equivalent, including tables, figures, references), and comprise:

(a)Title should be relevant and commensurate with the theme of the paper.

(b) A brief Summary, "Abstract" (less than 150 words) containing the major results and conclusions.

(c) Up to ten keywords, that precisely identifies the paper's subject, purpose, and focus.

(d) An Introduction, giving necessary background excluding subheadings; objectives must be clearly declared.

(e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition; sources of information must be given and numerical methods must be specified by reference, unless non-standard.

(f) Results should be presented concisely, by well-designed tables and/or figures; the same data may not be used in both; suitable statistical data should be given. All data must be obtained with attention to numerical detail in the planning stage. As reproduced design has been recognized to be important to experiments for a considerable time, the Editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned un-refereed;

(g) Discussion should cover the implications and consequences, not just recapitulating the results; conclusions should be summarizing.

(h) Brief Acknowledgements.

(i) References in the proper form.

Authors should very cautiously consider the preparation of papers to ensure that they communicate efficiently. Papers are much more likely to be accepted, if they are cautiously designed and laid out, contain few or no errors, are summarizing, and be conventional to the approach and instructions. They will in addition, be published with much less delays than those that require much technical and editorial correction.

The Editorial Board reserves the right to make literary corrections and to make suggestions to improve briefness.

It is vital, that authors take care in submitting a manuscript that is written in simple language and adheres to published guidelines.

Format

Language: The language of publication is UK English. Authors, for whom English is a second language, must have their manuscript efficiently edited by an English-speaking person before submission to make sure that, the English is of high excellence. It is preferable, that manuscripts should be professionally edited.

Standard Usage, Abbreviations, and Units: Spelling and hyphenation should be conventional to The Concise Oxford English Dictionary. Statistics and measurements should at all times be given in figures, e.g. 16 min, except for when the number begins a sentence. When the number does not refer to a unit of measurement it should be spelt in full unless, it is 160 or greater.

Abbreviations supposed to be used carefully. The abbreviated name or expression is supposed to be cited in full at first usage, followed by the conventional abbreviation in parentheses.

Metric SI units are supposed to generally be used excluding where they conflict with current practice or are confusing. For illustration, 1.4 l rather than 1.4 × 10-3 m3, or 4 mm somewhat than 4 × 10-3 m. Chemical formula and solutions must identify the form used, e.g. anhydrous or hydrated, and the concentration must be in clearly defined units. Common species names should be followed by underlines at the first mention. For following use the generic name should be constricted to a single letter, if it is clear.

Structure

All manuscripts submitted to Global Journals Inc. (US), ought to include:

Title: The title page must carry an instructive title that reflects the content, a running title (less than 45 characters together with spaces), names of the authors and co-authors, and the place(s) wherever the work was carried out. The full postal address in addition with the e-mail address of related author must be given. Up to eleven keywords or very brief phrases have to be given to help data retrieval, mining and indexing.

Abstract, used in Original Papers and Reviews:

Optimizing Abstract for Search Engines

Many researchers searching for information online will use search engines such as Google, Yahoo or similar. By optimizing your paper for search engines, you will amplify the chance of someone finding it. This in turn will make it more likely to be viewed and/or cited in a further work. Global Journals Inc. (US) have compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

Key Words

A major linchpin in research work for the writing research paper is the keyword search, which one will employ to find both library and Internet resources.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy and planning a list of possible keywords and phrases to try.

Search engines for most searches, use Boolean searching, which is somewhat different from Internet searches. The Boolean search uses "operators," words (and, or, not, and near) that enable you to expand or narrow your affords. Tips for research paper while preparing research paper are very helpful guideline of research paper.

Choice of key words is first tool of tips to write research paper. Research paper writing is an art.A few tips for deciding as strategically as possible about keyword search:

- One should start brainstorming lists of possible keywords before even begin searching. Think about the most important concepts related to research work. Ask, "What words would a source have to include to be truly valuable in research paper?" Then consider synonyms for the important words.
- It may take the discovery of only one relevant paper to let steer in the right keyword direction because in most databases, the keywords under which a research paper is abstracted are listed with the paper.
- One should avoid outdated words.

Keywords are the key that opens a door to research work sources. Keyword searching is an art in which researcher's skills are bound to improve with experience and time.

Numerical Methods: Numerical methods used should be clear and, where appropriate, supported by references.

Acknowledgements: Please make these as concise as possible.

References

References follow the Harvard scheme of referencing. References in the text should cite the authors' names followed by the time of their publication, unless there are three or more authors when simply the first author's name is quoted followed by et al. unpublished work has to only be cited where necessary, and only in the text. Copies of references in press in other journals have to be supplied with submitted typescripts. It is necessary that all citations and references be carefully checked before submission, as mistakes or omissions will cause delays.

References to information on the World Wide Web can be given, but only if the information is available without charge to readers on an official site. Wikipedia and Similar websites are not allowed where anyone can change the information. Authors will be asked to make available electronic copies of the cited information for inclusion on the Global Journals Inc. (US) homepage at the judgment of the Editorial Board.

The Editorial Board and Global Journals Inc. (US) recommend that, citation of online-published papers and other material should be done via a DOI (digital object identifier). If an author cites anything, which does not have a DOI, they run the risk of the cited material not being noticeable.

The Editorial Board and Global Journals Inc. (US) recommend the use of a tool such as Reference Manager for reference management and formatting.

Tables, Figures and Figure Legends

Tables: Tables should be few in number, cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g. Table 4, a self-explanatory caption and be on a separate sheet. Vertical lines should not be used.

Figures: Figures are supposed to be submitted as separate files. Always take in a citation in the text for each figure using Arabic numbers, e.g. Fig. 4. Artwork must be submitted online in electronic form by e-mailing them.

Preparation of Electronic Figures for Publication

Even though low quality images are sufficient for review purposes, print publication requires high quality images to prevent the final product being blurred or fuzzy. Submit (or e-mail) EPS (line art) or TIFF (halftone/photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Do not use pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings) in relation to the imitation size. Please give the data for figures in black and white or submit a Color Work Agreement Form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

For scanned images, the scanning resolution (at final image size) ought to be as follows to ensure good reproduction: line art: >650 dpi; halftones (including gel photographs) : >350 dpi; figures containing both halftone and line images: >650 dpi.

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6. AFTER ACCEPTANCE

Upon approval of a paper for publication, the manuscript will be forwarded to the dean, who is responsible for the publication of the Global Journals Inc. (US).

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The corresponding author will receive an e-mail alert containing a link to a website or will be attached. A working e-mail address must therefore be provided for the related author.

Acrobat Reader will be required in order to read this file. This software can be downloaded

(Free of charge) from the following website:

www.adobe.com/products/acrobat/readstep2.html. This will facilitate the file to be opened, read on screen, and printed out in order for any corrections to be added. Further instructions will be sent with the proof.

Proofs must be returned to the dean at dean@globaljournals.org within three days of receipt.

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TECHNIQUES FOR WRITING A GOOD QUALITY RESEARCH PAPER:

1. Choosing the topic: In most cases, the topic is searched by the interest of author but it can be also suggested by the guides. You can have several topics and then you can judge that in which topic or subject you are finding yourself most comfortable. This can be done by asking several questions to yourself, like Will I be able to carry our search in this area? Will I find all necessary recourses to accomplish the search? Will I be able to find all information in this field area? If the answer of these types of questions will be "Yes" then you can choose that topic. In most of the cases, you may have to conduct the surveys and have to visit several places because this field is related to Computer Science and Information Technology. Also, you may have to do a lot of work to find all rise and falls regarding the various data of that subject. Sometimes, detailed information plays a vital role, instead of short information.

2. Evaluators are human: First thing to remember that evaluators are also human being. They are not only meant for rejecting a paper. They are here to evaluate your paper. So, present your Best.

3. Think Like Evaluators: If you are in a confusion or getting demotivated that your paper will be accepted by evaluators or not, then think and try to evaluate your paper like an Evaluator. Try to understand that what an evaluator wants in your research paper and automatically you will have your answer.

4. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

5. Ask your Guides: If you are having any difficulty in your research, then do not hesitate to share your difficulty to your guide (if you have any). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work then ask the supervisor to help you with the alternative. He might also provide you the list of essential readings.

6. Use of computer is recommended: As you are doing research in the field of Computer Science, then this point is quite obvious.

7. Use right software: Always use good quality software packages. If you are not capable to judge good software then you can lose quality of your paper unknowingly. There are various software programs available to help you, which you can get through Internet.

8. Use the Internet for help: An excellent start for your paper can be by using the Google. It is an excellent search engine, where you can have your doubts resolved. You may also read some answers for the frequent question how to write my research paper or find model research paper. From the internet library you can download books. If you have all required books make important reading selecting and analyzing the specified information. Then put together research paper sketch out.

9. Use and get big pictures: Always use encyclopedias, Wikipedia to get pictures so that you can go into the depth.

10. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right! It is a good habit, which helps to not to lose your continuity. You should always use bookmarks while searching on Internet also, which will make your search easier.

11. Revise what you wrote: When you write anything, always read it, summarize it and then finalize it.

12. Make all efforts: Make all efforts to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in introduction, that what is the need of a particular research paper. Polish your work by good skill of writing and always give an evaluator, what he wants.

13. Have backups: When you are going to do any important thing like making research paper, you should always have backup copies of it either in your computer or in paper. This will help you to not to lose any of your important.

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15. Use of direct quotes: When you do research relevant to literature, history or current affairs then use of quotes become essential but if study is relevant to science then use of quotes is not preferable.

16. Use proper verb tense: Use proper verb tenses in your paper. Use past tense, to present those events that happened. Use present tense to indicate events that are going on. Use future tense to indicate future happening events. Use of improper and wrong tenses will confuse the evaluator. Avoid the sentences that are incomplete.

17. Never use online paper: If you are getting any paper on Internet, then never use it as your research paper because it might be possible that evaluator has already seen it or maybe it is outdated version.

18. Pick a good study spot: To do your research studies always try to pick a spot, which is quiet. Every spot is not for studies. Spot that suits you choose it and proceed further.

19. Know what you know: Always try to know, what you know by making objectives. Else, you will be confused and cannot achieve your target.

20. Use good quality grammar: Always use a good quality grammar and use words that will throw positive impact on evaluator. Use of good quality grammar does not mean to use tough words, that for each word the evaluator has to go through dictionary. Do not start sentence with a conjunction. Do not fragment sentences. Eliminate one-word sentences. Ignore passive voice. Do not ever use a big word when a diminutive one would suffice. Verbs have to be in agreement with their subjects. Prepositions are not expressions to finish sentences with. It is incorrect to ever divide an infinitive. Avoid clichés like the disease. Also, always shun irritating alliteration. Use language that is simple and straight forward. put together a neat summary.

21. Arrangement of information: Each section of the main body should start with an opening sentence and there should be a changeover at the end of the section. Give only valid and powerful arguments to your topic. You may also maintain your arguments with records.

22. Never start in last minute: Always start at right time and give enough time to research work. Leaving everything to the last minute will degrade your paper and spoil your work.

23. Multitasking in research is not good: Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. Think technically: Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page
- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.

- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

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Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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