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Intuitionistic Fuzzy Sets Approach in Appointment of Positions in an Organization Via Max-Min-Max Rule

By P. A. Ejegwa

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Abstract- From the myriads of research on the applications of intuitionistic fuzzy sets theory in decision making, one can easily say that intuitionistic fuzzy sets theory is of great importance in decision science due to its efficiency and reliability. Here, we proposed a new area of application of intuitionistic fuzzy sets theory in appointment of positions in an organisation using intuitionistic fuzzy sets approach via max-min-max rule.

Keywords: *appointments, fuzzy sets, intuitionistic fuzzy sets, max-min-max rule, organisation.*

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P. A. Ejegwa

Abstract- From the myriads of research on the applications of intuitionistic fuzzy sets theory in decision making, one can easily say that intuitionistic fuzzy sets theory is of great importance in decision science due to its efficiency and reliability. Here, we proposed a new area of application of intuitionistic fuzzy sets theory in appointment of positions in an organisation using intuitionistic fuzzy sets approach via max-min-max rule.

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1. INTRODUCTION

Zadeh [20] introduced the concept of fuzzy sets with the aim to model the vagueness and ambiguity in complex systems. Fuzzy sets theory is the generalization of classical or crisp set. It has greater flexibility to capture various aspects of incompleteness or imperfection in information about a situation. The key elements in the human thinking are not just the numbers but can be approximated to classes of objects in which the transition from membership to non-membership is gradual rather than abrupt. In dealing with vague notions, it is sometimes difficult to determine the exact boundaries of class, hence the decision that whether an element belongs to it or not is replaced by a measure from some scale. Each element of the class is evaluated by a measure which expresses its place and role in the class. This measure is called the grade of membership of the given class. This class in which each element is characterised by its membership grade is called a fuzzy set. These membership grades are very often represented by real number values ranging from the closed interval $[0, 1]$.

Subsequently, Atanassov [1] proposed the concept of intuitionistic fuzzy sets (IFSs) as the generalisation of fuzzy sets. Since fuzzy set is only concerned with the membership function without minding the significance of non-membership function and hesitation margin (which is integral in decision making), Atanassov then included the non-membership function and defined the hesitation margin as 1 minus the sum of the membership and non-membership functions. Lots have been done on IFSs theory; see [2, 5, 10, 15]. Sequel to the introduction of IFSs theory, many studies examined its applications in various areas such as in medical diagnosis, sustainable supplier

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evaluation, pattern recognition, medical imaging, electoral system [3-4, 6-9, 11-14, 16-19] etc.

In this paper, we shall propose a new application of our “famous” IFSs theory in appointment of positions in an organisation with the aid of max-min-max rule.

a) *Meaning of intuitionistic fuzzy sets*

Definition 1: Crisp set A of X is defined as the characteristic function of A and is denoted by $f_A(x)$, mathematically, $f_A(x): X \rightarrow \{0,1\}$

$$\text{where, } f_A(x) = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \notin A \end{cases}$$

Definition 2: Fuzzy set A of a set X is defined by the membership function of the set A s.t. $\mu_A(x): X \rightarrow [0,1]$,

$$\text{where, } \mu_A(x) = \begin{cases} 1, & \text{if } x \text{ is totally in } A \\ 0, & \text{if } x \text{ is not in } A \\ (0,1), & \text{if } x \text{ is partly in } A \end{cases}$$

The closer the membership value $\mu_A(x)$ to 1, the more x belongs to A, where the grades 1 and 0 represents full membership and full non-membership.

Definition 3/2/: Let X be nonempty set. An intuitionistic fuzzy set (IFS) A in X is an object having the form;

$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$, where the functions $\mu_A(x), \nu_A(x): X \rightarrow [0, 1]$ define the degree of membership and degree of non-membership of the element $x \in X$ to the set A. For every $x \in X, 0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Furthermore, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is the intuitionistic fuzzy set index or hesitation margin and is the degree of indeterminacy concerning the membership of x in A, then $0 \leq \mu_A(x) + \nu_A(x) + \pi_A(x) \leq 1$. Whenever $\pi_A(x) = 0$, IFS reduces automatically to fuzzy set.

b) *Appointment of positions in an organisation via max-min-max rule for IFSs*

Suppose an organisation wants to reshuffle its cabinet, the challenge is how to appoint suitable officers into different positions assuming we have more than enough candidates for the positions. IFSs approach provides the solution because of its competency in handling uncertainties in decision making.

Let A be an IFS of nonempty set X and let R be the intuitionistic fuzzy relation (IFR) from $X \rightarrow Y$, then the max-min-max composition B of X with the IF relation $R(X \rightarrow Y)$ is defined as $B = R \circ A$ with membership and non-membership function defined as

$$\mu_B(y) = \max_{x \in X} \{ \min [\mu_A(x), \mu_R(x, y)] \} \text{ and}$$

$$\nu_B(y) = \min_{x \in X} \{ \max [\nu_A(x), \nu_R(x, y)] \}.$$

Also let $Q = \{q_1, q_2, \dots, q_l\}; P = \{p_1, p_2, \dots, p_m\}; C = \{c_1, c_2, \dots, c_n\}$; be the finite set of qualifications, positions, and candidates respectively.

Suppose we have two IFRs $R(C \rightarrow Q)$ and $S(Q \rightarrow P)$ s.t.

$$R = \{ \langle (c, q), \mu_R(c, q), \nu_R(c, q) \rangle : (c, q) \in C \times Q \}$$

$S = \{(q, p), \mu_S(q, p), \nu_S(q, p)\}: (q, p) \in Q \times P\}$ where,
 $\mu_R(c, q)$ indicate the degree to which the candidate c possesses the qualification q and
 $\nu_R(c, q)$ indicate the degree to which the candidate c does not possess the qualification q .

Similarly,

$\mu_S(q, p)$ indicate the degree to which the qualification q determines the position p and
 $\nu_S(q, p)$ indicate the degree to which the qualification q does not determine the position p .

The composition T of the IFRs R and S is given as $T = R \circ S$. It describes the state in which the candidates c_i in terms of the qualifications fit the positions p_j . It is given by membership and non-membership degrees as:

$$\mu_T(c_i, p_j) = \max_{q_j \in Q} \{ \min [\mu_R(c_i, q_j), \mu_S(q_j, p_j)] \}$$
 and

$$\nu_T(c_i, p_j) = \min_{q_j \in Q} \{ \max [\nu_R(c_i, q_j), \nu_S(q_j, p_j)] \} \forall c_i \in C \text{ and } p_j \in P \text{ for } i, j \in \mathbb{N}.$$

From R and S , one can compute new measure of IFR T for which, the appointments of the candidates c_i for any position p s.t. the following are to be satisfied:

- (i) $S_T = \mu_T - \nu_T \cdot \pi_T$ is the greatest and
- (ii) The equality $T = R \circ S$ is retained.

To see the application of this method, we frame a hypothetical case study:

Let $C = \{c_1, c_2, c_3, c_4\}$ be the set of candidates to be appointed, let $P = \{p_1, p_2, p_3, p_4, p_5\}$ be the positions to be occupied and let $Q = \{\text{honesty, team spirit, hardworking, transparency, academic fitness}\}$ be the set of qualifications the expected candidates ought to possessed. We assume the candidates are score by impartial member of the organisation appointment committee (10-member committee) using IFSs values.

Suppose the IFR $R(C \rightarrow Q)$ is given hypothetically below as scored by the 10-member committee.

Table 1

R	honesty	team spirit	hardworking	transparency	acad. fitness
c_1	(0.5, 0.2)	(0.6, 0.3)	(0.7, 0.1)	(0.8, 0.1)	(0.6, 0.2)
c_2	(0.8, 0.1)	(0.5, 0.2)	(0.7, 0.2)	(0.6, 0.2)	(0.4, 0.5)
c_3	(0.5, 0.2)	(0.6, 0.1)	(0.4, 0.3)	(0.5, 0.3)	(0.8, 0.1)
c_4	(0.7, 0.1)	(0.5, 0.3)	(0.8, 0.1)	(0.6, 0.2)	(0.7, 0.2)

Note that, the first entry is the membership value and the second entry is the non-membership value.

Suppose the IFR $S(Q \rightarrow P)$ is given hypothetically below as stipulated by the organisation appointment committee as standing qualifications for the positions.

Table 2

S	p_1	p_2	p_3	p_4	p_5
Honesty	(0.7, 0.2)	(0.7, 0.1)	(0.6, 0.2)	(0.8, 0.0)	(0.6, 0.3)

team spirit	(0.8,0.1)	(0.7,0.2)	(0.8,0.0)	(0.7,0.2)	(0.8,0.1)
Hardworking	(0.8,0.2)	(0.8,0.1)	(0.8,0.1)	(0.6,0.2)	(0.8,0.1)
Transparency	(0.7,0.2)	(0.7,0.2)	(0.9,0.0)	(0.8,0.1)	(0.7,0.1)
acad. Fitness	(0.9,0.1)	(0.9,0.0)	(0.6,0.3)	(0.7,0.2)	(0.5,0.3)

The composition $T = R \circ S$ is as follows:

Table 3

T	p_1	p_2	p_3	p_4	p_5
c_1	(0.7,0.2)	(0.7,0.1)	(0.8,0.1)	(0.8,0.1)	(0.7,0.1)
c_2	(0.7,0.2)	(0.7,0.1)	(0.7,0.2)	(0.8,0.1)	(0.7,0.2)
c_3	(0.8,0.1)	(0.8,0.1)	(0.6,0.1)	(0.7,0.2)	(0.6,0.1)
c_4	(0.8,0.2)	(0.8,0.1)	(0.8,0.1)	(0.7,0.1)	(0.8,0.1)

Now, we calculate S_T as below:

Table 4

S_T	p_1	p_2	p_3	p_4	p_5
c_1	0.68	0.68	0.79	0.79	0.68
c_2	0.68	0.68	0.68	0.79	0.68
c_3	0.79	0.79	0.57	0.68	0.57
c_4	0.79	0.79	0.79	0.68	0.79

From Table 4 above, c_1 can fit into p_3 and p_4 ; c_2 can fit into p_4 only; c_3 can fit into p_1 and p_2 ; c_4 can fit into p_1, p_2, p_3 and p_5 .

II. CONCLUSION

We conclude that IFSs theory is a very suitable and decisive tool use in critical decision making problem like this. We observe that without IFSs theory, this exercise would have been compromised with a consequent effect on the organisation.

REFERENCES RÉFÉRENCES REFERENCIAS

1. K. T. Atanassov, *Intuitionistic fuzzy sets*, VII ITKR's Session, Sofia, 1983.
2. K. T. Atanassov, *Intuitionistic fuzzy sets: theory and application*, Springer (1999).
3. T. Chaira, Intuitionistic fuzzy set theory in medical imaging, *Int. J. of Soft Computing and Engineering* 1 (2011) 35-37.
4. S. K. De, R. Biswas, A. R. Roy, An application of intuitionistic fuzzy sets in medical diagnostic, *Fuzzy Sets and Systems* 117 (2) (2001) 209-213.
5. P. A. Ejegwa, S. O. Akowe, P. M. Otene, J. M. Ikyule, An overview on intuitionistic fuzzy sets, *Int. J. of Scientific & Technology Research* 3 (3) (2014) 142-145.
6. P. A. Ejegwa, B. S. Uleh, E. Onwe, Intuitionistic fuzzy sets in electoral system, *Int. J. of Science & Technology* 3 (4) (2014) 241-243.
7. P. A. Ejegwa, A. J. Akubo, O. M. Joshua, Intuitionistic fuzzy set in career determination, *Global J. of Sci. Frontier Research: Math. & Decision Sciences* 14 (3) (2014) 85-89.
8. P. A. Ejegwa, *Intuitionistic fuzzy sets theory in decision making*, LAMBERT Academic Publishing, 2015.

9. P. A. Ejegwa, J. T. Alabaa, On the Correspondence between intuitionistic fuzzy sets and democracy, *Intern. J. of Fuzzy Mathematical Archive* 5(2) (2014) 71-73.
10. P. A. Ejegwa, J. T. Alabaa, S. Yakubu, Two new algebraic properties defined over intuitionistic fuzzy sets, *Intern. J. of Fuzzy Mathematical Archive* 5(2) (2014) 75-78.
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13. P. A. Ejegwa, S. N. Chukwukelu, D. E. Odoh, Test of accuracy of some distance measures use in the application of intuitionistic fuzzy sets in medical diagnosis, *J. of Global Research in Mathematical Archives* 2(5) (2014) 55-60.
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15. A. M. Ibrahim, P.A. Ejegwa, Remark on some operations of intuitionistic fuzzy sets, *Int. J. of Science & Technology* 2 (1) (2013) 94-96.
16. D. Li, C. Cheng, New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions, *Pattern Recognition Letters* 23 (2002) 221-225.
17. E. Szmidt, J. Kacprzyk, Intuitionistic fuzzy sets in some medical applications, *Note on IFS* 7 (4) (2001) 58-64.
18. E. Szmidt, J. Kacprzyk, Medical diagnostic reasoning using a similarity measure for intuitionistic fuzzy sets, *Note on IFS* 10 (4) (2004) 61-69.
19. L. Wen, L. Xu, R. Wang, Sustainable supplier evaluation based on intuitionistic fuzzy sets group decision methods, *J. of Inf. and Computational. Sci.* 10(10) (2013) 3209-3220.
20. L.A. Zadeh, Fuzzy sets, *Information and Control* 8 (1965) 338-353.

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Existence of Solution of First order Differential Balance Equation in Interpolation

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Abstract- In this paper I derive the formula from the first order differential balance equation. In Numerical methods we are finding the missing values and derivatives from the available data by using different formula. One can solve problem directly using balance equation.

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GJSFR-F Classification : FOR Code : MSC 2010: 31B35 , 03C40



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Abstract- In this paper I derive the formula from the first order differential balance equation. In Numerical methods we are finding the missing values and derivatives from the available data by using different formula. One can solve problem directly using balance equation.

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I. INTRODUCTION

Interpolation is one of the concept in mathematics. In Numerical methods we are finding the missing values and derivatives from the available data by using different formula.

Here I have derived three formulae from the first order balance equation. The first two formulae denotes the process of computing the values of a function for any value of the Independent variable and also the value of a Independent variable for any value of a function either within an interval values from the available data.

Numerical differentiation is the process of computing the value of the derivatives for some particular value, from given data, when the actual relationship between X and Y is not known. The balance derivative formula to be used depends as usual on the particular value of X at which the value of first derivative is required.

II. BALANCE DIFFERENTIAL EQUATION AND AN APPLICATION

The balance differential equation is $dp = k\sqrt{p}dt$ -----(I)

This is a seperable equation whose solution can be developed as follows.

Integrating (I), we get

$$2\sqrt{p} = kt + c \text{ -----(i)}$$

where c is constant

When $p = p_1, t = t_1$, (i) becomes

$$2\sqrt{p_1} = kt_1 + c \text{ -----(ii)}$$

When $p = p_2, t = t_2$, (i) becomes

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$$2\sqrt{p_2} = kt_2 + c \text{ -----(iii)}$$

(iii)-(ii) gives $k = \frac{2(\sqrt{p_2}-\sqrt{p_1})}{t_2-t_1}$

From (ii) $c = 2\sqrt{p_1} - \frac{2(\sqrt{p_2}-\sqrt{p_1})}{(t_2-t_1)}t_1$

Substituting 'k' and 'c' in (i), we get

$$P = \left\{ \frac{(\sqrt{p_2} - \sqrt{p_1})(t - t_1)}{(t_2 - t_1)} + \sqrt{p_1} \right\}^2$$

From this formula we can find the dependent variable for any value of the independent variable. The other formula is as follows

$$t = \frac{(\sqrt{p} - \sqrt{p_1})(t_2 - t_1)}{(\sqrt{p_2} - \sqrt{p_1})} + t_1$$

From this formula we can calculate the independent variable for any value of the dependent variable.

III. APPLICATIONS

The following are data from the steam table

Temperature(t)	140	150	160	170	180
Pressure(p)	3.685	4.854	6.302	8.076	10.225

The solution is as follows

t ₁	t ₂	p ₁	p ₂	t	p
140	160	3.685	6.302	When t=150	P=4.9 app
140	160	3.685	6.302	t=150 app	When p=4.854

Now, we consider the unequal interval data

t	5	6	9	11
p	12	13	14	16

The solution is as follows

t ₁	t ₂	p ₁	p ₂	t	p
5	9	12	14	When t=6	P=12.4 8app
5	9	12	14	t=6 app	When p=13
5	11	12	16	When t=9	P=14 app

IV. BALANCE DIFFERENCE FORMULA TO COMPUTE THE DERIVATIVES

Consider

$$P = \left\{ \frac{(\sqrt{p_2} - \sqrt{p_1})(t - t_1)}{(t_2 - t_1)} + \sqrt{p_1} \right\}^2$$

$$\dot{p} = \frac{2\sqrt{p}(\sqrt{p_2} - \sqrt{p_1})}{t_2 - t_1}$$

Consider the table of values

t	1.00	1.05	1.10	1.15
p	1.00	1.02470	1.04881	1.07238

Here, when $p_1 = 1.00$ $t = 1.00$ $p_2 = 1.04881$ $t_2 = 1.10$ $P=1.02470$
 (\dot{p}) at $t = 1.05 = ?$

By using the above formula $\dot{p}=0.447$

V. CONCLUSION

These formulae we have used are to find out the values of both equal and unequal intervals. This can be applied for all types of data.

REFERENCES RÉFÉRENCES REFERENCIAS

1. C. H. Edwards Jr and David E. Penny-Differential equations and Boundary value problems.
2. R. W. Hamming –Numerical methods for scientists and Engineers.
3. Dr. M. K. Venkatraman-Numerical methods in Science and Engineering.



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Discriminant Analysis by Projection Pursuit

By Okeke, Joseph Uchenna, Onyeagu, Sidney & Okonkwo, Evelyn Nkiruka

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Abstract- A non-parametric discriminant analysis (projection pursuit by principal component analysis) is discussed and used to compare three robust linear discriminant functions that are based on high breakdown point (of location and covariance matrix) estimators. The major part of this paper deals with practical application of projection pursuit by principal component. In this study 10 simulated data sets that are binomially distributed and a real data set on the yield of two different progenies of palm tree were used for comparisons. From the findings we concluded that the non-parametric procedure (projection pursuit by principal component) have the highest predictive power among other procedures we considered. S-estimator performed better than the other two estimators when real data is considered, while MCD estimator performed better than MWCD estimator.

Keywords: *discriminant analysis (DA), principal component analysis (PCA), projection pursuit, minimum covariance determinant (MCD), minimum within covariance determinant (MWCD), and s-estimator.*

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Discriminant Analysis by Projection Pursuit

Okeke, Joseph Uchenna ^α, Okeke, Evelyn Nkiruka ^σ, and Onyeagu, Sidney ^ρ

Abstract- A non-parametric discriminant analysis (projection pursuit by principal component analysis) is discussed and used to compare three robust linear discriminant functions that are based on high breakdown point (of location and covariance matrix) estimators. The major part of this paper deals with practical application of projection pursuit by principal component. In this study 10 simulated data sets that are binomially distributed and a real data set on the yield of two different progenies of palm tree were used for comparisons. From the findings we concluded that the non-parametric procedure (projection pursuit by principal component) have the highest predictive power among other procedures we considered. S-estimator performed better than the other two estimators when real data is considered, while MCD estimator performed better than MWCD estimator.

Keywords: discriminant analysis (DA), principal component analysis (PCA), projection pursuit, minimum covariance determinant (MCD), minimum within covariance determinant (MWCD), and s-estimator.

I. INTRODUCTION

The problem of discriminant analysis arises when one wants to assign an individual into one of k groups on the basis of a p -dimensional characteristic vector. In classical discriminant analysis, the populations under study are, assumed to be, normally distributed with equal covariance matrices. But in some practical situations, problems with data that are categorical, mixed, sparse, and contaminated abound. In these situations, classical discriminant analysis will not be optimal in classifying objects into the existing populations. The need for robust methods that will be optimal in classifying objects becomes necessary.

Robust methods for discriminant analysis have been proposed by many authors; Todorov et al (1990) replaced the classical estimates of linear and quadratic discriminant functions by MCD estimates, Chork and Rousseeuw (1992) used MVE instead, Hawkins and McLachlan (1997) defined MWCD especially for the case of linear discriminant analysis, He and Fung (2000) and Croux and Dehon (2001) used S estimates, Hubert and Van Driessen (2004) applied the MCD estimates computed by the FAST MCD algorithm, Todorov and Pires (2007) used M-iteration described by Woodruff and Rocke (1996). Most of these works concentrated on replacing the classical mean vectors and covariance matrices by their robust counterpart. However when the covariance structure is singular or close to it the later methods may fail to be optimal. To solve singularity problem, projection pursuit approach has come up as a remedy. This method aimed at reducing a high dimensional data set to low dimension so that the statistical tool for the low dimensional data can be applied. Polzehl (1993) studied discriminant analysis based on projection pursuit density estimation and chose his projection to minimize estimates of the expected overall loss in each projection pursuit

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stage. According to him cross-validation techniques are used to avoid overfitting effect and at last he concluded that his procedure competes favorably with other classification methods in situations where parametric approaches are not flexible enough and when sample sizes are too small to use fully non-parametric procedure.

Pires and Branco (2010) studied projection pursuit estimator of the normalized discriminant vector induced by robust estimators of location, T , and the univariate estimator of scale, S and discovered that under contaminated data their method performed well and is strong competitor of other methods they studied.

Gunduz and Fokoue (2015) in their work explored and compared the predictive performance of robust classification and robust principal component analysis and applied it to variety of large small data sets. They also explored the performance of random forest by way of comparing and contrasting the differences of single model methods and ensemble method. Their work revealed that random forest although not robust to outlier substantially outperforms the existing techniques specifically design to achieve robustness.

In this paper we proposed projection pursuit by method of principal component because it allows prior standardization that is important for badly scaled data. This method was compared with the robust linear estimates: MCD and MWCD estimates obtained using Mahalanobis distance of the data points on 10 simulated data sets that are distributed binomially with a view of coming out with classifier with the highest predictive power.

II. PRINCIPAL COMPONENT AND PROJECTION PURSUIT (NONPARAMETRIC DA)

In many fields of research, principal component analysis (PCA) is used as an efficient tool for providing an informative and low-dimensional representation of multivariate data in which features in the data such as clustering, skewness and outliers can be easily detected (Bolton and Krzanowski 1999). PCA does not necessarily afford the ‘best’ view of the data structures and it may miss other interesting characteristics of the data. With this in mind, much research have been done in recent years on approaches to identifying projections that display, particularly, “interesting” features of the data. These techniques go under the generic name “projection pursuit” (Friedman and Tukey 1974). In projection pursuit the dimension that will give the required projection and the criterion that will find projections of the desired structure when optimized are studied.

Local optimization of the criterion over all projections of the required dimensionality yields “interesting” projections of the data that can be graphically displayed. The projection is usually chosen to be one, two, or three dimensional for convenience.

Projection pursuit indices are diverse but the construction of most of them is motivated by consideration of central limit theorem results. Diaconis and Freedman (1984) showed that projected subspaces of high-dimensional data converged, weakly in probability, to normality. Consequently, most projection pursuit indices are developed from the standpoint that normality represents the notion of “uninterestingness” (Huber 1985). These indices are thus optimized to find projections showing departures from normality.

PCA can be viewed as a particular case of projection pursuit in which the index of “interest” is the variance of the data, which is maximized over all unit length projections. In this paper, the first principal component was used to transform the p -

dimensional data space to one, so that we can apply the statistical tool for one dimensional data space.

a) *Procedures of PCA*

- Taking the whole dataset ignoring the class labels
- Computing the p -dimensional mean vector
- Computing the Covariance Matrix
- Computing eigenvectors and corresponding eigenvalues
- Sorting the eigenvectors by decreasing eigenvalues
- Choosing g eigenvectors with the largest eigenvalues
- Transforming the samples onto the new g subspace(s)

$$Y_i = a'X$$

Where $i = 1, \dots, g$ with $g \leq p$; a is the selected eigenvectors

Note: If the p variables of the original data are measured with different scales correlation matrix is used in place of covariance matrix.

b) *Allocation Based on Point-Group Transvariation*

This is a nonparametric allocation option suggested by Montanari (2004) which is based on the ranking of new observations among two samples used for classification. This utilizes the point group transvariation defined by Gini (1916) between the projected new observation and the projected X and Y . Allocate a new observation $Z = z$ into Π_x if $T_x(z) < T_y(z)$; otherwise, it is assigned to Π_y where

$$T_x(z) = \frac{1}{n} \sum_{i=1}^n I\{(u'_{opt} X_i - u'_{opt} z)\} [m_x(u'_{opt}) - u'_{opt} z] < 0\} \quad 2.1$$

The formula for $T_y(z)$ will be obtained when you replace all x in (2.1) with y .

$$u'_{opt} = \arg \min_{\|u\|=1} \left\{ \frac{1}{mn} \sum_{i=1}^m \sum_{j=1}^n I\{(u'x_i - u'y_j)(m_x(u) - m_y(u)) < 0\} \right\} \quad 2.2$$

where $m_x(u)$ and $m_y(u)$ are the locations of the two projected X and Y samples $u'x$ and $u'y$ respectively.

u'_{opt} is found using projection pursuit

The allocation scheme is based on ranks. It gets rid of the non optimality problem that transvariation distance (TD) has when skewed distributions are considered. Although the allocation scheme makes this method completely non parametric and works better than TD for skewed distributions, it does not perform as well for data with unequal sample sizes. This is due to the fact that an equal prior restriction is imposed by counting and we neglect group two (one) when we find the ranking of the new point in group one (two). So the priors are not necessarily taken into account and the effect shows in the misclassification error rate especially when the sample sizes are unequal.

III. METHODOLOGY

The discriminant procedures considered would be evaluated on 10 simulated data sets of two groups with different specifications distributed binomially and a real life

data. The real life data we used were obtained from Ph.D. seminar paper presented at the Department of Statistics, Nnamdi Azikiwe University, Awka, by Ekezie (2010). The data were from Nigeria Institute for Oil Palm Research and is on the characteristics and yield of two different progenies of palm tree. Table 1 below shows 10 simulated data sets and their specifications and optimal probability of misclassification.

Table 1 : Data Specifications and their Optimal Probability of Misclassification P(MC)

S/N	Sample Size	No. of variables	No. of trials		Probability of success		P(MC)
			Group X	Group Y	Group X	Group Y	
1	50	5	30	40	0.5, ..., 0.5	0.7, ..., 0.7	0.3300
2	45	3	60	70	0.4, ..., 0.4	0.5, ..., 0.5	0.426
3	40	4	50	100	0.4, ..., 0.4	0.3, ..., 0.3	0.5883
4	35	6	50	36	0.5, ..., 0.5	0.5, ..., 0.5	0.5000
5	30	6	30	50	0.5, ..., 0.5	0.5, ..., 0.5	0.5000
6	25	7	30	60	0.8, ..., 0.8	0.6, ..., 0.6	0.698
7	20	4	30	20	0.4, ..., 0.4	0.6, ..., 0.6	0.352
8	15	6	30	50	0.5, ..., 0.5	0.5, ..., 0.5	0.5000
9	10	4	20	30	0.4, ..., 0.4	0.6, ..., 0.6	0.352
10	5	2	25	30	0.3, ..., 0.3	0.6, ..., 0.6	0.365

The steps we follow in computing the projection pursuit discriminant procedure are explained in detail in section 3.1 and 3.2.

a) *Projection Pursuit (by Method of Principal Component)*

We started by pooling the two samples of X and Y. The pooled data was centered. The principal component analysis of pooled data was computed using Minitab computer package. From the computed result the first principal component was chosen for the final analysis. The coefficients of the variables \hat{u}_{opt} , which are the projection direction that maximizes the separation of the data between two groups were made to be orthogonal. The first principal component with orthogonal coefficient was used to sweep the p -dimensional data space R^p to one dimension R data space. With reduced data space, point-group transvariation probability that is univariate statistical tool was then used to cross validate the training samples.

b) *Probability of classification*

In order to evaluate the performance of this method in classification of future observations we estimate the overall probability of misclassification. A number of methods to estimate this probability exist in the literature but in this study we used apparent error rate (known also as resubstitution error rate or reclassification error rate). This is a straightforward estimator of the actual (true) error rate in discriminant analysis and is calculated by applying the classification criterion to the same data set from which it was derived and counting the number of misclassified observations. If there are plenty of observations in each class the error rate can be estimated by splitting the data into training and validation sets. The first one is used to estimate the discriminant rules and the second to estimate the misclassified error.

IV. RESULTS

The non-parametric discriminant method was evaluated with regard to its performance assessed by misclassification probabilities using 11 data sets. This method

competes favorably with robust linear estimators: MCD estimator, MWCD estimator, and S-estimator with the following misclassification probabilities.

Table 2.below contain the results of the 10 simulated data in terms of their misclassification probabilities,

Table 2 : Estimated Probability of Misclassification According to Sample Size

Sample size	MCD Estimator	MWCD Estimator	S-estimator	PP (PCA)
50	0.0000	0.0000	0.0000	0.0000
45	0.0111	0.0111	0.0111	0.0000
40	0.0000	0.0000	0.0000	0.0000
35	0.0142	0.0142	0.0142	0.0000
30	0.0000	0.0000	0.0000	0.0000
25	0.0000	0.0000	0.0000	0.0000
20	0.0000	0.0000	0.0000	0.0000
15	0.0000	0.0000	0.0000	0.0000
10	0.0000	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000	0.0000
Life Data	0.0750	0.1000	0.0500	0.0125

The result of real life data showed that projection pursuit has the highest predictive power with $P(\text{MC})$ of 0.0125, followed by S-estimator with $P(\text{MC})$ of 0.05, MCD with $P(\text{MC})$ of 0.075 and then, MWCD estimator which have $P(\text{MC})$ of 0.1.

Considering the computational ease and $P(\text{MC})$, projection pursuit (by PC) performed better than the other three procedure. S-estimator performed better than the other two estimators when real data is considered, while MCD estimator performed better than MWCD estimator. Although the quality of the estimates (for robust linear estimators) is important since it entirely determines the robustness of the discriminant rule towards outlier. In our study we only concentrated on the predictive power of the procedures leaving the other aspects for further work.

V. CONCLUSION

Based on our observations during iteration and our findings after the analysis, we conclude that nonparametric classification procedure (projection pursuit by principal component) has highest predictive power among other procedures we considered.

REFERENCES RÉFÉRENCES REFERENCIAS

1. Bolton, R. J., and Krzanowski, W.J. (1999). "A characterization of principal components for projection pursuit", *The American Statistician*, 53(2):108-109.
2. Chork, C. and Rousseeuw, P. J. (1992). "Integrating a high breakdown option into discriminant analysis in exploration geochemistry", *Journal of Geochemical Exploration*, 43, 191-203.
3. Croux, C., and Dehon, C. (2001). "Robust linear discriminant analysis using S-estimators", *Canadian Journal of Statistics*, 29(3):473-493.
4. Diaconis, P., and Freedman, D. (1984). "Asymptotic of graphical projection pursuit", *The Annals of Statistics*, 12:793-815.

5. Friedman, J. H., and Turkey, J. W. (1974). "Projection pursuit for exploratory data analysis", IEEE Transactions on Computers, 23(9): 881-890.
6. Gunduz, N, and Fokoue, E. (2015). Robust classification of high dimension low sample size data. arxiv.org e-print archive.
7. Hawkins, D. M., and McLachlan, G. J. (1997). "High-breakdown linear discriminant analysis" .Journal of American Statistical Association, 92(437): 136-143.
8. He, X., and Fung, W. K.(2000). "High breakdown estimation for multiple populations with application to discriminant analysis", Journal of Multivariate Analysis, 72(2):151-162.
9. Huber, P. J. (1985). "Projection pursuit", Annals of Statistics, 13920: 435-525.
10. Hubert, M. and Van Driessen, K. (2004). "Fast and robust discriminant analysis", Comput. Statist. Data Anal., 45(2):301-320.
11. Pires, A. M. and Branco, J. A. (2010). "Projection pursuit approach to robust linear discriminant analysis", Journal of Multivariate Analysis, 101(10): 2464-2485.
12. Polzehl, J., (1993). Projection pursuit discriminant analysis, Center for Operations Research and Economics.
13. Todorov, V. and Pires, A. M. (2007). "Comparative performance of several robust linear discriminant analysis methods", REVSTAT-Statistical Journal, 5(1):63-83.
14. Todorov, V, Neykov, N., and Neytchev, P. (1990). "Robust selection of variable in the discriminant analysis based on MVE and MCD estimators", In: "Proceedings in Computational Statistics", COMPSTAT", Physica Verlag, Heidelberg.
15. Woodruff, D. L. and Rocke, D.M. (1994). "Computable robust estimation of multivariate location and shape in high dimension using compound estimators", Journal of the American Statistical Association, 89, 888-896.



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Harmonic Univalent Function with Varying Arguments Defined by using Salagean Integral Operator with Fixed Point

By Dr. Poonam Dixit, Nikhil Kumar & Puneet Shukla

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Abstract- In this paper authors study a new class of harmonic functions defined by using Salagean Integral operator with varying arguments. We obtain coefficient inequalities, extreme points and distortion bounds.

Keywords: *harmonic, analytic, varying arguments, sense preserving, salagean integral operator, fixed point.*

GJSFR-F Classification : FOR Code : MSC 2010: 30F15



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Harmonic Univalent Function with Varying Arguments Defined by using Salagean Integral Operator with Fixed Point

Dr. Poonam Dixit ^α, Nikhil Kumar ^σ & Puneet Shukla ^ρ

Abstract In this paper authors study a new class of harmonic functions defined by using Salagean Integral operator with varying arguments. We obtain coefficient inequalities, extreme points and distortion bounds .

Keywords: harmonic, analytic, varying arguments, sense preserving, salagean integral operator, fixed point.

I. INTRODUCTION

A continuous complex-valued function $f = u + iv$ which is defined in a simply-connected complex domain D is said to be harmonic in D if both u and v are real harmonic in D . In any simply-connected domain we can write

$$f = h + \bar{g} \quad (1)$$

where h and g are analytic in D . We call h the analytic part and g the co-analytic part of f . A necessary and sufficient condition for f to be locally univalent and sense preserving in D is that $|h'(z)| > |g'(z)|, z \in D$ (see[7]).

Denote by S_H The class of functions f of the form (1) that the harmonic univalent and sense preserving in the unit disc $U = \{z \in C : |z| < 1\}$ for which

$$f(0) = h(0) = f'_z(0) - 1 = 0$$

then for $f = h + \bar{g} \in S_H$ we may express the analytic functions h and g as

$$h(z) = a_1 z + \sum_{k=2}^{\infty} a_k z^k, g(z) = \sum_{k=1}^{\infty} b_k z^k, |b_1| < 1 \quad (2)$$

In 1984 Clunie and Shell-Small [7] investigated the class S_H as well as its geometric subclasses and obtained some coefficient bounds . Since then , there have been several related papers on S_H and its subclasses.

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Salagean integral operator I^n is defined as follows (see[9])

(a) $I^0 f(z) = f(z)$

(b) $I^1 f(z) = If(z) = \int_0^z f(t)t^{-1}dt$

.....

(c) $I^n f(z) = I(I^{n-1} f(z))$ ($n \in N = \{1, 2, 3, \dots\}$).

In [4], Cotirla defined Salagean integral operator for harmonic univalent functions $f(z)$ such that $h(z)$ and $g(z)$ are given by (2) as follows

$$I^n f(z) = I^n h(z) + (-1)^n \overline{I^n g(z)}, \tag{3}$$

where

$$I^n h(z) = a_1 z + \sum_{k=2}^{\infty} k^{-n} a_k z^k$$

and

$$I^n g(z) = \sum_{k=1}^{\infty} k^{-n} b_k z^k$$

With the help of the modified Salagean integral operator we let $E_H(m, n, \gamma, \rho,)$ be the family of harmonic functions $f = h + \bar{g}$, which satisfy the following condition [11]

$$Re \left[(1 + \rho e^{i\alpha}) \frac{I^n f(z)}{I^m f(z)} - \rho e^{i\alpha} \right] \geq \gamma \tag{4}$$

($\alpha \in R, 0 \leq \gamma < 1, \rho \geq 0, m \in N, n \in N_0 = N \cup \{0\}, m > n$, and $z \in U$),

where $I^n f$ is defined by (3), we note that

(a) taking $\alpha = 0$, $E_H(n + 1, n; 2\beta - 1, 1) = H(n, \beta)$ ($0 \leq \beta < 1$) (see Cotirla[4])

(b) taking $m = n + q$, $E_H(n + q, n; \gamma, \rho) = H_{\rho,q}(n, \gamma)$ ($q \in N$) (see Guney and Sakar [5]).

also we note that, by the special choices of α, γ, ρ, m and n , we obtain the following special cases

(a) Taking $\alpha = 0$, then $E_H(m, n, 2\beta - 1, 1) = H(m, n; \beta) = \{f \in S_H :$

$$Re \left[\frac{I^n f(z)}{I^m f(z)} \right] > \beta (0 \leq \beta < 1; m \in N; n \in N_0; m > n; z \in U) \}$$

(b) $E_H(n + 1, n; \gamma, \rho) = E_H(n; \gamma, \rho) = \{f \in S_H :$

$$Re \left[(1 + \rho e^{i\alpha}) \frac{I^n f(z)}{I^{n+1} f(z)} - \rho e^{i\alpha} \right] \geq \gamma (\alpha \in R; 0 \leq \gamma < 1; \rho \geq 0; n \in N_0; z \in U) \}$$

5. Guney, H. O., Sakar, F.M., (2011), On uniformly starlike functions defined by an integral operator, Acta Universitatis Apulensis, 28, pp.293-301.

(c) $E_H(1, 0; \gamma, \rho) = E_H(\gamma, \rho) = \{f \in S_H :$

$$Re \left[(1 + \rho e^{i\alpha}) \frac{f(z)}{If(z)} - \rho e^{i\alpha} \right] \geq \gamma (\alpha \in R; 0 \leq \gamma < 1; \rho \geq 0; z \in U) \}$$

also we define the subclass $V_{\bar{H}}(m, n; \gamma, \rho)$ consists of harmonic functions $f_n = h + \bar{g}_n$ in $E_H(m, n; \gamma, \rho)$ such that h and g_n are the form

$$h(z) = a_1 z + \sum_{k=2}^{\infty} a_k z^k, g_n(z) = \sum_{k=1}^{\infty} b_k z^k \tag{5}$$

and there exists a real number ϕ such that , mod 2π ,

$$arg(a_k) + (k - 1)\phi \equiv \pi, k \geq 2 \text{ and } arg(b_k) + (k + 1)\phi \equiv (n - 1)\pi, k \geq 1. \tag{6}$$

also we note that, by the special choices of α, γ, m and n , we obtain:

- (a) taking $\alpha = 0$, $V_{\bar{H}}(n + 1, n; 2\beta - 1, 1) = V_{\bar{H}}(n, \beta)$;
- (b) taking $\alpha = 0$, $V_{\bar{H}}(m, n; 2\beta - 1, 1) = V_{\bar{H}}(m, n, \beta)$;
- (c) $V_{\bar{H}}(n + 1, n; \gamma, \rho) = V_{\bar{H}}(n; \gamma, \rho)$;
- (d) $V_{\bar{H}}(1, 0; \gamma, \rho) = V_{\bar{H}}(\gamma, \rho)$;

II. MAIN RESULT

Unless otherwise mentioned , we assume in the reminder of this paper that , $\alpha \in R, 0 \leq \gamma < 1, \rho \geq 0, m \in N, n \in N_0, m > n$ and $z \in U$. We begin with a sufficient coefficient condition for functions in the class $E_H(m, n; \gamma, \rho, z_0)$.

Theorem 2.1 . Let $f = h + \bar{g}$ be such that h and g are given by 2. Furthermore,

$$\sum_{k=2}^{\infty} \left[\frac{(1 + \rho)k^{-n} - (\gamma + \rho)k^{-m}}{1 - \gamma} |a_k| + \frac{(1 + \rho)k^{-n} - (-1)^{m-n}(\gamma + \rho)k^{-m}}{1 - \gamma} |b_k| \right] \leq 2a_1 \tag{7}$$

where $a_1 = 1$ Then $f \in E_H(m, n; \gamma, \rho, z_0)$. $0 \leq \gamma < 1, \rho \geq 0$

Proof. We need to show that if (7) holds the condition (4) is satisfied, then we want to prove that

$$Re \left[\frac{(1 + \rho e^{i\alpha}) I^n f(z) - \rho e^{i\alpha} I^m f(z)}{I^m f(z)} \right] = Re \frac{A(z)}{B(z)} \geq \gamma \tag{8}$$

Using the fact that $Re(\omega) > \gamma$ if and only if $|1 - \gamma + \omega| > |1 + \gamma - \omega|$, it suffices to show that

$$|A(z) + (1 - \gamma)B(z)| - |A(z) - (1 + \gamma)B(z)| \geq 0, \tag{9}$$

Where $A(z) = (1 + \rho e^{i\alpha})I^n f(z) - \rho e^{i\alpha}I^m f(z)$ and $B(z) = I^m f(z)$. Substituting for $A(z)$ and $B(z)$ in the left side of (9) we obtain,

$$\begin{aligned}
 & |(1 + \rho e^{i\alpha})I^n f(z) - \rho e^{i\alpha}I^m f(z) + (1 - \gamma)I^m f(z)| \\
 & - |(1 + \rho e^{i\alpha})I^n f(z) - \rho e^{i\alpha}I^m f(z) - (1 + \gamma)I^m f(z)| \\
 = & |(1 + \rho e^{i\alpha})(I^n h(z) + (-1)^n I^n g(z)) - \rho e^{i\alpha}(I^m h(z) + (-1)^n I^n g(z)) + (1 - \gamma)I^m f(z)| \\
 & - |(1 + \rho e^{i\alpha})(I^n h(z) + (-1)^n I^n g(z)) - \rho e^{i\alpha}(I^m h(z) + (-1)^n I^n g(z)) - (1 + \gamma)I^m f(z)| \\
 = & |a_1(z) + \sum_{k=2}^{\infty} k^{-n} a_k z^k + \rho e^{i\alpha} a_1(z) + \rho e^{i\alpha} \sum_{k=2}^{\infty} k^{-n} a_k z^k \\
 & + (-1)^n \sum_{k=1}^{\infty} k^{-n} b_k z^k + \rho e^{i\alpha} (-1)^n \sum_{k=1}^{\infty} k^{-n} b_k z^k - \rho e^{i\alpha} a_1(z) \\
 & - \rho e^{i\alpha} \sum_{k=2}^{\infty} k^{-m} a_k z^k - \rho e^{i\alpha} (-1)^m \sum_{k=1}^{\infty} k^{-m} b_k z^k + a_1(z) + \sum_{k=2}^{\infty} k^{-m} a_k z^k \\
 & + (-1)^m \sum_{k=1}^{\infty} k^{-m} b_k z^k - \gamma a_1(z) - \gamma \sum_{k=2}^{\infty} k^{-m} a_k z^k - (-1)^m \gamma \sum_{k=1}^{\infty} k^{-m} b_k z^k| \\
 & - |a_1(z) + \sum_{k=2}^{\infty} k^{-n} a_k z^k + \rho e^{i\alpha} a_1(z) + \rho e^{i\alpha} \sum_{k=2}^{\infty} k^{-n} a_k z^k \\
 & + (-1)^n \sum_{k=1}^{\infty} k^{-n} b_k z^k + \rho e^{i\alpha} (-1)^n \sum_{k=1}^{\infty} k^{-n} b_k z^k - \rho e^{i\alpha} a_1(z) \\
 & - \rho e^{i\alpha} \sum_{k=2}^{\infty} k^{-m} a_k z^k - \rho e^{i\alpha} (-1)^m \sum_{k=1}^{\infty} k^{-m} b_k z^k - a_1(z) \\
 & - \sum_{k=2}^{\infty} k^{-m} a_k z^k - (-1)^m \sum_{k=1}^{\infty} k^{-m} b_k z^k - \gamma a_1(z) - \gamma \sum_{k=2}^{\infty} k^{-m} a_k z^k - (-1)^m \gamma \sum_{k=1}^{\infty} k^{-m} b_k z^k| \\
 = & |(2 - \gamma)a_1(z) + \sum_{k=2}^{\infty} [(1 + \rho e^{i\alpha})k^{-n} + (1 - \gamma - \rho e^{i\alpha})k^{-m}]a_k z^k \\
 & + (-1)^n \sum_{k=1}^{\infty} [(1 + \rho e^{i\alpha})k^{-n} - (-1)^{m-n}(\rho e^{i\alpha} + \gamma - 1)k^{-m}] \times \overline{b_k z^k}| \\
 & - |\gamma a_1(z) - \sum_{k=2}^{\infty} [(1 + \rho e^{i\alpha})k^{-n} - (1 + \rho e^{i\alpha} + \gamma)k^{-m}]a_k z^k
 \end{aligned}$$

$$\begin{aligned}
 & - (-1)^n \sum_{k=1}^{\infty} [(1 + \rho e^{i\alpha})k^{-n} - (-1)^{m-n}(1 + \rho e^{i\alpha} + \gamma)k^{-m}] \overline{b_k z^k} \\
 & \geq 2(1-\gamma)a_1|z| - 2 \sum_{k=2}^{\infty} [(1+\rho)k^{-n} - (\gamma+\rho)k^{-m}] |a_k||z|^k - 2 \sum_{k=1}^{\infty} [(1+\rho)k^{-n} - (-1)^{m-n}(\gamma+\rho)k^{-m}] |b_k||z|^k \\
 & \geq 2(1-\gamma)|z| \left[a_1 - \sum_{k=2}^{\infty} \frac{(1+\rho)k^{-n} - (\gamma+\rho)k^{-m}}{(1-\gamma)} |a_k||z|^{k-1} - \sum_{k=1}^{\infty} \frac{(1+\rho)k^{-n} - (-1)^{m-n}(\gamma+\rho)k^{-m}}{1-\gamma} |b_k||z|^{k-1} \right]
 \end{aligned}$$

By using (7), then the last expression is non negative, then (9) is satisfied.

The harmonic function

$$f(z) = a_1(z) + \sum_{k=2}^{\infty} \frac{(1-\gamma)}{(1+\rho)k^{-n} - (\gamma+\rho)k^{-m}} x_k z^k + \sum_{k=1}^{\infty} \frac{1-\gamma}{(1+\rho)k^{-n} - (-1)^{m-n}(\gamma+\rho)k^{-m}} \overline{y_k z^k} \tag{10}$$

Where $\sum_{k=2}^{\infty} |x_k| + \sum_{k=1}^{\infty} |y_k| = 1$, shows that the coefficient bound given by (7) is sharp.

In the following theorem, it is shown that the condition (7) is also necessary for function $f_n = h + g_n$, where h and g_n are of the form(5).

Theorem 2.2. Let $f_n = h + g_n$, where h and g_n are given by (5). Then $f_n \in V_{\bar{H}}(m, n; \gamma, \rho, z_0)$, if and "only if" the coefficient condition (7) holds.

Proof. Since $V_{\bar{H}}(m, n; \gamma, \rho, z_0) \subseteq E_{\bar{H}}(m, n; \gamma, \rho, z_0)$ we only need to prove the only if part of the theorem. For functions $f_n = h + g_n$, where h and g_n are given by (5), the inequality (4) with $f = f_n$ is equivalent to

$$\begin{aligned}
 & R_e \left\{ \frac{(1+\rho e^{i\alpha}) \left(a_1 z + \sum_{k=2}^{\infty} k^{-n} a_k z^k + (-1)^n \sum_{k=1}^{\infty} k^{-n} \bar{b}_k \bar{z}^k \right)}{a_1 z + \sum_{k=2}^{\infty} k^{-m} a_k z^k + (-1)^m \sum_{k=1}^{\infty} k^{-m} \bar{b}_k \bar{z}^k} \right\} - \\
 & - R_e \left\{ \frac{(\gamma + \rho e^{i\alpha}) \left(a_1 z + \sum_{k=2}^{\infty} k^{-m} a_k z^k + (-1)^m \sum_{k=1}^{\infty} k^{-m} \bar{b}_k \bar{z}^k \right)}{a_1 z + \sum_{k=2}^{\infty} k^{-m} a_k z^k + (-1)^m \sum_{k=1}^{\infty} k^{-m} \bar{b}_k \bar{z}^k} \right\} > 0.
 \end{aligned}$$

The above condition holds for all values of $\alpha \in R$ and $z \in U$. Upon choosing ϕ according (6) and substituting $\alpha = 0$ and $z = r e^{i\phi}$ ($0 < r < 1$), we must have

$$\frac{E}{a_1 - \left[\sum_{k=2}^{\infty} k^{-m} |a_k| - (-1)^{m+n-1} \sum_{k=1}^{\infty} k^{-m} |b_k| \right] r^{k-1}} > 0, \tag{11}$$

Where

$$E = (1 - \gamma) - \left(\sum_{k=2}^{\infty} [(1 + \rho)k^{-n} - (\gamma + \rho)k^{-m}] |a_k| \right) r^{k-1} - (\sum_{k=1}^{\infty} [(1 + \rho)k^{-n} - (-1)^{m-n}(\gamma + \rho)k^{-m}] |b_k|) r^{k-1} .$$

If the inequality(7) does not hold , then E is negative for r sufficiently close to 1. Thus there exists $z_0 = r_0$ in(0, 1) for which the quotient in (11) is negative. But this is a contradiction, the proof of Theorem (2.2) is completed.

We now obtain the distortion bounds for functions in $V_{\bar{H}}(m, n; \gamma, \rho, z_0)$.

Theorem 2.3. Let $f_n = h + g_n$, where h and g_n are given by (5) and $f_n \in V_{\bar{H}}(m, n; \gamma, \rho, z_0)$.

Then for $|z| = r < 1$, we have

$$|f_n(z)| \leq (a_1 + |b_1|)r + \left[\frac{(1 - \gamma)a_1}{(1 + \rho)2^{-n} - (\gamma + \rho)2^{-m}} - \frac{(1 + \rho) - (-1)^{m-n}(\gamma + \rho)}{(1 + \rho)2^{-n} - (\gamma + \rho)2^{-m}} |b_1| \right] r^2 \quad (12)$$

and

$$|f_n(z)| \geq (a_1 + |b_1|)r + \left[\frac{(1 - \gamma)a_1}{(1 + \rho)2^{-n} - (\gamma + \rho)2^{-m}} - \frac{(1 + \rho) - (-1)^{m-n}(\gamma + \rho)}{(1 + \rho)2^{-n} - (\gamma + \rho)2^{-m}} |b_1| \right] r^2. \quad (13)$$

Proof. We prove the first inequality.

Let $f_n \in V_{\bar{H}}(m, n; \gamma, \rho, z_0)$, we have

$$\begin{aligned} (a) \cdot |f_n(z)| &\leq (a_1 + |b_1|)r + \sum_{k=2}^{\infty} (|a_k| + |b_k|)r^k \leq (a_1 + |b_1|)r + \sum_{k=2}^{\infty} (|a_k| + |b_k|)r^2 \\ &\leq (a_1 + |b_1|)r + \frac{(1-\gamma)}{(1+\rho)2^{-n}-(\gamma+\rho)2^{-m}} \sum_{k=2}^{\infty} \frac{(1 + \rho)2^{-n} - (\gamma + \rho)2^{-m}}{(1 - \gamma)} (|a_k| + |b_k|)r^2 \\ &\leq (a_1 + |b_1|)r + \frac{(1-\gamma)}{(1+\rho)2^{-n}-(\gamma+\rho)2^{-m}} \times \sum_{k=2}^{\infty} \left[\frac{(1 + \rho)k^{-n} - (\gamma + \rho)k^{-m}}{(1 - \gamma)} |a_k| + \frac{(1 + \rho)k^{-n} - (-1)^{m-n}(\gamma + \rho)k^{-m}}{(1 - \gamma)} |b_k| \right] r^2 \\ &\leq (a_1 + |b_1|)r + \frac{(1-\gamma)}{(1+\rho)2^{-n}-(\gamma+\rho)2^{-m}} \left[a_1 - \frac{(1+\rho)-(-1)^{m-n}(\gamma+\rho)}{(1-\gamma)} |b_1| \right] r^2 \\ &\leq (a_1 + |b_1|)r + \left[\frac{(1-\gamma)a_1}{(1+\rho)2^{-n}-(\gamma+\rho)2^{-m}} - \frac{(1+\rho)-(-1)^{m-n}(\gamma+\rho)}{(1+\rho)2^{-n}-(\gamma+\rho)2^{-m}} |b_1| \right] r^2 . \end{aligned}$$

The proof of the second inequality is similar, thus it is left.

The bounds given in Theorem (2.3) for functions $f_n = h + g_n$ such that h and g_n are given by (6) also hold for functions $f_n = h + g_n$ such that h and g_n are given by (2) if the coefficient condition (7) is satisfied .

Using the same technique used earlier by Aghalary [1] we introduce the extreme points of the class $V_{\bar{H}}(m, n; \gamma, \rho, z_0)$.

Ref

1. Aghalary, R., (2007), Goodman-Salagean-type harmonic univalent function with varying arguments. Int. Journal of math. Analysis, 1 (22), pp.1051-1057.

Theorem 2.4. The closed convex hull of the class $V_H(m, n; \gamma, \rho, z_0)$ (denoted by $clcoV_{\bar{H}}(m, n; \gamma, \rho, z_0)$) is

$$\{f(z) = a_1(z) + \sum_{k=2}^{\infty} a_k z^k + \overline{\sum_{k=1}^{\infty} b_k z^k} \in V_H(m, n; \gamma, \rho, z_0) :$$

$$\sum_{k=1}^{\infty} \left[\frac{(1+\rho)k^{-n} - (\gamma+\rho)k^{-m}}{(1-\gamma)} |a_k| + \frac{(1+\rho)k^{-n} - (-1)^{m-n}(\gamma+\rho)k^{-m}}{(1-\gamma)} |b_k| \right] \leq 2a_1 \}$$

where $a_1 = 1$. Set $\lambda_k = \frac{(1-\gamma)a_1}{(1+\rho)k^{-n} - (\gamma+\rho)k^{-m}}$ and $\mu_k = \frac{(1-\gamma)a_1}{(1+\rho)k^{-n} - (-1)^{m-n}(\gamma+\rho)k^{-m}}$

For b_1 fixed, $|b_1| \leq \frac{(1-\gamma)a_1}{(1+\rho)k^{-n} - (-1)^{m-n}(\gamma+\rho)k^{-m}}$, the extreme points of the class $V_{\bar{H}}(m, n; \gamma, \rho, z_0)$ are

$$\{z + \lambda_k x z^k + \bar{b}_1 z\} \cup \left\{ \overline{z + \mu_k x z^k + b_1 z} \right\} \tag{14}$$

where $k \geq 2$ and $|x| = 1 - \frac{(1+\rho) - (-1)^{m-n}(\gamma+\rho)}{(1-\gamma)}$.

Proof. Any function $f \in V_{\bar{H}}(m, n; \gamma, \rho, z_0)$ may be expressed as .

$$f(z) = a_1(z) + \sum_{k=2}^{\infty} |a_k| e^{i\beta_k} z^k + \overline{\sum_{k=2}^{\infty} |b_k| e^{i\delta_k} z^k} + \bar{b}_1 z ,$$

where the coefficients satisfy the inequality (7). Set

$$h_1(z) = z, g_1(z) = b_1 z, h_k(z) = z + \lambda_k e^{i\beta_k} z^k, g_k(z) = b_1 z + \mu_k e^{i\delta_k} z^k, k = 2, 3, \dots$$

Writing $X_k = \frac{|a_k|}{\lambda_k}, Y_k = \frac{|b_k|}{\mu_k}, k = 2, 3, \dots$ and $X_1 = 1 - \sum_{k=2}^{\infty} X_k, Y_1 = 1 - \sum_{k=2}^{\infty} Y_k$, we have

$$f(z) = \sum_{k=1}^{\infty} (X_k h_k(z) + \overline{Y_k g_k(z)})$$

In particular, setting $f_1(z) = z + \bar{b}_1 z$ and $f_k(z) = z + \lambda_k x z^k + \bar{b}_1 z + \overline{\mu_k y z^k}$,

$$\left(k \geq 2, |x| + |y| = 1 - \frac{(1+\rho) - (-1)^{m-n}(\gamma+\rho)}{(1-\gamma)} |b_1| \right)$$

we see that extreme points of the class $V_{\bar{H}}(m, n; \gamma, \rho, z_0)$ are contained in $\{f_k(z)\}$. To see that $f_1(z)$ is not an extreme point, note that $f_1(z)$ may be written as

$$f_1(z) = \frac{1}{2} \left[f_1(z) + \lambda \left(1 - \frac{(1+\rho)k^{-n} - (-1)^{m-n}(\gamma+\rho)k^{-m}}{(1-\gamma)} |b_1| \right) z^2 \right] + \frac{1}{2} \left[f_1(z) - \lambda \left(1 - \frac{(1+\rho)k^{-n} - (-1)^{m-n}(\gamma+\rho)k^{-m}}{(1-\gamma)} |b_1| \right) z^2 \right],$$

a convex linear combination of functions in the class $V_{\bar{H}}(m, n; \gamma, \rho, z_0)$. Next we will show if both $|x| \neq 0$ and $|y| \neq 0$, then f_k is not an extreme point.

Without loss of generality, assume $|x| \geq |y|$ choose $\epsilon > 0$ small enough so that $\epsilon < \frac{|x|}{|y|}$. Set $A = 1 + \epsilon$ and $B = 1 - \left| \frac{\epsilon x}{y} \right|$, we then see that both

$$t_1(z) = z + \lambda_k x A z^k + \overline{b_1 z + \mu_k y B z^k}$$

and

$$t_2(z) = z + \lambda_k x (2 - A) z^k + \overline{b_1 z + \mu_k y (2 - B) z^k}$$

are in the class $V_{\bar{H}}(m, n; \gamma, \rho, z_0)$ and note that

$$f_k(z) = \frac{1}{2} t_1(z) + t_2(z).$$

The extremal coefficient bounds shows that functions of the form (14) are the extreme points for the class $V_{\bar{H}}(m, n; \gamma, \rho, z_0)$, then the proof of Theorem (2.4) is completed.

Now we will examine the closure properties of the class $V_{\bar{H}}(m, n; \gamma, \rho, z_0)$ under the generalized Bernardi - Libera - Livingston integral operator (see[2, 7]) $L_c(f)$ which is defined by

$$L_c f(z) = \frac{c+1}{z^c} \int_0^z t^{c-1} f(t) dt \quad (c > -1) \tag{15}$$

Theorem 2.5. Let $f_n = h + g_n \in V_{\bar{H}}(m, n; \gamma, \rho, z_0)$, where h and g_n are given by (5). Then $L_c(f_n(z))$ belongs to the class $V_{\bar{H}}(m, n; \gamma, \rho, z_0)$.

Proof. From the representation of $L_c \{f_n(z)\}$, it follows that

$$\begin{aligned} L_c(f_n(z)) &= \frac{c+1}{z^c} \int_0^z t^{c-1} \{h(t) + \bar{g}_n(t)\} dt = \\ &= \frac{c+1}{z^c} \int_0^z t^{c-1} \left[t + \sum_{k=2}^{\infty} a_k t^k + \sum_{k=1}^{\infty} b_k t^k \right] dt = \\ &= a_1(z) + \sum_{k=2}^{\infty} A_k z^k + \sum_{k=1}^{\infty} B_k z^k, \end{aligned}$$

Where $A_k = \left(\frac{c+1}{c+k} \right) a_k$, $B_k = \left(\frac{c+1}{c+k} \right) b_k$. Therefore, we have,

$$\begin{aligned} &\sum_{k=2}^{\infty} \frac{(1+\rho)k^{-n} - (\gamma+\rho)k^{-m}}{(1-\gamma)} \frac{c+1}{c+k} |a_k| + \sum_{k=1}^{\infty} \frac{(1+\rho)k^{-n} - (-1)^{m-n}(\gamma+\rho)k^{-m}}{(1-\gamma)} \frac{c+1}{c+k} |b_k| \leq \\ &\leq \sum_{k=2}^{\infty} \frac{(1+\rho)k^{-n} - (\gamma+\rho)k^{-m}}{(1-\gamma)} |a_k| + \sum_{k=1}^{\infty} \frac{(1+\rho)k^{-n} - (-1)^{m-n}(\gamma+\rho)k^{-m}}{(1-\gamma)} |b_k| \leq 2a_1, \end{aligned}$$

and the proof of Theorem (2.5) is completed.

REFERENCES RÉFÉRENCES REFERENCIAS

1. Aghalary, R., (2007), Goodman- Salagian-type harmonic univalent function with varying arguments. *Int. Journal of math. Analysis*, 1 (22), pp.1051-1057.
2. Bernardi, S. D., (1969), convex and starlike univalent functions. *Trans. Amer. Math. Soc.*, 135, pp.429-446.
3. Clunie, J., Sheil-Small, T.,(1984), Harmonic univalent functions, *Ann. Acad. Sci. Fenn. Ser. A. I. Math.*, 9, pp.3-25.
4. Cotirla, L. I., (2009), Harmonic univalent functions defined by an integral operator, *Acta Universities Apulensis*, 17, pp.95-105.
5. Guney, H. O., Sakar, F.M.,(2011), On uniformly starlike functions defined by an integral operator, *Acta Universities Apulensis*, 28, pp.293-301.
6. Guney, H. O., Sakar, F.M.,(2012), A new class of harmonic uniformly starlike functions defined by an integral operator, *General Math.*, 20(1), pp.75-83.
7. Libera, R. J., (1965), Some classes of regular univalent functions, *Proc. Amer. Math. Soc.*, 16, pp.755-758.
8. R. M. EL-Ashwah, M. K. Aouf, A. A. M. Hassan, A. H. hassan, harmonic Univalent functions with varying arguments defined by using Salagean Integral operator with fixed point, *TWMS J. Pure Appl. Math.*V.4,N.1,2013,pp.95-102.
9. Salagean, G. S., (1983), Subclass of univalent functions, *Lecture Notes in Math.* Springer-Verlag, 1013, pp.362-372.



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Numerical Investigation of the Stability of Equilibrium Points for the Model of Three Mutually Competing, Symmetric and Continuous Time Reproducing Organism in a Fairly Stable Ecological Environment

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Abstract- In this paper, we examined the stability of the equilibrium points for the model of three mutually competing species within a stable ecosystem. For the stable ecosystem, we proposed that there are combinations of initial population densities and efficiency parameters that can balance the coexistence of these species for a very long time. This desirable biological property of a stable non-extinction equilibrium point for the ecosystem was obtained for all cases considered in this research work.

Keywords: *Stability, Equilibrium points, Ecosystem, Population density.*

GJSFR-F Classification : *FOR Code : MSC 2010: 010499*



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Numerical Investigation of the Stability of Equilibrium Points for the Model of Three Mutually Competing, Symmetric and Continuous Time Reproducing Organism in a Fairly Stable Ecological Environment

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Abstract- In this paper, we examined the stability of the equilibrium points for the model of three mutually competing species within a stable ecosystem. For the stable ecosystem, we proposed that there are combinations of initial population densities and efficiency parameters that can balance the coexistence of these species for a very long time. This desirable biological property of a stable non-extinction equilibrium point for the ecosystem was obtained for all cases considered in this research work.

Keywords: stability, equilibrium points, ecosystem, population density.

I. INTRODUCTION

In Obayomi and Oke (2015), we investigated the equilibrium state for the kind of system considered in this paper. We also confirmed the analytic equilibrium points and showed that all the numerical equilibrium points coincide with at least one of the analytic ones. This implies that the schemes may not possess numerical instabilities, Mickens (1994) and Mickens (2000). In this paper, we shall investigate the stability of these equilibrium points based on various combinations of parameters and initial values. Consider a general first order ordinary differential equation (ODE) of the form

$$y' = f(t, y), y(t_0) = y_0 \quad (1)$$

Suppose equation (1) possesses the properties of existence and uniqueness of solution in its domain U, then the following definitions holds.

Definition 1: The zeros of the function f in equation (1) is a critical point. A point $\underline{y} \in \mathbb{R}$ is called a fixed point or equilibrium point of the dynamical system defined by (1) if $f(\underline{y}) = 0$. If c is any critical point of f, then $y(x) = c$ is a constant solution of the differential equation.

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Remark 1: A fixed point y is stable if all nearby solutions stay nearby. It is asymptotically stable if all nearby solutions stay nearby and also tend to y or are attracted by y .

Definition 2: (Stability of equilibrium point)

Let $U \in \mathbb{R}^m$ be the domain of definition of f in equation (1), then any equilibrium point y of (1) is said to be stable if and only if, for every neighborhood Ω of y in U , there is an open neighborhood V of y such that any orbit starting globally in V remains in Ω for all $t \geq 0$. If $V = U$ then y is globally stable. If y is stable and there exists a domain V_0 such that any orbit originally in V_0 tends to y as t tends to infinity, then y is asymptotically stable, Beltrami (1986).

II. MATERIALS AND METHODS

a) Stability of Non-standard Finite Difference Schemes

We derived our standard reference for the qualitative property of non-standard schemes from the work of Anguelov and Lubuma (2003) and the references therein.

It is a common fact to write the functional dependence y_{n+1} on the quantities x_n , y_n and h in the form

$$y_{k+1} = y_k + h\varphi(x_k, y_k; h) \tag{2}$$

where $\varphi(x_k, y_k; h)$ is called the increment function.

Definition 3:

Let us denote (2) by the sequence

$$y_k = F(h; y_k) \tag{3}$$

and let us assume that the solution of equation (1) satisfies some property \mathcal{P} . The numerical scheme (3) is said to be qualitatively stable with respect to property \mathcal{P} or \mathcal{P} -stable, if for every value $h > 0$, the set of solutions of (3) satisfies \mathcal{P} , Anguelov and Lubuma(2003).

Definition 4: A set $G(\Omega)$ of real-valued functions defined on a subset Ω of $[t_0, \infty)$ monotonically depend on the initial value (t_0) if for every two functions $y, z \in G(\Omega)$, we have $y(t_0) \leq z(t_0)$ implies that $y(t) \leq z(t)$, for $t \in \Omega$, Anguelov and Lubuma(2003).

Definition 5: The finite difference scheme (3) is stable with respect to the property of monotonicity of solutions if for every $y_0 \in \mathbb{R}$, the solution y_k of (3) is an increasing or a decreasing sequence just as the $y(t)$ of equation (1) is increasing or decreasing.

Definition 6: Any fixed point \bar{y} of (1) is called hyperbolic fixed point if it satisfies the relationship $j \equiv f'(\bar{y}) \neq 0$, Anguelov and Lubuma(2003).

Remark 2: The asymptotic behavior of solutions of (1) with initial data near y may be reduced to the behavior of linear equation of the form

$$\epsilon' = J\epsilon. \tag{4}$$

Definition 7: A hyperbolic fixed point is called linearly-stable provided that the solution ϵ of equation (1) corresponding to any small initial data $\epsilon(0)$, for $|\epsilon(0)| \ll 1$ say,

satisfies $\lim_{t \rightarrow \infty} \epsilon(t) = 0$. Otherwise the fixed point is linearly unstable. The discrete analogue of (4) is given by

$$\epsilon_{k+1} = J_h \epsilon_k \quad \text{where } J_h = \frac{\partial F}{\partial y}(h; \bar{y}) \tag{5}$$

Definition 8: Assume that a hyperbolic fixed point \bar{y} of the differential equation (1) is a solution to the discrete method (3). We say that the constant solution \bar{y} is linearly stable provided that the solution ϵ_k of the equation (1) corresponding to any initial data $\epsilon(0)$, for $|\epsilon(0)| \ll 1$ say, satisfies $\lim_{k \rightarrow \infty} \epsilon_k = 0$. This is equivalent to saying that $|J_h| < 1$ in (5). Otherwise the fixed point is linearly unstable, Anguelov and Lubuma (2003).

Definition 9: The finite difference method (3) is called elementary stable if for any value of the step size h , its only fixed points \bar{y} are those of the differential equation (1), the linear stability property of each \bar{y} being the same for both the differential equation and the discrete method.

Theorems and conditions supporting these stability properties above may be found in Anguelov and Lubuma (2003).

Numerical experiments have been carried out in which several set of different initial values have been used to test the stability of the equilibrium points while fixing the efficiency parameters. The equilibrium points tested for stability are those that were obtained in Obayomi and Oke (2015).

III. NUMERICAL EXPERIMENTS

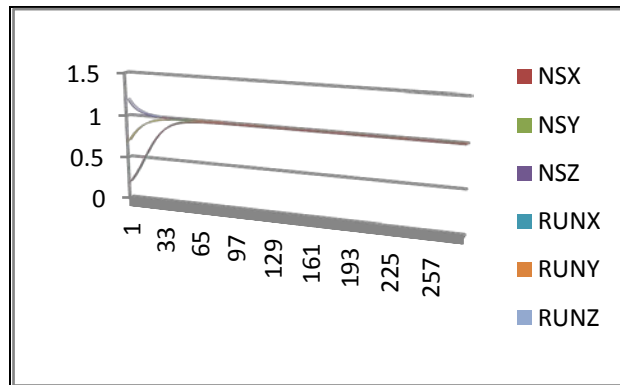


Figure 1 : Orbit of the schemes for $\alpha = 0, \beta = 0$

The equilibrium state is (1, 1, 1) for any non-zero initial values

Note: NS = Non-standard and RUN= Runge Kutta

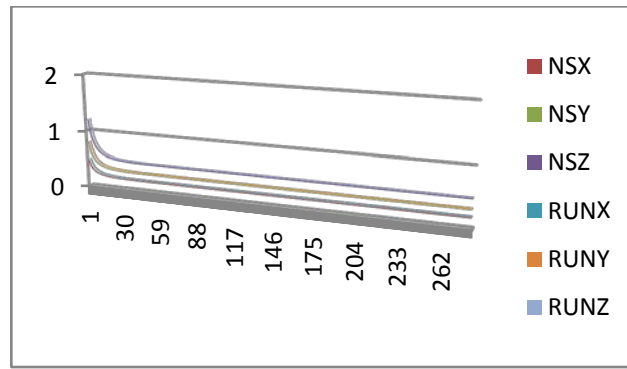


Figure 2 : Orbit of the schemes when $\alpha = 1, \beta = 1$

The equilibrium state is $(0.195, 0.317, 0.488)$ for initial value $(0.5, 0.8, 1.2)$ which is in the form (X, Y, Z) such that $X + Y + Z = 1$

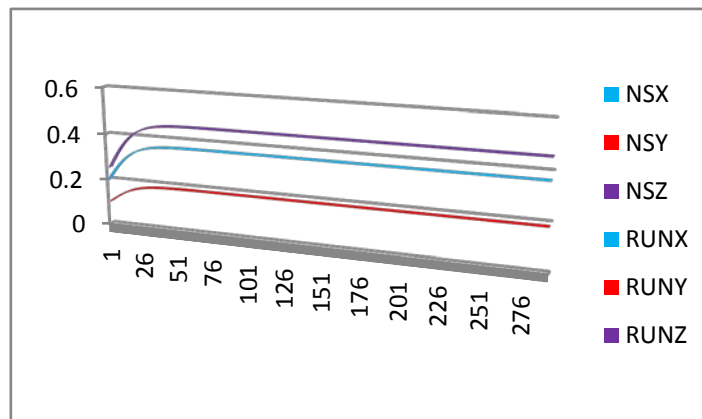


Figure 3 : Orbit of the schemes when $\alpha = 1, \beta = 1$

The equilibrium state is $(0.364, 0.183, 0.453)$ for initial value $(0.2, 0.1, 0.25)$ which is in the form (X, Y, Z) such that $X + Y + Z = 1$

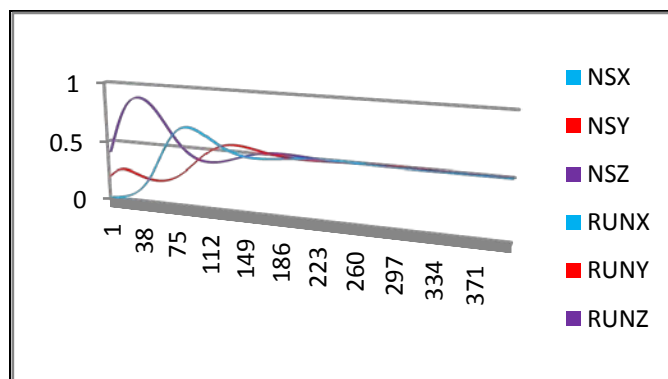


Figure 4 : Orbit of the schemes for $\alpha = 1, \beta = 0$ or $\alpha = 0, \beta = 1$

The equilibrium state is $(0.5, 0.5, 0.5)$ for any non-zero initial value

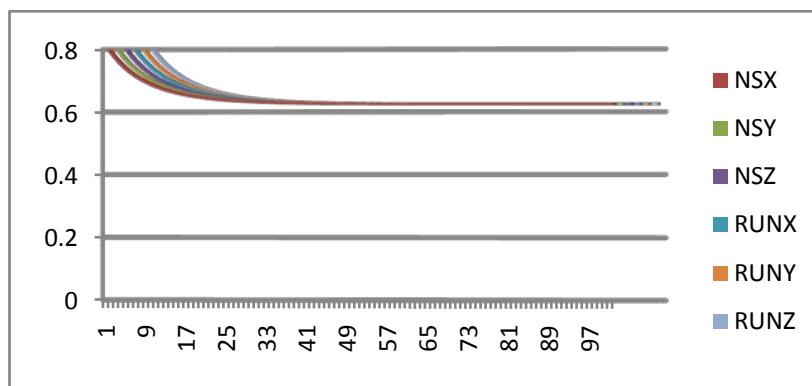


Figure 5 : Orbit of the schemes for $\alpha, \beta \in (0,1)$ and $\alpha \beta < 1$

The equilibrium point is $(0.625, 0.625, 0.625)$ when $(\alpha = 0.25, \beta = 0.35)$ with initial value $(0.8, 0.8, 0.8)$ which confirms (k, k, k) where $k = \frac{1}{1+\alpha+\beta}$.

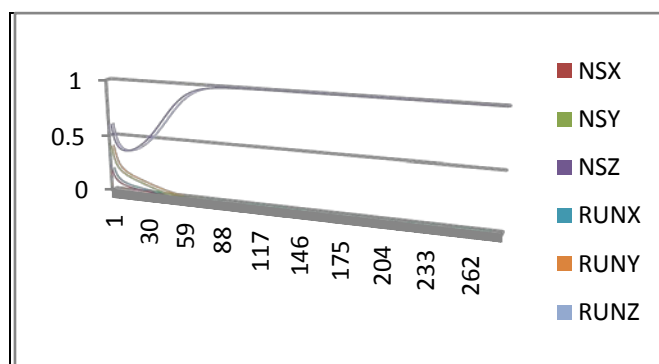


Figure 6 : Orbit of the schemes for $\alpha, \beta > 1$ i.e $\alpha \beta > 1$

The equilibrium point is $(0, 0, 1)$ for $(\alpha = \beta = 3)$ with initial value $(0.2, 0.4, 0.6)$.

Note: The equilibrium point is $(1,0,0)$, $(0,1,0)$ or $(0,0,1)$ depending on the initial values. The specie with the highest initial value will wipe out the other two species on the long run

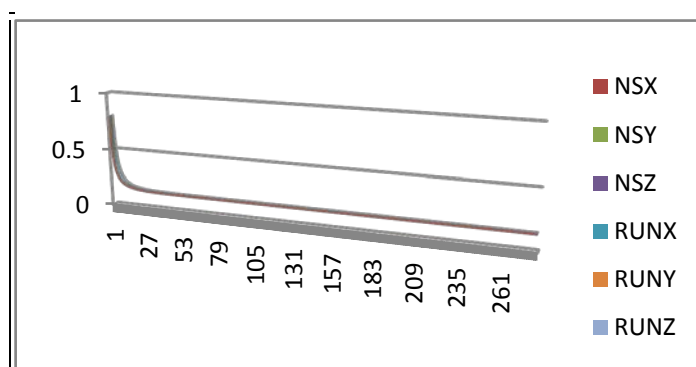


Figure 7 : Orbit of the schemes for $\alpha, \beta > 1$ i.e $\alpha \beta > 1$

The equilibrium point is $(0.14286, 0.14286, 0.14286)$ for $(\alpha = \beta = 3)$ with initial value $(0.8, 0.8, 0.8)$ which confirms (k, k, k) where $k = \frac{1}{1+\alpha+\beta}$

Note: The equilibrium point is (k, k, k) where $k = \frac{1}{1+\alpha+\beta}$ if the initial values are the same.

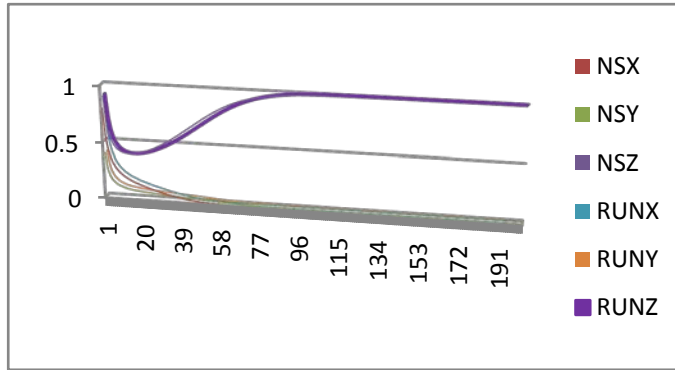


Figure 8 : Orbit of the schemes for $\alpha, \beta > 1, \alpha \neq \beta$ i.e $\alpha \beta > 1$

The equilibrium point is $(0, 1, 0)$ for $\alpha = 2, \beta = 3$

The equilibrium point is reduced to one of $(1, 0, 0)$, $(0, 1, 0)$ or $(0, 0, 1)$ in relation to the density at the initial time

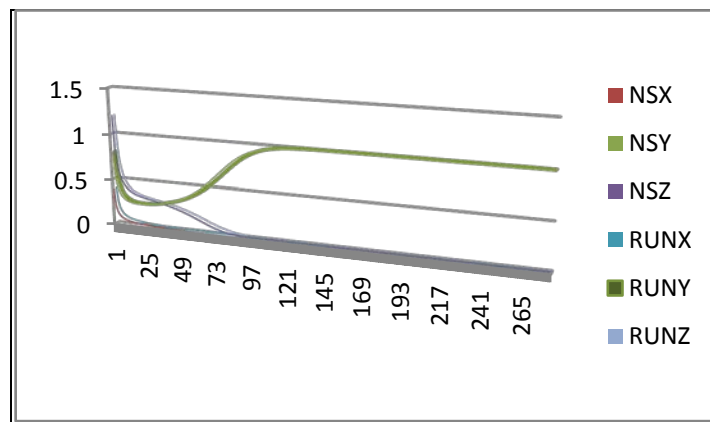


Figure 9 : Orbit of the schemes for $\alpha, \beta > 1, \alpha \neq \beta$ i.e $\alpha \beta > 1$

The equilibrium point is $(0,0,1)$ for $\alpha = 3, \beta = 2$, initial value is $(0.4, 0.8, 1.2)$.

The equilibrium point is reduced to one of $(1,0,0), (0,1,0)$ or $(0,0,1)$ in relation to the initial density.

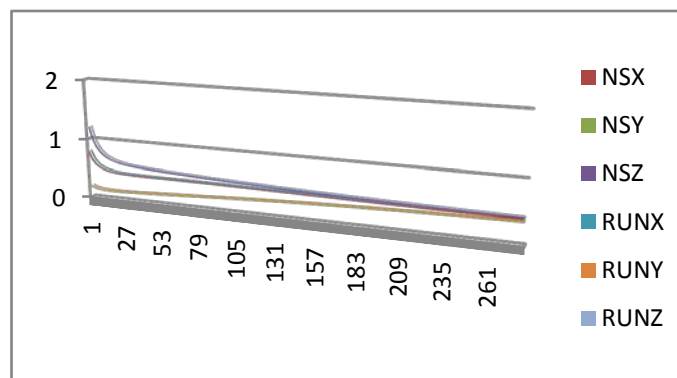


Figure 10 : Orbit of the schemes for $\alpha, \beta \in (0,1)$ and $\alpha \beta < 1$

The equilibrium point is $(0.4, 0.4, 0.4)$, $(\alpha = \beta = 0.75)$ with initial value $(0.8, 0.2, 1.2)$

which confirms (k, k, k) where $k = \frac{1}{1+\alpha+\beta}$.

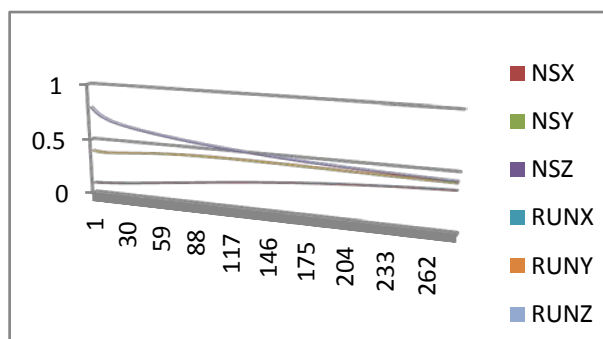


Figure 11 : Orbit of the schemes for $\alpha, \beta \in (0,1)$ and $\alpha \beta < 1$

The equilibrium point is $(0.4, 0.4, 0.4)$, $(\alpha = \beta = 0.75)$ with initial value $(0.1, 0.4, 0.8)$ which confirms (k, k, k) where $k = \frac{1}{1+\alpha+\beta}$.

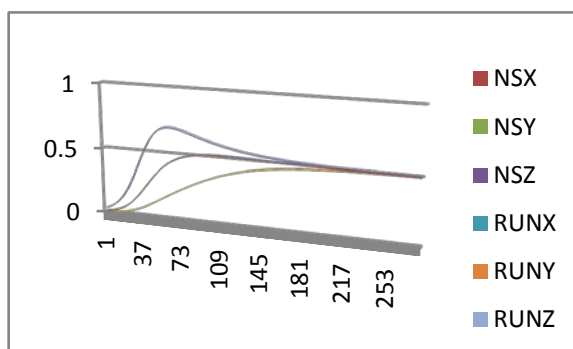


Figure 12 : Orbit of the schemes for $\alpha, \beta \in (0,1)$ and $\alpha \beta < 1$

The equilibrium point is $(0.5, 0.5, 0.5)$, $(\alpha = \beta = 0.5)$, initial value is $(0.01, 0.003, 0.03)$ which confirms (k, k, k) where $k = \frac{1}{1+\alpha+\beta}$.

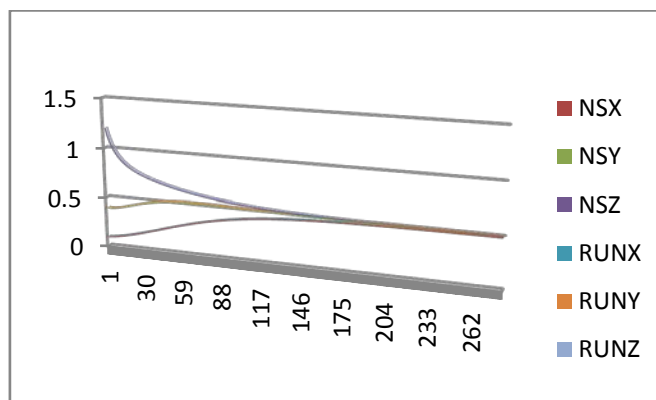


Figure 13 : Orbit of the schemes for $\alpha, \beta \in (0,1)$ and $\alpha \beta < 1$

The equilibrium point is $(0.5, 0.5, 0.5)$, $(\alpha = \beta = 0.5)$ with initial value $(0.1, 0.4, 1.2)$ which confirms (k, k, k) where $k = \frac{1}{1+\alpha+\beta}$.

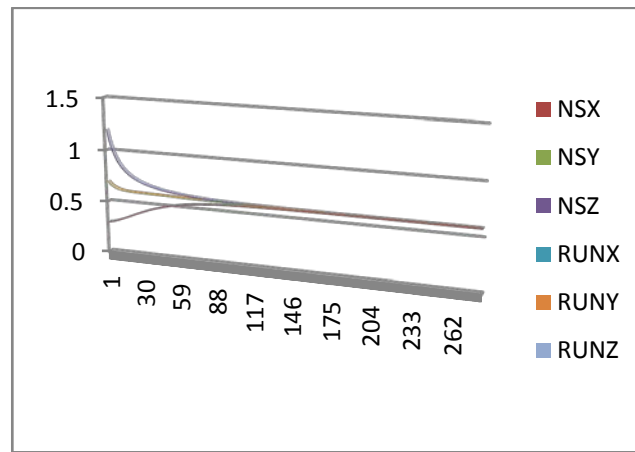


Figure 14 : Orbit of the schemes for $\alpha, \beta \in (0,1)$ and $\alpha \beta < 1$

The equilibrium point is $(0.588, 0.588, 0.588)$, ($\alpha = \beta = 0.35$) initial value is $(0.3, 0.7, 1.2)$ which confirms (k, k, k) where $k = \frac{1}{1+\alpha+\beta}$.

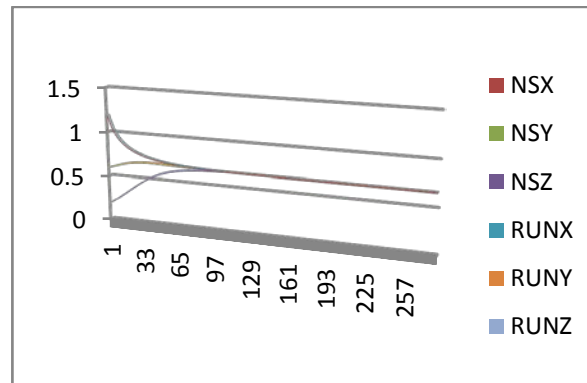


Figure 15 : Orbit of the schemes for $\alpha, \beta \in (0,1)$ and $\alpha \beta < 1$

The equilibrium point is $(0.67, 0.67, 0.67)$, ($\alpha = \beta = 0.25$) with initial value $(1.2, 0.6, 0.2)$ which confirms (k, k, k) where $k = \frac{1}{1+\alpha+\beta}$

IV. DISCUSSION OF RESULTS

The following were observed from the result of the numerical experiment.

- i For $\alpha = \beta = 0$, the equilibrium state is $(1,1,1)$. Therefore, for any initial value, it is expected that if the co-existent of the species are either mutually beneficial or of insignificant effect on each other then all species are expected to grow to the full carrying capacity of the ecological environment and the point $(1,1,1)$ is a stable equilibrium.
- ii For $\alpha = \beta = 1$, the equilibrium state is (X_1, X_2, X_3) such that $X_1 + X_2 + X_3 = 1$. It may however reduce the habitat to a partially extinct equilibrium state with only the specie with the largest initial value remaining and growing to its full capacity as competition fades away. This equilibrium points depends on the initial value and therefore it is not stable.

- iii For $\alpha = 1, \beta = 0$ or $\alpha = 0, \beta = 1$ the equilibrium state is $(0.5, 0.5, 0.5)$ for any initial value. The equilibrium point in this case is stable.
- iv For $\alpha, \beta \in (0, 1)$ and $\alpha\beta < 1$ the equilibrium point is (k, k, k) where $k = \frac{1}{1+\alpha+\beta}$ for any initial value. The equilibrium point here is also stable.
- v For $\alpha, \beta > 1$ i.e $\alpha\beta > 1$ the equilibrium point is (k, k, k) where $k = \frac{1}{1+\alpha+\beta}$. If the initial values are the same, the equilibrium points are unstable.
- vi For $\alpha, \beta > 1, \alpha = \beta$ i.e $\alpha\beta > 1$, the equilibrium point is (k, k, k) where $k = \frac{1}{1+\alpha+\beta}$. If the initial values are the same and the habitat is reduced to one of $(1, 0, 0)$, $(0, 1, 0)$ or $(0, 0, 1)$ depending on the specie with the largest initial value, then the equilibrium point is unstable.
- vii For $\alpha, \beta > 1, \alpha \neq \beta$ i.e $\alpha\beta > 1$. If the habitat is reduced to one of $(1, 0, 0)$, $(0, 1, 0)$ or $(0, 0, 1)$ depending on the initial values, then the equilibrium points are unstable.
- viii The computational results also confirm the trivial equilibrium points.

V. CONCLUSION

Our results support and confirm the earlier studies on this model and exposed the expected physical situation in the ecological environment. With a suitable control mechanism on the efficiency parameters, ecologists, zoologists etc can apply this model for research and ecological planning. The information obtained here may also be used for pest control in Agrarian science.

REFERENCES RÉFÉRENCES REFERENCIAS

1. Anguelov, R., & Lubuma, J.M.S. (2003). Nonstandard Finite Difference Method by Non-local Approximation: Mathematics and Computers in Simulation 6, 465-475.
2. Beltrami, E. (1986). Mathematics for Dynamic Modeling: Academic Press, New York.
3. Mickens, R. E. (1994). Nonstandard Finite Difference Models of Differential Equations: World Scientific, Singapore.
4. Mickens R.E. (2000). Applications of Nonstandard Methods to Initial Value Problems: World Scientific, Singapore.
5. Obayomi, A.A., & Oke, M.O. (2015). On the Equilibrium Points of Three Mutually Competing, Symmetric and Continuous Time Reproducing Organisms: European Journal of Mathematics and Computer Science, Birmingham, U.K. 2(1), 13 – 21.

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On Non-Invariant Hypersurfaces of δ -lorentzian Trans-Sasakian Manifolds

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Abstract- The object of the present paper is to study non-invariant Hypersurfaces of δ -Lorentzian trans-Sasakian Manifolds equipped with (f, g, u, v, λ) -structure and some properties obeyed by this structure are obtained also. The necessary and sufficient conditions have been obtained for totally umbilical non-invariant hypersurfaces with (f, g, u, v, λ) -structure of δ -Lorentzian trans-Sasakian Manifold to be totally geodesic.

Keywords: δ -Lorentzian trans-Sasakian, totally umbilical, totally geodesic.

GJSFR-F Classification : FOR Code : MSC 2000: 14J70, 53C20



Strictly as per the compliance and regulations of :





On Non-Invariant Hypersurfaces of δ -Lorentzian Trans-Sasakian Manifolds

Shyam Kishor ^α & Puneet Kumar Gupt ^σ

Abstract- The object of the present paper is to study non-invariant Hypersurfaces of δ -Lorentzian trans-Sasakian Manifolds equipped with (f, g, u, v, λ) -structure and some properties obeyed by this structure are obtained also. The necessary and sufficient conditions have been obtained for totally umbilical non-invariant hypersurfaces with (f, g, u, v, λ) -structure of δ -Lorentzian trans-Sasakian Manifold to be totally geodesic.

Keywords and phrases: δ -Lorentzian trans-Sasakian, totally umbilical, totally geodesic.

I. INTRODUCTION

Recently, many authors have studied Lorentzian α -Sasakian manifold [1] and Lorentzian β -Kenmotsu manifolds [7], [3]. S.S.Pujar and V.J.Khairnar [12] have initiated the study of Lorentzian Trans-Sasakian manifolds and studied the basic results with some of its properties. Earlier to this, S.S.Pujar [13] has initiated the study of δ -Lorentzian, α -Sasakian manifold [3] and δ -Lorentzian β -Kenmotsu manifolds [4].

In 2010, S.S.Shukla and D.D. Singh [14] have introduced the notion of ε -trans-Sasakian manifolds and studied its basic results and using these results some of its properties were studied. Earlier to this in 1969 Takahashi [16] had introduced the notaion of almost contact metric manifold equipped with pseudo Riemannian metric. In particular he studied the Sasakian manifolds equipped with Riemannian metric g . These indefinite almost contact metric manifolds and indefinite Sasakian manifolds are also known as ε -almost contact metric manifolds and ε -Sasakian manifolds respectively.

Recently, it has been observed that there does not exists a light like surface in the ε -Sasakian manifolds ([8], [16]). On the other hand in almost para contact manifold defined by Motsumoto [6], the semi Riemannian manifold has the index 1 and the structure vector field ξ is always a time like. This motivated Tripathi et. al [8] to introduce ε -almost para contact structure where the vector field ξ is space like or time like according as $\varepsilon = 1$ or $\varepsilon = -1$.

In 1970, S.I.Goldberg et. al [10] introduced the notion of a non-invariant hypersurfaces of an almost contact manifold in which the transform of a tangent vector of the hypersurface by the $(1, 1)$ structure tensor field ϕ defining the almost contact structure is never tangent to the hypersurface.

The notion of (f, g, u, v, λ) -structure was given by K.Yano [4]. It is well known that a hypersurface of an almost contact metric manifold always admits a (f, g, u, v, λ) -structure ([5] [2]). Goldberg et. al [10] proved that there always exists a (f, g, u, v, λ) -structure on a non-invariant hypersurface of an almost contact metric manifold.

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They also proved that there does not exist invariant hypersurface of a contact manifold. R.Prasad [9] studied the non-invariant hypersurfaces of a trans-Sasakian manifold. Non-invariant hypersurfaces of nearly trans-Sasakian manifold have been studied by S.Kishor et. al [11]. In the present paper, we study the non-invariant hypersurfaces of δ -Lorentzian trans-Sasakian manifolds.

II. PRELIMINARIES

A $(2n + 1)$ dimensional manifold \widetilde{M} , is said to be the δ -almost contact metric manifold if it admits a $(1, 1)$ tensor field ϕ , a structure tensor field ξ , a 1-form η , and an indefinite metric g such that

$$\phi^2 = I + \eta \otimes \xi, \eta(\xi) = -1, \phi \circ \xi = 0, \eta \circ \phi = 0 \quad (2.1)$$

$$g(\xi, \xi) = -\delta, \eta(X) = \delta g(X, \xi) \quad (2.2)$$

$$g(\phi X, \phi Y) = g(X, Y) + \delta \eta(X) \eta(Y) \quad (2.3)$$

$$g(X, \phi Y) = g(\phi X, Y) \quad (2.4)$$

for all $X, Y \in T\widetilde{M}$, where δ is such that $\delta^2 = 1$.

The above structure $(\phi, \xi, \eta, g, \delta)$ on \widetilde{M} is called the δ -Lorentzian structure on \widetilde{M} .

A δ -Lorentzian manifold with structure $(\phi, \xi, \eta, g, \delta)$ is said to be δ -Lorentzian trans-Sasakian manifold \widetilde{M} of type (α, β) if it satisfies the condition

$$(\widetilde{\nabla}_X \phi) Y = \alpha \{g(X, Y) \xi - \delta \eta(Y) X\} + \beta \{g(\phi X, Y) \xi - \delta \eta(Y) \phi X\} \quad (2.5)$$

for any vector fields X and Y on \widetilde{M} . where $\widetilde{\nabla}$ is the operator of covariant differentiation with respect to g . From above, we have

$$\widetilde{\nabla}_X \xi = \delta (-\alpha \phi X - \beta (X + \eta(X) \xi)) \quad (2.6)$$

and

$$(\widetilde{\nabla}_X \eta) Y = \alpha g(\phi X, Y) + \beta \{g(X, Y) + \delta \eta(X) \eta(Y)\} \quad (2.7)$$

A hypersurface of an almost contact metric manifold \widetilde{M} is called a non-invariant hypersurface, if the transform of a tangent vector of the hypersurface under the action of $(1, 1)$ tensor field ϕ defining the contact structure is never tangent to the hypersurface. Let X be a tangent vector on non-invariant hypersurface of an almost contact metric manifold \widetilde{M} , then ϕX is never tangent to the hypersurface.

Let M be a non-invariant hypersurface of an almost contact metric manifold. Now, if we define the following

$$\phi X = fX + u(X) \hat{N}, \quad (2.8)$$

$$\phi \hat{N} = -U, \quad (2.9)$$

$$\xi = V + \lambda \hat{N}, \lambda = \eta(\hat{N}), \quad (2.10)$$

$$\eta(X) = v(X), \quad (2.11)$$

where, f is a $(1, 1)$ tensor field, u, v are 1-form, \hat{N} is a unit normal to the hypersurface, $X \in TM$ and $u(X) \neq 0$, then we get an induced (f, g, u, v, λ) -structure on M satisfying the conditions

$$f^2 = -I + u \otimes U + v \otimes V, \tag{2.12}$$

$$fU = -\lambda V, fV = \lambda U, \tag{2.13}$$

$$u \circ f = \lambda v, v \circ f = -\lambda u, \tag{2.14}$$

$$u(U) = 1 - \lambda^2, u(V) = v(U) = 0, v(V) = 1 - \lambda^2, \tag{2.15}$$

$$g(fX, fY) = g(X, Y - u(X)u(Y) - v(X)v(Y)), \tag{2.16}$$

$$g(X, fY) = -g(fX, Y), g(X, U) = u(X), \tag{2.17}$$

$$g(X, V) = v(X), \tag{2.18}$$

for all $X, Y \in TM$, where $\lambda = \eta(\hat{N})$.

The Gauss and Weingarten formulae are given by

$$\tilde{\nabla}_X Y = \nabla_X Y + \sigma(X, Y)\hat{N}, \tag{2.19}$$

$$\tilde{\nabla}_X \hat{N} = -A_{\hat{N}}X, \tag{2.20}$$

for all $X, Y \in TM$, where $\tilde{\nabla}$ and ∇ are the Riemannian and induced Riemannian connections on \tilde{M} and M respectively and \hat{N} is the unit normal vector in the normal bundle $T^\perp M$. In this formula σ is the second fundamental form on M related to $A_{\hat{N}}$ by

$$\sigma(X, Y) = g(A_{\hat{N}}X, Y), \text{ for all } X, Y \in TM.$$

III. SOME PROPERTIES OF NON-INVARIANT HYPERSURFACES

Lemma 1. :Let M be a non-invariant hypersurface with (f, g, u, v, λ) -structure of δ -Lorentzian trans-Sasakian manifold \tilde{M} . Then

$$(\tilde{\nabla}_X \phi) Y = (\nabla_X f) Y - u(Y)(A_{\hat{N}}X) + \sigma(X, Y)U + ((\nabla_X u)Y + \sigma(X, fY))\hat{N} \tag{3.1}$$

$$(\tilde{\nabla}_X \eta) Y = (\nabla_X v) Y - \lambda \sigma(X, Y) \tag{3.2}$$

$$\tilde{\nabla}_X \xi = \nabla_X V - \lambda A_{\hat{N}}X + (\sigma(X, V) + X\lambda)\hat{N} \tag{3.3}$$

Proof. Consider

$$\begin{aligned} (\tilde{\nabla}_X \phi) Y &= (\tilde{\nabla}_X \phi Y) - \phi(\tilde{\nabla}_X Y) \\ &= \tilde{\nabla}_X (fY + u(Y)\hat{N}) - \phi(\nabla_X Y + \sigma(X, Y)\hat{N}) \\ &= \tilde{\nabla}_X (fY) + \tilde{\nabla}_X (u(Y)\hat{N}) - \phi(\nabla_X Y) - \sigma(X, Y)\phi(\hat{N}) \end{aligned}$$

$$= \nabla_X(fY) + \sigma(X, fY)\hat{N} + u(Y)(\tilde{\nabla}_X\hat{N}) + (\tilde{\nabla}_Xu(Y))\hat{N} - f(\nabla_XY) - u(\nabla_XY)\hat{N} + \sigma(X, Y)U$$

which gives

$$(\tilde{\nabla}_X\phi)Y = (\nabla_Xf)Y - u(Y)(A_{\hat{N}}X) + \sigma(X, Y)U + ((\nabla_Xu)Y + \sigma(X, fY))\hat{N}$$

Also we have,

$$\begin{aligned}(\tilde{\nabla}_X\eta)Y &= \tilde{\nabla}_X\eta(Y) - \eta(\tilde{\nabla}_XY) \\ &= \tilde{\nabla}_X(v(Y)) - \eta(\nabla_XY + \sigma(X, Y)\hat{N}) \\ &= \nabla_X(v(Y)) + \sigma(X, v(Y))\hat{N} - \eta(\nabla_XY) - \sigma(X, Y)\eta(\hat{N}) \\ &= \nabla_X(v(Y)) - v(\nabla_XY) - \lambda\sigma(X, Y)\end{aligned}$$

$$(\tilde{\nabla}_X\eta)Y = (\nabla_Xv)Y - \lambda\sigma(X, Y)$$

Further, consider

$$\begin{aligned}\tilde{\nabla}_X\xi &= \nabla_X\xi + \sigma(X, \xi)\hat{N} \\ &= \nabla_XV + \nabla_X\lambda\hat{N} + \sigma(X, V)\hat{N} \\ &= \nabla_XV + \lambda\nabla_X\hat{N} + (X\lambda)\hat{N} + \sigma(X, V)\hat{N}\end{aligned}$$

which gives

$$\tilde{\nabla}_X\xi = \nabla_XV - \lambda A_{\hat{N}}X + (\sigma(X, V) + X\lambda)\hat{N}$$

□

Theorem 1.: Let M be a non-invariant hypersurface with (f, g, u, v, λ) -structure of δ -Lorentzian trans-Sasakian manifold \tilde{M} . Then

$$\sigma(X, \xi)U = \alpha\delta f^2X - \alpha\delta u(X)U + \delta\beta f(X) + f(\nabla_X\xi) \quad (3.4)$$

$$u(\nabla_X\xi) = -\alpha\delta u(fX) - \beta\delta u(X) \quad (3.5)$$

Proof. :Consider

$$\begin{aligned}(\tilde{\nabla}_X\phi)\xi &= \tilde{\nabla}_X\phi\xi - \phi(\tilde{\nabla}_X\xi) \\ &= -\phi(\delta(-\alpha\phi X - \beta(X + \eta(X)\xi)))\end{aligned}$$

or

$$(\tilde{\nabla}_X\phi)\xi = \alpha\delta f^2X + \alpha\delta u(fX)\hat{N} - \alpha\delta u(X)U + \delta\beta f(X) + \beta\delta u(X)\hat{N}$$

and we know that the relation

$$\begin{aligned}(\tilde{\nabla}_X\phi)\xi &= \tilde{\nabla}_X\phi\xi - \phi(\tilde{\nabla}_X\xi) \\ &= -\phi(\nabla_X\xi + \sigma(X, \xi)\hat{N}) \\ &= -\phi(\nabla_X\xi) + \sigma(X, \xi)U \\ &= -f(\nabla_X\xi) - u(\nabla_X\xi)\hat{N} + \sigma(X, \xi)U\end{aligned}$$

from above two equation, we have

$$\begin{aligned}-f(\nabla_X\xi) - u(\nabla_X\xi)\hat{N} + \sigma(X, \xi)U &= \alpha\delta f^2X + \alpha\delta u(fX)\hat{N} - \alpha\delta u(X)U \\ &\quad + \delta\beta f(X) + \beta\delta u(X)\hat{N}\end{aligned}$$

Equating tangential and normal parts on both side, we get

$$\sigma(X, \xi)U = \alpha\delta f^2X - \alpha\delta u(X)U + \delta\beta f(X) + f(\nabla_X\xi)$$

and

$$u(\nabla_X\xi) = -\alpha\delta u(fX) - \beta\delta u(X)$$

□

Theorem 2.: Let M be a non-invariant hypersurface with (f, g, u, v, λ) -structure of δ -Lorentzian trans-Sasakian manifold \tilde{M} . Then

$$\begin{aligned} (\nabla_X f)Y &= u(Y)(A_{\hat{N}}X) - \sigma(X, Y)U + \alpha(g(X, Y)V - \delta v(Y)X) \\ &\quad + \beta(g(fX, Y)V - \delta v(Y)fX) \end{aligned} \quad (3.6)$$

$$(\nabla_X u)Y = \alpha\lambda g(X, Y) + \beta(\lambda g(fX, Y) - \delta u(X)v(Y)) - \sigma(X, fY) \quad (3.7)$$

$$\nabla_X V = \lambda A_{\hat{N}}X - \delta\alpha fX - \delta\beta(X + v(X)V) \quad (3.8)$$

$$\sigma(X, V) = -\delta\alpha u(X) - \delta\lambda\beta v(X) - X\lambda \quad (3.9)$$

$$(\nabla_X v)Y = \lambda\sigma(X, Y) + \alpha g(fX, Y) + \beta\{g(X, Y) - \delta v(X)v(Y)\} \quad (3.10)$$

Proof. : Using (2.8), (2.10) in (2.5) and (3.1) we obtain

$$\begin{aligned} &(\nabla_X f)Y - u(Y)(A_{\hat{N}}X) + \sigma(X, Y)U + ((\nabla_X u)Y + \sigma(X, fY))\hat{N} \\ &= \alpha g(X, Y)V + \alpha\lambda g(X, Y)\hat{N} - \alpha\delta v(Y)X + \beta g(fX, Y)V \\ &\quad + \beta\lambda g(fX, Y)\hat{N} - \beta\delta v(Y)fX - \beta\delta v(Y)u(X)\hat{N} \end{aligned}$$

Equating tangential and normal parts in the above equation, we get (3.6) and (3.7) respectively.

Using equation (2.6), (2.8) and (2.11) we get,

$$\tilde{\nabla}_X\xi = -\delta\alpha fX - \delta\alpha u(X)\hat{N} - \delta\beta X - \delta\beta v(X)V - \lambda\delta\beta v(X)\hat{N}$$

and also we have,

$$\tilde{\nabla}_X\xi = \nabla_X V - \lambda A_{\hat{N}}X + (\sigma(X, V) + X\lambda)\hat{N}$$

Equating the tangential and normal part of the above two equation, we get (3.8) and (3.9).

In last using (2.7), (2.8) and (3.2) we get (3.10) □

Theorem 3.: Let M be a non-invariant hypersurface with (f, g, u, v, λ) -structure of δ -Lorentzian trans-Sasakian manifold \tilde{M} . Then

$$\begin{aligned} (\tilde{\nabla}_X\phi)Y &= \alpha(g(X, Y)V - \delta v(Y)X) + \beta(g(fX, Y)V - \delta v(Y)fX) \\ &\quad + \{\alpha(\lambda g(X, Y)) + \beta(\lambda g(fX, Y) - \delta u(X)v(Y))\}\hat{N} \end{aligned} \quad (3.11)$$

Proof. :Consider

$$\begin{aligned} (\tilde{\nabla}_X\phi)Y &= (\tilde{\nabla}_X\phi)Y - \phi(\tilde{\nabla}_XY) \\ &= \tilde{\nabla}_X(fY + u(Y)\hat{N}) - \phi(\nabla_XY + \sigma(X, Y)\hat{N}) \\ &= \tilde{\nabla}_X(fY) + \tilde{\nabla}_X(u(Y)\hat{N}) - \phi(\nabla_XY) - \sigma(X, Y)\phi(\hat{N}) \\ &= \nabla_X(fY) + \sigma(X, fY)\hat{N} + u(Y)(\tilde{\nabla}_X\hat{N}) + (\tilde{\nabla}_Xu(Y))\hat{N} - f(\nabla_XY) \\ &\quad - u(\nabla_XY)\hat{N} + \sigma(X, Y)U \end{aligned}$$

which gives

$$(\tilde{\nabla}_X \phi) Y = (\nabla_X f) Y - u(Y) (A_{\hat{N}} X) + \sigma(X, Y) U + ((\nabla_X u) Y + \sigma(X, fY)) \hat{N} \quad (3.12)$$

and we have also from (3.6) and (3.7)

$$\begin{aligned} (\nabla_X f) Y &= u(Y) (A_{\hat{N}} X) - \sigma(X, Y) U + \alpha(g(X, Y) V - \delta v(Y) X) \\ &\quad + \beta(g(fX, Y) V - \delta v(Y) fX) \end{aligned} \quad (3.13)$$

and

$$(3.14) \quad (\nabla_X u) Y = \alpha \lambda g(X, Y) + \beta(\lambda g(fX, Y) - \delta u(X) v(Y)) - \sigma(X, fY)$$

now equation (3.12), (3.13), and (3.14) enables us to deduce (3.11) \square

Theorem 4. :Let M be a totally umbilical noninvariant hypersurface with (f, g, u, v, λ) -structure of δ -Lorentzian trans-Sasakian manifold. Then it is totally geodesic if and only if

$$(3.15) \quad \delta \alpha u(X) + \delta \lambda \beta v(X) + X \lambda = 0$$

Proof. :From equation (2.6) we have,

$$\tilde{\nabla}_X \xi = \delta(-\alpha \phi X - \beta(X + \eta(X) \xi))$$

Using (2.8) and (2.11) in above equation we get

$$\begin{aligned} \tilde{\nabla}_X \xi &= -\delta \alpha f X - \delta \alpha u(X) \hat{N} - \delta \beta X - \delta \beta v(X) V \\ &\quad - \delta \lambda \beta v(X) \hat{N} \end{aligned}$$

Equating the normal parts of above equation and equation (3.3) we obtain

$$(3.16) \quad \sigma(X, V) = -\delta \alpha u(X) - \delta \lambda \beta v(X) - X \lambda$$

If M is totally umbilical, then $A_{\hat{N}} = \zeta I$
where ζ is Kahlerian metric

$$\begin{aligned} \sigma(X, Y) &= g(A_{\hat{N}} X, Y) = g(\zeta X, Y) = \zeta g(X, Y) \\ \sigma(X, V) &= \zeta g(X, V) = \zeta v(X) \end{aligned} \quad (3.17)$$

Then, from (3.13) and (3.14) we get

$$\delta \alpha u(X) + \delta \lambda \beta v(X) + X \lambda + \zeta v(X) = 0$$

If M is totally geodesic, i.e. $\zeta = 0$
then,

$$\delta \alpha u(X) + \delta \lambda \beta v(X) + X \lambda = 0 \quad \square$$

Theorem 5. :Let M be a non-invariant hypersurface with (f, g, u, v, λ) -structure of δ -Lorentzian trans-Sasakian manifold \tilde{M} . If U is parallel, then we have

$$f(A_{\hat{N}} X) + \delta \lambda \alpha X + \delta \lambda \beta f X = 0 \quad (3.18)$$

Proof. :Consider

$$(\tilde{\nabla}_X \phi) \hat{N} = \tilde{\nabla}_X (\phi \hat{N}) - \phi (\tilde{\nabla}_X \hat{N})$$

Using (2.9), (2.19) and (2.20) we get

$$(\tilde{\nabla}_X \phi) \hat{N} = -\nabla_X U + f(A_{\hat{N}} X) \quad (3.19)$$

and from (2.5) we write

$$(\tilde{\nabla}_X \phi) \hat{N} = \alpha \{g(X, \hat{N}) \xi - \delta \eta(\hat{N}) X\} + \beta \{g(\phi X, \hat{N}) \xi - \delta \eta(\hat{N}) \phi X\}$$

Using (2.10) in above equation we get,

$$(\tilde{\nabla}_X \phi) \hat{N} = -\delta\alpha\lambda X - \beta\delta\lambda\phi X \quad (3.20)$$

Using (3.19) and (3.20), we get

$$\begin{aligned} \nabla_X U &= \delta\alpha\lambda X + \beta\delta\lambda\phi X + f(A_{\hat{N}}X) \\ &= \delta\alpha\lambda X + f(A_{\hat{N}}X) + \beta\delta\lambda fX + \beta\delta\lambda u(X)\hat{N} \end{aligned}$$

If U is parallel, then $\nabla_X U = 0$

$$\delta\alpha\lambda X + f(A_{\hat{N}}X) + \beta\delta\lambda fX + \beta\delta\lambda u(X)\hat{N} = 0$$

Now, equating the tangential part, we have the result. \square

REFERENCES RÉFÉRENCES REFERENCIAS

1. Ahmed Yildiz, On Lorentzian α -Sasakian manifolds, Kyungpook Math.J. 45(2005); 95–103:
2. D. E. Blair and G. D. Ludden, Hypersurfaces in Almost Contact Manifold, Tohoku Math. J., 22 (1969) 354 – 362:
3. G. T. Srinivas, Venkatesh and C.S. Bagewadi, On Lorentzian β - Kenmotsu manifolds, General Mathematics, 18(4)(2010), 61 – 69:
4. K. Yano and M. Okumura, On (f, g, u, v, λ) - Structures, Kodai Math. Sem. Rep., 22 (1970) 401 – 423:
5. K. Yano and M. Okumura, Invariant Submanifolds with (f, g, u, v, λ) - Structures, Kodai Math. Sem. Rep., 24 (1972) 75 – 90:
6. K. Motsumoto, On Lorentzian para contact manifolds, Bull. of the Yamagata Uni, 12(2)(1989); 151 – 156:
7. Mine Turan A.Y. and Eftal Acet A, On three dimensional Lorentzian α - Sasakian manifolds, Bulletin of Mathematical Analysis and Applications, SSN 1821 – 1291(2009); 90 – 98:
8. M.M. Tripathi, Erole Kilic and S.Y.Perktas, Inde.nite almost metric manifolds, Int. J. of Math. and Mathematical Sciences, (2010) Article ID 846195; doi.10; 1155=846195:
9. R. Prasad, On Non-invariant Hypersurfaces of Trans-Sasakian Manifold, J. Inter. Acad. Phys. Sci.
10. S. I. Goldberg and K. Yano, Non-invariant Hypersurfaces of Almost Contact Manifolds, J. of Math. Soc. Japan, 22(1970) 25 – 35:
11. S. Kishor, S. P. Yadav, On Non-invariant Hypersurfaces of Nearly Trans-Sasakian, Manifold, J. of Mathematics, Hindawi, Math. Sci., Art. ID 657690(2014), 5 pp, dx.doi.org/10:1155=2014=657690.
12. S. S. Pujar and V. J. Khairnar, On Lorentzian trans- Sasakian manifold-I, Int.J. of Ultra Sciences of Physical Sciences, 23(1)(2011); 53 – 66:
13. S. S. Pujar, On δ -Lorentzian α -Sasakian manifolds, to appear in Antactica J. of Mathematics 8(2012):
14. S. S. Shukla and D. D. Singh, On δ -trans- sasakian manifolds, Int.J. of math analysis, 4(2010); 2401 – 2414:
15. S. Tanno, The automorphism groups of almost contact Riemannian manifolds, Tohoku Math. J. 21(1969); 21 – 38:
16. T. Takahashi, Sasakian manifolds with Pseudo -Riemannian metric, Tohoku Math.J. 21(1969); 271 – 290:

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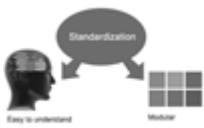
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<i>References</i>	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring



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