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Factor Analysis of Variance

Method of Direct Standardization

Highlights

Estimation of Adjusted Rate

Response of Transversely Isotropic

Discovering Thoughts, Inventing Future

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One Factor Analysis of Variance and Dummy Variable Regression Models

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Keywords: *dummy variable regression, analysis of variance, degrees of freedom, treatment, regression coefficient.*

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ONEFACTORANALYSISOFVARIANCEANDDUMMYVARIABLEREGRESSIONMODELS

Strictly as per the compliance and regulations of :

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Okeh UM [α] & Oyeka ICA [σ]

Abstract- This paper proposes and presents a method that would enable the use of dummy variable regression techniques for the analysis of sample data appropriate for analysis with the traditional one factor analysis of variance techniques with one, equal and unequal replications per treatment combination. The proposed method, applying the extra sum of squares principle develops F ratio-test statistics for testing the significance of factor effects in analysis of variance models. The method also shows how using the extra sum of squares principle builds more parsimonious explanatory models for dependent or criterion variables of interest. In addition, unlike the traditional approach with analysis of variance models, the proposed method easily enables the simultaneous estimation of total or absolute and the so-called direct and indirect effects of independent or explanatory variables on the dependent or criterion variables. The proposed methods are illustrated with some sample data and shown to yield essentially the same results as would the one factor analysis of variance techniques when the later methods are equally applicable.

Keywords: dummy variable regression, analysis of variance, degrees of freedom, treatment, regression coefficient.

I. Introduction

Analysis of variance and regression analysis whether single-factor or multi-factor, sometimes both in theory and applications have often been treated and presented as rather different concepts by various authors. In fact only limited attempts seem to have been made to present analysis of variance as a regression problem (Draper and Smith, 1966; Neter and Wasserman, 1974).

Nonetheless analysis of variance and regression analysis are actually similar concepts, especially when analysis of variance is presented from the perspective of dummy variable regression models. This is the focus of the present paper, which attempts to develop a method to use dummy variable regression models and apply the "extra sum of squares principle" in the analysis of one-factor analysis of variance models with unequal replications per treatment combination as a regression problem.

II. Proposed Method

Let $y_i = y_{ij}$ be the response, score, or observation for the ith subject in a random sample of size n_j randomly drawn from population Y_j or administered test or treatment T_j ,for i=1,2,…, n_j; $j = 1, 2, ..., k$ populations or treatments. It is for the moment assumed

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that population Y_j or treatment T_j are each continuous measurement on the ratio scale.

Then the usual one-way analysis of variance model reflecting the dependence of subjects' responses or scores as a function of the different effect of treatments may be expressed in the form

$$y_{ij} = \mu + \alpha_j + e_{ij} \tag{1}$$

Notes

Where μ is the grand or overall mean, α_j is the differential effect of treatment T_j and e_{ij} are independent error terms with $E(e_{ij}) = 0,\ for\ i = 1, 2, ..., n_j; and\ j = 1, 2, ..., k.$ The treatment differential effect α_j are also subject to the constraint

$$\sum_{j=1}^{k} \alpha_j = 0 \tag{2}$$

The expected value of y_{ij} of equation 1 which is also the mean value or mean score specific to subject, administered test or treatment T_j is

$$E(y_{ij}) = \mu_j = \mu + \alpha_j \tag{3}$$

For j=1,2,…,k.

The mean effect μ_j and the treatment effect α_j are estimated using the usual one-way analysis of variance techniques. The statistical significance of the differential effects α_j is also determined using the usual F-ratio with k-1 and n-k degrees of freedom where $n = \sum_{j=1}^{k} n_j$.

We here however obtain alternative methods for estimating these effects and determine their statistical significance using dummy variable regression techniques. To do this we will k-1 dummy variables of 1's 0's to represent the k level of treatments or populations. We have here used k-1 dummy variables of 1s and 0s to represent the k treatments in order to adjust for the constraints imposed on the α_{js} by equation (2) and to ensure that the variance-covariance matrices $X'X$ resulting from the designed metrics X for the regression model is of full column rank and hence non-singular. To obtain the design metrics X for the dummy variable regression model we may let

$$x_{ij} = \begin{cases} 1, if\ \ the\ response\ or\ score\ y_i = y_{ij}\ by\ the\ ith\ subject \\ is\ reported\ or\ observed\ \ for\ test\ or\ treatment\ T_j \\ o, otherwise \end{cases} \tag{4}$$

$$for\ i = 1, 2, ..., n_j;\ j = 1, 2, ..., k-1.$$

Then a dummy variable regression model expressing the dependent of the responses or scores y_i of all the $n = \sum_{j=1}^{k} n_j$ subjects on the k-1 dummy variables of 1s and 0s representing k treatments may be expressed as

Notes

$$y_i = \beta_0 + \sum_{j=1}^{k-1} \beta_j . x_{ij} + e_i \tag{5}$$

Where y_{is} are response scores by subjects, β_{js} are partial regression coefficients, $x_{ij}s$ are dummy variables of 1s and 0s,and ei's are error terms uncorrelated with x_{ij}, with $E(e_i) = 0,\ for\ i = 1, 2, ..., n = \sum_{j=1}^{k} n_j$. Now the expected value or mean value of y_i from equation (5).

$$E(y_i) = \beta_0 + \sum_{j=1}^{k-1} \beta_j x_{ij} . \tag{6}$$

In particular, note that the expected value or mean response or mean score by subjects at treatment T_j is obtained from equation (6) by setting $x_{ij} = 1$ and $x_{il} = 0,\ for\ all\ l \neq j;\ j = 1, 2, ..., k-1$ yielding

$$E(y_i)u_j = \beta_0 + \beta_j \tag{7}$$

Note from equations (3) and (7) that since $\beta_0 = \mu$ we will have that

$$\beta_j = \alpha_j \tag{8}$$

$$for\ \ j = 1, 2, .., k-1.$$

Equation can equivalently be expressed in its matrics form as

$$\underline{y} = X . \underline{\beta} + \underline{e} \tag{9}$$

Where $\underline{y}$ is an nx1 column vector of response scores, X is an nxk design metrics of 1s and 0s; $\underline{\beta}$ is a kx1 column vector of partial regression coefficient; and $\underline{e}$ is an nx1 column vector of error terms uncorrelated with X, $E(\underline{e}) = 0$.

Application of the usual least squares techniques to either equations (5) or (9) yields unbiased estimates of the partial regression coefficients, β as

$$\underline{\hat{\beta}} = \underline{b} = (X'X)^{-1} X'\underline{y} \tag{10}$$

Where $(X'X)^{-1}$ is the non-singular matrices inverse of the variance-covariance matrix $X'X$. This result will yield the fitted dummy variable regression model

$$\underline{\hat{y}} = X . \underline{b} \tag{11}$$

A null hypothesis that is usually of interest is that the regression models of either equations (5) or (9) kicks that is that not all the partial regression coefficients are zero. In other words the null hypothesis

$$H_0 : \underline{\beta} = \underline{0} \text{ versus } H_1 : \underline{\beta} \neq \underline{0} \quad (12)$$

This null hypothesis is tested using the usual F-ratio with k-1 and n-k degrees of freedom presented in the form of an analysis or variance table (Table 1)

Table 1 : Analysis of Variance Table for full Regression Model of equation (9)

Source of Variation	Sum of squares	Degree of freedom	Mean sum of squares	F-ratio
Regression	$SSR = \underline{b}'X'\underline{y} - n.\bar{y}^2$	k-1	$MSR = \frac{SSR}{k-1}$	$F = \frac{MSR}{MSE}$
Error	$SSE = \underline{y}'.\underline{y} - \underline{b}'.X'.\underline{y}$	n-k	$MSE = \frac{SSE}{n-k}$	
Total	$SST = \underline{y}'.\underline{y} - n.\bar{y}^2$	n-1		

If the null hypothesis fits, that is if not all the regression coefficient in $\underline{\beta}$ are equal to zero, then one may proceed to test further null hypothesis involving the β_{js} in a regression model, or equivalently α_{js} in the corresponding one-way analysis of variance order. In other words, one may proceed to test other null hypothesis concerning the $\beta_{js};$ *for some* $j = 1, 2, ..., k-1.$

These tests are usually conducted using the traditional students' t test statistic with n-k degrees of freedom. We will here however propose and present an alternative and perhaps more generalized method for the same purpose based on the so called extra sum of squares principle (Drapa and Smith,1996;Netter and Wassemann,1974;Oyeka and Okeh, 2014;Boyle,1974).

Now suppose the null hypothesis H_0 of equation 12 is rejected based on the full model of equation 9;that is suppose not all the β_{js} *in* $\underline{\beta}$ are equal to zero, then the regression coefficient $\underline{\beta}$ and hence its estimated value $\hat{\underline{\beta}}$ and the corresponding design matrix X can be partitioned into at most k-1 mutually exclusive portions or groups. However, for the present presentation and the application of the extra sum of squares principle, we will here partition the design matrix X into two mutually exclusive groups or sub-matrices, namely group A with sub-matrix X_A with 'a' dummy variables of 1s and 0s and group B with sub-matrix X_B with 'b' dummy variables of 1s and 0s such that $a = (k-1)$, *where* $a, b \geq 1$. The treatments in groups A and B may for some reasons be similar within but dissimilar between themselves. The regression vector $\underline{\beta}$ and its sample estimate $\underline{b}$ are similarly partitioned into $\underline{b}_A$ *and* $\underline{b}_B$ with a and b rows respectively.

In this situation the treatment sum of squares SST in analysis parlance which is also the regression sum of squares in regression models can similarly be partitioned as

$$SST = SSR = \underline{b}'.X'.\underline{y} - n.\bar{y}^2 = (X.b)'.\underline{y} - n.\bar{y}^2$$

Notes

or equivalently as

$$SSR = \left(X_A - X_A \begin{pmatrix} \hat{b}_A \\ \hat{b}_B \end{pmatrix} \right)' y - n\,\bar{y}^2 = \left(\underline{b}'_A\, X'_{A.\underline{y}} + \underline{b}'_B\, X'_{B.\underline{y}} \right) - n\,\bar{y}^2 \tag{13}$$

or

equivalently

$$SSR = \underline{b}'\, X'\, \underline{y} - n\,\bar{y}^2 = \left(\underline{b}'_A\, X'_{A.\underline{y}} - n\,\bar{y}^2 \right) + \left(\underline{b}'_B\, X'_{B.\underline{y}} - n\,\bar{y}^2 \right) + n\,\bar{y}^2 \tag{14}$$

Which when interpreted is the same as the statement

$$SST = SSR = SSA + SSB + SS(\bar{y} = \hat{\mu}) \tag{15}$$

Where SSR is the sum of square regression for the full model of equation 9 with design matrix X and with k-1 degrees of freedom; SSA is the sum of squares regression for group A for the reduced model with design matrix X_A with 'a' degrees of freedom; SSB is the sum of squares regression for the reduced model for group B with design matrix X_B with degrees of freedom b=(k-1)-a; and $SS(\bar{y} = \hat{\mu})$ is an additive correction factor due to mean effect.

These sums of squares namely SSR,SSA and SSB are obtained by separately fitting the full model of equation 9 with design matrix X, and the reduced regression model with design matrices X_A and X_B respectively again separately on the criterion or dependent variable $\underline{y}$.

Now if the full model of equation 9 fits, that is if the null hypothesis H_0 of equation 12 is rejected, then an additional null hypothesis such as the null hypothesis that treatment in group A on the average have different effects on subjects and also that treatments in group B on the average have different effects on subjects may be tested. These null hypothesis may be tested using the extra sum of squares principle (Drapa and Smith,1966;Netter and Wasserman,1974;Oyeka et al,2013).

Now if we denote the sums of squares due to the full model of equation 9 and the reduced models due to the fitting of the criterion variable $\underline{y}$ to any of the reduced design matrices X_A and X_B by SS (F) and SS(R) respectively, then following the extra sum of squares principle the extra sum of squares due to a given factor is calculated as

$$ESS = SS(F) - SS(R) \tag{16}$$

With degrees of freedom obtained as the difference between the degrees of freedom of SS(F) and SS(R) ,that is as

$$df(ESS) = df(F) - df(R) \tag{17}$$

Thus the extra sums of squares regression due to group A and group B are obtained as respectively.

$$ESSA = SSR - SSA; ESSB = SSR - SSB \tag{18}$$

Notes

With (k-1)-a=b degrees of freedom and (k-1)-b=a degrees of freedom respectively.

The corresponding extra sum of squares error for factors A and B are obtained from equation 16 using

$$ESSE = SSE(F) - SSE(R) \tag{19}$$

That is as

$$ESSEA = SSE - SSEA; ESSEB = SSE - SSEB \tag{20}$$

Notes

The corresponding degrees of freedom are obtained using equation 17 as

$$\left.\begin{array}{l} df\,(ESSEA) = \big((n-1)-a\big)-(n-k) = k-1-a = b; \\ df\,(ESSEB) = \big((n-1)-b\big)-(n-k) = k-1-b = a \end{array}\right\} \tag{21}$$

Table 2 : Analysis of variance table for multiple comparisons in dummy variable Regression models

Sources of variation	Sum of squares(SS)	df	MSE	F-ratio	ESS	df	EMSS	F-ratio
Full model								
Regression	$SSR = \underline{b}'X'.\underline{y} - n.\bar{y}^2$	$k-1$	$MSR = \frac{SSR}{k-1}$	$F = \frac{MSR}{MSE}$	ESSR = SSR	k-1		
Error	$SSE = \underline{y}'.\underline{y} - \underline{b}'X'.\underline{y}$	$n-k$	$MSE = \frac{SSR}{n-k}$		ESSE = SSE	n-k		
Reduced model								
Group A Regression	$SSA = \underline{b}'_A X'_A.\underline{y} - n.\bar{y}^2$	a	$MSR = \frac{SSA}{a}$	$F_A = \frac{MSA}{MSEA}$	ESSA = SSR-SSA	k-1-a=b		
Error	$SSEA = \underline{y}'.\underline{y} - \underline{b}'_A X'_A.\underline{y}$	$n-1-a$	$MSEA = \frac{SSEA}{n-1-a}$		ESSEA=SSEA-SSE=ESSA	k-n-a=b		
Group B Regression	$SSB = \underline{b}'_B X'_B.\underline{y} - n.\bar{y}^2$	b	$MSB = \frac{SSB}{b}$	$F_B = \frac{MSB}{MSEB}$	ESSB = SSR-SSB	k-1-b=a		
Error	$SSEB = \underline{y}'.\underline{y} - \underline{b}'_B X'_B.\underline{y}$	$n-1-b$	$MSEB = \frac{SSR}{n-1-b}$		ESSEB=SSEB-SSE=ESSB	n-1-b		
Total	$SST = \underline{y}'.\underline{y} - n.\bar{y}^2$	$n-1$						

The null hypothesis H_0 of equation 12 is tested using the usual F-ratio for the full model with k-1 and n-k degrees of freedom. If this null hypothesis is rejected, then one may apply the extra sum of squares principle to obtain the results of Table 2.The null hypothesis that treatments in group A have different effects on subject may be tested using the $F_A^* - ratio$ with b and n-k degrees of freedom while the null hypothesis that treatments in group B have different effects on subjects may be tested using the $F_B^* - ratio$ with a and n-k degrees of freedom.

These null hypothesis are each rejected at the α level of significance if

$$F_A^* \geq F_{1-\alpha;b,n-k;} F_B^* \geq F_{1-\alpha;a,n-k} \tag{22}$$

Otherwise the null hypothesis is accepted.

Notes

Note that if the null hypothesis to be tested is that the regression effect β_j in regression models or α_j in analysis of variance parlance is not different from zero, for some j=1,2,….,k-1,then design matrix X may be partitioned in such a way that X_A, say has only dummy variable of 1s and 0s so that only dummy variable of 1s and 0s so that a=1,and X_β has b=k-1-a=k-2,dummy variable of 1s and 0s.In this case the null hypothesis that a given regression coefficient is equal to zero may be tested and rejected if the F_A^* of equation 22 is satisfied with a-1 and b=k-2.Similarly,to test the null hypothesis

$$H_0 : \beta_j - \beta_l = 0 \; versus \; H_0 : \beta_j - \beta_l \neq 0 \quad (23)$$

$$for\ some\ j, l = 1, 2, ..., k-1;\ j \neq l$$

We may partition the design matrix X into X_A,say and X_B an nx2 and (k-3) design matrices of 1s and 0s respectively.

The null hypothesis H_0 of equation 23 is rejected at the α level of significance if the calculated $F^* - ratios$ satisfy the equation

$$F_A^* \geq F_{1-\alpha;k-3,n-k;} F_B^* \geq F_{1-\alpha;2,n-k} \quad (24)$$

Otherwise the null hypothesis is accepted.

An additional advantage of using dummy variable regression models in either two-way or one-way analysis of variance problems is that the method also more easily enables the simultaneous estimation of factor level or treatment effects separately of several factors on a specified dependent or criterion variable through the effects of their representative dummy variables of 1s and 0s. Specifically,the method enables the estimation of the direct effects of a given factor or independent variable on a dependent or criterion. This is a waited sum of the partial regression coefficients or effects of the set of dummy variables of 1s and 0s representing that independent variable(Wright,1973).That is the direct effect of an independent variable, referred to here as population or treatment $X_i\ or\ T$ on a dependent or criterion variable Y is the weighted sum of the partial regression coefficients β_j or effects of the set of dummy variables x_{ij} of 1s and 0s representing that independent variable. The weights θ_j are the simple regression coefficients of each representative dummy variable x_{ij} regressing on the specified independent variable or treatment X_j represented by numerical codes.

Thus suppose we here without loss of generality represent populations or treatments X_j with the numerical code $c+j$; for j=1,2,…,k-1, where c is any real number then a simple regression equation expressing the dependence of the dummy variable x_{ij} of 1s and 0s on its parent variable X_j may be written as

$$x_{ij} = \theta_0 + \theta_j X_j + e_i \quad (25)$$

Where θ_j a simple regression coefficient and ei is are error terms uncorrelated with X_j with $E(e_i) = 0,\ for\ j = 1, 2, ..., k-1; l = 1, 2, ..., n.$

Now taking the partial derivative of the expected value of equation (25) with respect to X_j, we have

$$\frac{dE(x_{ij})}{dX_j} = \theta_j \qquad (26)$$

Now taking the partial derivative of equation (6),the expected value of y_i with respect to X_j,and substituting equation (26) we obtain

$$\frac{dE(y_i)}{dX_{ij}} = \sum_{j=1}^{k-1} \beta_j . \frac{dE(x_{ij})}{dX_j} = \sum_{j=1}^{k-1} \theta_j . \beta_j = \beta_{dir} \qquad (27)$$

Where β_{dir} is the so-called direct effect of the treatment X and T on the dependent or criterion variable Y when these treatments are represented by set of dummy variable of 1s and 0s.

The corresponding sample estimate of β_{dir} is obtained in terms of the sample estimates of β_j namely b_j as

$$\hat{\beta}_{dir} = b_{dir} = \sum_{j=1}^{k-1} \theta_j b_j \qquad (28)$$

III. Illustrative Example

In a study of the effect of granulated starch on the disintegration time of tablets a random sample of four local sources of starch was chosen and five measurements of the disintegration time of a tablet were made for each source yielding the following results (in seconds) (Table 3)

Table 3 : Disintegration time (in seconds) of starch Tablets by source of starch

Maize	Cassava	Yam	Cocoyam
29	35	100	116
120	145	120	180
114	122	245	90
75	70	240	310
	55	180	
	75	246	

To use the proposed method to analyze the data, we would first use three dummy variables of 1s and 0s to represent the four different sources of starch by applying equation 4 to the data of Table 3 to obtain the design matrix X of Table 4.

Table 4 : Design Matrix X for the sample Data of table 3

S/N	y_i	X_{i0}	X_{i1}	X_{i2}	X_{i3}
1	29	1	1	0	0
2	120	1	1	0	0
3	114	1	1	0	0
4	75	1	1	0	0
5	35	1	0	1	0
6	145	1	0	1	0
7	122	1	0	1	0
8	70	1	0	1	0
9	55	1	0	1	0
10	75	1	0	1	0
11	100	1	0	0	1

Notes

12	120	1	0	0	1
13	245	1	0	0	1
14	240	1	0	0	1
15	180	1	0	0	1
16	246	1	0	0	1
17	116	1	0	0	0
18	180	1	0	0	0
19	90	1	0	0	0
20	310	1	0	0	0

Notes

Fitting the full model of Eqn 9 using the design matrix X of table 4, we obtain the fitted regression equation

$$\hat{y}_i = 33.322 + 20.242x_{i1} - 11.525x_{i2} + 8.122x_{i3}\,(Pvalue = 0.0000) \quad (29)$$

A P-value of 0.0000 clearly shows that the model fits.

The expected result of measurements of the disintegration time of a tablet for the first treatment is obtained by setting $x_{i1} = 1$, and all other $x_{is} = 0$ in equation (29) giving

$$\hat{y}_i = 33.322 + 20.242 = 53.564$$

The estimated response measurement result of the disintegration time of a tablet for the second treatment is estimated by setting

$x_{i2} = 1$ and all other $x_{i's} = 0$ in Equation (29) yielding

$\hat{y}_i = 33.322 - 11.525 = 21.797$

The estimated response measurement result of the disintegration time of a tablet for the third treatment is similarly estimated by setting

$x_{i3} = 1$ and all other $x_{i's} = 0$ in Equation (29) yielding

$\hat{y}_i = 33.322 + 8.122 = 41.444$

The corresponding analysis of variance table for the full model is presented in Table 5.

Table 5 : Anova Table for the Full Model of Equation (29)

Source of Variation	Sum of Squares (SS)	Degrees of freedom (Df)	Mean Sum of Squares (MS)	F-Ratio	P-Value
Regression (treatment)	3648.412	3	1216.137	9.859	0.0000
Error	1973.529	16	123.346		
Total	5621.941	19			

Having fitted the full model which is here seen to fit, we now proceed to fit the dependent variably y separately on each of the sub matrices x_{i1}, x_{i2} *and* x_{i3} each with two dummy variables of Is and 0s to obtain the corresponding sum of squares due to each of these factors. The sums of squares due to factor A and B are calculated following Equation (13). The results are summarized in a one factor analysis of variance Table with extra sums of squares (Table 6).

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Table 6 : One factor Analysis of Variance Table with Extra Sums of Squares for the Sample Data of Table 3

Source of Variation	Sum of Squares (SS)	Degrees of freedom (Df)	Mean of sum of squares MS	F-Ratio	Extra Sum of Squares (ESS)	Degrees of freedom (Df)	Extra mean sum of squares (EMS)	F- Ratio	Critical F value P- value
Full Model									
Regression	3648.412	3	1216.137	9.859	3648.412	3	1216.137	9.859	2.030*
Error	1973.529	16	123.346		1973.529	16	123.346		
Group A									
Regression	1834.132	4	458.533	2.609	1814.28	6	302.38	6.852	5.86*
Error	2635.451	15	175.697		661.922	15	44.128		
Group B									
Regression	952.112	6	158.69	0.704	2696.3	4	674.075	9.148	4.00*
Error	2931.421	13	225.49		957.892	13	73.684		
Total									

*Note: * indicates statistical significance at the 5 percent level*

These analyses indicate that the hypothesized model fits, that is that not all the factor level effects are zero. However group A and B have significant effects on the criterion variable Y.

Finally to estimate the direct effect or partial regression coefficient of group A and group B say, represented by the dummy variables x_{i1}, x_{i2} and x_{i3}, we first estimate the simple regression coefficient resulting when theses dummy variables are each regressed on A using Equation 26, yielding.

$$\alpha_{i1} = 2.13; \alpha_{i2} = 0.34; \alpha_{i3} = -0.23$$

Using these results with Equation (29) in (28), we obtain an estimate of the direct effect of A and B on 'y' as

$$b_{A\,and\,B}dir = (20.242\times 2.13) + (-11.525\times 0.34) + (8.122\times -0.23) = 37.3289$$

The estimated simple regression coefficient or effect of A and B on y is $b_{A\text{ and }B}$ =37.3289.

Hence the estimated indirect effect of A on'y' is

$$b_{A\,and\,B}\,ind = 37.3289 - 24.832 = 12.4969$$

IV. Summary and Conclusion

We have in this paper proposed and developed a method that enabled the use of dummy variable multiple regression techniques for the analysis of data appropriate for use with two factor analysis of variance models with unequal observations per treatment combination and with interactions. The proposed model and method employed the extra sum of squares principle to develop appropriate test statistics of F ratios to test for the significance of factor and interaction effects.

The method which was illustrated with some sample data was shown to yield essentially the same results as would the traditional two factor analysis of variance model with unequal observations per cell and interaction. However the proposed method is more generalized in its use than the traditional method since it can easily be

Notes

used in the analysis of two-factor models with one observation, equal, and unequal observations per cell as a rather unified analysis of variance problem.

Furthermore unlike the traditional analysis of variance models the proposed method is able to enable one using the extra sum of squares principle, to determine the relative contributions of independent variables or some combinations of these variables in explaining variations in a given dependent variable and hence build a more parsimonious explanatory model for any variable of interest. In addition, the method enables the simultaneous estimation of the total or absolute, direct and indirect effects of a given independent variable on a dependent variable, which provide additional useful information.

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Extended Exp($-\varphi(\xi)$)-Expansion Method for Solving the Generalized Hirota-Satsuma Coupled KdV System

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Abstract- In this research, The exact traveling wave solutions of the generalized Hirota-Satsuma couple KdV system is obtained as the first time in the framework of the extended exp(□'(_))-expansion method. When these parameters are taken special values, the solitary wave solutions are derived from the exact traveling wave solutions. It is shown that the extended exp(□'(_))-expansion method give a wide range of solutions and it provides an effective and a more powerful mathematical tool for solving nonlinear evolution equations in mathematical physics. Comparison between our results and the well-known results will be presented.

Keywords: the generalized hirota-satsuma coupled KdV system, the extended exp(□(_))-expansion method, traveling wave solutions, solitary wave solutions, kink and anti kink soliton solutions.

GJSFR-F Classification : FOR Code: MSC 2010: 35Q20 - 35K99 - 35P05.

EXTENDEDEXPEXPANSIONMETHODFORSOLVINGTHEGENERALIZEDHIROTASATSUMACOUPLEDKDVSYSTEM

Strictly as per the compliance and regulations of :

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Abstract- In this research, The exact traveling wave solutions of the generalized Hirota-Satsuma couple KdV system is obtained as the first time in the framework of the extended exp($-\varphi(\xi)$)-expansion method. When these parameters are taken special values, the solitary wave solutions are derived from the exact traveling wave solutions. It is shown that the extended exp($-\varphi(\xi)$)-expansion method give a wide range of solutions and it provides an effective and a more powerful mathematical tool for solving nonlinear evolution equations in mathematical physics. Comparison between our results and the well-known results will be presented.

Keywords: the generalized hirota-satsuma coupled KdV system, the extended exp($-\varphi(\xi)$-expansion method, traveling wave solutions, solitary wave solutions, kink and anti kink soliton solutions.

I. Introduction

No one can deny the important role which played by the nonlinear partial differential equations in the description of many and a wide variety of phenomena not only in physical phenomena, but also in plasma, fluid mechanics, optical fibers, solid state physics, chemical kinetics and geochemistry phenomena. So that, during the past five decades, a lot of method was discovered by a diverse group of scientists to solve the nonlinear partial differential equations. For examples tanh - sech method [12],[16] and [18], extended tanh - method [13], [6] and [20], sine - cosine method [19], [17] and [22], homogeneous balance method [4], the exp($-\varphi(\xi)$)-expansion Method [11], Jacobi elliptic function method [3], [5], [14] and [24], F-expansion method [2], [21] and [9], exp-function method [8] and [7], trigonometric function series method [32], $(\frac{G'}{G})-$ expansion method [10], [15], [29] and [26], the modified simple equation method [1], [27], [30], [28], [31] and [25] and so on.

The objective of this article is to apply the extended exp($-\varphi(\xi)$)-expansion method for finding the exact traveling wave solution of the generalized Hirota-Satsuma coupled KdV system [23], which play an important role in mathematical physics.

The rest of this paper is organized as follows: In section 2, we give the description of the e exp($-\varphi(\xi)$)-expansion method. In section 3, we use this method to find the exact solutions of the nonlinear evolution equations pointed out above. In section 4, conclusions are given.

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II. Description of Method

Consider the following nonlinear evolution equation

$$F(u, u_t, u_x, u_{tt}, u_{xx}, ...) = 0, \tag{2.1}$$

where F is a polynomial in $u(x,t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following,we give the main steps of this method
Step 1. We use the wave transformation

$$u(x,t) = u(\xi), \qquad \xi = x - ct, \tag{2.2}$$

where c is a positive constant, to reduce Eq.(2.1)to the following ODE:

$$P(u, u', u'', u''', ...) = 0, \tag{2.3}$$

where P is a polynomial in $u(\xi)$ and its total derivatives,while $u' = \frac{du}{d\xi}$.
Step 2. Suppose that the solution of ODE (2.3) can be expressed by a polynomial in $exp(-\varphi(\xi))$ as follows

$$u(\xi) = \sum_{i=-m}^{m} a_i \left(exp\left(-\varphi\left(\xi\right)\right)\right)^i, \tag{2.4}$$

Since a_m $(0 \leq m \leq n)$ are constants to be determined, such that $(a_m \, or \, a_{-m}) \neq 0$.
the positive integer m can be determined by considering the homogenous balance between the highest order derivatives and nonlinear terms appearing in Eq.(2.3). Moreover precisely, we define the degree of $u(\xi)$ as $D(u(\xi)) = m$, which gives rise to degree of other expression as follows:

$$D\left(\frac{d^q u}{d\xi^q}\right) = n + q, \; D\left(u^p \left(\frac{d^q u}{d\xi^q}\right)^s\right) = np + s(n+q).$$

Therefore, we can find the value of m in Eq.(2.3), where $\varphi = \varphi(\xi)$ satisfies the ODE in the form

$$\varphi'(\xi) = exp\left(-\varphi\left(\xi\right)\right) + \mu exp\left(\varphi\left(\xi\right)\right) + \lambda, \tag{2.5}$$

the solutions of ODE (2.3) are
when $\lambda^2 - 4\mu > 0, \mu \neq 0$,

$$\varphi(\xi) = ln\left(\frac{-\sqrt{\lambda^2 - 4\mu}\; tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\xi + C_1\right)\right) - \lambda}{2\mu}\right), \tag{2.6}$$

and

$$\varphi(\xi) = ln\left(\frac{-\sqrt{\lambda^2 - 4\mu}\; coth\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\xi + C_1\right)\right) - \lambda}{2\mu}\right), \tag{2.7}$$

when $\lambda^2 - 4\mu > 0, \mu = 0$,

$$\varphi(\xi) = -ln\left(\frac{\lambda}{exp\left(\lambda\left(\xi + C_1\right)\right) - 1}\right), \tag{2.8}$$

when $\lambda^2 - 4\mu = 0, \mu \neq 0, \lambda \neq 0$,

$$\varphi(\xi) = ln\left(-\frac{2\left(\lambda\left(\xi + C_1\right) + 2\right)}{\lambda^2\left(\xi + C_1\right)}\right), \tag{2.9}$$

Notes

when $\lambda^2 - 4\mu = 0, \mu = 0, \lambda = 0,$

$$\varphi(\xi) = ln\left(\xi + C_1\right), \tag{2.10}$$

when $\lambda^2 - 4\mu < 0,$

$$\varphi(\xi) = ln\left(\frac{\sqrt{4\mu - \lambda^2}\, tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\ (\xi + C_1)\right) - \lambda}{2\mu}\right), \tag{2.11}$$

and

$$\varphi(\xi) = ln\left(\frac{\sqrt{4\mu - \lambda^2}\, cot\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\ (\xi + C_1)\right) - \lambda}{2\mu}\right), \tag{2.12}$$

where $a_m, \ldots\ldots, \lambda, \mu$ are constants to be determined later,
Step 3. After we determine the index parameter m, we substitute Eq.(2.4) along Eq.(2.5) into Eq.(2.3) and collecting all the terms of the same power $exp\left(-m\varphi(\xi)\right)$, $m = 0, 1, 2, 3, \ldots.$ and equating them to zero, we obtain a system of algebraic equations, which can be solved by Maple or Mathematica to get the values of a_i.
Step 4. substituting these values and the solutions of Eq.(2.5) into Eq.(2.3) we obtain the exact solutions of Eq.(2.3).

III. Application

Here, we will apply the extended exp($-\varphi(\xi)$)-expansion method described in Sec.2 to find the exact traveling wave solutions and the solitary wave solutions of the generalized Hirota-Satsuma coupled KdV system[23]. We consider the generalized Hirota-Satsuma couple KdV system

$$\begin{cases} u_t = \frac{1}{4}u_{xxx} + 3uu_x + 3\left(-v^2 + w\right)_x, \\ v_t = -\frac{1}{2}v_{xxx} - 3uv_x, \\ w_t = -\frac{1}{2}w_{xxx} - 3uw_x. \end{cases} \tag{3.1}$$

When $w = 0$, Eq.(3.1) reduce to be the well known Hirota-Satsuma couple KdV equation. Using the wave transformation $u(x,t) = u(\xi)$, $v(x,t) = v(\xi)$, $w(x,t) = w(\xi)$, $\xi = k(x - \lambda_1 t)$ carries the partial differential equation (3.1) into the ordinary differential equation

$$\begin{cases} -\lambda_1\, k\, u' = \frac{1}{4}k^3u''' + 3\, k\, u\, u' \, + \, 3\, k\, \left(-v^2 + w\right)', \\ -\lambda_1\, k\, v' \, = -\frac{1}{2}k^3v''' - 3\, k\, u\, v', \\ -\lambda_1\, k\, w' = -\frac{1}{2}k^3w''' - 3\, k\, u\, w'. \end{cases} \tag{3.2}$$

Suppose we have the relations between $(u\, and\, v)$ and $(w\, and\, v) \Rightarrow \left(u = \alpha v^2 + \beta v + \gamma\right)$ and $(w = Av + B)$ where α, β, γ, A and B are arbitrary constants. Substituting this relations into second and third equations of Eq.(3.2) and integrating them , we get the same equation and integrate it once again we obtain

$$k^2v'^2 = -2\alpha v^4 - 2\beta v^3 + 2\left(\lambda_1 - 3\gamma\right)v^2 + 2c_1v + c_2, \tag{3.3}$$

where c_1 and c_2 is the arbitrary constants of integration, and hence, we obtain

$$\begin{aligned} k^2u'' &= 2\alpha k^2v'^2 + k^2\left(2\alpha v + \beta\right)v'' \\ &= 2\alpha\left[-\alpha v^4 - 2\beta v^3 + 2\left(\lambda_1 - 3\gamma\right)v^2 + 2c_1v + c_2\right] \\ &+ \left(2\alpha v + \beta\right)\left[-2\alpha v^3 - 3\beta v^2 + 2\left(\lambda_1 - 3\gamma\right)v + c_1\right]. \end{aligned} \tag{3.4}$$

Ref

23. Yan, Z. The extended jacobian elliptic function expansion method and its application in the generalized hirota Ü̈satsuma coupled kdv system. *Chaos, Solitons & Fractals 3*, 15(2003), 575–583.

So that, we have

$$P'' + lP - mP^3 = 0. \tag{3.5}$$

Where

$$c_1 = \frac{1}{2\alpha^2\left(\beta^2 + 2\lambda_1\alpha\beta - 6\alpha\beta\gamma\right)}, \quad v(\xi) = aP(\xi) - \frac{\beta}{2\alpha}, \quad \alpha = \frac{\beta^2 - 4}{4\left(\gamma - \lambda_1\right)}, \quad A = \frac{4\beta\left(\lambda_1 - \gamma\right)}{\beta^2 - 4},$$

$$B = \frac{1}{6\left(-\gamma + \lambda_1\right)\left(\beta^2 - 4\right)^2}(16c_3\lambda_1\beta^2 - 2c_3\lambda_1\beta^4 - 16c_3\gamma\beta^2 + 3c_3\gamma\beta^4 + 56\lambda_1^2\gamma\beta^2$$

$$- 48\gamma^2\lambda_1\beta^2 - 16c_2 + c_2\beta^6 - 12c_2\beta^4 + 12c_2\beta^2 - 16\gamma^2\lambda_1 - 32\lambda_1^2\gamma - 8\lambda_1^3\beta^2 + \beta^4\gamma^3$$

$$- 2\beta^4\lambda_1^3 + 32c_3\gamma - 32c_3\lambda_1 + 48\gamma^3 + \beta^4\gamma^2\lambda_1),$$

$$l = \frac{-a}{k^2}\left(\frac{3\beta^2}{2\alpha} + 2\lambda_1 - 6\gamma\right), \quad m = \frac{-2\alpha a^3}{k^2}.$$

Balancing between the highest order derivatives and nonlinear terms appearing in P'' and P^3 $\Rightarrow (N + 2 = 3N) \Rightarrow (N = 1)$. So that, by using Eq.(2.4) we get the formal solution of Eq.(3.5)

$$P(\xi) = a_{-1}exp(\varphi(\xi)) + a_0 + a_1exp(-\varphi(\xi)). \tag{3.6}$$

Substituting Eq.(3.6) and its derivative into Eq.(3.5) and collecting all term with the same power of $[exp(-3\varphi(\xi)), exp(-2\varphi(\xi)), ..., exp(+3\varphi(\xi))]$ we obtained:

$$2\,a_1 + m{a_1}^3 = 0, \tag{3.7}$$

$$3\,\lambda\,a_1 + 3\,ma_0{a_1}^2 = 0, \tag{3.8}$$

$$2\,\mu\,a_1 + \lambda^2a_1 + la_1 + 3\,ma_{-1}{a_1}^2 + 3\,m{a_0}^2a_1 = 0, \tag{3.9}$$

$$\lambda\,a_{-1} + \mu\,\lambda\,a_1 + la_0 + 6\,ma_{-1}a_0a_1 + m{a_0}^3 = 0, \tag{3.10}$$

$$2\,\mu\,a_{-1} + \lambda^2a_{-1} + la_{-1} + 3\,m{a_{-1}}^2a_1 + 3\,ma_{-1}{a_0}^2 = 0, \tag{3.11}$$

$$3\,\mu\,\lambda\,a_{-1} + 3\,m{a_{-1}}^2a_0 = 0, \tag{3.12}$$

$$2\,\mu^2a_{-1} + m{a_{-1}}^3 = 0. \tag{3.13}$$

Solving above system by using maple 16, we get:

Case 1.

$$l = 4\,\mu,\; m = \frac{-2}{{a_1}^{-2}},\; \lambda = 0,\; a_{-1} = \mu\,a_1,\; a_0 = 0,\; a_1 = a_1.$$

Case 2.

$$l = -8\,\mu,\; m = \frac{-2}{{a_1}^{-2}},\; \lambda = 0,\; a_{-1} = -\mu\,a_1,\; a_0 = 0,\; a_1 = a_1.$$

Thus the solution is

For Case 1.

$$p(\xi) = \mu\,a_1exp(\varphi(\xi)) + a_1exp(-\varphi(\xi)). \tag{3.14}$$

For Case 2.

$$p(\xi) = -\mu\,a_1exp(\varphi(\xi)) + a_1exp(-\varphi(\xi)). \tag{3.15}$$

Notes

Let us now discuss the following cases:
For Case 1. When $\lambda^2 - 4\mu > 0, \mu \neq 0$,

$$P(\xi) = \mu\, a_1 \left(\frac{-\sqrt{\lambda^2 - 4\mu}\, tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi + C_1)\right) - \lambda}{2\mu} \right) + a_1 \left(\frac{2\mu}{-\sqrt{\lambda^2 - 4\mu}\, tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi + C_1)\right) - \lambda} \right). \tag{3.16}$$

and

$$P(\xi) = \mu\, a_1 \left(\frac{-\sqrt{\lambda^2 - 4\mu}\, coth\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi + C_1)\right) - \lambda}{2\mu} \right) + a_1 \left(\frac{2\mu}{-\sqrt{\lambda^2 - 4\mu}\, coth\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi + C_1)\right) - \lambda} \right). \tag{3.17}$$

When $\lambda^2 - 4\mu > 0, \mu = 0$,

$$P(\xi) = \mu\, a_1 \left(\frac{exp\,(\lambda\,(\xi + C_1)) - 1}{\lambda} \right) + a_1 \left(\frac{\lambda}{exp\,(\lambda\,(\xi + C_1)) - 1} \right). \tag{3.18}$$

When $\lambda^2 - 4\mu = 0, \mu \neq 0, \lambda \neq 0$,

$$P(\xi) = \mu\, a_1 \left(-\frac{2\,(\lambda\,(\xi + C_1) + 2)}{\lambda^2\,(\xi + C_1)} \right) + a_1 \left(-\frac{\lambda^2\,(\xi + C_1)}{2\,(\lambda\,(\xi + C_1) + 2)} \right). \tag{3.19}$$

When $\lambda^2 - 4\mu = 0, \mu = 0, \lambda = 0$,

$$P(\xi) = \mu\, a_1\,(\xi + C_1) + a_1 \frac{1}{(\xi + C_1)}. \tag{3.20}$$

When $\lambda^2 - 4\mu < 0$,

$$P(\xi) = \mu\, a_1 \left(\frac{\sqrt{4\mu - \lambda^2}\, tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\,(\xi + C_1)\right) - \lambda}{2\mu} \right) + a_1 \left(\frac{2\mu}{\sqrt{4\mu - \lambda^2}\, tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\,(\xi + C_1)\right) - \lambda} \right). \tag{3.21}$$

and

$$P(\xi) = \mu\, a_1 \left(\frac{\sqrt{4\mu - \lambda^2}\, cot\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\,(\xi + C_1)\right) - \lambda}{2\mu} \right) \tag{3.22}$$

Notes

$$+ a_1 \left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\, cot \left(\frac{\sqrt{4\mu-\lambda^2}}{2} \ (\xi + C_1) \right) - \lambda} \right).$$

For Case 2. When $\lambda^2 - 4\mu > 0, \mu \neq 0,$

$$P(\xi) = -\mu\, a_1 \left(\frac{-\sqrt{\lambda^2-4\mu}\, tanh \left(\frac{\sqrt{\lambda^2-4\mu}}{2} (\xi + C_1) \right) - \lambda}{2\mu} \right) + a_1 \left(\frac{2\mu}{-\sqrt{\lambda^2-4\mu}\, tanh \left(\frac{\sqrt{\lambda^2-4\mu}}{2} (\xi + C_1) \right) - \lambda} \right). \tag{3.23}$$

and

$$P(\xi) = -\mu\, a_1 \left(\frac{-\sqrt{\lambda^2-4\mu}\, coth \left(\frac{\sqrt{\lambda^2-4\mu}}{2} (\xi + C_1) \right) - \lambda}{2\mu} \right) + a_1 \left(\frac{2\mu}{-\sqrt{\lambda^2-4\mu}\, coth \left(\frac{\sqrt{\lambda^2-4\mu}}{2} (\xi + C_1) \right) - \lambda} \right). \tag{3.24}$$

When $\lambda^2 - 4\mu > 0, \mu = 0,$

$$P(\xi) = -\mu\, a_1 \left(\frac{exp\,(\lambda\,(\xi + C_1)) - 1}{\lambda} \right) + a_1 \left(\frac{\lambda}{exp\,(\lambda\,(\xi + C_1)) - 1} \right). \tag{3.25}$$

When $\lambda^2 - 4\mu = 0, \mu \neq 0, \lambda \neq 0,$

$$P(\xi) = -\mu\, a_1 \left(-\frac{2\,(\lambda\,(\xi + C_1) + 2)}{\lambda^2\,(\xi + C_1)} \right) + a_1 \left(-\frac{\lambda^2\,(\xi + C_1)}{2\,(\lambda\,(\xi + C_1) + 2)} \right). \tag{3.26}$$

When $\lambda^2 - 4\mu = 0, \mu = 0, \lambda = 0,$

$$P(\xi) = -\mu\, a_1\,(\xi + C_1) + a_1 \frac{1}{(\xi + C_1)}. \tag{3.27}$$

When $\lambda^2 - 4\mu < 0,$

$$P(\xi) = -\mu\, a_1 \left(\frac{\sqrt{4\mu-\lambda^2}\, tan \left(\frac{\sqrt{4\mu-\lambda^2}}{2} \ (\xi + C_1) \right) - \lambda}{2\mu} \right) + a_1 \left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\, tan \left(\frac{\sqrt{4\mu-\lambda^2}}{2} \ (\xi + C_1) \right) - \lambda} \right). \tag{3.28}$$

Notes

and

$$P(\xi) = -\mu\, a_1 \left(\frac{\sqrt{4\mu-\lambda^2}\; cot\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\;(\xi + C_1) \right) - \lambda}{2\mu} \right) + a_1 \left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\; cot\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\;(\xi + C_1) \right) - \lambda} \right). \tag{3.29}$$

IV. Conclusion

The extended $\exp(-\varphi(\xi))$-expansion method has been successfully used to find the the wide range of exact and solitary traveling wave solutions for the generalized Hirota-Satsuma couple KdV system. Let us compare between our results obtained in the present article with the well-known results obtained by other authors using different methods as follows: Our results of the generalized Hirota-Satsuma couple KdV system are new and different from those obtained in [23]. It can be concluded that this method is reliable and propose a variety of exact solutions NPDEs. The performance of this method is effective and can be applied to many other nonlinear evolution equations. The solutions represent the solitary traveling wave solution for the generalized Hirota-Satsuma couple KdV system.

Competing interests

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors. The author did not have any competing interests in this research.

Author's contributions

All parts contained in the research carried out by the researcher through hard work and a review of the various references and contributions in the field of mathematics and the physical Applied

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Generalized Class of Exponential Chain Ratio-Cum-Chain Product Type Estimator for Finite Population Mean under Double Sampling Scheme in Presence of Non-Response

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Abstract- In this paper, a generalized class of exponential chain ratio-cum-chain product type estimator has been developed for estimating finite population mean and its properties have been studied in presence of non-response. The expressions for the bias and mean square error of the proposed estimator have been obtained in two different cases of non-response. The theoretical and empirical studies have been given to demonstrate the efficiency of the proposed estimator with respect to the other relevant estimators under consideration.

Keywords: *exponential, chain, estimator, mean, bias, mean square error.*

GJSFR-F Classification : *62D05*

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Yater Tato [α] & B. K. Singh [σ]

Abstract- In this paper, a generalized class of exponential chain ratio-cum-chain product type estimator has been developed for estimating finite population mean and its properties have been studied in presence of non-response. The expressions for the bias and mean square error of the proposed estimator have been obtained in two different cases of non-response. The theoretical and empirical studies have been given to demonstrate the efficiency of the proposed estimator with respect to the other relevant estimators under consideration.

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I. Introduction

Using auxiliary information in proposing selection procedure and the estimators for the population parameters was initiated by Bowely (1926), Neyman (1934, 1938), Watson (1937), Cochran (1940, 1942), Hansen et al. (1953) and Robson (1957).If the population mean $\bar{X}$ of the auxiliary variable x is not known but the population mean $\bar{Z}$ of an additional auxiliary variable z is known which is less correlated to the study variable y in comparison to the main auxiliary variable x (i.e. $\rho_{yx} > \rho_{yz}$, $\rho_{yx}, \rho_{yx} > 0$), then in such case Chand (1975), Kiregyera (1980, 1984) and Srivastava et al. (1990) proposed chain ratio type estimators using additional auxiliary variable with its known population mean.

Sometimes, it may not be possible to collect the complete information for all the units selected in the sample due to non-response. The missing observations due to non-response may occur during the investigation, which may be at random, and the ignorance of such missing observations may lead to biased estimator, though the amount of the bias may be very negligible. If the missing observation due to non-response is not at random then the amount of bias in the estimator will be large and may increase the error in the estimation, and the sampling error will also increase.

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Little and Rubin (2002) suggested to ignore the missing data completely if the percentage of incomplete cases are very low. This practice will reduce the sample size and may increase the bias and the variance of the estimator when the incomplete cases are large. However, some imputation techniques to replace the missing observation are considered by Rao and Toutenburg (1995) and Toutenburg and Srivastava (1998, 2003). Estimation of the population mean $\bar{Y}$ in sample surveys when some observations are missing due to non-response not at random was considered by Hansen and Hurwitz (1946), Rao (1986, 1987), Khare and Srivastava (1993, 1995).

Notes

Let $\bar{Y}$, $\bar{X}$ and $\bar{Z}$ be the population means of study character y, auxiliary character x and additional auxiliary character z. Let a finite population of size N is divided into N_1 responding units and N_2 not responding units and $W_1 = \frac{N_1}{N}$, $W_2 = \frac{N_2}{N}$ are their corresponding weights. According to Hansen and Hurwitz a sample of size n is taken from population of size N by using simple random sampling without replacement (SRSWOR) scheme of sampling and it has been observed that n_1 units respond and n_2 units do not respond. Again from the n_2 non-respondents, a sub sample of size $m = \left(n_2 f^{-1}\right)$ is drawn from n_2 non-responding units and information is collected on m units for study character y. Hence, the estimator for $\bar{Y}$ based on $(n_1 + m)$ units on study character y is given by:

$$\bar{y}^* = w_1 \bar{y}_1 + w_2 \bar{y}_{2m}$$

where $w_1 = \frac{n_1}{n}$, $w_2 = \frac{n_2}{n}$; $\bar{y}_1$ and $\bar{y}_{2m}$ denote the sample means of variable y based on n_1 and m units respectively. The estimators $\bar{y}^*$ is unbiased and has variance

$$V(\bar{y}^*) = \lambda S_y^2 + \lambda' S_{2y}^2 \tag{1}$$

where $\lambda = \left(\frac{1}{n} - \frac{1}{N}\right)$, $\lambda' = \frac{W_2\left(k-1\right)}{n}$, $W_2 = \frac{N_2}{N}$;

$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{Y})^2$ and $S_{2y}^2 = \frac{1}{N_2 - 1}\sum_{i=1}^{N_2}(y_i - \bar{Y}_2)^2$ are population mean squares of y for entire population and non-responding part of the population.

Similarly, the estimator $\bar{x}^*$ for population mean $\bar{X}$ in the presence of non-response based on corresponding $(n_1 + m)$ units is given by

$$\bar{x}^* = w_1 \bar{x}_1 + w_2 \bar{x}_{2m}$$

where $\bar{x}_1$ and $\bar{x}_{2m}$ denote the sample means of variable x based on n_1 and m units respectively. We have

$$V(\bar{x}^*) = \lambda S_x^2 + \lambda' S_{2x}^2$$

where

$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{X})^2$ and $S_{2x}^2 = \frac{1}{N_2 - 1}\sum_{i=1}^{N_2}(x_i - \bar{X})^2$ are population mean squares of x for the entire population and non-responding part of the population.

Notes

In case, when population mean $\bar{X}$ is not known, then, it is estimated by taking a preliminary sample of size n' $(n' < N)$ from the population of size N by using simple random sampling without replacement (SRSWOR) method of sampling. In this situation, Khare and Srivastava (1995) proposed conventional T_1 and alternative T_2 two phase sampling ratio estimators for population mean $\bar{Y}$ in the two different cases of non-response, i.e. when there is non-response on both the study variable y as well as on the auxiliary variable x and when there is non-response in the study variable y only, which are given as follows:

$$T_1 = \bar{y}^* \frac{\bar{x}'}{\bar{x}^*}$$

$$T_2 = \bar{y}^* \frac{\bar{x}'}{\bar{x}}$$

where $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, $\bar{x}' = \frac{1}{n'}\sum_{i=1}^{n'} x_i$.

The bias and MSE of T_1 and T_2 are given respectively as

$$B(T_1) = \frac{\lambda}{\bar{X}}(RS_x^2 - S_{xy}) + \frac{\lambda'}{\bar{X}}(RS_{2x}^2 - S_{2xy}) \qquad MSE(T_1) = \lambda S_d^2 + \lambda' S_{2d}^2,$$

$$B(T_2) = \frac{\lambda}{\bar{X}}(RS_x^2 - S_{xy})$$

$$MSE(T_2) = \lambda S_d^2 + \lambda' S_{2y}^2 .$$

where $R = \frac{\bar{Y}}{\bar{X}}$ is the population ratio of $\bar{Y}$ to $\bar{X}$,

$$S_d^2 = S_y^2 - 2RS_{xy} + R^2 S_x^2,$$

$$S_{2d}^2 = S_{2y}^2 - 2RS_{2xy} + R^2 S_{2x}^2$$

S_{xy}, S_{2xy} are the covariances for the whole population and the population of non-respondents respectively.

Now, when population mean $\bar{X}$ is not known, but the population mean $\bar{Z}$ of the additional auxiliary variable z closely related to x but compared to x remotely related to y i.e. $\rho_{yx} > \rho_{yz}$ is known, then we take a preliminary sample of size n' $(n' < N)$ from the population of size N with SRSWOR scheme and estimate the population mean $\bar{X}$ by using the sample means $\bar{x}'$ and $\bar{z}'$ based on n' units and the known additional population mean $\bar{Z}$. We observe that $\bar{X} = \frac{\bar{x}'}{\bar{z}'}\bar{Z}$ is more precise than preliminary sample mean $\bar{x}'$ if $\rho_{xz} > \frac{1}{2}\frac{C_x}{C_z}$, where $\bar{z}' = \frac{1}{n'}\sum_{i=1}^{n'} z_i$. Using available information on two auxiliary variables x

and z , Khare et al (2012) proposed chain ratio type estimators in the presence of non-response given as follows:

$$t_1 = \frac{\bar{y}^*}{\bar{x}^*}\frac{\bar{x}'}{\bar{z}'}\bar{Z}$$

$$t_2 = \frac{\bar{y}^*}{\bar{x}}\frac{\bar{x}'}{\bar{z}'}\bar{Z}$$

The MSE of t_1 is given by

$$MSE(t_1) = \bar{Y}^2\left[\lambda C_y^2 + \lambda' C_{2y}^2 + B' - 2A'\right] \quad (2)$$

Where

$$A' = \left(\frac{1}{n} - \frac{1}{n'}\right)k_{yx}C_x^2 + \left(\frac{1}{n'} - \frac{1}{N}\right)k_{yz}C_z^2 + \lambda' k_{2yx}C_{2x}^2$$

and

$$B' = \left(\frac{1}{n} - \frac{1}{n'}\right)C_x^2 + \left(\frac{1}{n'} - \frac{1}{N}\right)C_z^2 + \lambda' C_{2x}^2$$

$$k_{yx} = \rho_{yx}C_y/C_x \,, \quad k_{yz} = \rho_{yz}C_y/C_z \,,$$

$$k_{2yx} = \rho_{2yx}C_{2y}/C_{2x} \text{ and } k_{2yz} = \rho_{2yz}C_{2y}/C_{2z}$$

The MSE of t_2 is given by,

$$MSE(t_2) = \bar{Y}^2\left[\lambda C_y^2 + \lambda' C_{2y}^2 + B - 2A\right] \quad (3)$$

where

$$A = \left(\frac{1}{n} - \frac{1}{n'}\right)k_{yx}C_x^2 + \left(\frac{1}{n'} - \frac{1}{N}\right)k_{yz}C_z^2 ,$$

$$B = \left(\frac{1}{n} - \frac{1}{n'}\right)C_x^2 + \left(\frac{1}{n'} - \frac{1}{N}\right)C_z^2 ,$$

Bahl and Tuteja (1991) introduced an exponential ratio-type and exponential product-type estimators for population mean as

$$\bar{y}_{er} = \bar{y}\exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right]$$

$$\bar{y}_{ep} = \bar{y}\exp\left[\frac{\bar{x} - \bar{X}}{\bar{X} + \bar{x}}\right]$$

Singh and Choudhury (2012) suggested the exponential chain ratio and product type estimators for $\bar{Y}$ in double sampling respectively as

Notes

$$\bar{y}_{eR}^{dc} = \bar{y}\exp\left(\frac{\bar{x}'\frac{\bar{Z}}{\bar{z}'} - \bar{x}}{\bar{x}'\frac{\bar{Z}}{\bar{z}'} + \bar{x}}\right) \tag{4}$$

and

Notes

$$\bar{y}_{eP}^{dc} = \bar{y}\exp\left(\frac{\bar{x} - \bar{x}'\frac{\bar{Z}}{\bar{z}'}}{\bar{x} + \bar{x}'\frac{\bar{Z}}{\bar{z}'}}\right) \tag{5}$$

Motivated by Singh and Choudhury (2012) and Khare et al (2012), we suggest a generalized class of exponential chain ratio-cum-chain product type estimator in presence of non-response. The expressions for bias and mean square error of the proposed estimator have been obtained. Comparative studies of the proposed estimator have been made with the other relevant estimators and an empirical study has been given to illustrate its efficiency.

II. The Proposed Estimator

Utilizing information on the auxiliary variables x and z, using a scalar quantity α, we suggest a generalized class of exponential chain ratio-cum-chain product type estimator for the population mean $\bar{Y}$ in two different cases of non-response, which is given as follows.

$$\bar{y}_{eRP}^{dc} = \bar{y}\left[\alpha\exp\left(\frac{\bar{x}'\frac{\bar{Z}}{\bar{z}'} - \bar{x}}{\bar{x}'\frac{\bar{Z}}{\bar{z}'} + \bar{x}}\right) + (1-\alpha)\exp\left(\frac{\bar{x} - \bar{x}'\frac{\bar{Z}}{\bar{z}'}}{\bar{x} + \bar{x}'\frac{\bar{Z}}{\bar{z}'}}\right)\right] \tag{6}$$

The proposed estimator will be studied in two cases of non-response.

Case I: When non-response occurs only on y.

Case II: When non-response occurs both on y and x.

III. Case I: Non Response only on y

The estimator is

$$\bar{y}_{eRP}^{dc*} = \bar{y}^*\left[\alpha_1\exp\left(\frac{\bar{x}'\frac{\bar{Z}}{\bar{z}'} - \bar{x}}{\bar{x}'\frac{\bar{Z}}{\bar{z}'} + \bar{x}}\right)\right.$$

$$+\left(1-\alpha_1\right)\exp\left(\frac{\bar{x}-\bar{x}'\frac{\bar{Z}}{\bar{z}'}}{\bar{x}+\bar{x}'\frac{\bar{Z}}{\bar{z}'}}\right)\Bigg] \tag{7}$$

where α_1 is a scalar constant.

In order to obtain the expressions for the bias and MSE of $\bar{y}^{dc*}_{eRP}$, let

$$\bar{y}^* = \bar{Y}\left(1+e^*_0\right),\ \ \bar{x} = \bar{X}\left(1+e_1\right),$$

$\bar{x}' = \bar{X}\left(1+e_2\right)$ and $\bar{z}' = \bar{Z}\left(1+e_3\right)$ such that

$$E(e^*_0) = E(e_1) = E(e_2) = E(e_3) = 0.$$

$$\mathrm{E}(\mathrm{e}_0^{*2}) = \frac{V\left(\bar{y}^*\right)}{\bar{Y}^2} ,= \frac{1}{\bar{Y}^2}\left(\lambda S_y^2 + \lambda' S_{2y}^2\right)$$

$$\mathrm{E}(\mathrm{e}_0^*\mathrm{e}_1) = \frac{\mathrm{cov}\left(\bar{y}^*,\bar{x}\right)}{\bar{Y}\bar{X}} = \frac{\lambda S_{xy}}{\bar{Y}\bar{X}}.$$

$$\mathrm{E}(\mathrm{e}_1^2) = \frac{V\left(\bar{x}\right)}{\bar{X}^2} = \frac{\lambda S_x^2}{\bar{X}^2}, \tag{8}$$

Expressing $\bar{y}^{dc*}_{eRP}$ in terms of e's, we obtain

$$\bar{y}^{dc*}_{eRP} = \bar{Y}\left(1+e^*_0\right)\Big[\alpha_1 \exp\Big\{\left(1+e_2\right)\left(1+e_3\right)^{-1}$$

$$-\left(1+e_1\right)\Big\}\Big\{\left(1+e_2\right)\left(1+e_3\right)^{-1}+\left(1+e_1\right)\Big\}^{-1}\Big\}$$

$$+\left(1-\alpha_1\right)\exp\Big\{\left(1+e_1\right)-\left(1+e_2\right)\left(1+e_3\right)^{-1}$$

$$\Big\{\left(1+e_1\right)+\left(1+e_2\right)\left(1+e_3\right)^{-1}\Big\}^{-1}\Big\}\Big]$$

Expanding the right hand side of the above equation and retaining terms of e's up to second degree, we get

$$\bar{y}^{dc*}_{eRP} - \bar{Y} = \bar{Y}\Big[e^*_0 + \frac{1}{2}\Big\{e_1 - e_2 + e_3 + e^*_0 e_1 - e^*_0 e_2 + e^*_0 e_3$$

$$-\frac{e_1^2}{4} + \frac{3e_2^2}{4} - \frac{e_3^2}{4} - \frac{e_1 e_2}{2} - \frac{e_2 e_3}{4} + \frac{e_1 e_3}{4}\Big\}$$

$$+\alpha_1\Big\{-e_1 + e_2 - e_3 - e^*_0 e_1 + e^*_0 e_2 - e^*_0 e_3$$

$$+\frac{e_1^2}{2} - \frac{e_2^2}{2} + \frac{e_3^2}{2}\Big\}\Big] \tag{9}$$

Notes

Notes

The bias of the estimator $\overline{y}_{eRP}^{dc*}$ can be obtained by using the results of (8) in equation (9) as

$$B(\overline{y}_{eRP}^{dc*}) = \overline{Y}\left[\frac{A}{2} - \frac{B}{8} + \alpha_1\left(\frac{B}{2} - A\right)\right] \quad (10)$$

Squaring both the sides of equation (9), taking expectations and using the results of (8) we get the MSE of $\overline{y}_{eRP}^{dc*}$ to the first degree of approximation as

$$MSE\left(\overline{y}_{eRP}^{dc*}\right) = \overline{Y}^2\left[\lambda C_y^2 + \lambda' C_{2y}^2 + A + \frac{B}{4} - \alpha_1\left(2A + B\right) + \alpha_1^2 B\right] \quad (11)$$

Minimization of (11) with respect to α_1 yields its optimum as

$$\alpha_1 = \frac{1}{2} + \frac{A}{B} = \alpha_{1(opt)} \quad \text{(say)} \quad (12)$$

Substituting the value of α_1 from (12) in (7) gives the asymptotically optimum estimator (AOE) as

$$\overline{y}_{eRP(opt)}^{dc*} = \overline{y}^*\left[\alpha_{1(opt)} \exp\left(\frac{\overline{x}'\frac{\overline{Z}}{\overline{z}'} - \overline{x}}{\overline{x}'\frac{\overline{Z}}{\overline{z}'} + \overline{x}}\right) + \left(1 - \alpha_{1(opt)}\right)\exp\left(\frac{\overline{x} - \overline{x}'\frac{\overline{Z}}{\overline{z}'}}{\overline{x} + \overline{x}'\frac{\overline{Z}}{\overline{z}'}}\right)\right]$$

Thus, the resulting MSE of $\overline{y}_{eRP(opt)}^{dc*}$ is given as

$$MSE(\overline{y}_{eRP(opt)}^{dc*}) = \overline{Y}^2\left[\lambda C_y^2 + \lambda' C_{2y}^2 - \frac{A^2}{B}\right] \quad (13)$$

Remarks:

1. When $\alpha_1 = 1$, the proposed estimator $\overline{y}_{eRP}^{dc*}$ in (7) reduces to exponential chain ratio estimator $\overline{y}_{eR}^{dc*}$ when non-response occurs on y. The MSE of $\overline{y}_{eR}^{dc*}$ is obtained by putting $\alpha_1 = 1$ in (11) as

$$MSE(\overline{y}_{eR}^{dc*}) = \overline{Y}^2\left[\lambda C_y^2 + \lambda' C_{2y}^2 + \frac{B}{4} - A\right] \quad (14)$$

2. When $\alpha_1 = 0$ the proposed estimator $\overline{y}_{eRP}^{dc*}$ reduces to the exponential chain product estimator $\overline{y}_{eP}^{dc*}$ when non-response occurs on y. The MSE of $\overline{y}_{eP}^{dc*}$ is obtained by putting $\alpha_1 = 0$ in (11) as

$$MSE(\overline{y}_{eP}^{dc*}) = \overline{Y}^2\left[\lambda C_y^2 + \lambda' C_{2y}^2 + \frac{B}{4} + A\right] \tag{15}$$

Notes

IV. Efficiency Comparisons in Case I

a) Comparison with mean per unit estimator $\overline{y}^*$

From (1) and (13), we get

$$V\left(\overline{y}^*\right) - MSE(\overline{y}_{eRP(opt)}^{dc*}) = \overline{Y}^2\left[\frac{A^2}{B}\right] > 0 \tag{16}$$

b) Comparison with chain ratio estimator in double sampling t_2

From (3) and (13), we get

$$MSE(t_2) - MSE\left(\overline{y}_{eRP(opt)}^{dc*}\right) = \overline{Y}^2\left[\frac{A}{\sqrt{B}} - \sqrt{B}\right]^2 > 0 \tag{17}$$

c) Comparison with exponential chain ratio estimator in double sampling $\overline{y}_{eR}^{dc*}$

From (14) and (13), we get

$$MSE(\overline{y}_{eR}^{dc*}) - MSE\left(\overline{y}_{eRP(opt)}^{dc*}\right) = \overline{Y}^2\left[\frac{A}{\sqrt{B}} - \frac{\sqrt{B}}{2}\right]^2 > 0 \tag{18}$$

d) Comparison with exponential chain product estimator in double sampling $\overline{y}_{eP}^{dc*}$

From (15) and (13), we get

$$MSE(\overline{y}_{eP}^{dc*}) - MSE\left(\overline{y}_{eRP(opt)}^{dc*}\right) = \overline{Y}^2\left[\frac{A}{\sqrt{B}} + \frac{\sqrt{B}}{2}\right]^2 > 0 \tag{19}$$

V. Case II: Non Response on both y and x

Assuming that there is non-response on study variable y and auxiliary variable x. The proposed estimator is given as

$$\overline{y}_{eRP}^{dc**} = \overline{y}^*\left[\alpha_2 \exp\left(\frac{\overline{x}'\dfrac{\overline{Z}}{\overline{z}'} - \overline{x}^*}{\overline{x}'\dfrac{\overline{Z}}{\overline{z}'} + \overline{x}^*}\right) + (1-\alpha_2)\exp\left(\frac{\overline{x}^* - \overline{x}'\dfrac{\overline{Z}}{\overline{z}'}}{\overline{x}^* + \overline{x}'\dfrac{\overline{Z}}{\overline{z}'}}\right)\right] \tag{20}$$

Notes

where α_2 is a scalar constant and let $\bar{x}^* = \bar{X}\left(1+e_1^*\right)$.

In this case to obtain the bias (B) and MSE of $\bar{y}_{eRP}^{dc**}$, we have

$$E\left(e_1^{*2}\right)=\frac{V\left(\bar{x}^*\right)}{\bar{X}^2} = \frac{1}{\bar{X}^2}\left(\lambda S_x^2 + \lambda' S_{2x}^2\right)$$

$$E(e_0^* e_1^*)=\frac{\mathrm{cov}\left(\bar{y}^*, \bar{x}^*\right)}{\bar{Y}\bar{X}} = \frac{(\lambda S_{xy} + \lambda' S_{2xy})}{\bar{Y}\bar{X}}$$

Using the above results and following the procedure in case I, the bias and MSE of $\bar{y}_{eRP}^{dc**}$ are given as

$$B(\bar{y}_{eRP}^{dc**}) = \bar{Y}\left[\frac{A'}{2} - \frac{B'}{8} + \alpha_2\left(\frac{B'}{2} - A'\right)\right] \tag{21}$$

$$MSE\left(\bar{y}_{eRP}^{dc**}\right) = \bar{Y}^2\Big[\lambda C_y^2 + \lambda' C_{2y}^2 + A' + \frac{B'}{4}$$

$$-\alpha_2\left(2A' + B'\right) + \alpha_2^2 B'\Big] \tag{22}$$

Differentiating the equation (22) with respect to α_2 and equating it to zero, we get the optimum value of α_2 as

$$\alpha_2 = \frac{1}{2} + \frac{A'}{B'} = \alpha_{2(opt)} \tag{23}$$

Substitution of α_2 from (23) in (22) gives the optimum MSE of $\bar{y}_{eRP}^{dc**}$ as

$$MSE(\bar{y}_{eRP(opt)}^{dc**}) = \bar{Y}^2\left[\lambda C_y^2 + \lambda' C_{2y}^2 - \frac{A'^2}{B'}\right] \tag{24}$$

Remarks:

1. When $\alpha_2 = 1$, $\bar{y}_{eRP}^{dc**}$ in (20) reduces to Singh and Choudhury exponential chain ratio estimator $\bar{y}_{eR}^{dc**}$ when non-response is on both y and x. The MSE of (4)is obtained by putting $\alpha_2 = 1$ in (22) and is expressed as

$$MSE(\bar{y}_{eR}^{dc**}) = \bar{Y}^2\left[\lambda C_y^2 + \lambda' C_{2y}^2 + \frac{B'}{4} - A'\right] \tag{25}$$

2. When $\alpha_2 = 0$, the proposed estimator $\bar{y}_{eRP}^{dc**}$ reduces to Singh and Choudhury exponential chain product estimator $\bar{y}_{eP}^{dc**}$ .The MSE of (5) is obtained by putting $\alpha_2 = 0$ in (22) and is given by

$$MSE(\bar{y}_{eP}^{dc**}) = \bar{Y}^2\left[\lambda C_y^2 + \lambda' C_{2y}^2 + \frac{B'}{4} + A'\right] \tag{26}$$

VI. Efficiency Comparisons in Case II

a) Comparison with mean per unit estimator $\bar{y}^*$

From (1) and (24), we get

$$V\left(\bar{y}^*\right) - MSE(\bar{y}^{dc**}_{eRP(opt)}) = \bar{Y}^2\left[\frac{A'^2}{B'}\right] > 0 \tag{27}$$

b) Comparison with chain ratio estimator in double sampling t_1

From (2) and (24), we get

$$MSE(t_1) - MSE\left(\bar{y}^{dc**}_{eRP(opt)}\right) = \bar{Y}^2\left[\frac{A'}{\sqrt{B'}} - \sqrt{B'}\right]^2 > 0 \tag{28}$$

c) Comparison with exponential chain ratio estimator in double sampling $\bar{y}^{dc**}_{eR}$

From (25) and (24), we get

$$MSE(\bar{y}^{dc**}_{eR}) - MSE\left(\bar{y}^{dc**}_{eRP(opt)}\right) = \bar{Y}^2\left[\frac{A'}{\sqrt{B'}} - \frac{\sqrt{B'}}{2}\right]^2 > 0 \tag{29}$$

d) Comparison with exponential chain product estimator in double sampling $\bar{y}^{dc**}_{eP}$

From (26) and (24), we get

$$MSE(\bar{y}^{dc**}_{eP}) - MSE\left(\bar{y}^{dc**}_{eRP(opt)}\right) = \bar{Y}^2\left[\frac{A'}{\sqrt{B'}} + \frac{\sqrt{B'}}{2}\right]^2 > 0 \tag{30}$$

VII. Empirical Study

To illustrate the performances of the different estimators we have considered the data used by Khare and Sinha (2009). The description of the population is given below:

96 village wise population of rural area under Police-station-Singur, District-Hooghly, West Bengal has been taken under the study from the District Census Handbook 1981. The 25% villages (i.e. 24 villages) whose area is greater than 160 hectares have been considered as non-response group of the population. The number of agricultural labourers in the village is taken as study character (y) while the area (in hectares) of the village, the number of cultivators in the village and the total population of the village are taken as auxiliary characters x and z respectively. The values of the parameters of the population under study are as follows:

$$N = 96,\ n = 24,\ n' = 60,\ W_2 = 0.25$$

$$\bar{Y} = 137.9271,\ \bar{X} = 144.8720,\ \bar{Z} = 185.2188,\ S_y = 182.5012,\ S_{2y} = 287.4202,$$

$$C_x = 0.8115, C_{2x} = 0.9408,$$

$$C_z = 1.0529, C_{2z} = 1.4876,$$

$$\rho_{yx} = 0.773, \rho_{2yx} = 0.724,$$

$$\rho_{yz} = 0.786, \rho_{2yz} = 0.787,$$

$$\rho_{xz} = 0.819, \rho_{2xz} = 0.724.$$

Notes

Table 1 : Percentage relative efficiencies of different estimators with respect to $\overline{y}$

W_2	f	$\overline{y}^*$	t_2	$\overline{y}_{eR}^{dc*}$	$\overline{y}_{eP}^{dc*}$	$\overline{y}_{eRP}^{dc*}$
0.1	1.5	100	199.48	151.76	65.59	204.30
	2.0	100	177.53	142.59	68.52	180.86
	2.5	100	163.52	136.18	70.99	166.02
	3.0	100	153.79	131.44	73.10	155.78
0.2	1.5	100	177.53	142.59	68.52	180.86
	2.0	100	153.79	131.44	73.10	155.78
	2.5	100	141.18	124.92	76.52	142.58
	3.0	100	133.36	120.64	79.16	134.43
0.3	1.5	100	163.52	136.18	70.99	166.02
	2.0	100	141.18	124.92	76.52	142.58
	2.5	100	130.47	119.01	80.27	131.42
	3.0	100	124.18	115.36	82.99	124.90

Table 2 : Percentage relative efficiencies of different estimators with respect to $\overline{y}$

W_2	f	$\overline{y}^*$	t_1	$\overline{y}_{eR}^{dc**}$	$\overline{y}_{eP}^{dc**}$	$\overline{y}_{eRP}^{dc**}$
0.1	1.5	100	228.59	161.35	63.36	238.14
	2.0	100	221.50	158.04	64.38	232.28
	2.5	100	216.28	155.56	65.19	228.13
	3.0	100	212.27	153.62	65.86	225.06
0.2	1.5	100	221.50	158.04	64.38	232.28
	2.0	100	212.27	153.62	65.86	225.06
	2.5	100	206.52	150.81	66.88	220.90
	3.0	100	202.59	148.86	67.64	218.26
0.3	1.5	100	216.28	155.56	65.19	228.13
	2.0	100	206.52	150.81	66.88	220.90
	2.5	100	201.07	148.09	67.95	217.29
	3.0	100	197.58	146.33	68.67	215.22

VIII. Conclusion

The PRE of the suggested estimator has been compared with the usual unbiased estimator $\overline{y}^*$, estimators $(t_2, \overline{y}_{eR}^{dc*}, \overline{y}_{eP}^{dc*}, \overline{y}_{eRP}^{dc*})$ in Case I; and $\overline{y}^*$, estimators $(t_1, \overline{y}_{\mathrm{Re}}^{dc**}, \overline{y}_{Pe}^{dc**}, \overline{y}_{eRP}^{dc**})$ in Case II.

From tables 1 and 2, it is concluded that the proposed estimator in its optimality is performing better than the estimators taken for comparisons. Also, it has been observed that the percent relative efficiency of the suggested estimator decreases when non-response rate increases in its both the cases.

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Simultaneous Estimation of Adjusted Rate of Two Factors using Method of Direct Standardization

By Oyeka ICA & Okeh UM
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Abstract- This paper presents the use of standardization or adjustment of rates and ratios in comparing two populations using single indices rather than a series of specific rates or ratios. Here the overall adjusted crude rate or the unadjusted crude rate for two populations will have same estimate irrespective of the nature of the standard population distribution. These results are obtained in all cases whenever the two standard distributions are of the total sample. In these cases the overall adjusted crude rates based on the two sets of directly adjusted rates would be equal to each other, although not necessarily always equal to the overall unadjusted crude rate as is found to be the case here. However, if the standard population distribution chosen for a given population is different from that chosen for another, then the two resulting estimated adjusted or standardized crude rates would most likely not be equal to each other.

Keywords: *standardization, adjusted specific, unadjusted crude rate, adjusted crude rate, ratios.*

GJSFR-F Classification : MSC 2010: 97K80

SIMULTANEOUSESTIMATIONOFADJUSTEDRATEOFTWOFACTORSUSINGMETHODOFDIRECTSTANDARDIZATION

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Notes

Simultaneous Estimation of Adjusted Rate of Two Factors using Method of Direct Standardization

Oyeka ICA [α] & Okeh UM [σ]

Abstract- This paper presents the use of standardization or adjustment of rates and ratios in comparing two populations using single indices rather than a series of specific rates or ratios. Here the overall adjusted crude rate or the unadjusted crude rate for two populations will have same estimate irrespective of the nature of the standard population distribution. These results are obtained in all cases whenever the two standard distributions are of the total sample. In these cases the overall adjusted crude rates based on the two sets of directly adjusted rates would be equal to each other, although not necessarily always equal to the overall unadjusted crude rate as is found to be the case here. However, if the standard population distribution chosen for a given population is different from that chosen for another, then the two resulting estimated adjusted or standardized crude rates would most likely not be equal to each other.

Keywords: standardization, adjusted specific, unadjusted crude rate, adjusted crude rate, ratios.

I. Introduction

Standardization or adjustment of rates and ratios is often necessary because it is usually easier in comparing two populations, say, to make the comparison using single summary indices rather than a series of specific rates or ratios. This approach also helps avoid the problem of small imprecise and sometime non-existence of specific rates and ratios (Flies,1981;Pepe,2003; Greenberg et al,2001).

Standardization of rates and ratios may be done for only one factor or several factors of classification of a criterion variable of interest. In particular if a criterion variable or condition is associated with each of two factors of classification which may by themselves also be associated with each other, then standardization of rates or ratios may sometimes be necessary for a clearer analysis and inter-presentation of results to simultaneously standardize or adjust the rates for the two factors of classification, first specific to the levels or categories of one of the factors across the levels of the other factor, and then also specific to the levels of the second factor say holding constant the levels, that is for all levels or categories of the first factor(Cochran,1950,Gibbon,1971).

Research interest in this case would be to identify and measure the separate effects of the two factors of classification on the criterion variable or condition.

This paper proposes, develops and presents a formatted systematic statistical method for this purpose.

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a) *The proposed method*

Research interest here is using the direct method of standardization of rates to measure or estimate the separate effects of two factors of classification which may be associated on the variable being studied and to obtain sample estimate of unadjusted and adjusted crude rates specific to the levels of each of the factors holding the levels of the other factor of classification constant.

Now to develop the method of estimation of direct standardized or adjusted rate, suppose A and B are two variables of classification with 'a' and 'b' groups or levels respectively. Factors A and B may be associated or related. Research interest is to estimate the rates of occurrence of a criterion variable or condition specific to each of the levels of factor A across, that is for all levels of factor B and also the rates of occurrence of the specified condition specific to each of the levels of factor B for all levels of factor A as well as the corresponding marginal rates and overall rate.

Suppose a total random sample of size N=N.. of subject are randomly drawn from an antecedent or predisposing population C for all levels of factors A and B, of which N_{ij} is the number of subjects at the it h level of factor A and j th level of factor B, for i=1, 2,…a_j and j=1,2,…..,b.

Also suppose there are a total of n=n.. outcomes or cases in condition or set D of cases for all levels of factors A and B of which n_{ij} cases are at the ith level of factor A and jth level of factor B, for i=1,2,…a_j and j=1,2,…,b where population D is possibly a subset of population C.

Now the rate of occurrence of cases in population D as a function of cases in population C specific to the it h level of factor A and j th level factor B is

$$r_{ij} = \frac{n_{ij}}{N_{ij}} \quad \text{.........................(1)}$$

For i=1, 2, …a_j; j=1,2,…b.

Let

$$N_{i.} = \sum_{j=1}^{b} N_{ij}\,;\, N_{.j} = \sum_{i=1}^{a} N_{ij} \quad \text{.................(2)}$$

be respectively the total or marginal number of subjects or observations in population C at the ith level of factor A and jth level of factor B.

Similarly let

$$n_{i.} = \sum_{j=1}^{b} n_{ij}\,;\, n_{.j} = \sum_{i=1}^{a} n_{ij} \quad \text{.................(3)}$$

be respectively the total or marginal number of cases or outcomes in population D at the ith level of factor A and jth level of factor B. Then the estimated unadjusted crude rates of occurrence of cases or outcomes in population D as a function of outcomes in population C specific to the ith level of factor A for all levels of factor B for all levels of factor A are respectively the ratios

$$r_{i.;unadj} = \frac{n_{i.}}{N_{i.}}\,;\, r_{.j;unadj} = \frac{n_{.j}}{N_{.j}} \quad \text{.................(4)}$$

For i=1, 2,….a; j=1,2,…b.

Notes

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Note that

$$N = N_{..} = \sum_{i=1}^{a} n_{i.} = \sum_{j=1}^{b} N_{.j} = \sum_{j=1}^{b}\sum_{i=1}^{a} N_{ij} \tag{5}$$

and

$$n = n_{..} = \sum_{i=1}^{a} n_{i.} = \sum_{j=1}^{b} n_{.j} = \sum_{j=1}^{b}\sum_{i=1}^{a} n_{ij} \tag{6}$$

Therefore the overall unadjusted crude rate of occurrence of event D as a function of event C for all levels of factors A and B is

$$r_{unadj} = r = \frac{n_{..}}{N_{..}} \tag{7}$$

As noted above research interest is to obtain standardized or adjusted crude rate specific to each level of factor A for all levels of factor B and also specific to each level of factor B for all levels of factor A as well as the overall adjusted or standardized crude rate.

To obtain estimates of adjusted or standardized crude rates specific to each level of factor B for all levels of factor A we use the proportionate distribution of total number of observations $N_{..}$ across the 'a' levels or groups of factor A, namely P_{is} the waiting factor, for i=1,2,..,a.

Thus

$$P_{is} = \frac{N_{i.}}{N_{..}} \tag{8}$$

Similarly to obtain estimates of adjusted or standardized crude rate specific to each level of factor A for all levels of factor B we use the proportionate distributions N.. across the 'b' levels or groups of factor B, namely P_{sj} the waiting factor, for j=1,2,…,'b.' Thus

$$P_{sj} = \frac{N_{.j}}{N_{..}} \tag{9}$$

Hence the adjusted or standardized crude rate of condition D as a function of population C specific to the jth level of factor B for all levels of factor A is

$$r_{.j;adj} = \sum_{i=1}^{a} p_{is} r_{ij} \tag{10}$$

Similarly the adjusted or standardized crude rate of condition D as a function of population C specific to the ith level of factor A for all levels of factor B is

$$r_{i.;adj} = \sum_{j=1}^{b} p_{sj} r_{ij} \tag{11}$$

We then obtain the sample estimate of the overall adjusted crude rate of condition D as a function of population C for all levels of factors A and B as

$$r_{s;adj} = r_{..adj} = \sum_{i=1}^{a} p_{is}.r_{i.} = \sum_{j=1}^{b} p_{sj}.r_{.j} \tag{12}$$

These results are summarized in Table 1.

Table 1 : Data format for Estimation of Unadjusted and Adjusted Rates in two Factor Standardization by Direct method

	FACTOR B							
FACTOR A	**1**	**2**	**...........**	**b**	**Total**	**Proportion**	**Unadjust**	**adjust**
	$n_{11}(N_{11})$	$n_{12}(N_{12})$	....	$n_{1b}(N_{1b})$	$(n_{1.}(N_{1.}))$	(p_{sj})	$(r_{.j;adj})$	$(r_{.j;unadj})$
1								
$r_{.1;adj}$								
$\frac{n_{.1}}{N_{.1}}$								
2	$n_{21}(N_{21})$	$n_{22}(N_{22})$	....	$n_{2b}(N_{2b})$	$n_{2.}(N_{2.})$	p_{2s}	$\frac{n_{2.}}{N_{2.}}$	$r_{2.;adj}$
$r_{.2;adj}$	r_{21}	r_{22}	....	r_{2b}	$r_{2.}$	....	....	.....
$\frac{n_{.2}}{N_{.2}}$								
...								
a	$n_{a1}(N_{a1})$	$n_{a2}(N_{a2})$	......	$n_{ab}(N_{ab})$	$\frac{p_{sa.}}{n_{a.}(N_{a.})}$	p_{as}	$\frac{n_{a.}}{N_{a.}}$	$r_{a.;adj}$
$r_{.j;adj}$	r_{a1}	r_{a2}	....	r_{ab}	$r_{a.}$			
Total	$n_{.1}(N_{.1})$	$n_{.2}(N_{.2})$		$n_{.b}(N_{.b})$				
$(n_{.j}(N_{.j}))$	$r_{.1}$	$r_{.2}$		$r_{.b}$				
proportion								
(p_{sj})	p_{s1}	p_{s2}	....	p_{sb}	....	.....	....	
Unadjust $(r_{.j;adj})$	$\frac{n_{.1}}{N_{.1}}$	$\frac{n_{.1}}{N_{.1}}$	.....	$\frac{n_{.b}}{N_{.b}}$	......	...	$r_{.unadj}=\frac{n_{..}}{N_{..}}$	
Adjust $(r_{.j;unadj})$	$r_{.1;adj.}$	$r_{.2;adj}$	......	$r_{.b;adj}$				$r_{..;adj}$

In table 1 the entries in each of the cells are the number of cases in condition D,the number of observations in population D and the ratios of these numbers.

b) *Illustrative Example*

We now illustrate the proposed method with the sample data of Table 2 on premature and live births by birth order and age of mother in a certain population.

Table 2 : Sample Data on Premature and Live births by Birth order and Maternal age in a population

	Birth Order						
Maternal Age	**1**	**2**	**3**	**4**	**5+**	**Total $(n_{i.}(N_{1.}))$**	**Proportion of total births (p_{is})**
Under 20	11(23)	3(72)	3(32)	1(43)	0(33)	18(203)	
	0.478	0.042	0.094	0.023	0.000	0.089	0.066
20-24	14(329)	15(327)	7(176)	3(69)	8(67)	47(968)	
	0.043	0.046	0.040	0.043	0.119	0.049	0.012

Notes

25-29	6(115)	11(209)	11(207)	6(132)	6(123)	40(786)	
	0.052	0.053	0.053	0.045	0.049	0.051	0.254
30-34	4(78)	8(83)	10(117)	9(98)	12(150)	43(526)	
	0.051	0.096	0.085	0.092	0.080	0.082	0.170
35-39	4(42)	8(56)	11(90)	14(56)	3(104)	40(348)	
	0.095	0.143	0.122	0.050	0.029	0.115	0.112
40 and above	3(34)	4(457)	8(72)	10(48)	4(68)	29(267)	
	0.088	0.089	0.111	0.208	0.059	0.109	0.086
Total $\left(n_{.j}(N_{.j})\right)$	42(621)	49(792)	47(694)	45(446)	33(545)	217(3098)	
	0.068	0.010	0.068	0.096	0.060	0.070	0.070
Proportion of total births (p_{sj})	0.200	0.256	0.256	0.224	0.144	0.176	

The data of Table 2 is used to obtain estimates of the unadjusted and adjusted crude rate specific to each of the levels or groups of the two factors of classification.

Specifically to estimate adjusted or standardized crude rates specific to birth order, we apply the proportionate distribution of the total life births across maternal age as the standard population, namely p_{is} in the last column of Table 2 to each of the columns of rates, r_{ij} of the Table, for j=1,2,3,4,5. Similarly to estimate adjusted or standardize crude rate specific to Maternal age we apply the proportionate distribution of total life births across birth order as the standard population, namely p_{sj} in the last row of Table 2 to each of the rows of rates, r_{ij} of the Table, for i=1,2,3,4,5,6.The results are presented in Table 3.

Table 3 : Simultaneous Estimates of Unadjusted Adjusted Premature Birth rates by Maternal age and Birth order: Direct Standardization

	Birth Order												
Maternal Age	**Proportion of total birth** (p_{is})	r_{i1}	**1**	r_{i2}	**2**	r_{i3}	**3**	r_{i4}	**4**	r_{i5}	**5+**	**Unadjusted crude rate** $(r_{1.;unadj})$	**Adjusted crude rate** $(r_{1.;adj})$
less than 20	0.066	0.478		0.042		0.094		0.023		0.000		0.089	0.131
20-24	0.312	0.042		0.046		0.040		0.043		0.119		0.049	0.057
25-29	0.254	0.052		0.052		0.053		0.045		0.049		0.050	0.051
30-34	0.170	0.051		0.096		0.085		0.092		0.080		0.082	0.081
35-39	0.112	0.095		0.143		0.122		0.250		0.029		0.115	0.157
40 and over	0.086	0.088		0.008		0.111		0.208		0.059		0.109	0.594
Proportion of total birth $(p_{.j})$													
Unadjusted cruderate $(r_{.j;unadj})$		0.068		0.062		0.068		0.096		0.061		0.070	
Adjusted crude rate $(r_{.j;adj})$			0.086		0.070		0.069		0.088		0.068		0.070

II. Summary and Conclusion

The adjusted crude rate of premature births specific to birth order for all age groups shown in the last row of Table 3 are estimated using equation 10,while the corresponding adjusted crude rate specific to maternal age for all birth orders shown in the last column of Table 3 are estimated using equation 11.

Thus the last two rows of Table 3 show rates specific for birth order and directly adjusted for maternal age, with the standard maternal age distribution of births being that of the total sample of births. The last two columns of the Table show rates specific for maternal age and directly adjusted for birth order, with the standard birth order distribution of birth being that of total sample of births.

Notes

The estimated adjusted specific premature birth rate of Table 3 seem to indicate that incidence of premature births may not be strongly associated with birth order, but may probably be some how associated with increasing maternal age, especially from age 25 years.

The overall adjusted crude premature birth rate is estimated to be severally 70 per 1000 live births whether the standard population distribution is either the proportionate distribution of total birth by birth order or by maternal age. The unadjusted crude rate is also here estimated to be 70 per 1000 live births.

These results are usually the case whenever the two standard distributions are those of the total sample. In these cases the overall adjusted crude rates based on the two sets of directly adjusted rates would be equal to each other, although not necessarily always equal to the overall unadjusted crude rate as is found to be the case here.

However, if the standard population distribution chosen for population A (here maternal age)is different from that chosen for factor B(here birth order),then the two resulting estimated adjusted or standardized crude rates would most lively not be equal to each other.

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A Robust Regression Type Estimator for Estimating Population Mean under Non-Normality in the Presence of Non-Response

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Abstract- In sampling theory, regression type estimators are extensively used to estimate the population mean when the correlation between study and auxiliary variables is high. In this study, we incorporate robust modified maximum likelihood estimators (MMLEs) into regression type estimator in the presence of non-response and their properties have been obtained theoretically. For the support of the theoretical outcomes, simulations under several super-population models have been made. We study the robustness properties of these modified estimators. We show that utilization of MMLEs in estimating finite populations mean leads to robust estimates, which is very advantageous when we have non-normality or other common data anomalies such as outliers.

Keywords: *regression type estimator, modified maximum likelihood, robust linear regression, super-population, simulation study, non-response.*

GJSFR-F Classification : *MSC 2010: 93D21*

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A Robust Regression Type Estimator for Estimating Population Mean under Non-Normality in the Presence of Non-Response

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Abstract- In sampling theory, regression type estimators are extensively used to estimate the population mean when the correlation between study and auxiliary variables is high. In this study, we incorporate robust modified maximum likelihood estimators (MMLEs) into regression type estimator in the presence of non-response and their properties have been obtained theoretically. For the support of the theoretical outcomes, simulations under several super-population models have been made. We study the robustness properties of these modified estimators. We show that utilization of MMLEs in estimating finite populations mean leads to robust estimates, which is very advantageous when we have non-normality or other common data anomalies such as outliers.

Keywords: regression type estimator, modified maximum likelihood, robust linear regression, super population, simulation study, non-response.

I. Introduction

The use of auxiliary information in sample survey have been considered mainly in the field of agricultural, biological, medical and social sciences at the stage of planning, designing, selection of units and devising the estimation procedure. In sampling theory, the ratio method of estimation uses the auxiliary information which is correlated with the study variable to improve the precision which results in improved estimators when the regression of y on x is linear and passes through origin. When the regression of y on x is linear, it is not necessary that the line should always passes through origin. Under such conditions, it is more appropriate to use the regression type estimators and the correlation between study and auxiliary variables is high.

Sometimes, it may not be possible to collect complete information for all the units selected in the sample due to non-response. Estimation of the population mean in sample in the presence of non-response has been considered by Hansen and Hurwitz (1946), Rao (1986, 1987) and several other authors.

Let $\overline{Y}$ and $\overline{X}$ be the population mean of the main study variable y and the auxiliary variable x for the population $U:(U_1, U_2, \ldots . U_N)$. The population U is supposed to be composed of N_1 responding and N_2 non-responding units. From the population of size N, a sample of size n is selected by using SRSWOR method of sampling and it was observed that n_1 units respond and n_2 units don't respond. Further, by making extra

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effort, a sub-sample of size $r = \frac{n_2}{K} (K > 1)$ is drawn from n_2 non-responding units by using SRSWOR method of sampling. Hence, we have n_1 units from respondent group and r units from non-respondent group of the population in the sample for which the value of the y character is obtained. Hansen and Hurwitz (1946) proposed the unibiased estimator for $\overline{Y}$, which is given as follows:

$$\overline{y}^* = \frac{n_1}{n}\overline{y}_1 + \frac{n_2}{n}\overline{y}_2^{'}, \tag{1.1}$$

Notes

where $\overline{y}_1$ and $\overline{y}_2^{'}$ are the sample means based on n_1 and r units respectively.
The estimator $\overline{y}^*$ is unbiased and the $V(\overline{y}^*)$ is given by

$$V(\overline{y}^*) = \frac{f}{n} S_y^2 + \frac{W_2(K-1)}{n} S_{y(2)}^2 \tag{1.2}$$

where $f = \frac{N-n}{N}; W_i = \frac{N_i}{N} (i = 1,2)$, S_y^2 and $S_{y(2)}^2$ are the population mean squares of the character y for the whole population and for the non-responding part of the population.

The regression type estimator in the presence of non-response (Rao1990)when the population mean $\bar{X}$ is known, is given by

$$t_{lr} = \bar{y}^* + \hat{\theta}_L(\bar{X} - \bar{x}) , \tag{1.3}$$

where, $\hat{\theta}_L = \frac{\hat{S}_{yx}}{s_x^2}$is the regression coefficient obtained by least square estimation.$\hat{S}_{yx}$ and s_x^2(sample mean square) denote the unibiased estimates of S_{yx} and S_x^2 based on $n_1 + r$ observations and n observations respectively.

The bias and mean square error (MSE) of the traditional regression estimator is given by

$$B(t_{lr}) = -Cov(\bar{x}, \theta_L) \tag{1.4}$$

$$MSE(t_{lr}) = \left(\frac{1}{n} - \frac{1}{N}\right)\left(S_y^2 + \theta_L^2 S_x^2 - 2\theta_L S_{yx}\right) + \frac{W_2(K-1)}{n} S_{y(2)}^2, \tag{1.5}$$

where, $\theta_L = \frac{S_{yx}}{S_x^2}$, $S_x^2$is the population variance of the auxiliary variable, $S_{yx}$is the population covariance between the study variable and the auxiliary variable.

We know that the regression estimator is useful in estimating the finite population mean when the information on the auxiliary variable is available, however this is known to be quiet sensitive to outliersas studied by Farrell and Barrera(2006) and Gwet and Rivest (1992).

In sample survey studies, non-normal distributions are very common in practice as found in Cochran (1977), Jenkinset. al.(1977), Chambers (1986)and Farrell and Barrera(2007).

In this paper, we study robust modified maximum likelihood estimator (MMLE) into regression type estimator (Rao 1990)in the presence of non-response and provide their properties theoretically.

We specially focus on the situation where the error term is not normally distributed. We obtain the mean square error of the proposed regression estimator theoretically and found the conditions under which the proposed regression type estimator in the presence of non-response has less mean square error than the corresponding regression type estimator. We support the theoretical result with simulations under several super population models and study the robustness property of the modified regression estimator. We show that utilization of MMLE for estimating finite populations mean results to robust estimate, which is very fruitful when we have non-normality or other common data anomalies such as outliers.

II. Non-Normal Errors and Proposed Regression Estimator.

Notes

For the linear regression model,$y_i = \theta x_i + e_i$; $i = 1,2, \ldots, n$,let the distribution of the error term follows the long tailed symmetric family.

$$f\ (e) = LTS(p, \sigma) = \frac{\Gamma p}{\sigma\sqrt{K}\,\Gamma\left(\frac{1}{2}\right)\Gamma\left(p-\frac{1}{2}\right)}\left\{1+\frac{1}{K}\left(\frac{e}{\sigma}\right)^2\right\}^{-p} ; -\infty < e < \infty, \quad (2.1)$$

where, $K = 2p - 3, p \geq 2$ is the shape parameter ($p$is known) with $E(e_i) = 0$and $V(e_i) = \sigma^2$.

Here it can be obtained that the kurtosis of (2.1) is $\frac{\mu_4}{\mu_2^2} = 3K/(K-2)$.

The coefficients of kurtosis of the LTS family that we consider in this family are ∞, 6, 4.5, 4.0 for p= 2.5, 3.5, 4.5, 5.5 respectively.

We realize that when $p = \infty$, (2.1) reduces to a normal distribution. The likelihood equations obtained from the likelihood function of (2.1) are expressions in terms of the intractable functions.

$$g\ (z_i) = z_i / \left\{1 + \frac{1}{K}(z_i^2)\right\},$$

where, $z_i = \frac{e_i}{\sigma}$ $(i = 1,2, \ldots, n)$and do not have explicit solutions.

The robust MMLE which is known to be asymptotically equivalent to the MLE are obtained in following three steps:

1. The likelihood equations are expressed in terms of the ordered variate $z_{(i)} = \frac{e_{(i)}}{\sigma}$.
2. The function $g(z_i)$ are replaced by their linear approximations and
3. The resulting equations are solved for the parameters.

The solutions which are explicit functions of the concomitant observations$(y_{[i]}, x_{[i]}), i = 1,2, \ldots, n$ are

$$\widehat{\theta}_T = K + L\hat{\sigma}_T, \ \text{and}\hat{\sigma}_T = G + \frac{\sqrt{G^2 + 4nC}}{2\sqrt{n(n-2)}}, \quad (2.2)$$

where, $K = \sum_{i=1}^{n} \beta_i y_{[i]} x_{[i]} \,/\, \sum_{i=1}^{n} \beta_i x_{[i]}^2$

$$L = \frac{\sum_{i=1}^{n} \alpha_i x_{[i]}}{\sum_{i=1}^{n} \beta_i x_{[i]}^2}, G = (2p/K) \sum_{i=1}^{n} \alpha_i\ (y_{[i]} - K x_{[i]}), \quad (2.3)$$

$$C = (2p/K) \sum_{i=1}^{n} \beta_i (y_{[i]} - K x_{[i]})^2$$

$$\alpha_i = \left(\frac{2}{K}\right) \frac{t_{[i]}^3}{\{1 + (1/K)t_{[i]}^2\}^2} \text{and} \beta_i = \frac{1 - (1/K)t_{[i]}^2}{\{1 + (1/K)t_{[i]}^2\}^2}, \quad (2.4)$$

where, the approximate $t_{(i)}$ values are obtained from the equation

$$\int_{-\infty}^{t_{(i)}} h(z)dz = \frac{i}{n+1}\ ; 1 \leq i \leq n,$$

where $h(z)$ is the distribution of $z = e/\sigma$

In the same linear model, $y_i = \theta x_i + e_i$; $i = 1,2,\dots,n$, now we suppose that the error term has one of the distributions in the skewed family namely, generalised logistic distribution which is given by

$$f(e) = \frac{r}{\sigma} \frac{\exp(-e/\sigma)}{\{1+\exp(-e/\sigma)\}^{r+1}}; -\infty < e < \infty, \tag{2.5}$$

where, r is the shape parameter with $E(e_i) = \sigma\{\Psi(r) - \Psi(1)\}andV(e_i) = \sigma^2\{\Psi'(r) + \Psi'(1)\}$.

Here $\Psi(x) = \Gamma'(x)/\Gamma(x)$ is the psi function and $\Psi'(x)$ is its derivative.

For,$r < 1, r = 1,$ and$r > 1$,(2.5) represents negatively skewed, symmetric and positively skewed distribution respectively.

The coefficient of skewness and kurtosis of the generalised logistic distribution which we consider in this study are computed from the moment generating function $M_e(t) = \frac{r\,\Gamma(r+t\sigma)\Gamma(1-t\sigma)}{\Gamma(r+1)}$ and $r$is given below:

r-values	0.5	1.5	2.0	4.0	5.0
Skewness	- 0.855	0.380	0.577	0.868	0.924
Kurtosis	5.400	4.188	4.332	4.758	4.870

The likelihood equations obtained from (2.5) can be expressed in terms of the ordered variates $z_{(i)}, (i = 1,2,\dots,n)$, and in whole the intractable function $g(z_{(i)}) = \frac{1}{1+E\{z_{(i)}\}}$. These functions are linearised as we have done in the LTS family case. The solutions of the MMLE equations are the MMLEs which are given as follows:

$$\hat{\theta}_T = K - M\hat{\sigma}_T \text{ and } \hat{\sigma}_T = \frac{-D+\sqrt{D^2+4nE}}{2\sqrt{n(n-1)}}\ , \tag{2.6}$$

where, K can be calculated from the formula (2.3) by replacing $\alpha_i, \beta_i and t_i$with

$$\alpha_i = \frac{1+e^{t_{(i)}}+t_{(i)}e^{t_{(i)}}}{(1+e^{t_{(i)}})^2},\ \beta_i = \frac{e^{t_{(i)}}}{(1+e^{t_{(i)}})^2}\ , \tag{2.7}$$

and $t_{(i)} = -\log(q_i^{-\frac{1}{r}} - 1)$,$q_i = \frac{1}{n+1}$respectively.

In equation (2.6) for calculating $\hat{\theta}_T$ and $\hat{\sigma}_T$, M, D and Evalues are calculated from the following equations

$$M = \frac{\sum_{i=1}^{n} \Delta_i x_{[i]}}{\sum_{i=1}^{n} p_i x_{[i]}^2},\ D = (r+1)\sum_{i=1}^{n} \Delta_i (y_{[i]} - Kx_{[i]}) \tag{2.8}$$

and

$$E = (r+1)\sum_{i=1}^{n} \beta_i \left(y_{[i]} - Kx_{[i]}\right)^2, \tag{2.9}$$

where $\Delta_i = \alpha_i - (r+1)^{-1}\, for\ 1 \leq i \leq n$

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Islam et. al. (2001) showed that the MMLEs given in (2.6) are more efficient and robust then their corresponding least square estimators (LSEs) when the error term is from the skewed family (2.5).

In this study we calculate the MMLE $\hat{\theta}_T$ from (2.2) if the error term is from $LTS(p,\sigma)$or from (2.6) if the error is from $GL(r,\sigma)$and modify the traditional regression estimator in the presence of non-response as given in (1.1) to achieve efficient estimator under non-normality, which is given by

Notes

$$T_{lr} = \bar{y}^* + \hat{\theta}_T(\bar{X} - \bar{x}) \ , \tag{2.10}$$

The bias and mean square error of the proposed estimator (2.10) are given by

$$B(T_{lr}) = -Cov(\bar{x}, \theta_T) \tag{2.11}$$

$$MSE(T_{lr}) = \left(\frac{1}{n} - \frac{1}{N}\right)\left(S_y^2 + {\theta_T}^2 S_x^2 - 2\theta_T S_{xy}\right) + \frac{W_2(K-1)}{n} S_{y(2)}^2 \tag{2.12}$$

In order to compare the MSE of the proposed estimator in (2.10) with the MSE of the regression type estimator $t_{lr} = \bar{y}^* + \hat{\theta}_L(\bar{X} - \bar{x})$, we get the following conditions under which the proposed estimator is more efficient than the regression type estimatort_{lr}.

$$MSE(T_{lr}) \leq MSE(t_{lr}) \text{if} \theta_T \leq \theta_L \text{ or } \theta_T \geq \theta_L \tag{2.13}$$

NOTE:- In general the shape parameter p in (2.1) and r in (2.5) may not be known. Using the least square estimator and constructing q-q plots (with the observed values as in Hamilton(1992), one can easily determine the closest distribution for the error term. Since the families (2.1) and (2.5) include a very large variety of location scale distribution, one can easily determine an approximate distribution for the error by using one of the two families given in study.

III. Simulation Study

In this study for the simulation, we have used R-programming software. In the super population models generated, we use the model

$$y_i = \theta x_i + e_i, i = 1,2,\dots,N, \tag{3.1}$$

where, we generate e_i and x_i independently and calculate y_i for $i = 1,2,\dots,N$.

Let the errors $e_1, e_2, \dots, e_N$ be the random observations from a super population either from (2.1) or (2.5). Let U_N denotes the corresponding finite population consists of Npairs (x_1,y_1), (x_2,y_2), ... , (x_N, y_N). To calculate the MSE of the proposed estimator in (2.10), we calculate T_{lr} for all possible samples $\binom{N}{n}$ simple random samples of size $n$from U_N. Since $\binom{N}{n}$is extremely large, so we conduct all Monte-Carlo studies as follows.

We take $N = 500$ in each simulation and generate U_{500} pairs from an assumed super population. From the generated finite population U_{500}, we have selected a sample of size $(n = 14, 19, 26, 40, 70)$ by simple random sampling without replacement. From each selected sample, the last 43% (3, 4, 6, 8, 11 respectively) of units have been considered as non-responding units. Now, we choose at random S= 15000 samples for all the possible $\binom{500}{n}$ samples of size n $(n = 14, 19, 26, 40, 70)$, which gives 15000 values of T_{lr}. To compare the efficiency of the proposed estimator under different models for a given n, we calculate the values of mean square errors as follows:

$MSE(T_{lr}) = \frac{1}{S}\sum_{j=1}^{S}(T_{lrj} - \bar{Y})^2$,$MSE(t_{lr}) = \frac{1}{S}\sum_{j=1}^{S}(t_{lrj} - \bar{Y})^2$ and

$$MSE(\bar{y}^*) = \frac{1}{S}\sum_{j=1}^{S}(\bar{y}^* - \bar{Y})^2,$$

where, $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$

For setting the population correlation ρ_{yx} is sufficiently high, which choose the value of parameter θ in the model $= \theta x + e$, such that the correlation coefficient between study variable (y) and auxiliary variable (x) is ρ_{yx} to determine the value of θ that satisfied this condition, we follow a similar way given by Rao and Beegle (1967) and write the population correlation between the study variable (y) and the auxiliary variable (x) .For example if $X\sim U(0,1)$, the value of θ for which the population correlation between y and x becomes $\theta^2 = \frac{12\sigma^2\rho_{yx}^2}{1-\rho_{yx}^2}$ for the LTS family and $\theta^2 = \frac{12\sigma^2\left(\varphi'(r)+\varphi(1)\right)\rho_{yx}^2}{1-\rho_{yx}^2}$ for the skewed family. Similarly, if $x$is generated from $Exp\,(1)$, the value of θ for which the population correlation becomes $\theta^2 = \frac{\sigma^2\rho_{yx}^2}{1-\rho_{yx}^2}$ for the symmetric family and $\theta^2 = \frac{\sigma^2\left(\varphi'(r)+\varphi(1)\right)\rho_{yx}^2}{1-\rho_{yx}^2}$ for the skewed family.

Here we take $\sigma^2 = 1$, in all situations without loss of generality and calculate the require parameter θ for which $\rho_{yx} = 0.75$.

IV. Comparison of Efficiencies of the Proposed Estimator

We consider four different super-population models given below to see how much efficiency we gain with the proposed modified estimator, when the condition (2.13) is satisfied under
non-normality:

I. $x\sim U(0,1)$and$e\sim LTS(p,1)$ and independent of x.
II. $x\sim exp(1)$and$e\sim LTS(p,1)$ and independent of x.
III. $x\sim U(0,1)$and$e\sim GL(r,1)$ and independent of x.
IV. $x\sim exp(0.5)$and$e\sim GL(r,1)$ and independent of x.

For the models (1) to (4), the values of θwhich makes the population correlation $\rho_{yx} = 0.75$ are given in table 1.

Table 1 : Parameter values of θused in models (1)–(4) that give $\rho_{yx} = 0.75$

Population	p		
	2.5	4.5	5.5
Model (1)	3.928	3.928	3.928
Model (2)	1.133	1.133	1.133
Population	r		
	0.5	1.5	2.0
Model (3)	10.076	6.309	5.944
Model (4)	2.057	1.288	1.213

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Here, we note that for the LTS family (2.1), the value of θ does not depend on the shape parameter$p$.

To verify that the super-population are generated appropriately, we provide a scatter graph and error distribution of model to $p = 4.5$ for model (2) in the figure 1 and in the figure 2. Similarly for the model (3) with $r = 0.5$ in the figure 3 and in the figure 4, a scatter graph and error distribution is provided.

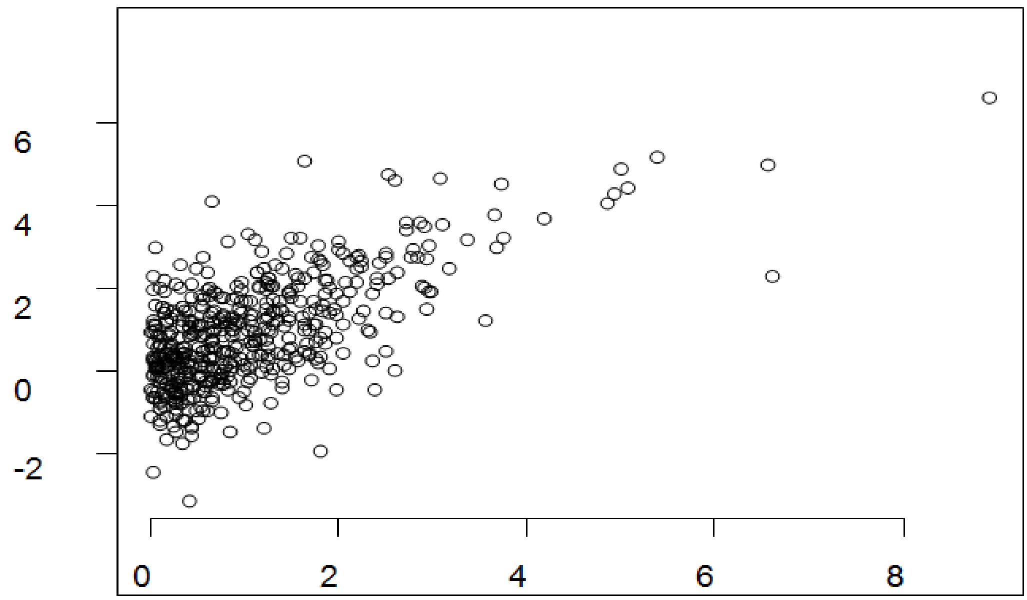

Figure 1 : A scatter graph of the study variable and auxiliary variable obtained from model (2) for $p = 4.5$.

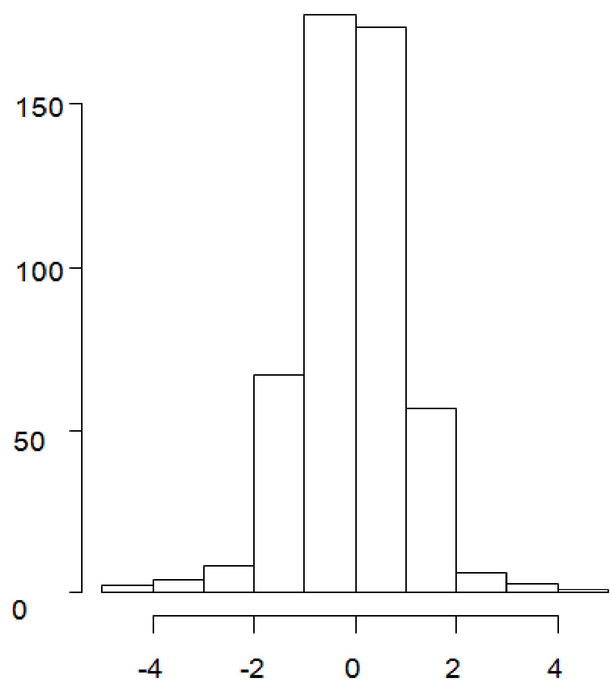

Figure 2 : Generated error distribution obtained from model (2) for $p = 4.5$

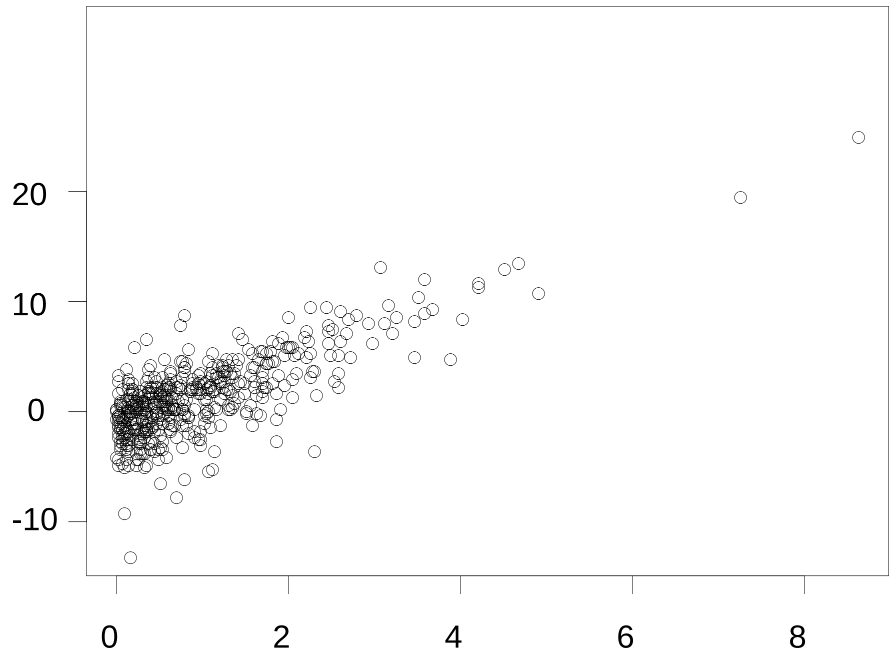

Figure 3 : A scatter graph of the study variable and auxiliary variable obtained from model (3) for $r = 0.5$.

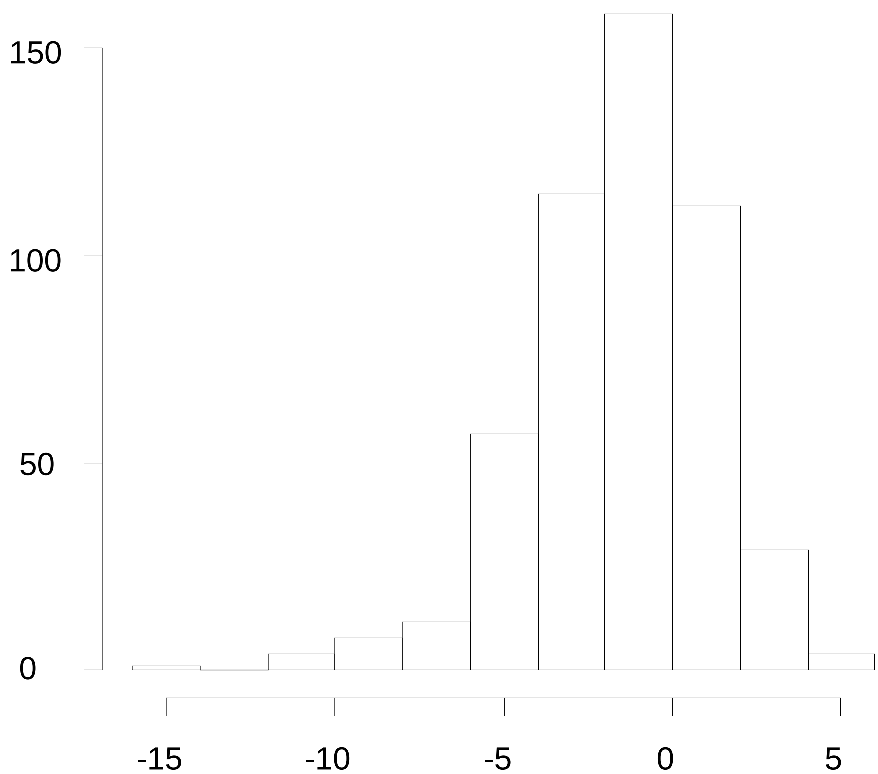

Figure 4 : Generated error distribution obtained from model (3) for $r = 0.5$.

Relative efficiencies are calculated as $RE = \frac{MSE(\bar{y}^*)}{MSE(.)} * 100$,

where, MSE(.)and relative efficiency (RE) are given in the table 2 for the model (1) and (2) and in the table 3 for the model (3) to and (4).

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From the table 2, we see that the proposed estimator T_{lr} is more efficient than the regression estimator t_{lr} in the presence of non-response because the theoretical condition is satisfied. We also observe that when sample size increases, mean square error decreases.

From the table 3, we also observe that the proposed estimator T_{lr} in the presence of non-response is more efficient than the regression estimator t_{lr} because the theoretical condition is satisfied. It is also clear that when sample size increases, mean square error decreases.

Table 2 : Mean square error and efficiencies of the estimators under super-populations (1-2).

$Model(1): x\sim U(0,1) and\ e\sim LTS(p,1)$						
	Est.	***n***				
		11	**15**	**21**	**31**	**51**
$p=2.5$	$\bar{y}^*$	0.1999 (100.00)*	0.1550 (100.00)	0.1339 (100.00)	0.0898 (100.00)	0.0518 (100.00)
	t_{lr}	0.1220 (163.85)	0.1041 (148.90)	0.0881 (151.99)	0.0626 (143.45)	0.0346 (149.71)
	T_{lr}	0.1177 (169.84)	0.1019 (152.11)	0.0862 (155.34)	0.0620 (144.84)	0.0345 (150.15)
$p=4.5$	$\bar{y}^*$	0.2212 (100.00)	0.1704 (100.00)	0.1208 (100.00)	0.0781 (100.00)	0.0541 (100.00)
	t_{lr}	0.1398 (158.79)	0.1081 (157.63)	0.0723 (167.08)	151.95 (151.95)	0.0382 (141.62)
	T_{lr}	0.1343 (164.71)	0.1058 (161.06)	0.0714 (169319)	0.0510 (153.14)	0.0381 (142.00)
$p=5.5$	$\bar{y}^*$	0.2598 (100.00)	0.1690 (100.00)	0.1146 (100.00)	0.0823 (100.00)	0.0533 (100.00)
	t_{lr}	0.1776 (146.28)	0.1077 (156.92)	0.0677 (169.28)	0.0549 (149.91)	0.0374 (142.51)
	T_{lr}	0.1723 (150.78)	0.1054 (160.34)	0.0667 (171.81)	0.0544 (151.29)	0.0372 (143.28)
$Model(2): x\sim exp(1) and\ e\sim LTS(p,1)$						
	Est.	n				
		11	15	21	31	51
$p=2.5$	$\bar{y}^*$	0.2211 (100.00)*	0.1811 (100.00)	0.1219 (100.00)	0.0639 (100.00)	0.0496 (100.00)
	t_{lr}	0.1630 (135.64)	0.1192 (151.93)	0.0719 (169.54)	0.0424 (150.71)	0.0329 (150.76)
	T_{lr}	0.1583 (139.67)	0.1184 (152.96)	0.0718 (169.78)	0.0420 (152.14)	0.0327 (151.68)
$p=4.5$	$\bar{y}^*$	0.1856 (100.00)	0.1719 (100.00)	0.1336 (100.00)	0.0823 (100.00)	0.0584 (100.00)
	t_{lr}	0.1346 (137.89)	0.1141 (150.66)	0.0803 (166.38)	0.0499 (164.93)	0.0430 (135.81)
	T_{lr}	0.1320 (140.61)	0.1130 (152.12)	0.0797 (167.63)	0.0498 (165.26)	0.0429 (136.13)
$p=5.5$	$\bar{y}^*$	0.2419 (100.00)	0.1747 (100.00)	0.1125 (100.00)	0.0808 (100.00)	0.0479 (100.00)
	t_{lr}	0.1697 (142.55)	0.1166 (149.83)	0.0720 (156.25)	0.0501 (161.28)	0.0351 (136.47)
	T_{lr}	0.1664 (145.37)	0.1161 (150.47)	0.0716 (157.12)	0.0500 (161.60)	0.0349 (137.25)

*(*Efficiencies are in the parenthesis)*

Table 3 : Mean square error and efficiencies of the estimators under super-populations (3-4).

$Model(3): x\sim U(0,1) and\ e\sim GL(r,1)$						
	Est.	***n***				
		11	**15**	**21**	**31**	**51**
$r=0.5$	$\bar{y}^*$	1.5083 (100.00)*	1.0495 (100.00)	0.6897 (100.00)	0.5187 (100.00)	0.3088 (100.00)

	Est.	11	15	21	31	51
	t_{lr}	0.9555 (157.85)	0.6606 (158.87)	0.4183 (164.88)	0.3257 (159.26)	0.2246 (137.49)
	T_{lr}	0.9457 (159.49)	0.6603 (158.94)	0.4161 (165.75)	0.3252 (159.50)	0.2242 (137.73)
$r = 1.5$	$\bar{y}^*$	0.5654 (100.00)	0.3904 (100.00)	0.2970 (100.00)	0.2071 (100.00)	0.1505 (100.00)
	t_{lr}	0.3881 (145.68)	0.2668 (146.33)	0.1655 (179.46)	0.1308 (158.33)	0.0988 (152.33)
	T_{lr}	0.3758 (150.45)	0.2630 (148.44)	0.1605 (185.05)	0.1293 (160.17)	0.0983 (153.10)
$r = 2.0$	$\bar{y}^*$	0.5506 (100.00)	0.4200 (100.00)	0.2795 (100.00)	0.1734 (100.00)	0.1228 (100.00)
	t_{lr}	0.3337 (165.00)	0.2544 (165.09)	0.1694 (164.99)	0.1032 (168.02)	0.0835 (147.07)
	T_{lr}	0.3155 (174.52)	0.2458 (170.87)	0.1652 (169.19)	0.1028 (168.68)	0.0830 (147.95)

$Model(4)$: $x \sim exp(0.5)$ and $e \sim GL(r,1)$

	Est.	n				
		11	15	21	31	51
$r = 0.5$	$\bar{y}^*$	2.2570 (100.00)	1.6369 (100.00)	1.0539 (100.00)	0.6654 (100.00)	0.4526 (100.00)
	t_{lr}	1.1878 (190.02)	0.8950 (182.89)	0.6039 (174.52)	0.4048 (164.38)	0.3119 (145.11)
	T_{lr}	1.1738 (192.28)	0.8803 (185.95)	0.5971 (176.50)	0.3982 (167.10)	0.3117 (145.20)
$r = 1.5$	$\bar{y}^*$	0.9732 (100.00)	0.7423 (100.00)	0.4968 (100.00)	0.3323 (100.00)	0.2227 (100.00)
	t_{lr}	0.5078 (191.65)	0.4077 (182.07)	0.2485 (199.92)	0.1788 (185.85)	0.1271 (175.22)
	T_{lr}	0.4963 (196.09)	0.4038 (183.83)	0.2463 (201.71)	0.1770 (187.74)	0.1270 (175.35)
$r = 2.0$	$\bar{y}^*$	0.7822 (100.00)	0.6629 (100.00)	0.3742 (100.00)	0.2942 (100.00)	0.2207 (100.00)
	t_{lr}	0.4033 (193.95)	0.3245 (204.28)	0.1720 (217.59)	0.1590 (185.03)	0.1280 (172.42)
	T_{lr}	0.3899 (200.62)	0.3240 (204.60)	0.1719 (217.69)	0.1573 (187.03)	0.1272 (173.51)

*(*Efficiencies are in the parenthesis)*

V. Robustness of the Proposed Estimator

The outliers in sample data are normally a in centered problem for survey statistician [1]. In practice, the shape parameters p in $LTS(p,\sigma)$, and r in $GL(r,\sigma)$ might be mis-specified. Therefore, it is very important for estimators to have efficiencies of robustness estimates such as an estimator is full efficient or nearly so for an assumed model and maintains high efficiencies for plausible to the assumed model.

Here, we take $N = 500$ and $\sigma^2 = 1$ without loss of generality and we study the robustness property of proposed estimator under different outlier models as follows.

We assume $x \sim U(0,1)$ and the error term $e \sim LTS(p = 3.5, \sigma^2 = 1)$. We determine our super-population model as follow:

(5). True model: $LTS(p = 3.5, \sigma^2 = 1)$

(6). Dixon's outliers model: $N - N_o$ observations from $LTS(3.5, 1)$ and N_o (we don't know which) form $LTS(3.5, 2.0)$

(7). Mis-specified model: $LTS(4.0, 1)$

Here, we realize that the model (5), the assumed super population model is given for the purpose of comparison and the models (6) and (7) are taken as its plausible alternatives. Here we have assumed the super population model $LTS(3.5, 1)$ for estimating θ_T. The coefficients (α_i, β_i) from (2.4) are calculated with $p = 3.5$ and are used in models (5) and (6). N_o in model (6) is calculated from the formula $(|0.5 + 0.1 *$

Ref

1. Chambers, R. L. (1986), Outlier robust finite population estimation. Journal of the American Statistical Association, 81, 1063–1069.

Notes

$N| = 50)$ for $N = 500$.We standardised the generated $e_i^{'}s, (i = 1,2, \dots, N)$ in all the models to have the same variance as that of $LTS(3.5, 1)$ i.e. it should be equal to 1. The simulated values of MSE and the relative efficiency are given in table 4. Here the estimators $\tilde{\theta}_L, \hat{\theta}_T$ are both location invariant estimators so that both of them are the estimators of θ under all the models described above. Here theoretical condition (2.13) is satisfied for model (5).

From the table 4, we see that in the presence of non-response, the proposed estimator T_{lr} is more efficient than the regression estimator t_{lr} because the theoretical condition is satisfied. We also observe that when sample size increases, mean square error decreases.

Now we assumed that the error term e is from the skewed family and $x$is from U (0,1). We assumed the model to be $GL(3,1)$ and determine our super population as

(8) True model: $GL(3,1)$

(9) Dixon outlier model: $N - N_o$observations from $GL(3,1)$ and N_o (we don't know which) from $GL(3, 2)$, where $(N_o = |0.5 + 0.1 * N| = 50)$.

(10) Mis-specified model: $GL(5,1)$.

The model (8) is assumed as a super population model and all other models (9) and(10) are taken as its plausible alternatives. The generated $e_i^{'}s, (i = 1,2, \dots, N)$ were standardized in the models (9) and (10) to have the same variance as that of $GL(3,1)$ i.e. $V(e_i) = \{\Psi^{'}(4) + \Psi^{'}(1)\}$,where $\Psi^{'}(x)$ is the derivative of the psi function. The simulated values of the MSEs and relative efficiencies of the estimators and relative efficiency under models (8) to(10) are given in the table 4. Also, from the table 4, we see that the proposed estimator T_{lr} is more efficient than the regression estimator t_{lr} since the theoretical condition is satisfied. Also, it is clear that when sample size increases, mean square error decreases.

Table 4 : Mean square errors and efficiencies under super-populations (5)–(7) for LTS family and under super-populations (8)–(10) for skewed family

Est.	n			n		
	11	15	21	5	11	15
	$True\ Model(5): x{\sim}Uni(0,1) and\ e{\sim}LTS(3.5,1)$			$Dixon\ outlier\ Model(6): x{\sim}Uni(0,1) and\ (N - N_0)\ e{\sim}LTS(3.5,1 + N_0 e{\sim}LTS(3.5,2)$		
$\bar{y}^*$	0.2189 (100.00)*	0.1742 (100.00)	0.1157 (100.00)*	0.2385 (100.00)	0.1684 (100.00)	0.1228 (100.00)
t_{lr}	0.1627 (134.54)	0.1081 (161.15)	0.0695 (166.48)	0.1521 (156.81)	0.1061 (158.72)	0.0629 (195.23)
T_{lr}	0.1609 (136.05)	0.1074 (162.20)	0.0691 (167.44)	0.1438 (165.86)	0.1048 (160.69)	0.0587 (209.20)
	$Mis-specified\ Model(7): x{\sim}Uni(0,1) and\ e{\sim}LTS(4.0,1)$			$True\ Model(8): x{\sim}Uni(0,1) and\ e{\sim}GL(3.0,1)$		
$\bar{y}^*$	0.7036 (100.00)	0.5112 (100.00)	0.2919 (100.00)	0.4989 (100.00)	0.3117 (100.00)	0.2416 (100.00)
t_{lr}	0.3866 (182.00)	0.2594 (197.07)	0.1653 (176.59)	0.2975 (167.70)	0.2167 (143.84)	0.1429 (169.07)
T_{lr}	0.3798 (185.26)	0.2566 (199.22)	0.1632 (178.86)	0.2878 (173.35)	0.2138 (145.79)	0.1416 (170.62)
	$Dixon\ outlier\ Model(9): x{\sim}Uni(0,1) and\ (N - N_0)\ e{\sim}GL(3.0,1 + N_0 e{\sim}GL(3.0,2)$			$Mis-specified\ Model(10): x{\sim}Uni(0,1) and\ e{\sim}GL(5.0,1$		
$\bar{y}^*$	0.6942 (100.00)	0.8314 (100.00)	0.7949 (100.00)	0.4725 (100.00)	0.2977 (100.00)	0.2232 (100.00)
t_{lr}	0.4563	0.6873	0.5410	0.3069	0.1970	0.1324

	(152.14)	(120.97)	(146.93)	(153.96)	(151.12)	(168.58)
T_{lr}	0.4373 (158.75)	0.6767 (122.86)	0.5302 (149.93)	0.3014 (156.77)	0.1949 (152.75)	0.1300 (171.69)

*(*Efficiencies are in the parenthesis)*

VI. Determination of Shape Parameter

Notes

In order to determine whether when a particular density is appropriate for the error term, a Q-Q plot of the ordered estimated residuals which are calculated using the LSEs (Least Square Estimation), $e_i = y_i + \tilde{\theta}_L x_i, (i = 1,2,\dots,n)$ are plotted against population quantiles for that density.

The population quantiles t_i are determined from the equation $\int_{-\infty}^{t(i)} t(u)du = \frac{i}{n+1}$; $1 \le i \le n$, where n is the sample size.

The Q-Q plot that closely approximates a straight line would be assumed to be the most appropriate.

VII. Conclusion

In this study, we show that when the error term is not normal which is applicable in most areas, MML integrated regression estimator(T_{lr})in the presence of non-response can improve the efficiency of regression estimatort_{lr}. We also show that the MML integrated regression estimator (T_{lr})(modified regression estimators) is robust to outliers as well as other data anomalies.

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Notes

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Non-Local Solution of Mixed Integral Equation with Singular Kernel

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Abstract- In this paper, we consider a non-local mixed integral equation in position and time in the space 2 *L* 1,1 *C* 0,*T* ;*T*. Then, using a quadratic numerical method, we have a system of Fredholm integral equations (SFIEs), where the existence of a unique solution is considered. Moreover, we consider Product Nystrom method (PNM), as a famous method to solve the singular integral equations, to obtain an algebraic system. Finally, some numerical results are considered, and the error estimate, in each case, is computed.

Keywords: non-local solution, fredholm-volterra integral equation, system of fredholm integral equations, weakly kernel, algebraic system.

GJSFR-F Classification : *45B05, 45G10, 60R*

NONLOCALSOLUTIONOFMIXEDINTEGRALEQUATIONWITHSINGULARKERNEL

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6. N. I. Muskhelishvili, Singular Integral Equations, Noordhoff (1953).

Non-Local Solution of Mixed Integral Equation with Singular Kernel

M. A. Abdou [α], S. A. Raad [σ] & W. Wahied [ρ]

Abstract- In this paper, we consider a non-local mixed integral equation in position and time in the space $L_2[-1,1]\times C[0,T];T<1$. Then, using a quadratic numerical method, we have a system of Fredholm integral equations (SFIEs), where the existence of a unique solution is considered. Moreover, we consider Product Nystrom method (PNM), as a famous method to solve the singular integral equations, to obtain an algebraic system. Finally, some numerical results are considered, and the error estimate, in each case, is computed.

Keywords: non-local solution, fredholm-volterra integral equation, system of fredholm integral equations, weakly kernel, algebraic system.

I. Introduction

The integral equations have received considerable interest of many applications in different mathematical areas of sciences. Therefore, the authors established many analytic and numeric methods to obtain the solutions of the integral equations. For some works, the reader can forward to the following references [1-5]. For the analytical methods, one can use degenerate kernel method, Cauchy method (singular integral method), Laplace transformation method, Fourier transformation method, potential theory method, and Krien's method. More information for the analytic methods can be found in Muskhelishvili [6], Popov [7], Tricomi [8], Hochstad [9] and Green [10]. More recently, since analytical methods on practical problems often fail, numerical solutions of these equations are much studied subjected of numerous works. The interested reader should consult the fine exposition by Atkinson [11], Delves and Mohamed [12], Golberg [13] and Linz [14] for some different numerical methods. In [15] a mixed integral equation in one-dimensional is considered, under certain conditions, and the solution in a series form is obtained. In addition, a mixed integral equation of the second kind, when the Fredholm kernel takes a logarithmic form is discussed and solved in [16]. In [17] Kauthen used a collection method to solve the mixed integral equation with continuous kernel, numerically.

Consider the mixed integral equation of the second kind:

$$\mu\phi(x,t)=f(x,y)-H(x,t,\phi(x,t))+\lambda\int_{-1}^{1}k(|x-y|)\phi(y,t)dy+\lambda\int_{0}^{t}F(|t-\tau|)\phi(x,\tau)d\tau. \quad (1)$$

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The given two continuous function $f(x,y)$ and $H(x,t,\phi(x,t))$ define in the Banach space $L_2[-1,1]\times C[0,T]$, $0\le t\le T<1$, where the first function is called the free term and the second function is known as the memory of the integral equation. The known functions $k(|x-y|)$ and $F(t,\tau)$ represent the kernels of Fredholm and Volterra integral terms, respectively. The unknown function $\phi(x,t)$ represents the solution of (1).The constant λ may be complex, has a physical meaning and μ, defines the kind of integral equation.

In order to guarantee the existence and uniqueness solution of Eq.(1), we assume the following conditions:

(i) The given function $H(x,t,\phi(x,t))$ with its partial derivatives with respect to position and time is continuous in the space $L_2[\Omega]\times C[0,T]$, for the constant $\mathrm{L}>\mathrm{L}_1$ and $\mathrm{L}>\mathrm{L}_2$ satisfies the following conditions:

$$(i.a)\left|H(x,t,\phi(x,t))\right|\le \mathrm{L}_1\left|\phi(x,t)\right| \quad (i.b)\left|H(x,t,\phi(x,t))-H(x,t,\psi(x,t))\right|\le \mathrm{L}_2\left|\phi(x,t)-\psi(x,t)\right|$$

Where the norm is defined as $\|\phi(x,t)\|=\max\limits_{0\le t\le T}\int_0^t\left[\int_{-1}^{1}|\phi(x,\tau)|^2dx\right]^{1/2}d\tau.$

(ii) The Fredholm kernel satisfies $\left[\int_{-1}^{1}\int_{-1}^{1}k(|x-y|)^2dydx\right]=M^2$, ($M$ is a constant).

(iii) The discontinuous function $F(|t-\tau|)$ is absolutely integrable with respect to τ for all

$0\le t\le T<1$, and satisfies $\int_0^t F(|t-\tau|)d\tau=N$, (N is constant).

(iv) The given function $f(x,t)$ with its partial derivatives with respect to position x and time t are continuous in the space $L_2[-1,1]\times C[0,T]$ and its norm is defined as

$$\|f(x,t)\|=\max_{0\le t\le T}\left|\int_0^t\left\{\int_{-1}^{1}f^2(x,\tau)dx\right\}^{\frac{1}{2}}d\tau\right|=V=\text{costant}.$$

a) Theorem 1 (without proof): If the conditions (i) –(iii) are satisfied, then Eq. (1) has a unique solution $\phi(x,t)$ in the space $L_2[-1,1]\times C[0,T]$ inside the sphere of radius ρ such that:

$$\rho=V/[\mu-(\mathrm{L}+|\lambda|M+|\lambda|TN)],\quad (\mathrm{L}+|\lambda|M+|\lambda|TN)<\mu.\ \blacksquare \tag{2}$$

In the remainder, of this paper a suitable quadratic numerical method is used to reduce the mixed integral equation into **SFIEs** of the second kind. Then using PNM, as a suitable numerical method to solve the singular integral equations, the **SFIEs** will reduce to an algebraic system. Finally, many numerical results are calculated when the kernel takes a logarithmic form and Carleman function forms. Moreover, the error estimate, in each case, is computed.

II. System of Fredholm Integral Equations

In this section, a quadratic numerical method is used; see Atkinson [11], Delves and Mohamed [12], to obtain **SFIEs** of the second kind, where the existence and uniqueness of the integral

Ref

11. K. E. Atkinson, A Survey of Numerical Method for the Solution of Fredholm Integral Equation of the Second Kind, SIAM, Philadelphia, 1976.

system are considered. Moreover, the equivalence between the **F-VIE** and the **SFIEs** is obtained.

For this, we divide the interval $[0,T]$ into m subintervals, by means of the points $0=t_0<t_1<...<t_m=T$, where $t=t_i, \tau=t_j, i, j=0,1,2,...,m$, then using the quadrature formula. The formula (1) can be adapted in the following form

Notes

$$\mu_i\phi_i(x)=f_i(x)-H_i(x,\phi_i(x))+\lambda\sum_{j=0}^{i-1}\omega_j F_{i,j}\phi_j(x)+\lambda\int_{-1}^{1}k(|x-y|)\phi_i(y)dy, \qquad i=0,1,2,...,m. \quad (3)$$

Here, we used the following nations:

$$\phi(x,t_i)=\phi_i(x),\ H(x,t_i,\phi(x,t_i))=H_i(x,\phi_i(x));\ F(|t_i-t_j|)=F_{i,j};\ \mu_i=(\mu-\omega_i F_{i,i}),$$

$$f(x,t_i)=f_i(x); \qquad \omega_j=\begin{cases} h_j/2 & j=0,\ j=i \\ h_j & 0<j<i \end{cases}.$$

Here, $h_i=\max\limits_{0\le j\le m}(t_{j+1}-t_j), h_i$ is the step size of integration, and ω_j are the weights,

The value of i and p depend on the number of derivative of $v(t,\tau)$ with respect to t for all $\tau\in[0,T]$. Here, we neglect the term of the error of the quadratic numerical method $0(h_i^{p+1})$.

If in (3) $\mu=\omega_i F_{i,i}$, we have homogeneous **SFIEs**. While, the system is nonhomogeneous if $\mu_i=(\mu-\omega_i F_{i,i})\ne 0$.

Definition 1: The estimate error $R_{m,i}$, of the quadratic method, is determined by the relation

$$R_{m,i}=\left|\int_0^t F(|t_i-t_j|)\phi(x,\tau)d\tau-\sum_{j=0}^{i}\omega_j F_{i,j}\phi_j(x)\right|, \qquad i=1,2,...,m\ .\blacksquare \quad (4)$$

Remark1: Consider $\Phi(x)=\{\phi_0(x),\phi_1(x),...,\phi_i(x),...\}$ be the set of all continuous functions in E, where $\phi_i(x)\in L_2[-1,1]$ for all i, and define on E the norm by

$$\|\phi\|_E=\max_i\left[\int_{-1}^{1}|\phi_i(x)|^2dx\right]^{\frac{1}{2}}=\max_i\|\phi_i\|_{L_2[\Omega]} \qquad ,\forall i. \quad (5)$$

Then, E is a Banach space.$\blacksquare$

In order to guarantee the existence of a unique solution of (3) in the Banach space E, we assume the following:

(1). $\max\limits_j\sum_{j=0}^{i-1}|\omega_j F_{i,j}|\le N^*$, (2). $\left[\int_{-1}^{1}\int_{-1}^{1}|k(|x-y|)|^2dydx\right]=M^2$, (3). $\|f\|_E=\max\limits_i\|f_i\|=V^*,\ \forall i.$

(4). The functions $H_i(x,\phi_i(x))$, for the constants $L^*>L_1^*$ and $L^*>L_2^*$ satisfies the following conditions:

(4.a). $|H_i(x,\phi_i(x))|\le L_1^*|\phi_i(x)|$, (4b). $|H_i(x,\phi_i(x))-H_i(x,\psi_i(x))|\le L_2^*|\phi_i(x)-\psi_i(x)|$.

b) Theorem 2 (without proof): The **SFIEs** (4) have a unique solution $\phi_i\left(x\right)$ in the Banach space E under the conditions**:**

$$\left|\mu^*\right| \le V^* / [1-\left(L^* + |\lambda| N^* + |\lambda| M\right)]; \quad \mu^* = \min_i |\mu_i|. \blacksquare \tag{6}$$

III. Product Nystrom Method

In this section, we discuss the numerical solution of Eq. (3), when the kernel of position has a singular term, using **PNM**, see [11, 12].

For this, write the singular kernel of (3) in the form

$$k\left(|x-y|\right) = \overline{k\left(|x-y|\right)}\,\ell(x,y) \tag{7}$$

Where, $\overline{k\left(|x-y|\right)}$ is a badly behavior, while $\ell(x,y)$ well behavior.

In view of (7), the **SFIEs** (3), can be adapted in the form

$$\mu_i \phi_{i,p} = f_{i,p} - H_{i,p}\left(\phi_{i,p}\right) + \lambda \sum_{j=0}^{i-1} \omega_j F_{i,j} \phi_{j,p} + \lambda \sum_{q=0}^{N} w_{p,q} \ell_{p,q} \phi_{i,q}, i = 1,2,\ldots,N. \tag{8}$$

Where, we use the following definitions $\phi_i\left(x_p\right) = \phi_{i,p}; H_i\left(x_p, \phi_i\left(x_p\right)\right) = H_{i,p}\left(\phi_{i,p}\right)$:

$\ell(x_p, x_q) = \ell_{p,q}\ x_p = y_p = -1 + ph_1, p = 0,1,\ldots,N$ with $h_1 = \frac{2}{N}, N$ even.

In addition, the weight terms $w_{p,q}$ are given by, see [12],

$$w_{p,0} = \beta_{p,1}, \quad w_{p,2q+1} = -2\gamma_{p,q+1}, \quad w_{p,2q} = \alpha_{pq} + \beta_{p,q+1}, \qquad w_{p,N} = \alpha_{p,N/2}. \tag{9}$$

$$\alpha_{p,q} = \frac{h_1}{2}\int_0^2 \xi(\xi-1)\overline{k\left(|y_p - (y_{2q-2} + \xi h_1)|\right)} d\xi, \quad \beta_{p,q} = \frac{h_1}{2}\int_0^2 (\xi-1)(\xi-2)\overline{k\left(|y_p - (y_{2q-2} + \xi h_1)|\right)} d\xi,$$

$$\gamma_{p,q} = \frac{h_1}{2}\int_0^2 \xi(2-\xi)\overline{k\left(|y_p - (y_{2q-2} + \xi h_1)|\right)} d\xi.$$

Here, in (9), we introduce the change of variable $y = y_{2q-2} + \xi h_1, \quad 0 \le \xi \le 2$

Definition 2: The relation between the estimate local error $R_{N,q}$ and Eq. (8) is

$$R_{N,q} = \left| \int_{-1}^{1} k\left(x,y\right)\phi_i\left(y\right)dy - \sum_{q=0}^{N} w_{pq} \ell_{p,q} \phi_{i,q} \right|. \blacksquare \tag{10}$$

Definition 3: The **PNM** is said to be convergent of order r in the interval $\left[-1,1\right]$, if and only if for sufficiently large N, there exists a constant $s > 0$ independent of N such that

$$\left\| \phi_i\left(x\right) - \left(\phi_i\left(x\right)\right)_N \right\|_\infty \le s N^{-r}. \blacksquare \tag{11}$$

In order to guarantee the existence of unique solution of the **NAS** (8) in the Banach space ℓ_∞, we assume the following conditions:

Ref

12. L. M. Delves and J.L. Mohamed, Computational Methods for Integral Equations, Phila-delphia, 1985.

$(a)\max_j \sum_{j=0}^{i-1} \left|\omega_j F_{i,j}\right| \le N^*$; $(b)\sup_q \sum_{q=0}^{N} \left|w_{p,q}\ell_{p,q}\right| \le c$; (c) $\|f\|_{\ell_\infty} = \sup_{i,p}\left|f_{i,p}\right| = V_1^*$ (N^*, V_1^*, c constants).

(d) For the constants $\mathrm{L}' > \mathrm{L}_1'$ and $\mathrm{L}' > \mathrm{L}_2'$, $H_{i,p}\left(\phi_{i,p}\right)$ satisfies the conditions:

$$(d.1)\ \left|H_{i,p}\left(\phi_{i,p}\right)\right| \le \mathrm{L}_1'\left|\phi_{i,p}\right|, \quad (d.2)\ \left|H_{i,p}\left(\phi_{i,p}\right) - H_{i,p}\left(\psi_{i,p}\right)\right| \le \mathrm{L}_2'\left|\phi_{i,p} - \psi_{i,p}\right|.$$

a) Theorem3 (without proof): The **NAS** (8) has a unique solution $\phi_{i,p}$ in the space ℓ_∞; $\|\phi\|_{\ell_\infty} = \sup_{i,p}\left|\phi_{i,p}\right|$, under the following conditions

$$\left\|\phi_{i,p}\right\| \le V^* / [\mu^* - \left(\mathrm{L}' + |\lambda| N^* + |\lambda| c\right)]; \quad \mu^* = \min_i |\mu_i|. \blacksquare \tag{12}$$

The equivalence between the **NAS** (8) and the **SFIEs** (3) is satisfies if:

$$\sum_{q=0}^{N} w_{p,q}\ell_{p,q}\phi_i\left(qh_1\right) \to \int_{-1}^{1} k\left(x,y\right)\phi_i\left(y\right)dy \ (N \to \infty).$$

Theorem 4: Under the conditions of theorem3, the sequence of functions, $\{\Phi_N\} = \left\{\left(\phi_{i,p}\right)_N\right\}$, of (10) convergence uniformly to the solution $\Phi = \left\{\phi_{i,p}\right\}$ of (3) in the space ℓ_∞.

Proof: From Eq. (8), we write

$$\left|\phi_{i,p} - \left(\phi_{i,p}\right)_N\right| \le \frac{1}{|\mu_i|}\left\{\left|H_{i,p}\left(\phi_{i,p}\right) - H_{i,p}\left(\left(\phi_{i,p}\right)_N\right)\right| + |\lambda| \sum_{j=0}^{i-1}\left|\omega_j F_{i,j}\right|\left|\phi_{j,p} - \left(\phi_{j,p}\right)_N\right| + \right.$$
$$\left. |\lambda| \sum_{q=0}^{N}\left|w_{p,q}\ell_{p,q}\right|\left|\phi_{i,q} - \left(\phi_{i,q}\right)_N\right|\right\}. \tag{13}$$

Using the conditions (a-d) of theorem 3, and using condition (3) of theorem 2, the above inequality can be adapted in the form

$$\sup_{i,p}\left|\phi_{i,p} - \left(\phi_{i,p}\right)_N\right| \le \sigma\left\|\Phi - \Phi_N\right\|_{\ell_\infty}, \quad \sigma = \frac{\left(\mathrm{L}' + |\lambda| N^* + |\lambda| c\right)}{\mu^*} < 1.$$

Hence, we have $\left\|\Phi - \Phi_N\right\|_{\ell_\infty} \to 0$ as $N \to \infty$. $\blacksquare$

Definition 4: The estimate total error of **PNM**, $R_{m,N} = R_{m,i} + R_{N,q}$ of Eq.(1) is determined by the following relation

$$R_{m,N} = \left|\int_{-1}^{1} k\left(x,y\right)\phi\left(y,t\right)dy + \int_0^t F\left(t,\tau\right)\phi\left(x,\tau\right)d\tau - \sum_{q=0}^{N} w_{p,q}\ell_{p,q}\phi_{i,q} - \sum_{j=0}^{i}\omega_j F_{i,j}\phi_{j,p}\right|. \tag{14}$$

●The equivalence between the NAS and F-VIE:

When $m, N \to \infty$, the sum

$$\sum_{q=0}^{N} w_{pq}\ell_{p,q}\phi_{i,q} + \sum_{j=0}^{i}\omega_j v_{ij}\phi_{j,p} \to \int_{-1}^{1} k\left(|x-y|\right)\phi\left(y,t\right)dy + \int_0^t F\left(t,\tau\right)\phi\left(x,\tau\right)d\tau.$$

Then the solution of the **NAS** (8) becomes the solution of **F-VIE** (1).

Theorem 5: **if** the conditions (i) and (ii) of theorem1 are satisfied, then the sequence of functions $\{\Phi_{m,N}\}=\{\phi_{m,N}(x,t)\}$ convergence uniformly to the exact solution $\Phi=\phi(x,t)$ of Eq. (1)) in the space $L_2[-1,1]\times C[0,T]$.

Proof: the formula (1) with its approximation solution gives

$$\mu\|\Phi-\Phi_{m,N}\|\le\|H(x,t,\phi(x,t))-H(x,t,\phi_{m,N}(x,t))\|+|\lambda|\left\|\int_{-1}^{1}|k(|x-y|)||\phi(y,t)-\phi_{m,N}(y,t)|dy\right\|+ \tag{19}$$

$$|\lambda|\left\|\int_0^t|F(t,\tau)||\phi(x,\tau)-\phi_{m,N}(x,\tau)|d\tau\right\|.$$

Using $|F(t,\tau)|\le N$, $F(t,\tau)$ is continuous, then with the aid of conditions (i-2), (ii) then, applying Cauchy Schwarz inequality, the above inequality becomes

$$\mu\|\Phi-\Phi_{m,N}\|\le\xi\|\Phi-\Phi_{m,N}\|,\quad \xi=(\mathrm{L}_2+|\lambda|NT+|\lambda|M)<1.$$

Hence, we have $\|\Phi-\Phi_{m,N}\|_{L_2[\Omega]\times C[0,T]}\to 0$ as $m,N\to\infty$.

Corollary 2: the total error $R_{m,N}$ satisfies $\lim_{m,N\to\infty} R_{m,N}=0$.

IV. APPLICATIONS

Assume the **NF-VIE** of the second kind:

$$\phi(x,t)=f(x,t)-H(x,t,\phi(x,t))+\lambda\int_{-1}^{1}k(|x-y|)\,\phi(y,t)\,dy+\lambda\int_0^t F(t,\tau)\,\phi(x,\tau)\,d\tau$$

Where the kernel of Fredholm term has Carleman $k(|x-y|)=|x-y|^{-\upsilon}$, $0<\upsilon<\frac{1}{2}$, and Logarithmic $k(|x-y|)=\log|x-y|$ kernel, the historical function $H(x,t,\phi(x,t))$ take a linear form $(x,t)\,t^2$, and a nonlinear form $\phi^2(x,t)$, the kernel of Volterra term $F(t,\tau)=t\,\tau^2$, $\lambda=0.01$ and the exact solution : $\phi(x,t)=x^2\,t^2$. Applying **PNM**, the results are obtained numerically by Maple 12 software, for $t=0.0008\,,0.05\,,0.8,$ with $\alpha=0\,.2\,,0.3$ and 0.4. The interval $[-1,1]$ is divided into $N=21$ unites.

Application 1: When the singular kernel takes the Carleman function form

$$k(|x-y|)=|x-y|^{-\upsilon}\ ,\ 0<\upsilon<\frac{1}{2}$$

Case 1: $H(x,t,\phi(x,t))=t\,\phi(x,t),$

Here, the integral equation (1) takes the linear form

$$\delta(t)\phi(x,t)=f(x,t)+\lambda\int_{-1}^{1}k(|x-y|)\,\phi(y,t)\,dy+\lambda\int_0^t F(t,\tau)\,\phi(x,\tau)\,d\tau;\quad \delta=(1-t)$$

Notes

Notes

Table (1)

(1-i) T=0.0008

X	EX.	$\alpha = 0.2$		$\alpha = 0.3$		$\alpha = 0.4$	
		APP	ERR	APP	ERR	APP	ERR
-1	6.400E-07	6.400E-07	0.000E+00	6.400E-07	4.000E-16	6.400E-07	5.000E-16
-0.8	4.096E-07	4.096E-07	1.000E-16	4.096E-07	3.000E-16	4.096E-07	2.000E-16
-0.6	2.304E-07	2.304E-07	3.000E-16	2.304E-07	2.000E-16	2.304E-07	1.000E-16
-0.4	1.024E-07	1.024E-07	1.000E-16	1.024E-07	0.000E-00	1.024E-07	1.000E-16
-0.2	2.560E-08	2.560E-08	5.000E-17	2.560E-08	4.000E-17	2.560E-08	1.000E-17
0	0.000E-00	5.200E-18	5.200E-18	2.200E-18	2.200E-18	-7.400E-18	7.400E-18
0.2	2.560E-08	2.560E-08	3.000E-17	2.560E-08	2.000E-17	2.560E-08	2.000E-17
0.4	1.024E-07	1.024E-07	1.000E-16	1.024E-07	0.000E-00	1.024E-07	1.000E-16
0.6	2.304E-07	2.304E-07	3.000E-16	2.304E-07	2.000E-16	2.304E-07	1.000E-16
0.8	4.096E-07	4.096E-07	0.000E+00	4.096E-07	0.000E-00	4.096E-07	0.000E+00
1	6.400E-07	6.400E-07	3.000E-16	6.400E-07	3.000E-16	6.400E-07	5.000E-16

Table (2)

(1-ii) T=0.05

X	EX.	$\alpha = 0.2$		$\alpha = 0.3$		$\alpha = 0.4$	
		APP	ERR	APP	ERR	APP	ERR
-1	2.500E-03	2.500E-03	5.000E-12	2.500E-03	6.000E-12	2.500E-03	4.000E-12
-0.8	1.600E-03	1.600E-03	2.000E-12	1.600E-03	4.000E-12	1.600E-03	2.000E-12
-0.6	9.000E-04	9.000E-04	1.800E-12	9.000E-04	1.600E-12	9.000E-06	8.910E-12
-0.4	4.000E-04	4.000E-04	9.000E-13	4.000E-04	8.000E-13	4.000E-04	8.000E-13
-0.2	1.000E-04	1.000E-04	2.000E-13	1.000E-04	2.000E-13	1.000E-04	3.000E-13
0	0.000E+00	3.060E-14	3.060E-14	3.160E-14	3.160E-14	3.450E-14	3.450E-14
0.2	1.000E-04	1.000E-04	2.000E-13	1.000E-04	2.000E-13	1.000E-04	3.000E-13
0.4	4.000E-04	4.000E-04	9.000E-13	4.000E-04	1.000E-12	4.000E-04	7.000E-13
0.6	9.000E-04	9.000E-04	1.900E-12	9.000E-04	1.700E-12	9.000E-04	1.500E-12
0.8	1.600E-03	1.600E-03	3.000E-12	1.600E-03	3.000E-12	1.600E-03	2.000E-12
1	2.500E-03	2.500E-03	5.000E-12	2.500E-03	6.000E-12	2.500E-03	4.000E-12

Table (3)

(1-iii) T=0.8

X	EX.	$\alpha = 0.2$		$\alpha = 0.3$		$\alpha = 0.4$	
		APP	ERR	APP	ERR	APP	ERR
-1	0.64	0.64006387	6.38739E-05	0.640063922	6.39213E-05	0.640063992	6.39914E-05
-0.8	0.4096	0.409641023	4.10232E-05	0.409641088	4.10877E-05	0.409641183	4.11831E-05
-0.6	0.2304	0.230423216	2.32163E-05	0.230423263	2.32632E-05	0.230423329	2.33291E-05
-0.4	0.1024	0.10241049	1.04902E-05	0.102410519	1.05193E-05	0.102410558	1.05578E-05
-0.2	2.56E-02	2.56E-02	2.85211E-06	2.56E-02	2.86893E-06	2.56E-02	2.88919E-06
0	0	3.06E-07	3.05785E-07	3.18E-07	3.18284E-07	3.32E-07	3.32246E-07
0.2	2.56E-02	2.56E-02	2.85211E-06	2.56E-02	2.86892E-06	2.56E-02	2.88918E-06
0.4	0.1024	0.10241049	1.04901E-05	0.102410519	1.05191E-05	0.102410558	1.05577E-05
0.6	0.2304	0.230423217	2.32164E-05	0.230423263	2.32633E-05	0.230423329	2.33293E-05
0.8	0.4096	0.409641023	4.10228E-05	0.409641088	4.10877E-05	0.409641183	4.11832E-05
1	0.64	0.640063874	6.38738E-05	0.640063922	6.39213E-05	0.640063992	6.39915E-05

Case 2: When the non-local term is nonlinear $H(x,t,\phi(x,t)) = \phi^2(x,t)$

Table (4)

(2-i) T=0.0008

X	EX.	$\alpha = 0.2$		$\alpha = 0.3$		$\alpha = 0.4$	
		APP	ERR	APP	ERR	APP	ERR
-1	6.400E-07	6.351E-07	4.901E-09	6.344E-07	5.620E-09	6.333E-07	6.680E-09
-0.8	4.096E-07	4.043E-07	5.290E-09	4.033E-07	6.263E-09	4.019E-07	7.688E-09
-0.6	2.304E-07	2.253E-07	5.085E-09	2.246E-07	5.781E-09	2.236E-07	6.753E-09
-0.4	1.024E-07	9.757E-08	4.828E-09	9.715E-08	5.249E-09	9.660E-08	5.804E-09
-0.2	2.560E-08	2.096E-08	4.640E-09	2.073E-08	4.875E-09	2.045E-08	5.153E-09
0	0.000E+00	4.573E-09	4.573E-09	4.742E-09	4.742E-09	4.925E-09	4.925E-09
0.2	2.560E-08	2.096E-08	4.640E-09	2.073E-08	4.874E-09	2.045E-08	5.152E-09
0.4	1.024E-07	9.758E-08	4.823E-09	9.716E-08	5.241E-09	9.661E-08	5.792E-09
0.6	2.304E-07	2.253E-07	5.064E-09	2.247E-07	5.748E-09	2.237E-07	6.708E-09
0.8	4.096E-07	4.044E-07	5.233E-09	4.034E-07	6.176E-09	4.020E-07	7.573E-09
1	6.400E-07	6.354E-07	4.563E-09	6.350E-07	4.959E-09	6.345E-07	5.485E-09

Table (5)

(2-ii) T=0.05

X	EX.	$\alpha = 0.2$		$\alpha = 0.3$		$\alpha = 0.4$	
		APP	ERR	APP	ERR	APP	ERR
-1	2.5E-03	2.481E-03	1.905E-05	2.478E-03	2.185E-05	2.474E-03	2.597E-05
-0.8	1.6E-03	1.579E-03	2.060E-05	1.576E-03	2.439E-05	1.570E-03	2.994E-05
-0.6	9.0E-04	8.802E-04	1.983E-05	8.775E-04	2.254E-05	8.737E-04	2.633E-05
-0.4	4.0E-04	3.812E-04	1.885E-05	3.795E-04	2.049E-05	3.773E-04	2.265E-05
-0.2	1.0E-04	8.188E-05	1.812E-05	8.096E-05	1.904E-05	7.988E-05	2.012E-05
0	0.0E+00	1.786E-05	1.786E-05	1.853E-05	1.853E-05	1.924E-05	1.924E-05
0.2	1.0E-04	8.188E-05	1.812E-05	8.096E-05	1.904E-05	7.988E-05	2.012E-05
0.4	4.0E-04	3.812E-04	1.882E-05	3.795E-04	2.046E-05	3.774E-04	2.261E-05
0.6	9.0E-04	8.803E-04	1.975E-05	8.776E-04	2.241E-05	8.738E-04	2.616E-05
0.8	1.6E-03	1.580E-03	2.038E-05	1.576E-03	2.405E-05	1.571E-03	2.949E-05
1	2.5E-03	2.482E-03	1.774E-05	2.481E-03	1.928E-05	2.479E-03	2.132E-05

Table (6)

(2-iii) T=0.8

X	EX.	$\alpha = 0.2$		$\alpha = 0.3$		$\alpha = 0.4$	
		APP	ERR	APP	ERR	APP	ERR
-1	0.64	0.63788999	0.00211002	0.637573604	0.002426397	0.637107531	0.00289247
-0.8	0.4096	0.40671993	0.00288007	0.406183063	0.003416937	0.405396058	0.004203942
-0.6	0.2304	0.22693220	0.00346780	0.226453153	0.003946847	0.225783412	0.004616588
-0.4	0.1024	0.09838890	0.00401110	0.098036574	0.004363426	0.097572594	0.004827406
-0.2	2.56E-02	2.12E-02	0.00443337	2.09E-02	0.004658686	2.07E-02	0.004925819
0	0	4.60E-03	0.00459856	4.77E-03	0.004769697	4.95E-03	0.004954065
0.2	2.56E-02	2.12E-02	0.00443282	2.09E-02	0.004657871	2.07E-02	0.004924744
0.4	0.1024	0.09839359	0.00400641	0.098043649	0.004356351	0.097582076	0.004817924
0.6	0.2304	0.22694706	0.00345294	0.226475929	0.003924072	0.225814449	0.004585551
0.8	0.4096	0.40675174	0.00284827	0.406231277	0.003368724	0.405460083	0.004139917
1	0.64	0.63803898	0.00196103	0.637865533	0.002134467	0.637635663	0.002364338

Notes

Notes

Application 2: When the singular kernel takes the logarithmic function

$$k(|x-y|) = \ln|x-y|$$

Table (7)

Case 3: $H(x,t,\phi(x,t)) = t^2\,\phi(x,t)$

X	T=0.0008			T=0.05			T=0.8		
	EX	APP	ERR	EX	APP	ERR	EX	APP	ERR
-1	6.400E-07	6.400E-07	6.000E-16	2.5E-03	2.500E-03	4.000E-12	6.400E-01	0.64006336	6.33562E-05
-0.8	4.096E-07	4.096E-07	2.000E-16	1.6E-03	1.600E-03	2.000E-12	4.096E-01	0.40964041	4.04056E-05
-0.6	2.304E-07	2.304E-07	3.000E-16	9.0E-04	9.000E-04	1.000E-12	2.304E-01	0.23042266	2.26594E-05
-0.4	1.024E-07	1.024E-07	1.000E-16	4.0E-04	4.000E-04	7.000E-13	1.024E-01	0.10241001	1.00122E-05
-0.2	2.560E-08	2.560E-08	0.000E-00	1.0E-04	1.000E-04	0.000E+00	2.560E-02	2.56E-02	2.43323E-06
0	0.000E-00	3.605E-18	3.605E-18	0.0E+00	2.703E-15	2.703E-15	0.000E-00	9.17E-08	9.17299E-08
0.2	2.560E-08	2.560E-08	1.000E-17	1.0E-04	1.000E-04	7.000E-14	2.560E-02	2.56E-02	2.43325E-06
0.4	1.024E-07	1.024E-07	2.000E-16	4.0E-04	4.000E-04	5.000E-13	1.024E-01	0.10241001	1.00122E-05
0.6	2.304E-07	2.304E-07	0.00E+00	9.0E-04	9.000E-04	1.000E-12	2.304E-01	0.23042266	2.26595E-05
0.8	4.096E-07	4.096E-07	1.000E-16	1.6E-03	1.600E-03	3.000E-12	4.096E-01	0.40964041	4.04057E-05
1	6.400E-07	6.400E-07	5.000E-16	2.5E-03	2.500E-03	4.000E-12	6.400E-01	0.64006336	6.33562E-05

Table (8)

Case 4: $H(x,t,\phi(x,t)) = \phi^2(x,t)$

X	T=0.0008			T=0.05			T=0.8		
	EX	APP	ERR	EX	APP	ERR	EX	APP	ERR
-1	6.400E-07	6.388E-07	1.189E-09	2.500E-03	2.495E-03	4.621E-06	6.400E-01	6.395E-01	4.816E-04
-0.8	4.096E-07	4.084E-07	1.229E-09	1.600E-03	1.595E-03	4.785E-06	4.096E-01	4.090E-01	6.464E-04
-0.6	2.304E-07	2.300E-07	4.412E-10	9.000E-04	8.983E-04	1.720E-06	2.304E-01	2.301E-01	2.804E-04
-0.4	1.024E-07	1.029E-07	4.730E-10	4.000E-04	4.018E-04	1.846E-06	1.024E-01	1.028E-01	4.066E-04
-0.2	2.560E-08	2.675E-08	1.149E-09	1.000E-04	1.045E-04	4.487E-06	2.560E-02	2.670E-02	1.097E-03
0	0.000E+00	1.394E-09	1.394E-09	0.000E+00	5.447E-06	5.447E-06	0.00E+00	1.394E-03	1.394E-03
0.2	2.560E-08	2.675E-08	1.149E-09	1.000E-04	1.045E-04	4.487E-06	2.560E-02	2.670E-02	1.097E-03
0.4	1.024E-07	1.029E-07	4.730E-10	4.000E-04	4.018E-04	1.846E-06	1.024E-01	1.028E-01	4.066E-04
0.6	2.304E-07	2.300E-07	4.412E-10	9.000E-04	8.983E-04	1.720E-06	2.304E-01	2.301E-01	2.804E-04
0.8	4.096E-07	4.084E-07	1.229E-09	1.600E-03	1.595E-03	4.785E-06	4.096E-01	4.090E-01	6.464E-04
1	6.400E-07	6.388E-07	1.189E-09	2.500E-03	2.495E-03	4.621E-06	6.400E-01	6.395E-01	4.816E-04

V. Conclusions

●The non-local term is called the histories of the problem and is considered with negative sign

I- For the Carleman kernel $k(|x-y|) = |x-y|^{-\upsilon}$ and for the linear non- local term $H(x,t,\phi(x,t)) = t\,\phi(x,t)$, *we* have E. $_{\text{Max.}}$ and E.Min. respectively, the following:

(i) <u>In Table (1) at T=0.0008</u>: for $\alpha = 0.2$ are ,respectively3.000E-16 , 0.000E-00. While $for\ \alpha = 0.3$ are 4.000E-16 and 0.000E-00. Finally at $\alpha = 0.4$ are 5.000E-16 and 0.000E-00.

(ii) <u>In Table (2) at T=0.005</u>: for $\alpha = 0.2$; 5.000E-12 and 2.000E-13. For $\alpha = 0.3$ are 6.000E-12 and 1.000E-12. While $\propto = 0.4$ are 8.910E-12 and 3.450E-14.

(iii) <u>In Table (3) at T=0.8</u>: for $\alpha = 0.2$; 6.38738E-05 and 3.05785E-07; for $\alpha = 0.3$ are 6.39213E-05 and 1.05193E-05. Finally for $\propto = 0.4$ are 6.39914E-05 and 1.05577E-05.

II- For the Carleman kernel and nonlinear non- local term $H\big(x,t,\phi(x,t)\big) = \phi^2(x,t)$

(iv) In Table (4) at T=0.0008: for $\alpha = 0.2$ we obtain 5.290E-09, 4.563E-09. Also, $for\ \alpha = 0.3$: we get 6.263E-09and 4.742E-09. Finally for $\alpha = 0.4$ we have 7.688E-09and 4.925E-09.

(v) In Table (5) at T=0.005: for $\alpha = 0.2$; 2.060E-05and 1.774E-05. For $\alpha = 0.3$ are 2.439E-05and 1.853E-05. For $\propto = 0.4$ are 2.994E-05 and 1.924E-05.

(vi) In Table (6) at T=0.8: for $\alpha = 0.2$; 0.00459856 and 0.00196103; for $\alpha = 0.3$ are 0.004769697and 0.002134467. For $\propto = 0.4$ are 0.004954065and 0.002364338.

III- For a logarithmic kernel $k(|x-y|) = \ln|x-y|$ and linear non- local term $H\big(x,t,\phi(x,t)\big) = t^2\,\phi(x,t)$ the $E_{Max.}$ and E. Min are given respectively, as the following:

(vii) In Table (7) at T=0.0008:, we have 6.000E-16 and 0.000E-00. At T=0.005 we have 4.000E-12; 0.000E-00. At T=0. 8, 6.33562E-05, 9.17299E-08.

IV- For a logarithmic and non linear non- local term $H\big(x,t,\phi(x,t)\big) = \phi^2(x,t)$ the $E_{Max.}$ and E.Min are respectively,

(viii) In Table (8) at T=0.0008:, we have 1.229E-09 and 4.412E-10. At T=0.005 5.447E-06; 1.720E-06. At T=0. 8, 4.816E-04, 1.097E-03.

From the above results, we deduce that the error in the linear non- local function is less than the error in the nonlinear case. This result is true, where the integral equation without non-local term in the linear case.

Notes

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Thermomechanical Response of Transversely Isotropic Thermoelastic Solids with Two Temperature and without Energy Dissipation Due to Time Harmonic Sources

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Abstract- The paper is concerned with two dimensional deformation in a homogeneous, transversely isotropic thermoelastic solids without energy dissipation and with two temperatures due to various sources. Assuming the disturbances to be harmonically time-dependent, the transformed solution is obtained in the frequency domain. The application of a time harmonic concentrated and distributed sources have been considered to show the utility of the solution obtained. The transformed components of displacements, stresses and conductive temperature distribution so obtained are inverted numerically using a numerical inversion technique. Effect of anisotropy and two temperature on the resulting expressions are depicted graphically.

Keywords: transversely isotropic, thermoelastic, laplace transform, fourier transform, concentrated force, distributed sources.

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Thermomechanical Response of Transversely Isotropic Thermoelastic Solids with Two Temperature and without Energy Dissipation Due to Time Harmonic Sources

Nidhi Sharma [α], Rajneesh Kumar [σ] & Parveen Lata [ρ]

Abstract- The paper is concerned with two dimensional deformation in a homogeneous, transversely isotropic thermoelastic solids without energy dissipation and with two temperatures due to various sources. Assuming the disturbances to be harmonically time-dependent, the transformed solution is obtained in the frequency domain. The application of a time harmonic concentrated and distributed sources have been considered to show the utility of the solution obtained. The transformed components of displacements, stresses and conductive temperature distribution so obtained are inverted numerically using a numerical inversion technique. Effect of anisotropy and two temperature on the resulting expressions are depicted graphically.

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I. Introduction

Thermoelasticity with two temperatures is one of the non classical theories of thermomechanics of elastic solids. The main difference of this theory with respect to the classical one is a thermal dependence. During the last few decades, an intense amount of attention has been paid to the theories of generalized thermoelasticity as they attempt to overcome the shortcomings of the classical coupled theory of thermoelasticity, i.e., infinite speed of propagation of thermoelasticity disturbances, unsatisfactory response of a solid body to short laser action, and poor description of thermoelastic behaviour at low temperature.

Green and Naghdi [5] and [6] proposed three new thermoelastic theories based on an entropy equality rather than usual entropy inequality and proposed three models which are subsequently referred to as GN-I, II, and III models. The linearised version of model-I corresponds to classical Thermoelastic model. In model -II, the internal rate of production entropy is taken to be identically zero implying no dissipation of thermal energy . This model admits un-damped thermoelastic waves in a thermoelastic material and is best known as theory of thermoelasticity without energy dissipation. The principal feature of this theory is in contrast to classical thermoelasticity associated with Fourier's law of heat conduction, the heat flow does not involve energy dissipation. This theory permits the transmission of heat as thermal waves at finite speed. Model-III includes the previous two models as special cases and admits

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dissipation of energy in general. In context of Green and Naghdi model many applications have been found. Chandrasekharaiah and Srinath [1] discussed the thermoelastic waves without energy dissipation in an unbounded body with a spherical cavity.

Youssef [18] constructed a new theory of generalized thermoelasticity by taking into account two-temperature generalized thermoelasticity theory for a homogeneous and isotropic body without energy dissipation. Youssef [22] also obtained variational principle of two temperature thermoelasticity without energy dissipation. Chen and Gurtin [2], Chen et al. [3] and [4] have formulated a theory of heat conduction in deformable bodies which depends upon two distinct temperatures, the conductive temperature φ and the thermo dynamical temperature T. For time independent situations, the difference between these two temperatures is proportional to the heat supply, and in absence of heat supply, the two temperatures are identical. For time dependent problems, the two temperatures are different, regardless of the presence of heat supply. The two temperatures T, φ and the strain are found to have representations in the form of a travelling wave plus a response, which occurs instantaneously throughout the body.

Warren and Chen [17] investigated the wave propagation in the two temperature theory of thermoelasticity. Quintanilla [16] proved some theorems in thermoelasticity with two temperatures. Youssef AI-Lehaibi [19] and Youssef AI -Harby [20] investigated various problems on the basis of two temperature thermoelasticity. Kumar and Deswal [8] studied the surface wave propagation in a micropolar thermoelastic medium without energy dissipation. Kaushal, Kumar and Miglani [9] discussed response of frequency domain in generalized thermoelasticity with two temperatures. Sharma and Kumar [12] discussed elastodynamic response and interactions of generalised thermoelastic diffusion due to inclined load. Sharma, Kumar and Ram[13] discussed dynamical behaviour of generalized thermoelastic diffusion with two relaxation times in frequency domain. Kumar and Kansal [10] discussed propagation of cylindrical Rayleigh waves in a transversely isotropic thermoelastic diffusive solid half-space. Kumar, Sharma and Garg [11] analyzed effect of two temperature on reflection coefficient in micropolar thermoelastic media with and without energy dissipation. No attempt has been made so far to examine the thermomechanical response in transversely isotropic thermoelastic solid with two temperature and without energy dissipation in frequency domain.

The deformation at any point of the medium is useful to analyze the deformation field around mining tremors and drilling into the crust of earth. It can also contribute to the theoretical consideration of the seismic and volcanic sources since it can account for the deformation field in the entire volume surrounding the source region. The purpose of the present paper is to determine the expression for components of displacement, normal stress, tangential stress and conductive temperature, when the time -harmonic mechanical or thermal source is applied, by applying Integral transform techniques. The present model is useful for understanding the nature of interaction between mechanical and thermal fields since most of the structural elements of heavy industries are often subjected to mechanical and thermal stresses at an elevated temperature.

II. Basic Equations

Following Youssef [21] the constitutive relations and field equations in absence of body forces and heat sources are

$$t_{ij} = C_{ijkl}\, e_{kl} - \beta_{ij}\, T \tag{1}$$

$$C_{ijkl}\, e_{kl,j} - \beta_{ij}\, T_{,j} = \rho \ddot{u}_i \tag{2}$$

$$K_{ij}\, \varphi_{,ij} = \beta_{ij}\, T_0 \ddot{e}_{ij} + \rho C_E \ddot{T} \tag{3}$$

Where

$$T = \varphi - a_{ij}\, \varphi_{,ij} \tag{4}$$

Ref

1. Chandrasekharaiah, D. S., and Srinath, K. S; Thermoelastic waves without energy dissipation in an unbounded body with a spherical cavity, The International Journal of Mathematics and Mathematical Sciences,(2000), 23 (8), 555-562.

$$\beta_{ij} = C_{ijkl}\,\alpha_{ij} \tag{5}$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \qquad i,j = 1,2,3 \tag{6}$$

Here

$C_{ijkl}\,(C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk})$ are elastic parameters, β_{ij} is the thermal tensor, T is the temperature, T_0 is the reference temperature, t_{ij} are the components of stress tensor, e_{kl} are the components of strain tensor, u_i are the displacement components, ρ is the density, C_E is the specific heat, K_{ij} is the materialistic constant, a_{ij} are the two temperature parameters, α_{ij} is the coefficient of linear thermal expansion.

III. Formulation and Solution of the Problem

We consider a homogeneous, transversely isotropic thermoelastic body initially at uniform temperature T_0. We take a rectangular Cartesian co-ordinate system (x_1, x_2, x_3) with x_3 axis pointing normally into the half space, which is thus represented by $x_3 \geq 0$. We consider the plane such that all particles on a line parallel to x_2 −axis are equally displaced, so that the field component $u_2 = 0$ and u_1, u_3 and φ are independent of x_2 .we have used appropriate transformations following Slaughter [14] on the set of equations (1)-(3) to derive the equations for transversely isotropic thermoelastic solid with two temperature and without energy dissipation and we restrict our analysis to the two dimensional problem with

$$\vec{u} = (u_1, 0, u_3) \tag{7}$$

$$c_{11}\frac{\partial^2 u_1}{\partial x_1^2} + c_{44}\frac{\partial^2 u_1}{\partial x_3^2} + (c_{13} + c_{44})\frac{\partial^2 u_3}{\partial x_1 \partial x_3} - \beta_1 \frac{\partial}{\partial x_1}\left\{\varphi - \left(a_1\frac{\partial^2 \varphi}{\partial x_1^2} + a_3\frac{\partial^2 \varphi}{\partial x_3^2}\right)\right\} = \rho\frac{\partial^2 u_1}{\partial t^2} \tag{8}$$

$$(c_{13} + c_{44})\frac{\partial^2 u_1}{\partial x_1 \partial x_3} + c_{44}\frac{\partial^2 u_3}{\partial x_1^2} + c_{33}\frac{\partial^2 u_3}{\partial x_3^2} - \beta_3 \frac{\partial}{\partial x_3}\left\{\varphi - \left(a_1\frac{\partial^2 \varphi}{\partial x_1^2} + a_3\frac{\partial^2 \varphi}{\partial x_3^2}\right)\right\} = \rho\frac{\partial^2 u_3}{\partial t^2} \tag{9}$$

$$k_1\frac{\partial^2 \varphi}{\partial x_1^2} + k_3\frac{\partial^2 \varphi}{\partial x_3^2} = T_0\frac{\partial^2}{\partial t^2}\left(\beta_1\frac{\partial u_1}{\partial x_1} + \beta_3\frac{\partial u_3}{\partial x_3}\right) + \rho C_E\frac{\partial^2}{\partial t^2}\left\{\varphi - \left(a_1\frac{\partial^2 \varphi}{\partial x_1^2} + a_3\frac{\partial^2 \varphi}{\partial x_3^2}\right)\right\} \tag{10}$$

$$t_{11} = c_{11}e_{11} + c_{13}e_{33} - \beta_1 T$$

$$t_{33} = c_{13}e_{11} + c_{33}e_{33} - \beta_3 T$$

$$t_{13} = 2c_{44}e_{13} \tag{11}$$

where

$e_{11} = \frac{\partial u_1}{\partial x_1}$, $\quad e_{33} = \frac{\partial u_3}{\partial x_3}$, $\quad e_{13} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right)$, , $\quad T = \varphi - \left(a_1\frac{\partial^2 \varphi}{\partial x_1^2} + a_3\frac{\partial^2 \varphi}{\partial x_3^2}\right)$

$\beta_{ij} = \beta_i \delta_{ij}$, $\quad K_{ij} = K_i \delta_{ij}$

$\beta_1 = (c_{11}+c_{12})\alpha_1 + c_{13}\alpha_3$, $\quad \beta_3 = 2c_{13}\alpha_1 + c_{33}\alpha_3$

In the above equations we use the contracting subscript notations $(1 \to 11, 2 \to 22, 3 \to 33, 4 \to 23, 5 \to 13, 6 \to 12)$ to relate

$$c_{ijkl} \ to \ c_{mn} \tag{12}$$

The initial and regularity conditions are given by

$u_1(x_1, x_3, 0) = 0 = \dot{u_1}(x_1, x_3, 0)$

$u_3(x_1, x_3, 0) = 0 = \dot{u_3}(x_1, x_3, 0)$

$$\varphi(x_1, x_3, 0) = 0 = \dot{\varphi}(x_1, x_3, 0) \qquad \text{For} \quad x_3 \geq 0, \quad -\infty < x_1 < \infty \tag{13}$$

$$u_1(x_1, x_3, t) = u_3(x_1, x_3, t) = \varphi(x_1, x_3, t) = 0 \; for \; t > 0 \; when \; x_3 \to \infty \tag{14}$$

Assuming the harmonic behaviour as

Ref

14. Slaughter, W. S; The linearised theory of elasticity,Birkhausar (2002).

$$(u_1, u_3, \varphi)(x_1, x_3, t) = (u_1, u_3, \varphi)(x_1, x_3)e^{i\omega t} \tag{15}$$

where ω is the angular frequency.
To facilitate the solution, following dimensionless quantities are introduced:

$$x_1' = \frac{x_1}{L}, \quad x_3' = \frac{x_3}{L}, \quad u_1' = \frac{\rho c_1^2}{L\beta_1 T_0}u_1, \quad u_3' = \frac{\rho c_1^2}{L\beta_1 T_0}u_3, \quad T' = \frac{T}{T_0}, \quad t' = \frac{c_1}{L}t, \quad t_{33}' = \frac{t_{33}}{\beta_1 T_0},$$

$$t_{31}' = \frac{t_{31}}{\beta_1 T_0}, \quad \varphi' = \frac{\varphi}{T_0}, \quad a_1' = \frac{a_1}{L}, \quad a_3' = \frac{a_3}{L}, \quad F_1' = \frac{F_1}{\beta_1 T_0}, \quad F_2' = \frac{F_2}{T_0}, \quad \omega' = \frac{\omega}{L} \tag{16}$$

where $c_1^2 = \frac{c_{11}}{\rho}$ and L is a constant of dimension of length.

The equations (8)-(10) with the aid of (12) ,(15) and (16) recast into the following form (after suppressing the primes)

$$\frac{\partial^2 u_1}{\partial x_1^2} + \delta_1 \frac{\partial^2 u_1}{\partial x_3^2} + \delta_2 \frac{\partial^2 u_3}{\partial x_1 \partial x_3} - \left[1 - \left(a_1 \frac{\partial^2}{\partial x_1^2} + a_3 \frac{\partial^2}{\partial x_3^2}\right)\right]\frac{\partial \varphi}{\partial x_1} = -\omega^2 u_1 \tag{17}$$

$$\delta_4 \frac{\partial^2 u_3}{\partial x_3^2} + \delta_1 \frac{\partial^2 u_3}{\partial x_1^2} + \delta_2 \frac{\partial^2 u_1}{\partial x_1 \partial x_3} - p_5\left[1 - \left(a_1 \frac{\partial^2}{\partial x_1^2} + a_3 \frac{\partial^2}{\partial x_3^2}\right)\right]\frac{\partial \varphi}{\partial x_3} == -\omega^2 u_3 \tag{18}$$

$$\frac{\partial^2 \varphi}{\partial x_1^2} + p_3 \frac{\partial^2 \varphi}{\partial x_3^2} + \zeta_1 \omega^2 \frac{\partial u_1}{\partial x_1} + \zeta_2 \omega^2 \frac{\partial u_3}{\partial x_3} = -\zeta_3 \left[1 - \left(a_1 \frac{\partial^2}{\partial x_1^2} + a_3 \frac{\partial^2}{\partial x_3^2}\right)\right]\omega^2 \varphi \tag{19}$$

where

$$\delta_1 = \frac{c_{44}}{c_{11}}, \quad \delta_2 = \frac{c_{13}+c_{44}}{c_{11}}, \quad \delta_4 = \frac{c_{33}}{c_{11}}, \quad p_5 = \frac{\beta_3}{\beta_1}, \quad p_3 = \frac{k_3}{k_1}, \quad \zeta_1 = \frac{T_0\beta_1^2}{k_1\rho}, \quad \zeta_2 = \frac{T_0\beta_3\beta_1}{k_1\rho}, \quad \zeta_3 = \frac{C_E c_{11}}{k_1}$$

Applying Fourier transform defined by

$$\hat{f}(\xi, x_3, \omega) = \int_{-\infty}^{\infty} \bar{f}(x_1, x_3, \omega)e^{i\xi x_1}dx_1 \tag{20}$$

on equations (17)-(19), we obtain a system of 3 homogeneous equations in terms of $\hat{u}_1$, $\hat{u}_3$ and $\hat{\varphi}$ which yield a non trivial solution if determinant of the coefficient ($\hat{u}_1$, $\hat{u}_3$,$\hat{\varphi}$) vanishes i.e. we obtain the following characteristic equation

$$\left(\frac{d^6}{dx_3^6} + Q\frac{d^4}{dx_3^4} + R\frac{d^2}{dx_3^2} + S\right)(\hat{u}_1, \hat{u}_3, \hat{\varphi}) = 0 \tag{21}$$

$Q = \frac{1}{P}(\xi^2 E + F)$

$R = \frac{1}{P}(G\xi^4 + H\xi^2 + I)$

$S = \frac{1}{P}(J\xi^6 + L\xi^4 + M\xi^2 + N)$

Where $P = \delta_1(-\delta_4\zeta_3 a_3\omega^2 - \delta_4 p_3 + \zeta_2 p_5 a_3\omega^2)$

$E = \omega^2\{\zeta_3 a_3(\delta_4 + \delta_1 b_1 - \delta_2^2) + \zeta_3 a_1 - \zeta_2(p_5 a_3 + \delta_1 p_5 a_1) + \delta_4 p_5 a_3\zeta_1 + \delta_4 a_3(\zeta_2 - \zeta_1)\} + p_3(\delta_4 + \delta_1 b_1 - \delta_2^2) + \delta_1\delta_4$

$F = \omega^4\{\zeta_3 a_3(\delta_4 - \delta_1) + \delta_1\delta_4\zeta_2 p_5 a_3\} + \omega^2\{p_3(\delta_4 - \delta_1) + \delta_1\delta_4(\zeta_3 - \delta_1\zeta_2 p_5)\}$

$G = \omega^2\{a_1(\zeta_2 p_5 - p_5\zeta_1\delta_2 - \delta_2\zeta_2 + \zeta_1\delta_4 - \delta_1^2\zeta_3 - a_3\delta_4 + \zeta_3\delta_2^2) + a_3(-\delta_1\zeta_3 + \delta_1\zeta_1) - \delta_1 p_3 - \delta_1^2 - \delta_4 + \delta_2^2\}$

$H = \omega^4\{a_3(\delta_1\zeta_3 + \zeta_3 - \zeta_1) + a_1(\zeta_3\delta_4 + \delta_1\zeta_3)\} + \omega^2\{p_5(\zeta_2 - \zeta_1\delta_2 - a_1\zeta_2\zeta_2) - \zeta_2\delta_2 + \zeta_1\delta_4 + p_3(\zeta_1 + 1) - \delta_1^2\zeta_3 + \delta_1 + \delta_4 - \zeta_3\delta_4 + \zeta_3\delta_2^2\}$

$I = -\omega^6\zeta_3 a_3 - \omega^4(\zeta_2 p_5 + p_3 - \delta_1\zeta_3 + \zeta_3\delta_4)$

$J = a_1\delta_1\omega^2(\zeta_3 - \zeta_1) + \delta_1$

$L = \omega^4(a_1\zeta_1 - a_1\zeta_3 - \delta_1\zeta_3 a_1) + \omega^2(\delta_1\zeta_3 - \delta_1 - \delta_1\zeta_1 - 1)$

Notes

Notes

$M = \omega^6\zeta_3 a_1 + \omega^4(-\zeta_3\delta_1 - \zeta_3 + 1 + \zeta_1)$

$N = \omega^6\zeta_3$

The roots of the equation (21) are $\pm\lambda_i$ ($i = 1,2,3$) satisfying the radiation condition that $\hat{u}_1, \hat{u}_3, \hat{\varphi} \to 0$ as $x_3 \to \infty$, the solution of the equation (21) can be written as

$$\hat{u}_1 = A_1 e^{-\lambda_1 x_3} + A_2 e^{-\lambda_2 x_3} + A_3 e^{-\lambda_3 x_3} \tag{22}$$

$$\hat{u}_3 = d_1 A_1 e^{-\lambda_1 x_3} + d_2 A_2 e^{-\lambda_2 x_3} + d_3 A_3 e^{-\lambda_3 x_3} \tag{23}$$

$$\hat{\varphi} = l_1 A_1 e^{-\lambda_1 x_3} + l_2 A_2 e^{-\lambda_2 x_3} + l_3 A_3 e^{-\lambda_3 x_3} \tag{24}$$

where

$$d_i = \frac{-\lambda_i^3 P^* - \lambda_i Q^*}{\lambda_1^4 R^* + \lambda_1^2 S^* + T^*} \quad i = 1,2,3 \tag{25}$$

$$l_i = \frac{\lambda_i^2 P^{**} + Q^{**}}{\lambda_1^4 R^* + \lambda_1^2 S^* + T^*} \quad i = 1,2,3 \tag{26}$$

Where $P^* = i\xi\{(\zeta_1 p_5 a_3\omega^2 - \delta_2(\zeta_3 a_3\omega^2 + p_3)\}$

$Q^* = \delta_2(\xi^2 + \zeta_3\omega^2 + \zeta_3\omega^2 a_1\xi^2) - p_5\zeta_1(1 + a_1\xi^2)\omega^2$

$R^* = \zeta_2 p_5 a_3\omega^2 - \delta_4(\zeta_3 a_3\omega^2 + p_3)$

$S^* = (\xi^2 + \zeta_3\omega^2 + \zeta_3\omega^2 a_1\xi^2)\delta_4 + (\delta_1\xi^2 - \omega^2)(a_3\zeta_3\omega^2 + p_3) - \zeta_2 p_5\omega^2(1 + a_1\xi^2)$

$T^* = -(\delta_1\xi^2 - \omega^2)(\xi^2 + \zeta_3\omega^2 + \zeta_3\omega^2 a_1\xi^2)$

$P^{**} = -(\zeta_2\delta_2 - \zeta_1\delta_4)\omega^2 i\xi$

$Q^{**} = -\zeta_1\omega^2(\delta_1\xi^2 - \omega^2)$

IV. Applications

On the half-space surface($x_3 = 0$) normal point force and thermal point source, which are assumed to be time harmonic, are applied. We consider two types of boundary conditions, as follows

Case 1. The normal force on the surface of half-space

The boundary conditions in this case are

(1) $t_{33}(x_1, x_3, t) = -F_1\psi_1(x)e^{i\omega t}$

(2) $t_{31}(x_1, x_3, t) = 0$

(3) $\frac{\partial\,\varphi(x_1,x_3,t)}{\partial x_3} = 0$ at $x_3 = 0$ (27)

where F_1 is the magnitude of the force applied, $\psi_1(x)$ specify the source distribution function along x_1 axis.

Case 2. The thermal source on the surface of half-space

When the plane boundary is stress free and subjected to thermal point source, the boundary conditions in this case are

(1) $t_{33}(x_1, x_3, t) = 0$

(2) $t_{31}(x_1, x_3, t) = 0$

(3) $\frac{\partial\,\varphi(x_1,x_3,t)}{\partial x_3} = F_2\psi_1(x)e^{i\omega t}$ at $x_3 = 0$ (28)

where F_2 is the constant temperature applied on the boundary, $\psi_1(x)$ specify the source distribution function along x_1 axis.

a) Green's function

To synthesize the Green's function, i.e. the solution due to concentrated normal force and thermal source on the half-space is obtained by setting

$$\psi_1(x) = \delta(x) \tag{29}$$

In equations (27) and (28). Applying the Fourier transform defined by (20) on the equation (29) gives

$$\hat{\psi}_1(\xi) = 1 \tag{30}$$

Subcase 1(a). Mechanical force

Substitute the values of $\hat{u}_1, \widehat{u}_3$ *and* $\hat{\varphi}$ from (22)-(24) in the boundary conditions (27) and with the aid of (1), (4)-(7), (12), (15) ,(16) and (20),we obtain the components of displacement, normal stress, tangential stress and conductive temperature as

$$\hat{u}_1 = \frac{F_1\hat{\psi}_1(\xi)}{\Delta}\left(-M_{11}e^{-\lambda_1 x_3} + M_{12}e^{-\lambda_2 x_3} - M_{13}e^{-\lambda_3 x_3}\right)e^{i\omega t} \tag{31}$$

$$\hat{u}_3 = \frac{F_1\hat{\psi}_1(\xi)}{\Delta}\left(-d_1M_{11}e^{-\lambda_1 x_3} + d_2M_{12}e^{-\lambda_2 x_3} - d_3M_{13}e^{-\lambda_3 x_3}\right)e^{i\omega t} \tag{32}$$

$$\hat{\varphi} = \frac{F_1\hat{\psi}_1(\xi)}{\Delta}\left(-l_1M_{11}e^{-\lambda_1 x_3} + l_2M_{12}e^{-\lambda_2 x_3} - l_3M_{13}e^{-\lambda_3 x_3}\right)e^{i\omega t} \tag{33}$$

$$\widehat{t_{33}} = \frac{F_1\hat{\psi}_1(\xi)}{\Delta}\left(-\Delta_{11}M_{11}e^{-\lambda_1 x_3} + \Delta_{12}M_{12}e^{-\lambda_2 x_3} - \Delta_{13}M_{13}e^{-\lambda_3 x_3}\right)e^{i\omega t} \tag{34}$$

$$\widehat{t_{31}} = \frac{F_1\hat{\psi}_1(\xi)}{\Delta}\left(-\Delta_{21}M_{11}e^{-\lambda_1 x_3} + \Delta_{22}M_{12}e^{-\lambda_2 x_3} - \Delta_{23}M_{13}e^{-\lambda_3 x_3}\right)e^{i\omega t} \tag{35}$$

where

$$M_{11} = \Delta_{22}\Delta_{33} - \Delta_{32}\Delta_{23}\,, \quad M_{12} = \Delta_{21}\Delta_{33} - \Delta_{23}\Delta_{31}, \quad M_{13} = \Delta_{21}\Delta_{32} - \Delta_{22}\Delta_{31}$$

$$M_{21} = \Delta_{12}\Delta_{33} - \Delta_{13}\Delta_{22}, \quad M_{22} = \Delta_{11}\Delta_{33} - \Delta_{13}\Delta_{31}, \quad M_{23} = \Delta_{11}\Delta_{32} - \Delta_{12}\Delta_{31}$$

$$\Delta_{1j} = \frac{c_{31}}{\rho c_1^2}i\xi - \frac{c_{33}}{\rho c_1^2}d_j\lambda_j - \frac{\beta_3}{\beta_1}l_j + \frac{\beta_3}{\beta_1 T_0}a_3 l_j{\lambda_j}^2 - \frac{\beta_3}{\beta_1}l_j a_1\xi^2, \;\; j = 1,2,3$$

$$\Delta_{2j} = -\frac{c_{44}}{\rho c_1^2}\lambda_j + \frac{c_{44}}{\rho c_1^2}i\xi d_j \quad j = 1,2,3$$

$$\Delta_{3j} = l_j\,\lambda_j \;\; j = 1,2,3$$

$$\Delta = \Delta_{11}M_{11} - \Delta_{12}M_{12} + \Delta_{13}M_{13}$$

Subcase 2(a). Thermal source on the surface of half-space

Making use of (1), (4)-(7), (12),(15) and (16)in B.C. (28) , and applying Fourier Transform defined by (20) and substituting the values of $\hat{u}_1, \widehat{u}_3$ *and* $\hat{\varphi}$ from (22)-(24) in the resulting equations, we obtain the components of displacement, normal stress, tangential stress and conductive temperature are as given by equations (31)-(35) with M_{11}, M_{12} *and* M_{13} replaced by M_{31}, M_{32} and M_{33} respectively and F_1 replaced by F_2.

Where $M_{31} = \Delta_{12}\Delta_{23} - \Delta_{13}\Delta_{22}, \; M_{32} = \Delta_{11}\Delta_{23} - \Delta_{13}\Delta_{21}, \; M_{33} = \Delta_{11}\Delta_{22} - \Delta_{12}$ (36)

b). Influence function

The method to obtain the half-space influence function, i.e. the solution due to distributed load applied on the half space is obtained by setting

Notes

$$\psi_1(x)=\begin{cases}1 \; if \; |x|\le m\\ 0 \; if \; |x|>m\end{cases} \tag{37}$$

In equations (27) and (28). The Fourier transforms of $\psi_1(x)$ with respect to the pair (x,ξ) for the case of a uniform strip load of non dimensional width 2m applied at origin of co-ordinate system $x_1=x_3=0$ in the dimensionless form after suppressing the primes becomes

$$\hat{\psi}_1(\xi)=\left[{}^{2\sin(\xi m)}\!/_{\xi}\right], \xi\neq 0 \tag{38}$$

The expressions for displacement, stresses and conductive temperature can be obtained for uniformly distributed normal force and thermal source by replacing $\hat{\psi}_1(\xi)$ from (38) respectively in equations (31)-(35) along with (36)

V. Particular Cases

(i): If $a_1=a_3=0$, from equations (31)-(35) ,we obtain the corresponding expressions for displacement, stresses and conductive temperature in thermoelastic solid without energy dissipation.

(ii) If we take $a_1=a_3=a$, $c_{11}=\lambda+2\mu=c_{33}$, $c_{12}=c_{13}=\lambda$, $c_{44}=\mu$, $\beta_1=\beta_3=\beta$, $\alpha_1=\alpha_3=\alpha$, $K_1=K_3=K$ in equations $(31)-(35)$ we obtain the corresponding expressions for displacements, stresses and conductive temperature for isotropic thermoelastic solid without energy dissipation.

VI. Inversion of the Transformation

To obtain the solution of the problem in physical domain, we must invert the transforms in equations (31)-(35) . Here the displacement components, normal and tangential stresses and conductive temperature are functions of x_3 and the parameters of Fourier transforms ξ and hence are of the form $f(\xi, x_3)$. To obtain the function $f(x_1,x_3)$ in the physical domain, we first invert the Fourier transform as used by Sharma, Kumar and Ram [13] using

$$f(x_1,x_3)=\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\xi x_1}\hat{f}(\xi,x_3)d\xi=\frac{1}{2\pi}\int_{-\infty}^{\infty}|\cos(\xi x_1)f_e-i\sin(\xi x_1)f_o|\,d\xi \tag{39}$$

Where f_e and f_o are respectively the even and odd parts of $\hat{f}(\xi,x_3)$. The method for evaluating this integral is described in Press et al. [15] . It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

VII. Numerical Results and Discussion

Copper material is chosen for the purpose of numerical calculation which is transversely isotropic. Physical data for a single crystal of copper is given by

$c_{11}=18.78\times10^{10}\,Kgm^{-1}s^{-2}$, $c_{12}=8.76\times10^{10}\,Kgm^{-1}s^{-2}$, $c_{13}=8.0\times10^{10}\,Kgm^{-1}s^{-2}$
$c_{33}=17.2\times10^{10}\,Kgm^{-1}s^{-2}$, $c_{44}=5.06\times10^{10}\,Kgm^{-1}s^{-2}$ $C_E=0.6331\times10^{3}JKg^{-1}K^{-1}$
$\alpha_1=2.98\times10^{-5}K^{-1}$, $\alpha_3=2.4\times10^{-5}K^{-1}$, $\rho=8.954\times10^{3}Kgm^{-3}$,
$k_1=0.02\times10^{2}Nsec^{-2}deg^{-1}$, $k_3=0.04\times10^{2}Nsec^{-2}deg^{-1}$

Following Dhaliwal and Singh [5], magnesium crystal is chosen for the purpose of numerical calculation(isotropic solid). In case of magnesium crystal like material for numerical calculations, the physical constants used are

$\lambda=2.17\times10^{10}Nm^2$, $\mu=3.278\times10^{10}Nm^2$, $K=0.02\times10^{2}Nsec^{-2}deg^{-1}$

Ref

15. Press,W.H., Teukolshy,S.A., Vellerling,W.T.,and Flannery, B.P; Numerical recipes in Fortran, Cambridge University Press, Cambridge,(1986).

$\omega_1 = 3.58 \times 10^{11} S^{-1}$ $\beta = 2.68 \times 10^6\ Nm^{-2}deg^{-1}$, $\rho = 1.74 \times 10^3 Kgm^{-3}$
$T_0 = 298\text{K}$, $C_E = 1.04 \times 10^3 Jkg^{-1}deg^{-1}$

The values of normal displacement u_3, normal force stress t_{33} , tangential stress t_{31} and conductive temperature φ for a transversely isotropic thermoelastic solid (TIT) and for isotropic thermoelastic solid (IT) are presented graphically for the non-dimensional frequencies ω=.25, ω=.5 and ω=.75. Two temperature parameter for (TIT) are taken as $a_1 = 0.03$ and $a_3 = 0.05$ whereas for (IT) ,the two temperature parameter are taken as $a_1 = a_3 = 0.04$

1). The solid line, small dashed line and long dashed line , respectively corresponds to isotropic solid with frequencies ω=.25, ω=.5 and ω=.75 respectively and $a_1 = a_3 = 0.04$

2). The solid line with centre symbol circle , the small dashed line with centre symbol diamond and the long dashed line with centre symbol cross respectively correspond to transversely isotropic solid with frequencies ω=.25, ω=.5 and ω=.75 respectively and $a_1 = 0.03$ and $a_3 = 0.05$

a) Normal force on the surface of half-space

i. *Concentrated force*

Fig.1 shows the variations of the normal displacement u_3. The values of u_3 (TIT), follow oscillatory pattern for ω=.75 and for ω=.5, whereas for ω=.25,variations are very small owing to scale of graph. For u_3(IT), corresponding to the three frequencies, behaviour is oscillatory with difference in the magnitude. Fig.2 depicts the values of normal stress t_{33}. Near the loading surface, the values of t_{33} (TIT) increase sharply corresponding to the three frequencies but away from the loading surface, these oscillate for ω=.5 and ω=.75, however for ω=.25, it is descending oscillatory. For t_{33} (IT), small variations are observed corresponding to three frequencies. Fig.3 describes the variations of tangential stress t_{31}. For both the mediums (i.e. IT and TIT), variations in t_{31} are oscillatory for ω=.5 and ω=.75 where as for ω=.25, it increases near the loading surface and then decreases i.e. somehow oscillates. Fig.4 interprets the variations of conductive temperature φ .The values of φ (IT), for ω=.25 and ω=.5 increase sharply near the loading surface and then decrease i.e. are oscillatory with difference in magnitude whereas for ω=.75 it also oscillates with small magnitude. φ (TIT) shows small oscillations for ω=.5 and ω=.75 whereas variations for ω=.25 are very small in the whole range.

ii. *Uniformly Distributed force*

Fig. 5-8 show the characteristics for uniformly distributed force. It is depicted from Fig.5-Fig.8 that the distribution curves for u_3, normal stress t_{33}, tangential stress t_{31} and conductive temperature φ for uniformly distributed force, follow same trends as in case of concentrated force for both the mediums with difference in magnitudes in their respective patterns.

b) Thermal source on the surface of half-space

i. *Concentrated Thermal Source*

Fig.9 shows the variations of normal displacement u_3 when concentrated thermal source is applied. It is depicted that the variations in u_3 for both the mediums follow oscillatory pattern corresponding to the three frequencies with difference in their magnitude, except for ω=.25 (TIT). In case ω=.25 (TIT), small variations are observed. Fig.10. explains variations of normal stress t_{33}, near the loading surface, values of t_{33} (IT) increase, whereas a decrease is seen in t_{33}(TIT) ,but away from the loading surface, behaviour is oscillatory in the whole range with difference in their magnitudes corresponding to the three frequencies. Fig.11 displays the picture about the behaviour of tangential stress t_{31}, here for t_{31} (IT) ,there is a sharp increase in the range $0 \leq x \leq 2$ for ω=.5 and ω=.75 and afterwards pattern is oscillatory, whereas for

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t_{31}(TIT), oscillatory pattern is observed in the whole range corresponding to these frequencies. For ω=.25, different type of variations are observed as compared with ω=.5 and ω=.75 .Fig. 12 shows the movements of conductive temperature φ , here for both the mediums oscillatory variations are depicted for ω=.5 and ω=.75 whereas there are small variations for ω=.25(TIT) and movements are oscillatory for ω=.25 (IT).

ii. *Uniformly Distributed thermal source*

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Fig.13 –Fig.16 show that variations in normal displacement u_3, normal stress t_{33}, tangential stress t_{31} and the conductive temperature φ for both the mediums are of similar pattern as in case of concentrated thermal source with change in magnitude. In some figures, these appear as the mirror image of the figures of concentrated thermal source.

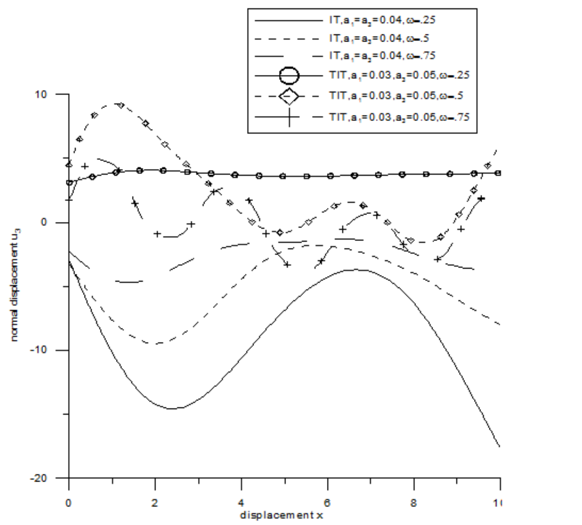

Figure 1 : Variation of Normal Displacement U_3 with Distance X(Concentrated Force)

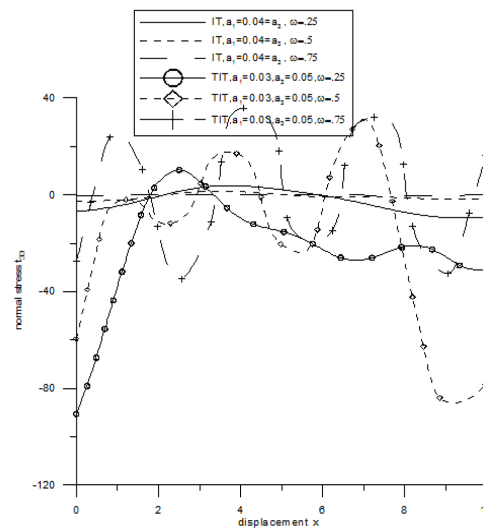

Figure 2 : Variation of Normal Stress t_{33} with Distance X(Concentrated Force)

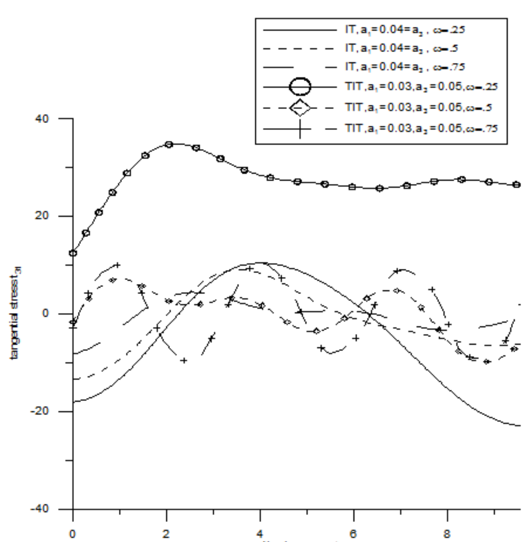

Figure 3 : Variation of Tangential Stress t_{31} with Distance X(Concentrated Force)

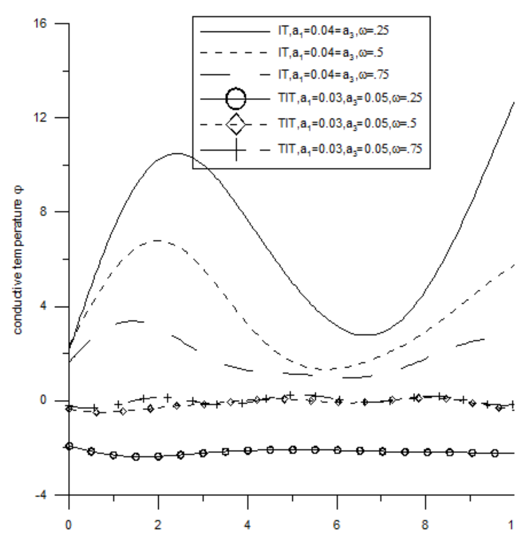

Figure 4 : Variation of Conductive Temperature φ with Distance X(Concentrated Force)

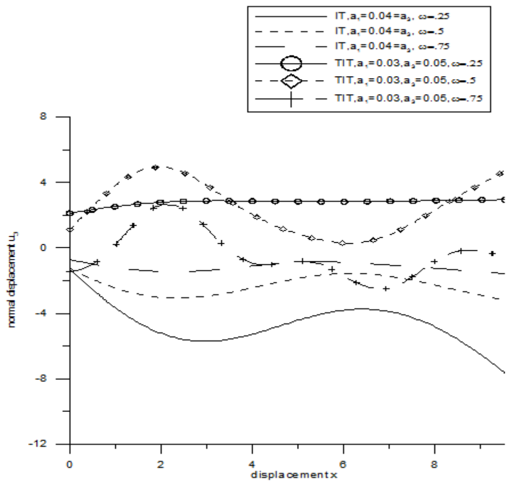

Figure 5 : *Variation of Normal Displacement U_3 with Distance X(Uniformly Distributed Force)*

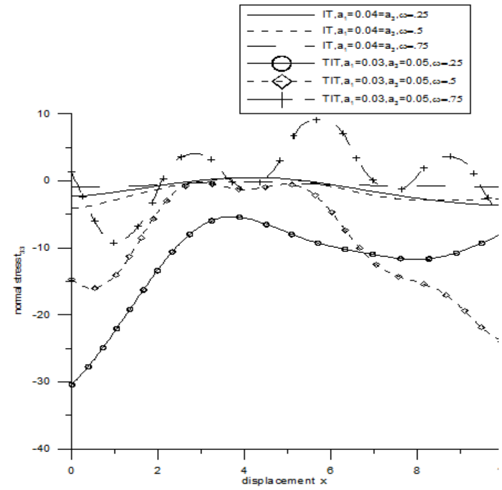

Figure 6 : *Variation of Normal Stress t_{33} with Distance X(Uniformly Distributed Force)*

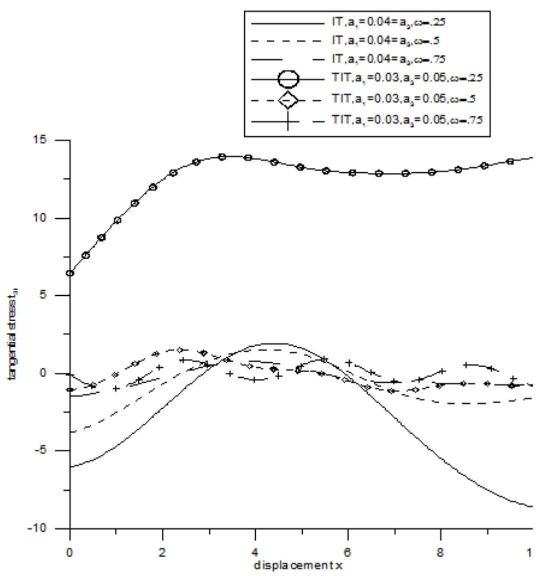

Figure 7 : *Variation of Tangential Stress t_{31} with Distance X(Uniformly Distributed Force)*

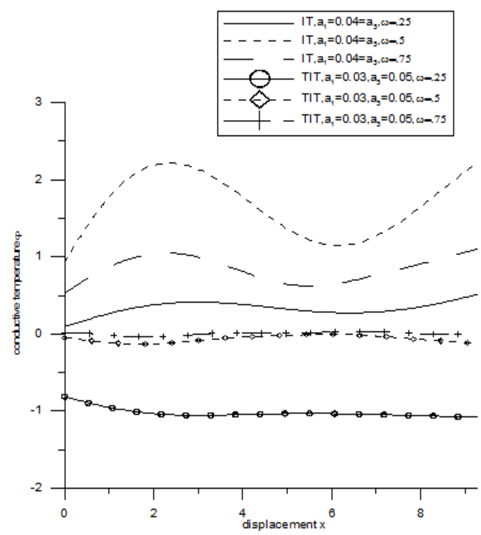

Figure 8 : *Variation of Conductive Temperature φ with Distance X(Uniformly Distributed Force)*

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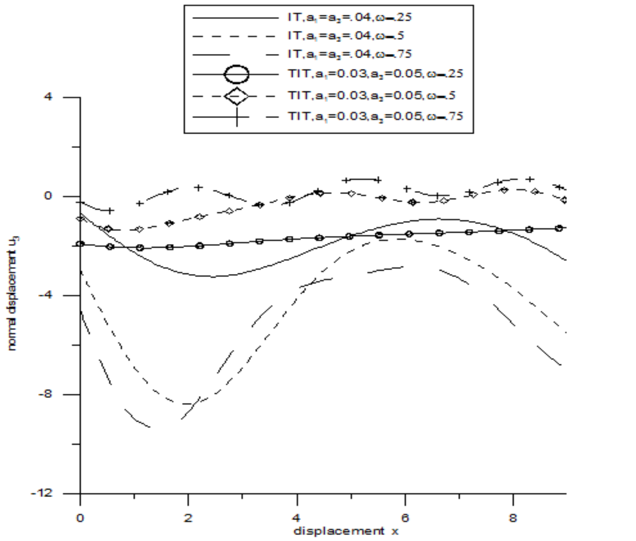

Figure 9 : Variation of Normal Displacement U_3 with Distance X(Concentrated Thermal Source)

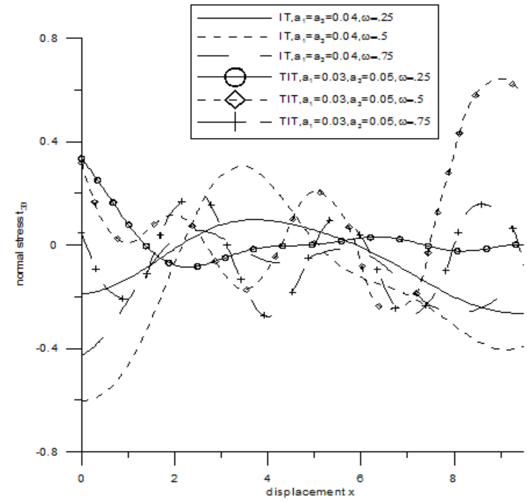

Figure 10 : Variation of Normal Stress t_{33} with Distance X(Concentrated Thermal Source)

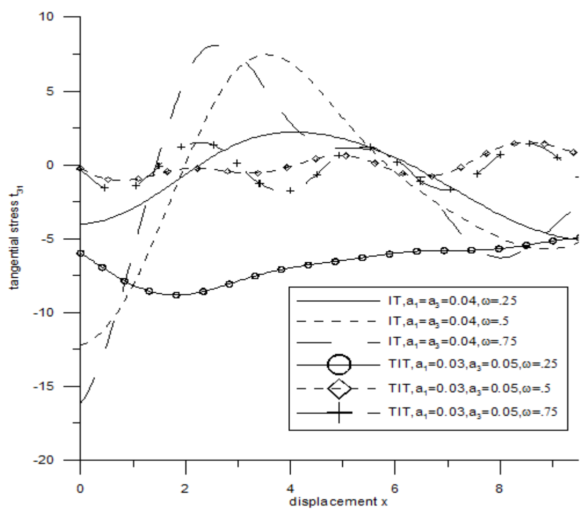

Figure 11 : Variation of Tangential Stress t_{31} with Distance X(Concentrated Thermal Source)

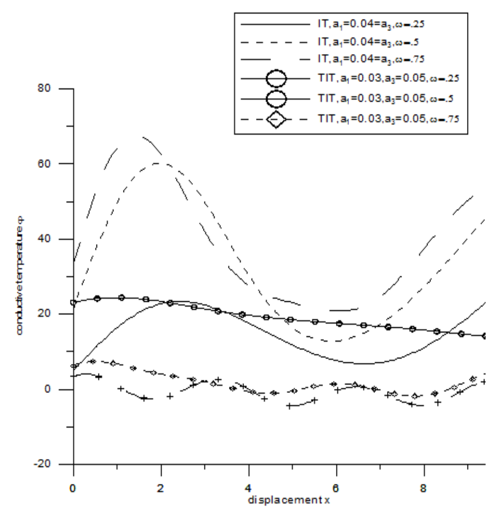

Figure 12 : Variation of Conductive Temperature φ with Distance X(Concentrated Thermal Source)

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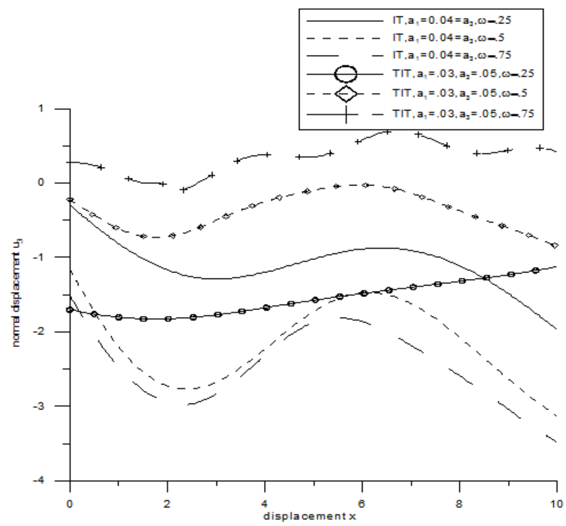

Figure 13 : *Variation of Normal Displacement U_3 with Distance X(Uniformly Distributed Thermal Source)*

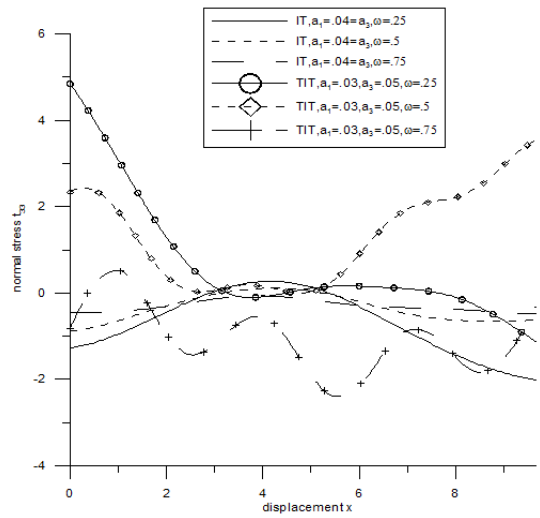

Figure 14 : *Variation of Normal Stress t_{33} with Distance X(Uniformly Distributed Thermal Source)*

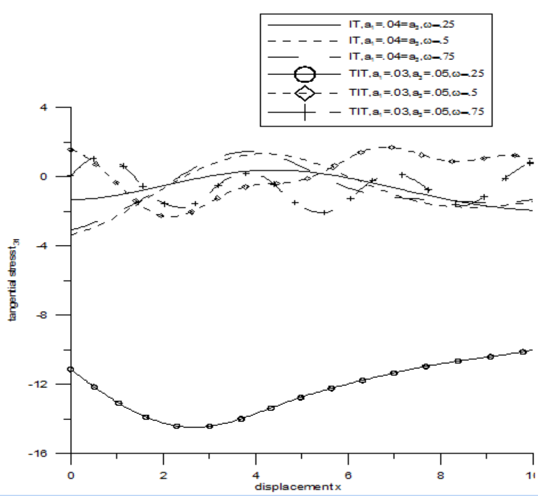

Figure 15 : *Variation of Tangential Stress t_{31} with Distance X(Uniformly Distributed Thermal Source)*

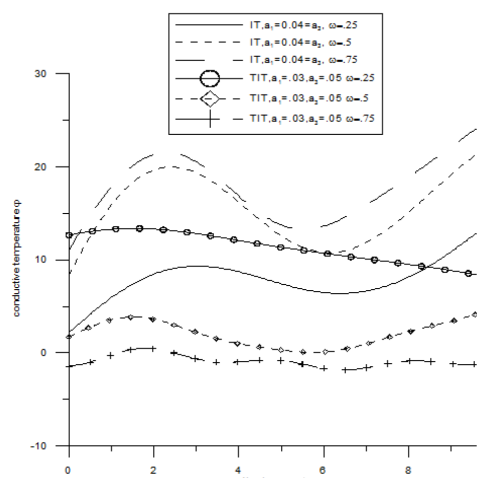

Figure 16 : *Variation of Conductive Temperature φ with Distance X(Uniformly Distributed Thermal Source)*

VII. Conclusion

From the graphs, it is observed that effect of anisotropy plays important role in the deformation of the body. As disturbance travels through the constituents of the medium, it suffers sudden changes resulting in an inconsistent/non uniform pattern of graphs. Anisotropy has significant impact on components of normal displacement, normal stress, tangential stress and conductive temperature. It is observed from the figures(1-8) that the trends in the variations of the characteristics mentioned are similar with difference in their magnitude when the mechanical forces (i.e. concentrated or distributed forces)are applied , where as the trends are also similar when thermal sources (i.e. concentrated or distributed forces) are applied as is observed in figures (8-16). The trend of curves exhibits the properties of the medium and satisfies requisite condition of the problem. It can also contribute to the theoretical considerations of the seismic and volcanic sources since it can account for deformation fields in the entire volume surrounding the source region.

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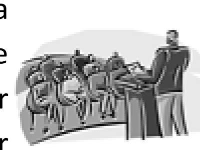

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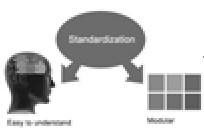

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Global Journals Incorporation (USA) is accredited by Open Association of Research Society, U.S.A (OARS) and in turn, affiliates research institutions as “Institutional Fellow of Open Association of Research Society” (IFOARS).

The “FARSC” is a dignified title which is accorded to a person’s name viz. Dr. John E. Hall, Ph.D., FARSC or William Walldroff, M.S., FARSC.

The IFOARS institution is entitled to form a Board comprised of one Chairperson and three to five board members preferably from different streams. The Board will be recognized as “Institutional Board of Open Association of Research Society”-(IBOARS).

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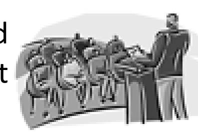
The IBOARS can initially review research papers of their institute and recommend them to publish with respective journal of Global Journals. It can also review the papers of other institutions after obtaining our consent. The second review will be done by peer reviewer of Global Journals Incorporation (USA) The Board is at liberty to appoint a peer reviewer with the approval of chairperson after consulting us.
The author fees of such paper may be waived off up to 40%.



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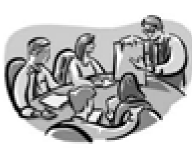

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The Board can also play vital role by exploring and giving valuable suggestions regarding the Standards of “Open Association of Research Society, U.S.A (OARS)” so that proper amendment can take place for the benefit of entire research community. We shall provide details of particular standard only on receipt of request from the Board.

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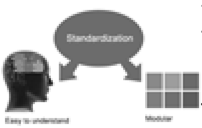

After nomination of your institution as “Institutional Fellow” and constantly functioning successfully for one year, we can consider giving recognition to your institute to function as Regional/Zonal office on our behalf.
The board can also take up the additional allied activities for betterment after our consultation.

The following entitlements are applicable to individual Fellows:

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Note :

"

- In future, if the board feels the necessity to change any board member, the same can be done with the consent of the chairperson along with anyone board member without our approval.
- In case, the chairperson needs to be replaced then consent of 2/3rd board members are required and they are also required to jointly pass the resolution copy of which should be sent to us. In such case, it will be compulsory to obtain our approval before replacement.
- In case of "Difference of Opinion [if any]" among the Board members, our decision will be final and binding to everyone.

"

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2. Ethical Guidelines,

3. Submission of Manuscripts,

4. Manuscript’s Category,

5. Structure and Format of Manuscript,

6. After Acceptance.

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The recommended size of original research paper is less than seven thousand words, review papers fewer than seven thousands words also.Preparation of research paper or how to write research paper, are major hurdle, while writing manuscript. The research articles and research letters should be fewer than three thousand words, the structure original research paper; sometime review paper should be as follows:

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17. Never use online paper: If you are getting any paper on Internet, then never use it as your research paper because it might be possible that evaluator has already seen it or maybe it is outdated version.

18. Pick a good study spot: To do your research studies always try to pick a spot, which is quiet. Every spot is not for studies. Spot that suits you choose it and proceed further.

19. Know what you know: Always try to know, what you know by making objectives. Else, you will be confused and cannot achieve your target.

20. Use good quality grammar: Always use a good quality grammar and use words that will throw positive impact on evaluator. Use of good quality grammar does not mean to use tough words, that for each word the evaluator has to go through dictionary. Do not start sentence with a conjunction. Do not fragment sentences. Eliminate one-word sentences. Ignore passive voice. Do not ever use a big word when a diminutive one would suffice. Verbs have to be in agreement with their subjects. Prepositions are not expressions to finish sentences with. It is incorrect to ever divide an infinitive. Avoid clichés like the disease. Also, always shun irritating alliteration. Use language that is simple and straight forward. put together a neat summary.

21. Arrangement of information: Each section of the main body should start with an opening sentence and there should be a changeover at the end of the section. Give only valid and powerful arguments to your topic. You may also maintain your arguments with records.

22. Never start in last minute: Always start at right time and give enough time to research work. Leaving everything to the last minute will degrade your paper and spoil your work.

23. Multitasking in research is not good: Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. Think technically: Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page
- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.

- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

Please carefully note down following rules and regulation before submitting your Research Paper to Global Journals Inc. (US):

Segment Draft and Final Research Paper: You have to strictly follow the template of research paper. If it is not done your paper may get rejected.

- The **major constraint** is that you must independently make all content, tables, graphs, and facts that are offered in the paper. You must write each part of the paper wholly on your own. The Peer-reviewers need to identify your own perceptive of the concepts in your own terms. NEVER extract straight from any foundation, and never rephrase someone else's analysis.
- Do not give permission to anyone else to "PROOFREAD" your manuscript.
- Methods to avoid Plagiarism is applied by us on every paper, if found guilty, you will be blacklisted by all of our collaborated research groups, your institution will be informed for this and strict legal actions will be taken immediately.)
- To guard yourself and others from possible illegal use please do not permit anyone right to use to your paper and files.

Criterion for Grading a Research Paper (Compilation) by Global Journals Inc. (US)

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Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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