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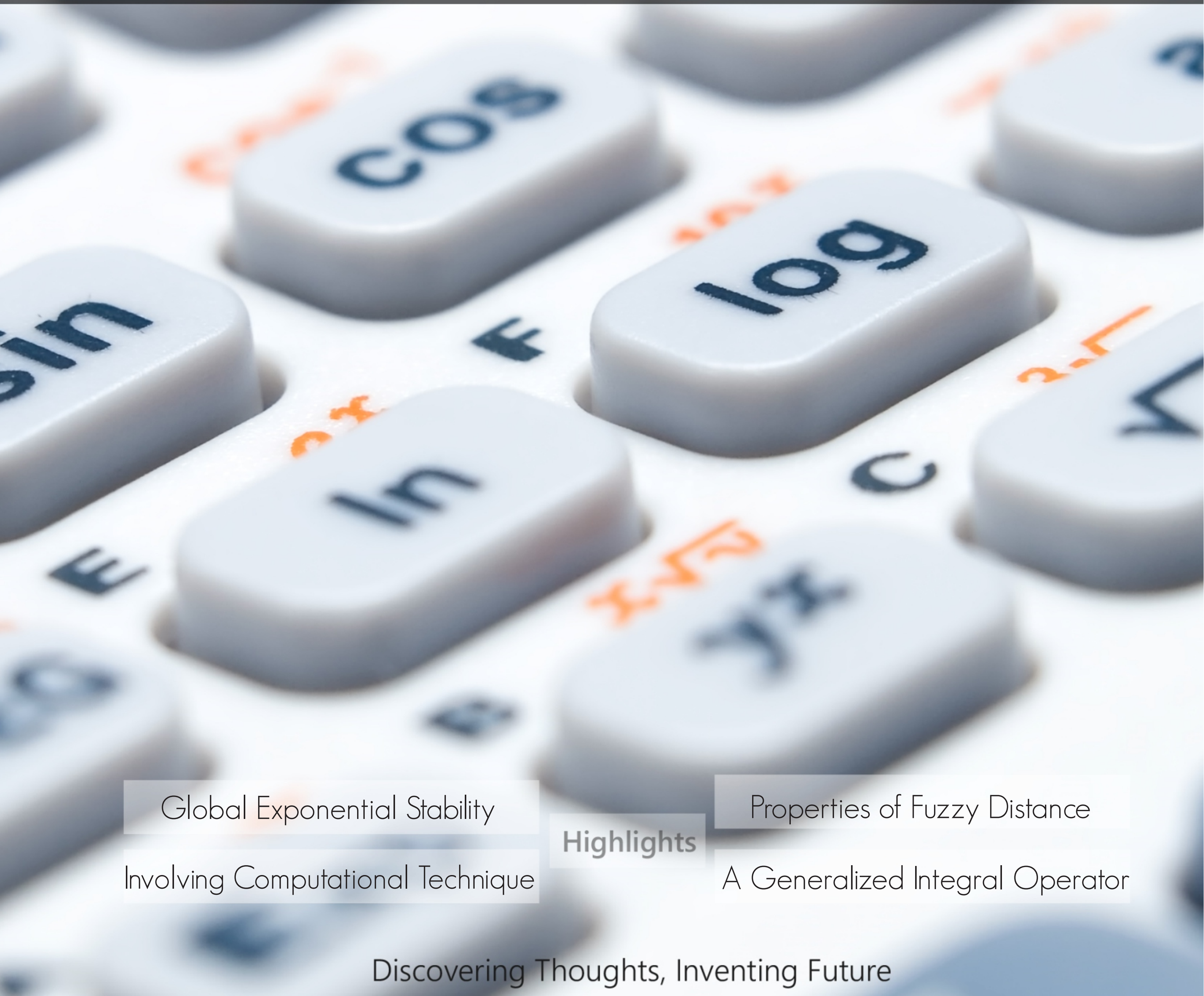

Highlights

Global Exponential Stability

Involving Computational Technique

Properties of Fuzzy Distance

A Generalized Integral Operator

Discovering Thoughts, Inventing Future

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On Certain Type Fractional Integration of Special Functions Via Pathway Operator

By Hemlata Saxena & Rajendra Kumar Saxena
Career Point University, India

Abstract- In this present paper we study product of some special functions via pathway fractional integral operator. Our results are quite general in nature .Some known and new results are also obtain here.

Keywords: *Pathway fractional integral operator, M-series, New generalized Mittag-Leffler function.*

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On Certain Type Fractional Integration of Special Functions Via Pathway Operator

Hemlata Saxena [α] & Rajendra Kumar Saxena [σ]

Abstract- In this present paper we study product of some special functions via pathway fractional integral operator. Our results are quite general in nature .Some known and new results are also obtain here.

Keywords: Pathway fractional integral operator, M-series, New generalized Mittag-Leffler function.

I. Introduction

Definition: Let $f(x) \in L(a,b)\,, \eta \in C\,, R(\eta) > 0, a > 0$ and let us take a "Pathway parameter" $\alpha < 1$. Then the pathway fractional integration operator is defined by Nair [8]

$$\left(P_{0^+}^{(\eta,\alpha)} f\right)(x) = x^{\eta} \int_0^{\left(\frac{x}{a(1-\alpha)}\right)} \left[1-\frac{a(1-\alpha)t}{x}\right]^{\frac{\eta}{(1-\alpha)}} f(t)dt \qquad ......(1.1)$$

when $\alpha = 0\,,\ a = 1$ *and* η is replaced by $\eta - 1$ in (1.1) it yields

$$\left(I_{0+}^{\eta} f\right)(x) = \frac{1}{\Gamma(\eta)} \int_0^x (x-t)^{\eta-1} f(t)dt \qquad ...(1.2)$$

which is the left – sided Riemann-Liouville fractional integral defined by Samko et. al.[9].

The pathway model is introduced by Mathai [5] and studied further by Mathai and Houbold [6] [7].

For $R(\alpha) > 0$, the pathway model for scalar random variables is represented by the following probability density function.

$$f(x) = c|x|^{\gamma-1}\left[1-a(1-\alpha)|x|^{\delta}\right]^{\frac{\beta}{1-\alpha}} \qquad ....(1.3)$$

$$\gamma > 0, \delta > 0, \beta \geq 0\,, \left\{1-a(1-\alpha)|x|^{\delta}\right\} > 0, -\infty < x < \infty,$$

where c is the normalizing constant and α is pathway parameter. For real α, the normalizing constant is as follows:

$$c = \frac{1}{2}\,\frac{\delta[a(1-\alpha)]^{\frac{\gamma}{\delta}}\,\Gamma\left(\frac{\gamma}{\delta}+\frac{\beta}{1-\alpha}+1\right)}{\Gamma\left(\frac{\gamma}{\delta}\right)\,\Gamma\left(\frac{\beta}{1-\alpha}+1\right)}\,, \qquad \alpha < 1 \qquad ....(1.4)$$

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$$= \frac{1}{2}\frac{\delta[a(1-\alpha)]^{\frac{\gamma}{\delta}}\,\Gamma(\frac{\beta}{1-\alpha})}{\Gamma(\frac{\gamma}{\delta})\,\Gamma(\frac{\beta}{1-\alpha}-\frac{\gamma}{\delta})}\ ,\ \text{for}\ \ \frac{1}{1-\alpha}-\frac{\gamma}{\delta}>0,\alpha>1 \qquad ......(1.5)$$

$$= \frac{1}{2}\frac{\delta[a\beta]^{\frac{\gamma}{\delta}}}{\Gamma(\frac{\gamma}{\delta})},\ \ \text{for}\ \ \alpha\to 1 \qquad ......(1.6)$$

For $\alpha<1$, it is a finite range density with $[1-a(1-\alpha)|x|^{\delta}]>0$ and (1.3) remains in the extended generalized type-1 beta family. For $\alpha<1$, the pathway density in (1.3) includes the extended type-1 beta density, the triangular density, the uniform density and many other p.d.f.

when $\alpha>1$, we write $1-\alpha=-(\alpha-1)$, then

$$\left(P_{0^+}^{(\eta,\alpha)}f\right)(x) = x^{\eta}\int_0^{\left(-\frac{x}{a(\alpha-1)}\right)}\left[1+\frac{a(\alpha-1)t}{x}\right]^{-\frac{\eta}{(\alpha-1)}} f(t)dt$$

$$f(x) = c\,|x|^{\gamma-1}\left[1+a(\alpha-1)|x|^{\delta}\right]^{-\frac{\beta}{\alpha-1}} \qquad ......\ (1.7)$$

Where $\alpha>1,\ \delta>0,\beta\geq 0,-\infty<x<\infty$,
which is extended generalized type-2 beta model for real x, It includes the type-2 beta density, the F density, the student-t density,the Cauchy density and many more.

Here, we consider only the case of pathway parameter $\alpha<1$. For $\alpha\to 1$ both (1.3) and (1.7) take the exponential form, since.

$$\lim_{\alpha\to 1} c\ |x|^{\gamma-1}\left[1-a(1-\alpha)|x|^{\delta}\right]^{\frac{\eta}{1-\alpha}}$$

$$= \lim_{\alpha\to 1} c\ |x|^{\gamma-1}\left[1+a(\alpha-1)|x|^{\delta}\right]^{-\frac{\eta}{\alpha-1}}$$

$$= c|x|^{\gamma-1}e^{-a\eta|x|^{\delta}} \qquad ....(1.8)$$

For $\alpha\to 1_{-}$, $\left[1-\frac{a(1-\alpha)t}{x}\right]^{\frac{\eta}{1-\alpha}}\to e^{-\frac{a\eta}{x}t}$, the operator (1.1) reduces to the following form

$$\left(P_{0^+}^{(\eta,1)}f\right)(x) = x^{\eta}\int_0^{\infty} e^{-\frac{a\eta}{x}t}\ f(t)dt$$

$$= x^{\eta}L_f\left(\frac{a\eta}{x}\right) \qquad ......(1.9)$$

It reduces to the Laplace integral transform of f with parameter $\frac{a\eta}{x}$.

In this paper we will integrate product of M-series, Fox's H-function and generalized Mittag-Leffler function by means of pathway model.

The generalized M-series is defined and studied by Sharma and Jain [10] as follows

$$_{\rho}M_{\sigma}^{\alpha',\beta'}(z) = \sum_{k=0}^{\infty}\frac{(a_1')_k\ \ldots\ldots(a_\rho')_k}{(b_1')_k\ \ldots\ldots(b_\sigma')_k}\ \frac{z^k}{\Gamma(\alpha' k+\beta')}$$

Ref

10. Sharma, Manoj and Jain, Renu, A note on a generalized M-series as a special function of fractional calculus, Fract. Cal. Appl. Anal. 12(4) (2009), 449-452.

$$= \psi_1(k) \qquad \ldots(1.10)$$

Where $z, \alpha', \beta' \in C, Re(\alpha') > 0$

Here $(a_j')_k, (b_j')_k$ are known as Pochammer symbols. The series (1.10) is defined when none of the parameters $b_{js}', j = 1,2, \ldots \sigma$ is negative integer or zero. The series in (1.10) is convergent for all z if $\rho \le \sigma$, it is convergent for $|z| < \delta = \alpha^{\alpha}$ if $\rho = \sigma + 1$ and divergent, if $\rho > \sigma + 1$. When $\rho > \sigma + 1$ and $|z| < \delta$, the series can converge on conditions depending on the parameters.

The series representation of Fox H- function studied by Fox C [2] as follows:

$$H_{P,Q}^{M,N}\left[z\middle|\begin{matrix}(e_p, E_p)\\(f_Q, F_Q)\end{matrix}\right] = \sum_{h=1}^{N}\sum_{v=0}^{\infty}\frac{(-1)^v X(\xi)}{v!\,E_h}, \left(\frac{1}{z}\right)^{\xi} \qquad \ldots(1.11)$$

where $\xi = \frac{e_h - v - 1}{E_h}$ and $(h = 1,2, \ldots ,N)$

and

$$X(\xi) = \frac{\prod_{j=1}^{M}\Gamma(f_j + F_j\xi)\prod_{\substack{j=1\\j\neq h}}^{N}\Gamma(1 - e_j + E_j\xi)}{\prod_{j=m+1}^{Q}\Gamma(1 - f_j - F_j\xi)\prod_{j=N+1}^{P}\Gamma(e_j + E_j\xi)} \qquad \ldots..(1.12)$$

Following are the convergence conditions :

$$T_1 = \sum_{i=1}^{N} E_i - \sum_{i=N+1}^{P} E_i + \sum_{i=1}^{M} F_i - \sum_{i=M+1}^{Q} F_i \qquad \ldots.(1.13)$$

$$T_2 = \sum_{i=1}^{n} \alpha_i - \sum_{i=n+1}^{q} \alpha_i + \sum_{i=1}^{m} \beta_i - \sum_{i=m+1}^{q} \beta_i \qquad \ldots(1.14)$$

Recently, a new generalization of Mittag-Leffler function was defined by Faraj and Salim [3] as follows:

$$E_{\alpha,\beta,p}^{\gamma,\delta,q}(z) = \sum_{n=0}^{\infty}\frac{(\gamma)_{qn}\, z^n}{\Gamma(\alpha n + \beta)\,(\delta)_{pn}} \qquad \ldots...(1.15)$$

Where $z, \alpha, \beta, \gamma, \delta \in C; Min\{Re(\alpha), Re(\beta), Re(\gamma), Re(\delta)\} > 0, p, q > 0$.

Further, generalization of Mittag- Leffler function was defined by Khan and Ahmed [4] as follows:

$$E_{\alpha,\beta,v,\sigma,\delta,p}^{\mu,\rho,\gamma,q}(z) = \sum_{n=0}^{\infty}\frac{(\mu)_{\rho n}\,(\gamma)_{qn}\, z^n}{\Gamma(\alpha n + \beta)(v)_{\sigma n}\,(\delta)_{pn}} \qquad \ldots(1.16)$$

Where $\alpha, \beta, \gamma, \delta, \mu, v, \rho, \sigma \in C; p, q > 0$ and $q \le Re(\alpha) + \rho p$, and

$$Min\{Re(\alpha), Re(\beta), Re(\gamma), Re(\delta), Re(\mu), Re(v), Re(\rho), Re(\sigma)\} > 0$$

If we take $\mu = v, \rho = \sigma$ in (1.16) it reduces to eq. (1.15).

Write generalized hypergeometric function was defined by Srivastava and Manocha [11] as follows:

Ref

2. Fox, C., The G and H-functions as symmetrical Fourier kernels, Trans. Amer. Math soc., 98 (1961), 395-429.

$${}_p\psi_q[(a_1,A_1),\ldots(a_p,A_p);(b_1,B_1),\ldots,(b_q,B_q);z]=\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma(a_i+A_in)z^n}{\prod_{j=1}^{q}\Gamma(b_j+B_jn)n!} \quad \ldots.(1.17)$$

II. Main Results

Theorem-1 Let $\eta,\gamma,\delta,q,p,\omega,\rho\in C,\ c,b\in R, Re(\beta)>0, Re(\delta)>0, Re(\eta)>0, Re(\gamma)>0, Re(\omega)>0, Re\left(1+\frac{\eta}{1-\alpha}\right)>0, Re(\rho)>0,\alpha<1, b\in R, c\in R, Re\left(\omega+\delta\frac{f_j}{F_j}\right)>0, |\arg c|<\frac{1}{2}T_1\pi, T_1T_2>0,\rho\le\sigma,|d|<\alpha'^{\alpha'},\beta^*>0, j=1,\ldots,Q$;

Then

$$P_{0^+}^{(\eta,\alpha)}\left\{t^{\omega-1}\,{}_{\rho}^{\alpha',\beta'}M_{\sigma}\,[dt^{-\beta^*}].\ H_{P,Q}^{M,N}\left[ct^{\delta'}\left|\begin{matrix}(e_p,E_p)\\(f_Q,F_Q)\end{matrix}\right.\right].E_{\alpha,\beta,p}^{\gamma,\delta,q}(bt^{\rho})\right\}$$

$$=\psi_1(k)\frac{d^kx^{\eta+\omega-\beta^*k}\,\Gamma\left(1+\frac{\eta}{1-\alpha}\right)\Gamma(\delta)}{\Gamma(\gamma)[a(1-\alpha)]^{\omega-\beta^*k}}\,{}_2\psi_3\left[\frac{bx^{\rho}}{[a(1-\alpha)]^{\rho}}\left|\begin{matrix}(\omega-\delta\xi-\beta^*k,\rho)(\gamma,q)\\(\omega,\rho)(\delta,p)\left(1+\omega+\frac{\eta}{1-\alpha}-\delta\xi-\beta^*k,\rho\right)\end{matrix}\right.\right].$$

$$H_{P,Q}^{M,N}\left[\frac{cx^{\delta'}}{[a(1-\alpha)]^{\delta'}}\left|\begin{matrix}(e_p,E_p)\\(f_Q,F_Q)\end{matrix}\right.\right] \quad \ldots\ldots\ldots..(2.1)$$

Proof: The theorem -1 can be evaluated by using the definitions (1.1),(1.10),(1.11) and (1.15) then by interchange the order of integrations and summations, evaluate the inner integral by making use of beta function formula, we arrive at the desired result (2.1).

Theorem-2 Let $\eta,\gamma,\delta,q,p,\beta,T_1,T_2>0,\mu,\rho,\gamma,\vartheta,\beta,v,\sigma,\delta\in C, Re(\eta)>0, Re(\gamma)>0, Re(\beta)>0, Re\left(1+\frac{\eta}{1-\alpha}\right)>0, b,c\in R,\ \alpha<1,\ Re\left(\omega+\delta\frac{f_j}{F_j}\right)>0,\ |\arg c|<\frac{1}{2}T_1\pi,\rho\le\sigma$ *and* $|d|<\alpha'^{\alpha'},\beta^*>0, j=1,\ldots,Q$ *and* $\min(Re(\vartheta), Re(\beta), Re(\gamma), Re(\delta), Re(\mu), Re(v), Re(\rho), Re(\sigma))>0$

Then

$$P_{0^+}^{(\eta,\alpha)}\left\{t^{\beta-1}\,{}_{\rho}^{\alpha',\beta'}M_{\sigma}\,[dt^{-\beta^*}].\ H_{P,Q}^{M,N}\left[ct^{\delta'}\left|\begin{matrix}(e_p,E_p)\\(f_Q,F_Q)\end{matrix}\right.\right].E_{\vartheta,\beta,v,\sigma,\delta,p}^{\mu,\rho,\gamma,q}(bt^{\vartheta})\right\}$$

$$=\psi_1(k)\frac{d^kx^{\eta+\beta-\beta^*k}\,\Gamma\left(1+\frac{\eta}{1-\alpha}\right)\Gamma(v)}{[a(1-\alpha)]^{\beta-\beta^*k}\Gamma(\mu)\Gamma(\gamma)}.$$

$${}_3\psi_3\left[\frac{bx^{\vartheta}}{[a(1-\alpha)]^{\vartheta}}\left|\begin{matrix}(\mu,\rho)\quad(\gamma,q)\ (\beta-\delta'\xi-\beta^*k,\vartheta)\\(\beta,\vartheta)(v,\sigma)\left(1+\beta+\frac{\eta}{1-\alpha}-\delta'\xi-\beta^*k,\vartheta\right)\end{matrix}\right.\right].$$

$$H_{P,Q}^{M,N}\left[\frac{cx^{\delta'}}{[a(1-\alpha)]^{\delta'}}\left|\begin{matrix}(e_p,E_p)\\(f_Q,F_Q)\end{matrix}\right.\right] \quad \ldots\ldots\ldots.(2.2)$$

Proof: The theorem -2 can be evaluated by using the definitions (1.1),(1.10) (1.11) and (1.16) then by interchange the order of integrations and summations, evaluate the inner integral by making use of beta function formula, we arrive at the desired result (2.2).

Notes

Notes

III. Special Cases

1. If we take $\delta = p = q = 1, \rho = \beta, \alpha = \beta,\ \beta = \omega$ and in H- function $\delta' = \delta$ in theorem -1 then we at once arrive at the known result of [1,Theorem-2].
2. If we take $\delta = p = 1$ in theorem -1 then we get the following particular case of the solution (2.1)
   Corollary-1

The following formula holds

$$P_{0^+}^{(\eta,\alpha)}\left\{t^{\omega-1}\ {}_{\rho}\overset{\alpha',\beta'}{M}_{\sigma}\ [dt^{-\beta^*}].\ H_{P,Q}^{M,N}\left[ct^{\delta'}\middle|\begin{matrix}(e_p,E_p)\\(f_Q,F_Q)\end{matrix}\right].E_{\rho,\omega}^{\gamma,q}(bt^{\rho})\right\}$$

$$=\psi_1(k)\frac{d^k x^{\eta+\omega-\beta^*k}\,\Gamma\left(1+\frac{\eta}{1-\alpha}\right)}{\Gamma(\gamma)[a(1-\alpha)]^{\omega-\beta^*k}}\ {}_2\psi_2\left[\frac{bx^{\rho}}{[a(1-\alpha)]^{\rho}}\middle|\begin{matrix}(\omega-\delta\xi-\beta^*k,\rho)(\gamma,q)\\(\omega,\rho)\left(1+\omega+\frac{\eta}{1-\alpha}-\delta\xi-\beta^*k,\rho\right)\end{matrix}\right]$$

$$H_{P,Q}^{M,N}\left[\frac{cx^{\delta'}}{[a(1-\alpha)]^{\delta'}}\middle|\begin{matrix}(e_p,E_p)\\(f_Q,F_Q)\end{matrix}\right]$$

Where $\eta,\gamma,q,\omega,\rho\in C,\ c,b\ \in R, Re(\beta)>0, Re(\delta)>0, Re(\eta)>0, Re(\gamma)>0, Re(\omega)>0, Re\left(1+\frac{\eta}{1-\alpha}\right)>0, Re(\rho)>0, \alpha<1, b\in R, c\in R, Re\left(\omega+\delta\frac{f_j}{F_j}\right)>0, |\arg c|<\frac{1}{2}T_1\pi, T_1T_2>0, \rho\le\sigma, |d|<\alpha'^{\alpha'}, \beta^*>0, j=1,\dots,Q;$

3. If we take $\mu = v, \rho = \sigma, \delta = p = q = 1$ and $\vartheta \to \beta, \beta \to \omega$ in H function $\delta' = \delta$ in theorem-2 then we at once arrive at the known result of [1, Theorem-1] .
4. If we take $\mu = v, \rho = \sigma$ then we at once arrive at the theorem-1.
5. Making $\beta^*, \delta' \to 0$ and $\delta = p = q = 1, \rho = \beta$ in the result (2.1) and $\beta^*, \delta' \to 0, \mu = v, \rho = \sigma, \delta = p = q = 1$ in result (2.2) then we at once arrive at the known result of Nair in[8].

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Robustness of the Sequential Test for the Scale Parameter of Nakagami Distribution

By Surinder Kumar & Mayank Vaish

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Abstract- In the present study, Sequential Probability Ratio Test (SPRT) is developed for the scale parameter of Nakagami Distribution and the robustness of scale parameter is studied when the shape parameter has undergone a change, for testing the hypothesis regarding the parameter of Nakagami Distribution. The expression for the Operating Characteristic (OC) and Average Sample Number (ASN) functions are derived and the results are presented through Graphs and Tables.

Keywords: *nakagami distribution, SPRT, OC and ASN functions, robustness.*

GJSFR-F Classification : *MSC 2010: 97K80*

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Robustness of the Sequential Test for the Scale Parameter of Nakagami Distribution

Surinder Kumar [α] & Mayank Vaish [σ]

Abstract- In the present study, Sequential Probability Ratio Test (SPRT) is developed for the scale parameter of Nakagami Distribution and the robustness of scale parameter is studied when the shape parameter has undergone a change, for testing the hypothesis regarding the parameter of Nakagami Distribution. The expression for the Operating Characteristic (OC) and Average Sample Number (ASN) functions are derived and the results are presented through Graphs and Tables.

Keywords: nakagami distribution, SPRT, OC and ASN functions, robustness.

I. Introduction

Nakagami distribution is a lifetime distribution, given by M. Nakagami (1960) has the probability density function (pdf)

$$f(x;\lambda,\beta)=\frac{2\lambda^{\lambda}}{\Gamma\lambda\,\beta^{\lambda}}x^{2\lambda-1}e^{-\frac{\lambda}{\beta}x^{2}}\;;\qquad x>0,\ \lambda,\beta>0\,, \qquad \ldots(1.1)$$

where λ is a shape parameter and β is scale parameter. Nakagami distribution is related to Rayleigh distribution and one-sided Gaussian distribution when $\lambda=1$, and $\lambda=1/2$, respectively.

In this paper, we have developed the SPRT for scale parameter of Nakagami distribution and studied the robustness of the scale parameter when there is change in the shape parameter. The robustness of SPRT has been studied by several authors. For a brief review, one may refer to Epstein and Sobel (1955), Johnson(1966), Barlow and Proschan (1967), Phatarfod (1971), Harter and Moore (1976), Montagne and Singpurwalla (1985), Joshi and Shah(1990), Chaturvedi, kumar and Kumar (1998), Chaturvedi, Kumar and Kumar (2000), Chaturvedi, Tiwari and Tomer (2002), Surinder and Naresh (2009).

In section 2, we state the problem, and develop SPRT for testing the simple null hypothesis against the simple alternative hypothesis. The expressions for OC and ASN functions are obtained in section 3. In section 4, robustness of the SPRT is studied and the results are discussed in section 5. Finally, the conclusions are given in section 6.

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II. Set-up of the Problem

Let the random variable X follows the Nakagami distribution given by the probability density function (pdf)

$$f(x;\lambda,\beta)=\frac{2\lambda^{\lambda}}{\Gamma\lambda\,\beta^{\lambda}}x^{2\lambda-1}e^{-\frac{\lambda}{\beta}x^{2}}\;;\qquad x>0,\ \lambda,\beta>0 \qquad ...(2.1)$$

Given a sequence of observations x_1, x_2, x_3,...from (2.1), suppose one wish to test the simple null hypothesis $H_0:\beta=\beta_0$ against the simple alternative $H_1:\beta=\beta_1(\beta_1>\beta_0)$. The expression for OC and ASN function is obtained and their behaviour is studied by plotting graph.

III. SPRT for Testing the Hypothesis Regarding 'β'

The SPRT for testing the null hypothesis $H_0:\beta=\beta_0$ against the simple alternative $H_1:\beta=\beta_1$ is defined as

$$z_i=\ln\frac{f(x_i;\lambda,\beta_1)}{f(x_i;\lambda,\beta_0)} \qquad ...(3.1)$$

Or,

$$z_i=\lambda\ln\left(\frac{\beta_0}{\beta_1}\right)-x^2\lambda\left(\frac{1}{\beta_1}-\frac{1}{\beta_0}\right) \qquad ...(3.2)$$

Or,

$$e^{z_i}=\left(\frac{\beta_0}{\beta_1}\right)^{\lambda}e^{-\lambda x^2\left(\frac{1}{\beta_1}-\frac{1}{\beta_0}\right)} \qquad ...(3.3)$$

Now, we choose two numbers A and B such that $0<B<1<A$. At the n^{th} stage, accept H_0, if $\sum_{i=1}^{n}z_i\le\ln B$, reject H_0 if $\sum_{i=1}^{n}z_i\le\ln A$, otherwise continue sampling by taking the $(n+1)^{th}$ observation.

If $\alpha\in(0,1)$ and $\beta\in(0,1)$ are TYPE I and TYPE II errors respectively, then according to Wald (1947), A and B are approximately given by

$$A\approx\frac{1-\beta}{\alpha}\quad\text{and}\quad B\approx\frac{\beta}{1-\alpha} \qquad ...(3.4)$$

The Operating Characteristic (OC) function $L(\theta)$ is given by

$$L(\theta)=\frac{A^{h}-1}{A^{h}-B^{h}}, \qquad ...(3.5)$$

where h is the non-zero solution of

$$\mathrm{E}\left[e^{hz}\right]=1 \qquad ...(3.6)$$

Or,

Notes

$$\int_0^{\infty}\left[\frac{f(x_i;\lambda,\beta_1)}{f(x_i;\lambda,\beta_0)}\right]^h f(x_i;\lambda,\beta)\,dx=1 \qquad ...(3.7)$$

From (2.1) and (3.3), since

$$E\left[e^z\right]^h=\frac{\left(\frac{\beta_0}{\beta_1}\right)^{\lambda h}}{\left[1+\beta h\left(\frac{1}{\beta_1}-\frac{1}{\beta_0}\right)\right]^{\lambda}}, \qquad ...(3.8)$$

we get from (3.6) that

$$\beta=\frac{1-\left(\frac{\beta_0}{\beta_1}\right)^h}{h\left(\frac{1}{\beta_1}-\frac{1}{\beta_0}\right)} \qquad ...(3.9)$$

The ASN function is approximately given by

$$E(N\,|\,\theta)=\frac{L(\theta)\ln B+\{1-L(\theta)\}\ln A}{E(Z)}, \qquad ...(3.10)$$

Provided $E(Z)\neq 0$, where

$$E(Z)=\lambda\left[\ln\left(\frac{\beta_0}{\beta_1}\right)-\beta\left(\frac{1}{\beta_1}-\frac{1}{\beta_0}\right)\right] \qquad ...(3.11)$$

From (3.11) ASN function under H_0 and H_1 are given by

$$E_0(N)=\frac{(1-\alpha)\ln B+\alpha\ln A}{\lambda\left[\ln\left(\frac{\beta_0}{\beta_1}\right)-\beta\left(\frac{1}{\beta_1}-\frac{1}{\beta_0}\right)\right]} \qquad ...(3.12)$$

and

$$E_1(N)=\frac{\beta\ln B+(1-\beta)\ln A}{\lambda\left[\ln\left(\frac{\beta_0}{\beta_1}\right)-\beta\left(\frac{1}{\beta_1}-\frac{1}{\beta_0}\right)\right]} \qquad ...(3.13)$$

IV. Robustness of SPRT for Parameter of Nakagami Distribution

Let us suppose that the parameter 'λ' has undergone a change then the probability distribution in (2.1) becomes $f(x;\lambda^*,\beta)$. To study the robustness of SPRT developed in section 3 with respect to OC function, consider 'h' as the solution of the equation

$$E_{\lambda^*}\left[e^z\right]^h=1 \qquad ...(4.1)$$

Notes

Or,
$$\int_0^\infty \left[\frac{f(x_i;\lambda,\beta_1)}{f(x_i;\lambda,\beta_0)}\right]^h f(x_i;\lambda^*,\beta)\,dx = 1 .$$

After solving, we get

$$\beta = \frac{1-\left(\dfrac{\beta_0}{\beta_1}\right)^{\left(\frac{\lambda}{\lambda^*}\right)h}}{h\dfrac{\lambda}{\lambda^*}\left(\dfrac{1}{\beta_1}-\dfrac{1}{\beta_0}\right)} \quad \ldots(4.2)$$

For different values of β, h is evaluated and the OC function is obtained. The Robustness of SPRT with respect to ASN can be studied by replacing denominator of (3.10) by

$$E_{\lambda^*}(z) = \int_0^\infty z\, f(x;\lambda^*,\beta)\,dx$$

$$= \lambda\left[\ln\left(\frac{\beta_0}{\beta_1}\right) - \beta\left(\frac{1}{\beta_1}-\frac{1}{\beta_0}\right)\right] \quad \ldots(4.3)$$

We consider the cases $\lambda > \lambda^*$ and $\lambda < \lambda^*$ to study the robustness of SPRT.

V. Results and Discussions

Consider the equation (4.2) and taking the logarithms of both sides, expanding and retaining the terms up to third degree in 'h', we get

$$\left\{\beta^3P^3\left(\frac{1}{\beta_1}-\frac{1}{\beta_0}\right)^3\right\}\frac{h^2}{3} - \left\{\beta^2P^2\left(\frac{1}{\beta_1}-\frac{1}{\beta_0}\right)^2\right\}\frac{h}{2} + \left\{\beta\left(\frac{1}{\beta_1}-\frac{1}{\beta_0}\right) - \ln\left(\frac{\beta_0}{\beta_1}\right)\right\}P = 0 \quad \ldots(5.1)$$

where $P = \dfrac{\lambda}{\lambda^*}$,

For testing $H_0:\beta=13$ verses $H_1:\beta=15$ and taking $\alpha=\beta=0.05$, the real roots of 'h' are obtained by using (5.1) for the different values of β. The OC and ASN functions are evaluated by using the equations (3.5) and (3.10) by considering the cases $\lambda>\lambda^*$, $\lambda=\lambda^*$ and $\lambda<\lambda^*$ respectively. The results are presented in *Table 1.* and *Table 2,* respectively. The graph for OC and ASN functions are plotted in *Fig.1* and *Fig.2*, respectively.

Table1 : Values of OC Function for Scale Parameter of Nakagami Distribution

β	L(β) P=0.5	P=1	P=1.5
12.8	0.999316	0.97451	0.919019
12.9	0.998652	0.964562	0.900481
13	**0.997371**	**0.951168**	**0.878632**
13.1	0.994931	0.933375	0.853186
13.2	0.990337	0.910101	0.823936
13.3	0.98182	0.880222	0.790789
13.4	0.966332	0.842704	0.753801
13.5	0.938946	0.796814	0.713205
13.6	0.892517	0.742377	0.669422
13.7	0.818716	0.680014	0.62306

Notes

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13.8	0.71206	0.611282	0.574883
13.9	0.57676	0.538609	0.525768
14	0.430317	0.464988	0.476637
14.1	0.296256	0.393507	0.428397
14.2	0.190791	0.326856	0.381868
14.3	0.117107	0.266968	0.337742
14.4	0.069694	0.214889	0.296546
14.5	0.040728	0.170848	0.258633
14.6	0.023563	0.134457	0.224191
14.7	0.013561	0.104946	0.193261
14.8	0.007784	0.081368	0.165766
14.9	0.004462	0.062748	0.141541
15	**0.002555**	**0.048174**	**0.120358**
15.1	0.001461	0.036846	0.101959
15.2	0.000834	0.028087	0.086067

Figure 1 : Graph of OC Function for Nakagami Distribution

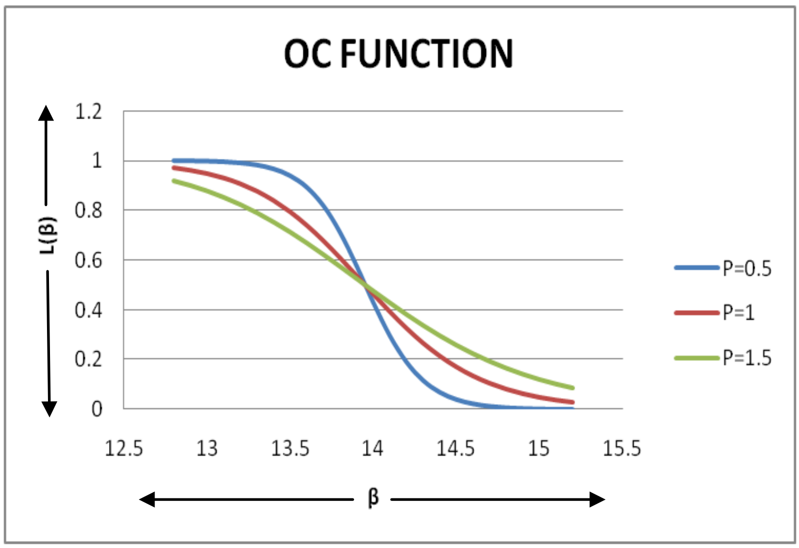
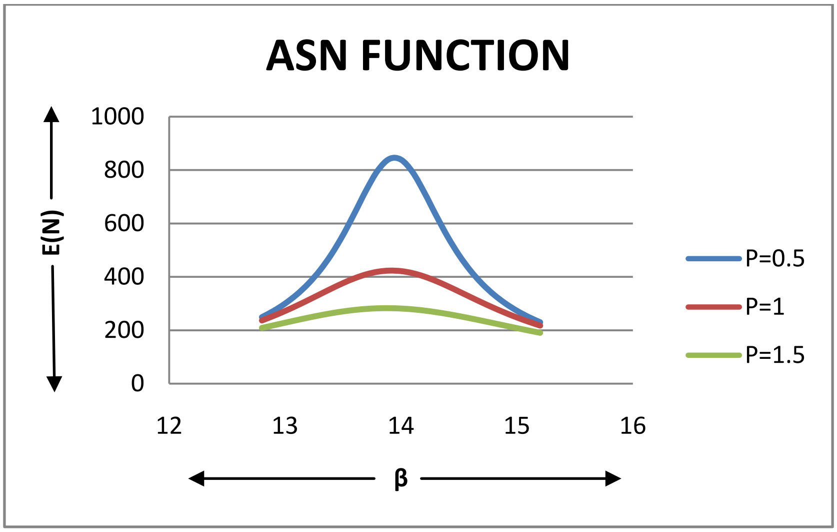


Table 2 : Values of ASN Function for Scale Parameter of Nakagami Distribution

	E(N)		
β	P=0.5	P=1	P=1.5
12.8	248.791298	236.431136	208.782162
12.9	272.070729	253.470573	218.507593
13	**299.867491**	**272.011541**	**228.279256**
13.1	333.40532	291.938749	237.9207
13.2	374.215811	312.981253	247.221902
13.3	424.084893	334.661168	255.944863
13.4	484.766036	356.250404	263.833837
13.5	557.173848	376.759353	270.630404
13.6	639.65168	394.980317	276.092553
13.7	725.216594	409.609487	280.01483
13.8	799.289775	419.441269	282.247663
13.9	842.175927	423.598729	282.7132
14	839.340213	421.72793	281.407782
14.1	792.201008	414.068829	278.409715
14.2	716.835926	401.397756	273.86376
14.3	632.339526	384.846574	267.964957
14.4	551.898184	365.674585	260.944299
14.5	481.492999	345.077459	253.045105
14.6	422.367225	324.058467	244.508307
14.7	373.556718	303.37858	235.557736
14.8	333.40091	283.559294	226.392386

14.9	300.232638	264.918963	217.180598
15	**272.620552**	**247.619408**	**208.059374**
15.1	249.413412	231.71082	199.13543
15.2	229.711533	217.169974	190.488238

Figure 2 : Graph of ASN Function for Nakagami Distribution



Notes

VI. Conclusions

The values of OC and ASN functions for the cases $\lambda < \lambda^*$, $\lambda = \lambda^*$ and $\lambda > \lambda^*$ are plotted in *Fig.1* and *Fig.2,* respectively. From the *Fig.1,* we observe that for $\lambda < \lambda^*$ ($\lambda > \lambda^*$), the OC curve shifts to the right side (left side) of the curve when $\lambda = \lambda^*$. From the *Fig.1,* it is clear that SPRT is non-robust for $\lambda^* = \lambda \pm 0.5$ as the deviation in OC function is significant. Again, from *Fig.2,* we observe that for $\lambda < \lambda^*$ ($\lambda > \lambda^*$), the ASN curve shifts above (below) of the curve when $\lambda = \lambda^*$. Both the curves are highly sensitive for the changes in λ. Thus we conclude that for the present model, the SPRT for testing the hypothesis regarding β, is highly non-robust for changes in λ.

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Notes

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Global Exponential Stability of Impulsive Functional Differential Equations with Effect of Delay at the Time of Impulses

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Abstract- This paper studies the global exponential stability of impulsive functional differential system with the effect of delay at the time of impulses by using Lyapunov functions and Razumikhin technique. This result extends some results existing in the literature. The obtained result also shows that the derivative of Lyapunov function may not be negative even then impulses can make the system globally exponentially stabilized.

Keywords: impulsive delay differential systems, lyapunov function, razumikhin technique, global exponential stability.

GJSFR-F Classification : MSC 2010: 35R50

GLOBALEXPONENTIALSTABILITYOFIMPULSIVEFUNCTIONALDIFFERENTIALEQUATIONSWITHEFFECTOFDELAYATTHETIMEOFIMPULSES

Strictly as per the compliance and regulations of :

Global Exponential Stability of Impulsive Functional Differential Equations with Effect of Delay at the Time of Impulses

Palwinder Singh [α], Sanjay K. Srivastava [σ], Kanwalpreet Kaur [ρ]

Abstract- This paper studies the global exponential stability of impulsive functional differential system with the effect of delay at the time of impulses by using Lyapunov functions and Razumikhin technique. This result extends some results existing in the literature. The obtained result also shows that the derivative of Lyapunov function may not be negative even then impulses can make the system globally exponentially stabilized.

Keywords: impulsive delay differential systems, lyapunov function, razumikhin technique, global exponential stability.

I. Introduction

The impulsive differential equations represent a framework for mathematical modeling of many real life situations in the field of engineering, biology, chemistry, physics, control systems, population dynamics and many more [1]. In last two decades the stability analysis of these have been extensively explored [4,5,7,15-19]. In [17] the criteria for global exponential stability for impulsive functional differential equations is obtained by using Lyapunov function and Razumikhin technique. Moreover, it has been shown that impulses may make the system exponentially stable even if derivative of Lyapunov function is not negative. It is supposed that the state variables on impulses are related to present state variables but it is also possible that state variables on impulses are related to time delay. The aim of this paper is to get global exponential stability criteria for impulsive functional differential equation when state variables are dependent on both present and past state variables.

This paper organized as follows. In section II, some notations and definitions are given. We proved some criteria of global exponential stability for impulsive functional differential equations in section III, At last some concluding remarks are given in section IV.

II. Preliminaries

Let R^n denotes the n-dimensional real space and N denotes the set of positive integers. For given constant $\tau > 0$, the linear space $PC([-\tau,0],R^n)$ with norm $\|.\|$ defined by

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$$\|\psi\| = \sup_{r\in[-\tau,0]} \|\psi(r)\|.$$

Consider the differential system :

$$x'(t) = f(t, x_t), t \ge t_0, t \ne t_k$$
$$\Delta x(t_k) = I_k(x(t_k^-)) + J_k(x(t_k^- - \tau)), k = 1,2,3,.....$$
$$x_{t_0} = \psi \qquad (1)$$

Where

$$f : R_+ \times PC([-\tau,0], R^n) \to R^n;$$

$$I_k, J_k \in C[R^n, R^n]; \psi \in PC([-\tau,0], R^n);$$

$$0 \le t_0 < t_1 < t_2 < .... < t_k < ...., \quad \text{with} \quad t_k \to \infty \text{ as } k \to \infty ;$$

$$\Delta x(t) = x(t) - x(t^-); x(t), x(t^-) \in R^n$$

Throughout in this paper, we assume that f, I_k, J_k, $k \in N$ satisfy all necessary conditions for the global existence and uniqueness of solutions for all $t \ge t_0$ [6]. For any $\psi \in PC([-\tau,0], R^n)$, there exists a unique solution of (1) denoted by $x(t) = x(t, t_0, \psi)$. We further assume that all solutions x(t) of (1) are continuous except at $t = t_k, k \in N$,at which x(t) is right continuous i.e. $x(t_k^+) = x(t_k), k \in N$ and left limit i.e. $x(t_k^-)$ exists.

Definition 1: The function $V : R_+ \times R^n \to R_+$ is said to belong to the class ν_0 if the following conditions hold:

1) V is continuous in each of the sets $[t_{k-1}, t_k) \times R^n$,and for each $x \in R^n, t \in [t_{k-1}, t_k)$ $k \in N$, $lim_{(t,w)\to(t_k^-,x)} V(t, w) = V(t_k^-, x)$ exists.

2) $V(t, x)$ is locally Lipschitzian in all $x \in R^n$, and for all $t \ge t_0$, $V(t, 0) \equiv 0$.

Definition 2: Given a function $V : R_+ \times R^n \to R_+$, the upper right-hand derivative of V with respect to system (1) is defined by

$$D^+V(t, \varphi(0)) = \lim_{\delta\to 0^+} \sup \frac{1}{\delta}[V(t+\delta, \varphi(0) + \delta f(t,\psi)) - V(t, \varphi(0))]$$

for $(t,\psi) \in R_+ \times PC([-\tau,0,], R^n)$.

Definition 3: The trivial solution of the system (1) is said to be globally exponentially stable if there exist some constants a >0 and M ≥1such that for any initial data $x_{t_0} = \psi$

$$\|x(t, t_0, \psi)\| \le M\|\psi\| e^{-a(t-t_0)}, t \ge t_0 \quad \text{, where } (t_0, \psi) \in R_+ \times PC([-\tau,0,], R^n).$$

III. Main Results

Now in this section, we shall establish criteria for global exponential stability of impulsive functional differential equation in which state variables on impulses are related to time delay. We have the followings results.

Ref

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Theorem 1: Assume that there exist a function $V \in \nu_0$ and some constants $p, b, b_1, b_2 > 0$ and $l > \tau, \lambda > b$ such that

(i) $b_1\|x\|^p \leq V(t,x) \leq b_2\|x\|^p$, for any $t \in R_+$ and $x \in R^n$

(ii) $D^+V(t, \varphi(0)) \leq bV(t, \varphi(0))$, for all $t \in [t_{k-1}, t_k)$ $k \in N$

Whenever $hV(t,\varphi(0)) \geq V(t+r,\varphi(r))$ for $r \in [-\tau, 0]$, where $h \geq e^{2l\lambda}$ is a constant

(iii) for all $\varphi \in PC([-\tau,0];R^n)$

$V(t_k,\varphi(0)+I_k(\varphi(0))+J_k(\varphi(r)) \leq z_k\left[V(t_k^-,\varphi(0))+\sup_{r\in[-\tau,0]} V(t_k^-+r,\varphi(r))\right]$, where $z_k > 0, k \in N$ are constants.

(iv) $\tau \leq t_k - t_{k-1} \leq l$ and $z_k < \dfrac{e^{-\lambda l}.e^{-\lambda(t_{k+1}-t_k)}}{1+e^{\lambda\tau}}$

Then the trivial solution of (1) is globally exponentially stable.

Proof :- Choose $M \geq 1$ such that

$$b_2\|\psi\|^p < M\|\psi\|^p e^{-\lambda(t_1-t_0)}e^{-lb} < M\|\psi\|^p e^{-\lambda(t_1-t_0)} \leq hb_2\|\psi\|^p \tag{2}$$

Let $x(t) = x(t,t_0,\psi)$ be any solution of (1) with $x_{t_0} = \psi$ and v(t) =V(t, x). We shall now show that

$$v(t) \leq M\|\psi\|^p e^{-\lambda(t_k-t_0)}, t \in [t_{k-1}, t_k), k \in N \tag{3}$$

We shall prove this by induction, so firstly we shall show that result is true for k = 1 i.e.

$$v(t) \leq M\|\psi\|^p e^{-\lambda(t_1-t_0)}, t \in [t_0, t_1) \tag{4}$$

From condition (i) and (2) for $t \in [t_0 - \tau, t_0]$

$$v(t) \leq b_2\|x\|^p \leq b_2\|\psi\|^p < M\|\psi\|^p e^{-\lambda(t_1-t_0)}e^{-lb}$$

If (4) is not true, then there exist some $\hat{t} \in (t_0, t_1)$ such that

$$v(\hat{t}) > M\|\psi\|^p e^{-\lambda(t_1-t_0)} > M\|\psi\|^p e^{-\lambda(t_1-t_0)}e^{-lb} > b_2\|\psi\|^p \geq v(t_0 + r) \tag{5}$$

where $r \in [-\tau, 0]$

Which implies that there exist $\overset{\#}{t} \in \left(t_0, \hat{t}\right)$ such that

$$v(\overset{\#}{t}) = M\|\psi\|^p e^{-\lambda(t_1-t_0)} \text{ and } v(t) \leq M\|\psi\|^p e^{-\lambda(t_1-t_0)}, t_0 - \tau \leq t \leq \overset{\#}{t} \tag{6}$$

Notes

Then there exist $\overset{\#\#}{t} \in \left(t_0, \overset{\#}{t}\right)$ such that

$$v(\overset{\#\#}{t}) = b_2\|\psi\|^p \text{ and } v(t) \geq b_2\|\psi\|^p, \overset{\#\#}{t} \leq t \leq \overset{\#}{t} \tag{7}$$

Then for any $t \in \left[\overset{\#\#}{t}, \overset{\#}{t}\right]$, we got

$$v(t+r) \leq M\|\psi\|^p e^{-\lambda(t_1-t_0)} \leq hb_2\|\psi\|^p \leq hv(t) \tag{8}$$

And therefore from condition (ii), we get

$D^+v(t) \leq bv(t)$, for $t \in \left[\overset{\#\#}{t}, \overset{\#}{t}\right]$ and then we have $v(\overset{\#\#}{t}) \geq v(\overset{\#}{t})e^{-lb}$ i.e.

$b_2\|\psi\|^p \geq M\|\psi\|^p e^{-\lambda(t_1-t_0)}e^{-lb}$ which contradicts (2) .Hence (4) holds that means result (3) is true for k=1

Now assume that result (3) holds for k =1,2,3,4….m

$$\text{i.e. } v(t) \leq M\|\psi\|^p e^{-\lambda(t_k-t_0)}, t \in [t_{k-1}, t_k), k = 1,2,3,\ldots\ldots.m \tag{9}$$

from condition (iii) and (9) ,we get

$$\begin{aligned}
v(t_m) &\leq z_m\left[V(t_m^-, \varphi(0)) + \sup_{r\in[-\tau,0]} V(t_m^- + r, \varphi(r))\right] \\
&\leq z_m\left[M\|\psi\|^p e^{-\lambda(t_m-t_0)} + M\|\psi\|^p e^{-\lambda(t_m+r-t_0)}\right] \\
&\leq z_m M\|\psi\|^p e^{-\lambda(t_m-t_0)}(1+e^{-\lambda r}) \\
&\leq z_m M\|\psi\|^p e^{-\lambda(t_m-t_0)}(1+e^{\lambda\tau}) \\
&< e^{-\lambda l}e^{-\lambda(t_{m+1}-t_m)}M\|\psi\|^p e^{-\lambda(t_m-t_0)} \\
&< e^{-\lambda l}M\|\psi\|^p e^{-\lambda(t_{m+1}-t_0)} \\
&< M\|\psi\|^p e^{-\lambda(t_{m+1}-t_0)}
\end{aligned} \tag{10}$$

next we shall show that (3) holds for k = m+1

$$\text{i.e. } v(t) \leq M\|\psi\|^p e^{-\lambda(t_{m+1}-t_0)}, t \in [t_m, t_{m+1}) \tag{11}$$

suppose that (11) is not true then we can define $\bar{t} = \inf\{t \in [t_m, t_{m+1}]; v(t) > M\|\psi\|^p e^{-\lambda(t_{m+1}, t_0)}\}$ from (11) we know that $\bar{t} \neq t_m$. By the continuity of v(t) in $[t_m, t_{m+1})$,we have

$$v(\bar{t}) = M\|\psi\|^p e^{-\lambda(t_{m+1}-t_0)} \text{ and } v(t) \leq M\|\psi\|^p e^{-\lambda(t_{m+1}-t_0)}, t \in [t_m, \bar{t}] \tag{12}$$

From (10) we have since $v(t_m) < e^{-\lambda l}M\|\psi\|^p e^{-\lambda(t_{m+1}-t_0)} < v(\bar{t})$ which implies that there exist some

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$\overset{*}{t} \in \left(t_m, \bar{t}\right)$ such that

$$v(\overset{*}{t}) = e^{-\lambda l} M \|\psi\|^p e^{-\lambda(t_{m+1}-t_0)} \quad \text{and} \quad v(\overset{*}{t}) \le v(t) \le v(\bar{t}), t \in [\overset{*}{t}, \bar{t}] \tag{13}$$

Since $\tau \le t_k - t_{k-1} \le l$ therefore $t + r \in [t_{m-1}, \bar{t}]$ for $t \in [\overset{*}{t}, \bar{t}], r \in [-\tau, 0]$ .By (9), (12) and (13), we get for $t \in [\overset{*}{t}, \bar{t}]$

$$\begin{aligned}
& v(t+r) \le M\|\psi\|^p e^{-\lambda(t_m - t_0)} \\
& = M\|\psi\|^p e^{-\lambda(t_{m+1}-t_0)} e^{\lambda(t_{m+1}-t_m)} \\
& \le e^{\lambda l} M \|\psi\|^p e^{-\lambda(t_{m+1}-t_0)} \\
& = e^{2\lambda l} v(\overset{*}{t}) \\
& \le h v(t)
\end{aligned}$$

Then from condition (ii), we get $D^+ v(t) \le b v(t)$, since $\lambda > b$ from (13) we have

$$v(\bar{t}) \le v(\overset{*}{t}) e^{lb} = e^{-\lambda l} M \|\psi\|^p e^{-\lambda(t_{m+1}-t_0)} e^{lb} < v(\bar{t})$$

Which is contradiction
Thus (3) also hold for k = m+1
Hence by principle of mathematical induction (3) holds and we have

$$v(t) \le M\|\psi\|^p e^{-\lambda(t-t_0)}, t \in [t_{k-1}, t_k)$$

Then by condition (i)

$$\begin{aligned}
& b_1 \|x\|^p \le v(t) \le M\|\psi\|^p e^{-\lambda(t-t_0)} \\
& \Rightarrow b_1 \|x\|^p \le M\|\psi\|^p e^{-\lambda(t-t_0)} \\
& \Rightarrow \|x\| \le \left(\frac{M}{b_1}\right)^{\frac{1}{p}} \|\psi\| \; e^{-\frac{\lambda}{p}(t-t_0)} \\
& \Rightarrow \|x\| \le M^* \|\psi\| \; e^{-\frac{\lambda}{p}(t-t_0)}
\end{aligned}$$

Where $M^* \ge \max\{1, \left(\frac{M}{b_1}\right)^{\frac{1}{p}}\}$

Therefore the trivial solution of system (1) is globally exponentially stable with rate of convergence $\frac{\lambda}{p}$

Remark 1: If we want to remove the restriction $\lambda > b$ in above theorem then we need to modify conditions (ii) and (iv) as follows:

Theorem 2: Assume that there exist a function $V \in \nu_0$ and some constants $p, b, b_1, b_2 > 0$ and $l > \tau$ such that

(i) $b_1 \|x\|^p \le V(t,x) \le b_2 \|x\|^p$, for any $t \in R_+$ and $x \in R^n$

(ii) $D^+V(t, \varphi(0)) \le bV(t, \varphi(0))$, for all $t \in [t_{k-1}, t_k)$ $k \in N$

Whenever $hV(t,\varphi(0)) \ge V(t+r,\varphi(r))$ for $r \in [-\tau, 0]$, where $h \ge \max\{e^{2l\lambda}, e^{lb}\}$ is a constant

(iii) for all $\varphi \in PC([-\tau,0]; R^n)$

$$V(t_k,\varphi(0)+I_k(\varphi(0))+J_k(\varphi(r)) \le z_k\left[V(t_k^-,\varphi(0))+\sup_{r\in[-\tau,0]} V(t_k^-+r,\varphi(r))\right], \text{ where}$$

$z_k > 0, k \in N$ are constants .

(iv) $\tau \le t_k - t_{k-1} \le l$ and $z_k < \dfrac{e^{-(\lambda+b)l}.e^{-\lambda(t_{k+1}-t_k)}}{1+e^{\lambda\tau}}$

Then the trivial solution of (1) is globally exponentially stable.

Proof:- The proof of this theorem is omitted as it is almost same as that of Theorem 1

Remark 2:- As we know that the derivative of the Lyapunov function should be negative for a delay differential system to be stable but in these theorems derivative may be positive which does not ensure the stability of the differential system . So it is clear that the impulses can contribute to make a system exponentially stable.

IV. Conclusion

In this paper, global exponential stability criteria for impulsive functional differential system have been extended to a system in which state variables on impulses are related to time delay. These results widen the scope of stability theory and are more general as compared to some existing results.

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The Univalence of A Generalized Integral Operator

By Dr. Poonam Dixit & Puneet Shukla
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Abstract- For analytic function $f_i, j=\overline{1,n}$, in the open disk U, an integral operator $K_{\alpha_1}, \cdots\alpha_n, \beta_1 \cdots \beta_n$ is introduced. In this paper we obtain the conditions of the univalence for the integral operator $K_{\alpha_1}, \cdots\alpha_n, \beta_1 \cdots \beta_n$.

Keywords: *fuzzy anti 2-banach space.*

GJSFR-F Classification : *MSC 2010: 11S23*

THEUNIVALENCEOFAGENERALIZEDINTEGRALOPERATOR

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The Univalence of a Generalized Integral Operator

Dr. Poonam Dixit [α] & Puneet Shukla [σ]

Abstract- For analytic function $f_i, j = \overline{1,n}$ in the open disk U, an integral operator $K_{\alpha_1, \ldots\alpha_n, \beta_1 \ldots \beta_n}$ is introduced. In this paper we obtain the conditions of the univalence for the integral operator $K_{\alpha_1, \ldots\alpha_n, \beta_1 \ldots \beta_n}$.

Keywords: *fuzzy anti 2-banach space.*

I. Introduction

Let A be the class of functions f of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k$$

which are analytic in the open disk $U = \{z \in C : |z| < 1\}$ with $f(0) = f'(0) - 1 = 0$. Let S denote the subclass of A consisting of the functions $f \in A$, which are univalent in U .We denote by P the class of functions p which are analytic in U, $p(0) = 1$ and $Rep(z) > 0$, for all $z \in U$. In this work, we introduce a new integral operator, which is given by

$$K_{\alpha_1, \ldots\alpha_n, \beta_1 \ldots \beta_n}(z) = \int_0^z \prod_{j=1}^{n} \left(\frac{D^m f_i(u)}{u} \right)^{\alpha_j} \left[(D^n f_j(u))' \right]^{\beta_j} du \qquad (1)$$

for α_j, β_j be complex numbers, $f_i \in A, f_j' \in P, \quad j = \overline{1,n}$.

For $m = 1 \quad \beta_j = 0 \quad j = \overline{1,n}$ we obtain the integral operator, which is defined in [4].

For $m = 1 \quad \alpha_j = 0 \quad j = \overline{1,n}$ we have the integral operator, which is defined in [5].

II. Preliminary Results

In order to prove our main results we will need the following lemmas .

Lemma 2.1 [1] If the function f is analytic in U and

$$(1 - |z|)^2 \left| \frac{z f''(z)}{f'(z)} - 1 \right| \leq 1 \qquad (2)$$

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for all $z \in U$, then the function f is univalent in U.

Lemma 2.2 [3] Let γ be a complex number $Re\gamma > 0$ and $f \in A$. If

$$\frac{1-|z|^{2Re\ \gamma}}{Re\ \gamma}\left|\frac{zf''(z)}{f'(z)}\right| \le 1 \tag{3}$$

for all $z \in U$, then for any complex number $\delta, Re\ \delta \ge Re\ \gamma$, the function

$$f_\delta(z) = \left[\delta \int_0^z u^{\delta-1} f'(u)du\right]^{1/\delta} \tag{4}$$

is regular and univalent in U .

Lemma 2.3 (Schwarz [2]) Let f be the function regular in the disk $U_R = \{z \in C : |z| < R\}$ with $|f(z)| < M, M$ fixed .

If f has in $z = 0$ one zero multiply $\ge m$ then

$$|f(z)| \le \frac{M}{R^m}|z|^m \qquad (z \in U_R) \tag{5}$$

the equality (in the inequality $(5) z \ne 0$) can hold only if .

$$f(z) = e^{i\theta}\tfrac{M}{R^m}z^m$$

where θ is constant.

III. Main Results

Theorem 3.1: Let α_j, β_j be the complex numbers M_j, L_j positive real numbers, $j = \overline{1,n}$ and the functions

$f_j \in A, f_j' \in P, f_j(z) = z + a_{2j}z^2 + a_{3j}z^3 + ...j = \overline{1,n}$ if .

$$\left|z\frac{D^{m+1}f_j(z)}{D^m f_j(z)} - 1\right| \le M_j \qquad (j = \overline{1,n} \quad : \ z \in U) \tag{6}$$

$$\left|z\frac{[D^n f_j(z)]''}{[D^n f_j(z)]'}\right| \le L_j \qquad (j = \overline{1,n} \quad : \ z \in U) \tag{7}$$

and

$$\sum_{j=1}^{n}[|\alpha_j|\, M_j + |\beta_j|\, L_j] \le \frac{3\sqrt{3}}{2} \tag{8}$$

Then the integral operator $K_{\alpha_1,....\alpha_n,\beta_1....\beta_n}$ defined by (1) is in the class S .

Proof : The function $K_{\alpha_1,....\alpha_n,\beta_1....\beta_n}(z)$ is regular in U and

$K_{\alpha_1,....\alpha_n,\beta_1....\beta_n}(0) = K'_{\alpha_1,....\alpha_n,\beta_1....\beta_n}(0) - 1 = 0$

Ref

2. O. Mayer, The functions theory of one variable complex, Bucuresti. 1981.

we have

$$\frac{zK''_{\alpha_1,\cdots\cdots\alpha_n,\beta_1\cdots\cdots\beta_n}(z)}{K'_{\alpha_1,\cdots\cdots\alpha_n,\beta_1\cdots\cdots\beta_n}(z)} = \sum_{j=1}^{n}\left[\alpha_j\left(\frac{D^{m+1}f_j(z)}{D^m f_j(z)}-1\right)\right] + \sum_{j=1}^{n}\left[\beta_j z\frac{[D^n f_j(z)]''}{[D^n f_j(z)]'}\right] \tag{9}$$

and hence we get

$$(1-|z|^2)\left|\frac{zK''_{\alpha_1,\cdots\cdots\alpha_n,\beta_1\cdots\cdots\beta_n}(z)}{K'_{\alpha_1,\cdots\cdots\alpha_n,\beta_1\cdots\cdots\beta_n}(z)}\right| \le (1-|z|^2)\sum_{j=1}^{n}\left[|\alpha_j|\left|z\frac{D^{m+1}f_j(z)}{D^m f_j(z)}-1\right| + |\beta_j|\left|z\frac{[D^n f_j(z)]''}{[D^n f_j(z)]'}\right|\right] \tag{10}$$

for all $z \in U$.

By (6), (7) and Lemma 2.3, we obtain

$$\left|z\frac{D^{m+1}f_j(z)}{D^m f_j(z)}-1\right| \le M_j\,|z| \qquad (j=\overline{1,n} \quad : \; z\in U) \tag{11}$$

$$\left|z\frac{[D^n f_j(z)]''}{[D^n f_j(z)]'}\right| \le L_j\,|z| \qquad (j=\overline{1,n} \quad : \; z\in U) \tag{12}$$

and from (10) we have

$$(1-|z|^2)\left|\frac{zK''_{\alpha_1,\cdots\cdots\alpha_n,\beta_1\cdots\cdots\beta_n}(z)}{K'_{\alpha_1,\cdots\cdots\alpha_n,\beta_1\cdots\cdots\beta_n}(z)}\right| \le (1-|z|^2)\,|z|\left\{\sum_{j=1}^{n}[|\alpha_j|\,M_j + |\beta_j|\,L_j]\right\} \tag{13}$$

for all $z \in U$.

Since

$$max_{|z|<1}[(1-|z|^2)\,|z|] = \frac{2}{3\sqrt{3}}$$

from (8) and (13) we get

$$(1-|z|^2)\left|\frac{zK''_{\alpha_1,\cdots\cdots\alpha_n,\beta_1\cdots\cdots\beta_n}(z)}{K'_{\alpha_1,\cdots\cdots\alpha_n,\beta_1\cdots\cdots\beta_n}(z)}\right| \le 1, \qquad (z\in U)$$

and by Lemma 2.1, it results that the integral operator $K_{\alpha_1,\cdots\cdots\alpha_n,\beta_1\cdots\cdots\beta_n}$ is in the class S.

Theorem 3.2 : Let α_j,β_j,γ be the complex numbers $j=\overline{1,n}$, $0 < Re\ \gamma \le 1$ and the functions

$$f_j \in A, f'_j \in P, f_j(z) = z + a_{2j}z^2 + a_{3j}z^3 + \ldots . j = \overline{1,n}$$

if

$$\left|z\frac{D^{m+1}f_j(z)}{D^m f_j(z)}-1\right| \le \frac{(2Re\ \gamma+1)^{\frac{2Re\ \gamma+1}{2Re\ \gamma}}}{2} \qquad (j=\overline{1,n} \quad : \; z\in U) \tag{14}$$

$$\left|z\frac{[D^n f_j(z)]''}{[D^n f_j(z)]'}\right| \leq \frac{(2Re\ \gamma+1)^{\frac{2Re\ \gamma+1}{2Re\ \gamma}}}{2} \qquad (j=\overline{1,n} \quad : \ z\in U) \tag{15}$$

and

$$\sum_{j=1}^{n}[|\alpha_j|+|\beta_j|] \leq 1 \tag{16}$$

Notes

then the integral operator $K_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ \ldots\ldots\beta_n}$ defined by (1) belong to the class S .

Proof : From (9) we obtain

$$\frac{zK''_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ \ldots\ldots\beta_n}(z)}{K'_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ \ldots\ldots\beta_n}(z)} = \sum_{j=1}^{n}\left[\alpha_j\left(\frac{zD^{m+1}f_j(z)}{D^m f_j(z)}-1\right)\right]+\sum_{j=1}^{n}\left[\beta_j\frac{z[D^n f_j(z)]''}{[D^n f_j(z)]'}\right]$$

and hence we get

$$\frac{1-|z|^{2Re\gamma}}{Re\ \gamma}\left|\frac{zK''_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ \ldots\ldots\beta_n}(z)}{K'_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ \ldots\ldots\beta_n}(z)}\right| \leq \frac{1-|z|^{2Re\gamma}}{Re\ \gamma}\sum_{j=1}^{n}\left[|\alpha_j|\left|z\frac{D^{m+1}f_j(z)}{D^m f_j(z)}-1\right|+|\beta_j|\left|z\frac{[D^n f_j(z)]''}{[D^n f_j(z)]'}\right|\right] \tag{17}$$

for all $z\in U$ by (14), (15) and Lemma 2.3 we have

$$\left|z\frac{D^{m+1}f_j(z)}{D^m f_j(z)}-1\right| \leq \frac{(2Re\ \gamma+1)^{\frac{2Re\gamma+1}{2Re\ \gamma}}}{2}|z| \qquad (j=\overline{1,n} \quad : \ z\in U) \tag{18}$$

$$\left|z\frac{[D^n f_j(z)]''}{[D^n f_j(z)]'}\right| \leq \frac{(2Re\ \gamma+1)^{\frac{2Re\gamma+1}{2Re\ \gamma}}}{2}|z| \qquad (j=\overline{1,n} \quad : \ z\in U) \tag{19}$$

and hence by (17) we get

$$\frac{1-|z|^{2Re\gamma}}{Re\ \gamma}\left|\frac{zK''_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ \ldots\ldots\beta_n}(z)}{K'_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ \ldots\ldots\beta_n}(z)}\right| \leq \frac{1-|z|^{2Re\gamma}}{Re\ \gamma}|z|\frac{(2Re\ \gamma+1)^{\frac{2Re\gamma+1}{2Re\ \gamma}}}{2}\sum_{j=1}^{n}[|\alpha_j|+|\beta_j|] \tag{20}$$

for all $z\in U$.

$$max_{|z|\leq 1}\left[\frac{1-|z|^{2Re\gamma}}{Re\ \gamma}|z|\right] = \frac{2}{(2Re\ \gamma+1)^{\frac{2Re\gamma+1}{2Re\ \gamma}}}$$

From (16) and (20) we obtain that

$$\frac{1-|z|^{2Re\gamma}}{Re\ \gamma}\left|\frac{zK''_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ \ldots\ldots\beta_n}(z)}{K'_{\alpha_1,\ldots\ldots\alpha_n,\beta_1\ \ldots\ldots\beta_n}(z)}\right| \leq 1 \tag{21}$$

for all $z \in U$ and by Lemma 2.2 for $\delta = 1$ and $f = K_{\alpha_1, \ldots\ldots \alpha_n, \beta_1 \ldots\ldots \beta_n}$ it results that the integral operator $K_{\alpha_1, \ldots\ldots \alpha_n, \beta_1 \ldots\ldots \beta_n}$ defined by (1) belongs to the class S.

Notes

IV. Corollaries

Corollary 4.1: Let α_j be the complex numbers M_j positive real numbers, $j = \overline{1,n}$ and the functions

$$f_j \in A, f_j' \in P, f_j(z) = z + a_{2j}z^2 + a_{3j}z^3 + \ldots . j = \overline{1,n} \text{ if } .$$

$$\left| z \frac{D^{m+1} f_j(z)}{D^m f_j(z)} - 1 \right| \leq M_j \qquad (j = \overline{1,n} \quad : \quad z \in U) \tag{22}$$

and

$$\sum_{j=1}^{n} [|\alpha_j|\, M_j] \leq \frac{3\sqrt{3}}{2} \tag{23}$$

then the function

$$G_{\alpha_1, \ldots\ldots \alpha_n}(z) = \int_0^z \prod_{j=1}^{n} \left(\frac{D^m f_i(u)}{u} \right)^{\alpha_j} du$$

is in the class S.

Corollary 4.2 : Let β_j be the complex numbers L_j positive real numbers, $j = \overline{1,n}$ and the functions

$$f_j \in A, f_j' \in P, f_j(z) = z + a_{2j}z^2 + a_{3j}z^3 + \ldots . j = \overline{1,n}$$

and

$$\left| z \frac{[D^n f_j(z)]''}{[D^n f_j(z)]'} \right| \leq L_j \qquad (j = \overline{1,n} \quad : \quad z \in U) \tag{24}$$

and

$$\sum_{j=1}^{n} [|\beta_j|\, L_j] \leq \frac{3\sqrt{3}}{2} \tag{25}$$

then the function

$$H_{\beta_1, \ldots\ldots \beta_n}(z) = \int_0^z \prod_{j=1}^{n} \left[\left(D^n f_i(u) \right)' \right]^{\beta_j} du$$

belongs to the class S.

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Two Incredible Summation Formula Involving Computational Technique

By Salahuddin, Upendra Kumar Pandit & M. P. Chaudhary
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Abstract- In this paper, we have established two summation formulae with the help of contiguous relation and some derived formulae of Salahuddin et al.

Keywords: contiguous relation, summation formulae.

GJSFR-F Classification : MSC 2010: 33C05, 33C20, 33C60

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Two Incredible Summation Formula Involving Computational Technique

Salahuddin [α], Upendra Kumar Pandit [σ] & M. P. Chaudhary [ρ]

Abstract- In this paper, we have established two summation formulae with the help of contiguous relation and some derived formulae of Salahuddin et al.

Keywords: *contiguous relation, summation formulae.*

I. Introduction and Results Required

Special functions and their applications are now incredible in their scope, variety and depth. Not only in their expedicious growth in pure Mathematics and its applications to the traditional fields of Physics, Engineering and Statistics but in new fields of applications like Behavioral Science, Optimization, Biology, Environmental Science and Economics, etc. they are emerging. Summation formulae for hypergeometric function has an important role in applied mathematics.

Prudnikov et al. [2; p.414] derived the following seven summation formulae

$$ {}_2F_1\left[\begin{matrix} a, & -a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c}}\Big[\frac{1}{\Gamma(\frac{c+a+1}{2})\,\Gamma(\frac{c-a}{2})} + \frac{1}{\Gamma(\frac{c+a}{2})\,\Gamma(\frac{c-a+1}{2})}\Big] \quad (1) $$

$$ {}_2F_1\left[\begin{matrix} a, & 1-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-1}}\Big[\frac{1}{\Gamma(\frac{c+a}{2})\,\Gamma(\frac{c-a+1}{2})}\Big] \quad (2) $$

$$ {}_2F_1\left[\begin{matrix} a, & 2-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{(a-1)\;2^{c-2}}\Big[\frac{1}{\Gamma(\frac{c+a-2}{2})\,\Gamma(\frac{c-a+1}{2})} - \frac{1}{\Gamma(\frac{c+a-1}{2})\,\Gamma(\frac{c-a}{2})}\Big] \quad (3) $$

$$ {}_2F_1\left[\begin{matrix} a, & 3-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{(a-1)(a-2)\;2^{c-3}}\Big[\frac{(c-2)}{\Gamma(\frac{c+a-2}{2})\,\Gamma(\frac{c-a+1}{2})} - \frac{2}{\Gamma(\frac{c+a-3}{2})\,\Gamma(\frac{c-a}{2})}\Big] \quad (4) $$

$$ {}_2F_1\left[\begin{matrix} a, & 4-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{(1-a)(2-a)(3-a)\;2^{c-4}}\Big[\frac{(a-2c+3)}{\Gamma(\frac{c+a-4}{2})\,\Gamma(\frac{c-a+1}{2})} + \frac{(a+2c-7)}{\Gamma(\frac{c+a-3}{2})\,\Gamma(\frac{c-a}{2})}\Big] \quad (5) $$

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$$ {}_2F_1\left[\begin{matrix} a, & 5-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-5}\{\prod_{\gamma=1}^{4}(\gamma-a)\}}\times $$

$$ \times\Big[\frac{\{2(c-2)(c-4)-(a-1)(a-4)\}}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-4}{2})}+\frac{(12-4c)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-5}{2})}\Big] \tag{6} $$

$$ {}_2F_1\left[\begin{matrix} a, & 6-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-6}\{\prod_{\delta=1}^{5}(\delta-a)\}}\times $$

$$ \times\Big[\frac{(4c^2+2ac-a^2-a-34c+62)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-5}{2})}-\frac{(4c^2-2ac-a^2+13a-22c+20)}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-6}{2})}\Big] \tag{7} $$

The contiguous relation is defined as Abramowitz et al. [1; p. 558]

$$ b\ {}_2F_1\left[\begin{matrix} a, & b+1 & ; & z \\ c & & ; & \end{matrix}\right] = (b-c+1)\ {}_2F_1\left[\begin{matrix} a, & b & ; & z \\ c & & ; & \end{matrix}\right] + (c-1)\ {}_2F_1\left[\begin{matrix} a, & b & ; & z \\ c-1 & & ; & \end{matrix}\right] \tag{8} $$

Salahuddin et al. [3 ; 4] derived the following eleven summation formulae

$$ {}_2F_1\left[\begin{matrix} a, & 7-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-7}\{\prod_{\varsigma=1}^{6}(\varsigma-a)\}}\times $$

$$ \times\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-6}{2})}(-3a^2c+12a^2+21ac-84a+4c^3-48c^2+158c-120)+ $$

$$ +\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-7}{2})}(2a^2-14a-8c^2+64c-108)\Big] \tag{9} $$

$$ {}_2F_1\left[\begin{matrix} a, & 8-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-8}\{\prod_{\xi=1}^{7}(\xi-a)\}}\times $$

$$ \times\Big[\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-7}{2})}(-a^3-4a^2c+30a^2+4ac^2-4ac-107a+8c^3-124c^2+576c-762)+ $$

$$ +\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-8}{2})}(-a^3+4a^2c-6a^2+4ac^2-68ac+181a-8c^3+92c^2-288c+210)\Big] \tag{10} $$

$$ {}_2F_1\left[\begin{matrix} a, & 9-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-9}\{\prod_{\varpi=1}^{8}(\varpi-a)\}}\times $$

$$ \times\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-8}{2})}(a^4-18a^3-8a^2c^2+80a^2c-85a^2+72ac^2-720ac+1494a+8c^4- $$

$$ -160c^3+1056c^2-2560c+1680)+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-9}{2})}\times $$

$$ \times(8a^2c-40a^2-72ac+360a-16c^3+240c^2-1072c+1360)\Big] \tag{11} $$

Ref

4. Salahuddin, Khola, R. K.:Certain new Hypergeometric Summation formulae arising from the sum-mation formulae of Salahuddin et al, (communicated).

Notes

$$
{}_2F_1\left[\begin{matrix} a, & 10-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-10}\{\prod_{\upsilon=1}^{9}(\upsilon-a)\}}\times
$$

$$
\times\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-10}{2})}(-a^4-4a^3c+42a^3+12a^2c^2-72a^2c-107a^2+8ac^3-252ac^2+
$$

$$
+1772ac-3054a-16c^4+312c^3-2000c^2+4704c-3024)+
$$

$$
+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-9}{2})}(a^4-4a^3c+2a^3-12a^2c^2+192a^2c-
$$

$$
-553a^2+8ac^3-12ac^2-868ac+3406a+16c^4-392c^3+3320c^2-11224c+12264)\Big] \tag{12}
$$

$$
{}_2F_1\left[\begin{matrix} a, & 11-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-11}\{\prod_{\varphi=1}^{10}(\varphi-a)\}}\times
$$

$$
\times\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-10}{2})}(5a^4c-30a^4-110a^3c+660a^3-20a^2c^3+360a^2c^2-1305a^2c-
$$

$$
-810a^2+220ac^3-3960ac^2+21010ac-31020a+16c^5-480c^4+
$$

$$
5240c^3-25200c^2+50544c-30240)+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-11}{2})}\times
$$

$$
\times(-2a^4+44a^3+24a^2c^2-288a^2c+530a^2-264ac^2+3168ac-8492a-32c^4+
$$

$$
768c^3-6352c^2+20928c-22320)\Big] \tag{13}
$$

$$
{}_2F_1\left[\begin{matrix} a, & 12-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-12}\{\prod_{\chi=1}^{11}(\chi-a)\}}\times
$$

$$
\times\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-12}{2})}(a^5-6a^4c+9a^4-12a^3c^2+300a^3c-1103a^3+
$$

$$
+32a^2c^3-408a^2c^2+46a^2c+6351a^2+16ac^4-800ac^3+10364ac^2-46852ac+
$$

$$
+62182a-32c^5+944c^4-10112c^3+47656c^2-93776c+55440)+
$$

$$
+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-11}{2})}(a^5+6a^4c-69a^4-12a^3c^2+12a^3c+769a^3-32a^2c^3+840a^2c^2-
$$

$$
-5662a^2c+8301a^2+16ac^4-32ac^3-4612ac^2+42380ac-96002a+
$$

$$
+32c^5-1136c^4+15104c^3-92536c^2+255392c-245640)\Big] \tag{14}
$$

$$
{}_2F_1\left[\begin{matrix} a, & 13-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-13}\{\prod_{\beta=1}^{12}(\beta-a)\}}\times
$$

$$
\times\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-12}{2})}(-a^6+39a^5+18a^4c^2-252a^4c+275a^4-468a^3c^2+6552a^3c
$$

$$
-18135a^3-48a^2c^4+1344a^2c^3-9834a^2c^2+5964a^2c+74246a^2+624ac^4-17472ac^3+
$$

$$+167388ac^2-631176ac+752856a+32c^6-1344c^5+$$

$$+21824c^4-172032c^3+674384c^2-1187424c+665280)+$$

$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-13}{2})}(-12a^4c+84a^4+312a^3c-2184a^3+64a^2c^3-1344a^2c^2+$$

$$+6620a^2c-2436a^2-832ac^3+17472ac^2-112424ac+216216a-64c^5+$$

$$+2240c^4-29312c^3+176512c^2-478752c+453600)\Big] \quad (15)$$

Notes

$${}_2F_1\left[\begin{matrix} a, & 14-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-14}\{\prod_{\gamma=1}^{13}(\gamma-a)\}}\times$$

$$\times\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-14}{2})}(a^6+6a^5c-87a^5-24a^4c^2+150a^4c+925a^4-32a^3c^3+1392a^3c^2-$$

$$-12706a^3c+24615a^3+80a^2c^4-1728a^2c^3+5368a^2c^2+58986a^2c-242486a^2+32ac^5-$$

$$-2320ac^4+47328ac^3-391568ac^2+1344076ac-1496568a-64c^6+2656c^5-42560c^4+$$

$$+330752c^3-1278144c^2+2222160c-1235520)+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-13}{2})}(-a^6+6a^5c-$$

$$-3a^5+24a^4c^2-570a^4c+2225a^4-32a^3c^3+48a^3c^2+7454a^3c-39225a^3-80a^2c^4+$$

$$+3072a^2c^3-35608a^2c^2+133626a^2c-68104a^2+$$

$$+32ac^5-80ac^4-19872ac^3+313808ac^2-1676564ac+$$

$$+2856228a+64c^6-3104c^5+59360c^4-566848c^3+2810304c^2-6724560c+5897520)\Big] \quad (16)$$

$${}_2F_1\left[\begin{matrix} a, & 15-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-15}\{\prod_{\varepsilon=1}^{14}(\varepsilon-a)\}}\times$$

$$\times\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-14}{2})}(-7a^6c+56a^6+315a^5c-2520a^5+56a^4c^3-1344a^4c^2+5103a^4c+$$

$$+16520a^4-1680a^3c^3+40320a^3c^2-271215a^3c+449400a^3-112a^2c^5+4480a^2c^4-54040a^2c^3+$$

$$+150080a^2c^2+845824a^2c-3383296a^2+1680ac^5-67200ac^4+999600ac^3-6787200ac^2+$$

$$+20482140ac-21070560a+64c^7-3584c^6+80864c^5-$$

$$-940800c^4+5987520c^3-20296192c^2+32464368c-17297280)+$$

$$+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-15}{2})}(2a^6-90a^5-48a^4c^2+768a^4c-$$

$$-1474a^4+1440a^3c^2-23040a^3c+77970a^3+160a^2c^4-5120a^2c^3+46640a^2c^2-$$

$$-90880a^2c-226192a^2-2400ac^4+76800ac^3-861600ac^2+3955200ac-6138120a-128c^6+$$

$$+6144c^5-116160c^4+1095680c^3-5363584c^2+12679168c-11009376)\Big] \quad (17)$$

Notes

$$
{}_2F_1\left[\begin{matrix} a, & 16-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-16}\{\prod_{\zeta=1}^{15}(\zeta-a)\}}\times
$$

$$
\times\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-16}{2})}(-a^7+8a^6c-12a^6+24a^5c^2-792a^5c+3710a^5-80a^4c^3+1080a^4c^2+
$$

$$
+6280a^4c-66600a^4-80a^3c^4+5280a^3c^3-85480a^3c^2+435480a^3c-458929a^3+192a^2c^5-
$$

$$
{}_6240a^2c^4+45200a^2c^3+271560a^2c^2-3746640a^2c+8942052a^2+64ac^6-6336ac^5+
$$

$$
+186000ac^4-2408160ac^3+15005072ac^2-42553152ac+41722740a-128c^7+7104c^6-
$$

$$
-158720c^5+1827360c^4-11505152c^3+38596416c^2-61194240c+32432400)+
$$

$$
+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-15}{2})}(-a^7-8a^6c+124a^6+24a^5c^2-24a^5c-2818a^5+
$$

$$
+80a^4c^3-3000a^4c^2+26360a^4c-40760a^4-80a^3c^4+160a^3c^3+45080a^3c^2-534760a^3c+
$$

$$
+1499471a^3-192a^2c^5+10080a^2c^4-175760a^2c^3+1189560a^2c^2-2226480a^2c-
$$

$$
-2760884a^2+64ac^6-192ac^5-75120ac^4+1782560ac^3-
$$

$$
-16394608ac^2+65703616ac-93008652a+128c^7-8128c^6+
$$

$$
+210944c^5-2878240c^4+22080512c^3-94015552c^2+202146816c-165145680)\Big] \tag{18}
$$

$$
{}_2F_1\left[\begin{matrix} a, & 17-a & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-17}\{\prod_{\vartheta=1}^{16}(\vartheta-a)\}}\times
$$

$$
\times\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-16}{2})}(a^8-68a^7-32a^6c^2+576a^6c-638a^6+
$$

$$
+1632a^5c^2-29376a^5c+101320a^5+160a^4c^4-5760a^4c^3+44640a^4c^2+129600a^4c-1341071a^4-
$$

$$
-5440a^3c^4+195840a^3c^3-2303840a^3c^2+9743040a^3c-9832052a^3-256a^2c^6+
$$

$$
+13824a^2c^5-246560a^2c^4+1411200a^2c^3+4297408a^2c^2-64103040a^2c+
$$

$$
+143207628a^2+4352ac^6-235008ac^5+4977600ac^4-52289280ac^3+
$$

$$
+282566656ac^2-727036416ac+670152240a+128c^8-9216c^7+275456c^6-4423680c^5+
$$

$$
+41249792c^4-224907264c^3+683065344c^2-1014128640c+518918400+
$$

$$
+\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-17}{2})}(16a^6c-144a^6-816a^5c+
$$

$$
+7344a^5-160a^4c^3+4320a^4c^2-22480a^4c-30960a^4+5440a^3c^3-
$$

$$
-146880a^3c^2+1157360a^3c-2484720a^3+384a^2c^5-17280a^2c^4+247840a^2c^3-
$$

$$
-1092960a^2c^2-1901760a^2c+15669504a^2-6528ac^5+293760ac^4-4999360ac^3+
$$

$$
+39804480ac^2-146267456ac+194890176a-256c^7+16128c^6-414976c^5+
$$

$$
+5610240c^4-42628864c^3+179788032c^2-383195904c+310867200)\Big] \tag{19}
$$

II. Main Summation Formulae

$$ {}_2F_1\left[\begin{array}{lll} a, & 18-a & ; \\ c & & ; \end{array} \; \frac{1}{2}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-18}\{\prod_{\eta=1}^{17}(\eta-a)\}}\times $$

$$ \times\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-18}{2})}(-a^8-8a^7c+148a^7+40a^6c^2-256a^6c-3362a^6+80a^5c^3-4440a^5c^2+ $$
$$ +49664a^5c-103400a^5-240a^4c^4+5520a^4c^3+18760a^4c^2-849520a^4c+3240271a^4-192a^3c^5+ $$
$$ +17760a^3c^4-440560a^3c^3+4091160a^3c^2-12923320a^3c+3622852a^3+448a^2c^6-20352a^2c^5+ $$
$$ +253360a^2c^4+576240a^2c^3-31091248a^2c^2+192701168a^2c-344444908a^2+128ac^7-16576ac^6+660032ac^5- $$
$$ -12228640ac^4+118499872ac^3-604789504ac^2+1488844864ac-1324543920a-256c^8+18304c^7- $$
$$ -542976c^6+8650240c^5-79993344c^4+432549376c^3-1303568384c^2+1923025920c-980179200)+ $$
$$ +\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-17}{2})}(a^8-8a^7c+4a^7-40a^6c^2+1264a^6c-6214a^6+80a^5c^3-120a^5c^2- $$
$$ -32416a^5c+213904a^5+240a^4c^4-12720a^4c^3+186440a^4c^2-743120a^4c-456391a^4- $$
$$ -192a^3c^5+480a^3c^4+216080a^3c^3-4278120a^3c^2+27569480a^3c-52277444a^3-448a^2c^6+ $$
$$ +30720a^2c^5-745840a^2c^4+7817520a^2c^3-30345632a^2c^2-19224224a^2c+253516684a^2+ $$
$$ +128ac^7-448ac^6-259264ac^5+8556320ac^4-118218848ac^3+813195488ac^2- $$
$$ -2692403360ac+3335839536a+256c^8-20608c^7+696192c^6-12817024c^5+ $$
$$ +139638144c^4-913535872c^3+3463541888c^2-6848013696c+5284782720)\Big] \tag{20} $$

$$ {}_2F_1\left[\begin{array}{lll} a, & 19-a & ; \\ c & & ; \end{array} \; \frac{1}{2}\right] = \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c-19}\{\prod_{\lambda=1}^{18}(\lambda-a)\}}\times $$

$$ \times\Big[\frac{1}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a-18}{2})}(9a^8c-90a^8-684a^7c+6840a^7-120a^6c^3+3600a^6c^2-14046a^6c-99540a^6+ $$
$$ +6840a^5c^3-205200a^5c^2+1664856a^5c-2968560a^5+432a^4c^5-21600a^4c^4+277080a^4c^3+ $$
$$ +327600a^4c^2-20793831a^4c+70898310a^4-16416a^3c^5+820800a^3c^4-14644440a^3c^3+ $$
$$ +111013200a^3c^2-315518940a^3c+131909400a^3-576a^2c^7+40320a^2c^6-992880a^2c^5+9324000a^2c^4+ $$
$$ +4429536a^2c^3-636886080a^2c^2+3695816316a^2c-6211091160a^2+10944ac^7- $$
$$ -766080ac^6+21827808ac^5-325310400ac^4+2707726176ac^3-12394025280ac^2+ $$
$$ +28254838896ac-23908836960a+256c^9-23040c^8+880512c^7-18627840c^6+238347264c^5- $$
$$ -1891123200c^4+9158978048c^3-25507261440c^2+35661692160c-17643225600)+ $$
$$ +\frac{1}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a-19}{2})}(-2a^8+152a^7+80a^6c^2-1600a^6c+3148a^6- $$

Notes

$$-4560a^5c^2+91200a^5c-371488a^5-480a^4c^4+19200a^4c^3-185680a^4c^2-126400a^4c+4559182a^4+$$
$$+18240a^3c^4-729600a^3c^3+9799440a^3c^2-50068800a^3c+73373288a^3+896a^2c^6-53760a^2c^5+$$
$$+1107680a^2c^4-8467200a^2c^3-743936a^2c^2+274718720a^2c-822056088a^2-$$
$$-17024ac^6+1021440ac^5-24338240ac^4+292569600ac^3-1853708096ac^2+$$
$$+5798641920ac-6885423072a-512c^8+40960c^7-1374464c^6+25123840c^5-271685888c^4+$$
$$+1764075520c^3-6639757056c^2+13042437120c-10013310720)\Big] \quad (21)$$

III. Derivation of the Main Formulae

Involving the contiguous relation (8) and the formula of Salahuddin et al. (19), one can established the result(20) and on the same way result(21) can be established.

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Properties of Fuzzy Ditance on Fuzzy Set

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Abstract- In this paper we introduce the definition of fuzzy distance space on fuzzy set then we study and discuss several properties of this space after some illustrative examples are given. Furthermore we introduce the definition of fuzzy convergence, fuzzy Cauchy sequence of fuzzy point and fuzzy bounded fuzzy distance space.

Keywords: fuzzy distance space on fuzzy, fuzzy convergence, fuzzy cauchy sequence of fuzzy point and fuzzy bounded fuzzy distance space.

GJSFR-F Classification : *MSC 2010: 03E72, 94D05*

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Properties of Fuzzy Ditance on Fuzzy Set

Dr. Jehad R. Kider [α] & Aisha J. Hassan [σ]

Abstract- In this paper we introduce the definition of fuzzy distance space on fuzzy set then we study and discuss several properties of this space after some illustrative examples are given. Furthermore we introduce the definition of fuzzy convergence, fuzzy Cauchy sequence of fuzzy point and fuzzy bounded fuzzy distance space.

Keywords: fuzzy distance space on fuzzy, fuzzy convergence, fuzzy cauchy sequence of fuzzy point and fuzzy bounded fuzzy distance space.

I. Introduction

In 1965 fuzzy sets was introduced by Zadeh [20] many authors used this concept to formulate the notion of fuzzy metric on ordinary set [1,2,3,7,8,9,10,13,15,16]. In 1975 Kramosil and Michalek [14] introduce the definition of fuzzy metric on ordinary set which was called later the KM- fuzzy metric space. In 1994 George and Veermani [5] modified the KM-fuzzy metric space by taking instead of minimum function the binary operation t-norm which was called later GV-fuzzy metric space on ordinary set. In this paper we modified the definition of GV-fuzzy metric to a fuzzy space on fuzzy set $\widetilde{A}$. This paper consist of three sections.

In section two we introduce basic properties of fuzzy set and explain the difference between continuous and discrete fuzzy sets by example. After that we introduce the definition of fuzzy distance on fuzzy set then we give some examples to illustrate this notion. In section three we define fuzzy open fuzzy ball, fuzzy convergence of sequence of fuzzy points, fuzzy closed fuzzy set, fuzzy bounded fuzzy set, fuzzy dense fuzzy set and fuzzy Cauchy fuzzy sequence. It was proved that, let$(\tilde{A},\widetilde{M},*)$ be a fuzzy distance space on the fuzzy set $\tilde{A}$ and let $\widetilde{B}$ be a subset of $\tilde{A}$. then $\widetilde{B}$ is a fuzzy dense in $\tilde{A}$ if and only if for every $a_\alpha \in \tilde{A}$ there is $b_\beta \in \widetilde{B}$ such that $\widetilde{M}(a_\alpha, b_\beta) > (1- \varepsilon\ )$ for some $0 < \varepsilon < 1$. (see Theorem 3.12).

II. Fuzzy Distance Space on Fuzzy Set

Definition 2.1:[20]

Let X be a nonempty set of elements, a fuzzy set $\widetilde{A}$ in X is characterized by a membership function, $\mu_{\widetilde{A}}(x)$: X$\rightarrow$ [0,1]. Then we can write

$$\widetilde{A} = -(x, \mu_{\widetilde{A}}(x)): x \in X,\ 0 \leq \mu_{\widetilde{A}}(x) \leq 1''.$$

We now recall an example of a continuous fuzzy set.

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Example 2.2:[17]

Let $X = \mathbb{R}$ and let $\tilde{A}$ be a fuzzy set in $\mathbb{R}$ with membership function by:

$$\mu_{\tilde{A}}(x) = \frac{1}{1+10x^2}.$$

Definition 2.3:[4]

Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy sets in X. then

1- $\tilde{A} \subseteq \tilde{B}$ if and only if $\mu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x)$ for all $x \in X$

2- $\tilde{A} = \tilde{B}$ if and only if $\mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x)$ for all $x \in X$

3- $\tilde{C} = \tilde{A} \cup \tilde{B}$ if and only if $\mu_{\tilde{C}}(x) = \mu_{\tilde{A}}(x) \vee \mu_{\tilde{B}}(x)$ for all $x \in X$

4- $\tilde{D} = \tilde{A} \cap \tilde{B}$ if and only if $\mu_{\tilde{D}}(x) = \mu_{\tilde{A}}(x) \wedge \mu_{\tilde{B}}(x)$ for all $x \in X$

5- $\mu_{\tilde{A}^c}(x) = 1- \mu_{\tilde{A}}(x)$ for all $x \in X$

Definition 2.4:[17]

If $\tilde{A}$ and $\tilde{B}$ are fuzzy sets in a nonempty sets X and Y respectively then the Cartesian product $\tilde{A} \times \tilde{B}$ of $\tilde{A}$ and $\tilde{B}$ is defined by:

$\mu_{\tilde{A} \times \tilde{B}}(x, y) = \mu_{\tilde{A}}(x) \wedge \mu_{\tilde{B}}(y)$ for all $(x, y) \in X \times Y$

Definition 2.5:[19]

A fuzzy point p in X is a fuzzy set with member

$$p(y) = \begin{cases} \alpha & \text{if } y=x \\ 0 & \text{Otherwise} \end{cases}$$

For all y in X where $0< \alpha <1$. We denote this fuzzy point by x_α. Two fuzzy points x_α and y_β are said to be distinct if and only if $x \neq y$.

Definition 2.6:[20]

Let x_α be a fuzzy point and $\tilde{A}$ be a fuzzy set in X. then x_α is said to be in $\tilde{A}$ or belongs to $\tilde{A}$ which is denoted by $x_\alpha \in \tilde{A}$ if and only if $\mu_{\tilde{A}}(x) > \alpha$.

Definition 2.7:[11]

Let f be a function from a nonempty set X into a nonempty set Y. If $\tilde{B}$ is a fuzzy set in Y then $f^{-1}(\tilde{B})$ is a fuzzy set in X defined by:

$\mu_{f^{-1}(\tilde{B})}(x) = (\mu_{\tilde{B}} \circ f)(x)$ for all x in X. Also if $\tilde{A}$ is a fuzzy set in X then $f(\tilde{A})$ is a fuzzy set in Y defined by:

$$\mu_{f(\tilde{A})}(y) = \begin{cases} \vee - \mu_{\tilde{A}}(x) \colon x \in f^{-1}(y) \} & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{Otherwise} \end{cases}$$

Proposition 2.8:[12]

Let $f\colon X \rightarrow Y$ be a function. Then for a fuzzy point x_α in X, $f(x_\alpha)$ is a fuzzy point in Y and $f(x_\alpha) = (f(x))_\alpha$.

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Definition 2.9:[5]

A binary operation $*$: $[0, 1] \times [0, 1] \to [0, 1]$ is a continuous triangular norm (or simply t-norm) if for all a, b, c, $e \in [0, 1]$ the following conditions hold:

1- $a*b = b*a$ (commutatively)

2- $a*1 = a$

3- $(a*b)*c = a*(b*c)$ (associativity)

4- If $a \le c$ and $b \le e$ then $a*b \le c*e$.

Example 2.10:[6]

Define $a*b = a.b$, for all a, $b \in [0,1]$, where a.b is the usual multiplication in [0,1] then $*$ is a continuous t-norm.

Example 2.11:[8]

Define $a*b = \min\{a,b\}$ for all a, $b \in [0,1]$, it follows that $*$ is a continuous t-norm.

Example 2.12:[10]

Define $a*b = \max\{0, a + b - 1\}$ for all a, $b \in [0,1]$, it follows that $*$ is a continuous t-norm.

Remark 2.13:[5]

For any $a > b$, we can find c such that $a*c \ge b$ and for any d we can find q such that $q*q \ge d$, where a, b, c, d and q belong to (0,1).

Now we introduce the definition of fuzzy distance space on fuzzy set

Definition 2.14:

be any set. The triple $(\tilde{A}, \widetilde{M}, *)$ is said to be a fuzzy distance space, where $\tilde{A}$ is an arbitrary fuzzy set in X, $*$ is continuous t-norm and $\widetilde{M}$ is a fuzzy set on $\tilde{A} \times \tilde{A} \to [0, 1]$ satisfying the following conditions :

(FM_1) $\widetilde{M}(x_\alpha, y_\beta) > 0$ for all $x_\alpha, y_\beta \in \tilde{A}$.

(FM_2) $\widetilde{M}(x_\alpha, y_\beta) = 1$ if and only if $x_\alpha = .y_\beta$

(FM_3) $\widetilde{M}(x_\alpha, y_\beta) = \widetilde{M}(y_\beta, x_\alpha)$ for all $x_\alpha, y_\beta \in \tilde{A}$.

(FM_4) $\widetilde{M}(x_\alpha, y_\beta)*\widetilde{M}(y_\beta, z_\sigma) \le \widetilde{M}(x_\alpha, z_\sigma)$ for all $x_\alpha, y_\beta, z_\sigma \in \tilde{A}$.

(FM_5) $\widetilde{M}(x_\alpha, y_\beta)$: is a continuous fuzzy set for all x_α, y_β.

Remark 2.15:

Condition (FM_2) means that $\widetilde{M}(x_\alpha, x_\alpha) = 1$ for all $x_\alpha \in \tilde{A}$ and $\widetilde{M}(x_\alpha, y_\beta) < 1$ for all $x_\alpha \ne y_\beta$ in $\tilde{A}$.

Remark 2.16:

$\widetilde{M}(x_\alpha, y_\beta)$ can be considered as the degree of nearness between x_α and y_β.

Definition 2.17:

Let $(\tilde{A}, \widetilde{M}, *)$ be a fuzzy distance space then $\widetilde{M}$ is continuous fuzzy set if whenever $(x_n, \alpha_n) \to x_\alpha$ and $(y_n, \beta_n) \to y_\beta$ in $\tilde{A}$ then $\widetilde{M}((x_n, \alpha_n),(y_n, \beta_n)) \to \widetilde{M}(x_\alpha, y_\beta)$ that is $\lim_{n\to\infty}\widetilde{M}((x_n, \alpha_n),(y_n, \beta_n)) = \widetilde{M}(x_\alpha, y_\beta)$.

Lemma 2.18:

Let $(\mathbb{R},|.|)$ be an ordinary metric space. Let $\tilde{A}$ be a fuzzy set in $\mathbb{R}$. Define $|x_\alpha|=|x|$ for all $x_\alpha \in \tilde{A}$. Then $(\tilde{A}, d)$ is a metric space where $d(x_\alpha, y_\beta)=|x_\alpha - y_\beta|=|x-y|$.

Example 2.19:

Let $X=\mathbb{R}$ and let $a*b=a.b$ for all a, b $\in [0,1]$. let $\tilde{A}$ be a fuzzy set in $\mathbb{R}$. Define $\widetilde{M}(x_\alpha, y_\beta)=\frac{1}{\exp|x_\alpha - y_\beta|}$ for all $x_\alpha, y_\beta \in \tilde{A}$. Then $(\tilde{A},\widetilde{M},.)$ is a fuzzy distance space on the fuzzy set $\tilde{A}$.

Proof:

(FM$_1$) It is clear that $\widetilde{M}(x_\alpha, y_\beta)>0$ for all $x_\alpha, y_\beta \in \tilde{A}$.

(FM$_2$) Assume that $x_\alpha = y_\beta$. Then this implies that $|x_\alpha - y_\beta| = 0$

Hence $\frac{1}{\exp|x_\alpha - y_\beta|}=1$ implies that $\widetilde{M}(x_\alpha, y_\beta)=1$

Conversely, assume that $\widetilde{M}(x_\alpha, y_\beta)=1$. So $\frac{1}{\exp|x_\alpha - y_\beta|}=1$, which implies that $\exp|x_\alpha - y_\beta| = e^0 = 1$. Hence $|x_\alpha - y_\beta| = 0$ it follows $x_\alpha = y_\beta$.

Therefore $\widetilde{M}(x_\alpha, y_\beta)=1$ if and only if $x_\alpha = y_\beta$

(FM$_3$) Since $|x_\alpha - y_\beta| = |y_\beta - x_\alpha|$ for all $x_\alpha, y_\beta \in \tilde{A}$ it follows that

$$\widetilde{M}(x_\alpha, y_\beta) = \widetilde{M}(y_\beta, x_\alpha) \text{ for all } x_\alpha, y_\beta \in \tilde{A}.$$

(FM$_4$) To prove $\widetilde{M}(x_\alpha, y_\beta) * \widetilde{M}(y_\beta, z_\sigma) \leq \widetilde{M}(x_\alpha, z_\sigma)$.

We know that for all x_α, y_β, and $z_\sigma \in \tilde{A}$.

$$|x_\alpha - z_\sigma| \leq |x_\alpha - y_\beta| + |y_\beta - z_\sigma|$$

Thus $\exp|x_\alpha - z_\sigma| \leq \exp|x_\alpha - y_\beta| \cdot \exp|y_\beta - z_\sigma|$

Since $\exp(x_\alpha)$ is an increasing function for all $x_\alpha > 0$

Therefore $\frac{1}{\exp|x_\alpha - z_\sigma|} \geq \frac{1}{\exp|x_\alpha - y_\beta|} * \frac{1}{\exp|y_\beta - z_\sigma|}$

Thus $\widetilde{M}(x_\alpha, z_\sigma) \geq \widetilde{M}(x_\alpha, y_\beta) * \widetilde{M}(y_\beta, z_\sigma)$

(FM$_5$) Let $\{x_n, \alpha_n)\}$ and $\{(y_n, \beta_n)\}$ be two sequences in $\tilde{A}$ such that

$$(x_n, \alpha_n) \rightarrow x_\alpha \text{ and } (y_n, \beta_n) \rightarrow y_\beta$$

$$\text{Therefore } \lim_{n\to\infty} \widetilde{M}((x_\alpha, \alpha_n), (y_n, \beta_n)) = \lim_{n\to\infty} \frac{1}{\exp|(x_n,\alpha_n)-(y_n,\beta_n)|}$$

$$= \frac{1}{\lim_{n\to\infty} \exp|(x_n,\alpha_n)-(y_n,\beta_n)|} = \frac{1}{\exp(\lim_{n\to\infty}|(x_n,\alpha_n)-(y_n,\beta_n)|)} = \frac{1}{\exp|x_\alpha - y_\beta|}$$

$$= \widetilde{M}(x_\alpha, y_\beta). \text{ That is } \widetilde{M}((x_n, \alpha_n),(y_n, \beta_n)) \rightarrow \widetilde{M}(x_\alpha, y_\beta).$$

Hence $\widetilde{M}$ is a continuous fuzzy set■

Remark 2.20:

1- In example 2.19 we can replace $\mathbb{R}$ by any nonempty set X and the usual metric on $\mathbb{R}$ by any metric d.

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2- Example 2.19 is also a fuzzy metric space with the t-norm defined by $a*b = \min\{a,b\}$ for all a, b $\in[0,1]$.

Example 2.21:

Let $X = \mathbb{N}$ and $a*b = a.b$ for all a, b $\in[0,1]$ and let $\tilde{A}$ be a fuzzy set in X

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$$\text{Define } \widetilde{M}(x_\alpha, y_\beta) = \begin{cases} \frac{x}{y} & \text{if } x \leq y \\ \frac{y}{x} & \text{if } y \leq x \end{cases}$$

for all $x_\alpha, y_\beta \in \tilde{A}$. Then $(\tilde{A}, \widetilde{M}, *)$ is a fuzzy distance space.

In the following example we show that not every fuzzy set on $\tilde{A}^2$ is a fuzzy metric space on the fuzzy set $\tilde{A}$.

Example 2.22:

Let $X = \mathbb{R}$ and let $\tilde{A} = [2,\infty]$ be a fuzzy set in X, consider the mapping $\widetilde{M} : \tilde{A} \times \tilde{A} \to [0, 1]$ is defined by :

$$\widetilde{M}(a_\alpha, b_\beta) = \begin{cases} 1 & \text{if } a = b \\ \left(\frac{1}{a}\right).\alpha + \left(\frac{1}{b}\right).\beta & \text{if } a \neq b \end{cases}$$

Where $\alpha * \beta = \alpha.\beta$ for all $\alpha, \beta \in[0,1]$

Proof:

(FM_4) We show that $\widetilde{M}(a_\alpha, c_\sigma) \geq \widetilde{M}(a_\alpha, b_\beta) * \widetilde{M}(b_\beta, c_\sigma)$ is not satisfied for all a_α, b_β, $c_\sigma \in \tilde{A}$. Let $a_\alpha = 10$, $b_\beta = 3$ and $c_\sigma = 100$ where $\alpha = \frac{1}{a}$, $\beta = \frac{1}{b}$, $\sigma = \frac{1}{c}$ Since $a \neq b \neq c$

Then $\widetilde{M}(a_\alpha, b_\beta) = \left(\frac{1}{a}\right).\alpha + \left(\frac{1}{b}\right).\beta = \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{100} + \frac{1}{9} = 0.01 + 0.111 = 0.121$

And $\widetilde{M}(b_\beta, c_\sigma) = \left(\frac{1}{b}\right).\beta + \left(\frac{1}{c}\right).\sigma = \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{9} + \frac{1}{10000} = 0.111 + 0.0001 = 0.1112$

$\widetilde{M}(a_\alpha, c_\sigma) = \left(\frac{1}{a}\right).\alpha + \left(\frac{1}{c}\right).\sigma = \frac{1}{a^2} + \frac{1}{c^2} = \frac{1}{100} + \frac{1}{10000} = 0.01 + 0.0001 = 0.0101$

There fore $\widetilde{M}(a_\alpha, b_\beta) * \widetilde{M}(b_\beta, c_\sigma) > \widetilde{M}(a_\alpha, c_\sigma)$

Thus $\widetilde{M}$ is not a fuzzy distance space ■

Lemma 2.23:

Let (X, d) be an ordinary metric space and let $\tilde{A}$ be a fuzzy set in X. define $d(x_\alpha, y_\beta) = d(x, y)$ for all $x_\alpha, y_\beta \in \tilde{A}$. Then $(\tilde{A}, d)$ is a metric space.

Proposition 2.24:

Let (X, d) be an ordinary metric space and let $a*b = a.b$ for all a, b$\in[0,1]$. Then by lemma 2.23, $(\tilde{A}, d)$ is a metric space. Define $\widetilde{M}_d(x_\alpha, y_\beta) = \frac{t}{t + d(x_\alpha, y_\beta)}$, then $(\tilde{A}, \widetilde{M}_d, *)$

is a fuzzy distance space and it is called the fuzzy metric on the fuzzy set $\tilde{A}$ induced by the metric d, where $t=\alpha \wedge \beta$.

Proof:

(FM_1) It is clear that $\tilde{M}_d(x_\alpha, y_\beta) > 0$ for all $x_\alpha, y_\beta \in \tilde{A}$.

(FM_2) Assume that $x_\alpha = y_\beta$ then $d(x_\alpha, y_\beta) = 0$ so $\tilde{M}_d(x_\alpha, y_\beta) = 1$.

Conversely, assume that $\tilde{M}_d(x_\alpha, y_\beta) = 1$

So, $\frac{t}{t+d(x_\alpha, y_\beta)} = 1$, implies $t = t+d(x_\alpha, y_\beta)$

Or $d(x_\alpha, y_\beta) = 0$, so $x_\alpha = y_\beta$, thus $\tilde{M}_d(x_\alpha, y_\beta) = 1 \Leftrightarrow x_\alpha = y_\beta$.

(FM_3) Since $d(x_\alpha, y_\beta) = d(y_\beta, x_\alpha)$ so $\tilde{M}_d(x_\alpha, y_\beta) = \tilde{M}_d(y_\beta, x_\alpha)$.

(FM_4) To prove $\tilde{M}_d(x_\alpha, y_\beta) * \tilde{M}_d(y_\beta, , z_\sigma) \leq \tilde{M}_d(x_\alpha, , z_\sigma)$ notice that for all x_α, y_β, $z_\sigma \in \tilde{A}$, we have

$$\tilde{M}_d(x_\alpha, z_\sigma) = \frac{t}{t+d(x_\alpha, z_\sigma)} \geq \frac{t}{t+d(x_\alpha, y_\beta)+d(y_\beta, z_\sigma)}$$

$$\geq \frac{t}{t+d(x_\alpha, y_\beta)} \cdot \frac{t}{t+d(x_\alpha, y_\beta)}$$

$$= \tilde{M}_d(x_\alpha, y_\beta) * \tilde{M}_d(y_\beta, z_\sigma)$$

(FM_5) Let $\{(x_n, \alpha_n)\}$ and $\{(y_n, \beta_n)\}$ be two sequences of fuzzy points in $\tilde{A}$ such that

$$(x_n, \alpha_n) \rightarrow x_\alpha, (y_n, \beta_n) \rightarrow y_\beta. \text{ Then } \lim_{n\to\infty} \tilde{M}_d((x_n, \alpha_n),(y_n, \beta_n)) =$$

$$\lim_{n\to\infty} \frac{t}{t+d((x_n,\alpha_n),(y_n,\beta_n))} = \frac{t}{t+\lim_{n\to\infty} d((x_n,\alpha_n),(y_n,\beta_n))} = \frac{t}{t+d(x_\alpha, y_\beta)} = \tilde{M}_d(x_\alpha, y_\beta)$$

That is $\tilde{M}_d((x_n, \alpha_n),(y_n, \beta_n)) \rightarrow \tilde{M}_d(x_\alpha, y_\beta)$. Hence $\tilde{M}$ is a continuous fuzzy set ■

Remark 2.25:

Let $(\tilde{A}, \tilde{M}, *)$ be a fuzzy distance space. Then

$$\tilde{M}((x_1,\alpha_1), (x_n, \alpha_n)) \geq \tilde{M}((x_1,\alpha_1), (x_2, \alpha_2)) * \tilde{M}((x_2,\alpha_2), (x_3, \alpha_3)) * \ldots\ldots * \tilde{M}((x_{n-1},\alpha_{n-1}), (x_n, \alpha_n)).$$

III. Fuzzy Convergence, Fuzzy Cauchy Sequences, Fuzzy Bounded, Fuzzy Open and Fuzzy Closed Fuzzy Sets

In this section ̃ will be a fuzzy set in the nonempty set X.

Definition 3.1:

Let $(\tilde{A}, \tilde{M}, *)$ be a fuzzy distance space on the fuzzy set $\tilde{A}$, we define $\tilde{B}(x_\alpha, r) = \{y_\beta \in \tilde{A}: \tilde{M}(x_\alpha, y_\beta) > (1- r)\}$ then $\tilde{B}(x_\alpha, r)$ is called an fuzzy open fuzzy ball with center the fuzzy point $x_\alpha \in \tilde{A}$ and radius $0 < r < 1$.

Proposition 3.2:

Let $\tilde{B}(x_\alpha, r_1)$ and $\tilde{B}(x_\alpha, r_2)$ be two fuzzy open fuzzy balls with the same center $x_\alpha \in \tilde{A}$ and with radiuses $r_1, r_2 \in (0,1)$. Then we either have $\tilde{B}(x_\alpha, r_1) \subseteq \tilde{B}(x_\alpha, r_2)$ or $\tilde{B}((x_\alpha, r_2) \subseteq \tilde{B}(x_\alpha, r_1)$.

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Proof:

Let $x_\alpha \in \tilde{A}$ and consider the fuzzy open fuzzy balls $\widetilde{B}(x_\alpha, r_1)$ and $\widetilde{B}(x_\alpha, r_2)$ with $r_1, r_2 \in (0,1)$. If $r_1 = r_2$ then the proposition holds.

Next, we assume that $r_1 \neq r_2$, we may assume without loss of generality that $r_1 < r_2$ this implies that $(1- r_2) < (1- r_1)$.

Now let $y_\beta \in \widetilde{B}(x_\alpha, r_1)$, it follows that $\widetilde{M}(y_\beta, x_\alpha) > (1- r_1)$. So $\widetilde{M}(y_\beta, x_\alpha) > (1- r_2)$. Hence $y_\beta \in \widetilde{B}(x_\alpha, r_2)$. This shows that $\widetilde{B}(x_\alpha, r_1) \subseteq \widetilde{B}(x_\alpha, r_2)$. By assuming that $r_2 < r_1$. We can similarly show that $\widetilde{B}(x_\alpha, r_2) \subseteq \widetilde{B}((x_\alpha, r_1)$ ■

Definition 3.3:

A sequence $\{(x_n, \alpha_n)\}$ of fuzzy points in a fuzzy distance space $(\tilde{A}, \widetilde{M}, *)$ is said to be fuzzy converges to a fuzzy point $x_\alpha \in \tilde{A}$ if for all $0 < \varepsilon < 1$, there exists a positive number N such that, $\widetilde{M}((x_n, \alpha_n), x_\alpha) > (1\text{-}\varepsilon)$ for all $n \geq N$.

Definition 3.4:

A sequence $\{(x_n, \alpha_n)\}$ of fuzzy points in a fuzzy metric space $(\tilde{A}, \widetilde{M}, *)$ is said to be fuzzy converges to a fuzzy point $x_\alpha \in \tilde{A}$ if $\lim_{n\to\infty} \widetilde{M}((x_n, \alpha_n), x_\alpha) = 1$.

Theorem 1.3.5:

Definition 3.3 and definition 3.4 are equivalent.

Proof:

Suppose that the sequence $\{(x_n, \alpha_n)\}$ fuzzy converges to x_α in sense of definition 3.3 then for all $0 < r < 1$ there exists a positive number N such that $\widetilde{M}((x_n, \alpha_n), x_\alpha) >$ $(1- r)$ for all $n \geq N$ and hence $[1\text{-}\widetilde{M}((x_n, \alpha_n), x_\alpha)] < r$. Therefore $\widetilde{M}((x_n, \alpha_n), x_\alpha)$ converges to 1 as n tends to ∞.

Conversely, assume that $\widetilde{M}((x_n, \alpha_n), x_\alpha)$ converges to 1 as n tends to ∞.

Then for $0 < r < 1$ there exists a positive integer N such that,

$[1- \widetilde{M}((x_n, \alpha_n), x_\alpha)] < r$ for all $n \geq N$.

It follows that $\widetilde{M}((x_n, \alpha_n), x_\alpha) > (1- r)$ for all $n \geq N$. Hence $\{(x_n, \alpha_n)\}$ fuzzy converges to x_α in sense of Definition of 3.4 ■

Proposition 3.6:

Let (X, d) be a metric space and let $(\tilde{A}, \widetilde{M}_d, *)$ be the fuzzy distance space induced by d. Let $\{(x_n, \alpha_n)\}$ be a sequence of fuzzy points in $\tilde{A}$. Then $\{(x_n, \alpha_n)\}$ converges to $x_\alpha \in \tilde{A}$ in $(\tilde{A}, d)$ if and only if $\{(x_n, \alpha_n)\}$ fuzzy converges to x_α in $(\tilde{A}, \widetilde{M}_d, *)$.

Proof:

Suppose that $\{(x_n, \alpha_n)\}$ converges to $x_\alpha \in \tilde{A}$ in $(\tilde{A}, d)$ it follows that

$$\lim_{n\to\infty} d((x_n, \alpha_n), x_\alpha) = 0$$

Now,

$$\lim_{n\to\infty} \widetilde{M}_d((x_n, \alpha_n), x_\alpha) = \lim_{n\to\infty} \frac{t}{t + d((x_n, \alpha_n), x_\alpha)} = \frac{t}{t + \lim_{n\to\infty} d((x_n, \alpha_n), x_\alpha)} = 1$$

Hence $\{(x_n, \alpha_n)\}$ fuzzy converges to x_α in $(\tilde{A}, \widetilde{M}_d, *)$, where $t = \min -\alpha, \alpha_n\}$

Conversely, assume that $\{(x_n,\alpha_n)\}$ fuzzy converge to x_α in $(\tilde{A},\widetilde{M}_d,*)$, it follows that $\lim_{n\to\infty}\widetilde{M}_d((x_n,\alpha_n),x_\alpha) = 1$

Now, $\lim_{n\to\infty}\frac{t}{t+d((x_n,\alpha_n),x_\alpha)} = 1$, where t=min–$\alpha,\alpha_n\}$ which implies

that $\frac{t}{t+\lim_{n\to\infty}d((x_n,\alpha_n),x_\alpha)} = 1$, so $t+\lim_{n\to\infty}d((x_n,\alpha_n),x_\alpha) = t$, it follows

that $\lim_{n\to\infty}d((x_n,\alpha_n),x_\alpha) = t\text{-}t = 0$

Hence$\{(x_n,\alpha_n)\}$ fuzzy converges to x_α in $(\tilde{A}, d)$ ■

Definition 3.7:

A fuzzy subset $\tilde{C}$ of a fuzzy distance space $(\tilde{A},\widetilde{M},*)$ is said to be fuzzy open if it contains a fuzzy ball about each of its fuzzy points. A fuzzy subset $\tilde{D}$ of $(\tilde{A},\widetilde{M},*)$ is said to be fuzzy closed if its complement is fuzzy open that is $\tilde{D}^c = \tilde{A} - \tilde{D}$ is fuzzy open.

Theorem 3.8:

Every fuzzy open fuzzy ball in a fuzzy distance space $(\tilde{A},\widetilde{M},*)$ on a fuzzy set $\tilde{A}$ is a fuzzy open fuzzy set.

Proof:

Consider a fuzzy open fuzzy ball $\widetilde{B}(x_\alpha, r)$ where $x_\alpha\in\tilde{A}$ and $0 < r < 1$. let $y_\beta\in\widetilde{B}(x_\alpha,r)$ implies $\widetilde{M}(x_\alpha,y_\beta) > (1-r)$, put $t = \widetilde{M}(x_\alpha,y_\beta) > (1-r)$, then we can find s, $0 < s < 1$, such that $t > (1-s) > (1-r)$. Now for a given t and s such that $t > (1-s)$, we can find $0 < r_1 < 1$ such that $(t * r_1) \geq (1-s)$ by Remark 2.13, now consider the fuzzy ball $\widetilde{B}(y_\beta,1-r_1)$, we claim $\widetilde{B}(y_\beta,1-r_1) \subseteq \widetilde{B}(x_\alpha,r)$. Let $z_\sigma\in\widetilde{B}(y_\beta, 1-r_1)$ so $\widetilde{M}(y_\beta, z_\sigma) > r_1$.

Therefore $\widetilde{M}(x_\alpha,z_\sigma)\geq \widetilde{M}(x_\alpha,y_\beta) * \widetilde{M}(y_\beta, z_\sigma)$

$\widetilde{M}(x_\alpha, z_\sigma)\geq (t * r_1) \geq (1-s) > (1-r)$

Hence $z_\sigma\in\widetilde{B}(x_\alpha, r)$ so $\widetilde{B}(y_\beta, 1-r_1) \subseteq \widetilde{B}(x_\alpha, r)$■

Definition 3.9:

Let $(\tilde{A},\widetilde{M},*)$ be a fuzzy distance space on a fuzzy set $\tilde{A}$ and let $\widetilde{C}\subset\tilde{A}$ then the fuzzy closure of $\widetilde{C}$ is denoted by $\overline{\widetilde{C}}$ or FCL($\widetilde{C}$) and is defined to be the smallest fuzzy closed fuzzy set contains $\widetilde{C}$.

Definition 3.10:

A fuzzy subset $\widetilde{C}$ of a fuzzy distance space $(\tilde{A},\widetilde{M},*)$ on a fuzzy set $\tilde{A}$ is said to be fuzzy dense in $\tilde{A}$ if $\overline{\widetilde{C}} = \tilde{A}$.

Lemma 3.11:

Let $\widetilde{C}$ be a fuzzy subset of $\tilde{A}$ and let $(\tilde{A},\widetilde{M},*)$ be a fuzzy distance space on the fuzzy set $\tilde{A}$ then $a_\alpha\in\overline{\widetilde{C}}$ if and only if there is a sequence $\{(a_n,\alpha_n)\}$ in $\widetilde{C}$ such that $(a_n,\alpha_n) \to a_\alpha$, where $\alpha,\alpha_n\in[0,1]$.

Notes

Proof:

Let $a_\alpha \in \overline{\overline{\widetilde{C}}}$, if $a_\alpha \in \widetilde{C}$ then we take sequence of that type is $(a_\alpha, a_\alpha, a_\alpha, \ldots, a_\alpha, \ldots)$.
If $a_\alpha \notin \widetilde{C}$, it is a limit fuzzy point of $\widetilde{C}$. Hence we construct the sequence $(a_n, \alpha_n) \in \widetilde{C}$ by $\widetilde{M}((a_n, \alpha_n), a_\alpha) > (1- \frac{1}{n})$ for each n = 1, 2, 3,.....

The fuzzy ball $\widetilde{B}(a_\alpha, \frac{1}{n})$ contains $(a_n, \alpha_n) \in \widetilde{C}$ and $(a_n, \alpha_n) \to a_\alpha$ because $\lim_{n\to\infty} \widetilde{M}((a_n, \alpha_n), a_\alpha) = 1$. Conversely if $\{(a_n, \alpha_n)\}$ in $\widetilde{C}$ and $(a_n, \alpha_n) \to a_\alpha$ then $a_\alpha \in \widetilde{C}$, or every neighborhood of a_α contains fuzzy points $(a_n, \alpha_n) \neq a_\alpha$, so that a_α is a fuzzy limit of $\widetilde{C}$, hence $a_\alpha \in \overline{\overline{\widetilde{C}}}$ by the definition of the fuzzy closure ■

Theorem 3.12:

Let $\widetilde{C}$ be a fuzzy subset of a fuzzy distance space $(\tilde{A}, \widetilde{M}, *)$ then $\widetilde{C}$ is fuzzy dense in $\tilde{A}$ if and only if for every $x_\alpha \in \tilde{A}$ there is $a_\beta \in \widetilde{C}$ such that $\widetilde{M}(x_\alpha, a_\beta) > (1- \varepsilon)$ for some $0 < \varepsilon < 1$.

Proof:

Suppose that $\widetilde{C}$ is fuzzy dense in $\tilde{A}$ and $x_\alpha \in \tilde{A}$ so $x_\alpha \in \overline{\overline{\widetilde{C}}}$ and by Lemma 3.11 there is a sequence $\{(a_n, \beta_n)\} \in \widetilde{C}$ such that $(a_n, \beta_n) \to x_\alpha$ that is for a given $0 < \varepsilon < 1$ there is a positive number N such that $\widetilde{M}((a_n, \beta_n), x_\alpha) > (1- \varepsilon)$ for all $n \geq N$. Take $a_\beta = a_N$, so $\widetilde{M}(a_\beta, x_\alpha) > (1- \varepsilon)$.

Conversely to prove $\widetilde{C}$ is fuzzy dense in $\tilde{A}$ we have to show that for each $x_\alpha \in \tilde{A}$ then there is $a_k \in \widetilde{C}$ such that $\widetilde{M}((a_k, \beta_k), x_\alpha) > (1- \frac{1}{k})$. Now take $0 < \varepsilon < 1$ such that $\frac{1}{k} < \varepsilon$ for each $k \geq N$ for some positive number N. Hence we have a sequence $((a_k, \beta_k)) \in \widetilde{C}$ such that $\widetilde{M}((a_k, \beta_k), x_\alpha) > (1- \frac{1}{k}) > (1- \varepsilon)$ for all $k \geq N$ that is $(a_k, \beta_k) \to x_\alpha$ so $x_\alpha \in \overline{\overline{\widetilde{C}}}$ ■

Definition 3.13:

A sequence $\{(x_n, \alpha_n)\}$ of fuzzy points in a fuzzy distance space $(\tilde{A}, \widetilde{M}, *)$ is said to be fuzzy Cauchy if for each $0 < \varepsilon < 1$ there is a positive number N such that $\widetilde{M}((x_n, \alpha_n), (x_m, \alpha_m)) > (1- \varepsilon)$ for all $n, m \geq N$.

Theorem 3.14:

In a fuzzy distance space every fuzzy convergent sequence of fuzzy points is fuzzy Cauchy.

Proof:

Let $\{(x_n, \alpha_n)\}$ be a sequence of fuzzy points in a fuzzy distance space $(\tilde{A}, \widetilde{M}, *)$ that is fuzzy converges to $x_\alpha \in \tilde{A}$, then for given $0 < \varepsilon < 1$ there is a positive number N such that $\widetilde{M}((x_n, \alpha_n), x_\alpha) > (1-\varepsilon)$. Now by Remark 2.13, there is $(1-r) \in (0,1)$ such that $(1- \varepsilon) * (1- \varepsilon) > (1- r)$. Now for each $m, n \geq N$, we obtain $\widetilde{M}((x_m, \alpha_m), (x_n, \alpha_n)) \geq \widetilde{M}((x_m, \alpha_m), x_\alpha) * \widetilde{M}(x_\alpha, (x_n, \alpha_n)) \geq (1- \varepsilon) * (1- \varepsilon) > (1- r)$. Hence $\{(x_n, \alpha_n)\}$ is a fuzzy Cauchy ■

Notes

Proposition 3.15:

Let (X, d) be a metric space and let $\widetilde{M}_d(x_\alpha, y_\beta) = \frac{t}{t+d(x_\alpha, y_\beta)}$ where t= min{α,β}. Then $\{(x_n, \alpha_n)\}$ is a Cauchy sequence in $(\tilde{A}, d)$ if and only if $\{(x_n, \alpha_n)\}$ is a fuzzy Cauchy sequence in $(\tilde{A}, \widetilde{M}_d, *)$.

Proof:

Suppose that $\{(x_n, \alpha_n)\}$ is a Cauchy sequence in $(\tilde{A}, d)$, then there is a positive number N such that $d((x_m, \alpha_m), (x_n, \alpha_n)) < \varepsilon$ for given ε and for all m, n≥ N.

Now $t + d((x_m, \alpha_m), (x_n, \alpha_n)) < t + \varepsilon$, implies $\frac{t}{t+d((x_m,\alpha_m),(x_n,\alpha_n))} > \frac{t}{t+\varepsilon}$ Put $\frac{t}{t+\varepsilon} = (1 - r)$ for some $0 < r < 1$. It follows that $\widetilde{M}_d((x_m, \alpha_m), (x_n, \alpha_n)) > (1 - r)$ for all m, n ≥ N. Hence $\{(x_n, \alpha_n)\}$ is a fuzzy Cauchy sequence in $(\tilde{A}, \widetilde{M}_d, *)$.

Conversely, assume that $\{(x_n, \alpha_n)\}$ is a fuzzy Cauchy sequence in $(\tilde{A}, \widetilde{M}_d, *)$ then given $0 < \varepsilon < 1$, there is a positive number N such that $\widetilde{M}_d((x_m, \alpha_m), (x_n, \alpha_n)) > (1 - \varepsilon)$. Put $(1 - \varepsilon) = r$ then $\frac{t}{t+d((x_m,\alpha_m),(x_n,\alpha_n))} > r$ for all n, m ≥ N.

This implies $t + d\big((x_m, \alpha_m), (x_n, \alpha_n)\big) < \frac{t}{r}$, it follows that $d\big((x_m, \alpha_m), (x_n, \alpha_n)\big) < (\frac{t}{r} - t)$ for all n, m ≥ N, put $\frac{t}{r} - t = k$. Then $d((x_m, \alpha_m), (x_n, \alpha_n)) < k$ for all n, m ≥N. Hence $\{(x_m, \alpha_m)\}$ is Cauchy sequence in $(\tilde{A}, d)$ ■

Definition 1.3.16:

Let $\{(x_n, \alpha_n)\}$ be a given sequence of fuzzy points in a fuzzy distance space $(\tilde{A}, \widetilde{M}, *)$ and let (n_k) be a sequence of positive integer such that $n_1 < n_2 < n_3 < \ldots$ Then the sequence $\{(x_{n_k}, \alpha_{n_k})\}$ is called a subsequence of $\{(x_n, \alpha_n)\}$. If (x_{n_k}, α_{n_k}) fuzzy converges, its limit is called a sub sequential limit of $\{(x_n, \alpha_n)\}$. It is clear that a sequence $\{(x_n, \alpha_n)\}$ in $\tilde{A}$ fuzzy converges to x_α if and only if every subsequence of it fuzzy converges to x_α.

Proposition 1.3.17:

If a fuzzy Cauchy sequence of fuzzy points in a fuzzy distance space $(\tilde{A}, \widetilde{M}, *)$ contains a fuzzy convergent subsequence, then the sequence fuzzy converges to the same fuzzy limit as the subsequence.

Proof:

Let $\{(x_n, \alpha_n)\}$ be a fuzzy Cauchy sequence in $(\tilde{A}, \widetilde{M}, *)$. Then for a given $0 < \varepsilon < 1$, there exists an integer N such that $\widetilde{M}((x_m, \alpha_m), (x_n, \alpha_n)) > (1 - \varepsilon)$ whenever m, n ≥N. Denote by $\{(x_{n_k}, \alpha_{n_k})\}$ a fuzzy convergent subsequence of $\{(x_n, \alpha_n)\}$ and its limit by x_α. It follows that $\widetilde{M}((x_{n_m}, \alpha_{n_m}), (x_n, \alpha_n)) > (1 - \varepsilon)$ whenever m, n ≥ N. Since (n_k) is strictly increasing sequence of positive integer.

Now $\widetilde{M}(x_\alpha, (x_n, \alpha_n)) \geq \widetilde{M}(x_\alpha, (x_{n_m}, \alpha_{n_m})) * \widetilde{M}((x_{n_m}, \alpha_{n_m}), (x_n, \alpha_n)) > \widetilde{M}(x_\alpha, (x_{n_m}, \alpha_{n_m})) * (1 - \varepsilon)$

Letting m→∞, we have $\widetilde{M}(x_\alpha, (x_n, \alpha_n)) \geq 1 * (1 - \varepsilon) = (1 - \varepsilon)$

So, the sequence $\{(x_n, \alpha_n)\}$ fuzzy converges to x_α ■

Notes

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Definition 1.3.18:

Let $(\tilde{A},\tilde{M},*)$ be a fuzzy distance space. A fuzzy subset $\tilde{C}$ of $\tilde{A}$ is said to be fuzzy bounded if there exists $0 < r < 1$ such that, $\tilde{M}(x_\alpha,y_\beta) > (1- r)$, for all $x_\alpha, y_\beta \in \tilde{C}$.

Proposition 1.3.19:

Let (X, d) be a metric space and let $\tilde{M}_d(x_\alpha,y_\beta)=\frac{t}{t+d(x_\alpha,y_\beta)}$ where $t=\alpha \wedge \beta$ then a fuzzy subset $\tilde{C}$ of $\tilde{A}$ is fuzzy bounded if and only if it is bounded.

Proof:

Assume that $\tilde{C}$ is fuzzy bounded then there is $0 < r < 1$ such that

$\tilde{M}_d(x_\alpha,y_\beta)> (1- r)$ for all $x_\alpha, y_\beta \in \tilde{C}$. Now put $(1- r) = \varepsilon$

Then $\tilde{M}_d(x_\alpha,y_\beta)= \frac{t}{t+d(x_\alpha,y_\beta)} > \varepsilon$. Implies $t+d(x_\alpha,y_\beta) < \frac{t}{\varepsilon}$, it follows that

$d(x_\alpha,y_\beta) < \frac{t}{\varepsilon} - t$, put $\frac{t}{\varepsilon} - t = k$. Therefore $d(x_\alpha,y_\beta) < k$ for all $x_\alpha, y_\beta \in \tilde{C}$.

Hence $\tilde{C}$ is bounded.

Conversely, suppose that $\tilde{C}$ is bounded then there is k such that

$d(x_\alpha,y_\beta) < k$ for all $x_\alpha, y_\beta \in \tilde{C}$. Implies $t+ d(x_\alpha,y_\beta) < t+k$, implies

$\frac{t}{t+d(x_\alpha,y_\beta)} > \frac{t}{t+k}$. Let $0 < \varepsilon < 1$ with $\frac{t}{t+k} = (1- \varepsilon)$.

Therefore $\tilde{M}_d(x_\alpha,y_\beta))> (1- \varepsilon)$ for all $x_\alpha, y_\beta \in \tilde{C}$

Hence $\tilde{C}$ is fuzzy bounded. ■

Lemma 1.3.20:

A fuzzy convergent sequence of fuzzy points in a fuzzy distance space $(\tilde{A},\tilde{M},*)$ is fuzzy bounded and its fuzzy limit is unique.

Proof:

Suppose that $\{(x_n,\alpha_n)\}$ fuzzy converges to x_α then given $0 < \varepsilon < 1$ we can find a positive number N such that $\tilde{M}((x_n,\alpha_n) , x_\alpha) > (1- \varepsilon)$ for all $n \geq N$

Let $t = \min\{\tilde{M}((x_1,\alpha_1), x_\alpha), \tilde{M}((x_2,\alpha_2), x_\alpha),\ldots, \tilde{M}((x_N,\alpha_N), x_\alpha)\}$. Then by Remark 1.2.5 there is $0 < r < 1$ such that $t * (1- \varepsilon) > (1- r)$. Now for all $n \geq N$

$$\begin{aligned}\tilde{M}((x_n,\alpha_n) , x_\alpha) &\geq \tilde{M}((x_n,\alpha_n) , (x_N,\alpha_N)) * \tilde{M}((x_N,\alpha_N) , x_\alpha)\\ &\geq t * (1- \varepsilon) > (1- r) .\end{aligned}$$

Hence $\{(x_n,\alpha_n)\}$ is fuzzy bounded.

Assume that $(x_n,\alpha_n) \to x_\alpha$ and $(x_n,\alpha_n) \to y_\beta$. So $\lim_{n\to\infty}\tilde{M}((x_n,\alpha_n), x_\alpha) = 1$ and $\lim_{n\to\infty}M((x_n,\alpha_n), y_\beta) = 1$. Now $\tilde{M}(x_\alpha,y_\beta) \geq \tilde{M}(x_\alpha, (x_n,\alpha_n)) * \tilde{M}((x_n,\alpha_n) , y_\beta)$ Taking limit to both sides, as n tends to ∞, we obtain

$\tilde{M}(x_\alpha,y_\beta) \geq 1*1 = 1$. So $\tilde{M}(x_\alpha,y_\beta) = 1$, hence $x_\alpha = y_\beta$ ■

Definition 1.3.21:

Let $(\tilde{A},\tilde{M},*)$ be a fuzzy distance space, then we define a fuzzy closed fuzzy ball with center $x_\alpha \in \tilde{A}$ and radius r, $0 < r < 1$ by $\tilde{B}[x_\alpha,r] = \{y_\beta \in X: \tilde{M}(x_\alpha,y_\beta) \geq (1- r)\}$.

Lemma 1.3.22:

Every fuzzy closed fuzzy ball in a fuzzy distance space $(\tilde{A},\widetilde{M},*)$ is fuzzy closed fuzzy set.

Proof:

Let $y_\beta \in \overline{\widetilde{B}[x_\alpha, r]}$ then by lemma 1.3.11 there exists a sequence $\{(y_n, \beta_n)\}$ in $\widetilde{B}[x_\alpha, r]$ such that (y_n, β_n) converges to y_β ,therefore $\lim_{n\to\infty}\widetilde{M}((y_n, \beta_n), y_\beta) = 1$

Now, $\widetilde{M}(x_\alpha, y_\beta) \geq \widetilde{M}(x_\alpha,(y_n, \beta_n)) * \widetilde{M}((y_n, \beta_n), y_\beta)$

$\geq \lim_{n\to\infty}\widetilde{M}(x_\alpha,(y_n, \beta_n)) * \lim_{n\to\infty}\widetilde{M}((y_n, \beta_n), y_\beta)$

$> (1- r) * 1 = (1- r)$

Hence $y_\beta \in \widetilde{B}[x_\alpha, r]$, therefore $\widetilde{B}[x_\alpha, r]$ is a fuzzy closed fuzzy set ■

Theorem 1.3.23:

A fuzzy distance space is a fuzzy topological space.

Proof:

Let $(\tilde{A},\widetilde{M},*)$ be a fuzzy distance space. Define $\tau_{\widetilde{M}} = \{\tilde{C} \subset \tilde{A}$: $x_\alpha \in \tilde{C}$ if and only if there exists $0 < r < 1$ such that $\widetilde{B}(x_\alpha, r) \subset \tilde{C}\}$. We prove now $\tau_{\widetilde{M}}$ is a fuzzy topology on $\tilde{A}$.

(i) Clearly ϕ and $\tilde{A}$ belong to $\tau_{\widetilde{M}}$.

(ii) Let $\tilde{C}_1, \tilde{C}_2, \ldots, \tilde{C}_n \in \tau_{\widetilde{M}}$ and put $U = \cap_{i=1}^{n} \tilde{C}_i$.We shall show that $U \in \tau_{\widetilde{M}}$.

Let $a_\alpha \in U$ then $a_\alpha \in \tilde{C}_i$ for each $1 \leq i \leq n$. Hence there exists $0 \leq r_i \leq 1$ such that $\widetilde{B}(a_\alpha, r_i) \subset \tilde{C}_i$.

Let $r = \min\{r_i: 1 \leq i \leq n\}$ thus $r \leq r_i$ for all $1 \leq i \leq n$ so $(1- r) \geq (1- r_i)$ for all $1 \leq i \leq n$. So $\widetilde{B}(a_\alpha, r) \subseteq \tilde{C}_i$ for all $1 \leq i \leq n$

Therefore $\widetilde{B}(a_\alpha, r) \subseteq \cap_{i=1}^{n} \tilde{C}_i = U$, this shows that $U \in \tau_{\widetilde{M}}$.

(iii) Let $\{\tilde{C}_i: i \in I\} \in \tau_{\widetilde{M}}$ and put $\widetilde{V} = \cup_{i\in I}\tilde{C}_i$. We shall show that $\widetilde{V} \in \tau_{\widetilde{M}}$.

Let $a_\alpha \in \widetilde{V}$ then $a_\alpha \in \cup_{i\in I}\tilde{C}_i$ which implies that $a_\alpha \in \tilde{C}_i$ for some $i \in I$ since $\tilde{C}_i \in \tau_{\widetilde{M}}$ there exists $0 < r < 1$ such that $\widetilde{B}(a_\alpha, r) \subset \tilde{C}_i$

Hence $\widetilde{B}(a_\alpha, r) \subset \tilde{C}_i \subseteq \cup_{i\in I}\tilde{C}_i = \widetilde{V}$, this shows that $\widetilde{V} \in \tau_{\widetilde{M}}$.

Hence $(\tilde{A}, \tau_{\widetilde{M}})$ is a fuzzy topological space.$\tau_{\widetilde{M}}$ is called the fuzzy topology induced by $\widetilde{M}$■

Proposition 1.3.24:

Let (X,d) be an ordinary metric space. Then $(\tilde{A},d)$ is a metric space and let $\widetilde{M}_d(x_\alpha, y_\beta) = \frac{t}{t+d(x_\alpha, y_\beta)}$ be the fuzzy distance space induced by d. Then the topology τ_d induced by d and the fuzzy topology $\tau_{\widetilde{M}_d}$ induced by $\widetilde{M}_d$ are the same. That is $\tau_d = \tau_{\widetilde{M}_d}$.

Proof:

Suppose that $\tilde{C} \in \tau_d$ then there exists $0 < \varepsilon < 1$ such that $\widetilde{B}_\varepsilon(x_\alpha) \subseteq \tilde{C}$ for every $x_\alpha \in \tilde{C}$, we obtain $\widetilde{M}_d(x_\alpha, y_\beta) = \frac{t}{t+d(x_\alpha, y_\beta)} > \frac{t}{t+\varepsilon}$ where $t = \alpha \wedge \beta$. Let $1- r = \frac{t}{t+\varepsilon}$, then $\widetilde{M}_d(x_\alpha, y_\beta) > (1- r)$. It follows that $\widetilde{B}(x_\alpha, r) \subseteq \tilde{C}$.

Hence $\tilde{C} \in \tau_{\widetilde{M}_d}$. This shows that $\tau_d \subseteq \tau_{\widetilde{M}_d}$.

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Conversely, suppose that $\tilde{C} \in \tau_{\tilde{M}_d}$ then there exists $0 < r < 1$ such that $\tilde{B}(x_\alpha, r) \subseteq \tilde{C}$ for every $x_\alpha \in \tilde{C}$.

Now $\tilde{M}_d(x_\alpha, y_\beta) = \frac{t}{t + d(x_\alpha, y_\beta)} > (1 - r)$ which implies that

$$t > t\,(1 - r) + (1 - r)\, d(x_\alpha, y_\beta). \quad \text{Then} \quad d(x_\alpha, y_\beta) < \frac{r}{1-r}$$

Let $\varepsilon = \frac{r}{(1-r)}$ then $d(x_\alpha, y_\beta) < \varepsilon$ and therefore $\tilde{B}_\varepsilon(x_\alpha) \subseteq \tilde{C}$.

Hence $\tilde{C} \in \tau_d$. This implies that $\tau_{\tilde{M}_d} \subseteq \tau_d$, therefore $\tau_d = \tau_{\tilde{M}_d}$ ■

Proposition 1.3.25:

Let (X, d) be an ordinary metric space. Then $(\tilde{A}, d)$ is a metric space and $\tilde{R}_d = \{(x_n, \alpha_n)\}$ and $\{(\acute{x}_n, '\alpha_n)\}$ fuzzy Cauchy sequences in (X,d), $(x_n, \alpha_n) \sim (\acute{x}_n, '\alpha_n) \Leftrightarrow \lim_{n\to\infty} d((x_n, \alpha_n), (\acute{x}_n, '\alpha_n)) = 0\}$. $\tilde{R}_{\tilde{M}_d} = \{(x_n, \alpha_n)\}$ and $\{(\acute{x}_n, '\alpha_n)\}$ fuzzy Cauchy sequences in $(X, \tilde{M}_d, *)$ such that $(x_n, \alpha_n) \sim (\acute{x}_n, '\alpha_n) \Leftrightarrow \lim_{n\to\infty} \tilde{M}_d((x_n, \alpha_n), (\acute{x}_n, '\alpha_n)) = 1\}$.Then $\tilde{R}_d = \tilde{R}_{\tilde{M}_d}$.

Proof:

The prove follows from the fact $\lim_{n\to\infty} d((x_n, \alpha_n), (\acute{x}_n, '\alpha_n)) = 0$

$$\Leftrightarrow \lim_{n\to\infty} \tilde{M}_d((x_n, \alpha_n), (\acute{x}_n, '\alpha_n)) = 1 \blacksquare$$

Theorem 1.3.26:

Every fuzzy distance space on a fuzzy set is a fuzzy Hausdorff space.

Proof:

Let $(\tilde{A}, \tilde{M}, *)$ be a fuzzy distance space and let $x_\alpha, y_\beta \in \tilde{A}$ with $x_\alpha \neq y_\beta$ Let $\tilde{M}(x_\alpha, y_\beta) = r$ for some $0 < r < 1$. Then for each t, $r < t < 1$, we can find r_1 such that $r_1 * r_1 \geq t$ by Remark 1.2.5. Now consider the two fuzzy open fuzzy balls $\tilde{B}(x_\alpha, 1 - r_1)$ and $\tilde{B}(y_\beta, 1 - r_1)$. Then $\tilde{B}(x_\alpha, 1 - r_1) \cap \tilde{B}(y_\beta, 1 - r_1) = \emptyset$ Since if there exists $z_\sigma \in \tilde{B}(x_\alpha, 1 - r_1) \cap \tilde{B}(y_\beta, 1 - r_1)$. Then $r = \tilde{M}(x_\alpha, y_\beta) \geq \tilde{M}(x_\alpha, z_\sigma) * \tilde{M}(z_\sigma, y_\beta) \geq r_1 * r_1 \geq t > r$, which is a contradiction, therefore $(\tilde{A}, \tilde{M}, *)$ is a fuzzy Hausdorff space ■

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Notes

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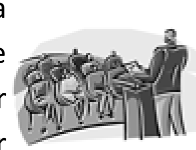

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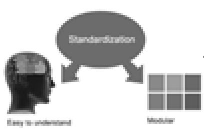

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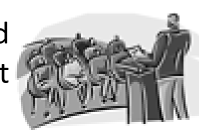

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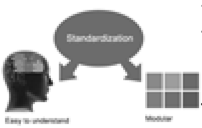

After nomination of your institution as “Institutional Fellow” and constantly functioning successfully for one year, we can consider giving recognition to your institute to function as Regional/Zonal office on our behalf.
The board can also take up the additional allied activities for betterment after our consultation.

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Note :

"

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"

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3. Submission of Manuscripts,

4. Manuscript’s Category,

5. Structure and Format of Manuscript,

6. After Acceptance.

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10. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right! It is a good habit, which helps to not to lose your continuity. You should always use bookmarks while searching on Internet also, which will make your search easier.

11. Revise what you wrote: When you write anything, always read it, summarize it and then finalize it.

12. Make all efforts: Make all efforts to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in introduction, that what is the need of a particular research paper. Polish your work by good skill of writing and always give an evaluator, what he wants.

13. Have backups: When you are going to do any important thing like making research paper, you should always have backup copies of it either in your computer or in paper. This will help you to not to lose any of your important.

14. Produce good diagrams of your own: Always try to include good charts or diagrams in your paper to improve quality. Using several and unnecessary diagrams will degrade the quality of your paper by creating "hotchpotch." So always, try to make and include those diagrams, which are made by your own to improve readability and understandability of your paper.

15. Use of direct quotes: When you do research relevant to literature, history or current affairs then use of quotes become essential but if study is relevant to science then use of quotes is not preferable.

16. Use proper verb tense: Use proper verb tenses in your paper. Use past tense, to present those events that happened. Use present tense to indicate events that are going on. Use future tense to indicate future happening events. Use of improper and wrong tenses will confuse the evaluator. Avoid the sentences that are incomplete.

17. Never use online paper: If you are getting any paper on Internet, then never use it as your research paper because it might be possible that evaluator has already seen it or maybe it is outdated version.

18. Pick a good study spot: To do your research studies always try to pick a spot, which is quiet. Every spot is not for studies. Spot that suits you choose it and proceed further.

19. Know what you know: Always try to know, what you know by making objectives. Else, you will be confused and cannot achieve your target.

20. Use good quality grammar: Always use a good quality grammar and use words that will throw positive impact on evaluator. Use of good quality grammar does not mean to use tough words, that for each word the evaluator has to go through dictionary. Do not start sentence with a conjunction. Do not fragment sentences. Eliminate one-word sentences. Ignore passive voice. Do not ever use a big word when a diminutive one would suffice. Verbs have to be in agreement with their subjects. Prepositions are not expressions to finish sentences with. It is incorrect to ever divide an infinitive. Avoid clichés like the disease. Also, always shun irritating alliteration. Use language that is simple and straight forward. put together a neat summary.

21. Arrangement of information: Each section of the main body should start with an opening sentence and there should be a changeover at the end of the section. Give only valid and powerful arguments to your topic. You may also maintain your arguments with records.

22. Never start in last minute: Always start at right time and give enough time to research work. Leaving everything to the last minute will degrade your paper and spoil your work.

23. Multitasking in research is not good: Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. Think technically: Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page
- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.

- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

The Administration Rules

Please carefully note down following rules and regulation before submitting your Research Paper to Global Journals Inc. (US):

Segment Draft and Final Research Paper: You have to strictly follow the template of research paper. If it is not done your paper may get rejected.

- The **major constraint** is that you must independently make all content, tables, graphs, and facts that are offered in the paper. You must write each part of the paper wholly on your own. The Peer-reviewers need to identify your own perceptive of the concepts in your own terms. NEVER extract straight from any foundation, and never rephrase someone else's analysis.
- Do not give permission to anyone else to "PROOFREAD" your manuscript.
- Methods to avoid Plagiarism is applied by us on every paper, if found guilty, you will be blacklisted by all of our collaborated research groups, your institution will be informed for this and strict legal actions will be taken immediately.)
- To guard yourself and others from possible illegal use please do not permit anyone right to use to your paper and files.

Criterion for Grading a Research Paper (Compilation) by Global Journals Inc. (US)

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Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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