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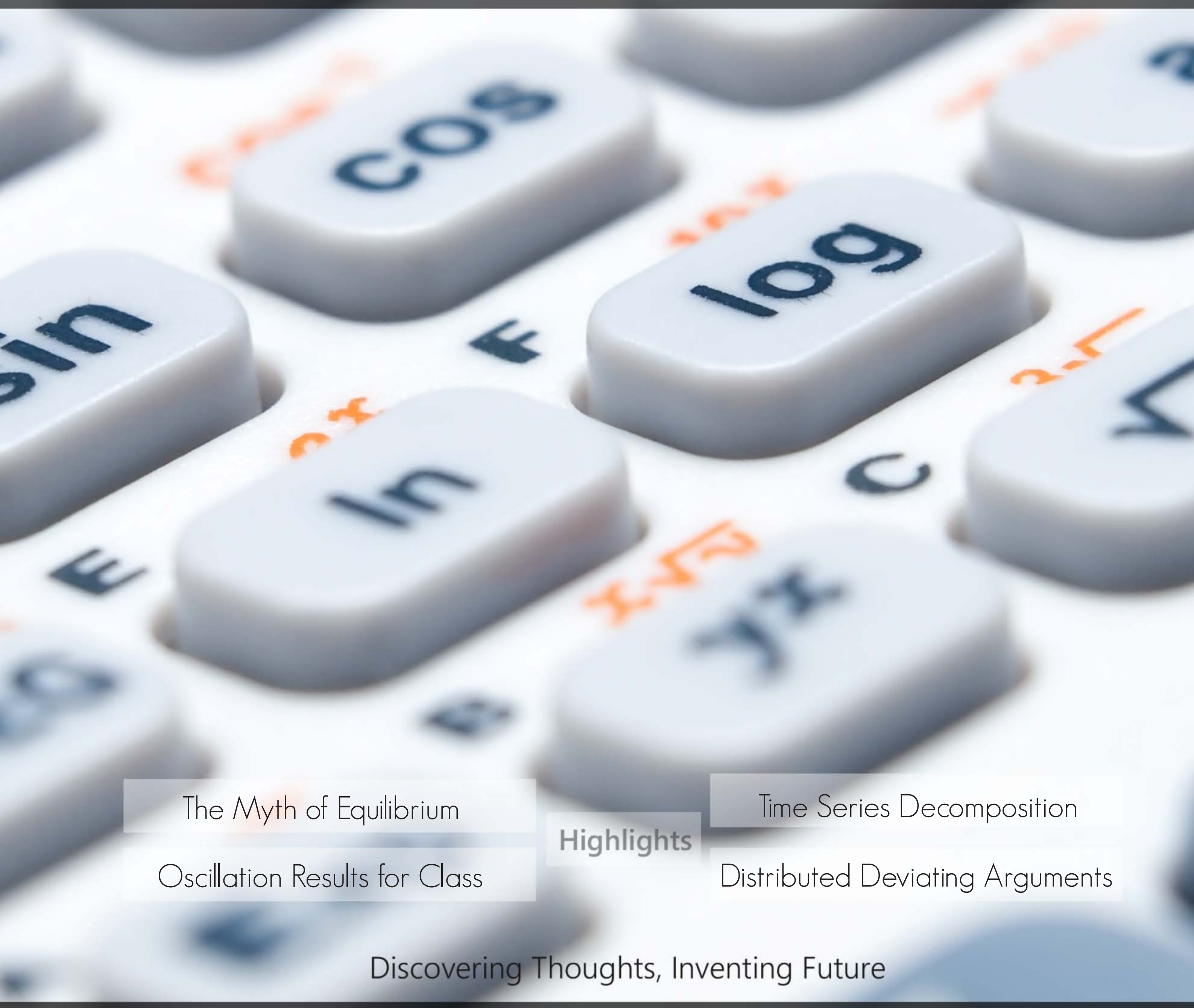

Highlights

The Myth of Equilibrium

Oscillation Results for Class

Time Series Decomposition

Distributed Deviating Arguments

Discovering Thoughts, Inventing Future

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New Oscillation Results for Class of Third Order Neutral Delay Differential Equations with Distributed Deviating Arguments

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Abstract- The purpose of this paper is to obtain the sufficient conditions which insure that solution of class of third order neutral delay differential equation is oscillatory or tended to zero. The results of this study basically generalize and improve the previous results. Examples given in the study to clarify the new results.

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New Oscillation Results for Class of Third Order Neutral Delay Differential Equations with Distributed Deviating Arguments

E. M. Elabbasy [α] & O. Moaaz [σ]

Abstract- The purpose of this paper is to obtain the sufficient conditions which insure that solution of class of third order neutral delay differential equation is oscillatory or tended to zero. The results of this study basically generalize and improve the previous results. Examples given in the study to clarify the new results.

Keywords and phrases: oscillation, third order, neutral delay, differential equations.

I. Introduction

In this scientific work we consider new class of third order neutral delay differential equations with distributed deviating arguments of the form

$$\left(r_2(t)\left(\left(r_1(t)\left(z'(t)\right)^{\alpha_1}\right)'\right)^{\alpha_2}\right)' + \int_a^b q(t,\xi)f(x(g(t,\xi)))d\sigma(\xi) = 0, t \geq t_0, \quad (1.1)$$

where $z(t) = x(t) + p(t)x(\tau(t))$ and we consider the following conditions

- **(A_1):** $p, \tau \in C(I, \mathbb{R}), 0 < p(t) \leq p < 1, \tau(t) \leq t, \lim_{t\to\infty} \tau(t) = \infty, \alpha_1$ and α_2 are a quotient of odd positive integers, $\alpha_1\alpha_2 = \beta$ and $I = [t_0, \infty)$,
- **(A_2):** $r_i \in C(I, (0, \infty)), \int_{t_0}^{\infty} (r_i(t))^{-1/\alpha_i} dt = \infty, i = 1, 2,$
- **(A_3):** $f \in C(\mathbb{R}, \mathbb{R}), xf(x) > 0$ for $t \geq t_0$,
- **(A_4):** $q \in C(I \times [a, b], [0, \infty)), q(t, \xi)$ is not zero on any half line $[t_\mu, \infty) \times [a, b], t_\mu \geq t_0$,
- **(A_5):** $g \in C(I \times [a, b], \mathbb{R}),\ g(t, \xi) \leq t$ for $t \geq t_0$ and $\xi \in [a, b]$, $g(t, \xi)$ is continuous, has positive partial derivative on $I \times [a, b]$ with respect to t, nondecreasing with respect to ξ and $\lim_{t\to\infty} g(t, \xi) = \infty$,
- **(A_6):** $\sigma \in C([a, b], \mathbb{R})$, σ is nondecreasing and the integral of Eq. (1.1) is in the sense Riemann-stieltijes.

We intend to a solution of Eq. (1.1) a function $x(t) : [t_x, \infty) \to \mathbb{R}, t_x \geq t_0$ such that $x(t), r_1(t)(z'(t))^{\alpha_1}$ and $r_2(t)((r_1(t)(z'(t))^{\alpha_1})')^{\alpha_2}$ are continuously differentiable for all $t \in [t_x, \infty)$ and $\sup\{|x(t)| : t \geq T\} > 0$ for any $T \geq t_x$. A solution of Eq. (1.1) is called oscillatory if it has arbitrary large zeros, otherwise it is called nonoscillatory.

In the last decades, there have been many research activity with regard to the oscillation of solutions of neutral delay differential equations. Significantly, this is due

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to recognition of the importance of differential equations in different applications, see [14].

Recently, there has been an growing interest in getting sufficient conditions for the oscillation of solutions of second/third order nonlinear neutral delay differential equations (see, for example [1]-[11], [13] and the references quoted therein). The oscillation problem for delay equation such as

$$(r_2(t)z''(t))' + f(t, z(t), z'(t)) = 0$$

and the half linear delay differential equation

$$(r_2(t)(z''(t))^{\alpha_2})' + q(t)x^{\alpha_2}(g(t)) = 0$$

have been discussed by many authors by different methods. Some results can be found in [11], [17] and also references therein. As well, [3] obtained some sufficient conditions of oscillation for neutral delay differential equation

$$(r_2(t)(z''(t))^{\alpha_2})' + q(t)f(x(g(t))) = 0$$

The aim of this paper is to discuss asymptotic behavior of solutions of class of third order neutral delay differential equation. By using Riccati transformation technique and new comparison principles, we established sufficient conditions which insure that solution of class of third order neutral delay differential equation is oscillatory or tended to zero. The results of this study basically generalize and improve the previous results. Examples given in the study to clarify the new results.

Let's recall the two sets of conditions that are commonly used, and we rely on:

(S_1): $\frac{f(x)}{x^\beta} \geq k > 0$ for $x \neq 0$ and $t \geq t_0$.
(S_2): $f'(x) > 0$ for $x \neq 0$ and $-f(-uv) \geq f(uv) \geq f(u)f(v)$ for $uv > 0$.

To discuss our main results, we review the following Theorem:
Consider the differential equation of the form

$$(-1)^n x^{(n)}(t) = F(t, x(v_1(t)), x(v_2(t)), ..., x(v_m(t))) \text{ for } t \geq t_0, \tag{E}$$

where $F \in C([t_0, \infty) \times [0, \infty)^m)$ and $v_j \in C([t_0, \infty))$ such that $\lim_{t\to\infty} v_j(t) = \infty$ for $j = 1, 2, ..., m$. The function $F = F(t, u_1, u_2, ..., u_m)$ is supposed to be increasing in each of $u_1, u_2, ..., u_m$. Furthermore, it is assumed that F is positive on $[t_0, \infty) \times [0, \infty)^m$ and $v_j(t) < t$ for every $t \geq t_0$ and $j = 1, 2, ..., m$.

Theorem 1.1. [16] *if y is a positive and strictly decreasing solution of the integral inequality*

$$y(t) \geq \int_{t_0}^{\infty} \frac{(s-t)^{n-1}}{(n-1)!} F(t, y(v_1(s)), y(v_2(s)), ..., y(v_m(s)))ds,$$

then there exists a positive solution $x(t)$ of the differential Equation (E) being such that $x(t) \leq y(t)$ for all large t and satisfying $\lim_{t\to\infty} x^{(i)}(t) = 0$ monotonically ($i = 1, 2, ..., n-1$).

Lemma 1.1. If X is nonnegative, $U \geq 0, V > 0$ and $\eta > 0$ then

$$UX - VX^{\frac{\eta+1}{\eta}} \leq \frac{\eta^\eta}{(\eta+1)^{\eta+1}} U^{\eta+1} V^{-\eta}.$$

Proof. Let

$$K(X) = UX - VX^{\frac{\eta+1}{\eta}}, X > 0.$$

Ref
3. B. Baculikova and J. Dzurina, On the asymptotic behavior of a class of third order nonlinear neutral differential equations, Cent. Eur. J. Math. 8(6) .2010, 1091-1103.

$K(X)$ obtains its maximum at $X = \left(\frac{\eta}{\eta+1}\right)^{\eta} U^{\eta+1}V^{-\eta}$ and

$$K(X) \leq \max_{X>0} K(X) = \frac{\eta^{\eta}}{(\eta+1)^{\eta+1}}U^{\eta+1}V^{-\eta}$$

The proof is complete. □

Notes

II. Main Results

In this section, we will establish new oscillation criteria for solutions of the Eq. (1.1). Assume that there exists a positive function $\rho(t)$. For the sake of convenience, we insert the next notation:

$$E_0(z(t)) = z(t), E_i(z(t)) = r_i(t)\left(\frac{d}{dt}E_{i-1}(z(t))\right)^{\alpha_i}, i = 1, 2,$$

$$R(t, t_0) = \left(\frac{1}{r_1(t)}\int_{t_0}^{t}\frac{1}{r_2^{1/\alpha_2}(s)}ds\right)^{1/\alpha_1}, \overline{R}(t, t_0) = \int_{t_0}^{t} R(u, t_0)du,$$

$$Q(t) = \int_a^b q(t,\xi)d\sigma(\xi), l(t) = \rho(t)\left(\beta R(g(t,a), t_0)\, g'(t,a)\right)^{-\beta}$$

$$\Theta_1(t) = k\rho(t)(1-p)^{\beta}Q(t), \Theta_2(t) = \int_a^b q(t,\xi)f(1-p(g(t,\xi)))d\sigma(\xi)$$

and

$$\mu = \frac{\beta^{\beta}}{(\beta+1)^{\beta+1}}.$$

Lemma 2.1. Let $x(t)$ be a positive solution of Eq. (1.1). Then $z(t)$ has only one of the following two properties eventually:

(P_1)**:** $z(t) > 0, z'(t) > 0$ and $\frac{d}{dt}E_1(z(t)) > 0$,

(P_2)**:** $z(t) > 0, z'(t) < 0$ and $\frac{d}{dt}E_1(z(t)) > 0$.

Proof. Let $x(t)$ be a positive solution of Eq. (1.1). From (A_1) and (A_5), there exists a $t_1 \geq t_0$ such that $x(t) > 0, x(\tau(t)) > 0$ and $x(g(t,\xi)) > 0$ for $t \geq t_1$. Then $z(t) > 0$ and Eq. (1.1) implies that $\frac{d}{dt}E_2(z(t)) \leq 0$. Hence, $E_2(z(t))$ is a non-increasing function and of one sign. We claim that $E_2(z(t)) > 0$ for $t \geq t_1$. Suppose that $E_2(z(t)) < 0$ for $t \geq t_2 \geq t_1$, then there exists a $t_3 \geq t_2$ and constant $K_1 > 0$ such that

$$\frac{d}{dt}E_1(z(t)) < -K_1(r_2(t))^{-1/\alpha_2},$$

for $t \geq t_3$. By integrating the last inequality from t_3 to t, we get

$$E_1(z(t)) < E_1(z(t_3)) - K_1\int_{t_3}^{t}(r_2(s))^{-1/\alpha_2}ds.$$

Letting $t \to \infty$, from (A_2), we have $\lim_{t\to\infty} E_1(z(t)) = -\infty$. Then there exists a $t_4 \geq t_3$ and constant $K_2 > 0$ such that

$$z'(t) < -K_2(r_1(t))^{-1/\alpha_1},$$

for $t \geq t_4$. By integrating this inequality from t_4 to t and using (A_2), we get $\lim_{t\to\infty} z(t) = -\infty$, which contradicts $z(t) > 0$. Now we have $E_2(z(t)) > 0$ for $t \geq t_1$. Therefore, $E_1(z(t))$ is increasing function. Thus (P_1) or (P_2) holds for $z(t)$, eventually. □

a) Oscillation results for f(x) without monotonicity. The purpose of this section is to study criteria of oscillation for solutions of the Eq. (1.1) by using a Riccati transformation technique.

Lemma 2.2. Let (S_1) holds, $x(t)$ be a positive solution of Eq. (1.1), and $z(t)$has the property (P_2). Assume that

$$\int_{t_0}^{\infty}\left(\frac{1}{r_1(v)}\int_v^{\infty}\left(\frac{1}{r_2(u)}\int_u^{\infty}Q(s)ds\right)^{1/\alpha_2}du\right)^{1/\alpha_1}dv=\infty. \tag{2.1}$$

Then the solution $x(t)$ is converges to zero as $t\to\infty$.

Proof. Let $x(t)$ be a positive solution of Eq. (1.1). Since $z(t)$ satisfies the property (P_2), we get

$$\lim_{t\to\infty} z(t)=\gamma.$$

Now. We shall prove that $\gamma=0$. Let $\gamma>0$, then we have $\gamma<z(t)<\gamma+\varepsilon$ for all $\varepsilon>0$ and t enough large. Choosing $\varepsilon<\frac{1-p}{p}\gamma$, we obtain

$$\begin{aligned} x(t) &= z(t)-p(t)x(\tau(t)) \qquad (2.2)\\ &> \gamma-pz(\tau(t))\\ &> L(\gamma+\varepsilon)>Lz(t), \end{aligned}$$

where $L=\frac{\gamma-p(\gamma+\varepsilon)}{\gamma+\varepsilon}>0$. Hence, from (1), (S_1) and (A_5), we have

$$\begin{aligned} \frac{d}{dt}E_2(z(t)) &< -kL^{\beta}\int_a^b q(t,\xi)z^{\beta}(g(t,\xi))d\sigma(\xi)\\ &< -kL^{\beta}z^{\beta}(t)Q(t)\\ &< -kL^{\beta}\gamma^{\beta}Q(t). \end{aligned}$$

By integrating two times from t to ∞, we get

$$-z'(t)>C\left(\frac{1}{r_1(t)}\int_t^{\infty}\left(\frac{1}{r_2(u)}\int_u^{\infty}Q(s)ds\right)^{1/\alpha_2}du\right)^{1/\alpha_1},$$

where $C=k^{1/\beta}L\gamma>0$. Integrating the last inequality from t_1 to ∞, we have

$$z(t_1)>C\int_{t_1}^{\infty}\left(\frac{1}{r_1(v)}\int_v^{\infty}\left(\frac{1}{r_2(u)}\int_u^{\infty}Q(s)ds\right)^{1/\alpha_2}du\right)^{1/\alpha_1}dv.$$

This contradicts to the condition (2.1), then $\lim_{t\to\infty} z(t)=0$, which implies that $\lim_{t\to\infty} x(t)=0$. □

Theorem 2.1. Let (S_1) and (2.1) hold. Assume that there exists a positive function $\rho(t)$such that

$$\limsup_{t\to\infty}\int_{t_0}^{t}\left(\Theta_1(s)-\mu\left(\frac{\rho'(s)}{\rho(s)}\right)^{\beta+1}l(s)\right)ds=\infty. \tag{2.3}$$

Then every solution of Eq. (1.1) is either oscillatory or tends to zero as $t\to\infty$.

Notes

Notes

Proof. Let x be a non-oscillatory solution of Eq. (1.1) on the interval I. Without loss of generality we may assume that $x(t) > 0$. Then there exists a $t_1 \geq t_0$ such that $x(t) > 0$, $x(\tau(t)) > 0$ and $x(g(t,\xi)) > 0$ for $t \geq t_1$. By Lemma **??**, we have that $z(t)$ has the property (P_1) or the property (P_2). If $z(t)$ has the property (P_2). From Lemma 2.2, we obtain $\lim_{t\to\infty} x(t) = 0$. Now, for $t \geq t_2 \geq t_1$, let $z(t)$ satisfies the property (P_1), then we get

$$x(t) = z(t) - p(t)x(\tau(t)) \geq (1-p(t))z(t) \geq (1-p)z(t). \tag{2.4}$$

Thus, from (1), (S_1) and (A_5), we have

$$\frac{d}{dt}E_2(z(t)) \leq -k(1-p)^{\beta} z^{\beta}(g(t,a))Q(t). \tag{2.5}$$

We define

$$\omega(t) = \rho(t)\frac{E_2(z(t))}{z^{\beta}(g(t,a))}.$$

By differentiating and using (2.5), we get

$$\omega'(t) \leq \frac{\rho'(t)}{\rho(t)}\omega(t) - \Theta_1(t) - \beta\rho(t)\frac{E_2(z(t))}{z^{\beta+1}(g(t,a))}z'(g(t,a))g'(t,a) \tag{2.6}$$

From (P_1), we have

$$\begin{aligned} E_1(z(t)) &= E_1(z(t_2)) + \int_{t_2}^{t} \frac{E_2^{1/\alpha_2}(z(s))}{r_2^{1/\alpha_2}(s)}ds \\ &\geq E_2^{1/\alpha_2}(z(t))\int_{t_2}^{t} \frac{1}{r_2^{1/\alpha_2}(s)}ds, \end{aligned} \tag{2.7}$$

for $t \geq t_2$. Since $\frac{d}{dt}E_2(z(t)) \leq 0$, we obtain

$$z'(g(t,a)) \geq E_2^{1/\beta}(z(t))R(g(t,a),t_2).$$

Hence, (2.6) implies

$$\omega'(t) \leq -\Theta_1(t) + \frac{\rho'(t)}{\rho(t)}\omega(t) - l^{-1/\beta}(t)\omega^{\frac{\beta+1}{\beta}}(t).$$

If $\eta = \beta, U = \frac{\rho'}{\rho}, V = l^{-1/\beta}$ and $X = \omega$, then from Lemma 1.1, we obtain

$$\frac{\rho'}{\rho}\omega - l^{-1/\beta}\omega^{\frac{\beta+1}{\beta}} \leq \mu\left(\frac{\rho'}{\rho}\right)^{\beta+1} l.$$

Therefore, we get

$$\omega'(t) \leq -\Theta_1(t) + \mu\left(\frac{\rho'(t)}{\rho(t)}\right)^{\beta+1} l(t).$$

By integrating the above inequality from t_2 to t we have

$$\omega(t) \leq \omega(t_2) - \int_{t_2}^{t}\left(\Theta_1(s) - \mu\left(\frac{\rho'(s)}{\rho(s)}\right)^{\beta+1} l(s)\right)ds.$$

Taking the superior limit as $t \to \infty$ and using (2.3), we get $\omega(t) \to -\infty$, which contradicts that $\omega(t) > 0$. This completes the proof of Theorem 2.1. □

Example 2.1. *Consider the third order neutral delay differential equation*

$$\left(t\left(\left(\frac{1}{t}\left(z'(t)\right)^{1/3}\right)'\right)^{5}\right)' + \int_1^2 \frac{t^2 e^{t^2(\xi-1)}}{e^{t^2}-1} x^{5/3}(\xi(t-1))\left(x^2(\xi(t-1))+2\right)d\xi = 0, \tag{2.8}$$

where $z(t) = x(t) + \frac{1}{2}x(t-1)$ *and* $t > 1$. *Choose* $\rho(t) = 1$ *and* $k = 2$. *It is easy to see that the conditions (2.1) and (2.3) are hold. Then, from Theorem 2.1, every nonoscillatory solution of Eq. (2.8) tends to zero as* $t \to \infty$.

Remark 2.1. *If* $\alpha_1 = \alpha_2 = 1$, $\tau(t) = t - \tau$ *and* $f(x) = x$, *then Theorem 2.1 extend and improve Theorem 2.1 in* [5].

b) Oscillation results for f(x) with monotonicity. In this section, we will establish some new criteria of oscillation for solutions of the Eq. (1.1) by using new comparison principles.

Lemma 2.3. *Let* (S_2) *holds,* $x(t)$ *be a positive solution of Eq. (1.1) and* $z(t)$ *has the property* (P_2). *If the condition (2.1) holds, then the solution* $x(t)$ *is converges to zero as* $t \to \infty$.

Proof. Proceeding as in the proof of Lemma 2.2, we see that (2.2) holds. Hence, from (1), (S_2) and (A_5), we have

$$\begin{aligned} \frac{d}{dt}E_2(z(t)) &\leq -\int_a^b q(t,\xi)f(Lz(g(t,\xi)))d\sigma(\xi) \\ &< -f(L)f(\gamma)Q(t). \end{aligned}$$

The rest of the proof runs as in Lemma 2.2. □

Theorem 2.2. *Let the condition* (S_2) *and (2.1) hold. If the first order delay differential equation*

$$y'(t) + \Theta_2(t) f\left(y^{1/\beta}(g(t,a))\right) f\left(\overline{R}(g(t,a),t_0)\right) = 0 \tag{2.9}$$

is oscillatory, then every solution of Eq. (1.1) is either oscillatory or tends to zero as $t \to \infty$.

Proof. Let x be a non-oscillatory solution of Eq. (1.1) on the interval I. Without loss of generality we may assume that $x(t) > 0$. As in the proof of Theorem 2.1, by Lemma **??**, we have that $z(t)$ has the property (P_1) or (P_2) for $t \geq t_2$. If $z(t)$ has the property (P_2). Then, from Lemma **??**, we obtain $\lim_{t\to\infty} x(t) = 0$. On the other hand, when $z(t)$ satisfies the property (P_1), we get that (2.4) and (2.7) hold. Thus, from (2.7), we obtain

$$z'(t) \geq E_2^{1/\beta}(z(t))\, R(t,t_2),$$

for $t \geq t_2 \geq t_1$. By integrating this inequality from t_2 to t, we get

$$z(t) \geq z(t_2) + \int_{t_2}^t E_2^{1/\beta}(z(s))\, R(s,t_2)\, ds.$$

Since $\frac{d}{dt}E_2(z(t)) < 0$, we obtain

$$z(g(t,a)) \geq E_2^{1/\beta}(z(g(t,a)))\, \overline{R}(g(t,a),t_2). \tag{2.10}$$

From (1), (S_2) and (2.4), we have

Notes

$$\begin{aligned} -\frac{d}{dt}E_2\left(z\left(t\right)\right) &\geq \int_a^b q\left(t,\xi\right) f\left(z\left(g\left(t,\xi\right)\right)\right) f\left(1-p\left(g\left(t,\xi\right)\right)\right) d\sigma\left(\xi\right) \\ &\geq f\left(z\left(g\left(t,a\right)\right)\right)\Theta_2\left(t\right), \end{aligned}$$

where $z'(t) > 0$. Hence, from (2.10), we get

$$-\frac{d}{dt}E_2\left(z\left(t\right)\right) \geq \Theta_2\left(t\right) f\left(E_2^{1/\beta}\left(z\left(g\left(t,a\right)\right)\right)\right) f\left(\overline{R}\left(g\left(t,a\right),t_2\right)\right).$$

By integrating above inequality from t to ∞ and Let $y(t) = E_2(z(t))$, we obtain

$$y\left(t\right) \geq \int_t^\infty \Theta_2\left(s\right) f\left(y^{1/\beta}\left(g\left(s,a\right)\right)\right) f\left(\overline{R}\left(g\left(s,a\right),t_2\right)\right) ds,$$

The function $y(t)$ is obviously strictly decreasing. Hence, by Theorem 1.1 there exists a positive solution of equation (2.9) with $\lim_{t\to\infty} y(t) = 0$ which contradicts that Equation (2.9) is oscillatory. This completes the proof of Theorem 2.2. □

Example 2.2. Consider the third order delay differential equation

$$\left(t\left(\left(\frac{1}{t}\left(z'\left(t\right)\right)^{1/3}\right)'\right)^3\right)' + \int_0^1 \frac{6\xi^2}{t} x\left(\frac{1}{2}\left(\xi+1\right)t\right) d\xi = 0, \qquad (2.11)$$

where $z(t) = x(t) + \frac{1}{2}x\left(\frac{t}{3}\right)$ *and* $t \geq 1$*, It easy to see that the Condition (2.1) holds and Eq. (2.9), reduces to*

$$y'\left(t\right) + \widehat{q}(t) y\left(\frac{t}{2}\right) = 0, \qquad (2.12)$$

where

$$\widehat{q}\left(t\right) = \frac{9}{896} + \frac{9}{1024}t^5 - \frac{243}{8192}2^{2/3}t^{13/3} + \frac{243}{3584}2^{1/3}t^{11/3} - \frac{27}{512}t^3.$$

On the other hand, Theorem 2.1.1 in [15] *guarantees the oscillation of (2.12). Since*

$$\liminf_{t\to\infty} \int_{t/2}^t \widehat{q}\left(t\right) ds > \frac{1}{e}.$$

Then, from Theorem 2.2, every nonoscillatory solution of Equation (2.11) tends to zero as $t \to \infty$*.*

In the following Theorem, we are concerned with the oscillation of solutions of Eq. (1.1) when $\tau(t) \geq t$.

Theorem 2.3. Let the condition (S_2) *holds, and there exists a function* $\zeta(t)$ *such that*

$$\zeta'\left(t\right) \geq 0, \zeta\left(t\right) > t \text{ and } \delta\left(t\right) = g\left(\zeta\left(\zeta\left(t\right)\right), b\right) < t$$

If the first order delay differential equation

$$z'\left(t\right) + Q_1(t) f^{1/\beta}\left(z\left(\delta\left(t\right)\right)\right) = 0, \qquad (2.13)$$

where

$$Q_1 = \left(\frac{1}{r_1(t)}\int_t^{\zeta(t)}\left(\frac{1}{r_2(u)}\int_u^{\zeta(t)}\Theta_2(s)ds\right)^{1/\alpha_2} du\right)^{1/\alpha_1},$$

Ref

15. G. S. Ladde, V. lakshmikantham and B. G. Zhang, Oscillation Theory of Differential Equations With Deviating Arguments, Macel Dekker, New York, 1987.

is oscillatory. Then every solution $x(t)$ *of Eq. (1.1) is either oscillatory or* $\limsup_{t\to\infty} |x(t)| = \infty$.

Proof. Let x be a non-oscillatory solution of Eq. (1.1) on the interval I. Then, without loss of generality, there exists a $t_1 \geq t_0$ such that $x(t) > 0$ and $x(g(t,\xi)) > 0$ for $t \geq t_1$. By Lemma **??**, we have $z(t)$ has only one of the two Cases (P_1) or (P_2). For the Case (P_1). Since $z(t) > 0$, $z'(t) > 0$ and $\frac{d}{dt}E_1(z(t)) > 0$, $\lim_{t\to\infty} z(t) = \infty$, and from definition of $z(t)$, we get

$$\limsup_{t\to\infty} |x(t)| = \infty.$$

In the Case (P_2). Since $z'(t) < 0$ and $\tau(t) \geq t$, we obtain

$$x(t) \geq z(t)(1 - p(t)),$$

for $t \geq t_2$. Thus, from (A_5), there exists a $t_3 \geq t_2$ with $g(t,\xi) \geq t_2$ for $t \geq t_3$ such that

$$x(g(t,\xi)) \geq z(g(t,\xi))(1 - p(g(t,\xi))).$$

Hence, Eq. (**??**) and (S_2) yield

$$-\frac{d}{dt}E_2(z(t)) \geq \int_a^b q(t,\xi) f(z(g(t,\xi))) f(1 - p(g(t,\xi))) d\sigma(\xi)$$

Then, from (A_5), we get

$$-\frac{d}{dt}E_2(z(t)) \geq \Theta_2(t) f(z(g(t,b)))$$

By integrating this inequality from t to $\zeta(t)$, we have

$$\frac{d}{dt}E_1(z(t)) \geq r_2^{-1/\alpha_2}(t) f^{1/\beta}(z(g(\zeta(t),b))) \left(\int_t^{\zeta(t)} \Theta_2(s) ds\right)^{1/\alpha_2}$$

Again, integrate the above inequality from t to $\zeta(t)$, we obtain

$$-z'(t) \geq Q_1(t) f^{1/\beta}(z(\delta(t))).$$

Hence, By Theorem 1.1, there exists a positive solution of Eq. (2.13) with $\lim_{t\to\infty} z(t) = 0$ which contradicts that Eq. (2.13) is oscillatory. This completes the proof of Theorem 2.3. □

Remark 2.2. If $\alpha_1 = 1, r_1(t) = 1, a = b = 1, q(t,\xi) = q(t), g(t,\xi) = g(t)$ *and* $\sigma(\xi) = \xi$ *then Theorems 2.2, 2.3 extend and improve Theorem 2.4, 2.10 in* [3].

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Time Series Decomposition and Seasonal Adjustment

By Maskurul Alam, Matiur Rahman, Sharmin Akter Sumy & Yasin Ali Parh
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Abstract- Many forecasting method are based on some notion that when an underlying pattern exists in a data series. That data can be distinguished from randomness by smoothing (averaging) past values. The effect of this smoothing is to eliminate randomness so the pattern can be projected into the future. It goes without saying that when a data is good enough and have nice pattern then forecast could be done more precisely. One of the main objectives for decomposition is to estimate seasonal effects that can be used to create and present seasonally adjusted values. A seasonally adjusted value removes the seasonal effect from a value so that trends can be seen more clearly. My main aim is to choose a best decomposition method and forecast the data more precisely.

Keywords: time series decomposition, decomposition models, seasonal adjustment, moving average smoother.

GJSFR-F Classification : *MSC 2010: 49M27*

TIMESERIESDECOMPOSITIONANDSEASONALADJUSTMENT

Strictly as per the compliance and regulations of :

Time Series Decomposition and Seasonal Adjustment

Maskurul Alam [α], Matiur Rahman [σ], Sharmin Akter Sumy [ρ] & Yasin Ali Parh [Ѡ]

Abstract- Many forecasting method are based on some notion that when an underlying pattern exists in a data series. That data can be distinguished from randomness by smoothing (averaging) past values. The effect of this smoothing is to eliminate randomness so the pattern can be projected into the future. It goes without saying that when a data is good enough and have nice pattern then forecast could be done more precisely. One of the main objectives for decomposition is to estimate seasonal effects that can be used to create and present seasonally adjusted values. A seasonally adjusted value removes the seasonal effect from a value so that trends can be seen more clearly. My main aim is to choose a best decomposition method and forecast the data more precisely.

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I. Introduction

In many instances the pattern of the data can be broken down (decomposed) into sub pattern that identify each component of the time series separately. Such break down of the data can give the better ideas about the understanding the behavior of the series which facilitates improves accuracy in forecasting. Decomposition method usually try to identify two separate useful components of the basic underline pattern that tend to characterize economic and business series. These are the trend cycle and seasonal factors. The seasonal factors relates to periodic fluctuations of constant length that are usually caused by such things as temperature, rainfall, month of the year, timing of holydays and corporate policies. The trend cycle represents the longer term changes in the level of the series. The trend cycle sometime could be separated into two components. These are trend and cycle components. But the distinction is somewhat artificial and most procedures leave the trend and cycle as a single component known as the trend-cycle.

II. Time Series Decomposition

Decomposition assume that the data are made as follows

$$Data = pattern + \ error$$

Now it is necessary to mention that the pattern of any data may form trend cycle and seasonality. This means a trend exists when there is a long-term increase or decrease in the data. It does not have to be linear. Sometimes we will refer to a trend "changing direction" when it might go from an increasing trend to a decreasing trend.

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On the other hand a seasonal pattern exists when a series is influenced by seasonal factors (e.g., the quarter of the year, the month, or day of the week). Seasonality is always of a fixed and known period.

So we could give a standard form of the decomposition time series on the basis of the pattern of the data.

$$Data = pattern + \ error$$

$$= f(trend - cycle, seasonality, error)$$

An element of the error or randomness is also assumed to be present in the data. It is actually the combined effect of the two sub patterns of the series. This means the combined effect of the trend-cycle, seasonality and the actual data.This is often called the “irregular” or the “reminder” component.

It goes without saying that there are several alternative approaches to decomposing a time series all of which aim to isolate each component of the series with great accuracy and precisely. Actually the main substance is to remove the trend cycle and then isolating the seasonal component.

Notes

III. Decomposition Models

As we discussed earlier about the decomposition we could give the following form

$$Yt = f(St, Tt, Et)$$

Where Yt it is the time series value (actual data) at period t,

St Is the seasonal component (or index) at period t,

Tt Is the trend cycle component at period t,

And Et is the irregular (or reminder) component at period t

There are two common method of decomposition these are

1. Additive decomposition and
2. Multiplicative decomposition

Additive decomposition:

A common approach is to assume the addition of seasonal component, trend cycle component and the irregular component.

$$Yt = St + Tt + Et$$

An additive model is usually appropriate if the magnitude or the span of the seasonal fluctuation doesn't vary with the level of the series. It actually means when the magnitude of the seasonal fluctuation remain same then additive decomposition is used.

Multiplicative decomposition

A common approach is to multiply the seasonal, trend cycle and irregular components together to give the observed series.

$$Yt = St * Tt * Et$$

A multiplicative decomposition is usually apply when the seasonal fluctuations increase and decrease proportionally with increase and decreases in the level of the series. Multiplicative decomposition is more apposite for the economic series because most seasonal economic series have seasonal variation and even it may vary for day, week, and month as well as for year.

Now either choosing an additive or multiplicative decomposition we could use a transformation. When the original data are not additive then logarithm transformation turns a multiplicative relationship into a additive relationship.

$$Yt = St * Tt * Et$$

Then $logYt = LogSt + LogTt + LogEt$

Notes

So we can fit a multiplicative relationship by fitting an additive relationship to the logarithm of the data.

IV. Decomposing Seasonal Data with the Help of Additive Decomposition Model

To estimate the trend component and seasonal component of a seasonal time series that can be described using an additive model, we can use the "decompose ()" function in R. This function estimates the trend, seasonal, and irregular components of a time series that can be described using an additive model.

We explain it though an example. We are considering a birth time series data of New York.

Time	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1946	26.663	23.598	26.931	24.740	25.806	24.364	24.477	23.901	23.175	23.227	21.672	21.870
1947	21.439	21.089	23.709	21.669	21.752	20.761	23.479	23.824	23.105	23.110	21.759	22.073
1948	21.937	20.035	23.590	21.672	22.222	22.123	23.950	23.504	22.238	23.142	21.059	21.573
1949	21.548	20.000	22.424	20.615	21.761	22.874	24.104	23.748	23.262	22.907	21.519	22.025
1950	22.604	20.894	24.677	23.673	25.320	23.583	24.671	24.454	24.122	24.252	22.084	22.991
1951	23.287	23.049	25.076	24.037	24.430	24.667	26.451	25.618	25.014	25.110	22.964	23.981
1952	23.798	22.270	24.775	22.646	23.988	24.737	26.276	25.816	25.210	25.199	23.162	24.707
1953	24.364	22.644	25.565	24.062	25.431	24.635	27.009	26.606	26.268	26.462	25.246	25.180
1954	24.657	23.304	26.982	26.199	27.210	26.122	26.706	26.878	26.152	26.379	24.712	25.688
1955	24.990	24.239	26.721	23.475	24.767	26.219	28.361	28.599	27.914	27.784	25.693	26.881
1956	26.217	24.218	27.914	26.975	28.527	27.139	28.982	28.169	28.056	29.136	26.291	26.987
1957	26.589	24.848	27.543	26.896	28.878	27.390	28.065	28.141	29.048	28.484	26.634	27.735
1958	27.132	24.924	28.963	26.589	27.931	28.009	29.229	28.759	28.405	27.945	25.912	26.619
1959	26.076	25.286	27.660	25.951	26.398	25.565	28.865	30.000	29.261	29.012	26.992	27.897

It is roughly could be mentioned from the above time series data the number of births per month in New York city is seasonal with a peak every summer and trough every winter, and can probably be described using an additive model since the seasonal and random fluctuations seem to be constant in size over time. This means the number of birth in New York per month roughly constant.

To see it visually we could use R program

```
birthstimeseries= ts (births, frequency=12, start=c(1946,1))
birthstimeseries
```

plot (birthstimeseries)

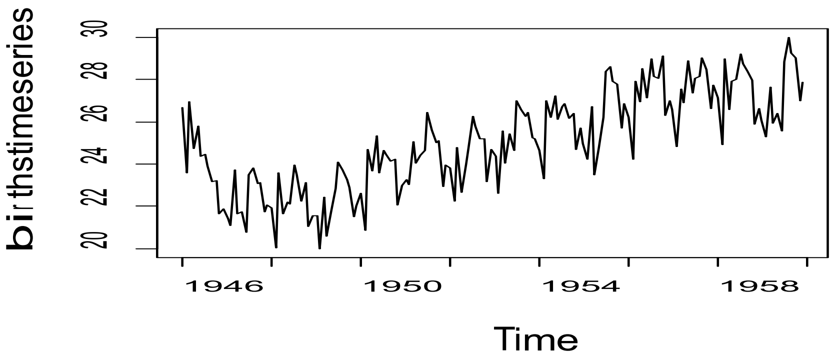

Figure : the plot of births time series from 1946 to 1959.

Now it is manifested to us from above figure that the number of birth per month in New York roughly constant but the data is seasonal in peak summer and trough every winter.

Now we estimate the trend-cycle, seasonal and irregular components in order to use additive model .

These could be estimated through R programming.

Births time series components=decompose(births time series) Births time series components

Now estimated all the components (trend-cycle, seasonal and irregular) are stored into the birth time series components variable.

We can plot the estimated trend, seasonal and irrigular compoents by using R command

Plot(

Plot(Births time series components)

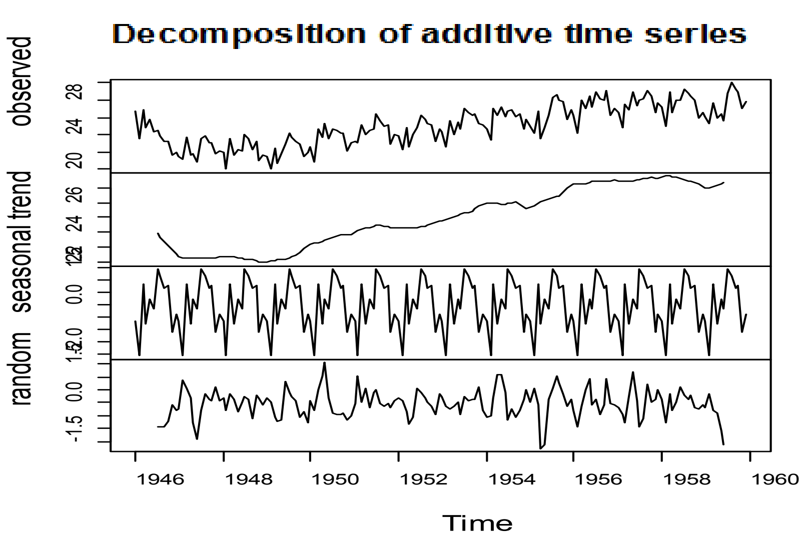

Figure : The plot births time series of Time series components

It could be mentioned from the above figure that the estimated seasonal component doesn't change much over time. The seasonal pattern at the start of the

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series is almost same as the pattern at the end of the series. On the other hand trend component shows a small decrease from about 24 in 1947 to about 22 in 1948, followed by a steady increase from then on to about 27 in 1959.

V. Seasonal Adjustment

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It is necessary to say that when a seasonal data is subtracted from the main data then the resulting values are referred to as "seasonal adjustment" data. The additive model is given by

$$Yt = St + Tt + Et$$

So the seasonal adjusted holds the following form

$$Yt - St = Tt + Et$$

This means leaving only trend cycle and irregular component.

And for multiplicative data seasonal data can be found by dividing the main data to the seasonal data. Mathematically it is given by

$$Yt/St = Tt * St$$

Most published economic data series are seasonally adjusted because seasonal variation is typically not primary interest. The seasonally adjusted data series shows the data after any seasonal variation has been removed.

For an example we consider the previous example of birth time series data.

The seasonal component can be found with the help of R programming. We type simply "decompose ()" in R and then subtract the seasonal component from the main time series data. It is usually done in order to remove the seasonal variation.

Birthstimeseriescomponents=decompose (birthstimeseries)
Birthstimeseriescomponents
Birthstimeseriesseasonallyadjusted=birthstimeseries-
birthstimeseriescomponents$seasonal
Birthstimeseriesseasonallyadjusted

We can plot the seasonally adjusted data in order to see the other components. we can type the following command in R

"plot(Birthstimeseriesseasonallyllyadjusted)"

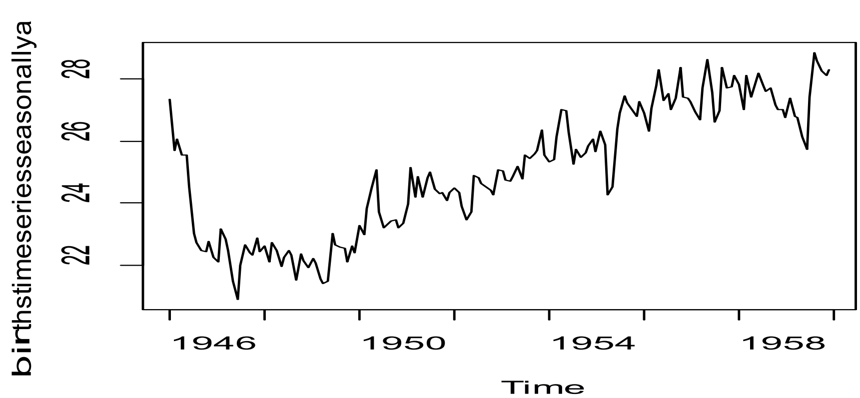

Figure : The plot of seasonal adjusted time series

It is manifested to us that the seasonal variation has been removed from the seasonally adjusted data. It is certainly say from here that the seasonally adjusted data just trend and irregular variation contain. Clearly saying first of all we had observed data, seasonal component, trend component and irregular component. After removing the seasonal variation from the adjusted data we have just trend and irregular component. It can be shown after observing the two figures.

VI. Moving Average Smoother

It is known to all that the trend cycle can be estimated by smoothing the series to reduce the random variation. There are number smoothers are available for reducing the random variation. The oldest and simplest method is moving average that could be used in order to reduce the random variation.
A moving average for m order can be written as

$$Tt=\frac{1}{k}\sum_{j=-m}^{km} yt+j$$

Where k is moving average of order k (or MA), k is an odd integer and it is defined as the average consisting of an observation and the m= (k-1)/2 points of either side. Observations that are nearby in time are also likely to be close in value, and the average eliminates some of the randomness in the data, leaving a smooth trend-cycle component. We call this an *k*-MA meaning a moving average of order *k*
We explain through an example
Suppose we have sales of detergent (in liters) over three year's period.

year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1989	266.0	145.9	183.1	119.3	180.3	168.5	231.8	224.5	192.8	122.9	336.5	185.9
1990	194.3	149.5	210.1	273.3	191.4	287.0	226.0	303.6	289.9	421.6	264.5	342.3
1991	339.7	440.4	315.9	439.3	401.3	437.4	575.5	407.6	682.0	475.3	581.3	646.9

Now our main purpose is to reduce the trend variation as much as possible. Taking average of the points near an observation will provide the reasonable estimate of the trend cycle at that observation. The average eliminates some of the randomness in the data. It is necessary to know how many data points to include in each average. Suppose we use the average of the three points, namely the observation at which we are calculating trend cycle and the points on either side. Clearly saying if we want to estimate the trend cycle of February month at 145.9 in 1989 then we just consider this value previous values(266.0) and and the next value(183.1) of it.This is called the moving average of order Three.
Mathematically it is given by

$$\text{T2}=\frac{1}{3}(Y1+Y2+Y3)=\frac{1}{3}(266.0+145.9+183.1)=198.3$$

Generally a moving average of order 3 centered at time t is $=\frac{1}{3}(Yt-1+Yt+Yt+1$

The 3MA can be determined with the help R programming. But it is bear in mind that first of all data must be read in R. We type the following command for reading as well as for determining 3 MA.

```
timeseries<- ts(mas, frequency=12, start=c(1989,1))
timeseries
plot(timeseries)
z=mav (timeseries,3)
z.
```

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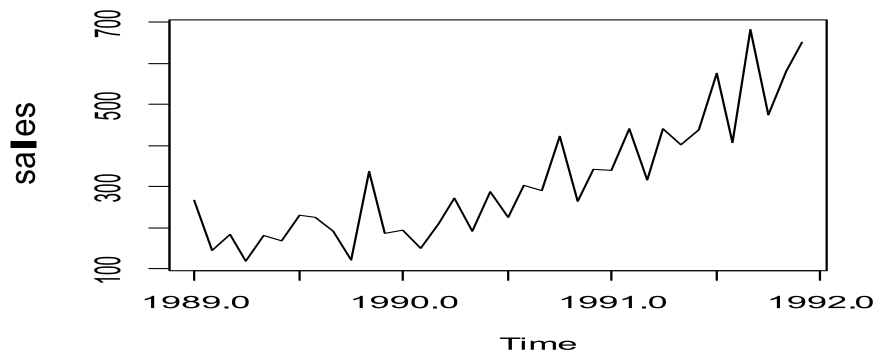

Figure : The sales of detergents over three years

The output of 3 MA in R

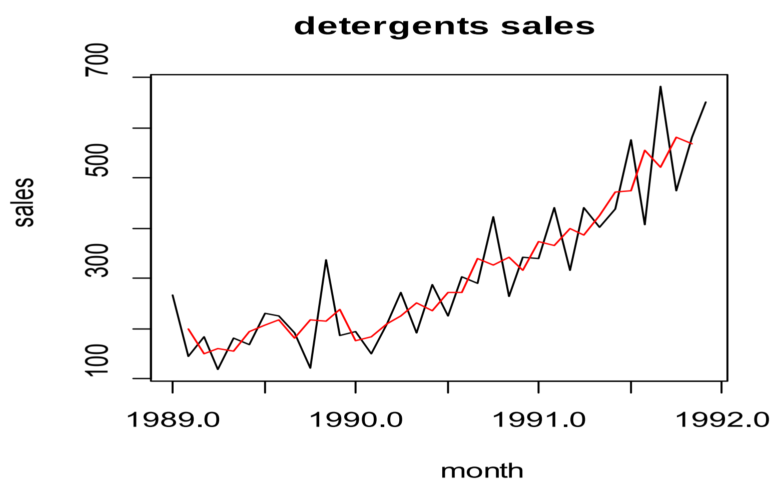

Figure : 3 MA smoother

[1,] NA

[2,] 198.3333

[3,] 149.4333to see what the trend estimate look like we plot it wth the original data.

[4,] 160.9000 we could make the above 3 order MA plot with the help of R command.

[5,] 156.0333plot(timeseries, main="detergents sales", ylab="sales",xlab="month")

[6,] 193.5333 lines(mav(timeseries,3),col="red") ……………..

[34,] 579.5333

[35,] 568.8333

[36,] NA

In the second column of this table a moving average of order 3 providing an estimate of trend cycle.

Notice how the trend (in red) is smoother than the original data and captures the main movement of the time series without all the minor fluctuations. It goes without saying that the order of the moving average determines the smoothness of the trend cycle estimate. It is certainly could be told that the higher order means a smoother curve. So we could make 5 order moving average in order make smoother. For the 5 order moving average we could determine the trend cycle with the help of R that saves our time compare to other methodology.

We could use the following command in order to see the how the trend look like and the

k=mav(timeseries,5)
k
plot(k)
plot(mas,main="detergents sales",ylab="sales", xlab="month")
lines(mav(mas,5),col="green")

output of the 5 order Moving average.

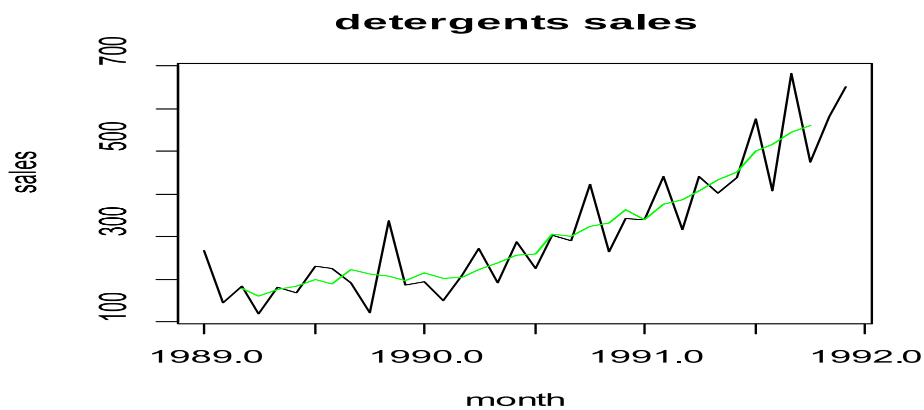

Figure : 5 MA smoother

[1,] NA

[2,] NA

[3,] 178.92

[4,] 159.42 so it could be mentioned from the above two 3 MA and 5 MA that

[5,] 176.603 MA is smoother leaves too much randomness in the trend cycle ……………. Estimate. It could be mentioned that the 5 MA smoother is better.

[34,] 559.22 but the trend cycle is probably smoother for other orders.

[35,] NA

[36,] NA

It should be bear in mind that determining the appropriate length of a moving average is an important task in decomposition method. As a rule a larger number of terms in the moving average increase the likelihood that randomness will be eliminated.

VII. Conclusion

Time series data can exhibit a huge variety of patterns and it is helpful to categorize some of the patterns and behaviors that can be seen in time series. It is also sometimes useful to try to split a time series into several components, each representing one of the underlying categories of pattern. Decomposition often plays the vital role to make time series better as well as improve the forecast.

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Neighborhood Properties of Generalized Bessel Function

By H. E. Darwish, A. Y. Lashin & B. F. Hassan
Mansoura University, Egypt

Abstract- Let A denote the class of functions of the form

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n \,(a_n \geq 0, n \in \mathbb{N}),$$

which are analytic in the open unit disk $U = \{z : |z| < 1\}$ In this paper, the new subclasses $Q_{n,c}(\gamma, k, \beta), H_{n,c}(\gamma, k, \beta; \mu), Q^{\alpha}_{n,c}(\gamma, k, \beta)$ and $H^{\alpha}_{n,c}(\gamma, k, \beta; \mu)$ of *A* which are de.ned by using generalized Bessel Function are introduced. Certain properties of neighborhood for functions belonging to these classes are studied.

Keywords: *univalent functions, neighborhoods, starlike functions, convex functions and bessel operator.*

GJSFR-F Classification : MSC 2010: 33C10

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Neighborhood Properties of Generalized Bessel Function

H. E. Darwish [α], A. Y. Lashin [σ] & B. F. Hassan [ρ]

Abstract- Let A denote the class of functions of the form

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n \quad (a_n \geq 0, n \in \mathbb{N}),$$

which are analytic in the open unit disk $U = \{z : |z| < 1\}$. In this paper, the new subclasses $Q_{n,c}(\gamma, k, \beta)$, $H_{n,c}(\gamma, k, \beta; \mu)$, $Q^{\alpha}_{n,c}(\gamma, k, \beta)$ and $H^{\alpha}_{n,c}(\gamma, k, \beta; \mu)$ of A which are defined by using generalized Bessel Function are introduced. Certain properties of neighborhood for functions belonging to these classes are studied.

Keywords: univalent functions, neighborhoods, starlike functions, convex functions and bessel operator.

I. Introduction

Let A denote the class of functions of the form:

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n \quad (a_n \geq 0, n \in \mathbb{N}). \tag{1.1}$$

which are analytic in the open unit disk $U = \{z : |z| < 1\}$.

For any function $f(z) \in A$, $z \in U$ and $\eta \geq 0$, we define

$$N_{n,\eta} f(z) = \left\{ g \in A : g(z) = z - \sum_{n=2}^{\infty} b_n z^n \text{ and } \sum_{n=2}^{\infty} n\,|a_n - b_n| \leq \eta \right\}, \tag{1.2}$$

which is the $(n, \eta)-$neighborhood of $f(z)$.

For $e(z) = z$, we see that

$$N_{n,\eta} e(z) = \left\{ g \in A : g(z) = z - \sum_{n=2}^{\infty} b_n z^n \text{ and } \sum_{n=2}^{\infty} n\,|b_n| \leq \eta \right\}. \tag{1.3}$$

The concept of neighborhoods was first introduced by Goodman [3] .

In this paper, we discuss certain properties of $(n, \eta)-$neighborhood results for functions in the classes $Q_{n,c}(\gamma, k, \beta)$, $H_{n,c}(\gamma, k, \beta; \mu)$, $Q^{\alpha}_{n,c}(\gamma, k, \beta)$ and $H^{\alpha}_{n,c}(\gamma, k, \beta; \mu)$ of A .

The subclass $S^{*}_{n}(\gamma)$[4] of A, is the class of functions of complex order γ satisfying

$$\operatorname{Re}\left\{ 1 + \frac{1}{\gamma}\left(\frac{zf'(z)}{f(z)} - 1 \right) \right\} > 0 \quad (z \in U, \gamma \in \mathbb{C}\setminus\{0\}).$$

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The subclass $K_n(\gamma)[4]$ of A, is the class of functions of complex order γ satisfying

$$\operatorname{Re}\left\{1+\frac{1}{\gamma}\frac{zf''(z)}{f'(z)}\right\}>0 \quad (z\in U,\gamma\in\mathbb{C}\backslash\{0\}).$$

The Hadamard product of two power series

$$f(z)=z+\sum_{n=2}^{\infty}a_nz^n \text{ and } g(z)=z+\sum_{n=2}^{\infty}b_nz^n.$$

is defined as $(f*g)(z)=z+\sum_{n=2}^{\infty}a_nb_nz^n$.

we recall here a generalized Bessel function $w(z)$ of the first kind of order γ, defined in [2] and given by

$$w_{\gamma,b,c}(z)=\sum_{n=0}^{\infty}\frac{(-c)^n}{n!\Gamma(\gamma+n+\frac{b+1}{2})}(\frac{z}{2})^{2n+\gamma} \quad (\ z\in U)$$

where stands for $\Gamma-$ Euler function. Which is the particular solution of the second - order homogeneous differential equation (see [5])

$$z^2w''(z)+bzw'(z)+\left[cz^2-\gamma^2+(1-b)\gamma\right]w(z)=0,$$

where $z\in U$. Now we consider the function $\varphi(z)$ defined by

$$\varphi_{\gamma,b,c}(z)=2^{\gamma}\Gamma(\gamma+\frac{b+1}{2})z^{1-\frac{\gamma}{2}}w\left(\sqrt{z}\right).$$

By using the well-know Pochhammer symbol $(x)_\mu$ defined for $x,\mu\in U$ and in the terms of the Euler gamma function, by

$$(x)_\mu=\frac{\Gamma(x+n)}{\Gamma(x)}\begin{cases}1 & (\mu=0)\\ x(x+1)\dots(x+n-1) & (\mu\in N=\{1,2,3,\dots\})\end{cases}$$

we can express $\varphi_{\gamma,b,c}(z)=\varphi_{k,c}(z)$ as

$$\varphi_{k,c}(z)=z+\sum_{n=1}^{\infty}\frac{\left(\frac{-c}{4}\right)^n}{(k)_n\,(n+1)}z^{n+1} \quad (k:=\gamma+\frac{b+1}{2}\notin z)$$

where $z_0=\{0,-1,-2,\dots\}$.

Now, by using idea of Dziok and Srivastava [1], and we introduced the B_k^c operator as follows:

$$B_k^cf(z)=\varphi(z)*f(z)=z-\sum_{n=2}^{\infty}\frac{(-c)^{n-1}a_nz^n}{4^{n-1}(k)_{n-1}\,(n-1)!}. \tag{1.4}$$

Definition 1. The subclass $Q_{n,c}\,(\gamma,k,\beta)$ of A is defined as the class of functions f such that

$$\left|\frac{1}{\gamma}\left(\frac{z\left[B_k^cf(z)\right]'}{B_k^cf(z)}-1\right)\right|<\beta \tag{1.5}$$

where , $\gamma\in\mathbb{C}\backslash\{0\}$, $0<\beta\le1$, $c\in N_0$ and $z\in U$.

Definition 2. Let the subclass $H_{n,c}(\gamma,k,\beta;\mu)$ of A is defined as the class of functions f such that

$$\left|\frac{1}{\gamma}\left[(1-\mu)\frac{B_k^cf(z)}{z}+\mu(B_k^cf(z))'-1\right]\right|<\beta \tag{1.6}$$

where , $\gamma\in\mathbb{C}\backslash\{0\}$, $0<\beta\le1$, $0\le\mu\le1$, $c\in N_0$ and $z\in U$.

Ref

2. E. Deniz, H. Orhan and H. M. Srivastava, Some sufficient conditions for univalence of certain families of integral operators involving generalized Bessel functions, Taiwanse J. Math. 15(2) (2011), 883-917.

II. Neighborhood for Classes $Q_{n,c}(\gamma, k, \beta)$ and $H_{n,c}(\gamma, k, \beta;\mu)$

In this section, we obtain inclusion relations involving $N_{n,\eta}$ for functions in the classes $Q_{n,c}\,(\gamma, k, \beta)$ and $H_{n,c}(\gamma, k, \beta; \mu)$.

Lemma 1. A function $f(z) \in Q_{n,c}\,(\gamma, k, \beta)$ if and only if

$$\sum_{n=2}^{\infty} \frac{(-c)^{n-1}}{4^{n-1}(k)_{n-1}(n-1)!}\left[n-1+\beta\left|\gamma\right|\right] a_n \leq \beta\left|\gamma\right|. \tag{2.1}$$

Proof. Let $f(z) \in Q_{n,c}\,(\gamma, k, \beta)$. Then , by (1.5) we can write,

$$\operatorname{Re}\left\{\frac{z\left[B_k^c f(z)\right]'}{B_k^c f(z)}-1\right\} > -\beta\left|\gamma\right| \qquad (z \in U). \tag{2.2}$$

Using (1.1) and (1.4), we have,

$$\operatorname{Re}\left\{\frac{-\sum_{n=2}^{\infty}\frac{(-c)^{n-1}}{4^{n-1}(k)_{n-1}(n-1)!}\left[n-1\right]a_n z^n}{z-\sum_{n=2}^{\infty}\frac{(-c)^{n-1}}{4^{n-1}(k)_{n-1}(n-1)!}a_n z^n}\right\} > -\beta\left|\gamma\right|, \quad (z \in U). \tag{2.3}$$

Letting $z \to 1$,through the real values, the inequality (2.3) yields the desired condition (2.1).

Conversely, by applying the hypothesis (2.1) and letting $|z| = 1$,we obtain,

$$\begin{aligned}\left|\frac{z\left[B_k^c f(z)\right]'}{B_k^c f(z)}-1\right| &= \left|\frac{\sum_{n=2}^{\infty}\frac{(-c)^{n-1}}{4^{n-1}(k)_{n-1}(n-1)!}\left[n-1\right]a_n z^n}{z-\sum_{n=2}^{\infty}\frac{(-c)^{n-1}}{4^{n-1}(k)_{n-1}(n-1)!}a_n z^n}\right| \\ &\leq \frac{\sum_{n=2}^{\infty}\frac{(-c)^{n-1}}{4^{n-1}(k)_{n-1}(n-1)!}\left[n-1\right]a_n}{1-\sum_{n=2}^{\infty}\frac{(-c)^{n-1}}{4^{n-1}(k)_{n-1}(n-1)!}a_n} \\ &\leq \beta\left|\gamma\right|.\end{aligned}$$

Hence, by the maximum modulus theorem, we have $f(z) \in Q_{n,c}(\gamma, k, \beta)$, which establishes the required result. □

On similar lines , we have the following Lemma.

Lemma 2. A function $f(z) \in H_{n.c}(\gamma, k, \beta; \mu)$ if and only if

$$\sum_{n=2}^{\infty} \frac{(-c)^{n-1}}{4^{n-1}(k)_{n-1}(n-1)!}\left[1+\mu(n-1)\right] a_n \leq \beta\left|\gamma\right|. \tag{2.4}$$

Theorem 1. Let $c < 0$. if

$$\eta = \frac{2\beta\left|\gamma\right|}{\frac{(-c)}{4(k)}\left[1+\beta\left|\gamma\right|\right]}, \qquad (\ |\gamma| < 1), \tag{2.5}$$

then $Q_{n,c}(\gamma, k, \beta) \subset N_{n,\eta}\,(e)$.

Proof. Let $f(z) \in Q_{n,k}(\gamma, k, \beta)$. By Lemma 1, we have,

$$\frac{(-c)}{4(k)}\left[1+\beta\left|\gamma\right|\right]\sum_{n=2}^{\infty} a_n \leq \beta\left|\gamma\right|,$$

which implies,

$$\sum_{n=2}^{\infty} a_n \leq \frac{\beta|\gamma|}{\frac{(-c)}{4(k)}[1+\beta|\gamma|]}. \tag{2.6}$$

Using (2.1) and (2.6) , we have,

$$\begin{aligned}\frac{(-c)}{4(k)}\sum_{n=2}^{\infty} na_n &\leq \beta|\gamma| + \frac{(-c)}{4(k)}[1-\beta|\gamma|]\sum_{n=2}^{\infty} a_n \\ &\leq \frac{2\beta|\gamma|}{[1+\beta|\gamma|]} = \eta.\end{aligned}$$

That is,

$$\sum_{n=2}^{\infty} na_n \leq \frac{2\beta|\gamma|}{\frac{(-c)}{4(k)}[1+\beta|\gamma|]} = \eta.$$

Thus, by the definition given by (1.3) , $f(z) \in N_{n,\eta}(e)$, which completes the proof. □

Theorem 2. Let $c < 0$. *If*

$$\eta = \frac{2\beta|\gamma|}{(1+\mu)\frac{(-c)}{4(k)}}, \qquad (|\gamma| < 1), \tag{2.7}$$

then $H_{n,c}(\gamma, k, \beta; \mu) \subset N_{n,\delta}(e)$.

Proof. Let $f(z) \in H_{n,c}(\gamma, k, \beta; \mu)$. Then, by Lemma 2, we have,

$$\frac{(-c)}{4(k)}(1+\mu)\sum_{n=2}^{\infty} a_n \leq \beta|\gamma|,$$

which gives the following coefficient inequality,

$$\sum_{n=2}^{\infty} a_n \leq \frac{\beta|\gamma|}{\frac{(-c)}{4(k)}(1+\mu)}. \tag{2.8}$$

Using (2.4) and (2.8) , we also have,

$$\begin{aligned}\mu\frac{(-c)}{4(k)}\sum_{n=2}^{\infty} na_n &\leq \beta|\gamma| + (\mu-1)\frac{(-c)}{4(k)}\sum_{n=2}^{\infty} a_n \\ &\leq \beta|\gamma| + (\mu-1)\frac{\beta|\gamma|}{(1+\mu)}.\end{aligned}$$

That is,

$$\sum_{n=2}^{\infty} na_n \leq \frac{2\beta|\gamma|}{(1+\mu)\frac{(-c)}{4(k)}} = \eta.$$

Thus, by the definition given by (1.3) , $f(z) \in N_{n,\eta}(e)$, which completes the proof. □

Notes

III. Neighborhood for Classes $Q^{\alpha}_{n,c}(\gamma, k, \beta)$ and $H^{\alpha}_{n,c}(\gamma, k, \beta; \mu)$

In this section, we define the subclasses $Q^{\alpha}_{n,c}(\gamma, k, \beta)$ and $H^{\alpha}_{n,c}(\gamma, k, \beta; \mu)$ of A and neighborhoods of these classes are obtained.

For $0 \le \alpha < 1$ and $z \in U$, a function $f(z) \in Q^{\alpha}_{n,c}(\gamma, k, \beta)$ if there exists a function $g(z) \in Q_{n,c}(\gamma, k, \beta)$ such that

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$$\left| \frac{f(z)}{g(z)} - 1 \right| < 1 - \alpha. \tag{3.1}$$

For $0 \le \alpha < 1$ and $z \in U$, a function $f(z) \in H^{\alpha}_{n,c}(\gamma, k, \beta; \mu)$ if there exists a function $g(z) \in H_{n,c}(\gamma, k, \beta; \mu)$ such that the inequality (3.1) holds true.

Theorem 3. If $g(z) \in Q_{n,c}(\gamma, k, \beta)$ and

$$\alpha = 1 - \frac{\eta \frac{(-c)}{4(k)} [1 + \beta |\gamma|]}{2 \left[\frac{(-c)}{4(k)} [1 + \beta |\gamma|] - \beta |\gamma| \right]}, \tag{3.2}$$

then $N_{n,\eta}(g) \subset Q^{\alpha}_{n,c}(\gamma, k, \beta)$.

Proof. Let $f(z) \in N_{n,\eta}(g)$. Then,

$$\sum_{n=2}^{\infty} n |a_n - b_n| \le \eta, \tag{3.3}$$

which yields the coefficient inequality,

$$\sum_{n=2}^{\infty} |a_n - b_n| \le \frac{\eta}{2}, \quad (n \in \mathbb{N}). \tag{3.4}$$

Since $g(z) \in Q_{n,c}(\gamma, k, \beta)$ by (2.6), we have ,

$$\sum_{n=2}^{\infty} b_n \le \frac{\beta |\gamma|}{\frac{(-c)}{4(k)} [1 + \beta |\gamma|]}, \tag{3.5}$$

so that,

$$\begin{aligned} \left| \frac{f(z)}{g(z)} - 1 \right| &< \frac{\sum_{n=2}^{\infty} |a_n - b_n|}{1 - \sum_{n=2}^{\infty} b_n} \\ &\le \frac{\eta}{2} \frac{\frac{(-c)}{4(k)} [1 + \beta |\gamma|]}{\frac{(-c)}{4(k)} [1 + \beta |\gamma|] - \beta |\gamma|} \\ &= 1 - \alpha. \end{aligned}$$

Thus, by definition, $f(z) \in Q^{\alpha}_{n,c}(\gamma, k, \beta)$ for α given by (3.2), which establishes the desired result. □

On similar lines, we can prove the following theorem .

Theorem 4. If $g(z) \in H_{n,c}(\gamma, k, \beta; \mu)$ and

$$\alpha = 1 - \frac{\eta \frac{(-c)}{4(k)} (1 + \mu)}{2 \left[\frac{(-c)}{4(k)} (1 + \mu) - \beta |\gamma| \right]} \tag{3.6}$$

then $N_{n,\delta}(g) \subset H^{\alpha}_{n,c}(\gamma, k, \beta; \mu)$.

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The Myth of Equilibrium and the Myth of Optimization Outside Natural Sciences: A Graduate Lecture

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Abstract- Both the optimization and equilibrium principles turn out to be more akin to common sense than to science. They have been postulated as describing markets, but lack the required empirical underpinning. Optimization is not a magic cure. In order to particularly circumvent some of the technical obstacles for a control problem , it turns out to be practically effective to reduce the system dynamics to a system of ordinary differential equations of considerably higher dimension, Such an approach might replace a theoretical difficulty by a greatly increased computational problem.

Keywords: *integrality condition, hamiltonian system, ad joint differential equation.*

GJSFR-F Classification : *MSC 2010: 74Gxx*

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The Myth of Equilibrium and the Myth of Optimization Outside Natural Sciences: A Graduate Lecture

Amaresh Das

Abstract- Both the optimization and equilibrium principles turn out to be more akin to common sense than to science. They have been postulated as describing markets, but lack the required empirical underpinning. Optimization is not a magic cure. In order to particularly circumvent some of the technical obstacles for a control problem , it turns out to be practically effective to reduce the system dynamics to a system of ordinary differential equations of considerably higher dimension, Such an approach might replace a theoretical difficulty by a greatly increased computational problem.

Keywords: integrality condition, hamiltonian system, ad joint differential equation.

I. Introduction

In his text book Intermediate Microeconomics. Hal Varian (1999) writes that much of the neo-classical theory in economics, finance and management is based on two principles: the optimization principle and the equilibrium principle. In the first people try to choose the best patterns of consumption they can afford. In the second, prices adjust until the amount people demand of something is equal to the amount that is supplied. Both of these principles may soundtrue. Both of these have been postulated as describing markets but lack the required empirical underpinning. This is because we do not know any universal laws of markets that could be used to explain even qualitatively correctly the phenomenon of economic growth, bubbles, recessions, depression, the lopsided distribution of wealth, the collapse of Marxism, and so on. Adam Smith long ago observed society qualitatively, as stated by Beinhocker (2006) and invented the notion of an Invisible Hand that hypothetically should match supply to demand in free markets.

Adam Smith's stabilizing Invisible Hand forms the theoretical basis of the neoclassical equilibrium market model. But, because of the lack of socioeconomic laws of nature and because of the non-uniqueness in explaining statistical data, we have more difficulties in explain equilibrium than in natural sciences. That is why attempts are being made as shown in Das ((2013), Das and Okpechi(2013) in recent days to replace the standard arguments about equilibrium with some empirically based non equilibrium dynamic models. The principle of optimization especially as it is used in management also lacks the dynamics of markets required empirical underpinning.

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II. Equilibrium and Associated Problems

As an example of how easy it is to violate the expectation of stable equilibrium within the confines of optimizing behavior, consider three agents with three assets. The model is defined by assuming individual utilities of the form

$$U_i \ (x) = \min \ (x_1 \ x_{2,}) \tag{1.1}$$

And an initial endowment for agent number 1

$$x_0 = (1, 0, 0) \tag{1.2}$$

The utilities and endowments of the other two agents are cyclic permutations on the above. Agent k has one item of asset k to sell and none of the other two assets. Recall that in neo-classical theory the excess demand equation $dp/dt = D(p.\ t) - S\ (p,\ t) = \varsigma\,(\mathrm{p},\ t)$, where p_k is the price of an asset, D is the demand at that price and S is the corresponding supply and the vector field ς is the excess demand. With demand assumed to be slaved to price in the form $x = D\ (p),$ the phase space is just the n-dimensional space of prices p. That phase space is flat means that global parallelization of flows is possible for integrable systems.

More generally, we could assume that $d\ p\,/\,d\,t = f\ \ (\varsigma\,(p,\ t)$ where f is any vector field with the same qualitative properties as the excess demand. Whatever the choice, we must be satisfied with studying topological classes of excess demand functions. Because the excess demand functions cannot be uniquely specified by the theory, given a model, equilibrium is determined by vanishing excess demand, i.e., by $\varsigma = 0$.[1] Stability of equilibrium, when equilibrium exist at all, is determined by the behavior of solutions displaced slightly from an equilibrium point. Note that dynamics require that we specify $x = D\ (p)$, not $p = f\,(x)$ and likewise for the supply schedule. Given a model of excess demand we can start by analyzing the number and character of equilibria and their stability.[2] Beyond that, one can ask whether is motion is integrable[3]. Typically, the notion for $n > 3$ is non integrable and may be chaotic or even complex, depending upon the topological class of model considered.[4]

We always assume that $x = D\ (p)$. if we relax the assumption and assume that demand is generated by a production function s

$$x = s\,(x, v, t) \tag{1.3}$$

where v denotes a set of unknown control functions. Assume a discounted utility functional

$$A = \int e^{-bt}\, u\ \ (x,\ \ v, t)\ \ dt \tag{1.4}$$

[1] The underlying reason for this constrain, called Walras Law, is just that capital and capital accumulation are not allowed in ne0-classical theory; neo-classical models assume a pure barter economy, so that the cost of the goods demanded can only equal the cost of the cost of the goods offered for sale. This condition simply means that the motion in the n-dimensional price space is confined to the surface of an n-1dimensional sphere. Therefore the motion is at most n -1 dimensional.

[2] The assumption of uniqueness of a single global equilibrium is equivalent to assuming the universality of the action of the Invisible Hand Independently of initial conditions. Here equilibrium would have to be an attractive fixed point with infinite basin of attraction in price space, see Jovanovic and Christopher (2013)

[3] See McCauley ((2004) (1997))

[4] What the motion looks like for n > 3 is a question that cannot be answered *a priori* without specifying a definite class of models, see Neftci (2000)

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where $u\ (x,v,t)$ is the discounted 'utility rate'. We maximize the utility functional A with respect to the set of instruments v. This is a problem in the calculus of variation.

$$\delta A = \int dt\, (\delta\, (e^{-bt}\, (u +$$

$$\bar{p}'\ \delta\, (s(x,\ v,\ t) - x))) = 0 \quad (1.5)$$

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where p_i' are the Lagrange multipliers?

We use the discounted utility rate $u\ (x,\ v,\ t) = e^{-bt}\, u\ (x,\ v,t)$ with $p = e^{-bt}\, p'$ to find

$$h\ (x,\ p,\ t) = \max\ (\ \varpi\ (x,\ v,\ t) +$$

$$\bar{p}\ S(x,\ v,\ and\ t)) \quad (1.6)$$

$$\bar{p}_i\ = -\, d\ h\ /\ dx_i \quad (1.7)$$

$$\dot{x}' = d\ h\ /\ d\ p_i = s\ (x,\ p,\ t) \quad (1.8)$$

which is a Hamiltonian system and h is generally time dependent and since h is dependent on time it is not conserved but integrability occurs if there are n global commuting conservation laws. The integrability condition due to n commuting global conservation laws can be written as

$$p = \nabla U\ \ (x) \quad (1.9)$$

where for bounded motion, the utility $U(x)$ is multivalued. U is just the reduced action

$$A = \int \bar{p}\, dx \quad (1.10)$$

In this scenario, a utility function cannot be chosen by the agent but is determined instead by the dynamics. When satisfied the integrality condition (1.9) eliminates chaotic motion (and complexity) from considerations because there is a global differentiable canonical transformation to a coordinate system where the motion is free particle motion described by n commuting constant speed translations on a flat manifold imbedded in the 2 n dimensional phase space. Conservation laws correspond to continuous symmetries of the Hamiltonian dynamical system. In the economic literature p is called the 'shadow price' but the condition (1.10) is just the neo-classical condition for price.

The generic case is that the motion in phase space is nonintegrable in which case it is typically chaotic. In this case the neo classical condition (1.9) does not exist and both the action

$$A = \int \varpi\, dt \quad (1.13)$$

and the reduced action (1.10) are path dependent functional in agreement with Mirowski (1989). In this case $p = f\,(x)$ does not exist. The main point is that chaotic dynamics,[5] which is more common than simple dynamics, makes it impossible to construct a utility function.

[5] For an excellent elucidation on chaotic dynamics, see Brock and Hommes (2006)

Exercise 1

If we assume that prices are determined by supply and Demand then the simplest model as we can see is

$$\frac{dp}{dt} = \varsigma\,(p.\ t) \tag{1.11}$$

where ς is excess demand. With the assumption that asset prices in liquid markets are random we have

$$D\,p = r(\mathrm{p}.t)dt + \mathrm{d}(\mathrm{p}.\ t)\ \mathrm{d}\ B\,(t) \tag{1.12}$$

where $B\,(t)$ is a Weiner process.[6] Write in a paragraph in the context of our equilibrium analysis, what does it mean? What will happen if financial prices appear to be random even on the shortest trading time scale?

III. Optimization and Decision Making

The current generation of decision makers has been led into thinking that the problem of effective decision making is an optimization problem. To illustrate, as in Casti (1977) one of the many things that can go wrong in optimal decision, assume the system dynamics are given by the scalar linear differential equation $dx/dt = fx + u$ where u is the decision function and f is a constant. Let it be required to choose u so as to minimize

$$æ\int u^2\ (\mathrm{t}) \tag{1.14}$$

Then it is a trivial exercise in the calculus of variations to see that the optimal system trajectory satisfies

$$\mathrm{A}_1\ \sin ft + \mathrm{A}_2\ \cos ft \tag{1.15}$$

where A_1 and A_2 are constants depending upon the initial and boundary conditions. Note, in particular that if $f = 0$, then x^* (t) = constant, while for any f =0 the system oscillates. Thus even a small change of the parameter f away from zero changes the entire qualitative character of the system trajectory. Furthermore, for any value of the system is not asymptotically stable when the so-called optimal decision is used.

In the example noted above, it is easy to see the technical factors that account for the instability of the optimal control but this is not the point. The real point is that if the objective is to choose u so as to class system stability, the criterion above does not reflect all of these factors. In fact, it reflects only on consideration: using as little control as possible. The situation is symptomatic in management, business and governmental environments, namely the optimization criterion imposed to simplify the decision problem account for only a limited number of desired system features and, furthermore, the resulting optimal description is generally a discontinuous function of changes in the problem data.

In essence, the problem is here is the problem of ‘attainability’, that is, the question is: can we get from where we are to where we want to be with the resources available within a specified time horizon.

[6] See DeBondt and Thaler(1985)

Consider a dynamical process described by the set of differential equations

$$dx / dt = f(x, u), \; x(t_0) = x_0 \tag{1.16}$$

where $x(\mathrm{t}) \in R^n$, $u(t)$ εR^m and f is a smooth function of its arguments. We assume that it is desired to transfer the system (1 .14).from x_0 to some state x^* at time T by application of input u belonging to some attainable Ω. Thus we have the attainability problem. Clearly, the solution to the attainability problem will depend upon the interrelationship of the problem data, i. e, the structure of f and Ω, together with the time T and the initial and terminal state x_0 and x^*, respectively[7].

Some problems of importance, however, are not smooth enough to possess gradients. If the non-smoothness is with respect to the state vector, a situation common in problems found in an economic setting, gradients do not exist, the value of the adjoint equation itself is lost, and the dynamic nature of perturbation behavior is destroyed. A new approach is required for problems of this type.[8]

Notes

Exercise 2

Consider a standard problem in management:

$$x^*(\mathrm{t}) = \mathrm{u}(t) - d(t), \mathrm{x}(0) = 0 \tag{1.17}$$

$$\mathrm{J} = \int_0^{t_1} \mathrm{I} - | x(\mathrm{t}) |'' + 1/2 \, u(\mathrm{t}^2) dt$$

$$u(\mathrm{t}) > 0 \tag{1.18}$$

this problem can be interpreted as production scheduling problem in which $x(t)$ represents inventory at hand (with negative inventory representing back orders) ; $d(\mathrm{t})$ is the demand rate at time t (it is assumed known) and $u(\mathrm{t})$ the control, is the production rate. There is a unit cost for storing inventory and a unit cost, for loss of goodwill in carrying back orders. The cost of production is quadratic.

Find a simple trial solution to the problem or simply see if you can get $u(\mathrm{t}) = d(\mathrm{t})$

IV. Conclusion

There are reasons against the notion that equilibrium exists, as is assumed explicitly by the intersecting supply-demand curves. It is shown here how easy it is to violate stability of equilibrium. The neo classical supply-demand curves cannot be expected to exist in the real markets. In order to circumvent some of the technical obstacles for a control problem, it turns out to be practically effective to reduce the system dynamics to a system of ordinary differential equations of considerably higher dimension. Such an approach, however, replaces the theoretical difficulties by a greatly increased computational problem.

[7] It takes little imagination to envision far worse surprises occurring when we try to regulate high-dimensional nonlinear processes unfolding in an uncertain environment – the typical sort of problem encountered in economics and management.

[8] Nonexistence of partial derivatives with respect tox more serious The adjoint equation breaks down and cannot easily be repaired by considering of two-sided derivatives or other simple measures. The fundamental dynamic property of the perturbation equations, at the root of the classical theory, breaks down and must be replaced by a new machinery

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Notes

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On the Dynamics of the Nonlinear Rational Difference Equation $x_{n+1} = Ax_n + Bx_{n-\ell} + \frac{ax_{n-\ell}+bx_{n-k}}{cx_{n-\ell}+dx_{n-k}}$

By E. M. Elabbasy & H. S. Alshawee
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Abstract- In this paper, we study the locally stability, global stability, the periodicity and the boundedness of the positive solutions of the following nonlinear difference equation $x_{n+1} = Ax_n + Bx_{n-\ell} + \frac{ax_{n-\ell}+bx_{n-k}}{cx_{n-\ell}+dx_{n-k}}$ where the coefficients A, B, a, b, c and d are positive real number and k, ℓ are positive integers. The initial conditions $x_{-j}, x_{-j+1}, \ldots, x_0$ are arbitrary positive real numbers such that $j = -\max\{k, \ell\}$.

Keywords: *stability, periodicity, boundedneec, global stable, difference equation.*

GJSFR-F Classification : *MSC 2010: 39A10, 39A11*

Ref

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On the Dynamics of the Nonlinear Rational Difference Equation

$$x_{n+1} = Ax_n + Bx_{n-\ell} + \frac{ax_{n-\ell}+bx_{n-k}}{cx_{n-\ell}+dx_{n-k}}$$

E. M. Elabbasy [α] & H. S. Alshawee [σ]

Abstract- In this paper, we study the locally stability, global stability, the periodicity and the boundedness of the positive solutions of the following nonlinear difference equation $x_{n+1} = Ax_n + Bx_{n-\ell} + \frac{ax_{n-\ell}+bx_{n-k}}{cx_{n-\ell}+dx_{n-k}}$ where the coefficients A, B, a, b, c and d are positive real number and $k,\ \ell$ are positive integers. The initial conditions x_{-j}, $x_{-j+1}, ..., x_0$ are arbitrary positive real numbers such that $j = -\max\{k, \ell\}$.

Keywords and phrases: stability, periodicity, boundedneec, global stable, difference equation.

I. Introduction

In this paper, we aim to achieve a qualitative study of some behavior and solutions in a non-linear differential equations

$$x_{n+1} = Ax_n + Bx_{n-\ell} + \frac{ax_{n-\ell}+bx_{n-k}}{cx_{n-\ell}+dx_{n-k}}, \quad n = 0, 1, 2, ..., \tag{1.1}$$

where the coefficients A, B, a, b, c and $d \in (0, \infty)$ while k and ℓ are positive integers. The initial conditions $x_{-j}, x_{-j+1}, ..., x_0$ are arbitrary positive real numbers such that $j = -\max\{k, \ell\}$. Qualitative analysis of difference equation is not only interesting in its own right, but it can provide insights into their continuous counterpararts, namely, differential equations.

There is a set of nonlinear difference equations, known as the rational difference equations, all of which consists of the ratio of two polynomials in the sequence terms in the same from .there has been many work about the global asympototic of solutions of rational difference equations [2][3][5][6][7][8][11].

There has been much recent investigation and interest in difference equations by several authors such Devault[4] has studied the global stability and the periodic character of solutions of the equation

$$x_{n+1} = \frac{ax_n + bx_{n-1}}{cx_n + dx_{n-2}}.$$

Kalabusic at al [12] investigated the global character of solutions of the nonlinear, third order, rational difference equation,

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$$x_{n+1} = \frac{ax_{n-1} + bx_{n-2}}{cx_{n-1} + dx_{n-2}}.$$

Zayed[14] have studied the recursive sequence

$$x_{n+1} = Ax_n + \frac{ax_n + bx_{n-\ell}}{cx_n + dx_{n-\ell}}$$

In the following we present some basic definitions and known results which will be useful in our study.

Definition 1. Consider a difference equation in the form

$$x_{n+1} = F\left(x_n, x_{n-k}, x_{n-\ell}\right) \tag{1.2}$$

where F is a continuous function, while k and ℓ are positive integers. An equilibrium point $\overline{x}$ of this equation is a point that satisfies the condition $\overline{x} = F\left(\overline{x}, \overline{x}, \overline{x}\right)$. That is, the constant sequence $\{x_n\}$ with $x_n = \overline{x}$ for all $n \geqslant -k \geqslant -\ell$ is a solution of that equation.

Definition 2. Let $\overline{x} \in (0, \infty)$ be an equilibrium point of Eq.(1.2). Then we have

(i) An equilibrium point $\overline{x}$ of Eq. is said to be locally stable if for every $\varepsilon > 0$ there exists $\sigma > 0$ such that, if $x_{-j}, ..., x_{-1}, x_0 \in (0, \infty)$ with $|x_{-j} - \overline{x}| + ... + |x_{-1} - \overline{x}| + |x_0 - \overline{x}| < \sigma$, then $|x_n - \overline{x}| < \varepsilon$ for all $n \geqslant -j$.

(ii) An equilibrium point $\overline{x}$ of Eq.(1.2) is said to be locally asymptotically stable if it is locally stable and there exists $y > 0$ such that, $x_{-j}, ..., x_{-1}, x_0 \in (0, \infty)$ with $|x_{-j} - \overline{x}| + ... + |x_{-1} - \overline{x}| + |x_0 - \overline{x}| < y$, then $\lim_{n\to\infty} x_n = \overline{x}$.

(iii) An equilibrium point $\overline{x}$ of Eq.(1.2) is said to be a global attractor if for every $x_{-j}, ..., x_{-1}, x_0 \in (0, \infty)$ we have $\lim_{n\to\infty} x_n = \overline{x}$.

(iv) An equilibrium point $\overline{x}$ of Eq.(1.2) is said to be globally asymptotically stable if it is locally stable and a global attractor.

(v) An equilibrium point $\overline{x}$ of Eq.(1.2) is said to be unstable if it is not locally stable.

Definition 3. The sequence $\{x_n\}$ is said to be periodic with period p if $x_{n+p} = x_n$ for $n = 0, 1, ...,$

Definition 4. Eq.(1.2) is said to be permanent and bounded if there exists numbers m and M with $0 < m < M < \infty$ such that for any initial conditions $x_{-j}, ..., x_{-1}, x_0 \in (0, \infty)$ there exists a positive integer N which depends on these initial conditions such that $m \leq x_n \leq M$ for all $n \geq N$.

Definition 5. The linearized equation of Eq.(1.2) about the equilibrium point $\overline{x}$ is defined by the equation.

$$y_{n+1} = p_0 y_n + p_1 y_{n-k} + p_2 y_{n-\ell} \tag{1.3}$$

$$p_0 = \frac{\partial f}{\partial x_n}\left(\overline{x}, \overline{x}, \overline{x}\right), p_1 = \frac{\partial f}{\partial x_{n-k}}\left(\overline{x}, \overline{x}, \overline{x}\right), p_2 = \frac{\partial f}{\partial x_{n-\ell}}\left(\overline{x}, \overline{x}, \overline{x}\right)$$

Ref

14. E.M.E. Zayed, M.A. El-Moneam, On the rational recursive sequence $x_{n+1} = Ax_n + (ax_n + bx_{n-k}) / (cx_n + dx_{n-k})$, Acta Appl. Math. 111 (2010) 287.301.

The characteristic equation associated with Eq. (1.3) is

$$p(\lambda) = \lambda^{\ell+1} - p_o\lambda^{\ell} - p_1\lambda^{\ell-k} - p_2 = 0 \qquad (1.4)$$

Theorem 1.1. [9]*Assume that* p_0, p_1 *and* $p_2 \in R$. *Then*

$$|p_0| + |p_1| + |p_2| < 1 \qquad (1.5)$$

is a sufficient condition for the locally stability of Eq.(1.2).

II. Local Stable of the Equilibrium Point

The equilibrium point of Eq. (1.1) is the positive solution of the equation

$$\overline{x} = A\overline{x} + B\overline{x} + \frac{a\overline{x} + b\overline{x}}{c\overline{x} + d\overline{x}}$$

which gives

$$\overline{x} = \frac{a+b}{(c+d)(1-A-B)}, \qquad A+B<1 \qquad (2.1)$$

Now let $f : (0,\infty)^3 \to (0,\infty)$ be a function defined by

$$f(u,v,w) = Au + Bv + \frac{av+bw}{cv+dw}.$$

Then, we have

$$\frac{\partial f}{\partial u} = A, \qquad (2.2)$$

$$\frac{\partial f}{\partial v} = B + \frac{(ad-bc)\,w}{(cv+dw)^2}, \qquad (2.3)$$

and

$$\frac{\partial f}{\partial w} = \frac{(bc-ad)\,v}{(cv+dw)^2}. \qquad (2.4)$$

In the following theorem we study the local stablity charaeter of the sulotion of (1.1).

Theorem 2.1. *If*

$$|B(c+d)(a+b) + (ad-bc)(1-A-B)| + |(bc-ad)(1-A-B)| < (1-A)(c+d)(a+b), \qquad (2.5)$$

then the equilibrium point $\overline{x} = (a+b)/((c+d)(1-A-B))$ *of eq (1.1) is local stable.*

Proof. From (2.2)-(2.4), we get

$$\frac{\partial f}{\partial u}(\overline{x},\overline{x},\overline{x}) = A = P_0$$

$$\frac{\partial f}{\partial v}(\overline{x},\overline{x},\overline{x}) = B + \frac{(ad-bc)(1-A-B)}{(a+b)(c+d)} = P_1$$

and,

$$\frac{\partial f}{\partial w}(\overline{x},\overline{x},\overline{x}) = \frac{(bc-ad)(1-A-B)}{(c+d)(a+b)} = P_2$$

Thus, the linearized equation associated with Eq. (1.1) about $\overline{x}$, is

$$y_{n+1} = p_0 y_n + p_1 y_{n-k} + p_2 y_{n-\ell}$$

It follows by Theorem References [9] that Eq. (1.1) is localy stable if

$$|A| + \left| B + \frac{(ad - bc)(1 - A - B)}{(a+b)(c+d)} \right| + \left| \frac{(bc - ad)(1 - A - B)}{(c+d)(a+b)} \right| < 1$$

By maltiplicating the last inequality by $(a+b)(c+d)$, we obtain

$$\begin{aligned} A(a+b)(c+d) + |B(a+b)(c+d) + (ad-bc)(1-A-B)| \\ + |(bc-ad)(1-A-B)| < (a+b)(c+d) \end{aligned}$$

which is true if

$$\begin{aligned} |B(c+d)(a+b) + (ad-bc)(1-A-B)| \\ + |(bc-ad)(1-A-B)| < |(1-A)(c+d)(a+b)|, \end{aligned}$$

The proof is completed □

Remark 2.1. *If $ad > bc$, then the condition (2.5) reduce to*

$$2(ad - bc) < (a+b)(c+d).$$

Example 2.1. *Fig. 1, shows that Eq. (1.1) has Local stable solutions if $a = \ell = 2$, $b = c = d = k = 1$, $A = 1/3$, $B = 1/6$, $x_0 = 1.5$, $x_{-1} = 5.4$, $x_{-2} = 1.3$, $x_{-3} = 0.0002$, $\overline{x} = 3$.*

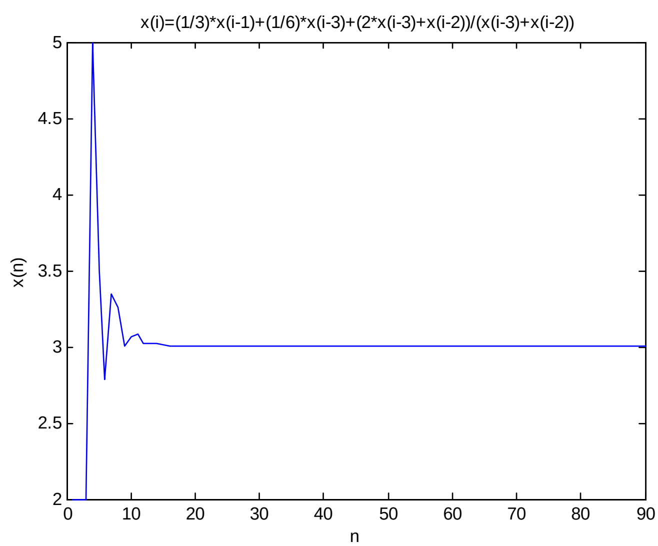

Fig. : 1

Also, if $ad < bc$ then the condition (2.5) reduce to $B(c+d)(a+b) > (bc - ad)(1 - A - B)$.

Example 2.2. *Fig. 2, shows that Eq. (1.1) has Local stable solutions if $a = b = d = k = 1$, $c = \ell = 2$, $A = 1/3$, $B = 1/4$, $\overline{x} = 1.587$, $x_0 = 1.5$, $x_{-1} = 5.4$, $x_{-2} = 1.3$, $x_{-3} = 0.0002$.*

Notes

9. Gibbons, C.H., Kulenovic, M.R.S., Ladas, G. and Voulov, H.D., On the trichotomy character of $x_{n+1} = (\alpha + \beta x_n + \gamma x_{n-1}) / (A + x_n)$, J. Difference Eqs. and Appl., 8 (2002) 75-92.

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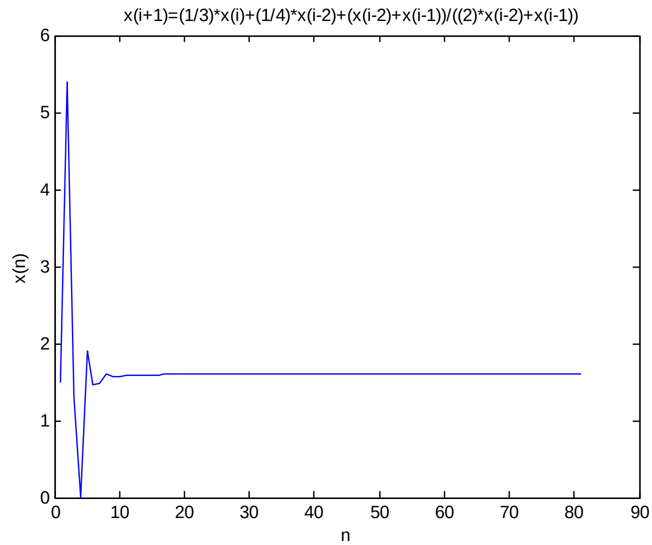

Fig. : 2

III. Periodic Solutions of Eq.(1.1)

In this part of the research we are studying the possibility of the existence of periodic solutions to the eq. (1.1).

Theorem 3.1. In the all following cases, Equation (1.1) has no positive prime period-two solutions:

(1) If k and ℓ are bath even positive numbers.
(2) If k and ℓ are bath odd positive numbers.

Proof. Case(1) Suppose that there exists a prime period-two solution

$$p, q, p, q, p, q, p$$

if k and ℓ are both even positive numbers, then we get $x_n = x_{n-k} = x_{n-\ell} = q, x_{n+1} = p$.

It follows from Eq. (1.1) that

$$p = Aq + Bq + \frac{a+b}{c+d},$$

and

$$q = Ap + Bp + \frac{a+b}{c+d}.$$

We get,

$$\begin{aligned} p - q &= A(q-p) + B(q-p) \\ (p-q)(1 + A + B) &= 0 \end{aligned}$$

since $1 + A + B \neq 0$ then, $p = q$

Similarly, we can prove other cases which is omitted here for convenience. Hence, the proof is completed. □

Theorem 3.2. *Eq. (1.1) has prime period two solution If*

$(i)k-even, \ell-odd$ and $(a-b)(d-Ac-Bc)+(B-1)(3b+a)> Ad(b-3a)$. (3.1)

$(ii)\ell-even, k-odd$ and $(A+B)[d(a-b)-c(a+3b)] > a(c+3d)$. (3.2)

Proof. For case (1) suppose that there exists a prime period-two solution

$$..., p, q, p, q, p, q, , ...$$

of (1.1). We will prove that condition (3.1) holds.

We see from (1.1) that if k even and ℓ odd, then $x_n = x_{n-k} = q$, $x_{n+1} = x_{n-\ell} = p$

$$p = Aq + Bp + \frac{ap+bq}{cp+dq}$$

and

$$q = Ap + Bq + \frac{aq+bq}{cq+dp}$$

Thus, we have

$$cp^2 + dpq = Acpq + Adq^2 + Bcp^2 + Bdpq + ap + bq \ , \tag{3.3}$$

and

$$cq^2 + dpq = Acpq + Adp^2 + Bcq^2 + Bdpp + aq + bq \tag{3.4}$$

By subtracting (3.3) and (3.4), we have

$$c\left(p^2 - q^2\right) = Ad\left(q^2 - p^2\right) + Bc\left(p^2 - q^2\right) + a\left(p - q\right) + b\left(q - p\right)$$

then

$$(p+q) = \frac{a-b}{c+Ad-Bc} \tag{3.5}$$

By Combining (3.3) and (3.4), we have

$$\begin{aligned} c\left(p^2+q^2\right)+2dpq &= 2Acpq + Ad\left(p^2+q^2\right) + Bc\left(p^2+q^2\right) \\ &\quad + 2Bdpq + a\left(p+q\right) + b\left(p+q\right) \end{aligned}$$

and,

$$2\left[d - Ac - Bd\right]pq = \left[-c + Ad + Bc\right]\left(p^2+q^2\right) + (a+b)(p+q) \tag{3.6}$$

then,

$$p^2 + q^2 = (p+q)^2 - 2pq \tag{3.7}$$

Form (3.5), (3.6) and (3.7), we get

Notes

$$pq = \frac{[-c+Ad+Bc](a-b)^2}{2(d-c)(1+A-B)(c+Ad-Bc)^2} + \frac{a^2-b^2}{2(c+Ad-Bc)(d-c)(1+A-B)} \quad (3.8)$$

Notes

Now it is clear from (3.5) and (3.8) that p and q are both two positive distince roots of the quadratic equation

$$u^2 + (p+q)u + pq = 0, \quad (3.9)$$

Thus, we obtain

$$\frac{(a-b)^2}{(c+Ad-Bc)^2} - \frac{2[-c+Ad+Bc](a-b)^2}{(d-c)(1+A-B)(c+Ad-Bc)^2} - \frac{2(a^2-b^2)}{(c+Ad-Bc)(d-c)(1+A-B)} > 0$$

which is equivalent to

$$(a-b)(d-Ac-Bc) + (B-1)(3b+a) > Ad(b-3a)$$

Hence, the proof is completed.

For case (2) suppose that there exists a prime period-two solution

$$p, q, p, q, p, q, p, q, p, q$$

of (1.1). We will prove that condition (2.3) holds.

We see from (1.1) that if k even and ℓ odd $x_n = x_{n-\ell} = q, x_{n+1} = x_{n-k} = p$

$$p = Aq + Bq + \frac{aq+bp}{cq+dp}$$

$$cpq + dp^2 = Acq^2 + Adpq + Bcq^2 + Bdpq + aq + bp \quad (3.10)$$

and,

$$q = Ap + Bp + \frac{ap+bq}{cp+dq}$$

$$cpq + dq^2 = Acp^2 + Adqq + Bcp^2 + Bdpq + ap + bq \quad (3.11)$$

By subtracting (3.10) and (3.11), we have

$$d\left(p^2-q^2\right) = Ac\left(q^2-p^2\right) + Bc\left(q^2-p^2\right) + a(q-p) + b(p-q)$$

then

$$(p+q)(d+Ac+Bc) = -a+b$$

$$(p+q) = \frac{-a+b}{d+Ac+Bc}$$

By Combining (3.3) and (3.4), we have

$$2cpq+d\left(p^2+q^2\right) = Ac\left(p^2+q^2\right)+2Adpq+Bc\left(p^2+q^2\right) + Bdpq+a\left(p+q\right)+b\left(p+q\right)$$

then

$$2\left[c-Ad-Bd\right]pq=\left[Ac+Bc-d\right]\left(p^2+q^2\right)\left(a+b\right)\left(p+q\right)$$

We have

$$p^2+q^2=\left(p^2+q^2\right)-2pq$$

$$2\left[c-Ad-Bd\right]pq = \left[Ac+Bc-d\right]\left[\frac{\left(-a+b\right)^2}{\left(d+Ac+Bc\right)^2}-2pq\right] + \left(a+b\right)\left[\frac{-a+b}{d+Ac+Bc}\right]$$

so that,

$$pq = \frac{\left[Ac+Bc-d\right]\left(-a+b\right)^2}{\left(c-d\right)\left[1+A+B\right]\left(d+Ac+Bc\right)^2} + \frac{b^2-a^2}{\left(d+Ac+Bc\right)\left(c-d\right)\left[1+A+B\right]}$$

We hava,

$$u^2+\left(p+q\right)u+pq=0 \quad \text{and} \quad \left(p+q\right)^2-4pq>0$$

also

$$\frac{\left(b-a\right)^2}{\left(d+Ac+Bc\right)^2}-\frac{2\left[Ac+Bc-d\right]\left(b-a\right)^2}{\left(c-d\right)\left[1+A+B\right]\left(d+Ac+Bc\right)^2} - \frac{2\left(b^2-a^2\right)}{\left(d+Ac+Bc\right)\left(c-d\right)\left[1+A+B\right]}>0$$

so we get,

$$\left(b-a\right)^2\left(c-d\right)\left[1+A+B\right]-2\left[Ac+Bc-d\right]\left(b-a\right)^2 -2\left(a^2-b^2\right)\left(d+Ac+Bc\right)>0$$

then,

$$\left(A+B\right)\left[d\left(a-b\right)-c\left(a+3b\right)\right]>a\left(c+3d\right)$$

Hence, the proof is completed. □

Example 3.1. Fig.3.1, shows that Eq. (1.1) has prime period two solutions if $k-$*even*, $\ell-$*odd*, $a=c=1$, $b=0.0003$, $d=3$, $A=B=(1/16)$, $x_0=0.1$, $x_{-1}=0.5$, $x_{-2}=0.3$, $x_{-3}=0.3$. (*see Table 3.1*)

Notes

Notes

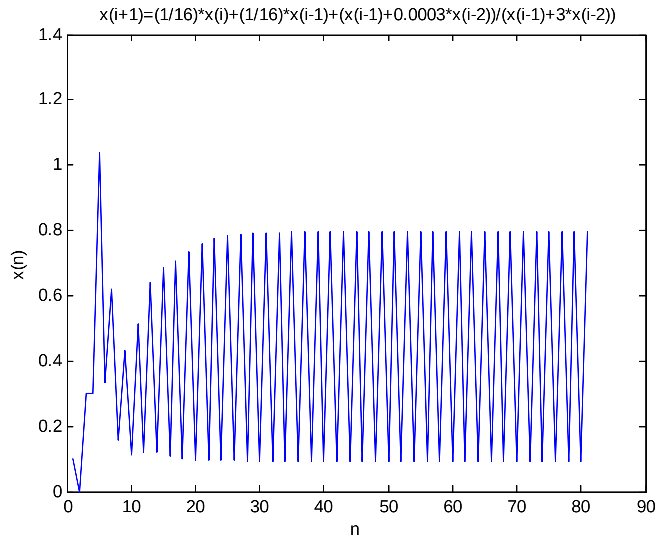

Fig. : 3.1

Table (3.1)

n	$x(n)$	n	$x(n)$	n	$x(n)$	n	$x(n)$
1	0.1000	16	0.1095	31	0.7928	46	0.0932
2	0	17	0.7054	32	0.0934	47	0.7953
3	0.3000	18	0.1015	33	0.7939	48	0.0932
4	0.3000	19	0.7328	34	0.0934	49	0.7954
5	1.0375	20	0.0980	35	0.7945	50	0.0932
6	0.3337	21	0.7584	36	0.0933	51	0.7954
7	0.6212	22	0.0963	37	0.7949	52	0.0932
8	0.1566	23	0.7740	38	0.0933	53	0.7954
9	0.4316	24	0.0951	39	0.7951	54	0.0932
10	0.1144	25	0.7825	40	0.0933	55	0.7954
11	0.5130	26	0.0943	41	0.7952	56	0.0932
12	0.1205	27	0.7877	42	0.0933	57	0.7954
13	0.6389	28	0.0938	43	0.7953	58	0.0932
14	0.1201	29	0.7909	44	0.0932	59	0.7954
15	0.6862	30	0.0936	45	0.7953	60	0.0932

IV. Global Stability

Theorem 4.1. If $1 - A - B \neq 0$ and $a \neq b$, then the equilibrium point $\overline{x}$ of Eq. (1.1) is global attractor.

Proof. We consider the following function

$$f(u, v, w) = Au + Bv + \frac{av + bw}{cv + dw}$$

where $ad > bc$

f non-decreasing for u, v and non-increasing for w

Let $m = f(m, m, M)$ and $M = f(M, M, m)$

$$f(u, v, w) = Au + Bv + \frac{av + bw}{cv + dw}$$

we have,

$$cm^2 + dmM = Acm^2 + AdmM + Bcm^2 + BdmM + am + bM \quad (4.1)$$

and,from (4.1)

$$cm^2 + dmM = AcM^2 + AdmM + BcM^2 + BdmM + aM + bm \quad (4.2)$$

By subtracting (3.10) and (3.11), we have

$$c\left(m^2 - M^2\right) = Ac\left(m^2 - M^2\right) + Bc\left(m^2 - M^2\right) + a\left(m - M\right) + b(M - m$$

so,

$$\left[c\left(1 - A - B\right)\left(m + M\right) + \left(b - a\right)\right]\left(m - M\right) = 0$$

then

$$\begin{aligned} 1 - A - B &\neq 0 \qquad \text{and} \qquad a \neq b \\ M &= m \end{aligned}$$

Hence, the proof is completed. □

V. Bounded

Theorem 5.1. Let $\{x_n\}_{n=-\max\{k,l\}}^{\infty}$ *be asolution of Eq (1.1), then the following statements are true :-*

(1) Assume that $a > c$ and let for some $N \geq 0, x_{N-l+1}, ..., x_{N-1}, x_N \in \left[\frac{a}{c}, 1\right]$ are valid, then we have

$$(A+B)\frac{a}{c} + \frac{(a+b)\frac{a}{c}}{c+d} \leq x_n \leq (A+B) + \frac{a+b}{(c+d)\frac{a}{c}}$$

(2) Assume that $a > c$ and for some $N \geq 0, x_{N-l+1}, ..., x_N \in \left[1, \frac{a}{c}\right]$ are valid, Then we have

$$(A+B) + \frac{a+b}{(c+d)\frac{a}{c}} \leq x_n \leq (A+B)\frac{a}{c} + \frac{(a+b)\frac{a}{c}}{c+d}$$

Proof. (1) if $a < c$ and
Then $x_{N-l+1}, ..., x_{N-1}, x_N \in \left[\frac{a}{c}, 1\right]$

$$x_{n+1} = Ax_n + Bx_{n-l} + \frac{ax_{n-l} + bx_{n-k}}{cx_{n-l} + dx_{n-k}}$$

then,

$$\begin{aligned} &\leq A + B + \frac{a+b}{(c+d)\frac{a}{c}} \\ &\leq A + B + \frac{a+b}{(c+d)\frac{a}{c}} \end{aligned}$$

and

$$x_{n+1} = Ax_n + Bx_{n-l} + \frac{ax_{n-l} + bx_{n-k}}{cx_{n-l} + dx_{n-k}}$$

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then,

$$\geq (A+B)\frac{a}{c}+\frac{(a+b)\frac{a}{c}}{c+d}$$

$$\geq (A+B)\frac{a}{c}+\frac{(a+b)\frac{a}{c}}{c+d}$$

Then

$$(A+B)+\frac{a+b}{(c+d)\frac{a}{c}}\leq x_n\leq (A+B)\frac{a}{c}+\frac{(a+b)\frac{a}{c}}{c+d}$$

Similarly, we can prove other cases which is omitted here for convenience. Hence, the proof is completed. □

Notes

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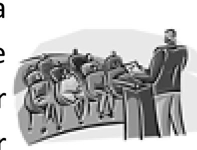

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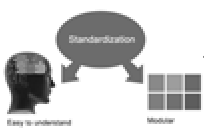

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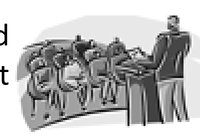

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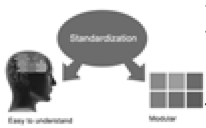

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Search engines for most searches, use Boolean searching, which is somewhat different from Internet searches. The Boolean search uses "operators," words (and, or, not, and near) that enable you to expand or narrow your affords. Tips for research paper while preparing research paper are very helpful guideline of research paper.

Choice of key words is first tool of tips to write research paper. Research paper writing is an art.A few tips for deciding as strategically as possible about keyword search:

- One should start brainstorming lists of possible keywords before even begin searching. Think about the most important concepts related to research work. Ask, "What words would a source have to include to be truly valuable in research paper?" Then consider synonyms for the important words.
- It may take the discovery of only one relevant paper to let steer in the right keyword direction because in most databases, the keywords under which a research paper is abstracted are listed with the paper.
- One should avoid outdated words.

Keywords are the key that opens a door to research work sources. Keyword searching is an art in which researcher's skills are bound to improve with experience and time.

Numerical Methods: Numerical methods used should be clear and, where appropriate, supported by references.

Acknowledgements: Please make these as concise as possible.

References

References follow the Harvard scheme of referencing. References in the text should cite the authors' names followed by the time of their publication, unless there are three or more authors when simply the first author's name is quoted followed by et al. unpublished work has to only be cited where necessary, and only in the text. Copies of references in press in other journals have to be supplied with submitted typescripts. It is necessary that all citations and references be carefully checked before submission, as mistakes or omissions will cause delays.

References to information on the World Wide Web can be given, but only if the information is available without charge to readers on an official site. Wikipedia and Similar websites are not allowed where anyone can change the information. Authors will be asked to make available electronic copies of the cited information for inclusion on the Global Journals Inc. (US) homepage at the judgment of the Editorial Board.

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Tables: Tables should be few in number, cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g. Table 4, a self-explanatory caption and be on a separate sheet. Vertical lines should not be used.

Figures: Figures are supposed to be submitted as separate files. Always take in a citation in the text for each figure using Arabic numbers, e.g. Fig. 4. Artwork must be submitted online in electronic form by e-mailing them.

Preparation of Electronic Figures for Publication

Even though low quality images are sufficient for review purposes, print publication requires high quality images to prevent the final product being blurred or fuzzy. Submit (or e-mail) EPS (line art) or TIFF (halftone/photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Do not use pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings) in relation to the imitation size. Please give the data for figures in black and white or submit a Color Work Agreement Form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

For scanned images, the scanning resolution (at final image size) ought to be as follows to ensure good reproduction: line art: >650 dpi; halftones (including gel photographs) : >350 dpi; figures containing both halftone and line images: >650 dpi.

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Before start writing a good quality Computer Science Research Paper, let us first understand what is Computer Science Research Paper? So, Computer Science Research Paper is the paper which is written by professionals or scientists who are associated to Computer Science and Information Technology, or doing research study in these areas. If you are novel to this field then you can consult about this field from your supervisor or guide.

TECHNIQUES FOR WRITING A GOOD QUALITY RESEARCH PAPER:

1. Choosing the topic: In most cases, the topic is searched by the interest of author but it can be also suggested by the guides. You can have several topics and then you can judge that in which topic or subject you are finding yourself most comfortable. This can be done by asking several questions to yourself, like Will I be able to carry our search in this area? Will I find all necessary recourses to accomplish the search? Will I be able to find all information in this field area? If the answer of these types of questions will be "Yes" then you can choose that topic. In most of the cases, you may have to conduct the surveys and have to visit several places because this field is related to Computer Science and Information Technology. Also, you may have to do a lot of work to find all rise and falls regarding the various data of that subject. Sometimes, detailed information plays a vital role, instead of short information.

2. Evaluators are human: First thing to remember that evaluators are also human being. They are not only meant for rejecting a paper. They are here to evaluate your paper. So, present your Best.

3. Think Like Evaluators: If you are in a confusion or getting demotivated that your paper will be accepted by evaluators or not, then think and try to evaluate your paper like an Evaluator. Try to understand that what an evaluator wants in your research paper and automatically you will have your answer.

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5. Ask your Guides: If you are having any difficulty in your research, then do not hesitate to share your difficulty to your guide (if you have any). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work then ask the supervisor to help you with the alternative. He might also provide you the list of essential readings.

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11. Revise what you wrote: When you write anything, always read it, summarize it and then finalize it.

12. Make all efforts: Make all efforts to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in introduction, that what is the need of a particular research paper. Polish your work by good skill of writing and always give an evaluator, what he wants.

13. Have backups: When you are going to do any important thing like making research paper, you should always have backup copies of it either in your computer or in paper. This will help you to not to lose any of your important.

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15. Use of direct quotes: When you do research relevant to literature, history or current affairs then use of quotes become essential but if study is relevant to science then use of quotes is not preferable.

16. Use proper verb tense: Use proper verb tenses in your paper. Use past tense, to present those events that happened. Use present tense to indicate events that are going on. Use future tense to indicate future happening events. Use of improper and wrong tenses will confuse the evaluator. Avoid the sentences that are incomplete.

17. Never use online paper: If you are getting any paper on Internet, then never use it as your research paper because it might be possible that evaluator has already seen it or maybe it is outdated version.

18. Pick a good study spot: To do your research studies always try to pick a spot, which is quiet. Every spot is not for studies. Spot that suits you choose it and proceed further.

19. Know what you know: Always try to know, what you know by making objectives. Else, you will be confused and cannot achieve your target.

20. Use good quality grammar: Always use a good quality grammar and use words that will throw positive impact on evaluator. Use of good quality grammar does not mean to use tough words, that for each word the evaluator has to go through dictionary. Do not start sentence with a conjunction. Do not fragment sentences. Eliminate one-word sentences. Ignore passive voice. Do not ever use a big word when a diminutive one would suffice. Verbs have to be in agreement with their subjects. Prepositions are not expressions to finish sentences with. It is incorrect to ever divide an infinitive. Avoid clichés like the disease. Also, always shun irritating alliteration. Use language that is simple and straight forward. put together a neat summary.

21. Arrangement of information: Each section of the main body should start with an opening sentence and there should be a changeover at the end of the section. Give only valid and powerful arguments to your topic. You may also maintain your arguments with records.

22. Never start in last minute: Always start at right time and give enough time to research work. Leaving everything to the last minute will degrade your paper and spoil your work.

23. Multitasking in research is not good: Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. Think technically: Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page
- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

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Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.

- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
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- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

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- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
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- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

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- Not at all, take in raw data or intermediate calculations in a research manuscript.
- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

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- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

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- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

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	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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