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Markov Switching Heteroscedasticity Model of Stock Return: A Test

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Markov Switching Heteroscedasticity Model of Stock Return: A Test

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I. INTRODUCTION

Although the ARCH process controls the short-run dynamics of stock return, the long-run dynamics are controlled by regime shifts in unconditional variance, while an unobserved Markov switching process drives the regime changes. Hamilton and Susmel (1994) propose a switching ARCH model in which they allow the parameters of the ARCH process to come from one set of several different regimes.¹ Regime switching models can match the tendency of financial markets to often change their behavior abruptly and the phenomenon that the new behavior of financial variables often persists for several periods after such a change. While the regimes captured by regime switching models are identified by an econometric procedure, they often correspond to different periods in regulation, policy, and other secular changes.^{2, 3} Suppose the variable u_t is governed by

$$u_t = \sigma_t v_t$$

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¹ Contagion plays a crucial role in the short-term transmission of a currency crisis. Its effects rely primarily on liquidity effects experienced by international investors. Thus, the drop in asset values after the Russian crisis represented a capital loss for investors, with the ensuing liquidity problems being countered by a reallocation of their respective portfolios

² For example, interest rate behavior markedly changed from 1979 through 1982, during which the Federal Reserve changed its operating procedure to targeting monetary aggregates. Other regimes identified in interest rates correspond to the tenure of different Federal Reserve Chairs.

³ The notion of regimes is closely linked to the familiar concept of good and bad states or states with low versus high risk, but surprising and somewhat counterintuitive results can be obtained from equilibrium asset pricing models with regime changes. Conventional linear asset pricing models imply a positive and monotonic risk-return relation. In contrast, changes between discrete regimes with different consumption growth rates can lead to increasing, decreasing, flat or non-monotonic risk-return relations as shown by, e.g., Backus and Gregory (1993), Whitelaw (2000), Ang and Liu (2007), and Rossi and Timmermann (2011).

where $\{ v_t \}$ is an i. i. d sequence with zero mean and unit variance. The conditional variance of u_t is specified to be a function of its past realization

$$\sigma^2 = g (u_{t-1}, u_{t-2} \dots)$$

Often it is assumed that $u_t \rightarrow (0, 1)$ and that $g (\cdot)$ depends linearly on the past squared realization of u .

$$\sigma^2 = \sum_{i=1}^q a_i u_{t-i}^2 + \sum_{i=1}^p b_i \sigma_{t-i}^2$$

This is a Gaussian GARCH (p, q) specification introduced by Belterstev (1986). When $p = 0$ it becomes ARCH (q) specification of Engle (1982). The popular approach to modelling stock volatility is the autoregressive conditional heteroscedasticity (ARCH) specification introduced by these authors. These authors argue that the variance ratio test that is often used for analyzing mean reversion⁴ may need to be modified to take into account the changes in variance due to changes in regimes⁵. The cause of the debate lies in the fact that testing for mean reversion is inherently difficult due to a lack of historical data on stock prices. Accurate estimation of the degree of long-run mean reversion requires very long stock price series, which are not available. For example, if stock prices were to revert back to their fundamental value every twenty years, one would need at least 1,000 to 2,000 yearly observations to obtain reliable estimations. Moreover, the likely structural breaks during long sample periods further complicate statistical analysis of mean reversion (Spierdijk et al. 2012). These methodological

⁴ After the seminal studies by Summers (1986), Poterba & Summers (1988), an ongoing debate has emerged in the literature as to whether stock prices and stock returns are mean-reverting or not. The substantial amount of recent publications in this field (Goyal & Welch 2008, Spierdijk et al. 2012) illustrates that the mean-reverting behavior of stocks is still an important issue

⁵ The standard sensitivity analysis shows that the choice of the variance ratio may have substantial impact on investment decisions. If the variance ratio is high – meaning that stock prices are strongly mean-reverting – stocks become relatively less risky in the long run, making it optimal to invest a relatively large share of wealth in stocks. However, if the true variance ratio is lower than the assumed value, the perceived risk exposure is lower than the actual risk exposure. Hence, too much wealth is allocated to stocks, resulting in a non-optimal overexposure to risk.

difficulties explain why mean reversion is a controversial issue in the economic literature.

Analyses suggest that the speed at which stocks revert to their fundamental value is faster in periods of high economic uncertainty, caused by major economic and/or political events. The highest mean reversion speed is found for the period including the Great Depression and the start of World War II. Furthermore, the early years of the Cold War and the period containing the Oil Crisis of 1973, the Energy Crisis of 1979 and Black Monday in 1987 are also characterized by relatively fast mean reversion.

II. THE MODEL

We will, to begin with, assume that the return series is drawn from a mixture of normal distributions as in Kim and Nelson (1998). These authors have shown that the Markov switching heteroscedasticity model of stock return is a good approximation of the underlying data generating process. This leads us to formulate the return series as follows:

$$r_t = \rho m_t + x_t$$

$$\rho m_t = \mu + (Q_0 + Q_1 \omega_{1,t}) \Psi_t$$

$$x_t = \phi x_{t-1} + (h_0 + h_1 \omega_{2,t}) \zeta_t$$

where $\zeta_t \rightarrow N(0, 1)$

In this model we use x_t to represent the temporary part of the return and not the prices directly. We include ϕ simply reflecting the fact that the temporary component of the return could be autocorrelated. $\omega_{1,t}$ and $\omega_{2,t}$ are unobserved state variables that evolve independently as two state Markov processes. These state variables determine the underlying regime at any given time. Their associated transitional probability matrices govern the evolution of these state variables. We define the transitional probability of the Markov process as follows

$$\begin{bmatrix} p_{11} & 1-p_{22} \\ 1-p_{11} & p_{12} \end{bmatrix}$$

The parameters h_1 and Q_1 help us identify any shift in variance during periods of uncertainty. The estimation of this model would allow us to comment on the time series behavior of the return volatility and how this is influenced by the switching probability of the transitory component.

The two Markov switching variables are independent of each other and the respective transition probabilities are defined as

$$\text{prob}(\omega_{1t} = 0 \mid \omega_{1t-1} = 0) = \rho_{00},$$

$$\text{prob}(\omega_{1t} = 1 \mid \omega_{1t-1} = 1) = \rho_{01}$$

$$\text{prob}(\omega_{2t} = 0 \mid \omega_{2t-1} = 0) = q_{00},$$

$$\text{prob}(\omega_{2t} = 1 \mid \omega_{2t-1} = 1) = q_{11}$$

In order to estimate such a model that involves unobserved components and is subject to Markov switching shocks, we use the procedure used by Kim and Nelson (1999). This involves generating a probability weighted likelihood function and a recursive algorithm to update the probabilities as new observations become available. This has been written with computer programming in mind. The parameters to be estimated are, therefore,

$$[\rho_{11}, \rho_{00}, \omega_0, \omega_1, \mu, q_{11}, q_{00}, h_0, h_1, \phi]$$

III. DATA

The stock price index is obtained from the Morgan Stanley Capital International Index, MSCI's All Country World Index (ACWI) is the industry's accepted gauge of global stock market activity. Composed of over 2,400 constituents, it provides a seamless, modern and fully integrated view across all sources of equity returns in 46 developed and emerging markets. The company has used eight factors in developing its indexes: momentum, volatility, value, size, growth, size nonlinearity, liquidity and financial leverage.

The rate of return on stocks for India is calculated as $x_t = (P_t - P_{t-1}) \times 100 / P_{t-1}$ where P_t is the stock price index at time t . The rates of return on stocks are obtained for the period from January 1980 to April 2010. Table 1 summarizes statistics on the rate of return in India, including descriptive statistics on the mean, standard deviation, skewness, kurtosis and the P -value of the Jarque – Bera test statistic (JB test) for testing the normality of the series. Under the null hypothesis, the Jarque- Bera statistic has a chi-square distribution with two degrees of freedom. When the required probability for the Jarque – Bera statistic is small, the null hypothesis of a normal distribution is rejected. The mean and standard deviation is quite high and the null hypothesis of normal distribution is rejected at the 5% significance level

Table 1.1: Summary Statistics

Mean(%)	Std.Dev	Skewness	Kurtosis	JB Test
1.786	7.453	2.312	9.654	.0000

Note: the hypothesis of normal distribution is rejected at the 5% level of significance if the P value for the JB test is less than .01

IV. RESULTS

Table 1.2 shows the parameter estimates of the Markov switching heteroscedasticity model for the

sample for our given time ⁶. The results are computed using the algorithm used by Kim and Nelson (1998). The initial values for the filter are obtained from the observations on January 1980 ending through December 1980.

Table 1. 2: Permanent and Transitory Components of Equity Return (Markov Switching Heteroscedasticity Model)

P_{11}	0.9898* (0.0004)
P_{00}	0.8767* (0.1034)
ω_0	1.5236* (0.2341)
ω_1	1.3458* (0.6451)
μ	1.5621 (0.6945)
q_{11}	0.7658* (0.1342)
q_{00}	0.9868* (0.0666)
h_0	0.0423 (0.0541)
h_1	6.7832* (3,4571)
ϕ	-0.3214 (0.3461)

Note: Standard errors given in parenthesis. Significance at the 5% level is indicated by*

The estimates of the transition probability P_{11} (high variance state of the permanent component) and the probability P_{00} (low variance state of the permanent component) are both highly significant for India. The low variance state estimate ω_0 appears to be statistically significant. In contrast the additional variance

⁶ A key issue in regime switching models is whether the same regimes repeat over time, as in the case of repeated recession and expansion periods, or if new regimes always differ from previous ones. If "history repeats" and the underlying regimes do not change, all regimes will recur at some time. With only two regimes this will happen if $P_{00} < 1$, $i = 0, 1$. Models with recurring regimes have been used to characterize bull and bear markets, calm versus turbulent markets, and recession and expansion periods. Alternative to the assumption of recurring regime is the change point process studied in the context of dynamics of stock returns by Pastor and Stambaugh (2001) and Perez- Quiros, and Timmermann, A (2012) This type of model is likely to be a good approximation of regime shifts related to technological change. Our model has abstracted from such technological changes.

(ω_1) of the permanent component due to the high volatility regime is also significant. It is also interesting to find that the magnitude of the overall variance of the permanent component during the high volatility state, i.e., $\omega_0 + \omega_1$ says very little for the Indian market. The parameters relate to the transitory components of our model. The transitional probabilities q_{11} (high variance state of the transitory component) and q_{00} (low variance state of the transitory component) are highly significant for India. This is an indication that the low volatility state dominates in India. The expected duration of the high volatility state is 4.41 months and the expected duration of the low volatility state is 26.12 months. The average duration of the low volatility state is 54.32 months while the average duration of the high volatility state is 11 months. This means that the high volatility transitory state fades in about 11 months on average for India. In order to check for the performance of the table, we analyze the residuals from the model using a variety of diagnostic tests.

The test results are presented in Table 1.3

Table 1.3: Residual Diagnostics Tests

Portmanteau	0.412
ARCH	0.333
KS	0.013
RB Test	0.041
MNR	0.889
Recursive T	0.771

Entries are P values for the respective statistics. The residuals in the portmanteau test is that the residuals are serially uncorrelated. The ARCH test residuals are for no serial correlation in the squared residuals up to lag 18. MNR is the Von Neuman ratio test using recursive residuals for model adequacy. If the model is correctly specified then Recursive T has a standard t- distribution. (Harvey (1990)). KS statistic represents the Kolmogorov Smirnov test statistic for normality. 95% confidence level in this test is .071. When KS statistic is less than 0.071 the null hypothesis of normality

Can't be rejected at the given level of significance. We also applied a pair of tests specifically designed for the recursive residuals produced by the state space system used in this study. The first, the modified Von Neuman ratio, test against serial correlations of the residuals; the second, the recursive T test, check for correct model specification. The adequacy of the model is overwhelmingly supported.

V. CONCLUSION

We applied the Markov switching heteroscedasticity model to stock returns in India. The modelling approach is superior to GARCH model. In particular the Markov switching model explicitly

considers the possibility of regime switch whereas the GARCH model does not. In terms of our estimate the high variance state of the transitory component lasts for an average of only 4 months.

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