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Bianchi Type V Universe Filled with Combination of Perfect Fluid and Scalar Field Coupled with Electromagnetic Fields in $f(R,T)$ Theory of Gravity

By D T Solanke & T M Karade
Amaravati University

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7. Nojiri S and Odintsov S D : Phys. Rev. D 68, 123512 (2003).

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D T Solanke [α] & T M Karade [σ]

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I. Introduction

Cosmological data from wide range of source have indicated that our universe is undergoing an accelerating expansion [2-8]. To explain this fact, two alternative theories are proposed: one concept of dark energy and other the amendment of general relativity leading to $f(R)$ and $f(R,T)$ theories [7, 10, 12] where R stands for Ricci scalar $R = g^{ij} R_{ij}$, R_{ij} being Ricci tensor and T stands for trace of energy momentum tensor and $T = g^{ij} T_{ij}$,T_{ij} being energy momentum tensor derived from Lagrangian L_m . The field equations of $f(R,T)$ theories due to Harko [10] are deduced by varying the action

$$s = \int f(R,T)\sqrt{-g} d^4 x + \int L_m \sqrt{-g} d^4 x \tag{1}$$

Where L_m is lagrangian and the other symbols have their usual meaning. Energy momentum tensor is given by

$$T_{ij} = L_m g_{ij} - 2\frac{\delta L_m}{\delta g^{ij}} \tag{2}$$

Varying the action (1) with respect to g^{ij} which yields as

$$\delta s = \frac{1}{2\chi}\int \left\{ f_R(R,T)\frac{\delta R}{\delta g^{ij}} + f_T(R,T)\frac{\delta T}{\delta g^{ij}} + \frac{f(R,T)}{\sqrt{-g}}\frac{\delta(\sqrt{-g})}{\delta g^{ij}} + \frac{2\chi}{\sqrt{-g}}\left(\frac{\delta(L_m\sqrt{-g})}{\delta g^{ij}}\right)\right\}\sqrt{-g}\, d^4x \tag{3}$$

We define $\theta_{ij} = g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{ij}}$ (4)

Author α σ: Department of Mathematics, Sant Gadge Baba, Amaravati University, Amaravati, Maharashtra state, India.
e-mail: solanke_dattarao@rediffmail.com

By defining the generalized kronecker symbol $\frac{\delta g^{\alpha\beta}}{\delta g^{ij}} = \delta_i^\alpha \delta_j^\beta$ we can reduce

$$\frac{\delta g^{\alpha\beta}}{\delta g^{ij}} T_{\alpha\beta} = \delta_i^\alpha \delta_j^\beta T_{\alpha\beta} \quad = g^{p\alpha} g_{pi} g^{q\beta} g_{qj} T_{\alpha\beta} = T_{ij}$$

Using above equations we can write

$$\frac{\delta T}{\delta g^{ij}} = \frac{\delta(g^{\alpha\beta} T_{\alpha\beta})}{\delta g^{ij}} = \frac{\delta g^{\alpha\beta}}{\delta g^{ij}} T_{\alpha\beta} + g^{\alpha\beta} \frac{\delta T_{\alpha\beta}}{\delta g^{ij}} = T_{ij} + \theta_{ij}$$

Integrating (3) we can obtain

$$f_R(R,T) R_{ij} - \frac{1}{2} f(R,T) g_{ij} + \left(g_{ij} \Box - \nabla_i \nabla_j\right) f_R(R,T) = \chi T_{ij} - f_T(R,T)\left[T_{ij} + \theta_{ij}\right] \tag{5}$$

This can be further rewritten as

$$f_R(R,T) G_{ij} + \frac{1}{2}[f_R(R,T) R - f(R,T)] g_{ij} + (g_{ij} \Box - \nabla_i \nabla_j) f_R(R,T) =$$

$$\chi T_{ij} - f_T(R,T)\left[T_{ij} + \theta_{ij}\right] \tag{6}$$

where $G_{ij} = R_{ij} - \frac{1}{2} R g_{ij}$

Taking trace of (5) we obtain

$$\Box f_R(R,T) = \frac{2}{3} f(R,T) - \frac{1}{3} f_R(R,T) R + \frac{\chi}{3} T - \frac{1}{3} f_T(R,T)[T + \theta] \tag{7}$$

Using (7) the equation (6) can be organized in the form

$$G_j^i = \frac{1}{f_R(R,T)}\left[g^{mi} \nabla_m \nabla_j f_R(R,T)\right] - \frac{1}{6 f_R(R,T)}[f_R(R,T) R + f(R,T)] g_j^i + \frac{\chi}{f_R(R,T)}\left[T_j^i - \frac{1}{3} T g_j^i\right] +$$

$$\frac{1}{3} \frac{f_T(R,T)}{f_R(R,T)} [T + \theta] g_j^i - \frac{f_T(R,T)}{f_R(R,T)}\left[T_j^i + \theta_j^i\right] \tag{8}$$

Let us now calculate the tensor θ_{ij} .Varying (2) with respect to metric tensor g^{ij} and using the definition (4) we obtain

$$\theta_{ij} = -T_{ij} + 2\left[\frac{\delta L_m}{\delta g^{ij}} - g^{\alpha\beta} \frac{\delta^2 L_m}{\delta g^{ij} \delta g^{\alpha\beta}}\right] \tag{9}$$

With this background, in this paper we discover the Bianchi type V space-time with combination of perfect fluid and scalar field interacting with electromagnetic one.

II. Matter field Lagrangian L_m

The electromagnetic field tensor is given by

$$F_{ij} = \frac{\partial V_i}{\partial x^j} - \frac{\partial V_j}{\partial x^i},$$

where V_i is electromagnetic four potential.

The aforesaid the matter Lagrangian L_m can be expressed as

$$L_m = \left[\frac{1}{4} F_{\eta\tau} F^{\eta\tau} - \frac{1}{2} \varphi_{,\eta} \varphi^{,\eta} \psi\right] \tag{10}$$

where $\psi = \psi(I), \quad I = V_i V^i$

Notes

The function ψ characterizes the interaction between the scalar φ and electromagnetic field [1].

Then the matter tensor in (2) can conveniently be expressed in the mixed form

$$T_j^i = \left(F_\alpha^i F_j^\alpha + \frac{1}{4} g_j^i F_{\alpha\beta} F^{\alpha\beta}\right) - \left[\frac{1}{2}\psi g_j^i - \dot{\psi} V^i V_j\right] \varphi_{,\eta} \varphi^{,\eta} + \psi \varphi^{,i} \varphi_{,j} \quad (11)$$

Similarly (9) is written as

$$\theta_j^i = -T_j^i - (\psi - I\dot{\psi})\varphi^{,i}\varphi_{,j} + I\ddot{\psi}\varphi_{,\eta}\varphi^{,\eta} V^i V_j \quad (12)$$

The equations (11) and (12), after contraction yield

$$T = -(\psi - I\dot{\psi})\varphi_{,\eta}\varphi^{,\eta} \quad (13)$$

$$\theta = I^2 \ddot{\psi} \varphi_{,\eta} \varphi^{,\eta} \quad (14)$$

III. Bianchi Type V Space-Time

The Bianchi type V metric of the space-time is specified by

$$ds^2 = dt^2 - A^2 dx^2 - e^{2\alpha x}(B^2 dy^2 + C^2 dz^2), \quad (15)$$

where A, B and C are functions of cosmic time t and α is non-zero constant

For this metric the non-vanishing components of the Ricci tensors are

$$R_1^1 = \frac{-2\alpha^2}{A^2} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC}, \quad R_2^2 = \frac{-2\alpha^2}{A^2} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC}, \quad R_3^3 = \frac{-2\alpha^2}{A^2} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC},$$

$$R_4^4 = \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C}, \quad R_4^1 = \frac{2\alpha\dot{A}}{A^3} - \frac{\alpha\dot{B}}{A^2 B} - \frac{\alpha\dot{C}}{A^2 C}, \quad R_1^4 = -\frac{2\alpha\dot{A}}{A^3} + \frac{\alpha\dot{B}}{A^2 B} + \frac{\alpha\dot{C}}{A^2 C},$$

and the non-vanishing components of the Einstein tensors are

$$G_1^1 = \frac{\alpha^2}{A^2} - \frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC}, \quad G_2^2 = \frac{\alpha^2}{A^2} - \frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} - \frac{\dot{A}\dot{C}}{AC}, \quad G_3^3 = \frac{\alpha^2}{A^2} - \frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB},$$

$$G_4^4 = \frac{3\alpha^2}{A^2} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{A}\dot{C}}{AC}, \quad G_4^1 = \frac{2\alpha\dot{A}}{A^3} - \frac{\alpha\dot{B}}{A^2 B} - \frac{\alpha\dot{C}}{A^2 C}, \quad G_1^4 = -2\alpha\frac{\dot{A}}{A} + \alpha\frac{\dot{B}}{B} + \alpha\frac{\dot{C}}{C}$$

a) *Electromagnetic field tensor* F_{ij}

To achieve the compatibility with the non-static space time (15), we assume the electromagnetic vector potential in the form

$$V_i = [u(x)V_1(t), V_2(t), V_3(t), V_4(t)] \quad (16)$$

$$I = V_i V^i = -\left[\frac{u^2 V_1^2}{A^2} + \frac{V_2^2}{B^2} e^{-2\alpha x} + \frac{V_3^2}{C^2} e^{-2\alpha x} - V_4^2\right] \quad (17)$$

Then it easy to deduce

$$F_{14} = u\dot{V}_1 \qquad F_{24} = \dot{V}_2 \qquad F_{34} = \dot{V}_3 \quad (18)$$

$$F_{ij} F^{ij} = -2\left[\frac{u^2 \dot{V}_1^2}{A^2} + \frac{\dot{V}_2^2}{B^2} e^{-2\alpha x} + \frac{\dot{V}_3^2}{C^2} e^{-2\alpha x}\right] \quad (19)$$

$$\varphi_i \varphi^i = \dot{\varphi}^2 \quad (20)$$

In reference to the above quantities at our disposal and space-time (15), the components of T_j^i in (11) assume the following values

Ref

1. Rybakov Yu. P, Shikin G N and Saha B : Int. J. Theor. Phys. **36**, 6,pp. 1475-1494, 1997.

$$T_1^1 = \frac{1}{2}u^2\frac{\dot{V_1}^2}{A^2} - \frac{1}{2}\frac{\dot{V_2}^2}{B^2}e^{-2\alpha x} - \frac{1}{2}\frac{\dot{V_3}^2}{C^2}e^{-2\alpha x} - \frac{1}{2}\psi\dot{\varphi}^2 - \dot{\psi}\dot{\varphi}^2u^2\frac{V_1^2}{A^2} \tag{21a}$$

$$T_2^1 = \frac{u\dot{V_1}\dot{V_2}}{A^2} - \dot{\psi}\dot{\varphi}^2\frac{uV_1V_2}{A^2} \tag{21b}$$

$$T_3^1 = \frac{u\dot{V_1}\dot{V_3}}{A^2} - \dot{\psi}\dot{\varphi}^2\frac{uV_1V_3}{A^2} \tag{21c}$$

$$T_2^2 = -\frac{1}{2}u^2\frac{\dot{V_1}^2}{A^2} + \frac{1}{2}\frac{\dot{V_2}^2}{B^2}e^{-2\alpha x} - \frac{1}{2}\frac{\dot{V_3}^2}{C^2}e^{-2\alpha x} - \frac{1}{2}\psi\dot{\varphi}^2 - \dot{\psi}\dot{\varphi}^2\frac{V_2^2}{B^2}e^{-2\alpha x} \tag{21d}$$

$$T_3^2 = \frac{\dot{V_2}\dot{V_3}}{B^2}e^{-2\alpha x} - \dot{\psi}\dot{\varphi}^2\frac{V_2V_3}{B^2}e^{-2\alpha x} \tag{21e}$$

$$T_3^3 = -\frac{1}{2}u^2\frac{\dot{V_1}^2}{A^2} - \frac{1}{2}\frac{\dot{V_2}^2}{B^2}e^{-2\alpha x} + \frac{1}{2}\frac{\dot{V_3}^2}{C^2}e^{-2\alpha x} - \frac{1}{2}\psi\dot{\varphi}^2 - \dot{\psi}\dot{\varphi}^2\frac{V_3^2}{C^2}e^{-2\alpha x} \tag{21f}$$

$$T_4^4 = \frac{1}{2}u^2\frac{\dot{V_1}^2}{A^2} + \frac{1}{2}\frac{\dot{V_2}^2}{B^2}e^{-2\alpha x} + \frac{1}{2}\frac{\dot{V_3}^2}{C^2}e^{-2\alpha x} + \frac{1}{2}\psi\dot{\varphi}^2 + \dot{\psi}\dot{\varphi}^2V_4^2 \tag{21g}$$

$$T = -(\psi - I\dot{\psi})\dot{\varphi}^2 \tag{21h}$$

Similarly the components of tensor θ_j^i from (12) can assume the following values

$$\theta_1^1 = -T_1^1 - I\ddot{\psi}\dot{\varphi}^2u^2\frac{V_1^2}{A^2} \tag{22a}$$

$$\theta_2^1 = -T_2^1 - I\ddot{\psi}\dot{\varphi}^2u\frac{V_1V_2}{A^2} \tag{22b}$$

$$\theta_3^1 = -T_3^1 - I\ddot{\psi}\dot{\varphi}^2u\frac{V_1V_3}{A^2} \tag{22c}$$

$$\theta_2^2 = -T_2^2 - I\ddot{\psi}\dot{\varphi}^2\frac{V_2^2}{B^2}e^{-2\alpha x} \tag{22d}$$

$$\theta_3^2 = -T_3^2 - I\ddot{\psi}\dot{\varphi}^2\frac{V_2V_3}{B^2}e^{-2\alpha x} \tag{22e}$$

$$\theta_3^3 = -T_3^3 - I\ddot{\psi}\dot{\varphi}^2\frac{V_3^2}{C^2}e^{-2\alpha x} \tag{22f}$$

$$\theta_4^4 = -T_4^4 - \psi\dot{\varphi}^2 + I\dot{\psi}\dot{\varphi}^2 + I\ddot{\psi}\dot{\varphi}^2V_4^2 \tag{22g}$$

$$\theta = I^2\ddot{\psi}\dot{\varphi}^2 \tag{22h}$$

Variation of Lagrangian in (10) with respect to the electromagnetic field gives us

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^j}\left(\sqrt{-g}F^{ij}\right) - \left(\varphi_{,j}\varphi^{,j}\right)\dot{\psi}V^i = 0$$

for $i = 1, j = 4 \Rightarrow \left(\frac{\dot{V_1}}{V_1}\right)^{\cdot} + \frac{\dot{V_1}^2}{V_1^2} + \frac{\dot{V_1}}{V_1}\left[\frac{\dot{B}}{B} + \frac{\dot{C}}{C} - \frac{\dot{A}}{A}\right] = \dot{\psi}\dot{\varphi}^2$ (23a)

for $i = 2, j = 4 \Rightarrow \left(\frac{\dot{V_2}}{V_2}\right)^{\cdot} + \frac{\dot{V_2}^2}{V_2^2} + \frac{\dot{V_2}}{V_2}\left[\frac{\dot{C}}{C} + \frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right] = \dot{\psi}\dot{\varphi}^2$ (23b)

for $i = 3, j = 4 \Rightarrow \left(\frac{\dot{V_3}}{V_3}\right)^{\cdot} + \frac{\dot{V_3}^2}{V_3^2} + \frac{\dot{V_3}}{V_3}\left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right] = \dot{\psi}\dot{\varphi}^2$ (23c)

for $i = 4, j = 1 \Rightarrow u(x) = k_1e^{-2\alpha x}$ (23d)

Notes

for $i = 4, j = 4 \Rightarrow V_4 = 0$ (23e)

where k_1 is constant of integration.

IV. Combination of Perfect Fluid and Scalar Field Coupled with Electromagnetic Field

Notes

Energy momentum tensor for perfect fluid is given by

$$T_j^i = (\rho + p)u_j u^i - p\delta_j^i \tag{24}$$

where $g_{ij} u^i u^j = 1$

$$T_1^1 = T_2^2 = T_3^3 = -p \ , T_4^4 = \rho$$

$$T_j^i = 0 \text{ for } i \neq j \tag{25}$$

We take combination of perfect fluid and scalar field coupled with electromagnetic field as

$$T_j^i = T_j^i(PF) + T_j^i(EF) \tag{26}$$

By using (25), (26) the equation (21) reduces to

$$T_1^1 = -p + \frac{1}{2}u^2\frac{\dot{V_1}^2}{A^2} - \frac{1}{2}\frac{\dot{V_2}^2}{B^2}e^{-2\alpha x} - \frac{1}{2}\frac{\dot{V_3}^2}{C^2}e^{-2\alpha x} - \frac{1}{2}\psi\dot{\varphi}^2 - \dot{\psi}\dot{\varphi}^2 u^2\frac{V_1^2}{A^2} \tag{27a}$$

$$T_2^1 = \frac{u\dot{V_1}\dot{V_2}}{A^2} - \dot{\psi}\dot{\varphi}^2\frac{uV_1V_2}{A^2} \tag{27b}$$

$$T_3^1 = \frac{u\dot{V_1}\dot{V_3}}{A^2} - \dot{\psi}\dot{\varphi}^2\frac{uV_1V_3}{A^2} \tag{27c}$$

$$T_2^2 = -p - \frac{1}{2}u^2\frac{\dot{V_1}^2}{A^2} + \frac{1}{2}\frac{\dot{V_2}^2}{B^2}e^{-2\alpha x} - \frac{1}{2}\frac{\dot{V_3}^2}{C^2}e^{-2\alpha x} - \frac{1}{2}\psi\dot{\varphi}^2 - \dot{\psi}\dot{\varphi}^2\frac{V_2^2}{B^2}e^{-2\alpha x} \tag{27d}$$

$$T_3^2 = \frac{\dot{V_2}\dot{V_3}}{B^2}e^{-2\alpha x} - \dot{\psi}\dot{\varphi}^2\frac{V_2V_3}{B^2}e^{-2\alpha x} \tag{27e}$$

$$T_3^3 = -p - \frac{1}{2}u^2\frac{\dot{V_1}^2}{A^2} - \frac{1}{2}\frac{\dot{V_2}^2}{B^2}e^{-2\alpha x} + \frac{1}{2}\frac{\dot{V_3}^2}{C^2}e^{-2\alpha x} - \frac{1}{2}\psi\dot{\varphi}^2 - \dot{\psi}\dot{\varphi}^2\frac{V_3^2}{C^2}e^{-2\alpha x} \tag{27f}$$

$$T_4^4 = \rho + \frac{1}{2}u^2\frac{\dot{V_1}^2}{A^2} + \frac{1}{2}\frac{\dot{V_2}^2}{B^2}e^{-2\alpha x} + \frac{1}{2}\frac{\dot{V_3}^2}{C^2}e^{-2\alpha x} + \frac{1}{2}\psi\dot{\varphi}^2 + \dot{\psi}\dot{\varphi}^2 V_4^2 \tag{27g}$$

$$T = -3p + \rho - (\psi - I\dot{\psi})\dot{\varphi}^2 \tag{27h}$$

By using (25), (26) the equation (22) reduces to

$$\theta_1^1 = -T_1^1 - p - I\ddot{\psi}\dot{\varphi}^2u^2\frac{V_1^2}{A^2} \tag{28a}$$

$$\theta_2^1 = -T_2^1 - I\ddot{\psi}\dot{\varphi}^2u\frac{V_1V_2}{A^2} \tag{28b}$$

$$\theta_3^1 = -T_3^1 - I\ddot{\psi}\dot{\varphi}^2u\frac{V_1V_3}{A^2} \tag{28c}$$

$$\theta_2^2 = -T_2^2 - p - I\ddot{\psi}\dot{\varphi}^2\frac{V_2^2}{B^2}e^{-2\alpha x} \tag{28d}$$

$$\theta_3^2 = -T_3^2 - I\ddot{\psi}\dot{\varphi}^2\frac{V_2V_3}{B^2}e^{-2\alpha x} \tag{28e}$$

$$\theta_3^3 = -T_3^3 - p - I\ddot{\psi}\dot{\varphi}^2 \frac{{V_3}^2}{C^2} e^{-2\alpha x} \tag{28f}$$

$$\theta_4^4 = -T_4^4 + \rho - \psi\dot{\varphi}^2 + I\dot{\psi}\dot{\varphi}^2 + I\ddot{\psi}\dot{\varphi}^2 {V_4}^2 \tag{28g}$$

$$\theta = I^2\ddot{\psi}\dot{\varphi}^2 \tag{28h}$$

Since the expression of the Einstein tensor in (8) is complicated, the solution of the Einstein's field equation in general cannot be obtained. With this reality we take recourse to the particular cases of the function $f(R,T)$ and there upon try to obtain the solution.

V. Sub Case $f(R,T) = f_1(R) + \lambda f_2(T)$

In this case we follow the notations

$$f_R(R,T) = \frac{\partial f(R,T)}{\partial R} = \dot{f}_1(R)\ , \qquad f_T(R,T) = \frac{\partial f(R,T)}{\partial T} = \lambda \dot{f}_2(T)$$

Then (8) reduces to the form

$$G_j^i = \frac{1}{\dot{f}_1(R)}\left[g^{mi}\nabla_m\nabla_j \dot{f}_1(R)\right] - \frac{1}{6\dot{f}_1(R)}\left[\dot{f}_1(R)R + f_1(R) + \lambda f_2(T)\right]g_j^i + \frac{\chi}{\dot{f}_1(R)}\left[T_j^i - \frac{1}{3}T g_j^i\right] + \frac{\lambda}{3}\frac{\dot{f}_2(T)}{\dot{f}_1(R)}[T+\theta]g_j^i - \frac{\lambda \dot{f}_2(T)}{\dot{f}_1(R)}\left[T_j^i + \theta_j^i\right] \tag{29}$$

Since for the space-time (15), $G_2^1 = 0, G_3^1 = 0, G_3^2 = 0$, by using (27) and (28), the field equations (29) yield

$$\frac{\dot{V}_1\dot{V}_2}{V_1V_2} = \dot{\psi}\dot{\varphi}^2 - \frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2 \tag{30a}$$

$$\frac{\dot{V}_1\dot{V}_3}{V_1V_3} = \dot{\psi}\dot{\varphi}^2 - \frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2 \tag{30b}$$

$$\frac{\dot{V}_2\dot{V}_3}{V_2V_3} = \dot{\psi}\dot{\varphi}^2 - \frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2 \tag{30c}$$

From (30) we can write

$$\frac{\dot{V}_1\dot{V}_2}{V_1V_2} = \frac{\dot{V}_2\dot{V}_3}{V_2V_3} = \frac{\dot{V}_1\dot{V}_3}{V_1V_3} = \dot{\psi}\dot{\varphi}^2 - \frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2 \tag{31}$$

or $\frac{\dot{V}_2}{V_2} = \frac{\dot{V}_3}{V_3} \equiv \frac{\dot{h}_3}{h_3}$, say (32)

where h_3 is some unknown function of t
Inserting (32) in (31) we get

$$\left(\frac{\dot{h_3}}{h_3}\right)^2 = \left(\frac{\dot{h_3}}{h_3}\right)^2 = \left(\frac{\dot{h_3}}{h_3}\right)^2 = \dot{\psi}\dot{\varphi}^2 - \frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2 \tag{33}$$

Upon the integration of equation (32), yield

$$V_1 = k_{38}h_3 \qquad V_2 = k_{39}h_3 \qquad V_3 = k_{40}h_3 \tag{34}$$

where $k's$ are constants of integration.

Now our plan is to express the components of T_j^i in (27) in terms of T_4^4. For this we consider the expression

Notes

$$\frac{u^2\dot{V_1}^2}{A^2}+\frac{\dot{V_2}^2}{B^2}e^{-2\alpha x}+\frac{\dot{V_3}^2}{C^2}e^{-2\alpha x}=\left[\frac{u^2{v_1}^2}{A^2}+\frac{{V_2}^2}{B^2}e^{-2\alpha x}+\frac{{V_3}^2}{C^2}e^{-2\alpha x}\right]\left(\frac{\dot{h}_3}{h_3}\right)^2 \quad \text{by (32)}$$

$$=-I\left(\frac{\dot{h}_3}{h_3}\right)^2 \quad \text{by (17) and (23e)}$$

$$=\frac{\lambda}{\chi}\dot{f}_2(T)I^2\ddot{\psi}\dot{\varphi}^2-I\dot{\psi}\dot{\varphi}^2 \quad \text{by (33) (35)}$$

Notes

We attempt to express the components of T_j^i in (27) in terms of T_4^4 by using (32), (33) and (35)

$$T_4^4=\rho+\frac{1}{2}\psi\dot{\varphi}^2-\frac{1}{2}I\dot{\psi}\dot{\varphi}^2+\frac{1}{2}\frac{\lambda}{\chi}\dot{f}_2(T)I^2\ddot{\psi}\dot{\varphi}^2 \tag{36a}$$

$$T_1^1=-T_4^4+\rho-p-\frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2u^2\frac{{V_1}^2}{A^2} \tag{36b}$$

$$T_2^1=-\frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2u\frac{V_1V_2}{A^2} \tag{36c}$$

$$T_3^1=-\frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2u\frac{V_1V_3}{A^2} \tag{36d}$$

$$T_2^2=-T_4^4+\rho-p-\frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2\frac{{V_2}^2}{B^2}e^{-2\alpha x} \tag{36e}$$

$$T_3^2=-\frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2\frac{V_2V_3}{A^2}e^{-2\alpha x} \tag{36f}$$

$$T_3^3=-T_4^4+\rho-p-\frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2\frac{{V_3}^2}{C^2}e^{-2\alpha x} \tag{36g}$$

$$T=-3p+\rho-(\psi-I\dot{\psi})\dot{\varphi}^2 \qquad (36f)$$

We consider the non-vanishing components of Einstein tensor G_1^1, G_2^2, G_3^3 from (29)

$$\frac{\alpha^2}{A^2}-\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}-\frac{\dot{B}\dot{C}}{BC}=-\frac{\dot{A}}{A}\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}-\frac{1}{6\dot{f}_1(R)}\left[\dot{f}_1(R)R+f_1(R)+\lambda f_2(T)\right]+\frac{\chi}{\dot{f}_1(R)}\left[T_1^1-\frac{1}{3}T\right]$$
$$+\frac{\lambda}{3}\frac{\dot{f}_2(T)}{\dot{f}_1(R)}[T+\theta]-\frac{\lambda\dot{f}_2(T)}{\dot{f}_1(R)}[T_1^1+\theta_1^1] \tag{37a}$$

$$\frac{\alpha^2}{A^2}-\frac{\ddot{C}}{C}-\frac{\ddot{A}}{A}-\frac{\dot{C}\dot{A}}{CA}=-\frac{\dot{B}}{B}\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}-\frac{1}{6\dot{f}_1(R)}\left[\dot{f}_1(R)R+f_1(R)+\lambda f_2(T)\right]+\frac{\chi}{\dot{f}_1(R)}\left[T_2^2-\frac{1}{3}T\right]$$
$$+\frac{\lambda}{3}\frac{\dot{f}_2(T)}{\dot{f}_1(R)}[T+\theta]-\frac{\lambda\dot{f}_2(T)}{\dot{f}_1(R)}[T_2^2+\theta_2^2] \tag{37b}$$

$$\frac{\alpha^2}{A^2}-\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}-\frac{\dot{A}\dot{B}}{AB}=-\frac{\dot{C}}{C}\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}-\frac{1}{6\dot{f}_1(R)}\left[\dot{f}_1(R)R+f_1(R)+\lambda f_2(T)\right]+\frac{\chi}{\dot{f}_1(R)}\left[T_3^3-\frac{1}{3}T\right]$$
$$+\frac{\lambda}{3}\frac{\dot{f}_2(T)}{\dot{f}_1(R)}[T+\theta]-\frac{\lambda\dot{f}_2(T)}{\dot{f}_1(R)}[T_3^3+\theta_3^3] \tag{37c}$$

Subtracting (37b) from (37a), (37c) from (37b) and (37a) from (37c) we get

$$\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{\dot{C}}{C}\left[\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right]+\left[\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right]\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}=\frac{\chi}{\dot{f}_1(R)}[T_1^1-T_2^2]+\frac{\lambda\dot{f}_2(T)}{\dot{f}_1(R)}[(T_2^2+\theta_2^2)-(T_1^1+\theta_1^1)] \tag{38a}$$

$$\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\frac{\dot{A}}{A}\left[\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right]+\left[\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right]\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}=\frac{\chi}{\dot{f}_1(R)}[T_2^2-T_3^3]+\frac{\lambda\dot{f}_2(T)}{\dot{f}_1(R)}[(T_3^3+\theta_3^3)-(T_2^2+\theta_2^2)] \tag{38b}$$

$$\frac{\ddot{C}}{C}-\frac{\ddot{A}}{A}+\frac{\dot{B}}{B}\left[\frac{\dot{C}}{C}-\frac{\dot{A}}{A}\right]+\left[\frac{\dot{C}}{C}-\frac{\dot{A}}{A}\right]\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}=\frac{\chi}{\dot{f}_1(R)}[T_3^3-T_1^1]+\frac{\lambda\dot{f}_2(T)}{\dot{f}_1(R)}[(T_1^1+\theta_1^1)-(T_3^3+\theta_3^3)] \quad (38c)$$

With the help of (28) and (36) the equation (38) reduces to

$$\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{\dot{C}}{C}\left[\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right]+\left[\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right]\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}=0 \quad (39a)$$

$$\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\frac{\dot{A}}{A}\left[\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right]+\left[\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right]\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}=0 \quad (39b)$$

$$\frac{\ddot{C}}{C}-\frac{\ddot{A}}{A}+\frac{\dot{B}}{B}\left[\frac{\dot{C}}{C}-\frac{\dot{A}}{A}\right]+\left[\frac{\dot{C}}{C}-\frac{\dot{A}}{A}\right]\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}=0 \quad (39c)$$

Integrating (39) we get

$$\frac{A}{B}=k_{42}exp\left\{k_{41}\int\frac{1}{ABC\dot{f}_1(R)}dt\right\} \quad (40a)$$

$$\frac{B}{C}=k_{44}exp\left\{k_{43}\int\frac{1}{ABC\dot{f}_1(R)}dt\right\} \quad (40b)$$

$$\frac{C}{A}=k_{46}exp\left\{k_{45}\int\frac{1}{ABC\dot{f}_1(R)}dt\right\} \quad (40c)$$

Where$k's$ are constants of integration, with the condition that $k_{42}k_{44}k_{46}=1$ and

$$k_{41}+k_{43}+k_{45}=0$$

From (40) we can express the values of A,B,C explicitly as

$$A=(ABC)^{\frac{1}{3}}k_{47}exp\left\{k_{48}\int\frac{1}{ABC\dot{f}_1(R)}dt\right\} \quad (41a)$$

$$B=(ABC)^{\frac{1}{3}}k_{49}exp\left\{k_{50}\int\frac{1}{ABC\dot{f}_1(R)}dt\right\} \quad (41b)$$

$$C=(ABC)^{\frac{1}{3}}k_{51}exp\left\{k_{52}\int\frac{1}{ABC\dot{f}_1(R)}dt\right\} \quad (41c)$$

where $k's$ are constants of integration.
Using (32) we can rewrite the equations (23) as

$$\left(\frac{\dot{h}_3}{h_3}\right)^{\cdot}+\frac{\dot{h}_3{}^2}{h_3{}^2}+\frac{\dot{h}_3}{h_3}\left[\frac{\dot{C}}{C}+\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right]=\dot{\psi}\dot{\varphi}^2 \quad (42a)$$

$$\left(\frac{\dot{h}_3}{h_3}\right)^{\cdot}+\frac{\dot{h}_3{}^2}{h_3{}^2}+\frac{\dot{h}_3}{h_3}\left[\frac{\dot{A}}{A}+\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right]=\dot{\psi}\dot{\varphi}^2 \quad (42b)$$

$$\left(\frac{\dot{h}_3}{h_3}\right)^{\cdot}+\frac{\dot{h}_3{}^2}{h_3{}^2}+\frac{\dot{h}_3}{h_3}\left[\frac{\dot{B}}{B}+\frac{\dot{C}}{C}-\frac{\dot{A}}{A}\right]=\dot{\psi}\dot{\varphi}^2 \quad (42c)$$

These equations further imply

$$\frac{\dot{C}}{C}+\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=\frac{\dot{A}}{A}+\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=\frac{\dot{B}}{B}+\frac{\dot{C}}{C}-\frac{\dot{A}}{A}$$

or

$$\frac{A^{\cdot}}{A}=\frac{B^{\cdot}}{B}=\frac{C^{\cdot}}{C} \quad (43)$$

Inserting (43) in (42) yield

Notes

$$\left(\frac{\dot{h}_3}{h_3}\right)^{\cdot} + \frac{\dot{h}_3{}^2}{h_3{}^2} + \frac{\dot{h}_3}{h_3}\left[\frac{\dot{A}}{A}\right] = \dot{\psi}\dot{\varphi}^2 \tag{44}$$

But from (33) we have $$\dot{\psi}\dot{\varphi}^2 = \left(\frac{\dot{h}_3}{h_3}\right)^2 + \frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2 \tag{45}$$

Inserting (45) in (44) we have

Notes

$$\left(\frac{\dot{h}_3}{h_3}\right)^{\cdot} + \frac{\dot{h}_3}{h_3}\left[\frac{\dot{A}}{A}\right] = \frac{\lambda}{\chi}\dot{f}_2(T)I\ddot{\psi}\dot{\varphi}^2 \tag{46}$$

If we confine to the linearity of ψ (i.e. $\psi = k_{53}I + k_{54}$) we can have the perfect solution of (46)

$$h_3 = k_{56}exp\left\{k_{55}\int\frac{1}{A}dt\right\} \tag{47}$$

With the help of (47) equations (34) reduces to

$$V_1 = k_{57}exp\left\{k_{55}\int\frac{1}{A}dt\right\} \tag{48a}$$

$$V_2 = k_{58}exp\left\{k_{55}\int\frac{1}{A}dt\right\} \tag{48b}$$

$$V_3 = k_{59}exp\left\{k_{55}\int\frac{1}{A}dt\right\} \tag{48c}$$

Where $k's$ are constants of integration.

VI. Combination of Perfect Fluid and Electromagnetic Field in $f(R,T) = f_1(R)f_2(T)$

In this case we follow the notations

$$f_R(R,T) = \frac{\partial f(R,T)}{\partial R} = \dot{f}_1(R)f_2(T)\ , \quad f_T(R,T) = \frac{\partial f(R,T)}{\partial T} = f_1(R)\dot{f}_2(T)$$

Then the field equation (8) reduces to

$$G_j^\mu = \frac{1}{\dot{f}_1(R)f_2(T)}\left[g^{\mu i}\nabla_\mu\nabla_j\dot{f}_1(R)f_2(T)\right] - \frac{1}{6\dot{f}_1(R)f_2(T)}\left[\dot{f}_1(R)f_2(T)R + f_1(R)f_2(T)\right]g_j^\mu +$$

$$\frac{\chi}{\dot{f}_1(R)f_2(T)}\left[T_j^\mu - \frac{1}{3}Tg_j^\mu\right] + \frac{1}{3}\frac{f_1(R)\dot{f}_2(T)}{\dot{f}_1(R)f_2(T)}[T+\theta]g_j^\mu - \frac{f_1(R)\dot{f}_2(T)}{\dot{f}_1(R)f_2(T)}\left[T_j^\mu + \theta_j^\mu\right] \tag{49}$$

Since for the space-time (15), $G_2^1 = 0, G_3^1 = 0, G_3^2 = 0$, from (49) and by using (27) and (28)

$$\frac{\dot{V}_1\dot{V}_2}{V_1V_2} = \dot{\psi}\,\dot{\varphi}^2 - \frac{f_1(R)\dot{f}_2(T)}{\chi}I\ddot{\psi}\dot{\varphi}^2 \tag{50a}$$

$$\frac{\dot{V}_1\dot{V}_3}{V_1V_3} = \dot{\psi}\,\dot{\varphi}^2 - \frac{f_1(R)\dot{f}_2(T)}{\chi}I\ddot{\psi}\dot{\varphi}^2 \tag{50b}$$

$$\frac{\dot{V}_2\dot{V}_3}{V_2V_3} = \dot{\psi}\,\dot{\varphi}^2 - \frac{f_1(R)\dot{f}_2(T)}{\chi}I\ddot{\psi}\dot{\varphi}^2 \tag{50c}$$

From (50) we can write

$$\frac{\dot{V}_1\dot{V}_2}{V_1V_2} = \frac{\dot{V}_2\dot{V}_3}{V_2V_3} = \frac{\dot{V}_1\dot{V}_3}{V_1V_3} = \dot{\psi}\,\dot{\varphi}^2 - \frac{f_1(R)\dot{f}_2(T)}{\chi}I\ddot{\psi}\dot{\varphi}^2 \tag{51}$$

Further we can rewrite it as

$$\frac{\dot{V}_1}{V_1}=\frac{\dot{V}_2}{V_2}=\frac{\dot{V}_3}{V_3}\equiv\frac{\dot{h}_8}{h_8},\text{ say} \tag{52}$$

Where h_8 is some unknown function of t
Inserting (52) in (51) yields

$$\left(\frac{\dot{h}_8}{h_8}\right)^2=\left(\frac{\dot{h}_8}{h_8}\right)^2=\left(\frac{\dot{h}_8}{h_8}\right)^2=\dot{\psi}\,\dot{\varphi}^2-\frac{f_1(R)\dot{f}_2(T)}{\chi}I\ddot{\psi}\dot{\varphi}^2 \tag{53}$$

Integrating (52) we get

$$V_1=m_{28}h_8 \qquad V_2=m_{29}h_8 \qquad V_3=m_{30}h_8 \tag{54}$$

Where m_{28}, m_{29}, m_{30} are constants of integration.

Now our plan is to express the components of T_j^i in (27) in terms of T_4^4 . For this we consider the expression

$$\frac{u^2\dot{V_1}^2}{A^2}+\frac{\dot{V_2}^2}{B^2}e^{-2\alpha x}+\frac{\dot{V_3}^2}{C^2}e^{-2\alpha x}=\left[\frac{u^2{V_1}^2}{A^2}+\frac{{V_2}^2}{B^2}e^{-2\alpha x}+\frac{{V_3}^2}{C^2}e^{-2\alpha x}\right]\left(\frac{\dot{h}_8}{h_8}\right)^2 \quad \text{by} \quad (52)$$

$$=-I\left(\frac{\dot{h}_8}{h_8}\right)^2 \quad \text{by (17) and (23e)}$$

$$=\frac{f_1(R)\dot{f}_2(T)}{\chi}I^2\ddot{\psi}\dot{\varphi}^2-I\dot{\psi}\dot{\varphi}^2 \quad \text{by} \quad (53) \tag{55}$$

We attempt to express the components of T_j^i in (27) in terms of T_4^4 by using (52), (53) and (55)

$$T_4^4=\rho+\frac{1}{2}\psi\dot{\varphi}^2-\frac{1}{2}I\dot{\psi}\dot{\varphi}^2+\frac{1}{2}\frac{f_1(R)\dot{f}_2(T)}{\chi}I^2\ddot{\psi}\dot{\varphi}^2 \tag{56a}$$

$$T_1^1=-T_4^4+\rho-p-\frac{f_1(R)\dot{f}_2(T)}{\chi}I\ddot{\psi}\dot{\varphi}^2u^2\frac{{V_1}^2}{A^2} \tag{56b}$$

$$T_2^1=-\frac{f_1(R)\dot{f}_2(T)}{\chi}I\ddot{\psi}\dot{\varphi}^2u\frac{V_1V_2}{A^2} \tag{56c}$$

$$T_3^1=-\frac{f_1(R)\dot{f}_2(T)}{\chi}I\ddot{\psi}\dot{\varphi}^2u\frac{V_1V_3}{A^2} \tag{56d}$$

$$T_2^2=-T_4^4+\rho-p-\frac{f_1(R)\dot{f}_2(T)}{\chi}I\ddot{\psi}\dot{\varphi}^2\frac{{V_2}^2}{B^2}e^{-2\alpha x} \tag{56e}$$

$$T_3^2=-\frac{f_1(R)\dot{f}_2(T)}{\chi}I\ddot{\psi}\dot{\varphi}^2\frac{V_2V_3}{A^2}e^{-2\alpha x} \tag{56f}$$

$$T_3^3=-T_4^4+\rho-p-\frac{f_1(R)\dot{f}_2(T)}{\chi}I\ddot{\psi}\dot{\varphi}^2\frac{{V_3}^2}{C^2}e^{-2\alpha x} \tag{56g}$$

$$T=-3p+\rho-(\psi-I\dot{\psi})\dot{\varphi}^2 \tag{56h}$$

We consider the non-vanishing components of Einstein tensor G_1^1, G_2^2, G_3^3 from (49)

$$\frac{\alpha^2}{A^2}-\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}-\frac{\dot{B}\dot{C}}{BC}=-\frac{1}{A^2}\left[\frac{\ddot{f}_2(T)}{f_2(T)}\left(\frac{dT}{dx}\right)^2+\frac{\dot{f}_2(T)}{f_2(T)}\frac{d^2T}{dx^2}\right]-\frac{\dot{A}}{A}\left[\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}+\frac{\dot{f}_2(T)}{f_2(T)}\frac{dT}{dt}\right]-\frac{1}{6\dot{f}_1(R)f_2(T)}\times$$

Notes

$$[\dot{f}_1(R)f_2(T)\mathrm{R}+\mathrm{f}_1(\mathrm{R})\mathrm{f}_2(\mathrm{T})]+\frac{\chi}{\dot{f}_1(R)f_2(T)}\left[T_1^1-\frac{1}{3}T\right]+\frac{1}{3}\frac{\mathrm{f}_1(\mathrm{R})\dot{\mathrm{f}}_2(\mathrm{T})}{\dot{f}_1(R)f_2(T)}[T+\theta]-\frac{\mathrm{f}_1(\mathrm{R})\dot{\mathrm{f}}_2(\mathrm{T})}{\dot{f}_1(R)f_2(T)}[T_1^1+\theta_1^1] \tag{57a}$$

$$\frac{\alpha^2}{A^2}-\frac{\ddot{C}}{C}-\frac{\ddot{A}}{A}-\frac{\dot{C}\dot{A}}{CA}=\frac{\alpha}{A^2}\left[\frac{\dot{f}_2(T)}{f_2(T)}\frac{dT}{dx}\right]-\frac{\dot{B}}{B}\left[\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}+\frac{\dot{f}_2(T)}{f_2(T)}\frac{dT}{dt}\right]-\frac{1}{6\dot{f}_1(R)f_2(T)}[\dot{f}_1(R)f_2(T)\mathrm{R}+\mathrm{f}_1(\mathrm{R})\mathrm{f}_2(\mathrm{T})]$$
$$+\frac{\chi}{\dot{f}_1(R)f_2(T)}\left[T_2^2-\frac{1}{3}T\right]+\frac{1}{3}\frac{\mathrm{f}_1(\mathrm{R})\dot{\mathrm{f}}_2(\mathrm{T})}{\dot{f}_1(R)f_2(T)}[T+\theta]-\frac{\mathrm{f}_1(\mathrm{R})\dot{\mathrm{f}}_2(\mathrm{T})}{\dot{f}_1(R)f_2(T)}[T_2^2+\theta_2^2] \tag{57b}$$

$$\frac{\alpha^2}{A^2}-\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}-\frac{\dot{A}\dot{B}}{AB}=\frac{\alpha}{A^2}\left[\frac{\dot{f}_2(T)}{f_2(T)}\frac{dT}{dx}\right]-\frac{\dot{C}}{C}\left[\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}+\frac{\dot{f}_2(T)}{f_2(T)}\frac{dT}{dt}\right]-\frac{1}{6\dot{f}_1(R)f_2(T)}[\dot{f}_1(R)f_2(T)\mathrm{R}+\mathrm{f}_1(\mathrm{R})\mathrm{f}_2(\mathrm{T})]$$
$$+\frac{\chi}{\dot{f}_1(R)f_2(T)}\left[T_3^3-\frac{1}{3}T\right]+\frac{1}{3}\frac{\mathrm{f}_1(\mathrm{R})\dot{\mathrm{f}}_2(\mathrm{T})}{\dot{f}_1(R)f_2(T)}[T+\theta]-\frac{\mathrm{f}_1(\mathrm{R})\dot{\mathrm{f}}_2(\mathrm{T})}{\dot{f}_1(R)f_2(T)}[T_3^3+\theta_3^3] \tag{57c}$$

Subtracting (57b) from (57a), (57c) from (57b) and (57a) from (57c) we obtain

$$\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{\dot{C}}{C}\left[\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right]+\left[\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right]\left[\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}+\frac{\dot{f}_2(T)}{f_2(T)}\frac{dT}{dt}\right]=-\frac{1}{A^2}\frac{\ddot{f}_2(T)}{f_2(T)}\left(\frac{dT}{dx}\right)^2-\frac{1}{A^2}\frac{\dot{f}_2(T)}{f_2(T)}\frac{d^2T}{dx^2}-\frac{\alpha}{A^2}\frac{\dot{f}_2(T)}{f_2(T)}\frac{dT}{dx}$$
$$\frac{\chi}{\dot{f}_1(R)f_2(T)}[T_1^1-T_2^2]+\frac{\mathrm{f}_1(\mathrm{R})\dot{\mathrm{f}}_2(\mathrm{T})}{\dot{f}_1(R)f_2(T)}[(T_2^2+\theta_2^2)-(T_1^1+\theta_1^1)] \tag{58a}$$

$$\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\frac{\dot{A}}{A}\left[\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right]+\left[\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right]\left[\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}+\frac{\dot{f}_2(T)}{f_2(T)}\frac{dT}{dt}\right]=\frac{\chi}{\dot{f}_1(R)f_2(T)}[T_2^2-T_3^3]+$$
$$\frac{\mathrm{f}_1(\mathrm{R})\dot{\mathrm{f}}_2(\mathrm{T})}{\dot{f}_1(R)f_2(T)}[(T_3^3+\theta_3^3)-(T_2^2+\theta_2^2)] \tag{58b}$$

$$\frac{\ddot{C}}{C}-\frac{\ddot{A}}{A}+\frac{\dot{B}}{B}\left[\frac{\dot{C}}{C}-\frac{\dot{A}}{A}\right]+\left[\frac{\dot{C}}{C}-\frac{\dot{A}}{A}\right]\left[\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}+\frac{\dot{f}_2(T)}{f_2(T)}\frac{dT}{dt}\right]=\frac{1}{A^2}\frac{\ddot{f}_2(T)}{f_2(T)}\left(\frac{dT}{dx}\right)^2+\frac{1}{A^2}\frac{\dot{f}_2(T)}{f_2(T)}\frac{d^2T}{dx^2}+\frac{\alpha}{A^2}\frac{\dot{f}_2(T)}{f_2(T)}\frac{dT}{dx}$$
$$+\frac{\chi}{\dot{f}_1(R)f_2(T)}[T_3^3-T_1^1]+\frac{\mathrm{f}_1(\mathrm{R})\dot{\mathrm{f}}_2(\mathrm{T})}{\dot{f}_1(R)f_2(T)}[(T_1^1+\theta_1^1)-(T_3^3+\theta_3^3)] \tag{58c}$$

Using (56) and (28) the equations (58) reduces to

$$\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{\dot{C}}{C}\left[\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right]+\left[\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right]\left[\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}+\frac{\dot{f}_2(T)}{f_2(T)}\frac{dT}{dt}\right]=-\frac{1}{A^2}\frac{\ddot{f}_2(T)}{f_2(T)}\left(\frac{dT}{dx}\right)^2-\frac{1}{A^2}\frac{\dot{f}_2(T)}{f_2(T)}\frac{d^2T}{dx^2}-\frac{\alpha}{A^2}\frac{\dot{f}_2(T)}{f_2(T)}\frac{dT}{dx} \tag{59a}$$

$$\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\frac{\dot{A}}{A}\left[\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right]+\left[\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right]\left[\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}+\frac{\dot{f}_2(T)}{f_2(T)}\frac{dT}{dt}\right]=0 \tag{59b}$$

$$\frac{\ddot{C}}{C}-\frac{\ddot{A}}{A}+\frac{\dot{B}}{B}\left[\frac{\dot{C}}{C}-\frac{\dot{A}}{A}\right]+\left[\frac{\dot{C}}{C}-\frac{\dot{A}}{A}\right]\left[\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}+\frac{\dot{f}_2(T)}{f_2(T)}\frac{dT}{dt}\right]=\frac{1}{A^2}\frac{\ddot{f}_2(T)}{f_2(T)}\left(\frac{dT}{dx}\right)^2+\frac{1}{A^2}\frac{\dot{f}_2(T)}{f_2(T)}\frac{d^2T}{dx^2}+\frac{\alpha}{A^2}\frac{\dot{f}_2(T)}{f_2(T)}\frac{dT}{dx} \tag{59c}$$

If we take the condition that $A=m_{31}C$ (60)

where m_{31} non-zero positive constant. Then we obtain

$$\frac{1}{A^2}\frac{\ddot{f}_2(T)}{f_2(T)}\left(\frac{dT}{dx}\right)^2+\frac{1}{A^2}\frac{\dot{f}_2(T)}{f_2(T)}\frac{d^2T}{dx^2}+\frac{\alpha}{A^2}\frac{\dot{f}_2(T)}{f_2(T)}\frac{dT}{dx}=0$$

And above equations reduces to

$$\frac{\ddot{C}}{C}-\frac{\ddot{B}}{B}+\frac{\dot{C}}{C}\left[\frac{\dot{C}}{C}-\frac{\dot{B}}{B}\right]+\left[\frac{\dot{C}}{C}-\frac{\dot{B}}{B}\right]\left[\frac{\ddot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}+\frac{\dot{f}_2T)}{f_2(T)}\frac{dT}{dt}\right]=0 \tag{61a}$$

$$\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\frac{\dot{C}}{C}\left[\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right]+\left[\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right]\left[\frac{\dot{f}_1(R)}{\dot{f}_1(R)}\frac{dR}{dt}+\frac{\dot{f}_2 T)}{f_2(T)}\frac{dT}{dt}\right]=0 \tag{61b}$$

Integrating (61) we get

$$\frac{B}{C}=m_{33}exp\left\{m_{32}\int\frac{1}{BC^2\dot{f}_1(R)f_2(T)}dt\right\} \tag{61a}$$

$$\frac{C}{B}=m_{35}exp\left\{m_{34}\int\frac{1}{BC^2\dot{f}_1(R)f_2(T)}dt\right\} \tag{61b}$$

Where $m's$ are constants of integration with the condition that $m_{33}m_{35}=1$ and $m_{32}+m_{34}=0$

We can express explicitly as

$$C=(BC^2)^{\frac{1}{3}}m_{36}exp\left\{m_{37}\int\frac{1}{BC^2\dot{f}_1(R)f_2(T)}dt\right\} \tag{62a}$$

$$B=(BC^2)^{\frac{1}{3}}m_{38}exp\left\{m_{39}\int\frac{1}{BC^2\dot{f}_1(R)f_2(T)}dt\right\} \tag{62b}$$

From (60) and (62a) we obtain

$$A=(BC^2)^{\frac{1}{3}}m_{40}exp\left\{m_{41}\int\frac{1}{BC^2\dot{f}_1(R)f_2(T)}dt\right\} \tag{62c}$$

By converting C in to A we get

$$A=(ABC)^{\frac{1}{3}}m_{42}exp\left\{m_{43}\int\frac{1}{ABC\dot{f}_1(R)f_2(T)}dt\right\} \tag{63a}$$

$$B=(ABC)^{\frac{1}{3}}m_{44}exp\left\{m_{45}\int\frac{1}{ABC\dot{f}_1(R)f_2(T)}dt\right\} \tag{63b}$$

$$C=(ABC)^{\frac{1}{3}}m_{46}exp\left\{m_{47}\int\frac{1}{ABC\dot{f}_1(R)f_2(T)}dt\right\} \tag{63c}$$

Where $m's$ are constant of integration.

Adjusting the constants in (41) and (63), the line element (15) assumes an isotropic form and hence we can generalize the results in the form of the following theorem.

Theorem 1: In $f(R,T)$theory of gravity the Bianchi type V space-time, filled with combination of perfect fluid and scalar field coupled with electromagnetic field, admits isotropy for the functional form $f(R,T)=f_1(R)+\lambda f_2(T)$, $f(R,T)=f_1(R)f_2(T)$

With the help of (52) we can rewrite the equation (23) as

$$\left(\frac{\dot{h}_8}{h_8}\right)^{\cdot}+\frac{\dot{h}_8{}^2}{h_8{}^2}+\frac{\dot{h}_8}{h_8}\left[\frac{\dot{C}}{C}+\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right]=\dot{\psi}\dot{\varphi}^2 \tag{64a}$$

$$\left(\frac{\dot{h}_8}{h_8}\right)^{\cdot}+\frac{\dot{h}_8{}^2}{h_8{}^2}+\frac{\dot{h}_8}{h_8}\left[\frac{\dot{A}}{A}+\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right]=\dot{\psi}\dot{\varphi}^2 \tag{64b}$$

$$\left(\frac{\dot{h}_8}{h_8}\right)^{\cdot}+\frac{\dot{h}_8{}^2}{h_8{}^2}+\frac{\dot{h}_8}{h_8}\left[\frac{\dot{B}}{B}+\frac{\dot{C}}{C}-\frac{\dot{A}}{A}\right]=\dot{\psi}\dot{\varphi}^2 \tag{64c}$$

Further these equations imply that

$$\frac{\dot{C}}{C}+\frac{\dot{A}}{A}-\frac{\dot{B}}{B}=\frac{\dot{A}}{A}+\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=\frac{\dot{B}}{B}+\frac{\dot{C}}{C}-\frac{\dot{A}}{A}$$

Notes

Notes

Or
$$\frac{\dot{A}}{A}=\frac{\dot{B}}{B}=\frac{\dot{C}}{C} \tag{65}$$

Inserting (65) in (64) we get
$$\left(\frac{\dot{h}_8}{h_8}\right)^{\cdot}+\frac{\dot{h}_8{}^2}{h_8{}^2}+\frac{\dot{h}_8}{h_8}\left[\frac{\dot{A}}{A}\right]=\dot{\psi}\dot{\varphi}^2 \tag{66}$$

But from (53) we have
$$\dot{\psi}\,\dot{\varphi}^2=\left(\frac{\dot{h}_8}{h_8}\right)^2+\frac{\mathrm{f}_1(\mathrm{R})\dot{\mathrm{f}}_2(\mathrm{T})}{\chi}I\ddot{\psi}\dot{\varphi}^2 \tag{67}$$

Inserting (67) in (66) we have
$$\left(\frac{\dot{h}_8}{h_8}\right)^{\cdot}+\frac{\dot{h}_8}{h_8}\left[\frac{\dot{A}}{A}\right]=\frac{\mathrm{f}_1(\mathrm{R})\dot{\mathrm{f}}_2(\mathrm{T})}{\chi}I\ddot{\psi}\dot{\varphi}^2 \tag{68}$$

If $\ddot{\psi}=0$ or $\psi=m_{37}I+m_{38}$ then equation (68) has perfect solution and its solution is
$$h_8=m_{49}exp\left\{m_{48}\int\frac{1}{A}dt\right\} \tag{69}$$

With the help of (69) the equation (54) convert in to
$$V_1=m_{50}exp\left\{m_{48}\int\frac{1}{A}dt\right\} \tag{70a}$$
$$V_2=m_{51}exp\left\{m_{48}\int\frac{1}{A}dt\right\} \tag{70b}$$
$$V_3=m_{52}exp\left\{m_{48}\int\frac{1}{A}dt\right\} \tag{70c}$$

where $m's$ are constants of integration.

Adjusting the constants in (48) and (70), the vector potential assumes the following form
$$V_i=[V_1,V_1,V_1,0]$$

Hence we generalize the result in the form of following theorem

Theorem 2: In $f(R,T)$ theory of gravity, the Bianchi type V space-time filled with combination of perfect fluid and scalar field coupled with electromagnetic field, admits the vector potential in the form $V_i=[V_1,V_1,V_1,0]$ for the functional form $f(R,T)=f_1(R)+\lambda f_2(T)$ and $f(R,T)=f_1(R)f_2(T)$.

VII. Sub Case $f(R,T)=F(R)$

In this case we follow the notations $f_R(R,T)=\frac{\partial f(R)}{\partial R}=\dot{f}(R)$, $f_T(R,T)=\frac{\partial f(R)}{\partial T}=0$

In this case equation (5) reduces to
$$G_j^i=\frac{1}{\dot{f}(R)}\left[g^{im}\nabla_m\nabla_j\dot{f}(R)\right]-\frac{1}{6\dot{f}(R)}\left[\dot{f}(R)R+f(R)\right]g_j^i+\frac{\chi}{\dot{f}(R)}\left[T_j^i-\frac{1}{3}Tg_j^i\right] \tag{71}$$

The computation for this case easily follows from those of the earlier case (section 5) by mere substitution of $f_1(R)=f(R)$, $\lambda=0$ or $f_2(T)=0$

We get the result as follows

$$A = (ABC)^{\frac{1}{3}} l_{53} exp\left\{l_{54} \int \frac{1}{ABC} dt\right\} \quad (72a)$$

$$B = (ABC)^{\frac{1}{3}} l_{55} exp\left\{l_{56} \int \frac{1}{ABC} dt\right\} \quad (72b)$$

$$C = (ABC)^{\frac{1}{3}} l_{57} exp\left\{l_{58} \int \frac{1}{ABC} dt\right\} \quad (72c)$$

Where $l's$ are constants of integration.

$$V_1 = l_{63} exp\left\{l_{61} \int \frac{1}{A} dt\right\} \quad (73a)$$

$$V_2 = l_{64} exp\left\{l_{61} \int \frac{1}{A} dt\right\} \quad (73b)$$

$$V_3 = l_{65} exp\left\{l_{61} \int \frac{1}{A} dt\right\} \quad (73c)$$

where $l's$ be constant of integration.

From section 5, 6 and 7 we observe that the result remain intact for $f(R,T) = f_1(R) + \lambda f_2(T)$, $f(R,T) = f_1(R) f_2(T)$ and $f(R,T) = f(R)$ only differ in constants of integration. Hence the equations (72) and (73) admit the theorem 1 and 2.

VIII. Sub Case $f(R,T) = R + \lambda T$

In this case we follow the notations $f_R(R,T) = \frac{\partial f(R,T)}{\partial R} = 1$, $f_T(R,T) = \frac{\partial f(R,T)}{\partial T} = \lambda$

In this case the field equation (5) reduces to

$$G_j^i = \chi T_j^i - \lambda\left[T_j^i + \theta_j^i\right] + \frac{\lambda}{2} T g_j^i \quad (74)$$

The consideration of this sub case follows from (section 5) $f(R,T) = f_1(R) + \lambda f_2(T)$ by taking $f_1(R) = R$ and $f_2(T) = T$.

We get the result as follows

$$A = (ABC)^{\frac{1}{3}} l_{53} exp\left\{l_{54} \int \frac{1}{ABC} dt\right\} \quad (75a)$$

$$B = (ABC)^{\frac{1}{3}} l_{55} exp\left\{l_{56} \int \frac{1}{ABC} dt\right\} \quad (75b)$$

$$C = (ABC)^{\frac{1}{3}} l_{57} exp\left\{l_{58} \int \frac{1}{ABC} dt\right\} \quad (75c)$$

Where $l's$ are constants of integration.

$$V_1 = l_{63} exp\left\{l_{61} \int \frac{1}{A} dt\right\} \quad (76a)$$

$$V_2 = l_{64} exp\left\{l_{61} \int \frac{1}{A} dt\right\} \quad (76b)$$

$$V_3 = l_{65} exp\left\{l_{61} \int \frac{1}{A} dt\right\} \quad (76c)$$

where $l's$ be constant of integration.

Notes

From section 5, 6 and 8 we observe that the result remain intact for $f(R,T)=f_1(R)+\lambda f_2(T)$, $f(R,T)=f_1(R)f_2(T)$ and $f(R,T)=R+\lambda T$ only differ in constants of integration. Hence the equations (75) and (76) admit the theorem 1 and 2.

IX. Sub Case $f(R,T)=RT$

In this case we follow the notations

$$f_R(R,T)=\frac{\partial f(R,T)}{\partial R}=T\ ,\quad f_T(R,T)=\frac{\partial f(R,T)}{\partial T}=R$$

Then the field equation (8) reduces to

$$G_j^i=\frac{1}{T}\left[g^{mi}\nabla_m\nabla_j T\right]-\frac{1}{3}Rg_j^i+\frac{\chi}{T}\left[T_j^i-\frac{1}{3}Tg_j^i\right]+\frac{R}{3T}[T+\theta]g_j^i-\frac{R}{T}\left[T_j^i+\theta_j^i\right] \tag{77}$$

The computation for this case easily follows from those of the earlier case (section 6) by mere substitution of $f_1(R)=R$, and $f_2(T)=T$

We get the result as follows

$$A=(ABC)^{\frac{1}{3}}n_{42}exp\left\{n_{43}\int\frac{1}{ABCT}dt\right\} \tag{78a}$$

$$B=(ABC)^{\frac{1}{3}}n_{44}exp\left\{n_{45}\int\frac{1}{ABCT}dt\right\} \tag{78b}$$

$$C=(ABC)^{\frac{1}{3}}n_{46}exp\left\{n_{47}\int\frac{1}{ABCT}dt\right\} \tag{78c}$$

Where $n's$ are constant of integration.

$$V_1=n_{52}exp\left\{n_{50}\int\frac{1}{A}dt\right\} \tag{79a}$$

$$V_2=n_{53}exp\left\{n_{50}\int\frac{1}{A}dt\right\} \tag{79b}$$

$$V_3=n_{54}exp\left\{n_{50}\int\frac{1}{A}dt\right\} \tag{79c}$$

Where $n's$are constants of integration.

From section 5, 6 and 9 we observe that the result remain intact for $f(R,T)=f_1(R)+\lambda f_2(T)$, $f(R,T)=f_1(R)f_2(T)$ and $f(R,T)=RT$ only differ in constants of integration. Hence the equations (75) and (76) admit the theorem 1 and 2.

X. Conclusion

1) In the present paper we have considered the Bianchi type V model in combination of perfect fluid and scalar field coupled with electromagnetic field in sub cases of $f(R,T)$ theory of gravity models

(i) $f(R,T)=f_1(R)+\lambda f_2(T)$
(ii) $f(R,T)=f(R)$
(iii) $f(R,T)=R+\lambda T$
(iv) $f(R,T)=R$
(v) $f(R,T)=f_1(R)f_2(T)$
(vi) $f(R,T)=RT$

Notes

We have derived the gravitational field equations corresponding to the general and particular cases of $f(R,T)$ theory of gravity.

2) It is observed that, even though the cases of $f(R,T)$ theory are distinct, the convergent, non-singular, isotropic solutions can be evolved in each case along with the components vector potential.
3) From finding of the $f(R,T)$ and $f(R)$ theory, general and particular cases, in this paper we believe firmly that the results of $f(R,T)$ and $f(R)$ depends on only R and not on T
4) From different cases of $f(R,T)$ we observe that the results remain intact only differ in constants of integration.

Notes

References Références Referencias

1. Rybakov Yu. P, Shikin G N and Saha B : Int. J. Theor. Phys. **36**, 6,pp. 1475-1494, 1997.
2. Perlmeter S et al. (Supernova Cosmological Project Collaboration); Nature **391**, 51, 1998.
3. Riess A G et al. (Supernova Search Team Collaboration); Astron. J., **116**, 1009, 1998.
4. Perlmeter S et al. (Supernova Cosmological Project Collaboration); Astrophys. J., **517**, 565,1999.
5. Tonry J L et al. (Supernova Search Team Collaboration); Astrophys. J., **594**,1,2003.
6. Perlmutter S et al. Astrophys. J., **598**,102, 2003.
7. Nojiri S and Odintsov S D : Phys. Rev. D 68, 123512 (2003).
8. Riess A G et al. (Supernova Search Team Collaboration); Astrophys. J., 607, **665**, 2004.
9. Clocchiatti A et al. (High Z S N Search Collaboration); Astrophys. J., **642**,1,2006.
10. Harko T, Lobo F S N, Nojiri S, Odintsov S D: Phys. Rev. D **84**,024020 (2011).
11. Bijan Saha, Int. Theor. Phys. DOI 10.1007/s 10773-015-2615-1, 31 March 2015.
12. Myrzakulov Ratbay,Ertan Gudekli, Razina Olga and Tsyba Pyotr : Int. J. Of Maths.Computational, Natural and Physical Engineering, **9,** 2, 2015.