Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 16 Issue 2 Version 1.0 Year 2016
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Improved Class of Ratio Estimators for Finite Population Variance

By Audu Ahmed, Adedayo Amos Adewara, Ran Vijay Kumar Singh
Usmanu Danfodiyo University Sokoto, Nigeria

Abstract- In this paper, we have suggested a class of improved ratio estimators for finite population variance. The proposed class of estimators is obtained by transforming both the sample variances of study and auxiliary variables. The MSE of the proposed estimators have been obtained and the conditions for their efficiency over some existing variance estimators have been established. The present family of finite variance estimator, having obtaining the optimal values of the constants, exhibit significant improvement over the estimators considered in the study. The empirical study is also conducted to corroborate the theoretical results and the results show that the proposed class of estimators is more efficient.

Keywords: *efficiency, mean square error, ratio estimator, finite population, variance.*

GJSFR-F Classification : *MSC 2010: 00A05*

IMPROVEDCLASSOFRATIOESTIMATORSFORFINITEPOPULATIONVARIANCE

Strictly as per the compliance and regulations of :

Improved Class of Ratio Estimators for Finite Population Variance

Audu Ahmed [α], Adedayo Amos Adewara [σ] & Ran Vijay Kumar Singh [ρ]

Abstract- In this paper, we have suggested a class of improved ratio estimators for finite population variance. The proposed class of estimators is obtained by transforming both the sample variances of study and auxiliary variables. The MSE of the proposed estimators have been obtained and the conditions for their efficiency over some existing variance estimators have been established. The present family of finite variance estimator, having obtaining the optimal values of the constants, exhibit significant improvement over the estimators considered in the study. The empirical study is also conducted to corroborate the theoretical results and the results show that the proposed class of estimators is more efficient.

Keywords: efficiency, mean square error, ratio estimator, finite population, variance.

I. Introduction

The variation of produce or yields in the manufacturing industries and pharmaceutical laboratories are sometime a matter of concern to researchers (Ahmed et al. [2]). The use of supplementary (auxiliary) information, being constant with unit (e.g. population mean, population standard deviation, e.t.c.) or unit free constant (e.g. Coefficient of variation, Kurtosis, e.t.c.), can enhance the efficiency at the estimation stage. In recent past, this concept has been utilized by several authors to improve the efficiency of ratio and product type estimators for estimating population mean as well population variance of study variable.

In this paper, an improved class of ratio estimators for estimating finite population variance has been proposed with objective to produce efficient estimators and their properties have established.

Let $\Omega = (1\ 2, 3 \ldots N)$ be a population of size N and Y, X be two real valued functions having values $(Y_i, X_i) \in \mathbb{R}^+ > 0$ on the i^{th} unit of $U(1 \leq i \leq N)$. We assume positive correlation $\rho > 0$ between the study variable Y and auxiliary variable X. Let S_y^2 and S_x^2 be the finite population variance of Y and X respectively and s_y^2 and s_x^2 be respective sample variances based on the random sample of size n drawn without replacement.

Singh et al. [8] defined the general family of estimators for estimating finite population variance S_y^2 of the study variable Y as

Author α: Department of Mathematics, Usmanu Danfodiyo University Sokoto, Nigeria. e-mail: ahmed.audu@udusok.edu.ng
Author σ: Department of Statistics, University of Ilorin, Kwara State, Nigeria. e-mail: aaadewara@gmail.com
Author ρ: Department of Mathematics, Kebbi State University of Sci. and Tech. Aliero, Nigeria. e-mail: singhrvk13@gmail.com

$$\eta = s_y^2\left[\frac{aS_x^2+b}{\alpha\left(as_x^2-b\right)+\left(1-\alpha\right)\left(aS_x^2-b\right)}\right] \quad (1.1)$$

where a and b are constants based on auxiliary variable X like coefficient of skewness, kurtosis and correlation coefficient etc. α is the constant that minimizes the mean square error (MSE) of the estimator. Table 1 shows some members of η-family for different values of a, b and α.

The MSEs/Variance of the estimators in Table 1 are given below:

$$Var\left(\eta_0\right) = \gamma S_y^2\left(\psi_{40}-1\right) \quad (1.2)$$

$$MSE\left(\eta_i\right) = \begin{cases} S_y^4\gamma\left[\left(\psi_{40}-1\right)+\left(\psi_{04}-1\right)-2\left(\psi_{22}-1\right)\right] & i=1 \\ S_y^4\gamma\left[\left(\psi_{40}-1\right)+h_i^2\left(\psi_{04}-1\right)-2h_i\left(\psi_{22}-1\right)\right] & i=2,3,4,5,6 \end{cases} \quad (1.3)$$

$$h_2=\frac{S_x^2}{S_x^2-C_x},\ h_3=\frac{S_x^2}{S_x^2-\beta_2(x)},\ h_4=\frac{S_x^2\beta_2(x)}{S_x^2\beta_2(x)-C_x},\ h_5=\frac{S_x^2C_x}{S_x^2C_x-\beta_2(x)},\ h_6=\frac{S_x^2}{S_x^2+\beta_2(x)}$$

where

$$s_y^2=\frac{1}{n-1}\sum_{i=1}^{n}\left(y_i-\bar{y}\right)^2,\ s_x^2=\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2,\ S_y^2=\frac{1}{N-1}\sum_{i=1}^{n}\left(Y_i-\bar{Y}\right)^2,\ S_x^2=\frac{1}{N-1}\sum_{i=1}^{n}\left(X_i-\bar{X}\right)^2$$

$$\bar{X}=\frac{1}{N}\sum_{i=1}^{N}X_i,\quad \bar{Y}=\frac{1}{N}\sum_{i=1}^{N}Y_i,\quad \bar{x}=\frac{1}{n}\sum_{i=1}^{n}x_i,\quad \bar{y}=\frac{1}{n}\sum_{i=1}^{n}y_i$$

$$\gamma=\frac{1}{n},\ \psi_{rs}=\frac{\lambda_{rs}}{\lambda_{20}^{r/2}\lambda_{02}^{s/2}}\ \text{and}\ \lambda_{rs}=\frac{1}{N-1}\sum_{i=1}^{N}\left(Y_i-\bar{Y}\right)^r\left(X_i-\bar{X}\right)^s$$

The MSE of η to second order approximation is given below:

$$MSE\left(\eta\right)=\gamma S_y^2\left[\left(\psi_{40}-1\right)+\alpha^2\theta^2\left(\psi_{04}-1\right)-2\alpha\theta\left(\psi_{22}-1\right)\right] \quad (1.4)$$

The $MSE\left(\eta\right)$ expression is minimized for the optimum values of α given by equation (1.5). This is obtained by partial differentiation of equation (1.4) with respect to α.

$$MSE\left(\eta\right)_{\min}=\gamma S_y^4\left[\left(\psi_{40}-1\right)-\frac{\left(\psi_{22}-1\right)^2}{\left(\psi_{04}-1\right)}\right] \quad (1.5)$$

Many other researchers including Kadilar and Cingi [6], Yadav and Kadilar [15], Singh and Solanki [10], Gupta and Shabir [3], Subramani and Kumarapandiyan [13], Singh and Vishwakarma [11], Singh, et al. [9], Sanaullah, et al. [7] and Solanki and Singh [12] have significantly contributed to the improvement of both ratio and product mean & variance estimators in sampling survey.

Ref

3. S. Gupta, J. Shabbir, Variance estimation in simple random sampling using auxiliary information, Hacettepe Journal of Mathematics and Statistics 37 (2008) 57–67.

II. Proposed Estimator

After studying the related finite population variance estimators stated in section 1 and motivated by the work of Yadav and Kadilar [16] and Adewara et al.[1] estimators for population mean in which the former (i.e. Yadav and Kadilar [16]) equals the latter (i.e. Adewara et al.[1]) when $k_1 = k_2 = k_3 = k_4 = k_5 = k_6 = 1$. Their estimators are defined as

$$\eta_1^* = k_1 \bar{y}^* \left(\frac{\bar{X}}{\bar{x}^*} \right),\ \eta_2^* = k_2 \bar{y}^* \left(\frac{\bar{x}^*}{\bar{X}} \right),\ \eta_3^* = k_3 \bar{y}^* \left(\frac{\bar{X} + C_x}{\bar{x}^* + C_x} \right),$$

$$\eta_4^* = k_4 \bar{y}^* \left(\frac{\bar{x}^* + C_x}{\bar{X} + C_x} \right),\ \eta_5^* = k_5 \bar{y}^* \left(\frac{\bar{X} + \rho}{\bar{x}^* + \rho} \right),\ \eta_6^* = k_6 \bar{y}^* \left(\frac{\bar{x}^* + \rho}{\bar{X} + \rho} \right)$$

Where $\bar{x}^*$ and $\bar{y}^*$ are the respective sample means of the auxiliary and study variables, having the relationship: (1) $\bar{X} = f\,\bar{x} + (1-f)\bar{x}^*$ (2) $\bar{Y} = f\,\bar{y} + (1-f)\bar{y}^*$ where $f = \frac{n}{N}$ is finite population correction, $k_i,\ i = 1,2,3,4,5,6,$ are real constants.

We proposed the following class of ratio estimator

$$\tau_1^* = k_1 s_y^{2*} \left(\frac{S_x^2}{s_x^{2*}} \right),\ \tau_2^* = k_2 s_y^{2*} \left(\frac{S_x^2 - C_x}{s_x^{2*} - C_x} \right),\ \tau_3^* = k_3 s_y^{2*} \left(\frac{S_x^2 - \beta_2(x)}{s_x^{2*} - \beta_2(x)} \right),\ \tau_4^* = k_4 s_y^{2*} \left(\frac{S_x^2 \beta_2(x) - C_x}{s_x^{2*} \beta_2(x) - C_x} \right),$$

$$\tau_5^* = k_5 s_y^{2*} \left(\frac{S_x^2 C_x - \beta_2(x)}{s_x^{2*} C_x - \beta_2(x)} \right),\ \tau_6^* = k_6 s_y^{2*} \left(\frac{S_x^2 + \beta_2(x)}{s_x^{2*} + \beta_2(x)} \right)$$

Where s_x^{2*} and s_y^{2*} are the respective sample finite variances of the auxiliary and study variables, having the relationship: (1) $S_x^2 = f\,s_x^2 + (1-f)s_x^{2*}$ (2) $S_y^2 = f\,s_y^2 + (1-f)s_y^{2*}$ with condition that $n < \frac{1}{2}N$.

In order to obtain the MSE, we defined $e_0 = \frac{s_y^2 - S_y^2}{S_y^2}$ and $e_1 = \frac{s_x^2 - S_x^2}{S_x^2}$ such that

$$\left. \begin{aligned} &E(e_0) = E(e_1) = 0,\ E(e_0^2) = \gamma(\psi_{40} - 1) \\ &E(e_1^2) = \gamma(\psi_{04} - 1),\ E(e_0 e_1) = \gamma(\psi_{22} - 1) \end{aligned} \right\} \quad (2.1)$$

Expressing $\tau_i^*,\ i = 1,2,3,4,5,6,$ in terms of e_0 and e_1, we have

$$\tau_i^* = \left. \begin{matrix} k_i S_y^2 (1 - v e_0)(1 - v e_1)^{-1} & i = 1 \\ k_i S_y^2 (1 - v e_0)(1 - v h_i e_1)^{-1} & i = 2,3,4,5,6 \end{matrix} \right\} \quad (2.2)$$

now assume that $|ve_1| < 1$ and $|vh_i e_1| < 1$ so that $(1 - ve_1)^{-1}$ and $(1 - vh_i e_i)^{-1}$ are expandable. Expanding the right hand side of (2.2) up to second degree approximation,

Ref

1. A. A. Adewara, R. Singh, M. Kumar, Efficiency of some modified ratio and product estimators using known value of some population parameters, International Journal of Applied Science and Technology 2 (2) (2012) 76-79.

subtract S_y^2 from its both sides, square the both sides and taking expectation using the results in equation (2.1), we obtain the MSEs of the proposed estimators as:

$$MSE\left(\tau_i^*\right)=\begin{cases} S_y^4\left\{\gamma v^2\begin{bmatrix} k_i^2\left(\psi_{40}-1\right)+\left(3k_i^2-2k_i\right)\left(\psi_{04}-1\right) \\ -2\left(2k_i^2-k_i\right)\left(\psi_{22}-1\right)\end{bmatrix}+\left(k_i-1\right)^2\right\} & i=1 \\ S_y^4\left\{\gamma v^2\begin{bmatrix} k_i^2\left(\psi_{40}-1\right)+\left(3k_i^2-2k_i\right)h_i^2\left(\psi_{04}-1\right) \\ -2\left(2k_i^2-k_i\right)h_i\left(\psi_{22}-1\right)\end{bmatrix}+\left(k_i-1\right)^2\right\} & i=2,3,4,5,6 \end{cases} \quad (2.3)$$

The $MSE\left(\eta_i\right)$, $i=1,2,3,4,5,6$ expressions are minimized for the optimum values of k given by

$$k_i^*=\frac{v^2\gamma\left[\left(\psi_{04}-1\right)-\left(\psi_{22}-1\right)\right]+1}{v^2\gamma\left[3\left(\psi_{04}-1\right)-4\left(\psi_{22}-1\right)+\left(\psi_{40}-1\right)\right]+1}=\frac{A_1}{B_1}, \quad i=1 \quad (2.4)$$

$$k_i^*=\frac{v^2\gamma\left[h_i^2\left(\psi_{04}-1\right)-h_i\left(\psi_{22}-1\right)\right]+1}{v^2\gamma\left[3h_i^2\left(\psi_{04}-1\right)-4h_i\left(\psi_{22}-1\right)+\left(\psi_{40}-1\right)\right]+1}=\frac{A_2}{B_2}, \quad i=2,3,4,5,6 \quad (2.5)$$

where $v=\dfrac{n}{N-n}$

Replacing k_i by k_i^*, $i=1,2,3,4,5,6$ in equation (2.3), we obtain the minimum MSE as

$$MSE_{\min}\left(\tau_i^*\right)=\begin{cases} S_y^4\left(1-\dfrac{A_1^2}{B_1}\right), & i=1 \\ S_y^4\left(1-\dfrac{A_2^2}{B_2}\right), & i=2,3,4,5,6 \end{cases} \quad (2.6)$$

III. Efficiency Comparisons

In this section efficiencies of the proposed estimators are compared with efficiencies of some estimators in the literature

The $\tau_i^*-family$ of estimators of the population variance is more efficient than η_0 if,

$$MSE_{\min}\left(\tau_i^*\right)<Var\left(\eta_0\right) \quad i=1,2,3,4,5,6$$

$$\left.\begin{matrix} \left(1-\dfrac{A_1^2}{B_1}\right)<\gamma\left[\left(\psi_{40}-1\right)\right] & i=1 \\ \left(1-\dfrac{A_2^2}{B_2}\right)<\gamma\left[\left(\psi_{40}-1\right)\right] & i=2,3,4,5,6 \end{matrix}\right\} \quad (3.1)$$

The $\tau_i^*-family$ of estimators of the population variance is more efficient than $\eta_i-family$ if,

Notes

$$MSE_{\min}\left(\tau_i^*\right) < MSE\left(\eta_i\right) \quad i = 1,2,3,4,5,6$$

$$\left.\begin{aligned} &\left(1-\frac{A_1^2}{B_1}\right) < \gamma\left[\left(\psi_{40}-1\right)+\left(\psi_{04}-1\right)-2\left(\psi_{22}-1\right)\right] && i=1 \\ &\left(1-\frac{A_2^2}{B_2}\right) < \gamma\left[\left(\psi_{40}-1\right)+h_i^2\left(\psi_{04}-1\right)-2h_i\left(\psi_{22}-1\right)\right] && i=2,3,4,5,6 \end{aligned}\right\} \qquad (3.2)$$

The $\tau_i^* - family$ of estimators of the population variance is more efficient than η if,

$$MSE_{\min}\left(\tau_i^*\right) < MSE_{\min}\left(\eta\right) \quad i = 1,2,3,4,5,6$$

$$\left.\begin{aligned} &\left(1-\frac{A_1^2}{B_1}\right) < \gamma\left[\left(\psi_{40}-1\right)-\left(\psi_{04}-1\right)\right] && i=1 \\ &\left(1-\frac{A_2^2}{B_2}\right) < \gamma\left[\left(\psi_{40}-1\right)-\left(\psi_{04}-1\right)\right] && i=2,3,4,5,6 \end{aligned}\right\} \qquad (3.3)$$

When conditions (3.1), (3.2) and (3.3) are satisfied, we can conclude that the family of proposed class of estimators is more efficient than the $\eta - family$ estimator.

IV. Empirical Study

In order to investigate the merits of the proposed estimators, we have considered the following two real populations given as:

Data 1: Subramani and Kumarapandiyan [13]

$$N = 49,\ n = 20,\ \bar{Y} = 116.1633,\ \bar{X} = 98.6765,\ \rho = 0.6904,\ S_y = 98.8286$$

$$S_x = 102.9709,\ C_y = 0.8508,\ C_x = 1.0435,\ \psi_{40} = 4.9245,\ \psi_{04} = 5.9878,\ \psi_{22} = 4.6977$$

Data 2: Subramani and Kumarapandiyan [13]

$$N = 80,\ n = 20,\ \bar{Y} = 51.8264,\ \bar{X} = 11.2646,\ \rho = 0.9413,\ S_y = 18.3569$$

$$S_x = 8.4563,\ C_y = 0.3542,\ C_x = 0.7507,\ \psi_{40} = 2.2667,\ \psi_{04} = 2.8664,\ \psi_{22} = 2.2209$$

The numerical demonstration to justify the appropriateness of the suggested class of variance estimator has been conducted using the two data sets. Table 2 and Table 3 show the mean square errors of proposed estimators and that of some existing estimators and their percentage relative efficiencies of different estimators with respect to s_y^2 respectively.

V. Conclusion

From section 3, the theoretical conditions obtained for the efficiencies of the proposed estimators supported by the numerical illustration in section 4 given in the Table 2 and 3. From the result, we infer that the suggested variance estimators produce a better estimate of finite population variance than the existing estimators in the sense of having higher percentage relative efficiency which implies lesser mean square error.

Ref

13. J. Subramani, G. Kumarapandiyan, Variance estimation using quartiles and their functions of an auxiliary variable, International Journal of Statistics and Applications 2 (5) (2012) 67–72.

References Références Referencias

1. A. A. Adewara, R. Singh, M. Kumar, Efficiency of some modified ratio and product estimators using known value of some population parameters, International Journal of Applied Science and Technology 2 (2) (2012) 76-79.
2. M.S. Ahmed, W.A. Dayyeh, A.A.O. Hurairah, Some estimators for finite population variance under two-phase sampling, Statistics in Transition 6 (1) (2003) 143-150.
3. S. Gupta, J. Shabbir, Variance estimation in simple random sampling using auxiliary information, Hacettepe Journal of Mathematics and Statistics 37 (2008) 57–67.
4. C.T. Isaki, Variance estimation using auxiliary information, Journal of the American Statistical Association 78 (1983) 117–123.
5. C. Kadilar, H. Cingi, Ratio estimators for population variance in simple and stratified sampling, Applied Mathematics and Computation 173 (2006) 1047–1059.
6. C. Kadilar, H. Cingi, Improvement in variance estimation in simple random sampling, Communications in Statistics – Theory and Methods 36 (2007) 2075–2081.
7. A. Sanaullah, H. Khan, A. Ali, R. Singh, Improved ratio-type estimators in survey sampling, Journal of Reliability and Statistical Studies 5 (2) (2012) 119–132.
8. R. Singh, P. Chauhan, N. Sawan, F. Smarandache, A general family of estimators for estimating population variance using known value of some population parameters, Far East Journal of Theoretical Statistics (2007).
9. R. Singh, P. Chauhan, N. Sawan, F. Smarandache, Improved exponential estimator for population variance using two auxiliary variables, Italian Journal of Pure and Applied Mathematics 28 (2011) 101–108.
10. H. P. Singh, R. S. Solanki, A new procedure for variance estimation in simple random sampling using auxiliary information, Statistical Papers 54(2) (2013) 479–497.
11. H. P. Singh, G. Vishwakarma, Some families of estimators of variance of stratified random sample mean using auxiliary information, Journal of Statistical Theory and Practice 2 (1) (2008) 21–43.
12. R. S. Solanki, H. P. Singh, An improved class of estimators for the population variance, Model Assisted Statistics and Applications 8(3) (2013) 229– 238.
13. J. Subramani, G. Kumarapandiyan, Variance estimation using quartiles and their functions of an auxiliary variable, International Journal of Statistics and Applications 2 (5) (2012) 67–72.
14. L.N. Upadhyaya, H. P. Singh, An estimator for population variance that utilizes the kurtosis of an auxiliary variable in sample surveys, Vikram Mathematical Journal, 19 (1999) 14-17.
15. S. K. Yadav, C. Kadilar, Improved exponential type ratio estimator of population variance, Revista Colombiana de Estadística 36(1) (2013) 145–152.
16. S. K. Yadav, C. Kadilar, Improved class of ratio and product estimators, Applied Mathematics and Computation 219 (2013) 10726–10731.

Notes

Table 1 : Some Member of η-family for different values of a, b and α

Estimator	a	b	α
$\eta_0 = s_y^2$ Sample variance	1	0	0
$\eta_1 = s_y^2\left(\frac{S_x^2}{s_x^2}\right)$ Isaki [4]	1	0	1
$\eta_2 = s_y^2\left(\frac{S_x^2 - C_x}{s_x^2 - C_x}\right)$ Kadilar and Cingi [5]	1	C_x	1
$\eta_3 = s_y^2\left(\frac{S_x^2 - \beta_2(x)}{s_x^2 - \beta_2(x)}\right)$	1	$\beta_2(x)$	1
$\eta_4 = s_y^2\left(\frac{S_x^2\beta_2(x) - C_x}{s_x^2\beta_2(x) - C_x}\right)$	$\beta_2(x)$	C_x	1
$\eta_5 = s_y^2\left(\frac{S_x^2 C_x - \beta_2(x)}{s_x^2 C_x - \beta_2(x)}\right)$	C_x	$\beta_2(x)$	1
$\eta_6 = s_y^2\left(\frac{S_x^2 + \beta_2(x)}{s_x^2 + \beta_2(x)}\right)$ Upadhyaya and Singh [14]	1	$-\beta_2(x)$	1

Table 2 : MSE of different estimators

Estimator	Data 1	Data 2	Estimator	Data 1	Data 2
η_0	187190	7191.859	η_{opt}	5643700	2657.427
$\eta - family$ **(Singh et al. [7] estimators)**			$\tau^* - family$ (Newly proposed estimators)		
η_1	723531	3924.948	τ_1^*	3053941	429.9105
η_2	723652	4003.906	τ_2^*	3054289	438.3024
η_3	724227	4249.508	τ_3^*	3055940	464.3576
η_4	723551	3952.035	τ_4^*	3053999	432.7903
η_5	724198	4372.12	τ_5^*	3055857	477.3385
η_6	722837	3658.1	τ_6^*	3051947	401.5034

Ref

14. L.N. Upadhyaya, H. P. Singh, An estimator for population variance that utilizes the kurtosis of an auxiliary variable in sample surveys, Vikram Mathematical Journal, 19 (1999) 14-17.

Table 3 : PRE of different estimators with respect to s_y^2

Estimator	Data 1	Data 2	Estimator	Data 1	Data 2
η_0	100.00	100.00	η_{opt}	331.6813	270.6324
$\eta - family$ (Singh et al. [7] estimators)			$\tau^* - family$ (Newly proposed estimators)		
η_1	258.7184	183.2345	τ_1^*	612.949	1672.873
η_2	258.6751	179.6211	τ_2^*	612.8791	1640.844
η_3	258.4697	169.2398	τ_3^*	612.5479	1548.776
η_4	258.7112	181.9786	τ_4^*	612.9373	1661.742
η_5	258.4801	164.4934	τ_5^*	612.5646	1506.658
η_6	258.9668	196.5956	τ_6^*	613.3494	1791.233

Notes