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On the Hazard Rate Functions of HIV/Aids using Weibull and Exponential Models

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I. Introduction

The hazard rate or mortality rates function $h(t)$ is usually obtained from the survival function and is defined as the probability that an individual who is under observation at a time t has an event at time t. It represents the instantaneous event rate for an individual who has already survival to time t.

The hazard rate $h(t)$, for a given distribution whose survival function is given by $S(t)$ is defined as

$$h(t) = \frac{-\frac{d}{dt}S(t)}{S(t)} \qquad \text{-----} (1)$$

II. Materials and Methods

The survival function of Weibull distribution is

$$\begin{aligned} S(t) &= P(T \geq t) \\ &= \int_t^\infty \lambda\gamma t^{\gamma-1} e^{-\lambda t^\gamma} dt \\ &= e^{-\lambda t^\gamma} \qquad \text{-----} (2) \end{aligned}$$

The hazard rate function is therefore

$$\begin{aligned} h(t) &= \frac{-\frac{d}{dt}e^{-\lambda t^\gamma}}{e^{-\lambda t^\gamma}} \\ &= \frac{\lambda\gamma t^{\gamma-1} e^{-\lambda t^\gamma}}{e^{-\lambda t^\gamma}} \\ &= \lambda\gamma t^\gamma \qquad \text{-----} (3) \end{aligned}$$

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For exponential distribution $S(t)$ is obtained as follows

$$S(t) = \int_t^\infty \lambda e^{-\lambda t}\, dt$$

$$= e^{-\lambda t} \quad \text{---------------------------------------} \quad (4)$$

$$h(t) = \frac{-\frac{d}{dt}e^{-\lambda t}}{e^{-\lambda t}}$$

$$= \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}}$$

$$= \lambda \quad \text{---------------------------------------} \quad (5)$$

Hence the hazard rate of exponential survival function is equal to the parameter and is always a constant.

In dynamics of epidemics such as HIV/AIDS the hazard rate is not constant. Hence the Weibull has assumption is preferred. In particular, the hazard rate increases as the period of infectiousness increases.

III. Results and Discussion

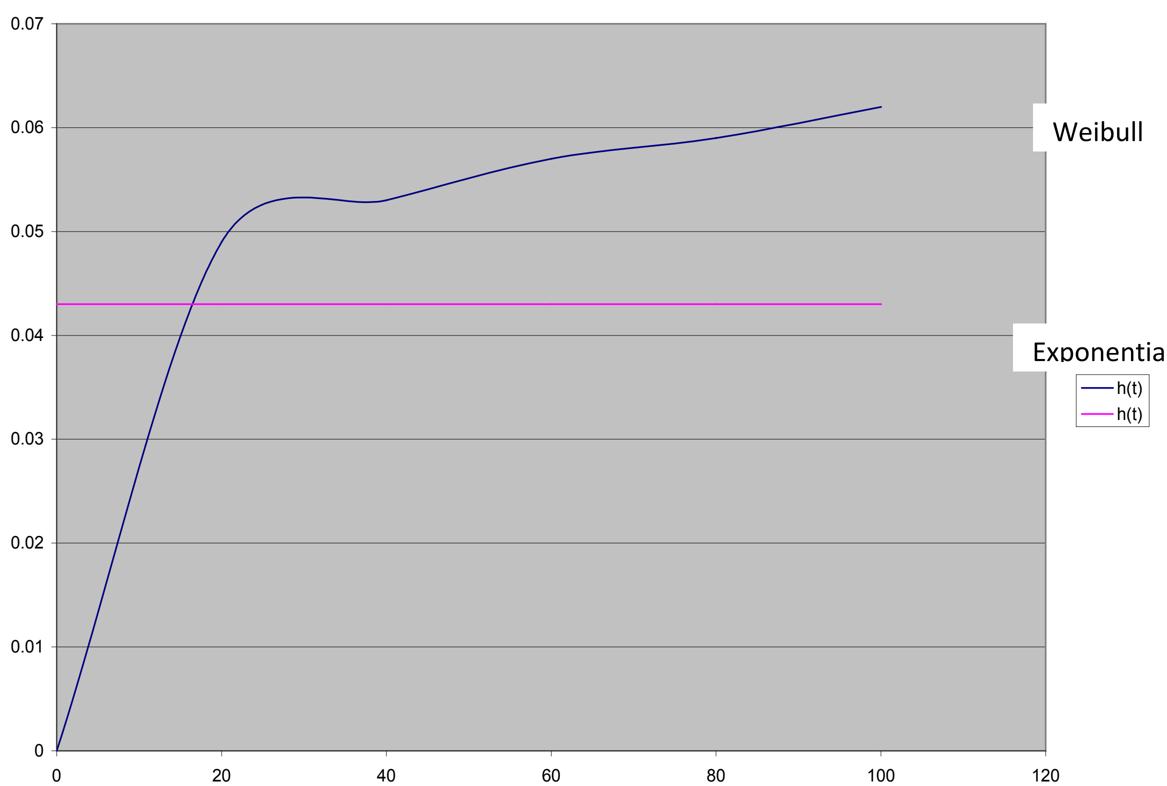
The Weibulll 7^{++} HIV/AIDS simulation for Weibull and Exponential Models with n = 500 yields for Weibull $\lambda = 0.0256$, $\gamma = 1.156$. For exponential $\lambda = 0.043$ With t (in months) = 0. 20, 40, 60, 80 and 100 yields the following tables For Weibull

T	0	20	40	60	80	100
h(t)	0	0.049	0.053	0.057	0.059	0.062

and for exponential

T	0	20	40	60	80	100
h(t)	0.043	0.043	0.043	0.043	0.043	0.043

The results are displayed in the following graph.



Notes

It could be observed from the graphical display that, the hazard rate using Exponential Survival Model is $\lambda = 0.043$ which is shown as a straight line with intercept 0.043 and is parallel to the t axis. Also looking at the display for Weibull's model, it increases sharply during the first 20 months of infectiousness and increases steadily from over 20 months.

Notes

IV. Conclusion

The hazard or mortality rate is the determiner of the number of years/months an individual diagnosed of HIV/AIDS or any other epidemic will survive before death.

The result obtained, for Exponential Function, show that the hazard rate is $\lambda = 0.043$ which is constant. This is not always the case with many epidemics like HIV/AIDS, for example, but the Weibull Model provided an in-depth information.

In the first 20 months of infection the failure rate increases sharply and this means that the immune system of an individual will breakdown devastatingly during this period this accounts for the fact that almost half of a cohort of infected individuals will die within the first 20 months approximately (Adeleke and Ogunwale 2013). The Weibull model is thus preferred to the Exponential Model because of this reason.

References Références Referencias

1. Anderson R.M, May R.M, (1991). Infectious Disease of Human: Dynamics and Control. Oxford U.P. Oxford
2. Anderson R.M, May R.M and Mclean A.R. (1988). Possible Demographic Consequences of AIDs in developing countries. Nature 332, 228-234.
3. Busenberg S, Cook K (1993). Vertically Transmitted Diseases: Models and Dynamics. Biomathematics 23, Spring, Berlin.
4. Odo Diemann and Heesterbek J.A.P (2000) Mathematical Epidemiology of Infectious Diseases.
5. Wiley series in Mathematical and Computational Biology. John Wiley & Sons, West Sub-Sex, England.