Abstract- This paper presents a model of the cosmic medium, which allows to describe the physical nature and microscopic structure of dark energy and has the macroscopic properties of both positive and negative pressure density. It is indicated that the cosmological constant \( \Lambda \) characterizes the elastic properties of the dark energy, and "antigravity universal law" is the law of elasticity Hooke. The evidence base is based on the results of experiments conducted in superfluid \(^3\)He-B in the p-state analogue projecting dark energy. In addition, for the construction of the resonance curves of the photoelectric effect in the near-Earth space environment, we used the results of observations obtained by space detector PAMELA, Fermi and detector AMS, installed on board the ISS.

Keywords: dark energy, dark matter, cosmological constant \( \Lambda \), gravitation, antigravitation, electron, positron, spin, mass, dipole, domain, pressure, density, polarization, superfluid \(^3\)He-B.

1. Introduction

Professor A.D. Chernin introduced into physics a new term "World antigravitation law" as the opposite of "law of gravitation" Isaac Newton \([1]\). Let's look at whether enough grounds for such a large-scale correction of physical laws.

Antigravitation cosmological theory of general relativity Einstein described by a linear force versus distance:

\[
FE = \left( \frac{c^2}{3} \right) \Lambda R,
\]

where \( \Lambda \) - the cosmological constant.

The interpretation of the cosmological constant in a spirit of understanding of the antigravitating environment with a constant density was the basis for the standard cosmological model \( \Lambda \)CDM (\( \Lambda \)- Cold Dark Matter).

The model \( \Lambda \)CDM dark energy is taken as an invisible space environment, physical nature and microscopic structure of which is unknown. However, it is assumed that the dark energy as a macroscopic medium has a number of special, peculiar only to her properties:

1) its density is positive and negative pressure and the energy density is equal in absolute value;

2) it does not create the attraction and antigravitation [1].

In this paper, based on the development of the theory of superfluid media are invited to expand the scope of the standard model and give a physical explanation of the cosmological acceleration, based on the structural features and the elastic properties of the space environment.

II. Experiments

a) Phase transitions in superfluid \(^3\)He-B as an analogue of dark energy and dark matter

In the early 21st century began to appear, which offers a model of the physical vacuum, with the properties of a superfluid liquid, consisting of a pair of oppositely electrically charged particles - fermions with zero total spin of the pair. This model describes the dielectric properties of the vacuum and the birth therein pairs of oppositely electrically charged particles (electron-positron) [4]. Further development of the theory of superfluid environment will consider the phase transitions in the model of the physical vacuum, similar to phase transitions in superfluid \(^3\)He-B [3]. L.B. Boldyrev in its superfluid model of the physical vacuum (SPV), has significantly expanded the analogy between the properties of superfluid \(^3\)He-B and the
In the space environment (dark energy and matter), mainly due to the account properties of vortices and the spin polarization of the medium in the vortex, the inertial properties of the vortex, and superfluid spin currents between them. [5] In this model SPV included the elements of the model of the physical vacuum G.I. Shipov [6] polarization model of heterogeneous physical vacuum V.L. Dyatlova [7] and the theory of ether A.V. Rykova [4].

First of all, it should be noted that the experimentally installed electric polarization of the medium in the core of the vortex in superfluid \(^3\)He-in is due to deformation of the atoms \(^3\)He consisting of electrically oppositely charged electrons and protons. A similar mechanism of electric dipole moment of the vortex core must exist in the cosmic medium, the microscopic structure of which are electrically oppositely charged electrons, positrons and protons, forming a dipole. Growth of the relative share of positrons in the total flux of positrons and electrons in the cosmic medium, since more than 30 GeV photon energy was discovered detector PAMELA, Fermi and others. Moreover, according to new data, the detector AMS, installed on board the ISS, positron spectrum with increasing energy to become more stringent, while the electron spectrum changes little. [8] It can serve as indirect evidence of the presence in near-Earth space environment of the two phase states, characterizing dark energy and dark matter as a unique two phases in \(^3\)He-B: A superconducting phase and spontaneously ferromagnetic \(\beta\) phase [15]. Further, in NASA experiments, conducted in 1989-1992, via spacecraft Cosmic Background Explorer (COBE), was established anisotropy of background radiation, which may also serve to confirm the heterogeneity of the structure in the space environment [10]. The relative motion of the particles that make up a Cooper pair in superfluid \(^3\)He-B corresponds to the p-state. In this state between the electrically charged particles of like with spins oriented in the same direction, there are forces of attraction, and between the electrically oppositely charged particles with spins oriented in the same direction of the force of repulsion.

Similarly, the interaction of particles in superfluid \(^3\)He-B must interact and microparticles in virtual space environment. In \(^3\)He-B holds the magnetization of the cores of the vortices along the axis of the vortex, that is, there is a spin polarization of superfluid in the core of the vortex. This phenomenon indicates that the superfluid \(^3\)He-acting effect in Barnett (there is a transfer of angular momentum coupled atoms \(^3\)He-B components whirlwind spins of atoms). This process is particularly significant in the core of the vortex. The existence of the effect in superfluid Barnett \(^3\)He-B is confirmed experimentally: vortices with opposite spins, the constituent atoms are characterized by the opposite direction of flow velocity of the liquid. By analogy, in the core of the vortex microparticles space environment holds spin polarization, ie, the orientation of the spins of the microparticles in the same direction. At the core of the vortex formed two spatially separated electrically oppositely charged "clusters" of microparticles. Therefore, one can speak of the electric dipole moment of the quantum object created by this vortex. We have said that in the district of state between the electrically oppositely charged microparticles with spin oriented along the same line, there are forces of repulsion.

The result of these forces is the appearance of an electric dipole moment of the vortex core. Since vortex cores in the space environment are electric dipoles, there is the electric polarization of the medium. This means that the pair of the microparticles constituting the space environment is "stretched" along the electric field. Thus, the space environment in a twist area can be characterized by a state of "full stretch". As part of the simulation model on the effect of superfluid vortex core can be mathematically described by introducing the pressure P at the boundary of the vortex core. Sign pressure depends on the nature of the internal stresses in the environment. If these internal stresses have the character of "comprehensive sprain", the pressure will be negative.

In addition, experimentally proved that when the photon energy \(W \geq 1\) MeV in the space environment (physical vacuum) takes place of a pair of elementary particles electrons and positrons with nonzero rest mass. This suggests that the density of the cosmic medium positive.

Thus, the proposed model of the space environment above (analog \(^3\)He-B) meets the characteristics of dark energy, and its microscopic structure does not contradict with modern physical notions.

As a result of astronomical telescope Planck universe is composed of:
- Dark energy (68.3%);
- Dark matter (26.8%);
- "Ordinary" (baryonic) matter (4.9%) [9].

Dark energy and dark matter form galactic and intergalactic medium, accounting for 95% of the average density of matter in the universe. This environment does not emit, absorb or reflect light, which is understandable, if we assume that it is a light-carrying medium. The key distinction of the dark energy of the dark matter is that dark matter attracts, possesses gravity, while dark energy, in a sense inherent anti-gravity. It causes the universe to expand rapidly. However, in the field of galaxies, where large masses of gravitating matter perturbs the vortex-wave processes, accompanied by spin precession microparticles in the vortex domain of the space environment, generate significant weight, a lot of large masses of particles in the medium.
Additional mass is created by the cosmic object in the medium (dark energy) due to inertial properties of the core of the vortex. The value added to the mass \( \Delta m \), is associated with the precession frequency of the spins of the microparticles in the core of the vortex, or, equivalently, with the frequency of the wave function \( \omega \).

The Schrödinger equation:

\[ \Delta m = \hbar \omega \text{sh} / c^2, \]

where \( c \) - velocity of light [5]

\[ F(\omega) = 6\pi \eta R [1 + R/\delta(\omega)] V(\omega) + 3\pi \rho R^2 \sqrt{2\pi \rho / \omega} [1 + 2R/9\delta(\omega)] \omega V(\omega), \]

\[ \delta(\omega) = (2\eta/\rho \omega)^{1/2} \]

where \( \rho \) - fluid density, \( \eta \) - the viscosity, \( V \) - velocity amplitude penetration sphere, \( \delta(\omega) \) - the so-called viscous penetration depth, which increases with an increase in viscosity and a decrease of the oscillation frequency.

The real part of the expression (2a) is a known Stokes force derived from the movement of fluid in the sphere. Imaginary component (coefficient of \( i\omega V \)) is naturally identified with the effective mass of the cluster added:

\[ M_{\text{eff}}(\omega R) = 2\pi \rho R^3 / 3 [1 + 9/2 \delta(\omega)/R] \]

Origin added (attached) mass \( M_{\text{eff}}(\omega R) \), depending on the frequency \( \omega \) and the radius \( R \) of the sphere of the cluster associated with the excitation of the field around a moving cluster of hydrodynamic velocity \( \omega (r) \) and the appearance in connection with this additional kinetic energy. In superfluid additional mass has two components: superfluid and normal [16].

Magnetic resonance experiment conducted with a rotating vortex cores. Also it found that in the case of superfluid \(^3\)He-B has an effect of Einstein - de Haas: this rotation liquid volume during magnetization. Since atom magnetization \(^3\)He means and their spin polarization, the effect of Einstein - de Haas - a volume of liquid during the rotation \( dS / dt \), where \( S \) is the total spin fluid volume selected. A similar effect should be observed, and in contact with the dipole vortex of dark energy in the magnetic field of the galaxy. Emerging with large domains have sufficient weight to gravitation and are the building blocks that make up dark matter.

In intergalactic space, where the disturbing factor of the masses of large space structures is absent, and there is no dark matter. RZG is the radius of zero gravity (outer space, where the force of gravity and repulsion are equal). When \( R < \text{RZG} \) predominant attraction, with \( R > \text{RZG} \) - repulsion. The paper A.Chernin [1] calculated the value of the radius around the local group RZG - gravitationally bound quasi-stationary system with a total mass \( M = (2 - 3) \times 10^{12} \) Me. This mass constitute the "normal" (baryonic) matter of stars and interstellar medium, and dark matter, which is about five times more. The value \( \text{RZG} = 1,3-1,4\text{Mpk.} \) [1].

A macroscopic approach, the hydrodynamic behavior of the added weight of spherical bodies of any nature (including those of charged clusters) in superfluid \(^3\)He-B (analogue of dark energy) is the primary source of job Stokes. It is a complex force \( F(\omega) \), exerted by the fluid on the sphere of radius \( R \), performs periodic oscillations with a frequency \( \omega \). Within the low Reynolds numbers we have:

\[ \text{LJ} = \text{cs} [\pi / (G\rho_0)]^{1/2} \]

This value depends on environmental parameters: velocity of acoustic vibrations in a medium (the speed of the longitudinal wave) \( \text{cs} \) and density \( \rho_0 \).

It defines the minimum scale perturbations, from which the elastic force of the medium are not able to withstand the forces of gravity, and that it leads to the gravitational instability of the medium [13]. In this small-size random packing medium grow in time if they cover an area of linear size \( L > \text{LJ} \). Perturbations with scales smaller than the Jeans length \( L < \text{LJ} \), are acoustic vibrations.

Since the space environment (dark energy) internal stresses are characterized by "extensive stretching", causing a negative pressure, the cosmological constant \( \Lambda \) in equation (1) describes the elastic properties of the medium, and the formula itself (1) in accordance with Hooke's law describes the repulsive forces between the structural elements forming a dark. Because of this, not some "universal law of anti-gravity" ie antigravity center corresponding to the center of gravity, cannot speak.
b) **Numerical values of the physical parameters that characterize how the microscopic structure of quantum objects medium (dark energy) and its macroscopic:**

The following are numerical parameters of structural objects of the space environment from the work A. Rykova "Fundamentals of the theory of ether" [4]. Structural objects of dark energy are electrically charged vortex dipole "kvazikollapsov" derived from virtual electrons and positrons.

When a size of a structure element of the ether is $r = 1.3988 \times 10^{-19}$ m, the ultimate deformation of the dipole (a destruction limit) would be $dr$ effects $= 1.0207 \times 10^{-17}$ m. At that, the distance between virtual charges $r$ of the electron and positron, forming the dipole, is 2.0145 times less than a classical electron radius. Dipole destruction occurs only when deformation is $1/137$ of its the integer value that says of extraordinary stability of the ether. Deformation in the ether, which is below this value, should have an electro elastic nature. A force of the elastic deformation of the ether has an enormous value $F_{\text{def}} = 1.1550 \times 10^{15}$ N/m². 

With regard to the macroscopic parameters of the space environment can be stated as follows:

The cosmological dark energy density is now measured with an accuracy of a few percent $\rho_{\text{v}} = (0.721 \pm 0.025) \times 10^{32}$ kg/m³ or $\rho_{\text{v}} = (0.721 \pm 0.025) \times 10^{32}$ kg/m³. 

The gravitational constant $G = 6.6720 \times 10^{-11}$ N·m²/kg²

The value of the cosmological constant $\Lambda$, and the negative pressure $P$ we find on the basis of representations about antigravitating environment with a constant density, laid down in the standard cosmological Model $\Lambda$CDM, from the following relations: [1]

$$\rho_{\text{v}} = c^2 \Lambda / (8\pi G) \quad \Lambda = (8\pi G \rho_{\text{v}}) / c^2$$  \hspace{1cm} (4)

$$P_{\text{v}} = -\rho_{\text{v}} c^2$$  \hspace{1cm} (5)

Substituting in the formula (4) a known density $\rho_{\text{v}} = (0.721 \pm 0.025) \times 10^{32}$ kg/m³ and $G = 6.6720 \times 10^{-11}$ N·m²/kg² find cosmological constant $\Lambda:

The absolute value of $\Lambda$ (s³) is $1.17 \times 10^{23}$ N/(kg·m)

Substituting into the formula (5) known density $\rho_{\text{v}}$ find the pressure.

The pressure is negative and the energy density equal to the absolute value [1]:

$$P_{\text{v}} = -(0.721 \pm 0.025) \times 10^{32} \text{ kg/m}^2$$

A similar expression for the negative pressure can be obtained by considering an analogue of dark energy - a superfluid $^3$He-B. As part of the simulation model a superfluid steady motion described by the equation [11]:

$$\rho_{\text{v}} u^2 / 2 + \rho_{\text{v}} \mu = \text{const}$$  \hspace{1cm} (6)

where $\mu$ - chemical potential,

$\rho_{\text{v}}$ - density of the medium,

$u$ - the propagation velocity of the liquid.

At the core of superfluid vortex $^3$He-B due to the orientation of the spins of the atoms in one direction $^3$He phase transition occurs with the formation of the superconducting phase A and spontaneously ferromagnetic $\beta$-phase. As a result of the phase transition within the core of the vortex potential $\mu$ inside the vortex is not [5]. In connection with this action superfluid at the vortex core can be analytically described pressure input $P$ on the border of the vortex core environment. Sign pressure depends on the nature of the internal stresses in the environment. If the internal voltage have the character of "comprehensive sprains" [11], the pressure will be negative. Thus it can be assumed that the pressure $P$ at the boundary of the vortex core of the dipole in the medium satisfies the equation:

$$\rho_{\text{v}} u^2 / 2 - P = \text{const.}$$  \hspace{1cm} (7)

which is identical to the expression (5), adopted in the standard model for dark energy. Considering that in hydrodynamics, pressure force $F_{\text{p}}$ are the integral:

$$F_{\text{p}} = - \int s' Pnds,$$  \hspace{1cm} (8)

where $n$ - waterproof external normal to the surface $S'$ $ds$ - an infinitesimal element of the surface

Using (6) we get:

$$F_{\text{p}} = - \frac{1}{2} \int s' \rho_{\text{v}} u^2 nds$$  \hspace{1cm} (9)

That is all the dynamic characteristics will have a sign opposite to that which they would have had for the usual ideal incompressible fluid with the same kinematic properties [5].

Strength $F_{\text{p}}$ - a repulsive force acting in the space environment (dark energy) between the structural elements of the environment (dipole vortices) for $u = c$. That it has the effect of anti-gravitation, and may cause the accelerated expansion of the universe.

c) **Resonance curves of the photoelectric effect in the cosmic medium:**

Consider the features of the phenomenon that is the photoelectric effect destruction process photons structural elements of the space environment and the birth of a pair of oppositely charged microparticles (electrons and positrons). The experimental curves of the relative growth of the flow of electrons and positrons in the space environment, since the photon energy (photoelectric threshold) $W_k = 1 \text{ MeV}$ energy and ending with cosmic radiation $200\text{GeV}$, suggest a resonant nature of the process of creation of electrons and positrons. For photons with energies below the $W_k = 1 \text{ MeV}$ photoelectric effect is observed. However, the photoelectric effect is not observed for gamma radiation. Moreover, it has been experimentally
established the presence of two photoelectric threshold and two resonant peaks, which may indicate the presence of near-Earth space in the two phase of the space environment: dark energy and dark matter. Resonance curves birth of electrons and positrons from the virtual micro-particles forming the dipole vortices are shown in Figure 1. The curves are plotted on the materials presented in [4,8].

The striking similarity between the two-humped dispersion curve of energy excitations in superfluid 4 He (Landau curve) and the double-humped resonance curve of the photoelectric effect at the birth of electrons and positrons in the near-Earth space environment the hypotheses that dark energy and dark matter are present therein as two phases of a cosmic medium.

![Resonance curves of photoelectric effect in near-Earth medium](image)

Figure 1: Resonance curves of photoelectric effect in near-Earth medium

For dark energy: \( W_{k(de)} \geq 1 \text{ MeV} = 1.6493 \cdot 10^{-13} \text{ J} \)

\[ \lambda_{k(de)} = 1.23 \cdot 10^{-13} \text{ m} \]

\[ W_{r(dm)} = 20 \text{ GeV} = 33 \cdot 0^{+10} \text{ J} \]

\[ v_{r(dm)} = 4.7 \cdot 10^{24} \text{ Hz} \]

For dark matter: \( W_{k(dm)} \geq 30 \text{ GeV} = 46.5 \cdot 10^{-10} \text{ J} \)

\[ \omega_{k(dm)} = 4.26 \cdot 10^{39} \text{ Hz} \]

\[ W_{r(dm)} = 200 \text{ GeV} = 330 \cdot 10^{-10} \text{ J} \]

\[ v_{r(dm)} = 4.78 \cdot 10^{25} \text{ Hz} \]

\[ \lambda_{r(dm)} = 0.6 \cdot 10^{-17} \text{ m} \]

The frequency corresponding to the critical energy of the photon (\( \nu \)) and wavelength (\( \lambda \)), and the precession frequency of the rod vortex dipole dark energy (\( \omega \)) (electron - positron) and the speed of the vortex dipole domain of dark matter (electron - positron + proton) define as the frequency of the wave function of Schrödinger and de Broglie (they describe the same probability density of finding a particle at any point in space):

\[ \nu = W / h \text{ or } \omega = W / \hbar \text{ and } \lambda = 2\pi s / \omega \] (10)

where \( W \) - the photon energy

\[ h \text{- Planck constant } h = 6.6260 \cdot 10^{-34} \text{ J / Hz} \]

\[ h = h / (2\pi) \text{ h} = 1.0546 \cdot 10^{-34} \text{ J / Hz} \]

\[ c \text{- the speed of light } c = 299792458 \text{ m / s} \]

An analysis of the resonance curves shown in Figure 1 can determine the frequency of the photon, the corresponding photoelectric threshold for dark energy and the natural frequency of rotation of the vortex dipole (photon resonance frequency) and its wavelength. Similarly, in Figure 1 we find the frequency photon corresponding photoelectric threshold for the dark matter and the natural frequency of rotation of the dipole domain, as well as the approximate boundaries of the frequency of electromagnetic radiation, in which the photoelectric effect in outer space is not available.

The natural frequency of the dipole dark energy solves the problem of the stability of its structural elements with the same classical position that the stability of atomic structures on the basis of nuclei and electrons. The length of the stable orbit dipole must fit an integer de Broglie waves.

The length of a circular orbit dipole dark energy:

\[ L_d = 2\pi r, \quad L_d(dm) = 8.7890 \cdot 10^{-15} \text{ m} \] (11)

where \( r \)-size structural element dipole equal to the distance between the virtual particles: an electron and a positron in the dipole \( r = 1.3988 \cdot 10^{-15} \text{ m} \) of dark energy. A ratio of the dipole orbit length \( L_d \) to the dipole own wavelength \( \lambda_r(dm) = 6.39 \cdot 10^{-17} \text{ m} \) is equal to 137.5335.

This approximate integer value of wavelengths' halves fits into the orbit length and is a quantum condition for stability in the dipole structure of the cosmic ether. Number 137.5335 agrees well the experimentally obtained value for a magnitude of the fine structure \( \alpha = 1/137.0355 \) of elementary particles. This fact underlines a deep connection between a structure of the structural unit within the cosmic ether (dipole) and a structure of elementary particles [4].

The resonant nature of the pair of elementary particles under the influence of external radiation is a fundamental process of the universe is formed in the space environment divergent flow or drain and source. Direct experimental determination of the resonance dependence of birth N elementary particle pairs of frequency \( \nu \) is almost completely silenced by modern physics. Following the deceptive logic of the modern theory, this dependence is drawn as a monotonically increasing curve [14].

The Earth has an electric charge, which, because of the Coulomb repulsion, tends to a spherical
surface of the planet. The electrification process of the near-Earth environment that behaves like the incompressible fluid looks like according to an expression by N. Tesla a yield state. At that, the energy is primarily transmitted along the curve - the shortest way between a source and a receiver on the Earth's surface. Distribution of currents of the "electric fluid" on the Earth's surface one describe analytically with the theory of the stationary, two-dimensional, ideal incompressible fluid on the Riemann surface. See Appendix A.

III. Conclusion

Rapid development of the theory of superfluid medium and high-precision astronomical data in recent years, obtained by Planck space telescopes and the HST, the detector PAMELA, Fermi, AMS, allows to fill the vacuum of space of the Universe physical content. In the framework presented in the new cosmological model, the space environment (dark energy and dark matter) has a quantum structure, and is seen as a nondissipative moving celestial bodies. This interpretation of the cosmological model allows us to give an answer on the nature of dark matter, the puzzle matter) has a quantum structure, and is seen as a nondissipative moving celestial bodies. This interpretation of the cosmological model allows us to give an answer on the nature of dark matter, the puzzle of the accelerated expansion of the universe and the role of the cosmological constant in the process.

Appendix A

Riemann Spaces and Modelling the Globe Electrisation Process

As interpreted by Helmholtz-Monastyrsky [18], the theory of analytic functions on the Riemann surface we can present as an issue of physics. We will show that the theory of the stationary two-dimensional ideal incompressible fluid on the surface entirely down to reduces to the theory of analytic functions. Let us consider a stationary fluid flow \( u \) on the plane \((x, y)\). The flow speed at each point has \( x \)-component \( P(x, y) \) and \( y \)-component \( Q(x, y) \). Through the cell with sides \( \Delta x \), \( \Delta y \) per a time unit the mass of liquid outflows (liquid density is constant and equals to

\[
\int_0^{\Delta y} \left[ P(x + \Delta x, y + h) - P(x, y + h) \right] dh + \int_0^{\Delta x} \left[ Q(x + l, y + \Delta y) - Q(x + l, y) \right] dl.
\]

Approximating an arbitrary domain \( \Omega \) with rectangles and applying the Green's formula, we obtain that the integral \((A.1)\) is equal to:

\[
\iint \left( \frac{dp}{dx} + \frac{dq}{dy} \right) dxdy.
\]  (A.2)

Since the fluid is incompressible and nowhere appears and disappears in the \( \Omega \) domain, it follows that the expression \((A.2)\) is zero. The stronger statement is also reasonable, i.e. flow divergence \( u \) is zero:

\[
\text{Div } U = \frac{dp}{dx} + \frac{dq}{dy} = 0 \quad (A.3)
\]

The flow circulation along the curve \( C \) is defined as the integral

\[
\int Pdx + Qdy.
\]

If this integral along any closed curve is zero, then the flow is called irrotational. For any single-bound domain, it follows that statement \( Pdx + Qdy \) is a complete differential of the function \( u(x, y) \). This function is harmonic.

The function \( U(x, y) \) is called the flow speed potential. Helmholtz introduced this concept. Curves \( U(x, y) = \text{const} \) are called equipotential lines. A tangent line to the equipotential line forms such an angle \( \alpha \) with the axis, that

\[
\tan \alpha = -\frac{\frac{dU}{dx}}{\frac{dU}{dy}}, \text{ if only } \Delta U \neq 0.
\]

The flow speed vector makes an angle \( \beta \) with the \( x \) axis,

\[
\tan \beta = \frac{\frac{dU}{dx}}{\frac{dU}{dy}},
\]

i.e. the flow goes orthogonally to equipotential lines in the direction of increasing \( U \) function.

As we remember, the harmonic function \( u(x, y) \) defines the function of

\[
f(z) = u + iv
\]

where \( v \) is a conjugate to the harmonic function \( u \), defined from Cauchy-Riemann equations \((A.4)\). Essentially, Cauchy solves the following problem, under which conditions for the complex function \( f(z) \) the integral \( \int f(z) \, dz \) in a closed loop \( l \) is zero. However, he does not speak explicitly of the complex function, but applying pairs of real functions \( P(x, y) \) and \( Q(x, y) \), gets his main result: the integral \( \int f(z) \, dz \) does not depend on the integration path if such conditions are met:

\[
\frac{dp}{dx} = \frac{dq}{dy}; \frac{dp}{dx} = -\frac{dQ}{dx} \quad (A.4)
\]

This condition is a \((A.4)\)-characteristic property of analyticity (holomorphicity) of function of a complex variable. In modern literature, the common name is the Cauchy-Riemann condition. The function \( f(z) \) people call the complex potential of the flow.

The tangent line to the curve \( u = \text{const} \) makes an angle \( \gamma \) with the \( x \) axis and

\[
\tan \gamma = -\frac{\frac{dv}{dx}}{\frac{dv}{dy}} = \frac{\frac{du}{dy}}{\frac{du}{dx}} = \tan \beta
\]

i.e. the \( u \) flow goes along the curve \( u = \text{const} \). These curves people call streamlines.

The condition \( \frac{dv}{dx} + \frac{du}{dy} = 0 \), equivalent to \( f'(z) \neq 0 \), indicates that streamlines are orthogonal to equipotential lines except at points where \( f'(z) = 0 \).
This physical analogy allows us to interpret any properties of analytic functions exceptionally clearly. For example, if the analytic function \( f(z) \) has at the point \( z_0 \) \( f'(z_0) = 0 \), then curves \( u = \text{const} \) and \( v = \text{const} \) do not cross at \( z_0 = x_0 + i y_0 \) at right angles. Such points are called stationary points, e.g. for the function

\[
f(z) = a_0 + a_2 z^2
\]

curves \( u = \text{const} \) and \( v = \text{const} \) intersect at an angle \( \pi / 4 \).

With the same success, we can explore arbitrary features of analytic functions.

Consider the flow with the potential \( f(z) \), a derivative \( f'(z) \) of which is the rational function, i.e. has only pole specifics \( (z-z_0)^{-n} \). Then the function \( f(z) \) itself we can represent in the neighbourhood of a specific point in the form of

\[
f(z) = A \log (z-z_0) + A_1 (z-z_0) + \ldots + \phi(z) \tag{A.5}
\]

where \( \phi(z) \) - function without specifics

Features of flows defined with the function \( f(z) \) are made from specifics of streams made by individual components (A.5).

Let us consider an influence of the logarithmic term. Let us at first assume that \( A \) is a real number. Let us choose a circle of the radius \( r \) around the point \( z_0 \):

\[
z = z_0 + re^{\iota \varphi}
\]

and assume that \( A \log (z-z_0) = u + i v \); separating the real and imaginary parts, we obtain \( A \log r = u, A \phi = \nu \).

Streamlines \( \nu = \text{const} \) will be radii going from the point \( z_0 \), while equipotential lines \( u = \text{const} \) - will be circles with a centre in \( z_0 \).

**Figure A.1:** Divergent flow or drain, source and curls in the near-Earth environment

Thus, the point \( z_0 \) will be either the source (Fig. A.1a) or the fluid outlet (Fig. A.1b), depending on the operator \( A \) (the liquid will either outflow, or flow into the point \( z_0 \)). If \( A \) is a purely imaginary value, then we obtain the conjugate stream \( A = i B, u = -B \phi, v = \log r \). Circles will be streamlines. Such streams are called curls. Direction of motion (clockwise or counter clockwise) depends on the B operator. We have obtained a great result. All features of the analytic function \( f(z) \) on the sphere we can describe in terms of the fluid flow with a defined number of sources, outlets, curls, etc.

**References Références Referencias**

