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### Bianchi Type VI Cosmological Model with Quadratic form of Time Dependent A Term in General Relativity

G. S. Khadekar <sup>a</sup>, Shilpa Samdurkar <sup>a</sup> & Shoma Sen <sup>p</sup>

Abstract- In this paper, we obtained solution of Einstein field equations for Bianchi type VI cosmological model with time dependant cosmological term  $\Lambda$  of the form  $\Lambda = \frac{\alpha}{R^2} + \beta H^2$ ,  $\Lambda = \frac{\alpha}{R^2} + \beta H^2 + \gamma$  and  $\Lambda = \Lambda_0 + \Lambda_1 H + \Lambda_2 H^2$ . It is observed that cosmological term  $\Lambda$  is decreasing function of time which is consistent with results from recent supernova la observations. Also, it is noted that the model approaches to isotropy for n = 1. All the physical parameters are calculated and discussed.

Keywords: bianchi type VI cosmological model  $\cdot$  stiff fluid  $\cdot$  cosmological term  $\Lambda$ .

#### I. INTRODUCTION

n the modern cosmological theories, one of the most theoretical parameter to calculate dark energy is cosmological constant (Weinberg [1], Sahni and Starobinsky [2], Peebles and Ratra The [3]). cosmological observations provide the existence of a positive cosmological constant with magnitude  $\Lambda \approx \left(\frac{Gh}{c^3}\right) = 10^{-123}$  by the High-Z Supernova Team and the Supernova Cosmological Project (Garnavich ([4], [5]), Perlmutter ([6], [7]), Riess [8], Schmidt [9]). It is found that there is a huge difference between observational and the particle physics prediction value for  $\Lambda$  which is known as cosmological constant problem. This dynamic cosmological term  $\Lambda(t)$  solves the cosmological constant problem in a natural way. There is significant observational evidence towards identifying Einsteins cosmological constant of the universe that varies slowly with time and space and so acts like.

It is observed that the dynamic  $\Lambda$  term decays with time (Gasperini ([10], [11]), Berman ([12], [13], [14]), Ozer and Taha [15], Freese [16], Peebles and Ratra [17], Chen and Wu [18], Abdussattar and Vishwakarma [19], Gariel and Le Dennat [20]). In the different context Berman ([12], [13], [14]), has argued in the favor of dependence  $\Lambda \propto \frac{1}{t^2}$ . The concept of decaying law helps to solve the cosmological problems very successfully. Further, the law of variation of scale

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factor useful to solve the field equations was proposed by Pavon [21]. In earlier literature, the dynamical  $\Lambda$  term is proportional to scale factor have been studied by Holye et al. [22], Olson et al. [23], Maia et al. [24], Silveria et al. ([25], [26]), Torres et al. [27].

Chen and Wu [18] considered  $\Lambda \propto R^{-2}$  where R is the scale factor was generalized by Carvalho et al. [28] by considering  $\Lambda = \alpha R^{-2} + \beta H^2$  and later on by Waga [29] by considering  $\Lambda = \alpha R^{-2} + \beta H^2 + \gamma$  where  $\alpha, \beta$  and  $\gamma$  are adjustable dimensionless parameters. Dwivedi and Tiwari [30] have investigated Bianchi Type V cosmological models with time varying G and by assuming the condition  $\Lambda = \frac{\beta}{R^2} + H^2$ . In cosmological models, stiff fluid creates more interest in the results. Barrow [31] has discussed the relevance of stiff equation of state  $\rho = p$  to the matter content of the universe in the early state of evolution of universe. Exact solution of Einsteins field equation with stiff equation of state has been investigated by Wesson [32].

At the early stages of evolution, the cosmological models play significant roles in the description of the universe. Bianchi I to IX spaces are very useful for constructing special homogeneous cosmological models. Homogeneous and anisotropic models have been widely studied in the frame work of general relativity by many authors viz. Wainwright et al. [33], Collins and Hawking [34], Ellis and MacCallum [35], Dunn and Tupper [36], MacCallum [37], Roy and Bali [38], Bali [39], Roy and Banerjee [40], Bali and Singh [41] to name only few. Raj Bali et al. [42] have studied Bianchi III cosmological model for barotropic fluid distribution with variable G and  $\Lambda$ . Singh and Beesham et al. [43] have investigated Bianchi V perfect fluid spacetime with variable G and  $\Lambda$ . Bianchi type V universe with bulk viscous matter and time varying gravitational and cosmological model have been studied by Baghel and Singh [44]. Saha et al. [45] have investigated Bianchi I cosmological model with time dependant gravitational and cosmological constants: An alternative approach. Singh et al. [46] have discussed bulk viscous anisotropic cosmological model with dynamical cosmological parameters G and  $\Lambda$ . Bianchi type V cosmological model with varying cosmological term have been studied by Tiwari and Singh [47].

Barrow [48] has pointed out that the Bianchi Type  $VI_0$  cosmological models give a better explanation

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of some of cosmological problems like primordial helium abundance and these models isotropize in special case. Chandel et al. [49] have discussed Bianchi Type V IO dark energy cosmological model in general relativity. Mishra et al. [50] have studied five dimensional Bianchi Type  $VI_0$  dark energy cosmlogical model in general relativity. Accelerating dark energy models with anisotropic fluid in Bianchi type  $VI_0$  space time has been discussed by Anirudh Pradhan [51]. Saha [52] have discussed Bianchi Type VI anisotropic dark enrgy model with varying EoS parameter. Verma and Shri Ram [53] have studied Bianchi type  $VI_0$  bulk viscous fluid models with variable gravitational and cosmological constants. Stability of viscous fluid in Bianchi type VI model with cosmological constant have been discussed by Sadeghi et al. [54]. Singh et al. [55] have discussed Bianchi Type  $VI_0$  viscous fluid cosmological models with time-dependent cosmological term  $\Lambda$ .

One of the motivations for introducing  $\Lambda$  term is to reconcile the age parameter and the density parameter of the universe with recent observational data. In this paper, we have discussed Bianchi type VI cosmological models with varying cosmological term  $\Lambda$  in the presence of stiff fluid. We have obtained the solutions of Einstein field equations for three different cases i.e case (i):  $\Lambda = \frac{\alpha}{R^2} + \beta H^2$ , case (ii):  $\Lambda = \frac{\alpha}{R^2} + \beta H^2 + \gamma$  and case (iii):  $\Lambda = \Lambda_0 + \Lambda_1 H + \Lambda_2 H^2$  by assuming the expansion  $\theta$  is proportional to shear  $\sigma$  which leads to the condition between metric potential  $A = B^n$ .

#### II. METRIC AND FIELD EQUATIONS

The spatially homogeneous and anisotropic Bianchi type VI spacetime is described by the line element

$$ds^{2} = -dt^{2} + A^{2}dx^{2} + B^{2}e^{-2x}dy^{2} + C^{2}e^{2x}dz^{2}, \qquad (1)$$

where A, B and C are functions of time t only.

We assume that cosmic matter is a perfect fluid given by energy momentum tensor

$$T_{ij} = (\rho + p)u_i u_j + pg_{ij},\tag{2}$$

satisfying equation of state (EoS)

$$p = \omega \rho, \quad 0 \le \omega \le 1, \tag{3}$$

where  $\rho$  being the matter density, p the isotropic pressure,  $u^i$  the flow vector of the fluid satisfying  $u_i u^i = -1$ .

Einstein field equations with time varying cosmological term  $\Lambda$  is given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi GT_{ij} + \Lambda g_{ij}.$$
 (4)

For the metric (1) and energy momentum tensor (2) in co-moving system of co-ordinates, the field equations (4) yields

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{1}{A^2} = 8\pi G\rho + \Lambda,$$
(5)

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{1}{A^2} = -8\pi Gp + \Lambda, \tag{6}$$

$$\frac{A}{A} + \frac{C}{C} + \frac{A}{A}\frac{C}{C} - \frac{1}{A^2} = -8\pi Gp + \Lambda, \qquad (7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{1}{A^2} = -8\pi Gp + \Lambda, \qquad (8)$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0. \tag{9}$$

Last equation gives

$$B = C.$$
 (10)

Then field equations (5) -(8) can be written as,

$$2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} = 8\pi G\rho + \Lambda,$$
(11)

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{A^2} = -8\pi Gp + \Lambda, \qquad (12)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{1}{A^2} = -8\pi Gp + \Lambda.$$
(13)

Taking into account the conservation equation  $div(T_i^j) = 0$ , we have

$$8\pi \dot{G}\rho + 8\pi G \dot{\rho} + 8\pi G \left(\rho + p\right) \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) - \dot{A} = 0, (14)$$

which leads to

$$8\pi \dot{G}\rho - \dot{\Lambda} = 0, \tag{15}$$

$$\dot{\rho} + (\rho + p) \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) = 0.$$
(16)

We define the average scale factor R and generalized Hubble parameter H for Bianchi VI universe as

$$V = R^3 = ABC = AB^2, \tag{17}$$

$$H = \frac{\dot{R}}{R} = \frac{1}{3}(H_1 + H_2 + H_3).$$
(18)

Adding (11) and (12) with the use of (3), we get,

$$\frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = 4\pi G \left(1 - \omega\right)\rho + \Lambda.$$
<sup>(19)</sup>

We assume that the expansion  $\theta$  is proportional to the shear  $\sigma$ . This condition leads to,

$$A = B^n. (20)$$

Then field equations (11)-(13) can be written as,

$$(2n+1)\frac{\dot{B}^2}{B^2} - \frac{1}{B^{2n}} = 8\pi G\rho + \Lambda,$$
(21)

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^{2n}} = -8\pi Gp + \Lambda,$$
 (22)

$$(n+1)\frac{\ddot{B}}{B} + n^2 \frac{\dot{B}^2}{B^2} - \frac{1}{B^{2n}} = -8\pi Gp + \Lambda.$$
 (23)

With the use of (20), for  $\omega = 1$ , Eq. (19) can be written as,

$$\frac{\ddot{B}}{B} + (n+1)\frac{\dot{B}^2}{B^2} = \Lambda.$$
 (24)

a) Case (i):

We assume the cosmological term in the form

$$\Lambda = \frac{\alpha}{R^2} + \beta H^2.$$
 (25)

Solving (24) with the use of (25), we get,

$$B\ddot{B} + \alpha_1 \dot{B}^2 = \frac{\alpha}{B^{\frac{2(n-1)}{3}}},$$
 (26)

where  $\alpha_1 = (n+1) - \frac{\beta(n+2)^2}{9}$ . Solving we get,

$$\dot{B}^{2} = \frac{9\alpha}{(n+2)\left[6 - \beta\left(n+2\right)\right]} \frac{1}{B^{\frac{2(n-1)}{3}}} + \frac{k_{1}}{B^{2\alpha_{1}}}.$$
 (27)

where  $k_1$  is integration constant. Integrating, we get

$$\int \frac{B^{\frac{b}{2}}dB}{\sqrt{k_1 + DB^{b-a}}} = \int dt + k_2,$$
(28)

where  $k_2$  is integration constant,

$$a = \frac{2(n-1)}{3}, b = 2(n+1) - \frac{2\beta(n+2)^2}{9}, D = \frac{9\alpha}{(n+2)[6-\beta(n+2)]}.$$

To solve Eq. (28), we assume that  $n \neq 1$ , a = bthen we get,

$$\int B^{\frac{b}{2}} dB = D_0 \left( \int dt + k_2 \right), \tag{29}$$

where  $D_0 = \sqrt{k_1 + D}$ . Solving, we get,

$$B = \left[\frac{(n+2)\,D_0\,(t+k_2)}{3}\right]^{\frac{3}{n+2}},\tag{30}$$

$$A = \left[\frac{(n+2) D_0(t+k_2)}{3}\right]^{\frac{3n}{n+2}}.$$
 (31)

The metric (1) reduces to,

$$ds^{2} = -dt^{2} + \left[\frac{(n+2)D_{0}(t+k_{2})}{3}\right]^{\frac{6n}{n+2}}dx^{2} + \left[\frac{(n+2)D_{0}(t+k_{2})}{3}\right]^{\frac{6}{n+2}}\left(e^{-2x}dy^{2} + e^{2x}dz^{2}\right).$$
 (32)

#### b) Some physical and geometrical aspects of the model

The energy density  $\rho$ , pressure p, cosmological parameter  $\Lambda$ , gravitational parameter G, expansion scalar  $\theta$ . Hubble parameter H, spatial volume V are given by,

$$\rho = p = \frac{k_3}{\left[\frac{(n+2)D_0(t+k_2)}{3}\right]^{2(n+2)}},$$
(33)

$$\Lambda = \left[\frac{9\alpha + (n+2)^2 \beta D_0^2}{9(n+2)^2 D_0^2}\right] \frac{1}{(t+k_2)^2},$$
 (34)

$$8\pi G = \left[\frac{9(2n+1) - \beta(n+2)^2 - 9\alpha}{9k_3}\right] \left[\frac{(n+2)D_0(t+k_2)}{3}\right]^2$$

$$-\frac{1}{k_3} \left[ \frac{(n+2) D_0 (t+k_2)}{3} \right]^{\frac{12}{n+2}}, \qquad (35)$$

$$\theta = \frac{3}{(t+k_2)}, \ \sigma^2 = \frac{3(n-1)^2}{(n+2)^2} \frac{1}{(t+k_2)^2},$$
 (36)

$$H = \frac{1}{(t+k_2)}, \quad V = \left[\frac{(n+2)D_0(t+k_2)}{3}\right]^3.$$
(37)

For  $n=1, \beta=\frac{5}{2}$ , we get

$$A = B = l_1 t^2 + l_2 t + l_3, \tag{38}$$

where  $l_1 = \frac{k_1}{4}, l_2 = \frac{k_1k_2}{2}, l_3 = \left(\frac{k_1k_2^2}{4} + \frac{2\alpha}{k_1}\right).$ The metric (1) reduces to,

$$ds^{2} = -dt^{2} + \left(l_{1}t^{2} + l_{2}t + l_{3}\right)^{2} \left(dx^{2} + e^{-2x}dy^{2} + e^{2x}dz^{2}\right).$$
 (39)

and accordingly get the values of energy density, pressure p, cosmological parameter  $\Lambda$ , gravitational parameter G, expansion scalar  $\theta$ , Hubble parameter H, spatial volume V which are similar to the values obtained as given by Eq. (33) to Eq. (37) by putting n = 1. For this case we get the isotropic model.

c) Case (ii):

We assume a relation,

$$\Lambda = \frac{\alpha}{R^2} + \beta H^2 + \gamma. \tag{40}$$

Solving Eq. (24) with the use of (40), we get,

$$B\ddot{B} + \alpha_1 \dot{B}^2 = \frac{\alpha}{B^{\frac{2(n-1)}{3}}} + \gamma B^2.$$
 (41)

Solving above Eq. we get the first integral,

$$\dot{B}^{2} = \frac{9\alpha}{(n+2)\left[6 - \beta\left(n+2\right)\right]} \frac{1}{B^{\frac{2(n-1)}{3}}}$$

$$+\frac{9\gamma}{(n+2)\left[6-\beta(n+2)\right]}B^{2}+\frac{k_{4}}{B^{2\alpha_{1}}}.$$
 (42)

where  $k_4$  is constant of integration.

To solve the above integration, we take  $k_4 = 0$  Eq. (42) can be written as,

$$\dot{B}^{2} = \frac{9\alpha}{(n+2)\left[6-\beta(n+2)\right]} \frac{1}{B^{\frac{2(n-1)}{3}}} + \frac{9\gamma}{(n+2)\left[6-\beta(n+2)\right]} B^{2}.$$
(43)

Integrating, we get

$$\int \frac{dB}{B\sqrt{m+DB^{-(a+2)}}} = \int dt + k_5, \qquad (44)$$

where  $k_5$  is constant of integration,

$$a = \frac{2(n-1)}{3}$$
,  $D = \frac{9\alpha}{(n+2)[6-\beta(n+2)]}$ ,  $m = \frac{9\gamma}{(n+2)[6-\beta(n+2)]}$ .

Solving Eq. (44), we get

$$B = \left\{ \sqrt{\frac{D}{m}} sinh\left[\frac{(n+2)\sqrt{m}\left(t+k_{5}\right)}{3}\right] \right\}^{\frac{3}{n+2}}, \quad (45)$$

$$A = \left\{ \sqrt{\frac{D}{m}} sinh\left[\frac{(n+2)\sqrt{m}\left(t+k_{5}\right)}{3}\right] \right\}^{\frac{3n}{n+2}}.$$
 (46)

The metric (1) reduces to,

$$ds^{2} = -dt^{2} + \left\{ \sqrt{\frac{D}{m}} sinh\left[\frac{(n+2)\sqrt{m}(t+C_{4})}{3}\right] \right\}^{\frac{n}{n+2}} dx^{2} + \left\{ \sqrt{\frac{D}{m}} sinh\left[\frac{(n+2)\sqrt{m}(t+k_{5})}{3}\right] \right\}^{\frac{6}{n+2}} (e^{-2x}dy^{2} + e^{2x}dz^{2}).$$
(47)

In this case, the energy density  $\rho$ , pressure p, cosmological parameter  $\Lambda$ , gravitational parameter G, expansion scalar  $\theta$ , Hubble parameter H, spatial volume *V* are given by,

$$\rho = p = \frac{k_3 m^3}{D^3} cosech^6 \left[ \frac{(n+2)\sqrt{m} (t+k_5)}{3} \right], \quad (48)$$
$$A = \left[ \frac{\left[ 9\alpha + \beta (n+2)^2 \right] m}{9D} \right] cosech^2 \left[ \frac{(n+2)\sqrt{m} (t+k_5)}{3} \right]$$

$$+\frac{\beta m \left(n+2\right)^2}{9} + \gamma, \tag{49}$$

$$8\pi G = \frac{1}{9k_3} \left[ \frac{9\left(2n+1\right) - \beta(n+2)^2}{9} \right] m \coth^2 \left[ \frac{(n+2)\sqrt{m}(t+k_5)}{3} \right]$$

$$\left\{ \sqrt{\frac{D}{m}} sinh\left[\frac{(n+2)\sqrt{m}(t+k_5)}{3}\right] \right\}^{6}$$
$$-\frac{1}{k_3} \left\{ \sqrt{\frac{D}{m}} sinh\left[\frac{(n+2)\sqrt{m}(t+k_5)}{3}\right] \right\}^{\frac{12}{n+2}}$$
$$-\frac{\alpha}{k_3} \left[ \left\{ \sqrt{\frac{D}{m}} sinh\left[\frac{(n+2)\sqrt{m}(t+k_5)}{3}\right] \right\}^{\frac{12}{n+2}} \right]^{4}, (50)$$

$$\theta = (n+2)\sqrt{m}\left\{ coth\left[\frac{(n+2)\sqrt{m}(t+k_5)}{3}\right] \right\}, \quad (51)$$

$$\sigma^{2} = \frac{(n-1)^{2}}{3} m coth^{2} \left[ \frac{(n+2)\sqrt{m}(t+k_{5})}{3} \right], \qquad (52)$$

$$\frac{\sigma}{\theta} = \frac{(n-1)}{\sqrt{3}(n+2)} = constant, \tag{53}$$

$$H = \frac{(n+2)}{3} \sqrt{m} \coth\left[\frac{(n+2)\sqrt{m}(t+k_5)}{3}\right],$$
 (54)

$$V = \left\{ \sqrt{\frac{D}{m}} sinh\left[\frac{(n+2)\sqrt{m}\left(t+k_{5}\right)}{3}\right] \right\}^{3}.$$
 (55)

*d)* Case (iii): We assume a relation,

$$\Lambda = \Lambda_0 + \Lambda_1 H + \Lambda_2 H^2.$$
(56)

Solving Eq. (24) with the use of (56), we get,

$$\frac{\ddot{B}}{B} - L_1 \frac{\dot{B}^2}{B^2} - L_2 \frac{\dot{B}}{B} = L_3,$$
(57)

where  $L_1 = \frac{\Lambda_2(n+2)^2}{9} - (n+1), \ L_2 = \frac{\Lambda_1(n+2)}{3}, \ L_3 = \Lambda_0.$ Solving, we get,

$$B = k_7 exp\left[\frac{-L_2 t}{2(L_1 - 1)}\right] \left[sec\left(\frac{L_4}{2}(k_6 + t)\right)\right]^{\frac{1}{L_1 - 1}}, \quad (58)$$

$$4 = \left[k_7 exp\left[\frac{-L_2 t}{2(L_1 - 1)}\right] \left[sec\left(\frac{L_4}{2}(k_6 + t)\right)\right]^{\frac{1}{L_1 - 1}}\right]^n.$$
 (59)

where  $L_4 = \sqrt{4(L_1 - 1)L_3 - L_2^2}$ ,  $k_6$  and  $k_7$  are constants of integration. The metric (1) reduces to,

$$ds^{2} = -dt^{2} + \left[k_{7}exp\left[\frac{-L_{2}t}{2(L_{1}-1)}\right]\left[sec\left(\frac{L_{4}}{2}(k_{6}+t)\right)\right]^{\frac{1}{L_{1}-1}}\right]^{2n}dx^{2} + \left[k_{7}exp\left[\frac{-L_{2}t}{2(L_{1}-1)}\right]\left[sec\left(\frac{L_{4}}{2}(k_{6}+t)\right)\right]^{\frac{1}{L_{1}-1}}\right]^{2}\left(e^{-2x}dy^{2}+e^{2x}dz^{2}\right).$$
(60)

In this case, the physical parameters are given by,

$$\rho = p = k_3 \left[ \frac{1}{k_7} exp \left[ \frac{L_2 t}{2(L_1 - 1)} \right] \left[ \cos \left( \frac{L_4}{2} \left( k_6 + t \right) \right) \right]^{\frac{1}{L_1 - 1}} \right]^{2(n+2)}, \tag{61}$$

$$\Lambda = \Lambda_0 + \frac{\Lambda_1 (n+2)}{6(L_1 - 1)} \left[ L_4 tan\left(\frac{L_4}{2} (k_6 + t)\right) - L_2 \right]$$
(62)

$$+\frac{\Lambda_{2}(n+2)^{2}}{36(L_{1}-1)^{2}}\left[L_{4}tan\left(\frac{L_{4}}{2}(k_{6}+t)\right)-L_{2}\right]^{2},$$

$$8\pi G = \frac{1}{k_3} \left[ k_7 exp \left[ \frac{-L_2 t}{2(L_1 - 1)} \right] \left[ \sec \left( \frac{L_4}{2} \left( k_6 + t \right) \right) \right]^{\frac{1}{L_1 - 1}} \right]^{2(n+2)}$$

$$\left\{ L_{5} \left[ L_{4} tan\left(\frac{L_{4}}{2} \left(k_{6} + t\right)\right) - L_{2} \right]^{2} - L_{6} \left[ L_{4} tan\left(\frac{L_{4}}{2} \left(k_{6} + t\right)\right) - L_{2} \right] \right\}$$

$$-\left\{\frac{1}{k_7}exp\left[\frac{L_2t}{2\left(L_1-1\right)}\right]\left[cos\left(\frac{L_4}{2}\left(k_6+t\right)\right)\right]\right\}^{2n}-\Lambda_0\},$$

where  $L_5 = \left[ (2n+1) - \frac{\Lambda_2(n+2)^2}{9} \right] \frac{1}{4(L_1-1)^2}, \ L_6 = \frac{\Lambda_1(n+2)}{6(L_1-1)}.$ 

$$\theta = \frac{(n+2)}{2(L_1-1)} \left[ L_4 tan\left(\frac{L_4}{2}(k_6+t)\right) - L_2 \right],$$
(64)

$$\sigma^{2} = \frac{(n-1)^{2}}{12(L_{1}-1)^{2}} \left[ L_{4} tan\left(\frac{L_{4}}{2}(k_{6}+t)\right) - L_{2} \right]^{2},$$

$$\frac{\sigma}{\theta} = \frac{(n-1)}{\sqrt{3}(n+2)} = constant,$$

$$H = \frac{(n+2)}{6(L_1-1)} \left[ L_4 tan\left(\frac{L_4}{2}(k_6+t)\right) - L_2 \right],$$
(67)

$$V = \left\{ k_7 exp \left[ \frac{-L_2 t}{2(L_1 - 1)} \right] \left[ sec \left( \frac{L_4}{2} (k_6 + t) \right) \right]^{\frac{1}{L_1 - 1}} \right\}^{(n+2)}.$$

(63)

(65)

(66)

(68)



*Fig.* 1: Case (i)  $\Lambda = \frac{\alpha}{R^2} + \beta H^2$ ,  $n \neq 1$ , a = b, Relation between *B* and time *t* for  $k_1 = 1$ ,  $k_2 = 2$ ,  $k_3 = 3$ , n = 2,  $\alpha = 1$ ,  $\beta = \frac{5}{2}$ , D = -0.56,  $D_0 = 0.6632$ .



*Fig. 2:* Case (i)  $\Lambda = \frac{\alpha}{R^2} + \beta H^2$ ,  $n \neq 1$ , a = b, Relation between  $\rho$  and time t for  $k_1 = 1$ ,  $k_2 = 2$ ,  $k_3 = 3$ , n = 2,  $\alpha = 1$ ,  $\beta = \frac{5}{2}$ , D = -0.56,  $D_0 = 0.6632$ .



*Fig.* 3: Case (i)  $\Lambda = \frac{\alpha}{R^2} + \beta H^2$ ,  $n \neq 1$ , a = b, Relation between  $\Lambda$  and time t for  $k_1 = 1$ ,  $k_2 = 2$ ,  $k_3 = 3$ , n = 2,  $\alpha = 1$ ,  $\beta = \frac{5}{2}$ , D = -0.56,  $D_0 = 0.6632$ .



*Fig.* 4: Case (i)  $\Lambda = \frac{\alpha}{R^2} + \beta H^2$ ,  $n \neq 1$ , a = b, Relation between *H* and time *t* for  $k_1 = 1$ ,  $k_2 = 2$ ,  $k_3 = 3$ , n = 2,  $\alpha = 1$ ,  $\beta = \frac{5}{2}$ , D = -0.56,  $D_0 = 0.6632$ .



*Fig.* 5: Case (ii)  $\Lambda = \frac{\alpha}{R^2} + \beta H^2 + \gamma$ , Relation between *B* and time *t* for  $k_3 = 3$ ,  $k_5 = 0.5$ , n = 2,  $\alpha = 1$ ,  $\beta = 1$ ,  $\gamma = 2$ , D = 1.125, m = 0.9.



*Fig.* 6: Case (ii)  $\Lambda = \frac{\alpha}{R^2} + \beta H^2 + \gamma$ , Relation between  $\rho$  and time t for  $k_3 = 3$ ,  $k_5 = 0.5$ , n = 2,  $\alpha = 1$ ,  $\beta = 1$ ,  $\gamma = 2$ , D = 1.125, m = 0.9.



Fig. 7: Case (ii)  $\Lambda = \frac{\alpha}{R^2} + \beta H^2 + \gamma$ , Relation between  $\Lambda$  and time t for  $k_3 = 3$ ,  $k_5 = 0.5$ , n = 2,  $\alpha = 1$ ,  $\beta = 1$ ,  $\gamma = 2$ , D = 1.125, m = 0.9.



Fig. 8: Case (ii)  $\Lambda = \frac{\alpha}{R^2} + \beta H^2 + \gamma$ , Kelation between H and time t for  $k_3 = 3$ ,  $k_5 = 0.5$ , n = 2,  $\alpha = 1$ ,  $\beta = 1$ ,  $\gamma = 2$ , D = 1.125, m = 0.9.



*Fig.* 9: Case (iii)  $\Lambda = \Lambda_0 + \Lambda_1 H + \Lambda_2 H^2$ , Relation between *B* and time *t* for  $k_3 = 3$ ,  $k_6 = 2$ ,  $k_7 = 2$ , n = 2,  $\Lambda_0 = 3$ ,  $\Lambda_1 = 2$ ,  $\Lambda_2 = 3$ ,  $L_1 = 2.33$ ,  $L_2 = 2.66$ ,  $L_3 = 3$ ,  $L_4 = 2.98$ .



Fig. 10: Case (iii)  $\Lambda = \Lambda_0 + \Lambda_1 H + \Lambda_2 H^2$ , Relation between  $\rho$  and time t for  $k_3 = 3$ ,  $k_6 = 2$ ,  $k_7 = 2$ , n = 2,  $\Lambda_0 = 3$ ,  $\Lambda_1 = 2$ ,  $\Lambda_2 = 3$ ,  $L_1 = 2.33$ ,  $L_2 = 2.66$ ,  $L_3 = 3$ ,  $L_4 = 2.98$ .



*Fig.* 11: Case (iii)  $\Lambda = \Lambda_0 + \Lambda_1 H + \Lambda_2 H^2$ , Relation between  $\Lambda$  and time t for  $k_3 = 3$ ,  $k_6 = 2$ ,  $k_7 = 2$ , n = 2,  $\Lambda_0 = 3$ ,  $\Lambda_1 = 2$ ,  $\Lambda_2 = 3$ ,  $L_1 = 2.33$ ,  $L_2 = 2.66$ ,  $L_3 = 3$ ,  $L_4 = 2.98$ .





*Our findings:* Here, we have obtained solutions of the Einstein field equations for three cases:

Case (i):  $\Lambda = \frac{\alpha}{R^2} + \beta H^2$ 

- it is seen that as  $t \to 0, V \to 0$  and as  $t \to \infty, V \to \infty$ .
- When  $t \to 0$ , the expansion scalar  $\theta$  and  $\rho$  tends to infinity and when  $t \to \infty$  the expansion scalar  $\theta$  and  $\rho$  tends to zero.
- Here,  $\frac{\sigma}{\theta}$  = constant.
- For  $\alpha > 0$ , it is observed that cosmological term  $\Lambda$  is decreasing function of time and approaches a small positive value at late time and for  $\alpha < 0$ , cosmological term  $\Lambda$  becomes positive.

- For n = 1, the model becomes isotropic.
- To illustrate the graph, we observe that the scale factor B(t) with respect to cosmic time t grows rapidly as shown in fig (1). Also from fig (2) to fig (4) we observe that the energy density  $\rho$ , cosmological term  $\Lambda$  and hubble parameter H goes on decreasing as time increases while they all become infinitely large as t approaches zero.

Case (ii):  $\Lambda = \frac{\alpha}{R^2} + \beta H^2 + \gamma$ 

- By taking suitable values of constant, it is observed that  $\rho$ ,  $\Lambda$ ,  $\theta$ ,  $\sigma$  and H are all infinite and at late time they become zero for n > 1.
- It is observed that  $\frac{\sigma}{\theta} \neq 0$ .

- Also, for  $t \to 0$ ,  $V \to 0$  and for  $t \to \infty$ ,  $V \to \infty$ .
- Also, for  $t \to 0$ ,  $V \to 0$  and for  $t \to \infty$ ,  $V \to \infty$ .
- In this case, we observe that the scale factor B(t) with respect to cosmic time t grows rapidly as shown in fig (5). Also from fig (6) to fig (8) we observe that the energy density  $\rho$ , cosmological term  $\Lambda$  and hubble parameter H decreases with time t and approches zero as t  $\rightarrow \infty$ .

Case (iii):  $\Lambda = \Lambda_0 + \Lambda_1 H + \Lambda_2 H^2$ 

- In this case also, the energy condition  $\rho \ge 0$  is satisfied.
- Also, the scale of expansion ( $\theta$ ) is infinite at t = 0 and  $\theta$  becomes zero when  $t \to \infty$ .
- It is seen that  $\frac{\sigma}{\theta} = \text{constant}$ .
- Cosmological term  $\Lambda$  is decreasing function of time.
- We observe that the scale factor B(t) and energy density  $\rho$  with respect to cosmic time t grows rapidly as shown in fig (9) and fig (10). Also from fig (11) and fig (12) cosmological term  $\Lambda$  and hubble parameter H decreases with time t and approches zero as  $t \to \infty$ .

#### III. DISCUSSION AND CONCLUSION

In this paper, we have investigated the role of  $\Lambda$  term in the evolution of Bianchi type-VI universe in the presence of stiff fluid ( $p = \rho$ ). Here, we have obtained exact solutions of Einstein field equations for three different cases depending upon the cosmological term  $\Lambda$  i. e

case (i):  $\Lambda = \frac{\alpha}{R^2} + \beta H^2$ ,

case (ii):  $\Lambda = \frac{\alpha}{B^2} + \beta H^2 + \gamma$  and

case (iii):  $\Lambda = \Lambda_0 + \Lambda_1 H + \Lambda_2 H^2$ .

- In the case (i), the universe starts expanding with zero volume and grows up at infinite past and future. By taking suitable values of constant, it is observed that  $\rho$ ,  $\Lambda$ ,  $\theta$ ,  $\sigma$  and H are all infinite and at late time they become zero. It is well known that a positive  $\Lambda$  corresponds to the universal repulsive force, while a negative one gives an additional gravitational force. Therefore, the new anisotropic cosmological model Eq. (32) represents expanding, shearing and non-rotating universe for n = 1. But for n = 1, the model becomes isotropic.
- In the case (ii) also, we have obtained an anisotropic cosmological model for n > 1. It is observed that the volume is increasing with respect to time.
- Hence, the graphs of energy density in case (*i*) and case (*ii*) shows the evolution of universe for some suitable values of constants.
- In the last case where we assume  $\Lambda$  in quadratic form, from the graph, it is clear that the cosmological term  $\Lambda$  is decreasing function of time. Thus, the model represents shearing and non-rotating universe. From Eq. (66), the Collins

condition [56] is satisfied i. e  $\frac{\sigma}{\theta}$  is constant for this case.

- Therefore, our constructed cosmological models are of great importance in the sense that the nature of decaying vacuum energy density is supported by recent cosmological observations. In all the cases, we have shown that our cosmological model, which is homogeneous and anisotropic at the early stage of universe becomes isotropic for n = 1. The result of this paper agree with the observational features of the universe.

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