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# New Generalization of Angular Displacement with Product of Certain Special Functions in A Shaft 

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# New Generalization of Angular Displacement with Product of Certain Special Functions in A Shaft 

Abstract-The object of this paper is to establish a new generalization of angular displacement in a shaft with a product of certain special function. A main result based upon the H -function of several complex variables, l-function of one variable, general polynomial of several variables, which provide unification and extension of numerous results in theory of special function. The special cases of the main result (which are also sufficiently general in nature and are of interested in themselves) have also been given.
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## I. Introduction

The importance of the H -function, I-function are realized by scientists, engineers and statisticians (caputo [16], Glockle and Nonnenmacher [17], Mainardi et al. [18], Hifer[19] etc) due to vast potential of their applications in diversified fields of science and engineering such as rheology, diffusion in porous media, fluid flow, turbulence, propagation of seismic waves etc.

In the view of importance and popularity of the I-function and H -function a large number of integral formulas involving these functions have been developed by many authors.

The series representation for the $\mathbf{H}$-function of several complex variables [3], is as follows:

$$
\begin{align*}
& H\left[z_{1}, \ldots, z_{k}\right]=H_{A, C:\left(B^{\prime}, D^{\prime}\right) ; \ldots ;\left(B^{K}, D^{K}\right)}^{0, \lambda\left(u^{\prime}, v^{\prime}\right) ;\left(u^{K}, K^{K}\right)}\left[\begin{array}{l}
{\left[(a): \theta^{\prime} ; \ldots ; \theta^{(K)}\right]:\left[b^{\prime}: \varphi^{\prime}\right] ; \ldots ;\left[b^{(k)}: \varphi^{(K)}\right]} \\
{\left[(c): \psi^{\prime} ; \ldots ; \psi^{(K)}\right]:\left[d^{\prime}: \delta^{\prime}\right] ; \ldots ;\left[d^{(K)}: \delta^{(K)}\right]}
\end{array} z_{1}, \ldots, z_{K}\right] \\
&=\sum_{e_{l}=0}^{u^{(l)}} \sum_{f_{l}=0}^{\infty} \phi_{1} \phi_{2} \frac{\prod_{l=1}^{K}\left(z_{l}\right)^{U_{l}}(-1)^{\sum_{l=1}^{K}\left(f_{l}\right)}}{\prod_{l=1}^{K}\left(\delta_{\left(e_{l}\right)}^{(l)} f_{l}!\right)} \tag{1....}
\end{align*}
$$

where

[^1]\[

$$
\begin{gather*}
\phi_{1}=\frac{\prod_{j=1}^{\lambda} \Gamma\left(1-a_{j}+\sum_{l=1}^{K} \theta_{j}^{(l)} U_{l}\right)}{\prod_{j=\lambda+1}^{A} \Gamma\left(a_{j}-\sum_{l=1}^{K} \theta_{j}^{(l)} U_{l}\right) \prod_{j=1}^{C} \Gamma\left(1-c_{j}+\sum_{l=1}^{K} \psi_{j}^{(l)} U_{l}\right)}  \tag{1....}\\
\phi_{2}=\frac{\prod_{j=1}^{u^{(l)}} \Gamma\left(d_{j}^{(l)}-\delta_{j}^{(l)} U_{l}\right) \prod_{j=1}^{v^{(l)}} \Gamma\left(1-b_{j}^{(l)}+\varphi_{j}^{(l)} U_{l}\right)}{\prod_{j=v^{(l)}+1}^{B^{(l)}} \Gamma\left(b_{j}-\varphi_{j}^{(l)} U_{l}\right) \prod_{j=u}^{D^{(l)}(l)+1} \Gamma\left(1-d_{j}+\delta_{j}^{(l)} U_{l}\right)} \tag{1.3}
\end{gather*}
$$
\]

where

$$
\begin{equation*}
U_{l}=\frac{d_{e_{l}}^{(l)}+f_{l}}{\delta_{e_{l}}^{(l)}} \quad \forall l=1, \ldots K \tag{1....}
\end{equation*}
$$

For the converges conditions and other detail of H -function of several complex variables, see [3].

The I-function, which is more general the Fox's H-function [5], defined by V.P. Sexena [4], by means of the following Mellin-Barns type contour integral:

$$
I[z]=I_{p_{i}, q_{i} ; r}^{m, n}[z]=I_{p_{i}, q_{i}, r}^{m, n}\left[z \left\lvert\, \begin{array}{l}
\left(a_{j}, \alpha_{j}\right)_{1, n}:\left(a_{j i}, \alpha_{j i}\right)_{n+1, p_{i}}  \tag{1.5}\\
\left(b_{j}, \beta_{j}\right)_{1, m}:\left(b_{j i}, \beta_{j i}\right)_{m+1, q_{i}}
\end{array}\right.\right]=\frac{1}{2 \pi i} \int_{L} \Theta(\zeta) z^{\zeta} d \zeta
$$

where

$$
\begin{equation*}
\Theta(\zeta)=\frac{\prod_{j=1}^{m} \Gamma\left(b_{j}-\beta_{j} \zeta\right) \prod_{j=1}^{n} \Gamma\left(1-a_{j}+\alpha_{j} \zeta\right)}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_{i}} \Gamma\left(1-b_{j i}+\beta_{j i} \zeta\right) \prod_{j=n+1}^{p_{i}} \Gamma\left(a_{j i}-\alpha_{j i} \zeta\right)\right\}} \tag{1.6}
\end{equation*}
$$

$p_{i,} q_{i}(i=1, \ldots, r), m, n$ are integer satisfying $0 \leq n \leq p_{i,} 0 \leq m \leq q_{i,} \alpha_{j}, \beta_{j}, \alpha_{j i}, \beta_{j i}$ are real and positive and $a_{j}, b_{j}, a_{j i}, b_{j i}$ are complex numbers. L is suitable contour of the MellinBarnes type running from $\gamma-i \alpha$ to $\gamma+i \alpha$ ( $\gamma$ is real) in the complex $\zeta$-plane. Detail regarding existence conditions and various parametric restriction of I-function, we may refer [4].

Remark: For $r=1,[4]$ reduce to the Fox's H-function

$$
I_{p_{i}, q_{i}, 1}^{m, n}\left[Z \left\lvert\, \begin{array}{l}
\left(a_{j}, \alpha_{j}\right)_{1, n}:\left(a_{j i}, \alpha_{j i}\right)_{n+1, p_{i}}  \tag{1.7}\\
\left(b_{j}, \beta_{j}\right)_{1, m}:\left(b_{j i}, \beta_{j i}\right)_{m+1, q_{i}}
\end{array}\right.\right]=H_{p, q}^{m, n}\left[z \left\lvert\, \begin{array}{l}
\left(a_{j}, \alpha_{j}\right)_{1, n}:\left(a_{j}, \alpha_{j}\right)_{n+1, p} \\
\left(b_{j}, \beta_{j}\right)_{1, m}:\left(b_{j}, \beta_{j}\right)_{m+1, q}
\end{array}\right.\right]
$$

The multidimensional analogue of a general class of polynomials $S_{n}^{m}(x)$ is defined by [6],

$$
\begin{equation*}
S_{n}^{m_{1}, \ldots, m_{s}}\left(x_{1}, \ldots, x_{s}\right)=\sum_{k_{1}, \ldots, k_{s}}^{m_{1} k_{1}+\cdots+m_{s} k_{s} \leq n}(-n)_{m_{1} k_{1}+\cdots+m_{s} k_{s} .} \mathcal{A}\left(n: k_{1}, \ldots, k_{s}\right) \frac{x_{1}^{k_{1}}}{k_{1}!} \ldots \frac{x_{s}^{k_{s}}}{k_{s}!} \tag{1.8}
\end{equation*}
$$

where $m_{1}, \ldots, m_{s}$ are arbitrary positive integers, $n=0,1,2 \ldots$ and the coefficients constant $\mathcal{A}\left(n: k_{1}, \ldots, k_{s}\right), k_{i} \geq 0, i=1, \ldots, s$ are arbitrary constants, real and complex. The order of highest degree of variables $x_{1}, \ldots, x_{s}$ of the multivariable polynomial (1.8) can be written as [7],

$$
\begin{equation*}
\mathbb{O}\left(S_{n}^{m_{1}, \ldots, m_{s}}\left(x_{1}, \ldots, x_{s}\right)\right)=\mathbb{O}\left(x_{1}^{\left[n / m_{1}\right]}, \ldots, x_{s}^{\left[n / m_{s}\right]}\right) \tag{1.9}
\end{equation*}
$$

where $[x]$ denotes the greatest integer $\leq x$.

## Remarks

1. For $m_{i}=1, i=1, \ldots, s$ and $\mathcal{A}\left(n: k_{1}, \ldots, k_{s}\right)=\left(1+\alpha_{1}+n_{1}\right)_{k_{1}} \ldots\left(1+\alpha_{s}+n_{s}\right)_{k_{s}}$, the multivariable polynomial reduces to a multivariable Bessel polynomial [8].
2. For $m_{i}=2, \sigma_{i}=1, i=1, \ldots, s, \mathcal{A}\left(n: k_{1}, \ldots, k_{s}\right)=1$ and replacing $t x_{1} \rightarrow \frac{1}{2\left(t x_{1}\right)^{2}}, t x_{i} \rightarrow$ $\frac{t x_{j}}{2\left(t x_{1}\right)^{2}}, j=1, \ldots, s$, the multivariable polynomial reduces to a multivariable Hermite polynomial [9].
3. For the case $r=1$ the multivariable polynomial (1.8) would give rise to the general class polynomials $S_{n}^{m}(x)$ denoted by Srivastava [10].

The following result is required in our present investigation, which is due to [Gradshteyn and Ryzhik 1965, p.375, eq. (2)]

$$
\begin{equation*}
\int_{0}^{\mu} \cos \left(\frac{\pi \varepsilon x}{\mu}\right)\left(\sin \frac{\pi x}{2 \mu}\right)^{2 \varepsilon-\sigma-1}\left(\cos \frac{\pi x}{2 \mu}\right)^{\sigma-1} d x=\frac{\mu .2^{2 \varepsilon-\sigma} \Gamma\left(\frac{2 \varepsilon-\sigma}{2}\right) \Gamma(\sigma)}{\sqrt{\pi} \Gamma\left(\frac{1-2 \varepsilon+\sigma}{2}\right) \Gamma(2 \varepsilon)} \tag{1.10}
\end{equation*}
$$

Provided $2 \varepsilon>\operatorname{Re}(\sigma)>0$.
Also by Churchill [11],

$$
\begin{equation*}
\Psi(x, t)=\frac{1}{2} a_{0}+\sum_{\tau=1}^{\infty} a_{\tau}\left(\cos \frac{\pi x \tau}{\mu}\right)\left(\cos \frac{\pi \tau R t}{\mu}\right) \tag{1.11}
\end{equation*}
$$

where $a_{\tau}(\tau=0,1,2 \ldots)$ are the coefficients in the Fourier cosine series for $\mathrm{f}(\mathrm{x})$ in the interval $(o, \mu)$.

## II. Statement of the Problem and Governing Equations

Here the continuous system and the equation of motion has been explained through a simple diagram it was earlier developed. There is the x -axis of the rod which is also called the side axis and is loaded with distributed moment. At the normal location of the rod, the angular displacement takes place is known as $x t$ and at the side view of the rod it can be pointed out is to be said point $p$. The motion of point can be seen in a large view, point p is present on to the shaft. Its radial position is r and if it got displaced, it reaches to the point p prime and this angle is called $\theta$ (theta).

It also can be assumed that the point $p$ moves in a same plane and it does not move towards the axial direction. Presently, the equation of motion for this type continuous system in which, the mass and stiffness both are distributed in a systematic way.


Figure 2.1


Figure 2.2


Figure 2.3

As an example of an application of one and multivariable special functions in applied mathematics, we shall consider the problem of determining the angular displacement. The solution of boundary value problem involving the partial differential equation of angular displacement in a shaft is given by

$$
\begin{equation*}
\frac{\partial^{2} \theta}{\partial t^{2}}=c^{2} \frac{\partial^{2} \theta}{\partial x^{2}} \tag{2.1}
\end{equation*}
$$

Let us consider the problem determine the angular displacement or twist $\theta(x, t)$ in a shaft of circular section with its axis along the x -axis. The ends $x=0$ to $x=\mu$ of the shaft are free, the $\theta(x, t)$ due to initial twist must satisfy the boundary value (2.1). Also the boundary conditions are

$$
\begin{equation*}
\frac{\partial}{\partial x} \theta(0, t)=0, \frac{\partial}{\partial x} \theta(\mu, t)=0, \frac{\partial}{\partial t} \theta(x, 0)=0 \tag{2.2}
\end{equation*}
$$

and $\theta(x, 0)=f(x)$, where c is a constant.

$$
\begin{gather*}
\text { Let } \begin{array}{c}
f(x)=\left(\sin \frac{\pi x}{2 \mu}\right)^{2 \delta-\lambda-1}\left(\cos \frac{\pi x}{2 \mu}\right)^{\lambda-1} S_{n_{1}, \ldots, n_{s}}^{m_{1}, \ldots, m_{s}}\left[\begin{array}{c}
t_{1}\left(\tan \frac{\pi x}{2 \mu}\right)^{2 k_{1}} \\
\vdots \\
t_{s}\left(\tan \frac{\pi x}{2 \mu}\right)^{2 k_{s}}
\end{array}\right] \cdot H\left[\begin{array}{c}
y_{1}\left(\tan \frac{\pi x}{2 \mu}\right)^{2 v_{1}} \\
\vdots \\
t_{K}\left(\tan \frac{\pi x}{2 \mu}\right)^{2 v_{K}}
\end{array}\right] \\
\times I_{p_{i}, q_{i}, r}^{m, n}\left[Z\left(\tan \frac{\pi x}{2 \mu}\right)^{2 h}\right]
\end{array} . .
\end{gather*}
$$

## iII. The Main Integral Formula

The main result to be established here as follows:

$$
\int_{0}^{\mu}\left(\cos \frac{\pi \delta x}{\mu}\right)\left(\sin \frac{\pi x}{2 \mu}\right)^{2 \delta-\lambda-1}\left(\cos \frac{\pi x}{2 \mu}\right)^{\lambda-1} S_{n_{1}, \ldots, n_{s}}^{m_{1}, \ldots m_{s}}\left[\begin{array}{c}
t_{1}\left(\tan \frac{\pi x}{2 \mu}\right)^{2 k_{1}} \\
\vdots \\
t_{s}\left(\tan \frac{\pi x}{2 \mu}\right)^{2 k_{s}}
\end{array}\right] \cdot H\left[\begin{array}{c}
y_{1}\left(\tan \frac{\pi x}{2 \mu}\right)^{2 v_{1}} \\
\vdots \\
t_{K}\left(\tan \frac{\pi x}{2 \mu}\right)^{2 v_{K}}
\end{array}\right]
$$

$$
\begin{gather*}
\times I_{p_{i}, q_{i} ; r}^{m, r}\left[z\left(\tan \frac{\pi x}{2 \mu}\right)^{2 h}\right] \\
=\frac{\mu}{\sqrt{2 \delta}} 2^{\delta \lambda-\lambda+2 \sum_{i=1}^{s} \alpha_{i} k_{i}+2 \sum_{l=1}^{K} V_{l} U_{l}} \sum_{e_{l}=1}^{U_{l}} \sum_{f_{l}=0}^{\infty} \phi_{1} \phi_{2} \cdot \frac{\prod_{l=1}^{K}\left(y_{l}\right)^{V_{l}}(-1)^{\sum_{l=1}^{K}\left(f_{l}\right)}}{\prod_{l=1}^{K}\left\{\delta_{\left(e_{l}\right)}^{(l)} f_{l}!\right\}} \sum_{\alpha_{i}=0}^{\left[n_{1} / m_{1}\right]} \cdots \sum_{\alpha_{s}=0}^{\left[n_{s} / m_{s}\right]} \\
\frac{(-1)_{m_{1} \alpha_{1}}}{\alpha_{1}!} \cdots \frac{(-1)_{m_{s} \alpha_{s}}}{\alpha_{s}!} B\left[n_{1} \alpha_{1} ; \ldots ; n_{s} \alpha_{s}\right] \cdot\left[t_{1}^{\alpha_{1}}, \ldots, t_{s}^{\alpha_{s}}\right] . I_{p_{i}+2, q_{i}+1: r}^{m+1, n+1} \\
{\left[\begin{array}{c}
\left(1-\delta+\frac{\lambda}{2}-\sum_{i=1}^{s} \alpha_{i} k_{i}-\sum_{l=1}^{K} V_{l} U_{l} ; h\right),\left(a_{j}, \alpha_{j}\right)_{1, n^{\prime}} ;\left(a_{j i}, \alpha_{j i}\right)_{n+1, p_{i}} \\
\left.\left(\lambda-2 \sum_{i=1}^{s} \alpha_{i} k_{i}-2 \sum_{l=1}^{K} V_{l} U_{l} ; 2 h\right),\left(b_{j}, \beta_{j}\right)_{1, m^{\prime}} ;\left(b_{j i}, \beta_{j i}\right)_{m+1, q_{i}}{ }^{\prime} \frac{1}{2}-\delta+\frac{\lambda}{2}-\sum_{i=1}^{s} \alpha_{i} k_{i}-\sum_{l=1}^{K} V_{l} U_{l} ; h\right)
\end{array}\right]} \tag{3.1}
\end{gather*}
$$

where $\quad k_{i}>0(i=1, \ldots, s), h>0, \mathfrak{R}\left(\lambda-2 k\left(\frac{\beta_{j}}{b_{j}}\right)\right)>0, j=1, \ldots m, m$ is an arbitrary positive integer and the coefficient $B\left[n_{1} \alpha_{1}, \ldots, n_{s} \alpha_{s}\right]$ are arbitrary constant, real or complex.

Proof. To prove the integral formula (3.1), we express the first multivariable H-function in series form by (1.1) and I-function in terms of Mellin-Barnes type of contour integral by (1.5) and the generalized polynomial given by (1.8), then interchanging the order summation and integration, which is justified due to the absolute converges of integral involved in the process. Evaluate the integral with help of (1.10), after straight calculation we finally arrive at (3.1).

## IV. Solution of the Problem

In this section we shell study the solution of the problem. So the solution of the problem posed to be obtained is

$$
\begin{gathered}
\Phi(x, t)=\frac{2^{2 \tau-\lambda+2 \sum_{i=1}^{S} \alpha_{i} k_{i}+2 \sum_{l=1}^{K} V_{l} U_{l}+1}}{\sqrt{\pi} \sqrt{2 \tau}} \sum_{e_{l}=1}^{U_{l}} \sum_{f_{l}=0}^{\infty} \phi_{1} \phi_{2} \cdot \frac{\prod_{l=1}^{K}\left(y_{l}\right)^{V_{l}}(-1)^{\sum_{l=1}^{K}\left(f_{l}\right)}}{\prod_{l=1}^{K}\left\{\delta_{\left(e_{l}\right)}^{(l)} f_{l}!\right\}} \\
\sum_{\alpha_{i}=0}^{\left[n_{1} / m_{1}\right]} \cdots \sum_{\alpha_{s}=0}^{\left[n_{s} / m_{s}\right]} \frac{(-1)_{m_{1} \alpha_{1}}}{\alpha_{1}!} \cdots \frac{(-1)_{m_{s} \alpha_{s}}}{\alpha_{s}!} B\left[n_{1} \alpha_{1}, \ldots, n_{s} \alpha_{s}\right] \cdot\left[t_{1}^{\alpha_{1}}, \ldots, t_{s}^{\alpha_{s}}\right] . I_{p_{i}+2, q_{i}+1: r}^{m+1, n+1}
\end{gathered}
$$

$$
\left[\begin{array}{c}
{\left[\begin{array}{rl} 
& \left(1-\tau+\frac{\lambda}{2}-\sum_{i=1}^{s h} \alpha_{i} k_{i}-\sum_{l=1}^{K} V_{l} U_{l} ; h\right),\left(a_{j}, \alpha_{j}\right)_{1, n} ;\left(a_{j i}, \alpha_{j i}\right)_{n+1, p_{i}} \\
& \left(\tau-2 \sum_{i=1}^{s} \alpha_{i} k_{i}-2 \sum_{l=1}^{K} V_{l} U_{l} ; 2 h\right),\left(b_{j}, \beta_{j}\right)_{1, m} ;\left(b_{j i}, \beta_{j i}\right)_{m+1, q_{i}},\left(1-\tau+\frac{\lambda}{2}-\sum_{i=1}^{s} \alpha_{i} k_{i}-\sum_{l=1}^{K} V_{l} U_{l} ; h\right) \\
& \times\left(\cos \frac{\pi x \tau}{\mu}\right)\left(\cos \frac{\pi \tau R t}{\mu}\right)
\end{array}\right.}
\end{array}\right.
$$

where $\quad k_{i}>0(i=1, \ldots, s), h>0, \mathfrak{R}\left(\lambda-2 k\left(\frac{\beta_{j}}{b_{j}}\right)\right)>0, j=1, \ldots m, m$ is an arbitrary positive integer and the coefficient $B\left[n_{1} \alpha_{1} ; \ldots ; n_{s} \alpha_{s}\right]$ are arbitrary constant, real or complex.
Proof. By eq. (1.11)

$$
\Psi(x, t)=\frac{1}{2} a_{0}+\sum_{\tau=1}^{\infty} a_{\tau}\left(\cos \frac{\pi x \tau}{\mu}\right)\left(\cos \frac{\pi \tau R t}{\mu}\right)
$$

where $a_{\tau}(\tau=0,1,2 \ldots)$ are the coefficients in the Fourier cosine series for $\mathrm{f}(\mathrm{x})$ in the interval $(o, \mu)$.
If $t=0$, then from (1.11) and (2.3), we have

$$
\begin{gather*}
\left(\sin \frac{\pi x}{2 \mu}\right)^{2 \delta-\lambda-1}\left(\cos \frac{\pi x}{2 \mu}\right)^{\lambda-1} S_{n_{1}, \ldots, n_{s}}^{m_{1}, \ldots, n_{s}}\left[\begin{array}{c}
t_{1}\left(\tan \frac{\pi x}{2 \mu}\right)^{2 k_{1}} \\
\vdots \\
t_{s}\left(\tan \frac{\pi x}{2 \mu}\right)^{2 k_{s}}
\end{array}\right] \cdot H\left[\begin{array}{c}
y_{1}\left(\tan \frac{\pi x}{2 \mu}\right)^{2 v_{1}} \\
\vdots \\
t_{K}\left(\tan \frac{\pi x}{2 \mu}\right)^{2 v_{K}}
\end{array}\right] \\
\times I_{p_{i} q_{i} ; r}^{m, n}\left[Z\left(\tan \frac{\pi x}{2 \mu}\right)^{2 h}\right]=\frac{1}{2} a_{0}+\sum_{\tau=1}^{\infty} a_{\tau}\left(\cos \frac{\pi x \tau}{\mu}\right) \tag{4.2}
\end{gather*}
$$

Now multiplying (4.1) both the sides by $\left(\cos \frac{\pi \delta x}{\mu}\right)$ and integrating with respect to x from 0 to $\mu$, we get

$$
\begin{align*}
& \int_{0}^{\mu}\left(\cos \frac{\pi \delta x}{\mu}\right)\left(\sin \frac{\pi x}{2 \mu}\right)^{2 \delta-\lambda-1}\left(\cos \frac{\pi x}{2 \mu}\right)^{\lambda-1} S_{n_{1}, \ldots, n_{s}}^{m_{1}, \ldots m_{s}}\left[\begin{array}{c}
t_{1}\left(\tan \frac{\pi x}{2 \mu}\right)^{2 k_{1}} \\
\vdots \\
t_{s}\left(\tan \frac{\pi x}{2 \mu}\right)^{2 k_{s}}
\end{array}\right] \cdot H\left[\begin{array}{c}
y_{1}\left(\tan \frac{\pi x}{2 \mu}\right)^{2 v_{1}} \\
\vdots \\
t_{K}\left(\tan \frac{\pi x}{2 \mu}\right)^{2 v_{K}}
\end{array}\right] \\
& \quad \times I_{p_{i}, q_{i}, r}^{m, n}\left[z\left(\tan \frac{\pi x}{2 \mu}\right)^{2 h}\right] d x=\frac{1}{2} a_{0} \int_{0}^{\mu}\left(\cos \frac{\pi \delta x}{\mu}\right) d x+\sum_{\tau=1}^{\infty} a_{\tau} \int_{0}^{\mu}\left(\cos \frac{\pi x \tau}{\mu}\right)\left(\cos \frac{\pi \delta x}{\mu}\right) d x \tag{4.3}
\end{align*}
$$

Now using result (3.1) along with orthogonal property of the cosine function, we obtain

$$
\begin{gathered}
a_{\tau}=\frac{2^{2 \tau-\lambda+2 \sum_{i=1}^{s} \alpha_{i} k_{i}+2 \sum_{l=1}^{K} V_{l} U_{l}+1}}{\sqrt{\pi} \sqrt{2 \tau}} \sum_{e_{l}=1}^{U_{l}} \sum_{f_{l}=0}^{\infty} \phi_{1} \phi_{2} \cdot \frac{\prod_{l=1}^{K}\left(y_{l}\right)^{V_{l}}(-1)^{\sum_{l=1}^{K}\left(f_{l}\right)}}{\prod_{l=1}^{K}\left\{\delta_{\left(e_{l}\right)}^{(l)} f_{l}!\right\}} \\
\sum_{\alpha_{i}=0}^{\left[n_{1} / m_{1}\right]} \cdots \sum_{\alpha_{s}=0}^{\left[n_{s} / m_{s}\right]} \frac{(-1)_{m_{1} \alpha_{1}}}{\alpha_{1}!} \cdots \frac{(-1)_{m_{s} \alpha_{s}}}{\alpha_{s}!} B\left[n_{1} \alpha_{1} ; \ldots ; n_{s} \alpha_{s}\right] \cdot\left[t_{1}^{\alpha_{1}}, \ldots, t_{s}^{\alpha_{s}}\right] \cdot I_{p_{i}+2, q_{i}+1: r}^{m+1, n+1}
\end{gathered}
$$

$$
\left[z 2^{2 h} \left\lvert\, \begin{array}{c}
\left(1-\tau+\frac{\lambda}{2}-\sum_{i=1}^{s} \alpha_{i} k_{i}-\sum_{l=1}^{K} V_{l} U_{l} ; h\right),\left(a_{j}, \alpha_{j}\right)_{1, n} ;\left(a_{j i}, \alpha_{j i}\right)_{n+1, p_{i}}  \tag{4.4}\\
\left(\tau-2 \sum_{i=1}^{s} \alpha_{i} k_{i}-2 \sum_{l=1}^{K} V_{l} U_{l} ; 2 h\right),\left(b_{j}, \beta_{j}\right)_{1, m} ;\left(b_{j i}, \beta_{j i}\right)_{m+1, q_{i}}{ }^{\prime}\left(1-\tau+\frac{\lambda}{2}-\sum_{i=1}^{s} \alpha_{i} k_{i}-\sum_{l=1}^{K} V_{l} U_{l} ; h\right)
\end{array}\right.\right]
$$

Now with help of (1.11) and (4.4), the solution (4.1) is obtained.
(A) If we set $S_{n_{i}}^{m_{i}}=1,(i=1, \ldots s)$ and $r=1$ in (3.1), then the results reduces to the result given by chaurasia and Gupta [12].
(B) For $r=1$, taking $s=2$ and $k_{i} \rightarrow 0$ in (3.1), we find the known result concluded by Chaurasia and Godika [13].
(C) Taking $\bar{H}$-function place of I-function and H-function of series form to unity, then we can easily obtained the known results given by Chaurasia and Shakhawat [14].
(D) For $r=2$ in multivariable H -function and $S_{n_{i}}^{m_{i}}=1,(i=1, \ldots s)$ and taking series representation of Fox's H-function place of I-function, we find the known result obtained by Mittal and Gupta [15]

## VI. Conculsion

In view of the generality of I-function and H -function of several complex variables, on specializing the various parameters, involved there in, we can obtain from our results, several Results involving a remarkably wide variety of useful function, which are expressible in terms of the H -function, $\bar{H}$-function, I-function of one variable and their special cases. Thus the results presented in this paper would at once yield a very large number of results involving a large variety of special function occurring in the problems of science, engineering and mathematical physics etc. Also results deduced in the present paper may provide better result of Angular displacement in a shaft and boundary value problem of some simpler functions.

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