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The Dance of the Spinning Top

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Following on from the foregoing, we suggest that planetary movement around the sun or that of artificial satellites around the Earth and, in general, all objects endowed with angular momentum on one of their rotating axes in translation and subject to non-coaxial external momenta, could also be interpreted in this way.

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The Dance of the Spinning Top

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I. Introduction and Grounds

The rotary movement of bodies is a phenomenon that has hitherto been studied insufficiently. Interest in this area dropped in 20th century only to focus on other aspects of physics that were considered to be of greater import at the time.

Notwithstanding, for some years now, the lecturer Gabriel Barceló Rico-Avello, Doctor in Industrial Engineering and Physics Graduate has been proposing a rotational mechanics that we believe better responds to the behaviour of bodies with intrinsic rotation. We refer to the *Theory of Dynamic Interactions* (TDI), which explains that the rotary movement of a body endowed with intrinsic rotation interacts with the translation of the same. *Dynamic interactions are understood as those inertial reactions that appear when a body is subject to accelerations* [1].

There is a host of rotating bodies that could serve as a subject of study. Nonetheless, this paper restricts its scope to that of the spinning top and, more exactly, to the *dance of the spinning top*, which consists of the curvilinear translation movement made by this body when rotating on its symmetrical axis, which in turn, forms a certain angle with the vertical axis perpendicular to the supporting plane. Moreover, we propose to show that the aforementioned *dance of the spinning top*, is in keeping with the basis of the *Theory of Dynamic Interactions* (TDI).

Intuition would suggest a similarity between this *dance of the spinning top* and the movement of any planet around the sun or that of any artificial satellite orbiting the Earth. Consequently, the conclusions to be gleaned from this behaviour will also be extrapolated to these cases of scientific interest.

Furthermore, other particular instances of the spinning top will be analysed to justify the generalisation of this theory and to show that the interaction is independent of the type of effect that provokes the movement, as long as there is rotation and translation.

II. Background

As stated by Dr. Barceló Rico-Avello [2], the peculiarity of a rotating body also warranted the attention of some 20th century scientists. Accordingly, G. Bruhat [3] defined it as the *apparently paradoxicalgyroscopic effect*. Likewise, Paul Appell [4] on referring to rotating bodies also frequently insisted on its paradoxical behaviour. Nonetheless, the principles of classical mechanics failed to delve into this aspect.

We believe the original error in the formulation of the theory of the scientific community as regards rotating bodies could lie in the studies of the scientist Louis Poinsot (1777-1859) from two perspectives.

Firstly, he studied the rotation movement with a fixed point (the case of the spinning top were translation not caused on its point of support on the surface) [5].

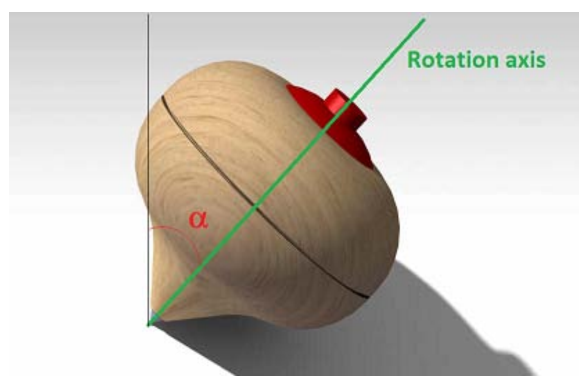

Figure 1 : Spinning top rotation axis and the angle it forms with the vertical

In this analysis he proposed that the *Instantaneous Axis of Rotation* defined by D'Alembert (1717-1783) [6] varied at each moment owing to the movement of the body, forming a cone with a fixed point vertex. However, he further claimed that the axis of rotation also varied inside the body itself. We hold that this starting point is erroneous. From experiments on the rotation movement of spinning top it can be observed that, by taking certain fixed axes on it, its axis of rotation does not shift or rotate in any way with respect to them.

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That is to say, as can be seen in figure 1, angle α remains constant.

Secondly, it must be pointed out that Poinsot shows that forces and momenta act on a solid body. However, he further states that "*each one of these dynamic elements produce their forces separately; force determines the translation movement of the body while torque generates rotation around its centre of gravity, without the force exercising any influence*" [7]. Such a reading had already been put forward by Lagrange [8].

This idea, which comes from Classical Mechanics, is the one that has established the method to calculate paths that is still used today. Notwithstanding, we suggest that dissociating translation and rotation in a body where both phenomena occur jointly is mistaken and that the *Theory of Dynamic Interactions* [9] better reflects what really happens, as indicated by Julio Cano in his paper on the pendulum of dynamic interactions. [10]

Other antecedents are known to have questioned the idea of separation. Such is the case of Dr. Shipov, who claims in the context of dynamic energy transfer:

> *Consequently, the law of conservation establishes a connection between translational and rotational forms of movement and makes it possible to transform the translation momentum in a rotational one and vice versa. This means that the law of conservation of momentum in Newtonian mechanics is limited and cannot be generalised to the case of the torsion interactions of colliding bodies.* [11]

Classical Mechanics uses differential equations to calculate paths, the well known "Euler Equations" [12]. Nevertheless, these expressions cannot be integrated even for the case of the movement of the spinning top and, therefore, from this limitation, the need to propose other new ones to provide results for this case arises. Moreover, it must be borne in mind that Newtonian Classical Mechanics is limited to inertial cases. Accordingly, to be able to apply its formula, Coriolis found it necessary to introduce the *Coriolis force* and the *centrifugal force* as fabricated forces for non-inertial cases [13]. Nonetheless, in the *Theory of Dynamic Interactions* we are proposing, there is no need to incorporate fabricated or apparent forces, as the dynamic behaviour of bodies is defined by the coupling of the dynamic fields generated.

Lagrange managed to express the components of angular velocity along the main axes and in function of Euler angles, but he only solved equations for a particular case and was unable to obtain a generalisation or to resolve the case of the spinning top, which is the one that concerns us here. [14].

Subsequently, Sofía Kovalévskaya in her paper entitled *On the rotation of a solid body about a fixed point* [15] attempted to resolve the Euler equations. However, the equations generated four algebraic integrals that it proved impossible to resolve by integration, so she proceeded to resolve them numerically by means of an approximation procedure, thus leaving the general resolution of the equations unresolved.

Some recent studies have been found on the *dance of the spinning top* which in our opinion fall short. Christopher G. Provativis of the Technical University of Athens, published a paper that studies this movement based on the Euler equations and Lagrange's Analytical Mechanics from which he obtained equation systems that cannot be integrated analytically and which he resolved by means of numerical integration, thus arriving at mere approximations. [16]

We came across another thesis in which, on the basis of Lagrange equations and by means of variation calculation, the path of a spinning top in its dance is obtained and visualised. This work compares the different results obtained by different discretizations, but always from the starting point of Lagrangian mechanics [17].

Given the impossibility of resolving the movement equations with the mathematical tools at our disposal and which prevent the confirming of their truthfulness and the relationship between rotation and orbiting, Dr. Barceló proposed the Theory of Dynamic Interactions and a mathematical formula that explains the behaviour of rotating bodies [18].

Indeed, on studying the reactions of a solid rigid body subject to non-coaxial torques, M. Hirn had already concluded in the 19th century that these torques were generating non-homogeneous velocity fields [19]. The *Theory of Dynamic Interactions* also covers the distribution of non-homogeneous fields.

Lastly, it is noteworthy that the Spanish physicist, Miguel A. Catalán, also interested himself in the movement of rotating bodies in the middle of the 20th century, comparing the rotating of an electron around the atom nucleus to spinning top movement [20]. Indeed, he is the lecturer that inspired Dr. Barceló Rico-Avello to develop the theory set forth in this paper.

III. "The Dance of the Spinning Top" and the Theory of Dynamic Interactions

When a spinning top starts to move, it does so owing to a torque acting on it that provokes its rotation.

If the axis of rotation of the spinning top remains perpendicular to the support surface this body stays spinning on a fixed point without shifting. However, what happens when this perpendicularity no longer holds?

By observation, it can be deduced that the spinning top will continue its rotating movement with respect to its rotation axis, which coincides with its symmetrical axis, and will move on the surface tracing a curved path. In this case, it is possible to speak in terms of displacement physics, given that the spinning top point is not fixed on the surface, i.e., it does not have zero speed but rather shifts position.

This curved path, under surface support and smooth displacement conditions (uniform friction coefficient) will be closed and, the more ideal the conditions in which the experiment is being conducted (absence of air) the closer it will come to tracing a circular path.

We have come across historical descriptions of this so-called *spinning top dance* behaviour accompanied by illustrations reflecting this movement (figure 2), as well as descriptions that explain that the spinning top would rest on the support surface if it was not for its rotation (figure 3). [21].

According to Classical Mechanics, a body in movement traces a curved path if there is a force that modifies its straight movement towards its inside at each point of movement (centripetal force). However, the forces acting on the spinning top are weight and the normal reaction with the surface, both of which are perpendicular to the support plane. Neither of them are, therefore, central forces.

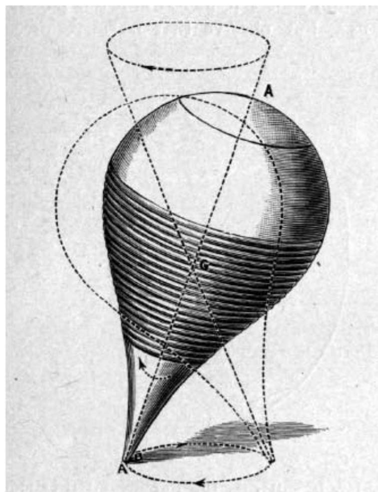

Figure 2 : Dance of the spinning top.

Source: Perry, J. (2010). Spinning Tops. (pg. 21)

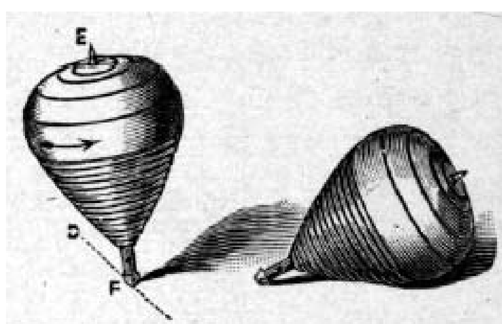

Figure 3 : Spinning top with and without rotation

Source: Perry, J. (2010). Spinning Tops. (pg. 21)

What, therefore, causes this path? We suggest that the reason for it is to be found in the momentum generated by the weight of the spinning top with respect to the point of support, which will provoke the overturning of the spinning top but this, instead of falling, if its support point allows it to slide, will start this dance. (See video: www.advanceddynamics.net/spinning-top-video/). [22] This idea can be seen in figure 4. It is the same effect as produced on the flight of the boomerang, the rotational and orbital movements of which have also been studied. Dr. Barceló suggests in this case:

The weight, a force applied at the boomerang's centre of mass, will not coincide with the resulting lift force, determining a couple that acts at the same time with the rotation, but without being coaxial with it. […] The couple will generate a rotation momentum that tends to tilt the boomerang around its flight direction axis. […] The weight couple and that resulting from the lift forces, which is non-coaxial with the intrinsic rotation of the boomerang, will be the dynamic interaction couple that generates the new path. [23]

This situation (see video) is possible owing to the intrinsic rotation of the body, given that were there no rotation, the spinning top would fall onto the surface.

Accordingly, the interaction between the rotation movement and the translation of a body has been explained.

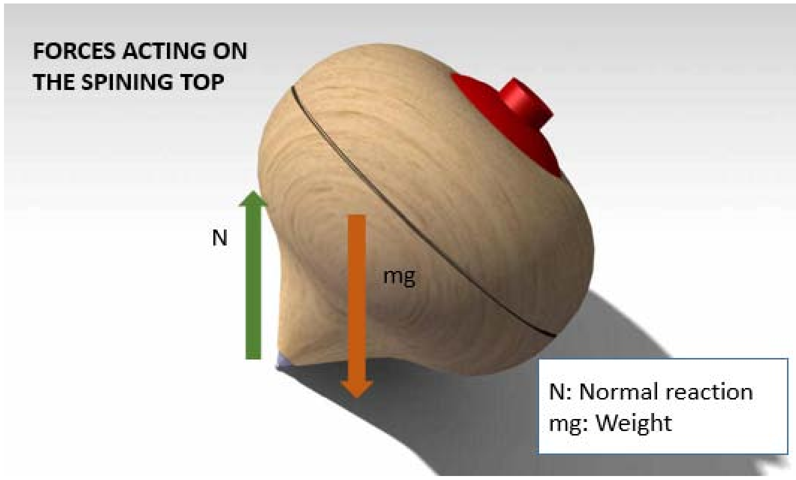

Figure 4 : Forces acting on the spinning top

IV. Similarity with the Planets Orbiting the Sun

Euler [24] was the first to announce the similarity between the Earths' movement around the sun and that of the spinning top. What we intend to do in this paper is not solely to reaffirm this similarity, but rather to explain that the reason behind the Earth's orbit resembles that of the "dance of the spinning top".

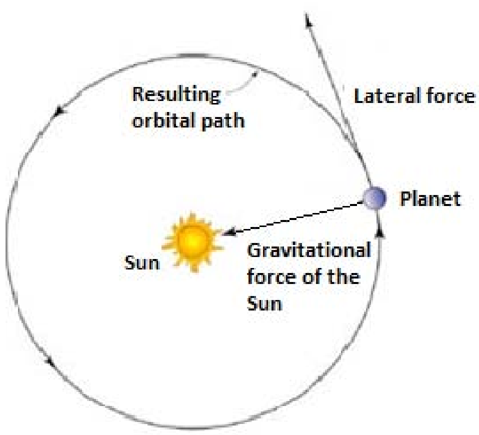

Figure 5 : Forces acting on the Earth according to Newton

Source: http://spaceplace.nasa.gov/review/dr-marc-solar-system/planet-orbits.sp.html

Isaac Newton [25] explained that the Earth's orbit around the sun was caused, on the one hand, by a tangential acceleration and, on the other, by a central force, the sun's force of attraction that acts as a centripetal force and which causes the body to deviate from its straight path, thus converting it into a curved one. In this way, Newton also explained why the Earth, or any of the planets, is at a certain distance from the sun and not stuck to it by virtue of its gravitational force. This idea can be seen in figure 5.

However, Newton's explanation of this orbit is brought into question by the *Theory of Dynamic Interactions.* This theory holds that, as is the case with the spinning top, the orbit of any planet around the sun is not necessarily caused by a central force, but rather by the momenta of forces. This is so based on the first corollary of the tenth law that underpins this theory: "*It is possible to infer the existence of inertial dynamic interactions in rotation and orbital phenomena from the momenta of forces.*"[1]and[26]

Obviously the dance of the spinning top is not caused by any central force, owing to the inexistence of the same, as has been explained in the previous section. However, this idea is not as obvious in the case of a solar system planet because the existence of the force of attraction of the sun is a reality. What follows is an attempt to explain and justify it.

The Earth or any planet is, as are the majority of bodies in space, rotating while at the same time moving in translation. As indicated in the principle of inertia formulated in Newton's laws [27], if there is no effect to impede it, such bodies would continue to move in a straight line. The planets orbit around the sun, thus obviously there must be something that provokes the curved path. Accordingly, the *Theory of Dynamic Interactions* is being proposed as a conceptual framework within which to explain the mechanics of celestial bodies understood as rigid solids subjected to successive non-coaxial rotations. [28]

The *Theory of Dynamic Interactions* holds that the orbit is not caused by the gravitational force of the sun, given that, were this the case, the planets would be stuck to it, but rather by the torque exerted by the sun on the centre of the mass of planets, as a result of its position not coinciding with the centre of the same.

On account of the non-homogeneous constitution of planets, the centre of mass of these is displaced as regards their geometrical centre. Accordingly, the force of the sun's attraction, which will have the direction of a vector that passes through the centre of mass of the sun and that of any planet, will provoke a momentum on the geometrical centre in such a way that the planet, with intrinsic rotation, is subjected to a new non-coaxial momentum. The TDI accepts that the velocity field generated by this new, non-coaxial momentum couples with the translation velocity field thus causing the new orbital path around the sun.

Hitherto, the resemblances have been shown between the spinning top movements and that of the planets. An important difference must now be highlighted: the rotation axis of the dance of the spinning top is inclined with respect to the path. This means that, owing to the effect of the weight, the rotation axis of the spinning top varies with respect to the inertial reference system by tracing a cone. In the case of the Earth, its axis of rotation maintains practically the same inclination with respect to the inertial system along any point of its orbital path. Of course, there will be variations, but much slower and imperceptible, taking some 25,800 years to trace a complete cone. This idea can be seen in figures 6 and 7.

We claim that the oscillation of the Earth's axis, a phenomenon known as *Chandler wobble*, is caused by gravitational forces, as in the case of the spinning top. It must basically be the gravitational force of the moon that makes the axis of rotation of the Earth so stable and showing such extremely slow variations.

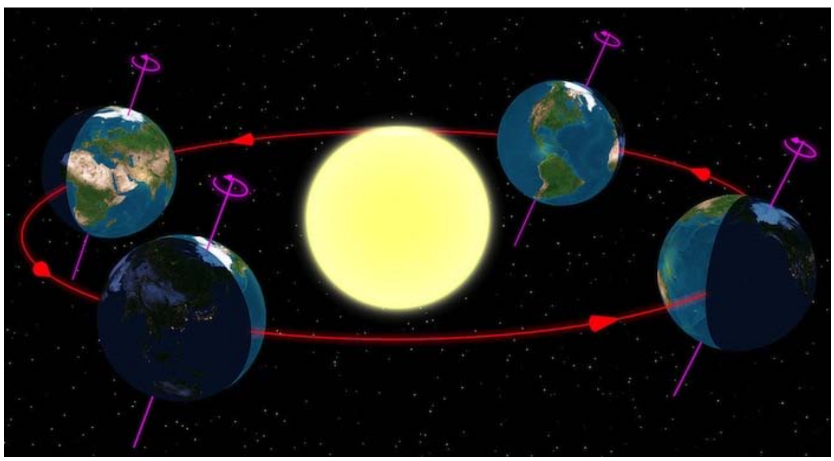

Figure 6 : Situation of the Earth in orbit at several positions.

Source: www.geoenciclopedia.com/movimientos-de-la-tierra/

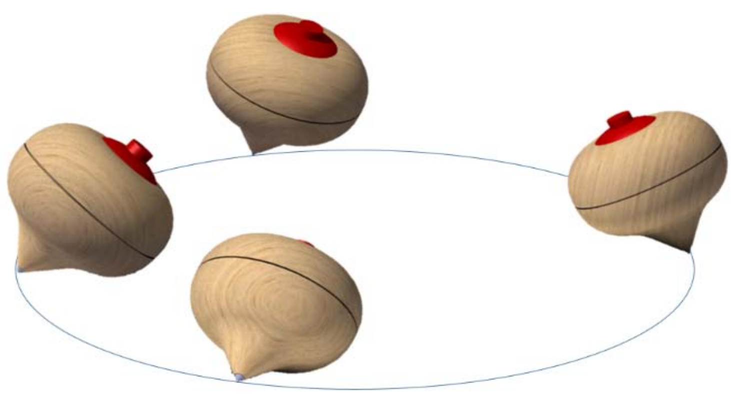

Figure 7 : Situation of the spinning top during its "dance" at several positions

V. Similarity with Artificial Satellites Round the Earth

Even though the case of an artificial satellite orbiting around the Earth resembles that of the Earth around the sun and, therefore, that of the spinning top dance, there will be differences as regards the mechanism that keeps the satellite orbiting around the Earth.

Dr. Barceló Rico-Avello already expounded his intention to extend the Theory of Dynamic Interactions in *Un mundo en rotación* (A Rotating World) [18] to communication satellites which, having an initial speed and endowed with intrinsic rotation if subject to a non-coaxial torque, will assume an orbiting movement.

Obviously, for a satellite to orbit around the Earth at a certain height over its surface, a launch vehicle is required to generate the thrust that will position it at that height. Moreover, it is also clear that, if at that height the thrust no longer exists, the satellite will return to the surface of the Earth, impacting against it, owing to the Earth's force of gravity. Consequently, some type of propellant is needed to generate a force parallel to the Earth's surface that will be sufficient not to represent a simple launch that eventually "spends itself" and returns to the Earth's surface on account of the gravitational action. This force is called thrust.

The thrust will suffice if it provides the satellite with the required orbital speed at that height to orbit the Earth and will be determined by the energy equation. This speed will depend on the Earth's mass [29].

The thrust can be exerted in two different ways, depending on whether it is produced by a motor located on the axis of symmetry of the satellite or from several situated around the same and structurally attached to it. In both cases, a non-coaxial momentum will be caused with respect to the axis of symmetry of the satellite, given that its centre of mass is not going to be located exactly on this axis owing to the heterogeneity peculiar to all satellites, which is inevitable. This will make the satellite rotate owing to the momentum caused and, in turn, endow it with a translation movement on account of the thrust, thus generating a path that will be curved; once again accounting for the *Theory of Dynamic Interactions* and clearly showing that, as is the case with the dance of the spinning top, the movements are not caused by a central force, even though in this case the Earth is indeed the centre of the movement.

Lastly, it must be pointed out that satellites are stabilised by different mechanisms to undertake the mission for which they have been launched and thus are not orbiting with the described characteristics.

VI. Interaction of Fields on the Spinning Top

In all movements there is another physical vectorial magnitude that acts about which no mention has yet been made, namely, translation velocity. In this section, the velocity fields are analysed that are present in the "dance" movement of the spinning top along with inertial fields and, with them, it will be explained how the moving spinning top maintains its position as it does. (See video).

The foregoing is based on the following idea: *"orbital movement is an inertial reaction caused by the distribution of speeds and accelerations that are generated by the action of the second external torque that is acting."* [18]

This idea is based on the four axioms [1] that were announced after long years of research and on which the later field analysis is based:

1. The rotation of space determines the generation of fields.
2. Result of the action of non-coaxial momenta.
3. Inertial fields cause dynamic interactions.
4. The resulting action of the different successive, non-coaxial torques that act on a rigid body cannot be determined by an algebraic sum or be calculated by means of an alleged resulting torque.

The *Theory of Dynamic Interactions* holds that when a rigid solid is subjected to the successive couplings of forces, the first momentum will cause its intrinsic rotation (in this case the rotation of the spinning top on its axis of symmetry) while the subsequent momenta will generate the non-homogenous velocity fields [23].

If the spinning top is placed vertically on its axis and it is endowed with a rotary movement, it will be left rotating on its axis of symmetry and will take longer to fall the faster it rotates. When the spinning top starts to rotate it will also start its dance, even though it has not been endowed with a thrust that provides it with a quantity of movement. It can be understood that at the initial moment, the axis of symmetry undertakes a falling movement endowing the centre of mass with an initial linear velocity, and finally, the spinning top will rotate and orbit at the same time. It can also be supposed that the spinning top's (or those of any rotating body) angular momenta do not add up algebraically or vectorially, resulting in a final momentum and therefore, no general law of the composition of momenta is obtained [18].

Firstly, (see video), a study is made of the fields to which the spinning top is subject at a particular instant of its movement, that is to say, on certain fixed axes to the same, without considering translation.

Figure 8 shows in blue the instantaneous inertial field generated initially on a spinning top that is tilted owing to the effect of the torque action of the forces present on the spinning top: the weight and reaction of the ground on the spinning top. This field will be perpendicular to the radius of the tilt and tangential to the tilt path.

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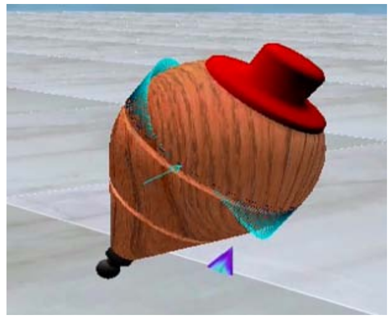

Figure 8 : Inertial fields on the spinning top (without considering its translation movement).

Nevertheless, were only this inertial field to exist, the spinning top would fall and impact against the ground. Therefore, the velocity and inertial fields are coupled thus causing its new path. This has also been observed and mentioned in other studies on bodies in which dynamic interactions are to be found [18] [30].

The body is endowed with intrinsic rotation on its axis of symmetry, which is going to cause a linear velocity on each point of the spinning top. As we are dealing with a rigid solid, this linear velocity value will be constant at each one of its points on the same disc at the same time.

Moreover, the tilt velocity generates a velocity that makes the spinning top fall onto its support surface. The velocity field in yellow in figure 9 that is tilted 90º with respect to the instantaneous tilt velocity is precisely the one that is going to prevent the overturning of the spinning top.

The maximum tilting velocity in the downward direction will correspond to the closest point to the surface (red point in figure 10). The maximum upward tilting velocity will be at 180º from this. Nevertheless, in the first revolution of the spinning top, the amounts of movement will be summed up in successive instants at each point on it. Accordingly, the initial maximum tilting velocity at the following instant will have been shifted by the rotation of the spinning top and the tilting velocity corresponding to its new position will be added to this tilting velocity leading to a change in velocity value. Therefore, on completing the first revolution, the tilt velocity field will have been displaced 90º thus giving rise the situation in figure 9. This situation will hold as long as the spinning top is in a state of dynamic stability. This succession of states until attaining stability (figure 9) can be seen in figure 10.

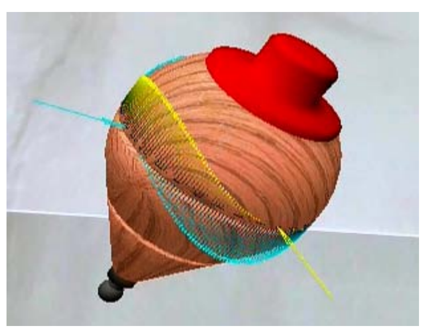

Figure 9 : Resulting velocities fields (yellow) and tilt velocities fields (blue) on the spinning top (without considering its translation movement)

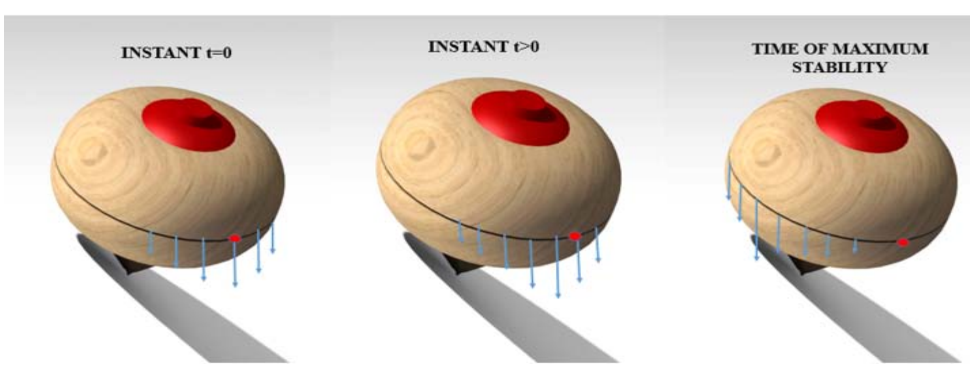

Figure 10 : Spinning top tilt velocities from the initial instant to maximum stability

It must be pointed out that, owing to friction with the surface and the Magnus effect, the linear velocity that caused rotation will begin to fall. The translation of the velocities field will, then, be contrary to that which was seen when it was increasing, reaching a moment in time in which it coincides with the situation at the initial instant and which causes, therefore, the spinning top to overturn.

In the light of the foregoing, it is understood that the spinning top movement passes from a transitory state in which the linear velocity of rotation of the spinning top increases to a dynamic stability in which this linear velocity is constant, until this speed falls entering once again in a transitory state until it comes to nothing and the spinning top is left overturned on the surface. The states of equilibrium correspond to states of spinning top stability, on the understanding that the concept of equilibrium is not restricted to "static equilibrium". According, Gabriel Barceló claims as follows: "*In accordance with the Theory of Dynamic Interactions proposed, we conceive a universe in dynamic equilibrium in which a constant orbit movement acts inside a closed path.*" [1]

However, it must not be overlooked that the linear translation velocity acts at the same time as the tilting velocity. The yellow velocities field couples with the system's translation velocities field thus generating the movement that is observed. The variation in the time of the two velocities fields gives rise to acceleration fields that serve to generate a force field. The force of these two fields is the effective force produced on the system.

Take the following axes (see video): the green triad symbolises the translation reference system of the solid, where the vector T indicates the direction of translation from the centre of mass of the solid, while R indicates the radial direction that points towards the centre of the curve traced by the dancing spinning top. The yellow triad symbolises the reference system without translation with respect to the foregoing, but tilted in accordance with the inclination of the body to the vertical plane of the Earth. These systems of axes are represented in figure 11.

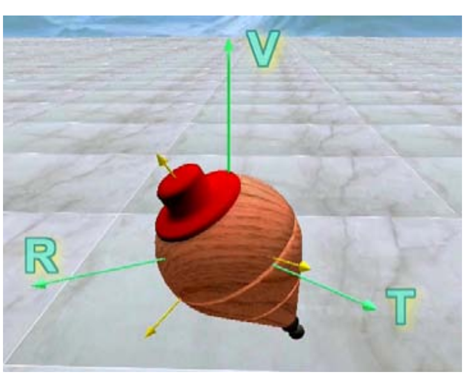

Figure 11 : Systems of axes on the spinning top

Based on the analysis conducted and the observation of the rotary and translation movement of the spinning top, (see video) the proposed axioms are duly corroborated and in the light of the coupling of the fields of velocities it can be concluded that: "*the body will keep to a closed path (e.g., a an orbital movement), without any need for a central force, and will simultaneously intrinsically rotate around its initial axis.*"[31].

VII. Magnetic Spinning Top

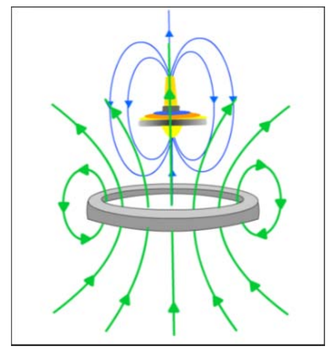

Figure 12 : Magnetic spinning tops

Source: http://cienceandoenelsulayr.blogspot.com.es/2014/09/inicio-del-curso-2014-2015.html

The magnetic spinning top, save for those that can levitate on account of the magnetic repulsion effect, behaves similarly to the conventional spinning top in terms of movement and, therefore, can only be distinguished from it because of the way in which it is activated and in the variations of angular velocity to which its rotation may be subject, owing to the variations in the magnetic forces applied. The spinning top must be magnetised to apply the external forces. These ideas about the magnetic spinning top have already been expounded by Gabriel Barceló in his book *Un mundo en rotación* (A Rotating World) [18]:

> *The disc would be activated remotely by a magnet. Magnetic spinning tops subjected to different external magnetic forces confirm the action of external magnetic torques in the variation of the rotating and orbiting velocity owing to the magnetic induction effect. This is also the case of a magnetic spinning top that keeps a constant rotating movement owing to the effect of an electromagnet acting by impulses, or which levitates in a magnetic field. In these cases, the orbital path generated has been confirmed for different rotating velocities and different levels of friction with the support surface.*

The foregoing, therefore, leads to the following conclusion: it is possible to propose a generalisation of the *Theory of Dynamic Interactions* for any body endowed with intrinsic rotation, regardless of the way in which this rotation is activated or the type of the external forces and momenta acting on it. We have now seen several cases: the conventional spinning top, planets, satellites and magnetic spinning top in which this theory is confirmed and in different circumstances the reason why the intrinsic rotation was different.

VIII. TDI Equations on the Spinning Top

The paths of several, different sized spinning tops with different masses have been traced to check compliance with the mathematical formula for the *Theory of Dynamic Interactions* in the case of the spinning top; paths that have been calculated applying the TDI. The results obtained are shown below.

The mathematical formula for the *Theory of Dynamic Interactions* is as follows[30]:

$$\vec{v} = \Psi \vec{V_0}$$

Where Ψ is a rotation matrix defined as follows:

$$\Psi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

Where $\alpha = t\frac{M'}{I\omega}$, M' is the external momentum, I the body's inertial momentum, in this case of the spinning top and ω the angular velocity of the rotating body [18].

Figure 13 shows two images of the first experiment. The image on the left shows the real path followed by the spinning top, while the one on the right, the path calculated according to TDI, considering the deceleration caused by dissipative forces (surface friction). This first experiment was done as follows: a spinning top was activated on a wooden table surface by means of a torsion spring. A video was recorded of the spinning top in movement and later images were captured at intervals of a second. Accordingly, the different positions of the spinning top were located during movement, which are the points plotted on the graph to the left of figure 13.

It can be seen that both paths are similar though the second one will be more perfect on having considered the dissipative forces causing an homogenous deceleration, when in reality this will depend on the friction coefficient at each surface point, which may not be homogenous.

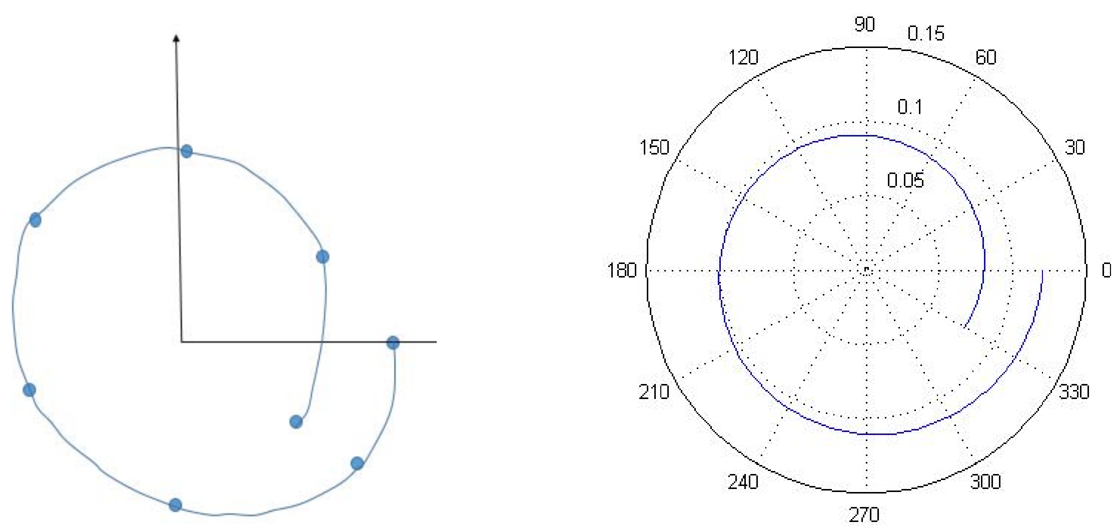

Figure 13 : Real path of the spinning top and the path calculated by TDI

Another experiment was conducted to assess the TDI results. In this experiment it was possible to obtain a high number of path points at halfway along the revolution of the dancing spinning top. These points were used to produce a graph that was superimposed on the one obtained by using the TDI. The result of this can be seen in figure 14.A second experiment was designed to obtain a greater number of points. This time, the spinning was activated as before, but it was placed on a sheet covered with powder to trace the path taken.

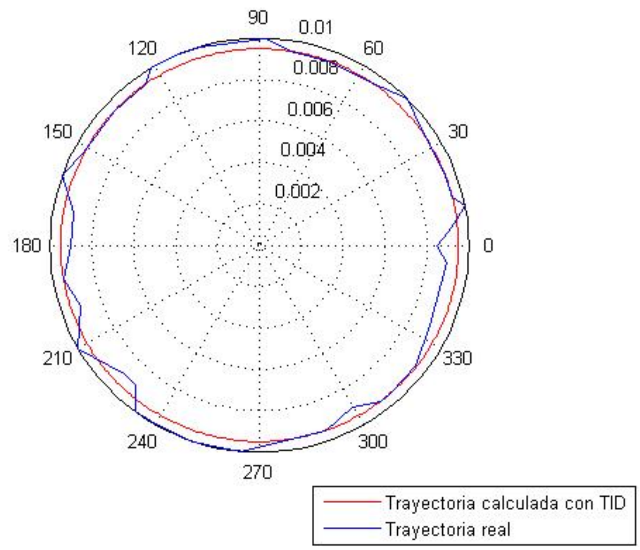

Figure 14 : Superimposition of real spinning top path graphs and the one calculated in accordance with the TDI (I)

Figure 14 shows the real movement of the spinning top at the start of the same. At this moment in time, the linear velocity begins increasing, which is reflected in the increase of the radius until such time as the dissipative effects cause it to diminish as can be seen in the graph. The difference between the real path and the calculated one arises from the difference in the coefficient of friction on the support surface, because as stated above, this was a sheet covered in powder. Moreover, this gap between the real path and the calculated one came about because the surface had a slight inclination that caused the increase of the radius.

A third experiment was done to reduce the difference among friction coefficients throughout the surface. This obtained the path of a complete revolution owing to the spinning top being able to paint on the surface travelled. However, this spinning top was of greater mass than the previous one so its path radius was smaller. This explains why less points were obtained and greater accuracy attained as regards the tracing of the path. Consequently, in figure 15, a path can be perceived that resembles the one calculated by the TDI, but which has gaps owing to the paucity of the data sample.

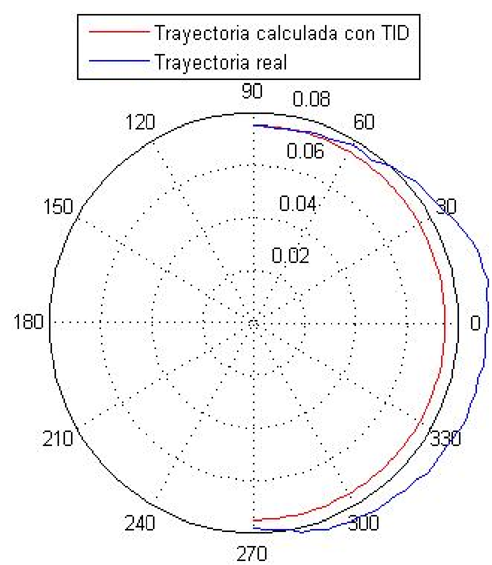

Figure 15 : Superimposition of real spinning top path graphs and the one calculated in accordance with the TDI (II)

In the light of the foregoing, it has been seen that the *Theory of Dynamic Interactions* manages to closely approximate the path of the spinning top and surpasses that of the Euler equations that did not allow for integration with respect to the case of the spinning top.

IX. Angular Momentum on the Dance of the Spinning Top

We also analyse the conservation of angular momentum on the movement of the spinning top. The angular momentum is transferred to the spinning top by means of a thrust that is generated on applying a torque in several ways [32].

As regards this issue, the following reflection by Dr. Barceló Rico-Avello must be mentioned:

> *This behaviour of nature can also be interpreted in the field of physics by the fact that, under such circumstances,* ***angular momenta are not added up****, so if a body is subject to an angular momentum* ***L****, its variation* ΔL*, due to the effects of external forces, will not necessarily be added to the already existing* ***L****, but rather can generate a new movement, different and simultaneous to the existing one, which we have called* ***precession movement*** *or, where appropriate,* ***orbital movement.*** *This would be the case of the spinning top, and in the case of a body in space without any restraint, like a boomerang, or numerous other like objects, the body will begin to orbit without the necessary existence of a central force.* [18]

In the case of the spinning top it is possible to differentiate three rotations (figure 16).

The inertial momentum of the spinning top does not vary given that we are dealing with a rigid body and, therefore, it will always have the same mass and be distributed in the same way. Consequently, the variation in the angular momentum is going to be determined by the angular velocity and will be constant in each axis system if the angular velocity in these is also constant.

The conservation of the angular momentum in the rotating of a spinning top (with respect to 0 axes) has already been expounded by A. Fernández-Rañada:

> *When the aforementioned angular velocity is high, the inertia of the spinning top due to its spinning on its own axis is very high and the external torque does not suffice to appreciably change I* ω *(...). Therefore, the* ***L*** *value of the angular momentum remains constant. As must be the conclusion that the angle formed by the axis of rotation and the Z axis must also be constant. In order for these two conditions to be met in the presence of the M torque, the axis of rotation of the spinning top has to trace a cone solid of revolution thus causing a precession movement around the Z axis.* [33]

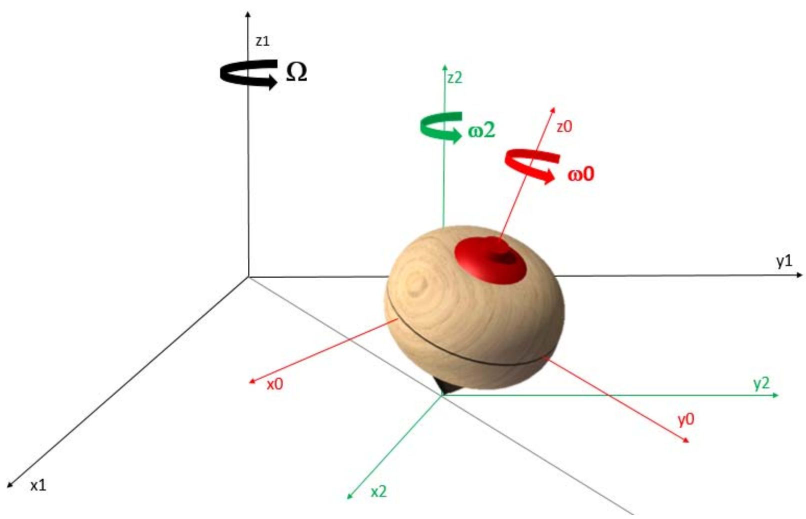

Figure 16 : Illustration of rotations in the "dance of the spinning top"

X. Future Lines of Research

The most important issues have been dealt with throughout this paper that are considered essential to showing how the *Theory of Dynamic Interactions* explains the dance of the spinning top and other bodies that behave similarly in reality and that may be of interest to the reader. We now deal with other issues related to the subject matter discussed, albeit outside the scope of this paper, but which point to future lines of research.

We know that the Earth makes the following movements: rotation on its own axis, orbiting around the sun, equinoctial precession, along with another we have had cause to mention: the Chandler wobble [34]. It would be interesting were future research to study the latter movement in relation to the others. In the "*El vuelo del bumerán*" (The Flight of the Boomerang) [2], Gabriel Barceló has already hinted at this subject:

> *In 1758, Euler predicted that the axis of rotation of the Earth also had a further movement with respect to a fixed frame of reference. In 1891, Chandler determined the time interval of this free polar movement. The oscillation radius of the Chandler wobble on the Earth's rotation axis is around 6 m.*

It is also proposed that research be done to confirm or refute Poisont's [5] claim that there is an instantaneous axis of rotation that is different at every instant of the movement, both on inertial axes, as well as on those axes bound up with the solid. This seems to coincide in the first case, but not in the latter, given that this would mean having a different axis of rotation to the axis of symmetry at every moment of rotation, whereas observation seems to reflect otherwise.

We also suggest research be done into the technological applications that could be developed on the basis of the *Theory of Dynamic Interactions* proposed by Dr. Barceló in previous papers. [35].

Lastly, we suggest that the reader have another look at the video to better understand the behaviour of the spinning top: www.advanceddynamics.net/spinning-top-video/

You can consult the following links for further information on the *Theory of Dynamic Interactions:*

http://advanceddynamics.net/
http://dinamicafundacion.com/

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